Contrails

MEMBRANE ANALYSIS OF PRESSURIZED THIN SPHEROID SHELLS COMPOSED OF FLAT GORES, AND ITS APPLICATION TO ECHO II

by

Hossein Bahiman
and
John M. Thole

INTRODUCTION

Computation of the shape of a pressurized thin spheroid shell composed of a large number of identical flat gores requires solution of five nonlinear partial differential equations of equilibrium and boundary conditions for three displacements of a gore. The position of each point on the gore could be determined as a function of two independent variables, namely, the distance of the point from the center line of the gore and the latitude of the point with respect to the equatorial plane of the spheroid. For an approximate solution to these nonlinear partial differential equations each displacement is expanded into a power series of the distance of the point from the gore center line such that the coefficients of the power series are at the most functions of the latitude. By equating to zero the terms of the lowest power in each equation, five new nonlinear differential equations for the six coefficients (two for each displacement) are obtained. The approximate solution of these equations by perturbation technique led to three displacement functions of the gore with one unknown constant. Application of variational method to the total elastic strain and potential energy of the system led to the evaluation of the above mentioned constant.

In this analysis, it is assumed that the displacement of the spheroid due to the gravity is negligible compared to the one caused by the constant internal pressure. It is also assumed that the material of the spheroid is homogeneous and isotropic and it obeys Hooke's law.

Approved for Public Release

Contrails

Finally this method of analysis is applied to Echo II, the 135 foot diameter passive communication satellite. Since the 3-ply aluminum-mylar-aluminum material of the satellite does not satisfy the requirements implied in the above mentioned assumptions, therefore the analytical results obtained here are expected to be only an approximation of the actual case.

THEORY

In order to determine the shape of a thin spheroid shell composed of identical flat gores it is sufficient to compute the true shape of a gore in the pressurized state. To do this, it is assumed that the material of the spheroid is homogeneous isotropic, and linear elastic. It is also assumed that the effect of gravity upon the shape of the spheroid is negligible in comparison with that of the constant internal pressure.

Let Cartesian coordinate axes be drawn in such a way that the origin O be located at the center of the balloon, y axis be along the polar axis, and Z axis be extended through the center of a gore, as shown in Figure 1. Draw X'Y'Z axes from a point P of the gore in such a way that X' axis be parallel to X axis, Z' axis be in the direction OP, and Y' (or θ) axis be tangent to the meridian at point P (Figure 1).

In the pressurized state of equilibrium, the resultant of all the forces acting on an element of the shell in the direction X', θ and Z'

