ASD TECHNICAL REPORT 61-467
PART I

INTERFACE DAMPING AT RIVETED JOINTS

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FOREWORD

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ABSTRACT

In this paper a theoretical examination is made of the potential structural damping increments that could be obtained by the insertion of a linear visco-elastic interfacial layer between the plates of riveted joints. Consideration is given to a lap joint having anti-symmetry about its single rivet, and being subjected to harmonic longitudinal loading.

Certain simplifying assumptions have been made which effectively reduce the analysis to that of a one dimensional system.

An expression is obtained for the energy dissipated in the layer per cycle of load. The magnitude of this energy dissipation has been computed for a wide range of joint dimensions, dynamic properties of the layer, and rivet stiffnesses. It has been found that as the thickness (or shear modulus) of the layer is varied, a maximum value of the energy dissipation occurs. The conditions for this maximum are examined, and a simple design rule is established whereby the maximum damping may be achieved in a joint using a given material.

It is shown that the elastic deformation of the plates has a significant effect only when high values of rivet stiffness are considered. For lower values of rivet stiffness the energy dissipation may be found to a sufficient degree of accuracy by neglecting altogether the plate flexibility.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

W. J. Trapp

W. J. TRAPP
Chief, Strength and Dynamics Branch
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Directorate of Materials and Processes

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LIST OF SYMBOLS

b	plate width
d	thickness of the visco-elastic interfacial layer
f	non-dimensional energy dissipation function
h	plate thickness
i	$\sqrt{-1}$
$k = (k_r + k_i)$	the joint stiffness
k_i	the real part of the complex shear stiffness of the interfacial layer
k_r	rivet stiffness (force per unit deflection)
ℓ	plate overlap length
$u_o(t)$	relative displacement of two inextensible plates
$u_1(x,t)$, $u_2(x,t)$	displacement of plates 1 and 2 respectively
$u(x,t) = u_1(x,t)-u_2(x,t)$	relative displacement of plates
u', u''	the inphase and quadrature components of u
t	time
E	modulus of elasticity of plate material
$G^* = G(1 + i\beta)$	complex shear modulus
P	amplitude of harmonic load applied to the system
β	loss factor (i.e. normalized imaginary part) of complex shear modulus of the interfacial layer
ϕ	shear parameter (see equation (9))
η	joint loss factor (see equation (15))
$\tau(x,t)$	shear stress acting in damping material
ω	circular frequency
Δ	energy dissipated per load cycle (elastic plates)
Δ_s	energy dissipated per load cycle (rigid plates)
Δ_r	maximum amount of energy dissipated per load cycle (rigid plates)

1. Introduction

It is well known that structural damping is an important parameter in controlling vibration amplitudes of aircraft structural systems. This type of damping originates at bolted and rivet joints in such structures. In the joint the mechanisms of elastic hysteresis, plastic deformation, and relative slipping of the joint plates against dynamic friction, all cause energy to be dissipated (a detailed description of the damping mechanism is given in reference 1). This damping could be increased by permitting greater dynamic friction and rivet slip. However, it is obvious that control of this damping is strictly limited, since any reduction in rivet stiffness and increased fretting enhances the probability of fatigue failure.

In 1946 Cooper (Ref. 2) showed experimentally that an increase in damping of several times could be obtained by the insertion of a thin layer of a suitable visco-elastic material between the joint surfaces of a riveted structure. It is evident that if an interfacial layer of visco-elastic material is inserted the former frictional shear stress will be replaced by shear stress in this layer. Being visco-elastic, the material will absorb and dissipate energy. Until recently the possible potentialities of this method had not been explored but detailed investigations are now being carried out at the University of Southampton.

In this paper a theoretical assessment is made of the energy dissipation per cycle of load in a single riveted "anti-symmetric" lap joint containing a linear visco-elastic interfacial layer (see Fig. 1). The joint is assumed to be subjected to a simply-harmonic longitudinal force. The problem is simplified by reducing it effectively to a one dimensional system. In order to do this certain assumptions have to be made:

(i) Bending of the plates is not considered

(ii) The rivet action is assumed to occur at the joint centre on a transverse plane $x = o$ (see Fig. 1). The rivet flexibility ($\propto$ 1/stiffness) is assumed to include the shearing and bending deformations of the rivet together with the "bearing" deformations of the plate adjacent to the rivet. Using this line rivet system at $x = o$ the stress distribution across the width of each plate is considered uniform for all x.

