Part of ADA241312 Digitized 5/4/2021

AN ANALYTICAL MODEL FOR THE VIBRATION OF VISCOELASTICALLY DAMPED CURVED SANDWICH BEAMS

Mohan D. Rao* and Shulin He**
ME-EM Department
Michigan Technological University
Houghton, MI 49931
(906) 487-2551

This paper describes an analytical model for the coupled flexural and longitudinal vibration of a curved sandwich beam system. The system consists of a pair of parallel and identical composite sandwich beams with a viscoelastic damping material forming the core. The ends of the beams may have any physically realizable boundary conditions, but, in this case are assumed to be simply-supported. The governing equations of motion for the forced vibration of the system are derived using the energy method and Hamilton's principle. Both shear and thickness deformation in the adhesive layer is included in the analysis. The solution of the governing equations for the system resonance frequencies and loss factors are obtained in closed form using the Rayleigh-Ritz method. A parametric study has been conducted to evaluate the effects of curvature, core-thickness and adhesive shear modulus on the system natural frequencies and loss factors. The implications of this parametric study on the damping effectiveness of the system along with some design guidelines are included in the paper.

* Assistant Professor
** Graduate Student

Confirmed public via DTIC 5/4/2021

Part of ADA241312 Digitized 5/4/2021

I. INTRODUCTION

The use of viscoelastic material as a core material in constrained layer and sandwich layer arrangement is an effective way of damping vibration and noise in structures subjected to dynamic loading. Many structural components such as the wing of an aircraft, body panel of an automobile are curved in shape. In the analysis and design of passive damping treatments of such panels, it has been customary to use straight beam theory approximations, because of lack of better analysis methods that are simple and accurate. The objective of this research effort is to develop simple yet accurate analysis and design procedures for passive vibration control of curved sandwich panels.

Considerable research work has been done on the vibration of straight sandwich beams. The classic work in this area is reported by Ross, Ungar and Kerwin [1], and Kerwin [2]. They have derived an expression for an effective complex flexural stiffness for the system consisting of a damping layer sandwich between two face-plates. Mead and Markus [3] have studied the forced vibration of a sandwich beam with arbitrary boundary conditions. They have derived the sixth-order differential equation of motion for the flexural vibration and obtained solutions. Rao [4] has studied the frequency and loss factors of sandwich beams under various boundary conditions using the energy approach. Miles and Reinhall [5] have described an analytical model for the vibration of laminated beams including the effects of both shear and thickness deformation in the adhesive layer. Some work has been done on the vibration of curved sandwich beams. Petyt and Fleischer [6] have investigated the free vibration of a curved homogeneous beam. Ahmed [7] has studied the free vibration of curved sandwich beams by the finite element method. Vaswani, Asnavi and Nakra [8] have derived a closed-form solution for the system loss factors and resonance frequencies for a curved sandwich beams with a

Confirmed public via DTIC 5/4/2021

Part of ADA241312 Digitized 5/4/2021

viscoelastic core by the Ritz method. But the thickness of the adhesive core was not considered in their analysis. Furthermore their analysis does not include the longitudinal inertia effects.

In this papper, the authors have used the energy method and Hamilton's principle to derive the governing equation of motion for the coupled flexural and longitudinal vibation of a curved snadwich beam system. Both shear and thickness deformations of the adhesive core are included. The longitudinal kinetic energy of the beam system is also considered. A closed-form solution of the system modal loss facotrs, resonance frequencies and mode shapes is derived for a system having simply supported ends by the Ritz method. The effects of curvature, core-thckness and adhesive shear modulus on the system natural frequencies and loss factors are also studied.

II. THEORY

Fig. 1 shows the curved sandwich beam system chosen for study. Both the upper and lower beams are constrained in the radial (transverse) direction at the two ends, and the upper beam is free to deform in the circumferential (longitudinal) direction. The governing equations of motion for the coupled flexural and longitudinal vibration under radial distribution load are derived by Hamilton's Princple. The following basic assumptions are made in the analysis.

1. Both upper and lower beams are elastic and isotropic. The adhesive core is viscoelastic whose elastic and shear moduli are modeled using the complex modulus approach.
2. Both beams and the adhesive core are assumed to be under plane stress state. The shear strain in the adhesive is proportional to $\left(\frac{R_c}{R_c + z_c}\right)$, where $-h_c/2 \le z_c \le h_c/2$, h_c being the adhesive thickness and R_c the radius of curvature of the middle surface of the adhesive layer. But

Confirmed public via DTIC 5/4/2021

the radial deformation of the adhesive is assumed to be constant through its thickness.

3. The transverse shear strains in both beams and the longitudinal deformation of the adhesive are neglected. Furthermore, rotational kinetic energies of both beams are also neglected.

Let the transverse and longitudinal displacements of a fiber in the middle surface of the upper beam be w_1 and u_1 respectively. The same variables with subscript 2 refer to those of the lower beam. The radial and circumferential displacements of the fiber at any point $p(\theta,z_1)$ in the upper beam are given by

$$w_p = w_1 , \qquad u_p = \frac{R_1+z_1}{R_1} u_1 - \frac{z_1}{R_1}\frac{\partial w_1}{\partial \theta} , \quad -h_1/2 \le z_1 \le h_1/2 \tag{1}$$

respectively. The strain in the fiber is given by

$$\varepsilon_{1,\theta} = \frac{1}{R_1+z_1}\left(\frac{\partial u_p}{\partial \theta} + w_p\right)$$

or

$$\varepsilon_{1,\theta} = \frac{1}{R_1} u_1' + \frac{1}{(R_1+z_1)}\left(w_1 - \frac{z_1}{R_1} w_1''\right) \tag{2}$$

where $u_1' = \frac{\partial u_1}{\partial \theta}$ and $w_1'' = \frac{\partial^2 w_1}{\partial \theta^2}$.

Strain energy of the upper beam is $U_1 = \frac{1}{2}\int_{v_1} E_1 \varepsilon_{1,\theta}^2 dv_1$. As $dv_1 = (R_1+z_1)dz_1 d\theta$, for unit width, we have

$$U_1 = \frac{E_1}{2}\int_0^{\phi} \frac{h_1}{R_1}\left[\left(u_1'+w_1\right)^2 + \left(\frac{h_1^2}{12R_1^2}\right)\left(w_1+w_1''\right)^2\right] d\theta \tag{3}$$

where E_1, R_1 and h_1 are the elastic moduli, radius of curvature of the middle surface and thickness of the upper beam respectively. z_1 is the radial coordinate from the middle surface.

Kinetic energy of the upper beam is approximated as

$$T_1 = \frac{\rho_1 h_1 R_1}{2}\int_0^{\phi}\left(\dot{u}_1^2 + \dot{w}_1^2\right) d\theta \tag{4}$$

where ρ_1 is the density of the upper beam material and $\dot{u}_1 = \frac{\partial u_1}{\partial t}$, $\dot{w}_1 = \frac{\partial w_1}{\partial t}$.
Similarly, kinetic energy T_2 and strain energy U_2 of lower beam are given by

$$T_2 = \frac{\rho_2 h_2 R_2}{2}\int_0^{\phi}\left(\dot{u}_2^2 + \dot{w}_2^2\right)d\theta \tag{5}$$

$$U_2 = \frac{E_2}{2}\int_0^{\phi}\frac{h_2}{R_2}\left[\left(u_2' + w_2\right)^2 + \left(\frac{h_2^2}{12R_2^2}\right)\left(w_2 + w_2''\right)^2\right]d\theta \tag{6}$$

where E_2, R_2, h_2 and ρ_2 are the young's moduli, radius of curvature of the middle surface, thickness and density of the lower beam respectively. Also, $u_2' = \frac{\partial u_2}{\partial \theta}$, $w_2'' = \frac{\partial^2 w_2}{\partial \theta^2}$, $\dot{u}_2 = \frac{\partial u_2}{\partial t}$ and $\dot{w}_2 = \frac{\partial w_2}{\partial t}$.

The shear strain in the middle fiber of the adhesive core is approximated as

$$\gamma_{mid} = \frac{u_B - u_A}{h_c} + \frac{1}{2}\left[\frac{1}{R_1}\frac{\partial w_1}{\partial \theta} + \frac{1}{R_2}\frac{\partial w_2}{\partial \theta}\right]$$

or

$$\gamma_{mid} = \frac{1}{h_c}\left[(a_2 u_2 - a_1 u_1) + \frac{h_1 + h_c}{2}\frac{w_1'}{R_1} + \frac{h_2 + h_c}{2}\frac{w_2'}{R_2}\right] \tag{7}$$

where $a_1 = 1 + h_1/(2R_1)$ and $a_2 = 1 - h_2/(2R_2)$.

Then the shear strain of the adhesive core is

$$\gamma_c = \frac{R_c}{R_c + z_c}\frac{1}{h_c}\left[(a_2 u_2 - a_1 u_1) + \frac{h_1 + h_c}{2}\frac{w_1'}{R_1} + \frac{h_2 + h_c}{2}\frac{w_2'}{R_2}\right], \quad -h_c/2 \le z_c \le h_c/2 \tag{8}$$

The radial strain of the adhesive core is $\varepsilon_{c,z} = \frac{w_2 - w_1}{h_c}$.