Approved for Public Release

Contrails

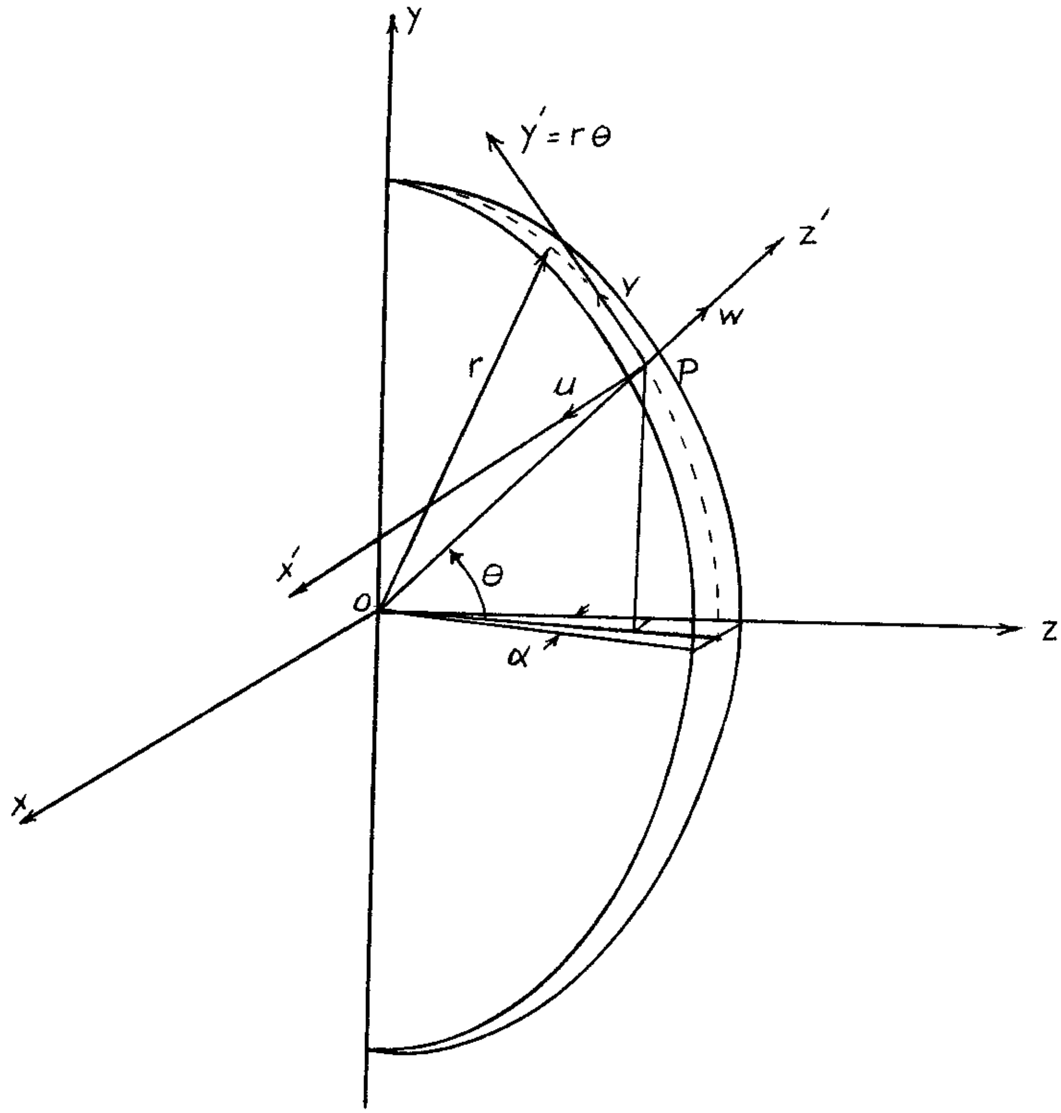

Figure 1 - Coordinate Axes and Displacement Components

Approved for Public Release

Contrails

vanishes. Let N_x, N_θ, and $N_{x\theta}$ be nromal and shear tractions per unit length of membrane, θ be the angle between the position vector and the equatorial plane, and p be the internal pressure of the spheroid, as shown in Figure 2. Hence,

$$\begin{cases} \dfrac{\partial N_x}{\partial x} + \dfrac{\partial N_{x\theta}}{r\partial\theta} = 0 \\ \dfrac{\partial N_{x\theta}}{\partial x} + \dfrac{\partial N_\theta}{r\partial\theta} = 0 \\ \dfrac{N_\theta}{r} - N_\theta \dfrac{\partial^2 w}{r^2\partial\theta^2} - N_x \dfrac{\partial^2 w}{\partial x^2} - 2N_{x\theta}\dfrac{\partial^2 w}{r\partial x\partial\theta} = p \end{cases} \tag{1}$$

Let u, v, and w be displacement components at a point P in the directions X', Y', and Z' respectively (Figure 2). The elastic strains are given by:

$$\begin{cases} \epsilon_x = \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \epsilon_\theta = \dfrac{1}{r}\dfrac{\partial v}{\partial\theta} + \dfrac{w}{r} + \dfrac{1}{2r^2}\left(\dfrac{\partial w}{\partial\theta}\right)^2 \\ \epsilon_{x\theta} = \dfrac{1}{2}\left(\dfrac{1}{r}\dfrac{\partial u}{\partial\theta} + \dfrac{\partial v}{\partial x} + \dfrac{1}{r}\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial\theta}\right) \end{cases} \tag{2}$$