The visco-elastic layer is assumed to have a linear dynamic shear modulus and its direct stress carrying capacity is ignored.

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It is evident that the dynamic properties of the layer, the joint dimensions, and the rivet stiffness will all have an effect on the energy dissipated at the joint. The magnitude of the rivet stiffness will in general be fixed by static strength and fatigue life requirements. For the same reasons, the possible variation in the joint dimensions will also be restricted. In order to investigate the maximum possible amount of energy that can be dissipated per load cycle for a given rivet stiffness, a wide range of the independent parameters have been examined and the optimum joint configurations established.

In the theoretical work a general solution is obtained for the energy dissipated per cycle, including the effects of the elastic deformation of the plates. It is also shown that for a wide range of rivet stiffness ($k_r/Eb < 0.003$), a further approximation - that the joint plates are rigid ($E(\text{plates}) \to \infty$) - enables simple formulae to be established for obtaining the correct joint configuration to give the maximum energy dissipation. The formulae are shown to give the energy dissipation to a sufficient degree of engineering accuracy when compared with the results computed using the theory for elastic plates.

Certain problems attend the use of such inserts. The damping properties of the interfacial layer should not be greatly affected by changes of temperature nor of the frequency of the exciting force, since large variations in the damping in a system are obviously undesirable. The fatigue life of the joint should not be impaired by the presence of the interfacial layer.

A second paper is to be published comparing experimental results with the theoretical work and examining closely the properties of the interfaces and the fatigue lives of various joint configurations.

2.1. The Energy Dissipation at a Riveted Joint with a Visco-elastic Interfacial layer (Elastic plates).

A diagram of the lap joint to be considered is shown in Fig. 1. The problem is effectively reduced to a one-dimensional case by ignoring non-uniform strains distributed across the width 'b' of the plate which would normally be caused by stress concentrations around the rivet. We further ignore any bending of the plates and therefore assume that the strain and displacement across the depth of the plates are uniform. Under a longitudinal exciting force $P\exp(i\omega t)$, the relative displacement of the plates $u(x, t)$ is taken as $(u_1 - u_2)$ where

$$u_1 = \left\{u_1'(x) + u_1''(x)\right\} \times \exp(i\omega t)$$

and $$u_2 = \left\{u_2'(x) + u_2''(x)\right\} \times \exp(i\omega t)$$

The dynamic shear modulus of the layer is represented by the usual complex relationship:

$$G^* = G(1 + i\beta)$$

where β is the material loss factor.

The forces acting on the section of an elemental length of the plate are now considered (Fig. 2). It is readily seen that for equilibrium of the element of plate 1:

$$Ebh(\partial u_1/\partial x + \partial^2 u_1/\partial x^2.dx) - \tau b.dx - Ebh\,\partial u_1/\partial x = 0$$

$$\text{i.e.}\quad Ebh.\,\partial^2 u_1/\partial x^2 - \tau b = 0 \qquad (1)$$

and for the element of plate 2:

$$Ebh(\partial u_2/\partial x + \partial^2 u_2/\partial x^2.dx) - \tau b.dx - Ebh\,\partial u_2/\partial x = 0$$

$$\text{i.e.}\quad Ebh\,\partial^2 u_2/\partial x^2 + \tau b = 0 \quad . \qquad (2)$$

We now assume that the layer carries no direct stress. The shear stress and hence the shear strain across the layer are therefore constant. The shear stress is then given by:

$$\tau = G^*(u_1 - u_2)/d \quad . \qquad (3)$$

Substituting (3) into (1) and (2) we now obtain:

$$\partial^2 u_1/\partial x^2 - (G^*/Ebh)u_1 + (G^*/Ebh)u_2 = 0 \qquad (4)$$

$$\partial^2 u_2/\partial x^2 - (G^*/Ebh)u_2 + (G^*/Ebh)u_1 = 0 \qquad (5)$$

The general solutions of (4) and (5) are:

$$u_1 = A_1 \operatorname{Cosh} \nu x + B_1 \operatorname{Sinh} \nu x + C_1 x + D_1 \qquad (6)$$

$$u_2 = A_2 \operatorname{Cosh} \nu x + B_2 \operatorname{Sinh} \nu x + C_2 x + D_2 \qquad (7)$$

where $\nu = (2G^*/Ehb)^{\frac{1}{2}}$.