Strain energy of the adhesive core is $U_c = \frac{1}{2}\int_{v_c}\left[G_c^* \gamma_c^2 + kE_c^* \varepsilon_{c,z}^2\right]dv_c$, i.e.

$$U_c = \int_0^{\phi}\left\{\frac{G_c^* R_c}{2h_c}\left(1 + \frac{h_c^2}{12R_c^2}\right)\left[(a_2 u_2 - a_1 u_1) + \left(\frac{h_1 + h_c}{2}\right)\frac{w_1'}{R_1} + \left(\frac{h_2 + h_c}{2}\right)\frac{w_2'}{R_2}\right]^2 + \frac{kE_c^* R_c}{2h_c}\left(w_2 - w_1\right)^2\right\}d\theta \tag{9}$$

where $k = \frac{1}{(1-\nu)(1+\nu)}$. ν is Poisson's ratio. E_c^* and G_c^* are the complex elastic and shear moduli of the adhesive layer which are applicable only under harmonic vibration. $E_c^* = E_c'(1+\eta_{c1})$ and $G_c^* = G_c'(1+\eta_{c2})$, where E_c' and G_c' are the storage moduli, and η_{c1} and η_{c2} are the loss factors corresponding to extersional and shear deformation of the viscoelastic material. Values of E_c^*

and G_c^* corresponding to a frequnecy and temperature can be obtained from material data sheets supplied by the manufacturers in the form of a nomogram.

Kinetic energy of the adhesive layer is approximated as

$$T_c = \frac{1}{8}\rho_c \int_{v_c} \left[(\dot{u}_A+\dot{u}_B)^2 + (\dot{w}_A+\dot{w}_B)^2\right] dv_c, \quad \text{or}$$

$$T_c = \frac{\rho_c h_c R_c}{8} \int_0^{\phi} \left\{ \left[a_1\dot{u}_1 + a_2\dot{u}_2 + \frac{1}{2}\left(\frac{h_2}{R_2}\dot{w}_2' - \frac{h_1}{R_1}\dot{w}_1'\right)\right]^2 + (\dot{w}_1 + \dot{w}_2)^2 \right\} d\theta \tag{10}$$

where ρ_c is the density of adhesive and $\dot{w}_i' = \frac{\partial^2 w_i}{\partial t \partial \theta}$, i = 1, 2.

Work done by external forces is

$$W = \int_0^{\phi} w_1 q_1 (R_1 - h_1/2) d\theta \tag{11}$$

Let $\int_0^{\phi} f d\theta = T_1+T_2+T_c-U_1-U_2-U_c+W$. Hamilton's Principle $\delta\int_{t_1}^{t_2}(T-V+W)dt = 0$ gives

$$\int_{t_1}^{t_2} \left\{ \int_0^{\phi} \delta f(u_1,u_2,u_1',u_2',\dot{u}_1,\dot{u}_2,w_1,w_2,w_1',w_2',w_1'',w_2'',\dot{w}_1',\dot{w}_2',\dot{w}_1,\dot{w}_2) d\theta \right\} dt = 0 \tag{12}$$

From the principle of calculus of variations, we get the following governing equations of motion of the system.

$$\frac{E_1 h_1}{R_1}\left(\frac{\partial^2 u_1}{\partial \theta^2} + \frac{\partial w_1}{\partial \theta}\right) + \frac{a_1 G_c^* R_c}{h_c}\left(1+\frac{h_c^2}{12R_c^2}\right)\left[(a_2u_2-a_1u_1) + \frac{\partial}{\partial\theta}\left(\frac{h_1+h_c}{2R_1}w_1 + \frac{h_2+h_c}{2R_2}w_2\right)\right]$$

$$- \rho_1 h_1 R_1 \frac{\partial^2 u_1}{\partial t^2} - \frac{a_1\rho_c h_c R_c}{4}\left[\frac{\partial^2}{\partial t^2}(a_1u_1 + a_2u_2) + \frac{1}{2}\frac{\partial^3}{\partial t^2\partial\theta}\left(\frac{h_2}{R_2}w_2 - \frac{h_1}{R_1}w_1\right)\right] = 0 \tag{13}$$

$$\frac{E_2 h_2}{R_2}\left(\frac{\partial^2 u_2}{\partial \theta^2} + \frac{\partial w_2}{\partial \theta}\right) - \frac{a_2 G_c^* R_c}{h_c}\left(1+\frac{h_c^2}{12R_c^2}\right)\left[(a_2u_2-a_1u_1) + \frac{\partial}{\partial\theta}\left(\frac{h_1+h_c}{2R_1}w_1 + \frac{h_2+h_c}{2R_2}w_2\right)\right]$$

$$- \rho_2 h_2 R_2 \frac{\partial^2 u_2}{\partial t^2} - \frac{a_2\rho_c h_c R_c}{4}\left[\frac{\partial^2}{\partial t^2}(a_1u_1 + a_2u_2) + \frac{1}{2}\frac{\partial^3}{\partial t^2\partial\theta}\left(\frac{h_2}{R_2}w_2 - \frac{h_1}{R_1}w_1\right)\right] = 0 \tag{14}$$

$$\frac{E_1 h_1^3}{12R_1^3}\left(\frac{\partial^4 w_1}{\partial\theta^4} + 2\frac{\partial^2 w_1}{\partial\theta^2} + w_1\right) + \frac{E_1 h_1}{R_1}\left(\frac{\partial u_1}{\partial\theta} + w_1\right) - \frac{kE_c^* R_c}{h_c}(w_2 - w_1)$$

$$- \frac{G_c^* R_c}{h_c}\left(1+\frac{h_c^2}{12R_c^2}\right)\left[\frac{\partial}{\partial\theta}(a_2u_2-a_1u_1)+\frac{\partial^2}{\partial\theta^2}\left(\frac{h_1+h_c}{2R_1}w_1+\frac{h_2+h_c}{2R_2}w_2\right)\right]\left(\frac{h_1+h_c}{2R_1}\right) + \rho_1 h_1 R_1\frac{\partial^2 w_1}{\partial t^2}$$

$$+ \frac{\rho_c h_c R_c}{4}\left\{\frac{\partial^2}{\partial t^2}(w_1+w_2) + \frac{h_1}{2R_2}\frac{\partial^3}{\partial t^2\partial\theta}\left[a_1u_1+a_2u_2+\frac{1}{2}\left(\frac{h_2}{R_2}w_2-\frac{h_1}{R_1}w_1\right)\right]\right\} = \left(R_1-\frac{h_1}{2}\right)q_1 \qquad (15)$$

$$\frac{E_2h_2^3}{12R_2^3}\left(\frac{\partial^4 w_2}{\partial\theta^4}+2\frac{\partial^2 w_2}{\partial\theta^2}+w_2\right) + \frac{E_2h_2}{R_2}\left(\frac{\partial u_2}{\partial\theta}+w_2\right) + \frac{kE_c^*R_c}{h_c}(w_2-w_1)$$

$$- \frac{G_c^* R_c}{h_c}\left(1+\frac{h_c^2}{12R_c^2}\right)\left[\frac{\partial}{\partial\theta}(a_2u_2-a_1u_1)+\frac{\partial^2}{\partial\theta^2}\left(\frac{h_1+h_c}{2R_1}w_1+\frac{h_2+h_c}{2R_2}w_2\right)\right]\left(\frac{h_2+h_c}{2R_2}\right) + \rho_2 h_2 R_2\frac{\partial^2 w_2}{\partial t^2}$$

$$+ \frac{\rho_c h_c R_c}{4}\left\{\frac{\partial^2}{\partial t^2}(w_1+w_2) - \frac{h_2}{2R_2}\frac{\partial^3}{\partial t^2\partial\theta}\left[a_1u_1+a_2u_2+\frac{1}{2}\left(\frac{h_2}{R_2}w_2-\frac{h_1}{R_1}w_1\right)\right]\right\} = 0 \qquad (16)$$

The natural boundary conditions are given by

$$\left[\frac{\partial f}{\partial u_1'}\delta u_1 + \frac{\partial f}{\partial u_2'}\delta u_2+\left(\frac{\partial f}{\partial w_1'} - \frac{\partial}{\partial\theta}\frac{\partial f}{\partial w_1''} - \frac{\partial}{\partial t}\frac{\partial f}{\partial \dot{w}_1'}\right)\delta w_1 + \left(\frac{\partial f}{\partial w_2'} - \frac{\partial}{\partial\theta}\frac{\partial f}{\partial w_2''} - \frac{\partial}{\partial t}\frac{\partial f}{\partial \dot{w}_2'}\right)\delta w_2\right.$$

$$\left. + \frac{\partial f}{\partial w_1''}\delta w_1' + \frac{\partial f}{\partial w_2''}\delta w_2'\right]\Bigg|_0^{\phi} = 0 \qquad (17)$$

We will first consider the free vibration of the system and solve the above four equations to find the system loss factors, resoance frequencies and mode shapes. By using the forced and natural boundary conditions, it is possible to set up the frequency equation which is a matrix equation with complex elements. Using a numerical search technique, we can find the system resonance frequencies, loss factors, etc. The disadvantage of this method, however, is that the natural frequencies and loss factors are not obtained in closed form as it requires a numerical searching technique to obtain the results. Since we are interested in finding only the first several modes of vibration of the system, other approximate methods that are fairly accurate would yield closed form solution. One such method is the the Rayleigh-Ritz method in which the mode shapes are first assumed to set up the eigenvalue problem. The details of the application of Rayleigh-Ritz method to the present problem are presented in the following sections.

