DYNAMICS OF A CLASS OF VISCOUSLY DAMPED STRUTS

Y. C. Yiu
Structural Dynamics, Space Systems Division, Dept. 62-18, B/104
Lockheed Missiles & Space Company, 1111 Lockheed Way, Sunnyvale, CA 94089
Telephone : 408-742-4048

S.D. Ginter
Honeywell Inc., Satellite Systems Operation
P.O. Box 52199, Phoenix, AR 85022
Telephone : 602-561-3244

ABSTRACT

The dynamics of a class of struts with one viscous chamber at one end of the strut is developed using formulation and methods consistent with finite element dynamic analysis of structural system. This technique is developed to enable consistent and systematic design and analysis of large truss structures passively damped by viscous struts. Modeling and model reduction methods for accurate analysis with a minimum number of design parameters are developed. Design parameters for optimum damping characteristic, and the associated dynamic stiffness and bandwidth characteristics are derived. A design procedure and design curves to size the struts for system level integration are presented.

INTRODUCTION

Large flexible structures are characterized by many flexible modes within the disturbance and control bandwidth. For most precision structures, the performance requirements are very stringent. However, this class of structures often has very low intrinsic damping, less than 0.1% equivalent viscous damping ratio[1], which results in significant dynamic responses. For truss type structures, a strut with good stiffness and damping characteristics will significantly enhance the structural performance.

Struts with viscoelastic materials have been designed, tested and implemented in demonstration test articles and structures[2]. Modal Strain Energy method is often used in the design and analysis of this type of struts and structures[3]. The mathematical problem of struts and structures are posed in a frequency dependent form. Results from this approximate solution technique matched quite well with test data[2].

Viscous energy dissipation is a well understood damping mechanism. Incorporating a damping chamber in a strut can provide the necessary damping characteristics. An effective design of this type of viscously damped struts has been implemented by Honeywell[4]. In order to successfully integrate the viscous struts into a system level design, the dynamics of the struts must be totally understood. The same analysis method should be used to study the strut dynamics and system level dynamics so that the integrated design and analysis can be performed consistently and systematically[5]. Also, in order to understand the behavior of the struts as contributing members of a large structure, the problem must be simplified to a few key design parameters by applying engineering assumptions. Simplified design procedure with design curves are presented to compute the kdy strut design parameters. However, the details of the mechanical design is not the subject of this paper.

VISCOUS STRUT CONFIGURATION

The viscous strut is a mechanical device comprised of three basic elements: an outer tube, an inner tube and a small viscous damper. A typical strut configuration is shown in Figure 1[4]. The damper is placed in series with the inner tube. The outer tube is placed in parallel with the damper/inner tube. An axial displacement across the strut produces a displacement across the damper. The damper forces fluid through a small diameter orifice, thereby causing a shear flow in the fluid. For Newtonian viscous fluids, the fluid shear is actually proportional to the displacement rate across the damper and thus, a velocity dependent viscous damping force is obtained. Under quasi-static load, the fluid flows and provides no resistance and the outer tube provides the static stiffness to the strut. The stiffness of the inner tube is important to impart sufficient displacement/velocity to the damper. The damping coefficient of the damper is a function of the fluid material properties and the geometry of the viscous chamber. Since the strut has other small components, they will introduce additional flexibility to the strut and degrade the performance. It is important to account for these flexible elements accurately.

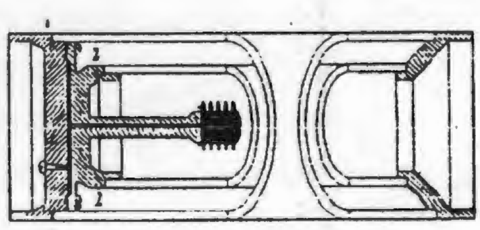

Figure 1 Configuration of Viscously Damped Strut[4]

The formulation presented here is applicable to a general class of viscously damped struts which are axially symmetric with the viscous chambers rigidly attached to one end of the struts. This considerably simplifies the mathematics and lead to a design model with a minimum number of key parameters.