The following conditions are necessary and sufficient to determine the constants of integration in these solutions and hence to find u(x, t):

(i) The total load across any cross section of the joint (e.g. Y-Y in Fig.1) must be equal to the externally applied load i.e.

$$P \exp(i\omega t) = Ebh(\partial u_1/\partial x + \partial u_2/\partial x) \text{ for all } x, t.$$

(ii) When $x = \ell$, $Ebh\,\partial u_1/\partial x = P \exp(i\omega t)$ and $\partial u_2/\partial x = 0$ for all t. This condition must also be satisfied when $x = -\ell$, but this is automatically satisfied if we consider only a joint which possesses 'anti-symmetry' about the rivet.

(iii) The total load transmitted by the damping interface and the rivet must be equal to the applied load P, i.e.

$$P \exp(i\omega t) = b. \int_{-\ell}^{+\ell} \tau \, dx + P(t)_{rivet}$$

where $P(t)_{rivet} = k_r(u_1 - u_2)_{x=0}$

and k_r is the rivet stiffness (load per unit shearing deflection) which includes any reduction in the stiffness due to bearing stress concentration in the plates at the rivet location.

Using these conditions to find the constant of integration, we find that the relative deflection of the two plates at any point is:

$$u(x, t) = P \exp(i\omega t) \quad \frac{\text{Cosh}\,\nu x - \text{Tanh}\,\nu\ell.\text{Sinh}\,\nu x}{Ebh\nu\,\text{Sinh}\,\nu\ell + k_r} + \frac{\text{Sinh}\,\nu x}{Ebh\nu\,\text{Cosh}\,\nu\ell} \cdot \quad (8)$$

The expression for the energy dissipated per cycle of harmonic loading may readily be shown to be:

$$\Delta = 2\pi P[u_1'']_{x=\ell} \quad .$$

Now ν in equation (8) is complex. The quadrature component of displacement must therefore be found using the relationships:

$$\text{Cosh}\,\nu\ell = \text{Cosh } n \text{ Cos } m + i \text{ Sin } m \text{ Sinh } n$$

$$\text{Sinh}\,\nu\ell = \text{Cos } m \text{ Sinh } n + \text{Sin } m \text{ Cosh } n$$

in which m and n are both real and are given by:

$$m = V\phi\ell/h; \qquad n = W\phi\ell/h \quad .$$

In these $V = \left\{\frac{(1+\beta^2)^{\frac{1}{2}} - 1}{2}\right\}^{\frac{1}{2}}; \quad W = \left\{\frac{(1+\beta^2)^{\frac{1}{2}} + 1}{2}\right\}^{\frac{1}{2}}$ (9)

and $\phi = (2Gh/Ed)^{\frac{1}{2}}$.

It is now possible to write the expression for the energy dissipation in the form:

$$\Delta = \pi P^2 f/Eb \tag{10}$$

where f is a non-dimensional function given by:

$$f = \alpha_6 \left\{ \frac{\phi \alpha_1 + (k_r/Eb)\alpha_2}{(k_r/Eb)^2 + 2\phi(k_r/Eb)\alpha_3 + \phi^2\alpha_4} + \frac{\alpha_5}{\phi} \right\} \tag{11}$$

This will be referred to as the "Energy Dissipation Function".

$$\alpha_1 = \tfrac{1}{2}\left\{W \text{ Sin } 2m \text{ Cosh } 2n - V \text{ sinh } 2n \text{ Cos } 2m\right\}$$

$$\alpha_2 = \text{Sin } m \text{ Sinh } n$$

$$\alpha_3 = \left\{W \text{ Cos } m \text{ Sinh } n - V \text{ Sin } m \text{ Cosh } n\right\}$$

$$\alpha_4 = (W^2 + V^2)\left\{\text{Cosh}^2 n - \text{Cos}^2 m\right\}$$

$$\alpha_5 = \frac{\text{Sinh } 2n - \text{Sin } 2m}{2(W^2 + V^2)}$$

$$\alpha_6 = \frac{1}{\text{Cos}^2 m + \text{Sinh}^2 n}$$

From the above expression it is seen that the energy dissipated per cycle of unit load on unit width of plate is dependent on the parameters:

(ℓ/h)- - - - - - - - - - - the 'Overlap Parameter',

ϕ - - - - - - - - - - - the 'Shear Parameter',

β - - - - - - - - - - - the Material Loss Factor

and (k_r/Eb)- - - - - - - - - - - the 'Rivet Stiffness Parameter'.