III. NUMERICAL RESULTS

To find the system resonance frequencies ω_r and the modal loss factors η corresponding to ω_r, we consider the free vibration of the system and assume $w_1(\theta,t)=W_1(\theta)e^{i\omega t}$, $u_1(\theta,t)=U_1(\theta)e^{i\omega t}$, $w_2(\theta,t)=W_2(\theta)e^{i\omega t}$ and $u_2(\theta,t)=U_2(\theta)e^{i\omega t}$, where $\omega = \omega_r(1 + i\,\eta)$. From the variational integral $\delta\int_{t_1}^{t_2}(T-V)dt = 0$, we get

$$\int_0^{\phi}\Bigg\{\bar{\omega}^2\Big[(U_1\delta U_1+W_1\delta W_1)+\bar{\rho}_2\bar{h}_2\bar{R}_2(U_2\delta U_2+W_2\delta W_2)+\frac{\bar{\rho}_c\bar{h}_c\bar{R}_c}{4}\Big[(a_1U_1+a_2U_2+\frac{h_2}{2R_2}W_2'-\frac{h_1}{2R_1}W_1')(a_1\delta U_1$$

$$+a_2\delta U_2+\frac{h_2}{2R_2}\delta W_2'-\frac{h_1}{2R_1}\delta W_1')+(W_1+W_2)(\delta W_1+\delta W_2)\Big]\Big] + \Big[(U_1'+W_1)(\delta U_1'+\delta W_1)+\frac{h_1^2}{12R_1^2}(W_1$$

$$+W_1'')(\delta W_1+\delta W_1'')+\frac{\bar{E}_2\bar{h}_2}{\bar{R}_2}\Big[(U_2'+W_2)(\delta U_2'+\delta W_2)+\frac{h_2^2}{12R_2^2}(W_2+W_2'')(\delta W_2+\delta W_2'')\Big]+\frac{k\bar{E}_cR_1R_c}{2h_1h_c}\delta\big(W_2$$

$$-W_1\big)^2+\frac{\bar{G}_cR_1R_c}{h_1h_c}\left(1+\frac{h_c^2}{12R_c^2}\right)\Big[a_2U_2-a_1U_1+\frac{h_1+h_c}{2R_1}W_1'+\frac{h_2+h_c}{2R_2}W_2'\Big]\Big[a_2\delta U_2-a_1\delta U_1$$

$$+\frac{h_1+h_c}{2R_1}\delta W_1'+\frac{h_2+h_c}{2R_2}\delta W_2'\Big]\Big]\Bigg\}\,d\theta = 0 \qquad (18)$$

where $\bar{h}_2 = \frac{h_2}{h_1}$, $\bar{\rho}_2 = \frac{\rho_2}{\rho_1}$, $\bar{R}_2 = \frac{R_2}{R_1}$, $\bar{h}_c = \frac{h_c}{h_1}$, $\bar{\rho}_c = \frac{\rho_c}{\rho_1}$, $\bar{R}_c = \frac{R_c}{R_1}$, $\bar{E}_2 = \frac{E_2}{E_1}$, $\bar{E}_c = \frac{E_c^*}{E_1}$, $\bar{G}_c = \frac{G_c^*}{E_1}$, $\bar{\omega} = \frac{\omega}{\omega_o}$ and $\omega_o = (\sqrt{E_1/\rho_1})/R_1$.

To define to mode shape of the curved sandwich beam system, it is assumed that both the lower and upper beams are simply supported in the transverse direction but the upper beam is free to move in longitudinal direction. The forced boundary conditions will then be $W_1(0) = W_2(0) = 0$, $U_2(0) = 0$, $W_1(\phi) = W_2(\phi) = 0$. Also the mode shape functions of the curved beams are approximated with the mode shape functions of straight Euler-Bernoulli beams. Then the mode shape functions can be chosen as

$$W_1(\theta) = \sum_{r=1}^{N} C_r \operatorname{Sin}\left(\frac{r\pi\theta}{\phi}\right) \tag{19}$$

$$U_1(\theta) = \sum_{r=1}^{N} C_r \operatorname{Cos}\left(\frac{r\pi\theta}{\phi}\right) \tag{20}$$

$$W_2(\theta) = \sum_{r=1}^{N} C_r \operatorname{Sin}\left(\frac{r\pi\theta}{\phi}\right) \tag{21}$$

$$U_2(\theta) = \sum_{r=1}^{N} C_r \operatorname{Sin}\left[\left(r - \frac{1}{2}\right)\frac{r\pi\theta}{\phi}\right] \tag{22}$$

which satisfy all the forced (essential) boundary conditions. The coefficient C_r for the shape functions are chosen to be the same for all four functions so that the resulting eigenvalue problem would yield the required N resonance frequencies and N loss factors of the system. Substituting the expressions of W_1, U_1, W_2 and U_2 into equation (18), we get,

$$\sum_{j=1}^{N}\left\{\frac{1}{2}\left\{\left(\frac{j\pi}{\phi}-1\right)^2+\frac{h_1^2}{12R_1^2}\left[1-\left(\frac{j\pi}{\phi}\right)^2\right]^2+\frac{\bar{E}_2\bar{h}_2}{\bar{R}_2}\left[\left(\frac{j\pi}{\phi}\right)^2\left((j-\tfrac{1}{2})^2+1\right)+\frac{h_2^2}{12R_2^2}\left(1-\left(\frac{j\pi}{\phi}\right)^2\right)^2\right]\right.\right.$$

$$\left.+\frac{\bar{G}_c R_1 R_c}{h_1 h_c}\left(1+\frac{h_c^2}{12R_c^2}\right)\left[a_1^2+a_2^2-\frac{a_1 j\pi}{\phi}\left(\frac{h_1+h_c}{R_1}+\frac{h_2+h_c}{R_2}\right)+\frac{1}{4}\left(\frac{j\pi}{\phi}\right)^2\left(\frac{h_1+h_c}{R_1}+\frac{h_2+h_c}{R_2}\right)^2\right]\right\}C_j$$

$$+\sum_{i=1}^{N}\left\{\frac{\bar{E}_2\bar{h}_2}{\phi\bar{R}_2}\left[\frac{i(j-\frac{1}{2})}{i^2-(j-\frac{1}{2})^2}+\frac{j(i-\frac{1}{2})}{j^2-(i-\frac{1}{2})^2}\right]+\frac{\bar{G}_c R_1 R_c}{h_1 h_c}\left(1+\frac{h_c^2}{12R_c^2}\right)\left[\frac{a_2}{2\phi}\left(\frac{h_1+h_c}{R_1}+\frac{h_2+h_c}{R_2}\right)\left(\frac{i(j-\frac{1}{2})}{(j-\frac{1}{2})^2-i^2}\right.\right.\right.$$

$$\left.\left.\left.\left.+\frac{i(i-\frac{1}{2})}{(i-\frac{1}{2})^2-j^2}\right)-\frac{a_1 a_2}{\pi}\left(\frac{(j-\frac{1}{2})}{(j-\frac{1}{2})^2-i^2}+\frac{(i-\frac{1}{2})}{(i-\frac{1}{2})^2-j^2}\right)\right]\right\}C_i\right\}\delta C_j + \bar{\omega}^2\sum_{j=1}^{N}\left\{\left\{(1+\bar{\rho}_2\bar{h}_2\bar{R}_2)\right.\right.$$

$$\left.+\frac{\bar{\rho}_c\bar{h}_c\bar{R}_c}{8}\left[(a_1^2+a_2^2)+\frac{a_1\pi j}{\phi}\left(\frac{h_2}{R_2}-\frac{h_1}{R_1}\right)+\frac{1}{4}\left(\frac{j\pi}{\phi}\right)^2\left(\frac{h_2}{R_2}-\frac{h_1}{R_1}\right)^2+4\right]\right\}C_j+\sum_{i=1}^{N}\frac{\bar{\rho}_c\bar{h}_c\bar{R}_c}{8}\left\{\frac{2a_1 a_2}{\pi}\left[\frac{(i-\frac{1}{2})}{(i-\frac{1}{2})^2-j^2}\right.\right.$$

$$\left.\left.\left.+\frac{(j-\frac{1}{2})}{(j-\frac{1}{2})^2-i^2}\right]+\frac{a_2}{\phi}\left(\frac{h_2}{R_2}-\frac{h_1}{R_1}\right)\left[\frac{i(j-\frac{1}{2})}{(j-\frac{1}{2})^2-i^2}+\frac{j(i-\frac{1}{2})}{(i-\frac{1}{2})^2-j^2}\right]\right\}C_i\right\}\delta C_j = 0 \tag{23}$$

Writing the above equation in matrix form, we have

$$\left[K_{ij}\right]\left\{C\right\} = \lambda\left[M_{ij}\right]\left\{C\right\} \tag{24}$$

where $[K_{ij}]$ and $[M_{ij}]$ are N × N matrices, called stiffness and inertia matrix respectively. $\lambda = \omega^2 = \omega_r^2(1+i\eta)$ and $\{C\} = [C_1\ C_2\ \cdots\ C_N]^T$. Here ω_r is the system resonance frequency and η is the modal loss factor corresponding to ω_r. A computer program has been written to solve this eigenvalue problem. Numerical results were generated to observe the effects of curvture, core-thickness and adhesive shear modulus on the system resonance frequencies ω_r and modal loss factors η for the first four modes.

