

THE NEED FOR PASSIVE DAMPING IN FEEDBACK CONTROLLED FLEXIBLE STRUCTURES *

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ABSTRACT

It is well established that an undamped infinite dimensional *mathematical model* of a flexible structure cannot be stabilized with finite bandwidth control; the required gain stabilization beyond the bandwidth is negated by the infinite structural gain at each resonance. Thus, even a mathematical model with no modeling uncertainty will show that passive damping is critical to enabling active control. What is less well known are the benefits of passive damping for the robust control of *real structures*. There has been a tendency, in the research literature, to define the research problem to consist of developing control approaches for the broadband control (many modes in the control bandwidth) of poorly modeled, lightly damped, modally dense structures. There is ample reason to believe that such control is practically unachievable and that the attribute "lightly damped" is one of the most easily and readily remedied characteristics of such a problem structure.

This paper reports upon the enabling effect of passive damping in the control of uncertain flexible structures, particularly with non–collocated actuators and sensors. Quantative results are all single–input single–output and the benefits of passive damping are then understandable in terms of classical ideas of gain and phase stabilization. The paper derives approximate expressions for the minimum acceptable level of passive damping in terms of modeling uncertainty and desired bandwidth. These relationships can then be interpreted as specifying either a minimum level of passive damping or a minimum level of modeling fidelity. If the requirement is not met, robust control with the bandwidth including uncertain flexible dynamics is not possible with linear time invariant (LTI) compensation.

INTRODUCTION

In this paper, "bandwidth" implies "that frequency range over which the loop transfer function magnitude ratio varies by no more than 3dB." This definition is sensible for reference command following and for disturbance rejection when the respective input command and output referenced disturbance signals are expected to be broadband. It is possible that neither condition is met in an application of feedback control to a lightly damped flexible structure. A common goal for flexible structures is disturbance rejection rather than command following. Typical output–referenced disturbances are narrow band, either because they were generated by a time periodic process (typically an operating machine) or because they have been filtered through the resonant structure. With reference to rejection of such disturbances, another definition of bandwidth may be appropriate.

This paper addresses the problem statement: "The control bandwidth must include many poorly modeled, lightly damped, closely spaced modes." This problem statement is figuratively depicted in Figure 1a. The fundamental point made in the paper is that no linear time invariant compensation exists which will robustly (in the sense of stability

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robustness as opposed to performance robustness) achieve what is suggested by Figure 1a, unless a significant amount of passive damping is present. The required level of passive damping is sketched in Figure 1b.

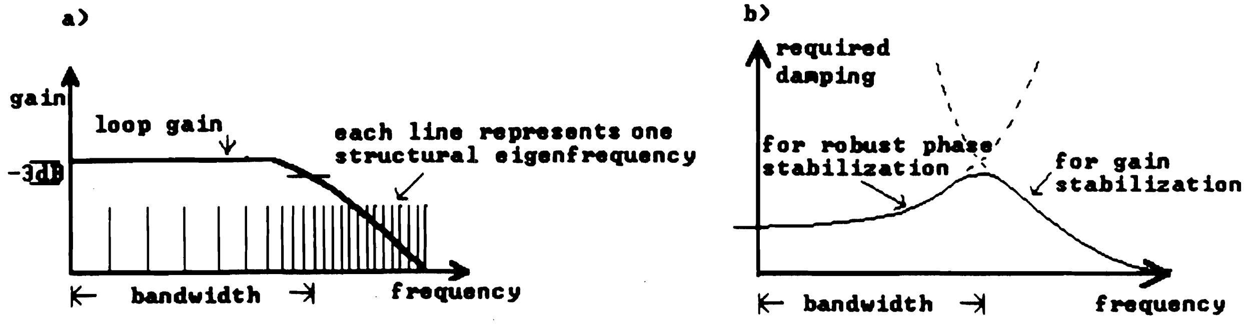

Figure 1. (a) Figurative depiction of problem statement for bandwidth to include many poorly modeled, lightly damped, closely spaced modes. (b) Required level of passive damping to meet problem specification.