Also,

$$\begin{cases} N_x = K(\epsilon_x + \mu\,\epsilon_\theta) \\ N_\theta = K(\epsilon_\theta + \mu\,\epsilon_x) \\ N_{x\theta} = K(1-\mu)\,\epsilon_{x\theta} \end{cases} \tag{3}$$

where $K = \dfrac{Et}{1-\mu^2}$ is membrane stiffness, and E, t, and μ are Young's modulus, thickness of the shell and Poisson's ratio, respectively. Substitution of Eqs. (2) into Eqs. (3) leads to

$$\begin{cases} N_x = K\left[\dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 + \mu\dfrac{\partial v}{r\partial\theta} + \mu\dfrac{w}{r} + \dfrac{\mu}{2}\left(\dfrac{\partial w}{r\partial\theta}\right)^2\right] \\ N_\theta = K\left[\dfrac{\partial v}{r\partial\theta} + \dfrac{w}{r} + \dfrac{1}{2}\left(\dfrac{\partial w}{r\partial\theta}\right)^2 + \mu\dfrac{\partial u}{\partial x} + \dfrac{\mu}{2}\left(\dfrac{\partial w}{\partial x}\right)^2\right] \\ N_{x\theta} = \dfrac{K(1-\mu)}{2}\left(\dfrac{\partial u}{r\partial\theta} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{r\partial\theta}\right) \end{cases} \tag{4}$$