The adhesive shear modulus plays a very important role on the damping of the sandwich system. The variations of the normalized resonance frequency $\bar{\omega}_r$ $(= \frac{\omega_r}{\omega_0})$ and modal loss factor η with respect to the normalized shear modulus G_{c1}(= real part of $\bar{G}_c$) are plotted in Figures 2 and 3 for the first four modes. The input data used here were $\eta_c = \eta_{c1} = \eta_{c2} = 0.1$, $h_1 = h_2 = 4$ mm, $\phi = 1.0$, $h_c = 2$ mm and $R_c = 1.2$ m. It can be observed from Figure 2 and Figure 3 that when $G_{c1} < 10^{-5}$ (soft adhesive material), the system resonance frequencies $\bar{\omega}_r$ increase very slowly (or almost constant) with G_{c1}, but the system loss factors η vary almost linearly with G_{c1}. For values of G_{c1} such that $10^{-5} < G_{c1} < 10^{-3}$, both $\bar{\omega}_r$ and η increase rapidly with G_{c1}, which means we can increase the system damping capacity without sacrificing the stiffness of system. Usually this is what the designers require in the design of constrained layer damping treatment for the system. When $G_{c1} > 10^{-3}$ (hard adhesive material), the system loss factors η remain almost constant with G_{c1}, but the resonance frequencies $\bar{\omega}_r$ increase linearly with G_{c1}.

The effects of the adhesive thickness h_c on the system resonance frequencies and loss factors are also studied. The input data in this case were $G_{c1} = 10^{-4}$, $\eta_{c2} = 0.1$, $h_1 = h_2 = 4$ mm, $R_1 = 1.2$ m and $\phi = 1.0$. The thickness h_c was increased from 0.5 mm to 6 mm in steps of 0.5 mm. The variations of $\bar{\omega}_r$ and η with h_c are plotted in Figures 4 and 5. It can be seen from these two figures that both $\bar{\omega}_r$ and η decrease with h_c. The decrease of $\bar{\omega}_r$ with h_c is obvious as the system stifness will decrease when h_c is increased. Furthermore, from section II, we know that the shear strain of the adhesive layer is mainly contributed by the term $(u_2 - u_1)/h_c$. Hence, increasing h_c will reduce the shear deformation of the adhesive layer and thus decrease the energy dissipation capacity of the sandwich system. This concludes that an increase in adhesive thickness does not always increase the system loss factor. It is therefore the combination of G_{c1} and h_c that needs to be optimized to maximize the system damping capacity.

The third parameter which affect the system resonance frequencies and modal loss factors is the the radius of curvature R_c of the middle surface of the adhesive layer. There are two ways of changing R_c as discussed below.

In the first case, the angle ϕ is kept constant, while changing R_c. This means the total length of the sandwich beam system will change with R_c. Figures 6 and 7 shows the variations of $\bar{\omega}_r$ and η with R_c. The input data were $\phi = 1.0$, $G_{c1} = 1 \times 10^{-4}$, $\eta_c = \eta_{c1} = \eta_{c2} = 0.1$, $h_1 = h_2 = 4$ mm, $h_c = 2$ mm and $\omega_o = \sqrt{E_1/\rho_1}$. R_c was varied from 0.1 m to 2.0 m in steps of 0.1 m. It can be seen that $\bar{\omega}_r$ decreases but η increases with R_c, especially when $R_c <$ 0.7 m. The variations of $\bar{\omega}_r$ and η with R_c are obvious as the total length of the curved sandwich beam system increases with an increase in R_c.

In the second case, the total length of the curved sandwich beam is kept constant at 1.2 m, which means changing R_c will result in a change of angle ϕ. Figures 8 and 9 show the variations of $\bar{\omega}_r$ and η with R_c. G_{c1}, η_c, h_1, h_2, h_c and ω_o were kept same as case one, but $\phi = 1.2/R_C$. R_c was varied from 0.3 m to 2.2 m in steps of 0.1 m. It is observed from Figure 8 that $\bar{\omega}_r$ is almost constant with R_c. In fact the change of $\bar{\omega}_r$ with R_c is very insignifcant for the first and second mode when $R_c > 0.6$ m. For the third and fourth modes, however, $\bar{\omega}_r$ change slightly with R_c when $R_c < 0.9$ m, but after $R_c > 0.9$ m the change of $\bar{\omega}_r$ with R_c is not very obvious. In fact, in this range, as R_c increases, the curved beam turns to a straight beam and so large values of R_c will have little effect on the system resonance frequencies. The variation of η with R_c has a somewhat decreasing trend as seen in Figure 9. In summary, when the radius of curvature R_c is relatively small, R_c has some effect on $\bar{\omega}_r$ and η, but the effects are negligible for large values of R_c.

IV. CONCLUSIONS

The coupled transverse and longitudinal vibration of a curved sandwich beam system is investigated using the energy method in this paper. A closed form solution for the system resonance frequencies and modal loss factors is derived by the Rayleigh-Ritz method. Numerical results show that high values of adhesive shear modulus will influence the resonance frequenies much more greatly than the modal loss factors. Relatively small radius of curvature of the beam system will affect the system resonance frequencies and modal loss factors. But large values of radius of curvature will have very little effect on the resonance frequencies and modal loss factors. Furthermore, an increase in the thickness of the adhesive layer will decrease both the system resonance frequencies and modal loss factors. The analytical model presented here can be used as an effective tool in the design of constrained layer damping treatments for passive vibration control of curved sandwich systems.

V. ACKNOWLEDGEMENT

The authors would like to acknowledge the Natural Science Foundation, Washington, D.C. for financial support for this project through a research grant.

VI. REFERENCES

1. D. Ross, E. Ungar and E. M. Kerwin, Jr. 1959, *Structural Damping, ASME, NewYork, ed. by J. E. Ruzicka* Damping of Plate Flexural Vibrations by

Means of Viscoelastic Laminae

2. E. M. KERWIN, Jr. 1959 *The Journal of the Acoustical Society of America* **31(7)**, 952-962. Damping of Flexural Waves by a Constrained Visco-elastic Layer.

3. D. J. MEAD and S. MARKUS 1970 *Journal of Sound and Vibration* **12(1)**, 99-112. Loss Factors and Resonant Frequencies of Encastre Damped Sandwich Beams.

4. D. K. RAO 1978 *Journal of Mechanical Engineering Science* **20(5)**, 271-282. Frequency and Loss Factors of Sandwich Beams Under Various Boundary Conditions.

5. R. N. MILES and P. G. REINHALL 1986 *Journal of Vibration, Acoustics, Stress and Reliability in Design* **108**, 56-64. An Analytical Model for the Vibration of Laminated Beams Including the Effects of Both Shear and Thickness Deformation in the Adhesive Layer.

6. M. PETYT and C. C. FLEISCHER 1971 *Journal of Sound and Vibration* **18(1)**, 17-30. Free Vibration of A Curved Beam.

7. K. M. AHMED 1971 *Journal of Sound and Vibration* **18**, 61-74. Free Vibration of Curved Sandwich Beam by Finite Element Method.

8. J. VASWANI, N. T. ASNANI and B. C. NAKRA 1988 *Composite Structures* **10**, 231-245. Vibration and Damping Analysis of Curved Sandwich Beams with a Visco-elastic Core.