The obvious approach to eliminating undesirable dynamics from the controlled plant is to generate control signals which cancel these motions [Rossi]. This "plant inversion" philosophy is what all control design techniques, both linear and nonlinear, SISO and MIMO [Williams], strive to realize when the aim is to achieve bandwidth greater than the frequency of the dominant modes. In benign systems, the effect of reasonable levels of model uncertainty is not such as to yield instability and is thus tolerable. For lightly damped oscillatory systems which typify the broadband structure control problem, however, the undesirable result of high gain/performance plant inversion, is instability of the closed loop system when sufficient model mismatch ocurs. A question which arises is how much damping is required to maintain closed loop stability if the structural dynamics are known only with a given level of uncertainty?

Some specialized literature dealing with design of controllers for uncertain lightly damped structural dynamics exists [ACC, Bryson, Bontsema, Rosenthal, Rossi, Wedell, Wie, et al], but the authors assume a plant given and unable to be modified, leading to a disregard of prescribing increased passive damping as part of the control solution. "Modern" (LQG, H_2, H^∞, L_1) control design techniques revolve around parameterizing of the uncertainty into the design model [Bontsema, Bryson, Wie, Wedell], often leading to rather convoluted design and robustness evaluation procedures. Notable for also ignoring passive damping is a large body of literature on combined optimal design of the structural dynamics and the active control system [JPL, Venkayya, Rao, Belvin]. Much of this literature implicitly defines structural dynamic design as the appropriate selection of stiffness and mass parameters. Passive damping levels are assumed to be very low and beyond specification by the designer. This leads to elegant mathematics and numerics, but also leads to results of questionable engineering relevance.

Novel nonlinear techniques, not LTI, such as that due to [Balas] or [Kopf], where use is made of phase–locked loops to track modal frequency and adjust compensation accordingly, also exist. These techniques may offer much promise for the broadband control of poorly modeled lightly damped structural dynamics, but are beyond the scope of this paper.

This paper makes the strong quantitative statement that sufficiently lightly damped and poorly modeled structural dynamics can never be within the bandwidth of an LTI control system. This statement is supported with some SISO case studies, which serve to quantify the level of passive damping required to permit robust phase stabilization of uncertain structural poles. Uncertainty in eigenfrequencies and in mode shapes is considered. Mode shape uncertainty is recognized as more serious since it is the mode shapes which govern the location of plant zeros. In the common situation of lightly damped poles and zeros such mode shape uncertainty makes the location of the zeros highly uncertain, leading to the strong possibility of transfer function phase uncertainty of 180^0 or greater for dislocated sensor/actuator pairs.

This paper implicitly assumes that the poles of the plant transfer function are spaced much more widely than their bandwidth; $\omega_i - \omega_{i+1} >> \pi\xi\omega_i$. In this case of low modal overlap, each pole can be considered practically in isolation, interacting at most with local plant zeros. The uncertainty in structural dynamic transfer function gain and phase as a function of modal overlap has been studied from a statistical perspective by [Lyon], and is inversely related to modal overlap.

GAIN STABILIZATION BEYOND THE BANDWIDTH

The result we report here is well known and has been reported elsewhere in many different forms [Gran, Spanos, Hughes, von Flotow]. The basic idea is that a poorly modeled or unmodeled flexible mode beyond the bandwidth must be gain stabilized. Since the gain of a flexible structure (from applied force or torque to displacement, rotation etc.) is maximized near each resonance at a value inversely proportional to damping ratio, the conclusion emerges that an undamped flexible mode can never be gain stabilized. How much damping ratio is required to ensure gain stabilization of modes beyond the control bandwidth depends then upon the gain roll–off of the loop, upon the spectral separation between the modal natural frequency and the control bandwidth and upon the modal participation or residue. The relationship is rather obvious and has been reported elsewhere [Gran, Spanos, Hughes, von Flotow]. It is this requirement that leads to one curve sketched in Figure 1b.

PHASE STABILIZATION WITHIN THE BANDWIDTH

If, for the purposes of disturbance rejection or command following, the control bandwidth must be broad and include one or several flexible modes, then each of these flexible modes must be robustly phase stabilized. In the absence of modeling error, this presents no particular difficulty; notch compensation or some other equivalent inversion of the structural dynamics is then an acceptable approach. Unfortunately, modeling errors are always present. In single–input, single–output LTI systems, the effect of these modeling errors upon control design can be summarized in terms of uncertainty in the location of the structural dynamic transfer function poles and zeros. Broadband control is then enabled by a sufficient level of passive damping, since approximate plant inversion is then feasible.