Approved for Public Release

Contrails

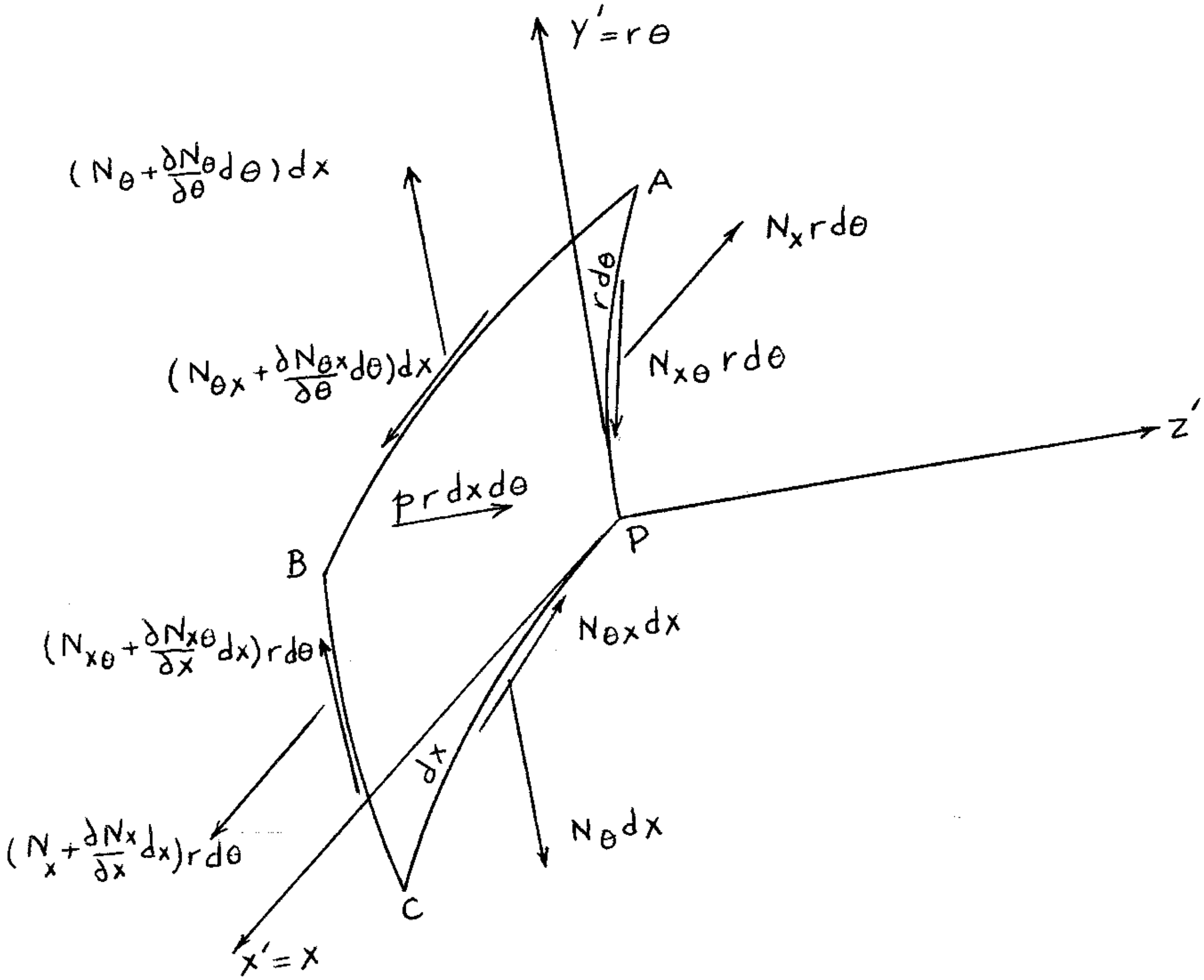

Figure 2 - Forces Acting on an Element of Gore

Approved for Public Release

Substitution of Eqs. (4) into Eqs. (1) yields

$$
\left\{
\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 v}{r\,\partial x\,\partial\theta} + \frac{\mu}{r}\frac{\partial w}{\partial x} + \frac{\mu}{r^2}\frac{\partial^2 w}{\partial x\,\partial\theta}\frac{\partial w}{\partial\theta} + \\
&\frac{1-\mu}{2r}\left(\frac{1}{r}\frac{\partial^2 u}{\partial\theta^2} + \frac{\partial^2 v}{\partial x\,\partial\theta} + \frac{1}{r}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial\theta^2} + \frac{1}{r}\frac{\partial^2 w}{\partial x\,\partial\theta}\frac{\partial w}{\partial\theta}\right) = 0 \\
&\frac{1-\mu}{2}\left(\frac{1}{r}\frac{\partial^2 u}{\partial x\,\partial\theta} + \frac{\partial^2 v}{\partial x^2} + \frac{1}{r}\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial\theta} + \frac{1}{r}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\,\partial\theta}\right) + \\
&\frac{\partial^2 v}{r^2\,\partial\theta^2} + \frac{1}{r^2}\frac{\partial w}{\partial\theta} + \frac{1}{r^3}\frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial\theta^2} + \frac{\mu}{r}\frac{\partial^2 u}{\partial x\,\partial\theta} - \frac{\mu}{r}w\frac{\partial^3 w}{\partial x^2\,\partial\theta} \\
&-\frac{\mu}{r}\frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial x^2} + \frac{\mu}{r}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\,\partial\theta} = 0 \\
&\left[\frac{1}{r^2}\frac{\partial v}{\partial\theta} + \frac{w}{r^2} + \frac{1}{2r}\left(\frac{\partial w}{r\,\partial\theta}\right)^2 + \frac{\mu}{r}\frac{\partial u}{\partial x} - \frac{\mu}{r}w\frac{\partial^2 w}{\partial x^2} + \frac{\mu}{2r}\left(\frac{\partial w}{\partial x}\right)^2\right] \\
&\left(1 - \frac{1}{r}\frac{\partial^2 w}{\partial\theta^2}\right) - \frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2} - \frac{\mu}{r}\frac{\partial v}{\partial\theta}\frac{\partial^2 w}{\partial x^2} \\
&-\frac{\mu}{r}w\frac{\partial^2 w}{\partial x^2} - \frac{\mu}{r^2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial\theta}\right)^2 - (1-\mu)\left(\frac{1}{r^2}\frac{\partial u}{\partial\theta}\frac{\partial^2 w}{\partial x\,\partial\theta} + \frac{1}{r}\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x\,\partial\theta}\right. \\
&\left. + \frac{1}{r^2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial\theta}\frac{\partial^2 w}{\partial x\,\partial\theta}\right) = \frac{p}{k}
\end{aligned}
\right.
\tag{5}
$$

Approved for Public Release

Contrails

The boundary conditions are:

$$\begin{cases} N_{x\theta} = 0 & \text{on} \quad x = r\alpha \cos\theta \\ u = \alpha W \cos\theta - \alpha V \sin\theta & \text{on} \quad x = r\alpha \cos\theta \end{cases} \tag{6}$$

along the seam, where $\alpha = \frac{\pi}{n}$. The first expression indicates that due to symmetry shear stress vanishes along the seam where any two adjacent gores meet, and the second expression implies that the points along each seam move in the meridional plane. Since the coefficients of only the first two terms of the power series expansion of displacements will be determined by this method, continuity of slope in the plane normal to the seam cannot be used as a third boundary condition. The use of such a boundary condition would lead to an erroneous solution, namely, that the cross section of the spheroid with any plane normal to its polar axis is a true circle, no matter what the magnitude of the internal pressure. Instead, a virtual displacement is given to the pressurized spheroid in the state of equilibrium and change of the total elastic strain and potential energy of the spheroid and the load is equated to zero.