9. W. FLUGGE 1962 *Stress in Shells*. Springer-Verlag.

Figure 1. Configuration of the curved sandwich beam system

Figure 2. Variation of system resonance frequency with adhesive shear modulus

Figure 3. Variation of system loss factor with adhesive shear modulus

Figure 4. Variation of system resonance frequency with adhesive thickness

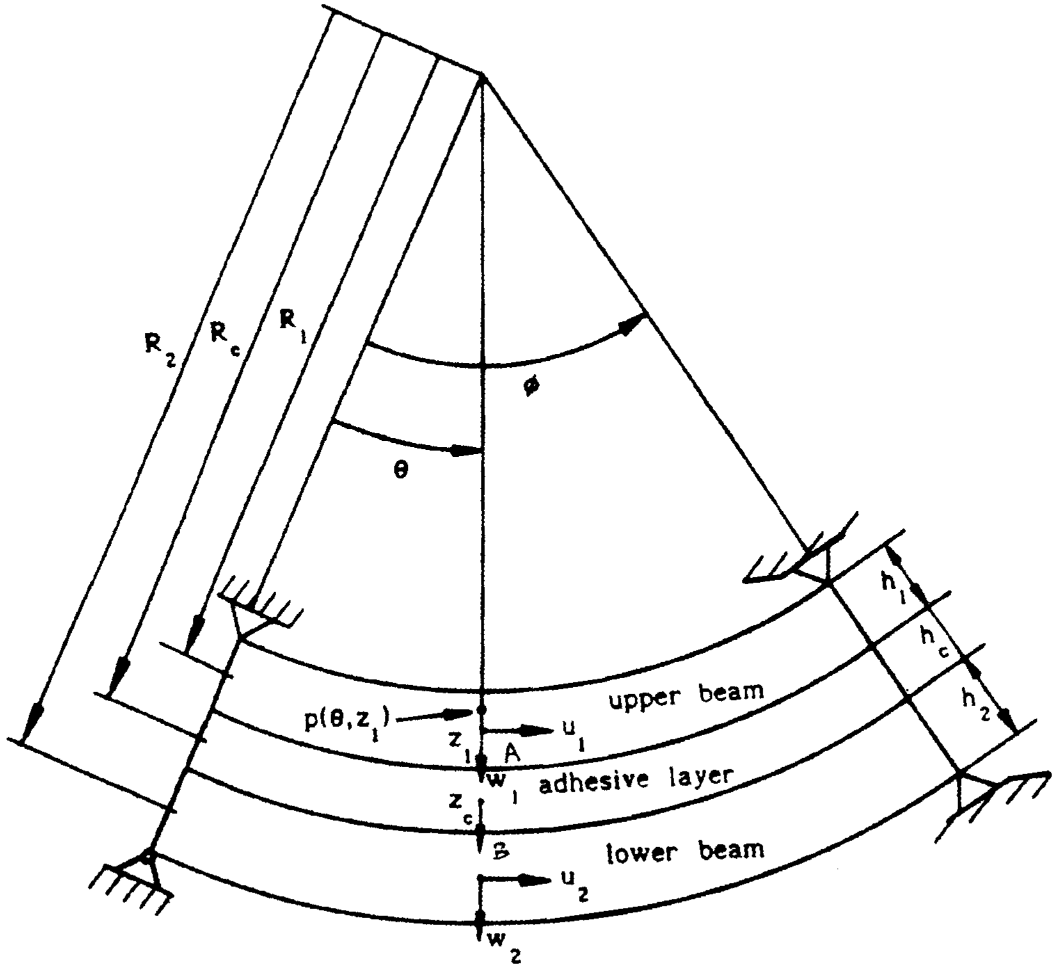
Figure 5. Variation of system loss factor with adhesive thickness

Figure 6. Variation of system resonance frequency with radius of curvature of the central surface of the adhesive layer ($\phi = 1.0$)

Figure 7. Variation of system loss factor with radius of curvature of the central surface of the adhesive layer ($\phi = 1.0$)

Figure 8. Variation of system resonance frequency with radius of curvature of the central surface of the adhesive layer ($\phi = 1.2/R_c$)

Figure 9. Variation of system loss factor with radius of curvature of the central surface of the adhesive layer ($\phi = 1.2/R_c$)



R2
Rc
R1
ø
θ
h1
hc
h2
upper beam
p(θ,z1)
u1
z1
A
w1
adhesive layer
zc
B
lower beam
u2
w2

Figure 1

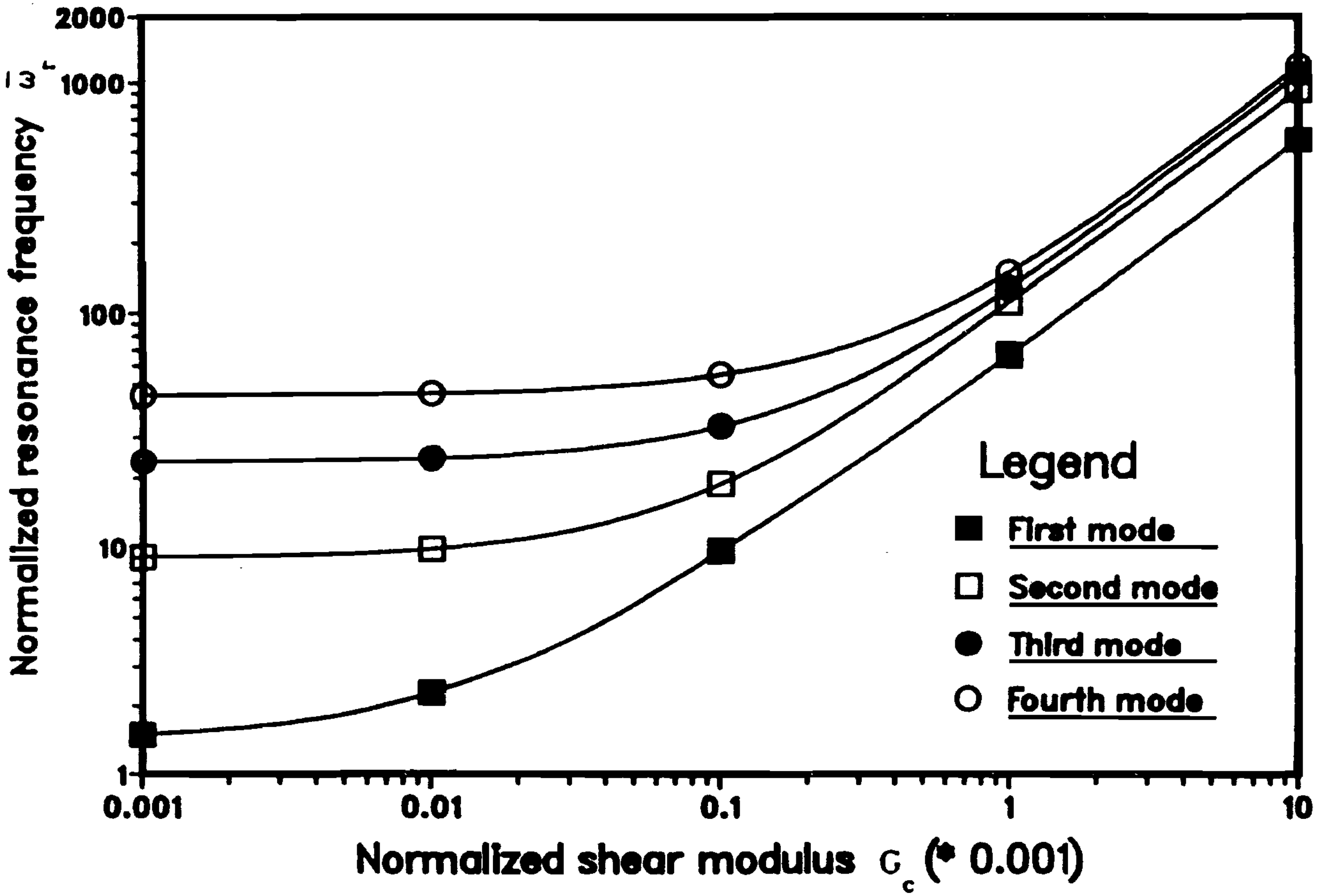

Normalized resonance frequency
2000
1000
100
10
1
0.001
0.01
0.1
1
10
Normalized shear modulus G_c (* 0.001)
Legend
First mode
Second mode
Third mode
Fourth mode