STRUT ANALYTICAL MODEL

A viscous strut is a structural component which can be analyzed by standard structural analysis methods. As such, it can be analyzed using conventional structural analysis techniques and tools. For a complex strut design, a finite element model can be developed easily using a combination of beam, plate, solid and viscous elements. The analysis is quite straight forward except for the viscous element which is not often used in conventional structural analysis. In general, the governing differential equation for a strut can be expressed as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{p}\, g(t) \qquad (1)$$

The damping matrix has contributions from two sources: the intrinsic material and joint damping, and damping from the viscous dashpot. The intrinsic damping is insignificant compared with the contribution from the viscous dashpot and hence ignored. Equation (1) is normally cast in the first order form for solution:

$$\begin{bmatrix} C & M \\ M & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \end{bmatrix} = \begin{bmatrix} p \\ 0 \end{bmatrix} g(t) \qquad (2)$$

The strut can be modeled by many structural nodes to provide a general description of its dynamic behavior in 3 dimensional space. Let one end of the strut be fixed, the displacement vector of the end node be $\mathbf{u}_1$, and the displacement vector at the viscous chamber be $\mathbf{u}_2$. Many other interior structural nodes may be needed to model the stiffness distribution in the finite element model (see Figure 2).

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ u_n \end{bmatrix} \qquad (3)$$

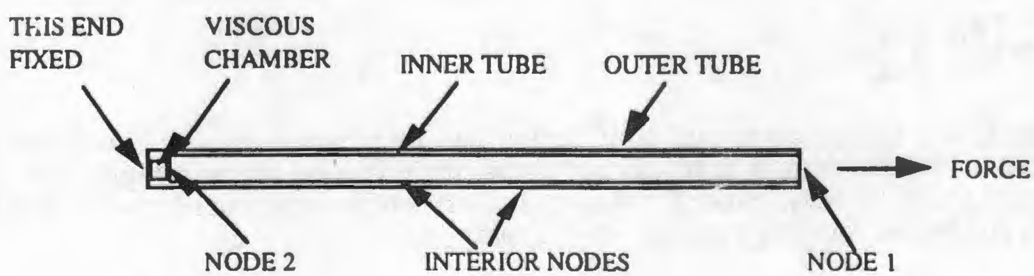

Figure 2 An Idealized Viscous Strut

The lumped mass matrix, damping matrix, and stiffness matrix can be expressed in the following form:

$$\mathbf{M} = \begin{bmatrix} M_{11} & 0 & \cdot\,\cdot & 0 \\ 0 & M_{22} & \cdot\,\cdot & 0 \\ \cdot & \cdot & \cdot\,\cdot & \cdot \\ \cdot & \cdot & \cdot\,\cdot & \cdot \\ 0 & 0 & \cdot\,\cdot & M_{nn} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & \cdot\,\cdot & 0 \\ 0 & C_{22} & \cdot\,\cdot & 0 \\ \cdot & \cdot & \cdot\,\cdot & \cdot \\ \cdot & \cdot & \cdot\,\cdot & \cdot \\ 0 & 0 & \cdot\,\cdot & 0 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} K_{11} & K_{12} & \cdot\,\cdot & K_{1n} \\ K_{21} & K_{22} & \cdot\,\cdot & K_{2n} \\ \cdot & \cdot & \cdot\,\cdot & \cdot \\ \cdot & \cdot & \cdot\,\cdot & \cdot \\ K_{n1} & K_{n2} & \cdot\,\cdot & K_{nn} \end{bmatrix} \tag{4}$$

Since there is no applied force at the interior nodes, the force vector is given by:

$$\mathbf{p} = \begin{bmatrix} p_1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{5}$$

This finite element model is capable of predicting all the details of the global and local strut behavior. However, the strut is normally designed to act only as an axial load carrying member providing strength, stiffness and damping to meet the design requirements. The analysis model in this form also does not explicitly express the relationship between the essential dynamic characteristics and the key parameters. It should only be used if the detailed local dynamics is important or as a verification model after the strut parameters are selected by other means.

STRUT MODEL REDUCTION

In order to understand the dynamic characteristics of the strut, the analytical model should be simplified to a small set of parameters. The reduction of the component level model will also significantly reduce the complexity of the system level model. For design purpose, only axial behavior of the struts are considered. Consequently, the analysis model is constrained to have displacement only in the axial direction. At each node, only the axial degree of freedom and two rotations are retained. For structural problem, the internal dynamics is generally not important and the internal inertial effect is ignored.