ROBUST POLE–ZERO CANCELLATION

If the closed loop bandwidth (bandwidth as defined in Figure 1a) is to extend past the eigenfrequency of a number of flexible modes, the only solution is pole–zero cancellation. In the absence of modeling errors, all flexible modes may be canceled and the closed loop bandwidth extended arbitrarily. When uncertainty regarding the eigenfrequencies exists, however, extra care needs to be taken in this strategy. A root locus departure angle argument serves well to explain the effect of uncertainty in eigenfrequency on the stability of the closed loop system where pole–zero cancellation is employed in compensation. Here, a single oscillatory mode system is considered and the damping is considered to be very small. Figure 2 assumes that the loop phase due to all other dynamics is $\cancel{-90^0}$ at the nominal value of the eigenfrequency in question.

-180

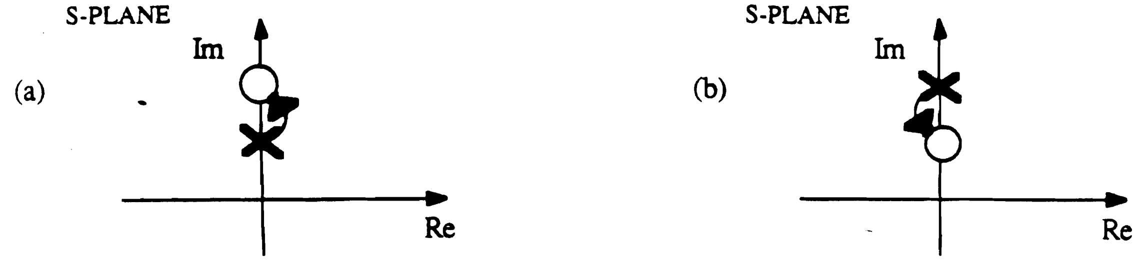

Figure 2. Departure angles due to pole–zero cancellation of single oscillatory mode.

Clearly, Figure 2b does not yield instability for small gains, as the departure angle is away from the imaginary axis. The phase lead added by the zero at frequency lower than the modal frequency ensures this. In Figure 2a, however, the departure angle is toward the imaginary axis, such that for sufficient uncertainty and sufficiently lightly damped poles, the closed loop system will be unstable.

CRITICAL DAMPING FOR ROBUST POLE–ZERO CANCELLATION

Considering the simple case of a single oscillatory mode, it is desirable to implement pole–zero cancellation as compensation in order to achieve a desired closed loop bandwidth. This, however, leads to possible instability if insufficient passive damping exists for a given level of uncertainty in modal frequency. This section serves to quantify ,to first order in relative uncertainty of the eigenfrequency, how much passive damping is necessary for stability (when employing this form of compensation) over the range of eigenfrequency uncertainty.

Phase Gradient with respect to Frequency

First, it is necessary to evaluate the "phase gradient" with respect to frequency, ω. Consider the single oscillatory mode represented by the transfer function

$$G(s) = \frac{1}{(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

setting $s = j\omega$ and evaluating the phase at frequency ω

$$\phi(\omega) = -\tan^{-1}\left(\frac{2\xi\omega_n\omega}{\omega_n^2 - \omega^2}\right)$$

The gradient of this phase evaluated at $\omega = \omega_n$ can be shown to be

$$\frac{d\phi}{d\omega} = \frac{-1}{\xi\omega_n} \qquad (1)$$

representing the phase gradient with respect to frequency at the frequency of maximum amplification of the amplitude ratio (vis–a–vis the Bode magnitude plot) as illustrated in Figure 3. Now, if the eigenfrequency is unknown to an uncertainty $\delta\omega = \omega_n - \omega_{actual}$, then the phase at the nominal eigenfrequency, ω_n, will be uncertain by as much as

$$\delta\phi = \frac{-\delta\omega}{\xi\omega_n} \qquad (1a)$$

to first order in $\delta\omega/\omega_n$.