Let r be the radius of the centerline of a gore before any load is applied, and a, b, c, d, f and g be functions of variable angle θ. Displacements W, U, and V may be expanded into power series as following:

$$\begin{cases} W = ar + b\,\frac{x^2}{r} + \cdots \\ u = cx + d\,\frac{x^3}{r^2} + \cdots \\ V = fr + g\,\frac{x^2}{r} + \cdots \end{cases} \tag{7}$$

Approved for Public Release

Contrails

Substitution of Eqs. (7) and the last one of Eqs. (4) into Eqs. (5) and (6) leads to

$$\begin{cases}
6d + (1+\mu)\dfrac{dg}{d\theta} + 2\mu b + (1+\mu)\dfrac{da}{d\theta}\dfrac{db}{d\theta} + \dfrac{1-\mu}{2}\dfrac{d^2c}{d\theta^2} + (1-\mu)b\dfrac{d^2a}{d\theta^2} + 4b^2 = 0 \\
\dfrac{1+\mu}{2}\dfrac{dc}{d\theta} + (1-\mu)g + (1-3\mu)b\dfrac{da}{d\theta} + \dfrac{d^2f}{d\theta^2} + \dfrac{da}{d\theta} + \dfrac{da}{d\theta}\dfrac{d^2a}{d\theta^2} - 2\mu a\dfrac{db}{d\theta} = 0 \\
\left[\dfrac{df}{d\theta} + a + \dfrac{1}{2}\left(\dfrac{da}{d\theta}\right)^2 + \mu c - 2\mu ab\right]\left(1 - \dfrac{d^2a}{d\theta^2}\right) - 2bc - 2\mu b - 2\mu ab \\
\quad - \mu b\left(\dfrac{da}{d\theta}\right)^2 = \dfrac{pr}{K} \\
\dfrac{dc}{d\theta} + 2g + 2b\dfrac{da}{d\theta} = 0 \\
c - a + f\tan\theta = (b-d)\alpha^2\cos^2\theta
\end{cases} \tag{8}$$

Neglecting small terms of higher order in Eqs. (8) leads to

$$\begin{cases}
6d + 2\mu b - \mu\dfrac{d^2c}{d\theta^2} - 2\mu b\dfrac{d^2a}{d\theta^2} + 4b^2 = 0 \\
\mu\dfrac{dc}{d\theta} - 2\mu b\dfrac{da}{d\theta} + \dfrac{d^2f}{d\theta^2} + \dfrac{da}{d\theta} - 2\mu a\dfrac{db}{d\theta} = 0 \\
\dfrac{df}{d\theta} + a + \mu c - 4\mu ab - 2bc - 2\mu b\dfrac{df}{d\theta} = \dfrac{pr}{K} \\
c - a + f\tan\theta = (b-d)\alpha^2\cos^2\theta \\
g = -\dfrac{1}{2}\dfrac{dc}{d\theta} - b\dfrac{da}{d\theta}
\end{cases} \tag{9}$$

Approved for Public Release

Contrails

Since n, the number of gores, is assumed to be large therefore α^2 is very small. Thus perturbation technique can be applied to the functions a, b, c, d, f, and g in the following fashion:

$$\begin{cases} a(\theta) = a_0(\theta) + \alpha^2 a_1(\theta) + \cdots \\ b(\theta) = b_0(\theta) + \alpha^2 b_1(\theta) + \cdots \\ c(\theta) = c_0(\theta) + \alpha^2 c_1(\theta) + \cdots \\ d(\theta) = d_0(\theta) + \alpha^2 d_1(\theta) + \cdots \\ f(\theta) = f_0(\theta) + \alpha^2 f_1(\theta) + \cdots \\ g(\theta) = g_0(\theta) + \alpha^2 g_1(\theta) + \cdots \end{cases} \tag{10}$$

