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DISTRIBUTED PARAMETER NONLINEAR DAMPING MODELS FOR FLIGHT STRUCTURES

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ABSTRACT

Experimental results on single mode free response using the SCOLE configuration at NASA Langley Flight Research Center exhibit significant nonlinear damping effects. To account for the observed behavior a class of nonlinear damping models is proposed for energy conservation systems. The Krylov-Bogoliubov technique provides a remarkably good approximation at all reasonable damping constants. It also shows that many of the known damping models such as constant friction, air-damping in flow at high Reynolds numbers among others cannot be distinguished from one generic model -- the "energy" model -- appropriately specialized, based on free response alone. This in turn raises the question whether forced response -- in particular response to random white noise -- could help resolve the ambiguity. Some exact analytical results for the non-Gaussian distributions that arise are presented based on the Fokker-Planck equations. Finally, several nonlinear damping models for distributed-parameter systems are suggested which would exhibit the observed single-mode free response.

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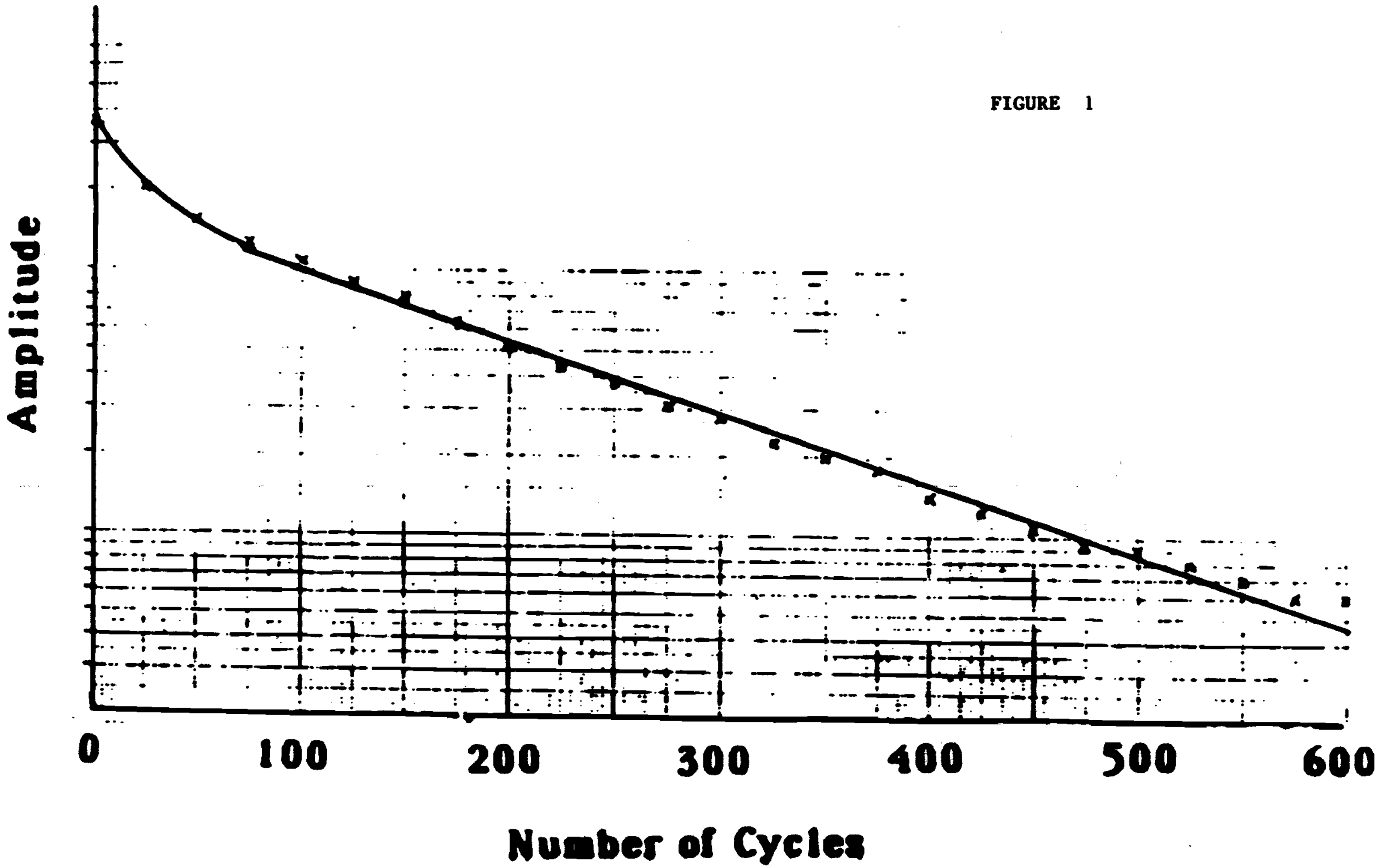
SCOLE DAMPING