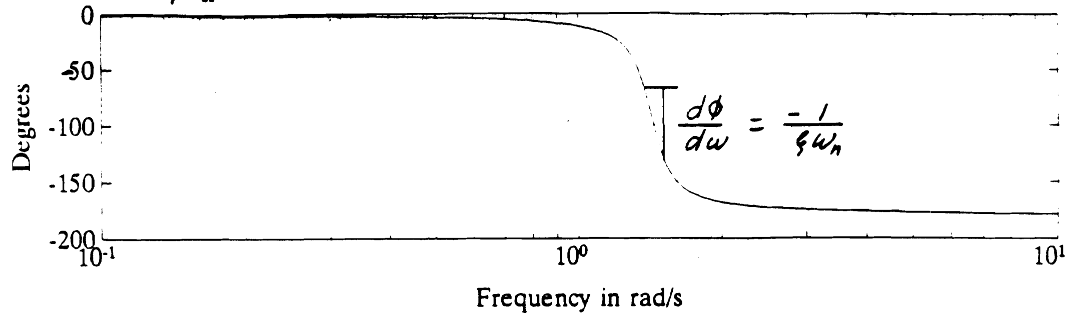

Figure 3. Bode plot illustrating the phase gradient wrt ω.

This allows estimates of local phase contributions due to oscillatory modes. Clearly, the phase gradient due to non–minimum phase oscillatory zeros of frequency ω_n and damping ξ (evaluated at ω_n), is also exactly (1), while that due to minimum phase zeros is the absolute value of (1).

Phase Excursions in Approximate Pole–Zero Cancellation

Using the relation (1a) it is now a simple matter to determine the relationship between the phase excursion due to inexact pole–zero cancellation. For the first uncertain mode in the system to be controlled, the Bode phase plot appears as in Figure 4, where the phase contribution due to the mode as well as the canceling zeros is depicted. Since $\delta\omega$ is assumed positive, the actual pole occurs at a frequency lower than that of the canceling zero and local phase lag is introduced.

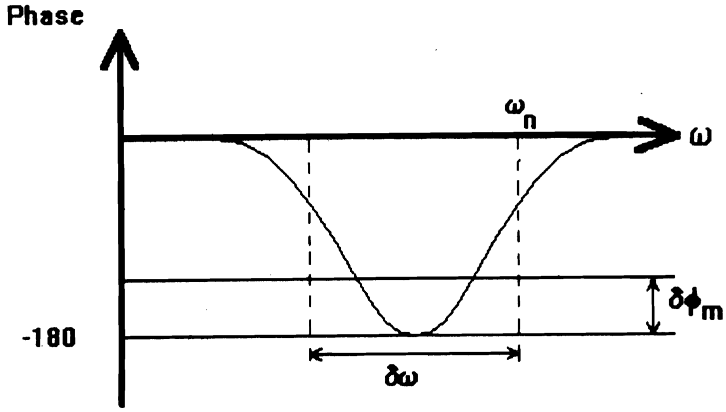

Figure 4. Bode phase plot of uncertain mode approximately canceled by compensator zeros in broadband structural control problem.

If $\delta\phi_m$ represents the "phase margin" at the canceled mode: i.e. the amount of unmodeled phase lag tolerable at this frequency, and if the linear approximation to the phase gradient (1) is assumed, a slightly pessimistic estimate of allowable $\delta\omega$ for a known level of passive damping, is given by this relation

$$\delta\omega \leq (\delta\phi_m)\xi\omega_n$$

and if $\delta\phi_m = 1\text{rad}$

$$\delta\omega \leq \xi\omega_n$$

This defines a first estimate of permissible modal eigenfrequency uncertainty in terms of passive damping for any system with flexibility between the sensors and actuators and required bandwidth including flexible eigenfrequencies.

A more precise expression for the maximum phase excursion due to pole–zero cancellation can be verified to be [Garcia]

$$\delta\phi = 2\tan^{-1}\left(\frac{p_i - z_i}{2\xi_i p_i}\right)$$

This expression is valid for large relative pole–zero spacing and correctly yields phase excursions limited to the range $\delta\phi \in (-180^0, 180^0)$. Since the zero frequency (z_i) is nominally chosen to cancel the pole, $z_i = \omega_n$ and the actual pole location is $p_i = \omega_{actual}$, then for small angles

$$\delta\phi = -\frac{z_i - p_i}{\xi_i p_i}$$

$$\simeq \frac{-\delta\omega}{\xi_i \omega_n}$$

which is the expression (1a).