2.2 The Energy Dissipation at a Riveted Joint with a Visco-Elastic Interfacial Layer (Rigid Plates).

We now consider the case where $E \to \infty$. The relative displacement of the plates is now the same for all x. Let this be $u_o(t)$. The complex shear force being transmitted at any instant by the rivet and interfacial layer is now clearly:

$$u_o . k_r + (u_o/d) . 2\ell bG(1 + i\beta) = P \exp(i\omega t) \tag{12}$$

This may be written in the form:-

$$k(1 + i\eta)\, u_o e^{i\omega t} = P \exp(i\omega t) \tag{13}$$

where $k = k_r + k_i$ (14a)

$$k_i = (2\ell b/d)\, G \quad \text{(shear stiffness of layer)} \tag{14b}$$

and $\eta = \beta k_i/(k_r + k_i)$. (15)

η may be referred to as the "joint loss factor". It is readily shown that the energy dissipated per cycle of load P is given by:

$$\Delta_s = \frac{\pi P^2}{k} \cdot \frac{\eta}{(1 + \eta^2)} \tag{16}$$

which shows that for maximum energy dissipation η must be unity, i.e.

$$\Delta_{max} = \pi P^2/2k = \pi P^2/2(k_r + k_i) \, . \tag{17}$$

For η to be unity, the corresponding value of β (β_{opt}) is obtained from equation 15, i.e.

$$\beta_{opt} = 1 + k_r/k_i \quad . \tag{18}$$

This indicates the optimum loss factor for a given layer stiffness k_i. Equation (17) shows that the maximum possible energy dissipation occurs when $k_i \rightarrow 0$. From (18), this implies that

$$\beta_{opt} \cdot k_i \rightarrow k_r \quad . \tag{19}$$

These are the properties of a purely "viscous"* material, which can sustain no steady shear stress. The corresponding maximum amount of energy dissipated is given by:

$$\Delta_r = P^2/2k_r \tag{20}$$

and is dependent only upon the rivet stiffness, and not upon the other joint dimensions. The properties of the layer are, however, dependent upon the interfacial area and thickness.

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* In fact, the material is not viscous but "hysteretic" since the damping stress is not proportional to the rate of strain but to the strain.

Now it has been established in equation (10) that the energy dissipated per load cycle in a joint of finite stiffness is given by the expression:

$$\Delta = (\pi P^2/Eb) . \ f \ (k_r/Eb, \ \phi, \ \ell/h, \ \beta \ ).$$

Dividing this equation by Δ_r we obtain the useful dimensionless quantity:

$$\Delta / \Delta_r = 2(k_r/Eb) . f \qquad (21)$$

This expresses the energy dissipation at the joint as a fraction of the maximum amount of energy that could be dissipated at the same load amplitude in the joint, having optimum layer properties and infinite E.

Substituting now for η and k (equations 14 and 15) into equation (16) we have:

$$\Delta_s = \pi P^2 \beta k_i / \left\{ (k_r + k_i)^2 + \beta^2 k_i^2 \right\} \qquad (22)$$

Supposing now that the loss factor of the material is specified, an optimum of k_i may be obtained to maximise Δ_s. Differentiating equation (22) with respect to k_i and equating to zero, we find that:

$$(k_i)_{opt} = k_r/(1 + \beta^2)^{\frac{1}{2}} \qquad (23)$$

We notice that, as β becomes very large, this equation becomes:-

$$\beta . (k_i)_{opt} \rightarrow k_r \qquad (24)$$

which is complimentary to equation (19). Substituting from equation (14b) into equation (23) we have:

$$(2G\ell/d)_{opt} = \frac{k_r/b}{(1 + \beta^2)^{\frac{1}{2}}} \qquad (25)$$

The foregoing theory relates to the case $E = \infty$. It is clearly a good approximation to the rigorous theory of par.(3.1), if E is large. Assuming now that E is very large but not infinite, we may multiply both sides of equation (25) by 1/E, whereupon we have:

$$2 \left(\frac{Gh}{Ed}\right)_{opt} \left(\ell/h\right)_{opt} = \frac{k_r/Eb}{(1 + \beta^2)^{\frac{1}{2}}}$$

which is recognized as:

$$\phi^2_{opt} \left(\frac{\ell}{h}\right)_{opt} = \frac{k_r/E_b}{(1 + \beta^2)^{\frac{1}{2}}} \qquad (26)$$

The values of ϕ and ℓ/h given by this equation are the optimum values required to maximise the energy dissipation when β , k_r, E and b are specified. Normally, of course, if the interfacial layer material is given, then G will also be known. Since ℓ is likely to be determined by static strength or manufacturing considerations, equation (26) gives the optimum thickness of the layer.