Fig. 2

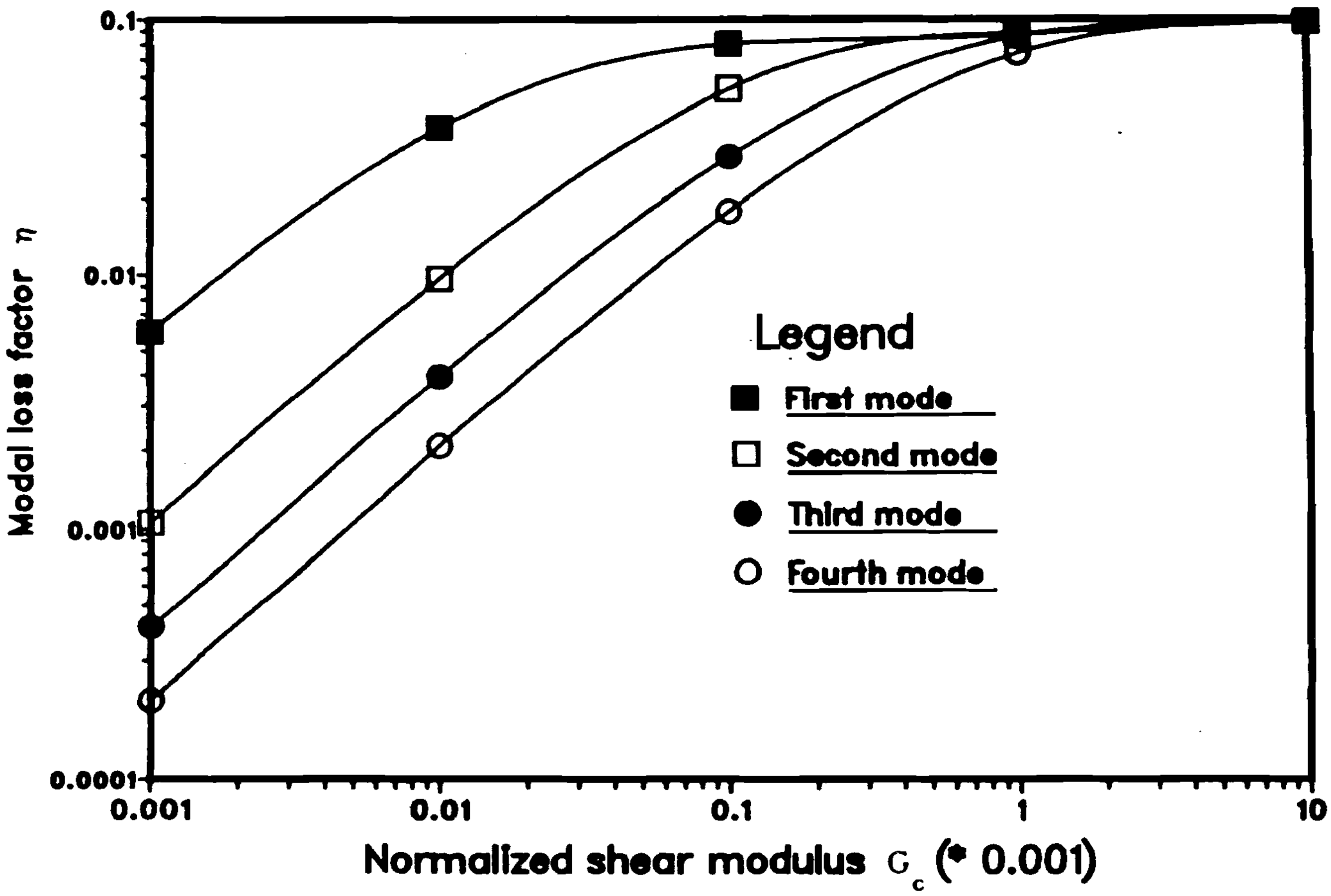

Modal loss factor η
0.1
0.01
0.001
0.0001
0.001
0.01
0.1
1
10
Normalized shear modulus G_c (* 0.001)
Legend
First mode
Second mode
Third mode
Fourth mode

Fig. 3

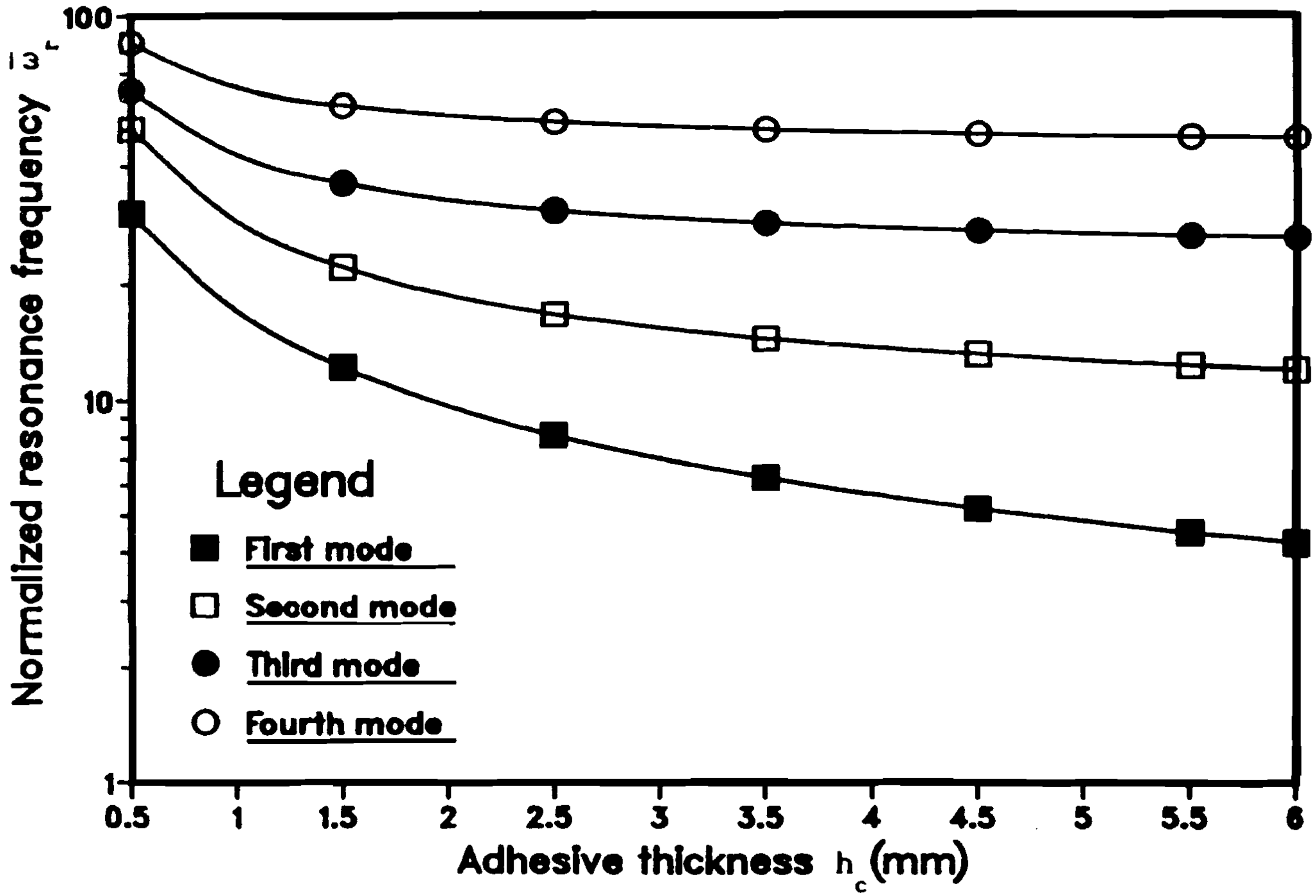

100
10
1
Normalized resonance frequency $\bar{\omega}_r$
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
Adhesive thickness h_c (mm)
Legend
First mode
Second mode
Third mode
Fourth mode