There are only two degrees of freedom necessary to characterize the strut: u_1 - the axial degree of freedom at the strut end for connectivity and u_2 - the axial degree of freedom at the dashpot for damping. The standard static condensation reduces the stiffness matrix to a symmetric 2x2 matrix with only 3 independent terms:

$$\mathbf{k} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{6}$$

Therefore, any complex viscous strut design can be reduced to only 3 equivalent stiffness constants. For the same 3 stiffness constants, there can be many designs having the same condensed characteristics. Since for the class of struts of interest, the dashpot is at the supported end, the condensed damping matrix is very simple:

$$\mathbf{c} = \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix} \tag{7}$$

As for the mass matrix, normally a simple lumping procedure is sufficient since the inertia effect of the strut is considered not important.

STRUT DESIGN MODEL

Static condensation of a relatively complex strut design allows a simple equivalent mechanical modeling of the strut for understanding its dynamics. Due to the design details, many strut configurations also have an additional characteristic[4] that $k_{12} \approx -k_{22}$. This allows a further simplification such that the abstract 2x2 stiffness matrix of Equation (6) can be represented by an equivalent lumped parameter model as shown in Figure 2. A viscously damped strut can now be represented by 3 frequency independent parameters, k_1 - the outer spring, k_2 - the inner spring and c - the dashpot.

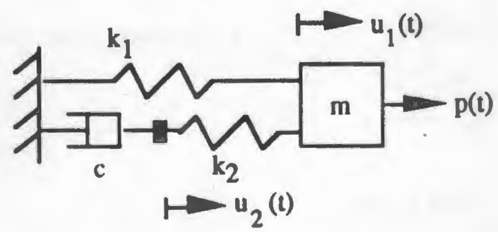

Figure 2 3-Parameter 2 DOFs Viscous Strut Model

The equation of motion of the 3-parameter viscous strut model can be written as:

$$\mathbf{m\ddot{u} + c\dot{u} + ku = p}\,g(t) \tag{8}$$

where,

$$\mathbf{m} = \begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix}, \ \mathbf{k} = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix}, \ \mathbf{p} = \begin{bmatrix} p \\ 0 \end{bmatrix} \tag{9}$$

If the strut is used to support a rigid mass which is include in the mass matrix, the characteristics of this structural system is given by the free vibration problem:

$$\mathbf{m\ddot{u} + c\dot{u} + ku = 0} \tag{10}$$

or, in the first order form[6]:

$$\begin{bmatrix} \mathbf{c} & \mathbf{m} \\ \mathbf{m} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{\dot{u}} \\ \mathbf{\ddot{u}} \end{bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{0} \\ \mathbf{0} & \mathbf{-m} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{\dot{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{11}$$

For this three parameter model, Equation (11) can be written explicitly as:

$$\begin{bmatrix} 0 & 0 & m & 0 \\ 0 & c & 0 & 0 \\ m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 & 0 & 0 \\ 0 & 0 & -m & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{12a}$$

The eigenvalue problem is therefore given by:

$$\left\{ \lambda_i \begin{bmatrix} 0 & 0 & m \\ 0 & c & 0 \\ m & 0 & 0 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2 & 0 \\ 0 & 0 & -m \end{bmatrix} \right\} \psi_i = 0 \tag{12b}$$

The eigenvalues, λ_i, and eigenvectors, ψ_l, are generally complex. For a lightly damped system, there is one pair of complex eigenvalues which represent the under-damped modes and one real eigenvalue which represents the over-damped mode. Eigensolvers used in structural codes normally assume the structures to be lightly damped and solve for complex pairs only. However, solving the eigenvalue problem does not give any physical insight into the design of struts. Therefore, a simpler design approach is more appropriate.