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1. INTRODUCTION

Although the inherent damping phenomena in flight structures are still little understood, many studies of free response -- such as the recent experiments with SCOLE [1] -- would indicate nonlinear behavior especially at high amplitudes. A typical free response curve is shown in Figure 1, which shows the characteristic "convex-downward" or CUP feature. Many nonlinear models have been proposed in the literature [2]. Thus in terms of the basic second-order dynamics for the displacement variable $x(t)$ in free response:

$$\frac{d^2x}{dt^2} + \omega^2 x(t) + \gamma D(x(t), \dot{x}(t)) = 0 \tag{1.1}$$

where ω is the mode frequency and γ the (small) damping constant, we have for "Coulomb Friction" Damping:

$$D(x(t), \dot{x}(t)) = \text{sign}\ \dot{x}(t) . \tag{1.2}$$

For high Reynolds number flow:

$$D(x(t), \dot{x}(t)) = |\dot{x}(t)|\ \dot{x}(t) \tag{1.3}$$

modifications of which include "nonanalytic" functions (see [2]):

$$D(x(t), \dot{x}(t)) = |\dot{x}(t)|^{\alpha}\dot{x}(t) , \qquad 0 < \alpha < 1 . \tag{1.4}$$

Our purpose here is to examine these models particularly from the point of view of System Identification (from orbit data, for example). We shall show that these models cannot be distinguished based on single-mode free response data. In particular we also suggest a new class of models based on the instantaneous total energy in the system. We also present some explicit results on response to random excitation using the Fokker-Planck equations, still in the single-mode case. Finally we present a variety of models for distributed parameter systems such as in vibrating strings and beams which can exhibit the kind of single-mode response discussed.

2. NONLINEAR DAMPING MODELS: SINGLE MODE RESPONSE

The basic dynamics are described by:

$$\frac{d^2x}{dt^2} + \omega^2 x(t) + \gamma D(x(t), \dot{x}(t)) = 0 . \tag{2.1}$$

To determine possible nonlinear damping models, we note first that the instantaneous energy $E(t)$ is given by

$$E(t) = \frac{1}{2}\left(\omega^2 x(t)^2 + \dot{x}(t)^2\right) . \tag{2.2}$$

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The rate of change is

$$\frac{d}{dt} E(t) = \left[\omega^2 x(t) + \ddot{x}(t)\right] \dot{x}(t)$$
$$= -\gamma D(x(t), \dot{x}(t)) \dot{x}(t) . \qquad (2.3)$$

Hence, since we are only considering energy conservative systems, we must have that

$$D(x(t), \dot{x}(t)) \dot{x}(t) \geq 0 . \qquad (2.4)$$

In terms of independent variables x, y, the function $D(\cdot, \cdot)$ must be such that

$$D(x, y)y \geq 0 \qquad \text{for all } x, y .$$

Note that this condition is satisfied by the models (1.2), (1.3), (1.4) in Section 1, taken from the literature. We may generalize these to:

$$D(x, y) = x^{2m}|x|^{\alpha} y^{2n+1} |y|^{\beta} \qquad (2.5)$$

where m and n are positive integers and $0 \leq \alpha, \beta \leq 1$, first presented in [1]. The primary question is whether we can identify the parmaeters involved m, n, α, β from free response data. For this purpose since the damping is small (small γ) it is convenient to use the Krylov-Bogoliubov approximation [3] to determine the solution of (2.1), which we shall now rewrite separating out the linear damping part:

$$\ddot{x}(t) + \omega^2 x(t) + 2\zeta\omega\dot{x}(t) + \gamma D(x(t), \dot{x}(t)) = 0 . \qquad (2.6)$$

The Krylov-Bogoliubov approximation is the slow varying sinusoid:

$$x(t) = A(t) \sin (\omega t + \phi)$$

where, defining

$$A_n = A(nT)$$

where T is the period:

$$T = \frac{2\pi}{\omega}$$

we have:

$$\log A_{k+1} = \log A_k - 2\pi\zeta - \gamma \frac{2\pi}{\omega} \cdot \frac{K(A_k)}{\omega A_k} \qquad (2.7)$$

where the function $K(\cdot)$ is determined by:

$$K(A) = \frac{1}{2\pi} \int_0^{2\pi} D(A \sin \theta, A\omega \cos \theta) \cos \theta \, d\theta . \qquad (2.8)$$