Fig. 4

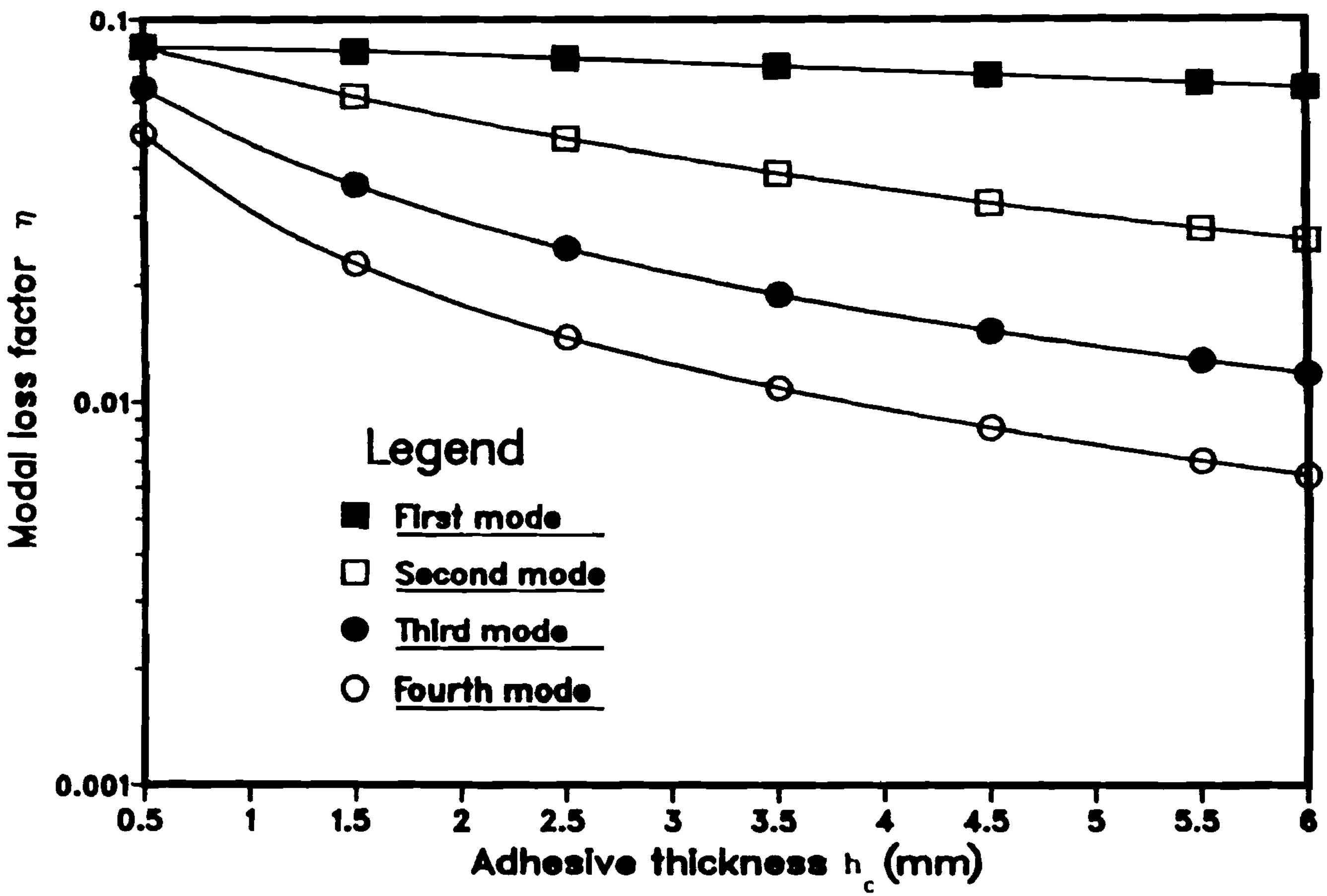

Modal loss factor η
0.1
0.01
0.001
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
Adhesive thickness h_c (mm)
Legend
First mode
Second mode
Third mode
Fourth mode

Fig. 5

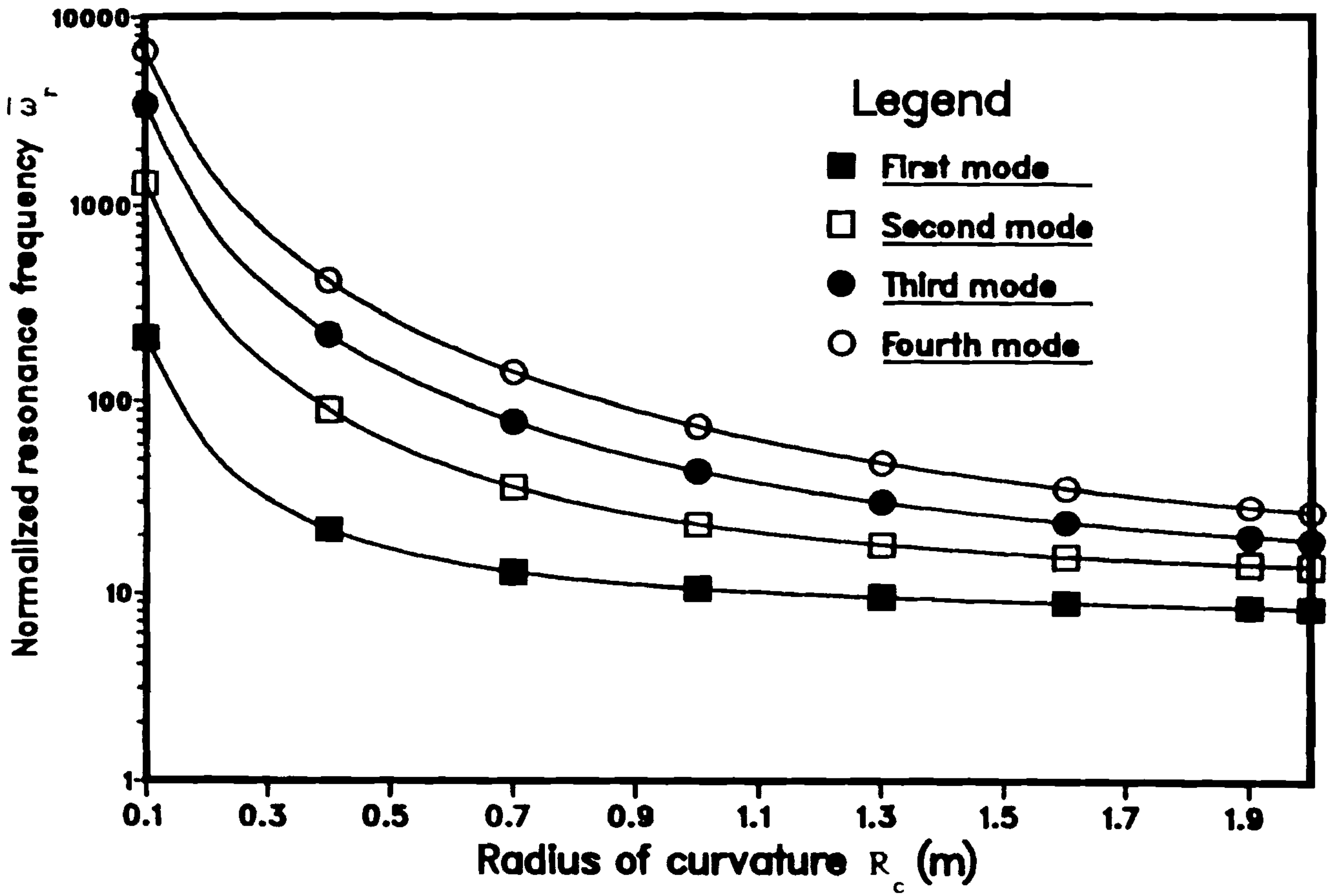

10000
1000
100
10
1
Normalized resonance frequency
0.1
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
Radius of curvature R_c (m)
Legend
First mode
Second mode
Third mode
Fourth mode

Fig. 6

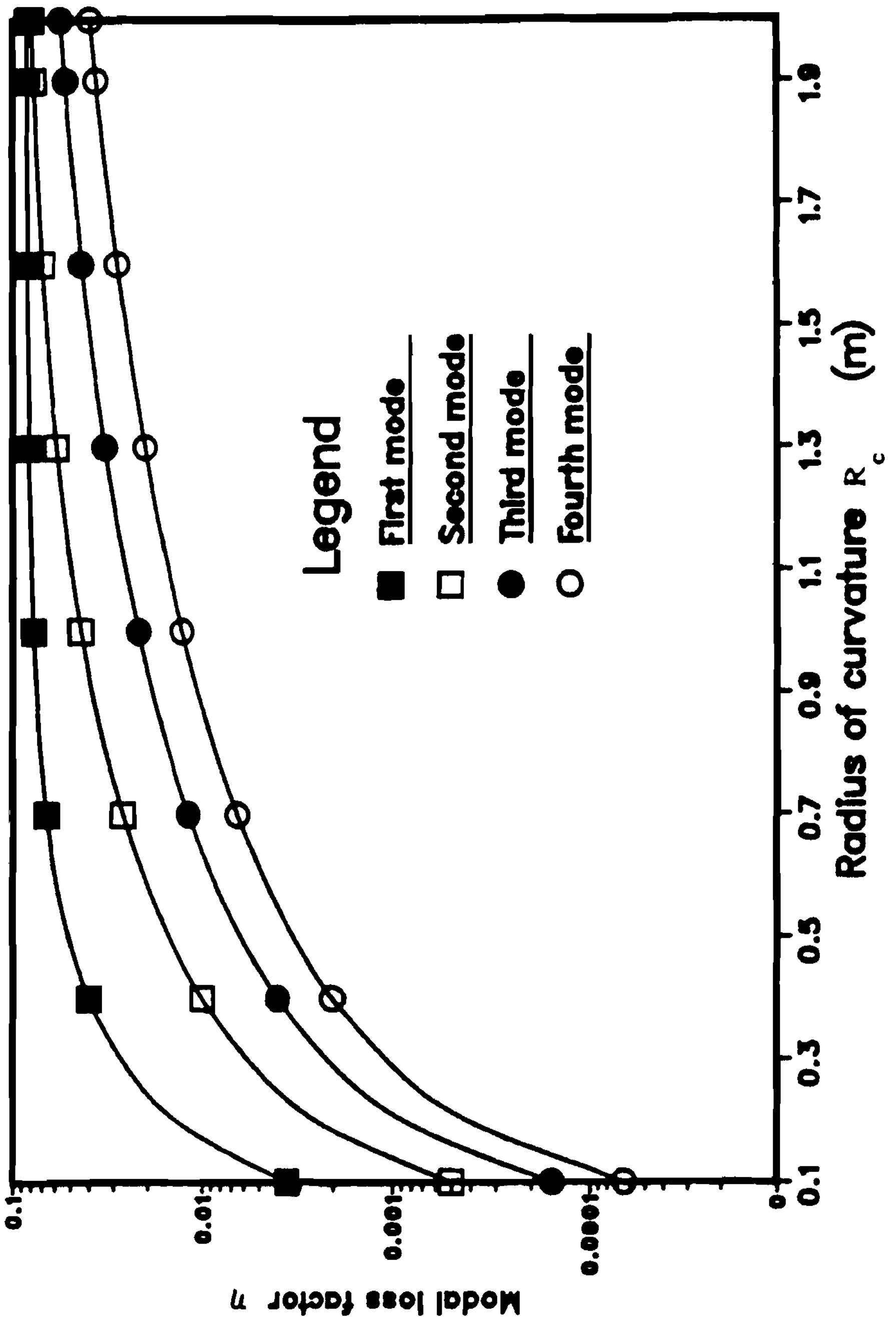

Legend
First mode
Second mode
Third mode
Fourth mode
Modal loss factor η
0.1
0.01
0.001
0.0001
0
0.1
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
Radius of curvature R c (m)