HOW MUCH ξ IS SUFFICIENT IN THE BANDWIDTH?

For a system compensated by pole–zero cancellation, an estimate (to first order) of the passive damping ($\xi_{required}$) necessary to maintain stability over $\delta\omega$ may be determined.

Assuming $\delta\omega << \omega_n$ and also $\delta\omega << (\omega_i - \omega_{i-1})$ ("i" indicates i'th mode) ,ensures that the phase plot is approximately linear over this range ($\delta\omega$) about ω_n. Assuming further that pole–zero cancellation is attempted and $\delta\omega$ is small enough that the effect of neighbouring poles and zeros may be ignored (if modal separation is too small, the effect of other contributing modes to the total phase gradient is easily accounted for by including these extra terms), then the local phase excursion due to the pole–zero pair under consideration is approximately (by equation 1a)

$$\delta\phi = \frac{\delta\omega}{\xi\omega_n}$$

If the design provides a phase margin, $\delta\phi_m$, then for a pole–zero mismatch of $\delta\omega$ a damping ratio of

$$\xi \geq \frac{1}{\delta\phi_m}\frac{\delta\omega}{\omega_n} \qquad (2)$$

will suffice.

An alternative viewpoint is available by considering the root locus in the vicinity of a nearly canceling pole–zero pair:

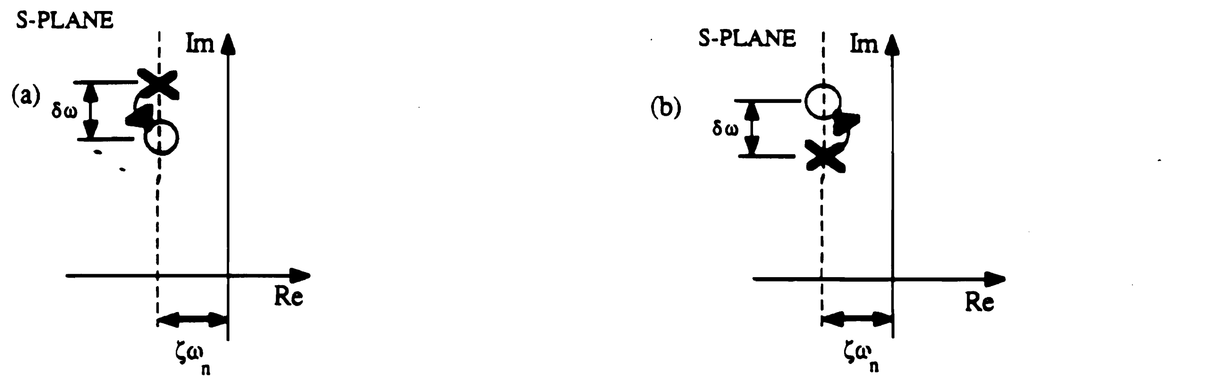

Figure 5. Root locus description of minimum damping ratio required to ensure stability of near pole–zero cancellation when assumed pole location is in error by $\delta\omega$.

PLANT ZERO UNCERTAINTY AND APPROXIMATE POLE–ZERO CANCELLATION

The preceding sections have implied that the plant contributes only poles which must be compensated with compensator zeros. Often the plant will contribute zeros, although their location, depending upon mode shapes, is typically more poorly known than the pole locations. Figure 6, from Rosenthal's PhD [Rosenthal], shows the possibility of a pole–zero "flip" when the inertia of one of the disks in this four degree of freedom system is changed by a factor of two. Near 20Hz, the plant phase is uncertain to ± 180^0. In such a situation phase stabilization is impossible.

Although the precise sequence of poles and zeros in a transfer function through a flexible structure is very uncertain, their approximate location can be known with more certainty [Lyon]. The plant phase can thus be viewed as an average phase related to a pole/zero density (with respect to frequency) and local phase excursions from this average phase. These excursions must be either small or well modeled if the plant is to be phase stabilized. If a close pole/zero pair is identified, but the sequence is uncertain (as in the case of [Rosenthal]) then a sufficient level of passive damping is required.