APPROXIMATE ANALYSIS OF DAMPED STRUTS

When a strut is functioning as a member of a large structure or as an individual member under a harmonic force given by:

$$g(t) = e^{i\omega t} \tag{13}$$

the steady state solution takes the form:

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} e^{i\omega t} \tag{14}$$

Assuming tthat he mass at the internal degree of freedom, u_2, is small, and the internal dynamics of the strut is not important to the problem, the governing differential equation is given by:

$$\left\{ \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} - \omega^2 \begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix} + i\,\omega \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix} \right\} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} p \\ 0 \end{bmatrix} \tag{15}$$

The equations of motion are described by frequency independent coefficient matrices. The internal degree of freedom, u_2, is not subject to any external force. Again, the static condensation technique is used to reduce the internal degree of freedom by considering the second equation of Equation (15):

$$-k_2\, u_1 + (k_2+i\omega c)\, u_2 = 0 \tag{16a}$$

$$u_2 = \frac{k_2}{k_2+i\omega c}\, u_1 \tag{16b}$$

Therefore, the effective strut dynamics is given by:

$$\left(-\omega^2 m + (k_1+k_2) - \frac{k_2^{\,2}}{k_2+i\omega c} \right) u_1 = p \tag{17a}$$

The term in parenthesis is the strut dynamic impedance which is frequency dependent. However it is more useful to describe the strut in terms of complex stiffness (i.e., $k(\omega) = k^R(\omega) + ik^I(\omega)$):

$$\left(-\omega^2 m + k^R + ik^I \right) u_1 = p \tag{17b}$$

where,

$$k^R = \frac{k_1 k_2^2 + (k_1+k_2)(c\omega)^2}{k_2^2 + (c\omega)^2} \tag{18a}$$

$$k^I = \frac{k_2^2\,(c\omega)}{k_2^2 + (c\omega)^2} \tag{18b}$$

The complex stiffness can further be expressed in a different form in terms of the real part of the stiffness and the loss factor as:

$$k = k^R\,(1+ i\eta) \tag{19a}$$

where,

$$\eta = \frac{k_2^2\,(c\omega)}{k_1 k_2^2 + (k_1+k_2)(c\omega)^2} \tag{19b}$$

These relationships can be presented in a more useful form for design purposes in terms of normalized parameters. Define the stiffness ratio as:

$$\kappa = \frac{k_2}{k_1} \tag{20a}$$

the strut frequency constant as:

$$\omega_c = \frac{k_1}{c} \tag{20b}$$

and the normalized excitation frequency as:

$$\beta = \frac{\omega}{\omega_c} \tag{20c}$$

Rewrite the strut real stiffness and loss factor in terms of the normalized ratios:

$$\frac{k^R}{k_1} = \left[\frac{\kappa^2 + (1+\kappa)\beta^2}{\beta^2 + \kappa^2}\right] \tag{21a}$$

$$\eta = \frac{\kappa^2\beta}{\kappa^2 + (1+\kappa)\beta^2} \tag{21b}$$

In this normalized form, useful design curves can be generated to aid damping design. The damping and frequency relationships of a few selected stiffness ratios are shown in Figure 4. The loss factor has a slope of one and negative one at the low and high frequency range on the log-log scale and has a distinct maximum at the mid frequency range. The damping loss factor increases with the stiffness ratio.

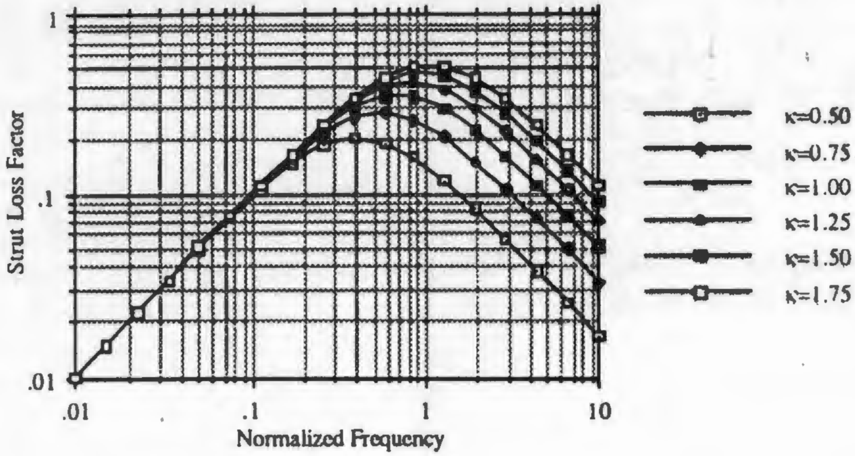

Figure 4 Normalized Design Curves

For design purposes, it is important to understand the frequency and damping characteristics of the damped strut in terms of an equivalent single degree of freedom (SDOF) system. This approximation bypasses the eigenvalue problem of Equation (12). An equivalent SDOF system is shown in Figure 5. The equation of motion of this system subject to steady state force is given by:

$$\left(-\omega^2 m + k + i\omega c\right) u = p \tag{22}$$

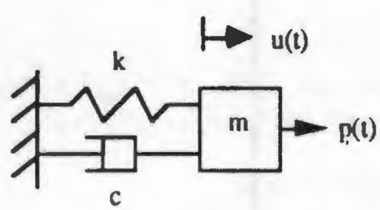

Figure 5 Equivalent Single Degree of Freedom System

Comparing Equation (22) to Equation (17), for lightly damped systems, say $\xi < 0.2$, the equivalent natural frequency of the damped strut system can be approximated by:

$$\omega_{eq} = \sqrt{\frac{k^R(\omega)}{m}} \tag{23}$$

The equivalent viscous damping ratio can be found by equating the energy loss of the strut to that of an equivalent SDOF viscous system. The energy dissipated per cycle of the strut as described by Equation (19) under a harmonic force is given by[7]:

$$D_\eta = \pi \eta k^R u_1^2 \tag{24}$$

The damping of an equivalent SDOF viscous system is given by:

$$D_\xi = \pi \xi_{eq} (2m\omega_{eq}) \omega u^2 \tag{25}$$

Equating the energy dissipation at resonance, $\omega = \omega_{eq}$:

$$\xi_{eq} = \frac{\eta(\omega_{eq})}{2} \tag{26}$$

OPTIMUM STRUT BEHAVIOR

For a given a design, i.e. k_1, k_2 and c, the strut dynamic stiffness and damping can be computed using Equations (21a) and (21b). A typical plot of the stiffness and loss factor of a strut is shown in Figure 6.

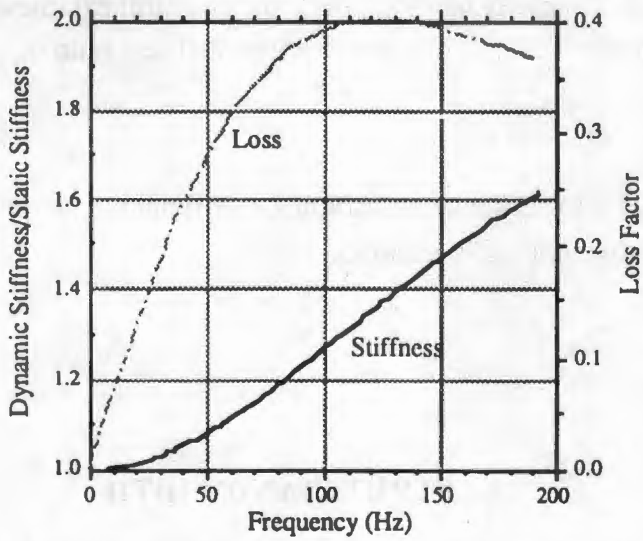

Figure 6 Typical Viscous Strut Stiffness and Loss Factor vs. Frequency

For design purposes, it is important to find the optimum performance region of the strut so that the strut can be designed to perform effectively, i.e. high damping at the desired frequency range. The maximum loss factor with respect to frequency can be found by setting the derivation of Equation (19b) to be zero:

$$\frac{\partial\eta}{\partial\omega} = 0 \tag{27}$$

The condition at which the damping is at maximum is denoted by the subscript op.