Substitution of Eqs. (10) into Eqs. (9) yields a set of five nonlinear differential equations in terms of a_o, b_o, c_o, d_o, f_o, g_o, a_1, b_1, c_1, d_1, etc. containing even powers of α . Terms containing each power of α in each differential equation must independently vanish. The terms independent of α must vanish in the following way:

$$\begin{cases} 6d_0 + 2\mu b_0 - \mu \dfrac{d^2 c_0}{d\theta^2} - 2\mu b \dfrac{d^2 a_0}{d\theta^2} + 4 b_0^2 = 0 \\ \mu \dfrac{dc_0}{d\theta} - 2\mu b_0 \dfrac{da_0}{d\theta} + \dfrac{d^2 f_0}{d\theta^2} + \dfrac{da_0}{d\theta} - 2\mu a_0 \dfrac{db_0}{d\theta} = 0 \\ \dfrac{df_0}{d\theta} + a_0 + \mu c_0 - 4\mu a_0 b_0 - 2 b_0 c_0 - 2\mu b_0 \dfrac{df_0}{d\theta} = 0 \\ c_0 - a_0 + f_0 \tan\theta = 0 \\ g_0 = -\dfrac{1}{2} \dfrac{dc_0}{d\theta} - b_0 \dfrac{da_0}{d\theta} \end{cases} \tag{11}$$

The solution of the above equations could be given as the following:

$$\begin{cases} b_0 = \text{Constant} = B \\ d_0 = -\dfrac{2}{3} B^2 - \dfrac{\mu}{3} B \\ a_0 = c_0 = f_0 = g_0 = 0 \end{cases} \tag{12}$$

Approved for Public Release

Contrails

The coefficients of α^2 in Eqs. (9) must vanish in the following fashion:

$$\begin{cases} 6d_1 + 2\mu b_1 - \mu \dfrac{d^2c_1}{d\theta^2} - 2\mu B \dfrac{d^2a_1}{d\theta^2} + 8Bb_1 = 0 \\ \mu \dfrac{dc_1}{d\theta} - 2\mu B \dfrac{da_1}{d\theta} + \dfrac{d^2f_1}{d\theta^2} + \dfrac{da_1}{d\theta} = 0 \\ \dfrac{df_1}{d\theta} + a_1 + \mu c_1 - 4\mu B a_1 - 2Bc_1 - 2\mu B \dfrac{df_1}{d\theta} = \dfrac{pr}{k\alpha^2} \\ c_1 - a_1 + f_1 \tan\theta = \left[\frac{2}{3}B^2 + (1 + \frac{\mu}{3})B\right]\cos^2\theta \\ g_1 = -\frac{1}{2}\dfrac{dc_1}{d\theta} - B\dfrac{da_1}{d\theta} \end{cases} \tag{13}$$

The solution of Eqs. (13) could be written as:

$$\begin{cases} a_1 = A_1 \cos 2\theta + A_2 \\ b_1 = 0 \\ c_1 = C_1 \cos 2\theta + C_2 \\ d_1 = D_1 \cos 2\theta \\ f_1 = F_1 \sin 2\theta \\ g_1 = G_1 \sin 2\theta \end{cases} \tag{14}$$

in which the constant coefficients are given by

$$\begin{cases} \gamma = \frac{1}{3}B^2 + (\frac{1}{2} + \frac{\mu}{3})B \\ F_1 = \dfrac{(2\mu B + 1 + \mu^2)\gamma}{-4\mu^2 B - 2\mu B + \mu^2 - 1} \\ A_1 = \dfrac{\mu\gamma + (2+\mu)F_1}{2\mu B - \mu - 1} \\ C_1 = \gamma + A_1 + F_1 \\ A_2 = \dfrac{(\mu - 2B)(F_1 - \gamma) + \frac{pr}{k\alpha^2}}{1 + \mu - 4\mu B - 2B} \\ C_2 = A_2 - F_1 + \gamma \\ D_1 = -\frac{2\mu}{3}C_1 - \frac{4}{3}\mu B A_1 \\ G_1 = C_1 + 2BA_1 \end{cases} \tag{15}$$

Approved for Public Release

Contrails

Knowing the value of Poisson's ratio, one can easily evaluate in an orderly manner γ, F_1, A_1, C_1, A_2, C_2, D_1, and G_1 as a function of Constant B.