In this section the arguments of the preceding section are directly applied to such pole–zero cancellation, suggesting that approximate plant pole–zero cancellation is effectively perfect cancellation if sufficient levels of passive damping are present.

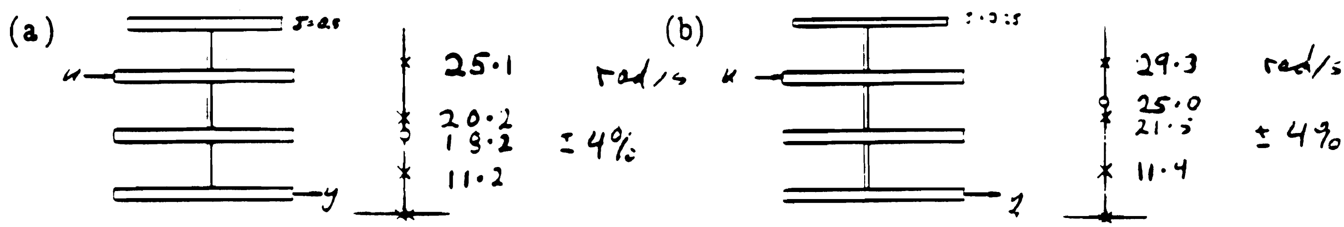

Figure 6. 4–Disk example by Rosenthal.

Poles and zeros in the loop transfer function are indistinguishable with respect to their origin; plant or compensator. We can thus apply the results of the preceding sections directly. If the plant pole–zero pair are known to be within $\delta\omega$ of one another, then a passive damping level of

$$\xi \simeq \frac{\delta\omega}{\omega_n}\frac{1}{\delta\Phi_m}$$

will suffice to ensure that their contribution will not destabilize the loop. For Rosenthal's system, the pole and zero frequencies were such that a passive damping ratio of 10 to 23 percent (for cases (a) and (b) of Figure 6, respectively) would have been sufficient. Such damping levels would then have permitted robust control of this plant with arbitrary bandwidth. For the intentionally low damping levels of Rosenthal's experiment ($\xi \simeq 0.004$), the closed loop bandwidth could never be made to approach this uncertain region near 20 Hz. Note that even for this laboratory system, the exact location of the plant poles and zeros was never known. For Rosenthal, this measurement uncertainty amounted to 1 to 4 percent. This implies that even for carefully identified lightly damped structures, a minimum passive damping level of 1 to 4 percent is needed to permit robust plant inversion in feedback control.

An Example. Bong Wie benchmark problem with pole–zero cancellation

The Bong Wie benchmark problem [ACC] has transfer function

$$G(s) \quad = \quad \frac{K\left(\xi\omega_n + \omega_n^2/2\right)}{s^2(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

with uncertainty in modal eigenfrequency such that ω_{actual} may lie in the range $1.0 < \omega_{actual} < 2.0$ (ω in rad/s) and nominally $\omega_n = \sqrt{2}$ rad/s.

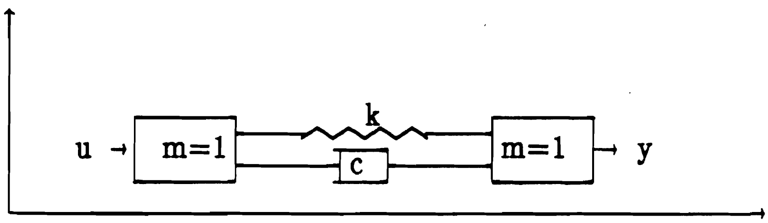

Figure 7. Bong Wie benchmark problem with dashpot providing passive damping.

For this eigenfrequency uncertainty, the estimate (1a) of necessary damping is evaluated as

$$\xi \geq (\sqrt{2} - 1)/\sqrt{2} \qquad = 0.29$$

Note that this assumes $\delta\phi_m = 1$ rad. The rigid body compensation, however, yields a phase margin of $\delta\phi \simeq 1.3$ rad at the nominal flexible mode eigenfrequency, yielding

$$\xi \geq 0.22.$$

The guidelines of the previous section, linear in $\delta\omega/\omega_n$, thus suggest that a passive damping level of 22 percent is needed for this huge uncertainty, if the nominal bandwidth is extended to beyond ω_n while maintaining stability.