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For $D(\cdot, \cdot)$ given by (2.5) we have

$$\log A_{k+1} = \log A_k - 2\pi\zeta - 2\pi\gamma\mu\omega^{2n+\beta-1} A_k^{2n+2m+\alpha+\beta} \qquad (2.9)$$

where

$$\mu = \frac{1}{2\pi} \int_0^{2\pi} D(\sin\theta, \cos\theta)\, d\theta = f(m, n, \alpha, \beta)$$

and is easily evaluated -- see [1]. But the exact value is not important from the identification point of view since it is multiplied in (2.9) by the unknown damping constant γ. Comparison between the solution given by (2.9) and that obtainable by numerical solution of (2.6) using multi-step (Runge-Kutta) techniques has been examined in [4], especially the behavior for high γ. It should be also noted that the exponent of A_k involves a combination of m, n, α, β and makes apparent the difficulty in resolving them.

An unexpected consequence of the validity of the Krylov-Bogoliubov approximation is that we can now present a new class of damping models -- "energy" models -- which can yield the same Krylov-Bogoliubov approximation. Thus let

$$D(x(t), \dot{x}(t)) = \dot{x}(t)\, E(t)^q = \dot{x}(t) \left[\frac{(\omega^2 x(t)^2 + \dot{x}(t)^2)}{2}\right]^q \qquad (2.10)$$

where $q > 0$. This clearly satisfies (2.4) and moreover the Krylov-Bogoliubov approximation is (see [5])

$$\log A_{k+1} = \log A_k - 2\pi\zeta - 2\pi\gamma\omega^{2(q-1)} \frac{1}{2^{q+1}} A_k^{2q} . \qquad (2.11)$$

Thus, as first observed in [5], we can obtain the same kind of response as in (2.7) by choosing q appropriately, viz. taking

$$2q = 2n + 2m + \alpha + \beta . \qquad (2.12)$$

This clearly underscores the difficulty in identifying nonlinear dmaping models from flight data. Hence we may want to examine the possibility of using forced response -- in particular response to random (white noise) excitation.

3. FORCED RESPONSE WITH RANDOM NOISE EXCITATION

Especially in the Civil Engineering oriented literature there is considerable work reported on the response to random noise excitation applied to nonlinear damping models in one dimension (single mode) [6]. One particular tool used is that of equivalent linearization because of the difficulty in obtaining exact distributions which cannot of course be Gaussian. The analytical tool for evaluating the first and/or second order

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steady state distributions of the response is of course provided by (and only by!) the Fokker-Planck partial differential equations. For the energy model (2.10), exact solutions have been presented in [5] for the first order steady state density, indicating explicitly the non-Gaussian nature. In particular it can be used to test validity of the equivalent linearization technique, at least for this example. Thus the first order steady state density of the forced response:

$$\ddot{x}(t) + \omega^2 x(t) + 2\zeta\dot{x}(t) + \dot{x}(t)\left(\omega^2 x(t)^2 + \dot{x}(t)^2\right)^q = N(t) \qquad (3.1)$$

where $N(\cdot)$ is white Gaussian with spectral density σ^2 given by:

$$y \sim \dot{x}$$

$$p_{x,\dot{x}}(x, y) = c \exp. \left\{\frac{-2\zeta\omega}{\sigma^2}\left(\omega^2 x^2 + y^2\right) - \gamma \frac{(\omega^2 x^2 + y^2)^{q+1}}{(q+1)}\right\} \qquad (3.2)$$

where c is a normalizing constant:

$$1 = \frac{\pi c}{\omega} \int_0^\infty \exp - \left(\frac{2\zeta\omega r}{\sigma^2} - \frac{\gamma r^{q+1}}{q+1}\right) dr \qquad (3.3)$$

where the integrand in (3.3) is actually the density of the energy $2E(t)$. The second term in the exponent in (3.2) clearly indicates the non-Gausssian nature. Unlike the linear case, $x(t)$ and $\dot{x}(t)$ are no longer independent. We can calculate that the steady state covariance of the displacement $x(t)$ is given by

$$E[x(t)^2] = \frac{1}{2\omega^2} \cdot \frac{\int_0^\infty r e^{-\lambda r - \gamma_1 r^{q+1}} dr}{\int_0^\infty e^{-\lambda r} e^{-\gamma_1 r^{q+1}} dr}$$

where

$$\lambda = \frac{2\zeta\omega}{\sigma^2} ; \qquad \gamma_1 = \frac{\gamma}{\sigma^2(q+1)} .$$