$$\eta_{op} = \frac{\kappa}{2\sqrt{1+\kappa}} \tag{28a}$$

$$k^R{}_{op} = \frac{2(1+\kappa)}{2+\kappa} k_1 \tag{28b}$$

$$\omega_{op} = \frac{\kappa}{\sqrt{1+\kappa}} \omega_c \tag{28c}$$

$$\beta_{op} = \frac{\kappa}{\sqrt{1+\kappa}} = 2\,\eta_{op} \tag{28d}$$

It is important to note that the maximum loss factor is governed by κ, the ratio of the inner and outer stiffnesses. A flexible inner tube is not effective in providing force to the damper to activate energy dissipation. A stiff inner tube is very desirable for high damping but the strut will also be heavier. It is also important to note that there is not much damping at low and high frequency. The stiffness corresponding to maximum loss factor is at the transition between the static stiffness, k_1, and asymptotic stiffness, k_1+k_2. The frequency at which the maximum loss occurs is proportional to the damper non-dimensional frequency, ω_c. As a matter of fact, the normalized optimum frequency is twice the maximum loss factor. Using these relationships, frequency independent parameters can be computed easily to match the key points of test data in order to characterize the dynamic behavior. Comparisons between analytical and test data were excellent.

These relationships can easily be used to size the key strut parameters. For a desired level of damping, η_r, use Equation (28a) to find the required stiffness ratio, κ_r.

$$\kappa_r = 2\eta_r^2 + 2\eta_r\sqrt{\eta_r^2 + 1} \tag{29}$$

Then use Equation (28c) to compute the damping coefficient, c_r, required to locate the frequency, ω_r, where the maximum damping is required.

$$c_r = \frac{\kappa_r}{\sqrt{1+\kappa_r}}\frac{k_1}{\omega_r} \tag{30}$$

STRUT BANDWIDTH

Another important performance parameter is the bandwidth of the strut over which there is significant amount of damping. The effective bandwidth can influence the design of struts for a large structure with a wide range of natural frequencies.

The bandwidth of the strut can be defined as the frequency range over which the strut has a damping efficiency γ:

$$\gamma = \frac{\eta_1}{\eta_{op}} \tag{31}$$

The bandwidth can be found by solving Equation (21b). For a given damping efficiency, there are two frequency points:

$$\beta_{1,2} = \frac{\kappa}{\gamma}\frac{1 \pm \sqrt{1 - \gamma^2}}{\sqrt{1+\kappa}} \tag{32}$$

The corresponding normalized frequency bandwidth is given by:

$$\Delta\beta = \frac{2\kappa\sqrt{1 - \gamma^2}}{\gamma\sqrt{1+\kappa}} \tag{33}$$

The actual frequency bandwidth is given by:

$$\Delta\omega = \frac{2\kappa\sqrt{1-\gamma^2}}{\gamma\sqrt{1+\kappa}}\,\frac{k_1}{c} \tag{34}$$

The damping bandwidth is shown graphically in Figure 7. The damping within the bandwidth is guaranteed to be higher then the specified efficiency. The bandwidth concept can be used for many other design reasons. The recipricol of damping efficiency can be interpreted as a safety factor for damping design. The bandwidth can be used to cover the uncertainty in the natural frequencies of a large structure.

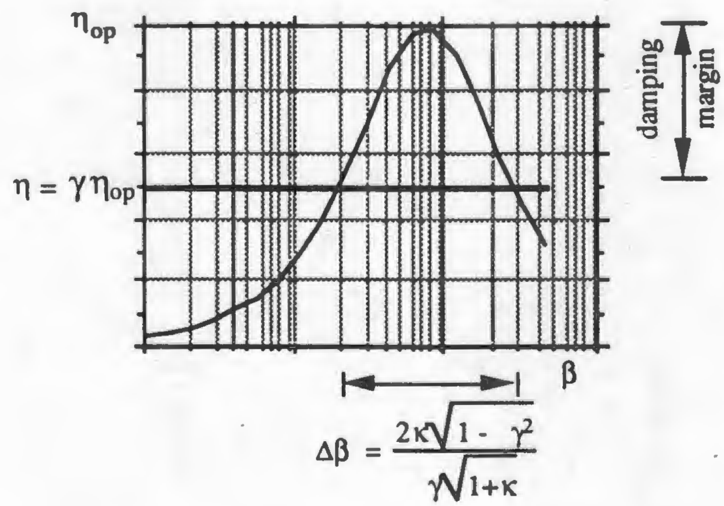

Figure 7 Strut Damping Bandwidth

DESIGN EXAMPLE

The method developed can be used to size the key parameters of a strut. Only simple algebraic equations are solved and an eigenvalue problem is totally avoided. Suppose a 20-pound weight is supported by a strut. The system is required to have 20 Hertz natural frequency and 5% viscous damping. By using the design equations, the strut parameters were computed to be: $k_1 = 758.8$ lb/in, $k_2 = 166.86$ lb/in, $c = 1.2$ lb-sec/in. The frequency and damping characteristic of the system with these parameters were checked with an exact eigensolution. The results compare favorably and are summarized in Table 1.