It is necessary to verify if this estimate of the required level of passive damping is adequate. To this end, Figure 8 displays a root locus versus gain of the closed loop dynamics where broadband compensation using pole–zero cancellation is used. In the spirit of classical control design, the rigid body modes are first compensated by means of a low frequency zero. The flexible mode is then canceled by zeros at the nominal location of the flexible poles, with suitable high frequency poles yielding the desired roll–off, for a compensator transfer function

$$K(s) = \frac{(6.66s + 1)(s^2 + 2\xi_z\omega_n s + \omega_n^2)16}{(s^2 + 5.656s + 16)\omega_n^2}$$

where the damping ratio of the compensator zero is

$$\xi_z = 0.19$$

The loop gain is chosen to yield closed loop bandwidth of 4 rad/s, greater than the uncertain mode frequency. The system is evaluated for stability when the modal frequency is $\omega_{actual} = 1$ rad/s (worst case, but including 22% passive damping ratio) and found to be stable (figures (8a and 8b)). Clearly the estimate yields a good first guess; the root locus of figure 8b doesn't quite cross into the right–half plane. Unfortunately, the lightly damped closed loop dynamics remaining due to imperfect pole–zero cancellation in Figure 8b limit the bandwidth to less than ω_n (bandwidth defined as in the introduction).

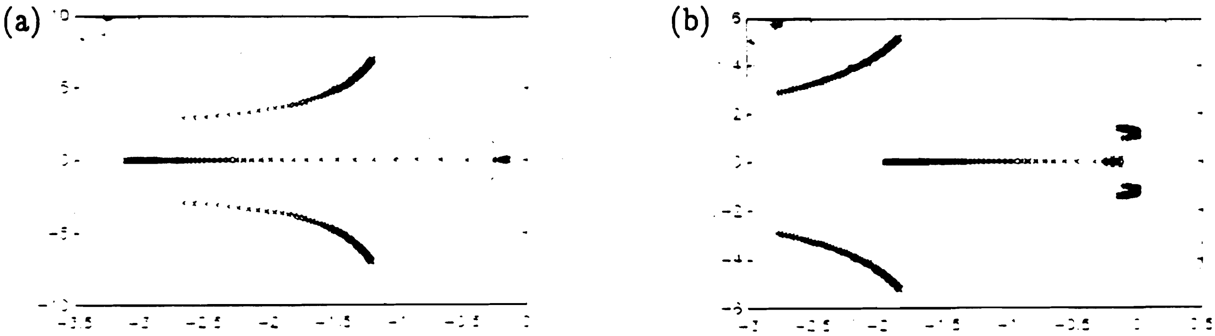

Figure 8. Root loci for Bong Wie problem. (a) shows the nominal case and (b) the nominal compensator implemented for the plant with eigenfrequency at the lower end of the uncertainty range.

Since it may not be practicable to implement the required amount of passive damping to achieve stability robustness for perfect cancellation of the nominal plant flexible modes, it is possible to exploit whatever passive damping is guaranteed by placing the compenastor zero somewhere inside the uncertainty bound for the achievable damping and ensuring stability over the entire uncertainty range as well as improving the nominal performance over conservative compensation with the zero at the lower end of the uncertainty range.

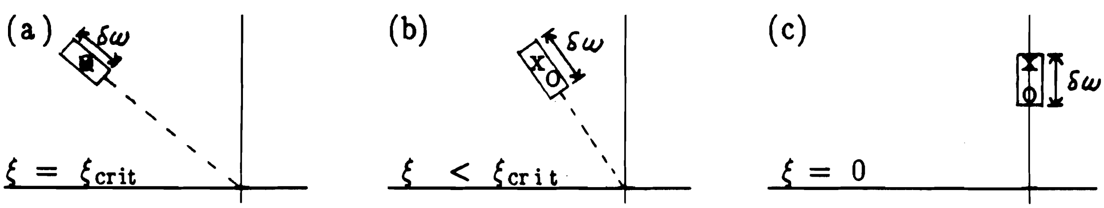

Figure 9. S–plane plots showing intermediate pole–zero cancellation achieving stability robustness and improved nominal performance over undamped uncertain design.