Fig. 7

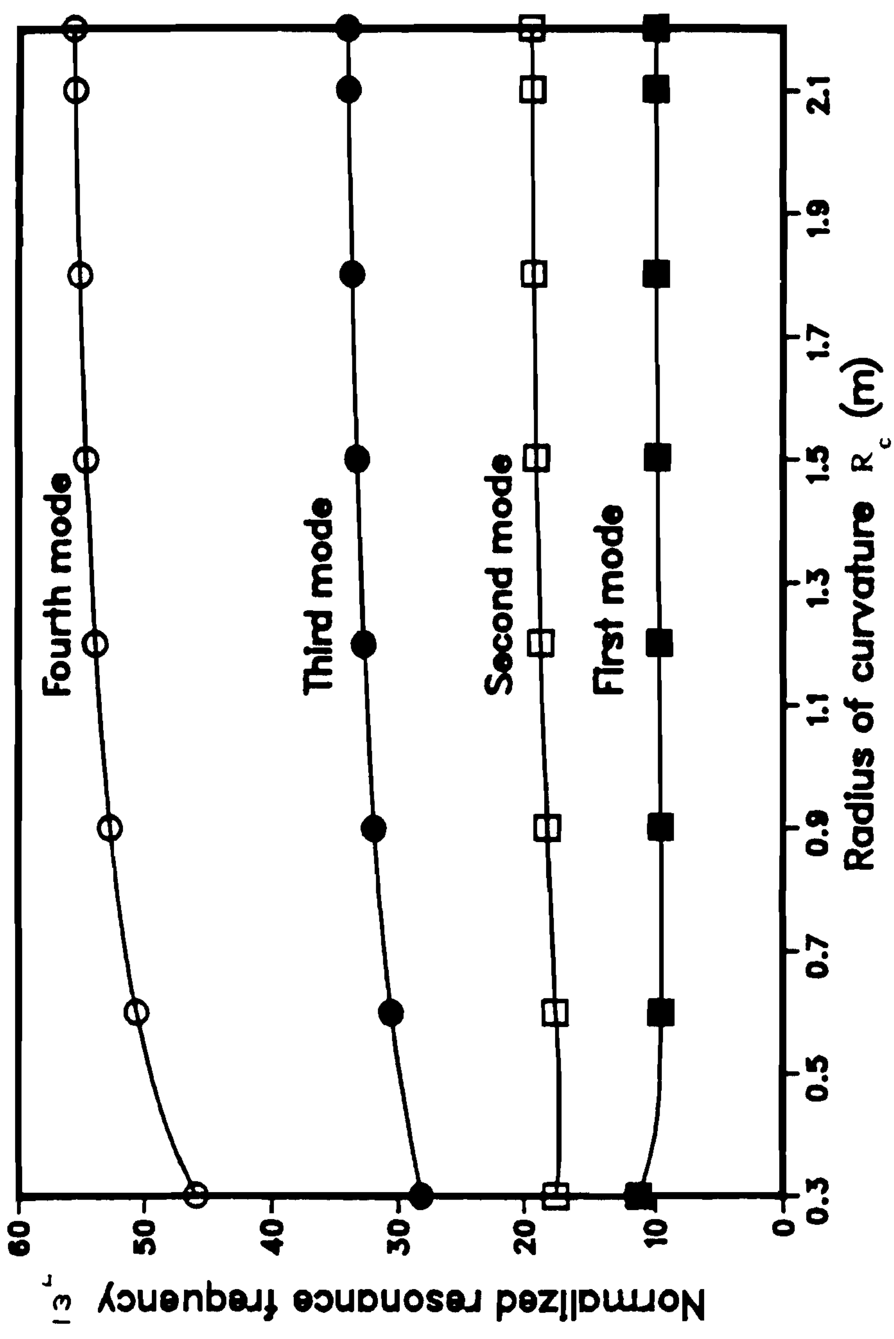

Fourth mode
Third mode
Second mode
First mode
Normalized resonance frequency
Radius of curvature R_c (m)
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
0
10
20
30
40
50
60

Fig. 8

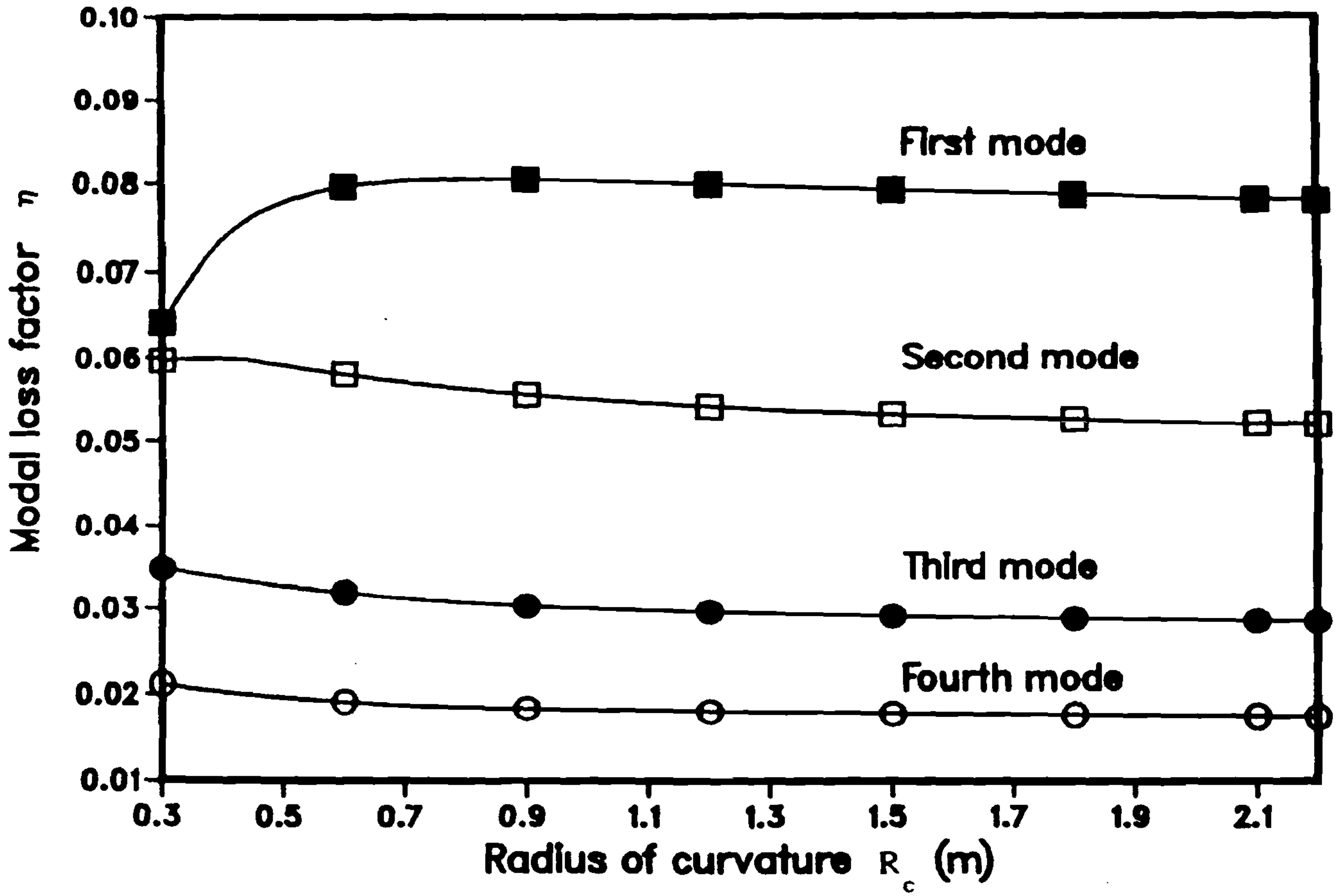

0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
Modal loss factor η
First mode
Second mode
Third mode
Fourth mode
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
Radius of curvature R c (m)

Fig. 9