For small enough γ we have the approximation

$$E[x(t)^2] = \left[1 - \frac{\gamma}{\sigma^2}(q+1)!\ \lambda^{q+1}\right] \frac{\sigma^2}{4\zeta\omega^3} . \qquad (3.4)$$

In particular the goodness of equivalent linearization can be assessed from (3.4). The density corresponding to the first model, (2.5), would appear to be more complicated than would be indicated by using the equivalent value for q. Thus the possibility of distinguishing between models using forced response to random excitation is yet to be explored.

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4. NONLINEAR DAMPING MODELS FOR DISTRIBUTED PARAMETER SYSTEMS

At the present time no nonlinear damping models for energy conservative distributed parameter systems are available. Here we propose several models for the beam torsion as well as beam bending modes for a uniform Bernoulli beam. To yield the model (2.5) for the single mode response we suggest for beam bending for a beam of length 2ℓ:

$$\ddot{u}(s,t) + \lambda u''''(s,t) - 2\zeta\sqrt{\lambda}\,\dot{u}''(s,t) - \gamma\left[\int_{-\ell}^{\ell} u'(s,t)\,\dot{u}'(s,t)\,ds\right]^{2(n+\beta)+1} u''(s,t) = 0\,, \tag{4.1}$$

$$-\ell < s < \ell; \quad 0 < t$$

where s denotes the spatial variable, super-dots represent derivatives with respect to time t and the primes derivatives with respect to s, and

$$0 \leqq \beta < \frac{1}{2}$$

and λ is the appropriate structure constant. And corresponding to the energy model (2.10), we propose:

$$\ddot{u}(t,s) - 2\zeta\sqrt{\lambda}\,u''(t,s) + \lambda u''''(t,s) - \gamma\left[\int_{-\ell}^{\ell}\left(\lambda u''(t,\sigma)^2 + \dot{u}(t,\sigma)^2\right)d\sigma\right]^q \dot{u}''(t,s) = 0\,. \tag{4.2}$$

For a proof that the single-mode response behavior corresponding to (4.1) with clamped end conditions is given by taking

$$D(x(t),\,\dot{x}(t)) = x(t)^{2n}\,\dot{x}(t)^{2n+1}\,|x(t)|^{\beta}\,|\dot{x}(t)|^{\beta} \tag{4.3}$$

reference may be made to [7]. A similar argument suffices also for (4.2). The beam torsion mode case is more complicated because the linear model for proportional damping is no longer a differential (local) operator (see [8]). Thus the nonlinear damping model proposed is:

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$$\ddot{u}(t, s) - \lambda u''(t, s) + 2\zeta\sqrt{\lambda} \int_{-\ell}^{\ell} \frac{\cos(\pi s/2\ell)}{(\sin(\pi s/2\ell) - \sin(\pi\sigma/2\ell))} \dot{u}'(t, \sigma)\, d\sigma$$

$$+ \gamma \left[\int_{-\ell}^{\ell} \int_{-\ell}^{\ell} u(t,s) \frac{\cos(\pi s/2\ell)}{(\sin(\pi s/2\ell) - \sin(\pi\sigma/2\ell))} \dot{u}'(t,\sigma)\, d\sigma\, ds \right]^{2(n+\beta)+1}$$

$$\times \int_{-\ell}^{\ell} \frac{\cos(\pi s/2\ell)}{(\sin(\pi s/2\ell) - \sin(\pi\sigma/2\ell))} u'(t, \sigma)\, d\sigma = 0 . \quad (4.4)$$

The "energy" model is obtained by replacing the term containing γ in (4.4) by

$$+ \gamma \left[\int_{-\ell}^{\ell} \left(\lambda u'(t,s)^2 + \dot{u}(t,s)^2\right) ds \right]^q \int_{-\ell}^{\ell} \frac{\cos(\pi s/2\ell)}{\sin(\pi s/2\ell) - \sin(\pi\sigma/2\ell)} \dot{u}'(t, \sigma)\, d\sigma . \quad (4.5)$$

It will take us too far afield to show that the corresponding single mode dynamics of (4.4) correspond to (4.3) and that of (4.5) to (2.10). It should be noted that (4.1), (4.2), (4.4), (4.5) are fairly complicated nonlinear partial differential equations requiring sophisticated mathematical techniques for analysis -- see [9] for example. Computer simulation may be an attractive alternative.

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