To evaluate constant B, the method of virtual displacement may be utilized. The elastic strain energy of one quarter of a gore is given by

$$T = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{x=0}^{x=r\alpha\cos\theta} \frac{1}{2}\left(N_x \epsilon_x + N_\theta \epsilon_\theta + 2 N_{x\theta} \epsilon_{x\theta} \right) r\,dx\,d\theta \tag{16}$$

Substitution of stress and strain values in Eq. (16) yields

$$\begin{aligned} T = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{x=0}^{x=r\alpha\cos\theta} \frac{K}{2}\Big[& \left(c + 3d\frac{x^2}{r^2} - 2ab\right)^2 + \Big(\frac{df}{d\theta} + \frac{x^2}{r^2}\frac{dg}{d\theta} + a + b\frac{x^2}{r^2} + \frac{1}{2}\left(\frac{da}{d\theta}\right)^2 \\ & + \frac{1}{2}\frac{x^4}{r^4}\left(\frac{db}{d\theta}\right)^2 + \frac{x^2}{r^2}\frac{da}{d\theta}\frac{db}{d\theta}\Big)^2 + 2\mu\left(c + 3d\frac{x^2}{r^2} - 2ab\right)\Big(\frac{df}{d\theta} + \frac{x^2}{r^2}\frac{dg}{d\theta} + \\ & a + b\frac{x^2}{r^2} + \frac{1}{2}\left(\frac{da}{d\theta}\right)^2 + \frac{1}{2}\frac{x^4}{r^4}\left(\frac{db}{d\theta}\right)^2 + \frac{x^2}{r^2}\frac{da}{d\theta}\frac{db}{d\theta}\Big) + \frac{1-\mu}{2}\Big(\frac{x}{r}\frac{dc}{d\theta} + \frac{x^3}{r^3}\frac{dd}{d\theta} + \\ & + \frac{2x}{r}g + \frac{2x}{r}b\frac{da}{d\theta} + 2\frac{x^3}{r^3}b\frac{db}{d\theta}\Big)^2\Big] r\,dx\,d\theta \end{aligned} \tag{17}$$

In a virtual displacement corresponding to change of B by δB the work done by the internal pressure for a quarter of a gore is given by

$$\frac{\partial \overline{W}}{\partial B}\delta B = p \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{x=0}^{x=r\alpha\cos\theta} \frac{\partial W}{\partial B} r\,dx\,d\theta \;\delta B \tag{18}$$

in which $\overline{W}$ is the total work done by the internal pressure for a quarter gore, and W is the radial displacement given by Eqs. (7). Hence, the Constant B could be determined from

$$\frac{\partial T}{\partial B} = p \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{x=0}^{x=r\alpha\cos\theta} \frac{\partial W}{\partial B} r\,dx\,d\theta \tag{19}$$

Thus for a given pressure p, Eq. (19) in conjunction with Eq. (17), (7), (10), (12), (14) and (15) furnishes B which upon substitution into Eq.

Approved for Public Release

Contrails

(15) gives coefficients A_1, A_2, etc. Substitution of these coefficients into Eq. (14) in conjunction with Eqs. (7), (10) and (12) gives the three displacement functions u, v, and w.

APPLICATION OF THEORY TO ECHO II

The 135 foot diameter Echo II is composed of 106 identical gores with a maximum width of 4 feet which occurs in the equatorial plane of the balloon. The 3-ply material of the balloon is composed of 0.00035 in. thick mylar sandwiched between two layers of aluminum of 0.00018 in. thickness each. The adjoining gores are located edge to edge with each other with one inch wide tape of the same material sealing the seams lengthwise. When separation of the two half-canisters takes place in the initial phase of orbiting, the residual air and water vapor inside the folded balloon which is of the order of magnitude of one millimeter inflates the satellite. While initially flat, each gore forms a narrow transverse portion of a half circular cylindrical surface of 135 feet diameter. Thus, the initial shape of the balloon is a spheroid with 0.356 in. maximum deviation from a sphere which occurs in the equatorial plane.

Due to the solar radiation skin temperature increases and as a result inflation material sublimes and builds up pressure until a maximum internal pressure of 225 microns is reached. This maximum pressure corresponds to a membrane stress of 1.7 lb/in.

Although the material of the balloon is rather nonhomogeneous and anisotropic and it does not quite obey Hooke's law, nevertheless for the purpose of approximation, it is assumed that the requirements of homogeneity, isotropy and linear elasticity necessitated by this analysis are met.

Approved for Public Release

Contrails

The average value of K corresponding to 1.70 lb/in. skin traction (given by Tests No. 1, 3, 4, and 6 of Reference 3) is 1050 lb/in.