Figure 9 shows the transition in designs from exact pole–zero cancellation of the nominal flexible modes to approximate cancellation at the lower end of the eigenfrequency uncertainty range. For each case the uncertainty is the value $\delta\omega$ and the zero frequency for cancellation may be estimated using equation 1a and the guaranteed level of passive damping ξ. Notice that in the case of this example, the rigid body compensation yields "phase margin" of approximately 75^0 ($\simeq$ 1.3 rad) in the frequency region near the uncertain poles. The frequency of the compensator zero should then be

$$\omega_z \leq \omega_{lower} + \delta\phi\xi \tag{3}$$

where ω_{lower} is the lower bound on the eigenfrequency range. A sequence of such designs for the Bong Wie benchmark problem yield Figure 10, showing the limiting value for the compensator zero as a function of plant damping ratio. This suggests that use of equation 3 is warranted for estimating the required zero frequency when a fixed level of passive damping is available and stability robustness with near pole–zero cancellation is desired.

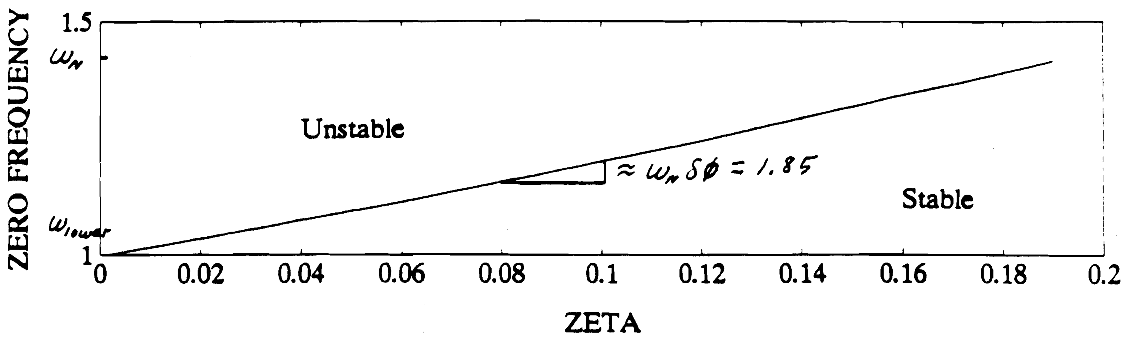

Figure 10. Stability boundary on ω_z vs ξ curve showing that equation 3 yields a good estimate of the zero frequency (ω_z) for near pole–zero cancellation of an uncertain mode where the minimum guaranteed level of passive damping is ξ.

INFERENCES FOR MIMO CONTROL

The preceding sections have presented simple arguments supporting the claim that broadband control of uncertain structural dynamics is impossible with LTI compensation unless a sufficient level of passive damping is present. Although it is not possible to extrapolate these arguments to MIMO systems with confidence, there appears to be little reason to believe that such systems are more benign. Each scalar transfer function of a MIMO system has pole–zero patterns like those described in the preceding sections. Worse, the mapping from actuators to sensors is a strong function of frequency, changing drastically in the near vicinity of each zero of any of the individual scalar transfer functions. The rate of change with frequency of these directions varies inversely with passive damping ratio.

Thus, in MIMO systems both the "directionality" and the phase behaviour of individual channels are comparably influenced by passive damping. Absent a better guideline, one might suggest that equation (2) yields a reasonable rule of thumb for the passive damping levels required for such systems. Here, relative uncertainty should perhaps be relative uncertainty in the zeros of the individual scalar transfer functions. This is not obviously correct and much remains to be understood about such MIMO situations.

SUMMARY

This paper has argued, simply but quantitatively, that a critical level of passive damping can be specified which will permit robust non–collocated LTI control of structural dynamics with the control bandwidth including many flexible eigenfrequencies. We have suggested that this level is as high as 1 to 4 percent, even for carefully identified laboratory structures and much higher for uncertain structures. Figure 1 perhaps summarizes the thesis of this paper most succintly.

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