Table 1 Comparison of Results

Parameters	Design Goal	Eigensolution	Error
Frequency	20.0 Hz	20.2 Hz	1%
Damping	5%	5.25%	5%

CONCLUSION

The dynamics of a class of viscously damped struts is presented. The derivation is based on the principles of structural dynamics and governing equations of motion of a finite element model. This approach is consistent with the system level analysis methods. The use of condensation technique allows a complex strut design to be reduced to 3 stiffness parameters which are further reduced to 2 lumped stiffness parameters. The dynamics of the struts can be understood through non-dimensional design variables. Design curves can be used to facilitate component sizing. The bandwidth characteristics of the struts provide further insight into the performance of this class of struts. Results from using this method compared favorably with the exact solution from a complex eigenvalue problem. Therefore, a 3-parameter model can be used to characterize the performance of a viscously damped strut for system level design and analysis. The method can be used to derive component specification to meet system level design requirements[5].

NOMENCLATURE

Symbols

C,**c**,c	=	viscous damping matrix or scalar
D	=	energy dissipation per cycle
g	=	forcing function
i	=	imaginary unit, $\sqrt{-1}$
K,**k**,k	=	stiffness stiffness or scalar
M,**m**,m	=	mass matrix or scalar
p	=	spatial force vector
u, u	=	displacement vector and axial displacement degree of freedom
β	=	non-dimensional forcing frequency
γ	=	damping efficiency factor
Δ	=	change/bandwidth
λ	=	complex eigenvalue
ψ	=	complex eigenvectors
η	=	loss factor
κ	=	stiffness ratio of inner spring to outer spring
ξ	=	damping ratio
ω	=	frequency, radian/second

Subscripts

eq	=	equivalent
c	=	pertaining to damping
i	=	for the i-th mode
op	=	condition at maximum loss factor
r	=	pertaining to the required conditions
ξ	=	pertaining to viscous damping
η	=	pertaining to viscoelastic (hysteretic) damping

Superscripts

I	=	Imaginary
R	=	Real
T	=	matrix transpose

REFERENCES

1. J.L. Fanson, G.H. Blackwood and C.-C. Chu, 'Active Member Control of Precision Structures', *AIAA/ASME,ASCE,AHS/ASC 30-th Structures, Structural Dynamics, and Materials (SDM) Conference*, 1989, Mobile, Alabama.

2. Johnson, C.D., and Kienholz, D.A., 'Design and Testing of a Sixty-Foot Damped Generic Space Truss', *Damping 1986*, AFWAL-TR-86-3059, Flight Dynamics Laboratory, Air Force Wight Aeronautical Laboratories (1986).

3. Johnson, C.D., and Kienholz, D.A., 'Finite Element Prediction of Damping in Structures with Constrained Viscoelastic Layers', *AIAA Journal* Vol 20, No. 9 (Sept. 1982).

4. E. Anderson, M. Trubert, J. Fanson, and P. Davis, 'Testing and Application of a Viscous Passive Damper for Use in Precision Truss Structure', *AIAA/ASME,ASCE,AHS/ASC 32-th Structures, Structural Dynamics, and Materials (SDM) Conference*, 1991, Baltimore, Maryland.

5. Y.C. Yiu, 'System Level Design and Analysis of Truss Structures Damped by Viscous Struts', *Damping '91 Conference*, February 1991, San Diego, California.

6. Hurty, W.C. and Rubinstein, M.F., *Dynamics of Structures*, Prentice-Hall, Inc., 1964.

7. Nashif, A.D., D.I.G. Jones, and J.P. Henderson, *Vibration Damping*, John Wiley & Sons (New York, 1985).