Since nominal Young's modulus for aluminum is 10.5×10^6 psi and for mylar is $.55 \times 10^6$ psi, therefore it is expected that aluminum should carry most of the load. The nominal value of K for aluminum layers is given by:

$$K_n = \frac{Et}{1-\mu^2} = 4240 \text{ lb/in.}$$

in which $\mu = .33$ has been inserted. This value of K_n is much larger than the experimental value K = 1050 lb/in. Therefore, the actual Young's modulus and perhaps Poisson's ratio for thin layers of aluminum are smaller than the nominal ones.

Following the steps suggested in the last paragraph of the previous section one could get an Eigen value equation for B whose coefficients are functions of parameter $m = \frac{pr}{K\alpha^2}$. For different values of maximum pressure the fundamental Eigen value for B is computed and listed in Table 1. Hence for p = 225 micron (or .000435 lb/in^2).

$$m = \frac{.000435 \times 810}{1050 \times \left(\frac{\pi}{106}\right)^2} = 3.81$$

which corresponds to B = -.319 in Table 1. Thus, the maximum differential radial displacement at the midgore and at the seam which occurs in the equatorial plane is given by

$$w\Big|_{\substack{\theta=0 \\ x=0}} - w\Big|_{\substack{\theta=0 \\ x=24^{in}}} = B\,\frac{x^2}{r}\Big|_{x=24^{in}} = 0.227^{\text{ in}}$$

This corresponds to a maximum radial difference

Δ = 0.356 - 0.227 = 0.129 in which 0.356 is in its initial unpressurized value.

Approved for Public Release

Contrails

TABLE I

Values of B versus m

m	B	m	B	m	B	m	B	m	B
0	0	2.0	-.294	4.0	-.320	6.0	-.320	8.0	-.317
.1	-.043	2.1	-.298	4.1	-.320	6.1	-.320	8.1	-.317
.2	-.079	2.2	-.300	4.2	-.320	6.2	-.320	8.2	-.317
.3	-.109	2.3	-.303	4.3	-.320	6.3	-.319	8.3	-.317
.4	-.135	2.4	-.305	4.4	-.320	6.4	-.319	8.4	-.317
.5	-.157	2.5	-.307	4.5	-.320	6.5	-.319	8.5	-.317
.6	-.176	2.6	-.309	4.6	-.320	6.6	-.319	8.6	-.316
.7	-.193	2.7	-.311	4.7	-.320	6.7	-.319	8.7	-.316
.8	-.207	2.8	-.312	4.8	-.321	6.8	-.319	8.8	-.316
.9	-.220	2.9	-.313	4.9	-.321	6.9	-.319	8.9	-.316
1.0	-.213	3.0	-.314	5.0	-.321	7.0	-.319	9.0	-.316
1.1	-.241	3.1	-.315	5.1	-.321	7.1	-.318	9.1	-.316
1.2	-.250	3.2	-.316	5.2	-.321	7.2	-.318	9.2	-.316
1.3	-.258	3.3	-.317	5.3	-.321	7.3	-.318	9.3	-.316
1.4	-.265	3.4	-.318	5.4	-.320	7.4	-.318	9.4	-.316
1.5	-.272	3.5	-.318	5.5	-.320	7.5	-.318	9.5	-.315
1.6	-.277	3.6	-.319	5.6	-.320	7.6	-.318	9.6	-.315
1.7	-.282	3.7	-.319	5.7	-.320	7.7	-.318	9.7	-.315
1.8	-.287	3.8	-.319	5.8	-.320	7.8	-.317	9.8	-.315
1.9	-.291	3.9	-.320	5.9	-.320	7.9	-.317	9.9	-.315

Approved for Public Release

Contrails

CONCLUSIONS

The maximum differential radial displacement of a pressurized spheroid, which is composed of identical flat gores, in a zero gravity field, computed by this technique is believed to be a very good approximation to the true one, provided that the material is homogeneous, isotropic, and linearly elastic.

REFERENCES

1. Timoshenko, S. and Woinowsky, Krieger S., "Theory of Plates and Shells," McGraw-Hill Book Co., Inc., New York, 1959.
2. Schuerch, H., "Structural Analysis of Echo II," ARC-R-91, Astro Research Corp., Santa Barbara, California, May 2, 1963.

Approved for Public Release