Substituting from (23) into (22) we find:

$$\Delta_s = \frac{\pi P^2}{2k_r} \left\{ \frac{\beta}{(1 + \beta^2)^{\frac{1}{2}} + 1} \right\} \qquad (27)$$

and from (20) it is evident that

$$\frac{\Delta_s}{\Delta_r} = \frac{\beta}{(1 + \beta^2)^{\frac{1}{2}} + 1} \qquad (28)$$

3. Computed Results

In order to obtain a comprehensive picture of the dependence of the dissipation function f upon the independent parameters ϕ, ℓ/h, and k_r/Eb, a wide range of these parameters has been considered. The corresponding values of f have been calculated using a digital computer. Some typical curves indicating the variation of f with ℓ/h and ϕ are shown in Figures 3 to 6. Each of these figures corresponds to particular values of k_r/Eb and β . Only curves for low values of (k_r/Eb) are considered in these figures.

An obvious characteristic of these curves is that for each value of ℓ/h there is an optimum value of ϕ which gives a true maximum to f. The maximum values of f in any one figure are to a close approximation the same, i.e. independent of ℓ/h for a given k_r/Eb and β . This would be expected if the plates were rigid, as we see from equation (27). Figures (7) and (8) show the maximum values of f that can be obtained for values of $k_r/Eb \leq 0.003$ as functions of the interfacial layer loss factor. These figures also show the corresponding optimum overlap and shear parameters.

The curves plotted in Fig. 9 are the same as the 'f' curves of Figs. 7 and 8, but on a log-log basis and covering a much higher range of β . If the values of Δ_s/Δ_r obtained from equation (28) are plotted on Fig. 10, it is found that the points fall very closely to the curve of Δ/Δ_r obtained from the previous figures. The energy dissipated in the joint within the range of parameters considered in Figs. 3 to 9 therefore corresponds very closely with that of the rigid plate approximation. However, at higher values of the rivet stiffness parameter, the rigid plate approximation is inadequate and the energy dissipation in the "optimized" configuration depends upon the overlap parameter, the rivet stiffness parameter and β. This dependence is shown in Fig. 11. Tables I to IV show the corresponding values of the optimum shear parameter. Fig. 12 shows the curves of Fig. 11 normalized as before, in effect comparing the energy dissipated in a joint with flexible plates with the maximum dissipated in a rigid plate joint.

4. Discussion of Results

It has already been pointed out that the maximum values of energy dissipation exist if particular values of ϕ and ℓ/h are chosen, i.e. particular values of the interface stiffness. The existence of a maximum as the shear stiffness varies is suggested by the following argument:

When the shear stiffness of the layer is small compared with that of the rivet most of the load is transmitted by the rivet. If the layer stiffness is slightly increased, the load transmitted by the layer increases, and the energy dissipated per cycle increases. If now the shear stiffness of the layer is much larger than that of the rivet, most of the load is transmitted by the layer. If the layer stiffness is increased now (β being kept constant) deflections will be reduced and so also will be the energy dissipated, i.e. at low layer stiffness, increasing the stiffness increases the energy dissipated, whereas at high stiffnesses, increasing the stiffness decreases it. Between these regions, a maximum value must, therefore, exist. This is investigated in greater detail in the Appendix.

As has already been shown when the rivet stiffness parameter is small ($\leqslant 0.003$), the maximised energy dissipation in the joint with elastic plates is virtually the same as that in the rigid plate joint, to a sufficient degree of engineering accuracy. At these low values of rivet stiffness, the shear strain in the interfacial layer due to the rivet deflection is very much greater than that arising from the unequal stretching of the plates. The latter effect is therefore small enough to be ignored, implying that the rigid plate theory is sufficiently accurate when $k_r/Eb \leqslant 0.003$. In this range, therefore, the simple design rule of equation (25) and (26) may be applied in order to find the optimum layer thickness (G, β and k_r being specified).

Equation (25) (rigid plate theory) rewritten in the form:

$$(2Gb\ell/d) \quad (1 + \beta^2)^{\frac{1}{2}} = k_r$$

states that the modulus of the complex shear stiffness of the interfacial layer should be equal to the elastic stiffness of the rivet for maximum energy dissipation. A reason for this is sought in the Appendix. Equation 26 is the alternative form of equation (25, and shows that if ℓ/h is kept constant while the rivet stiffness parameter is increased, then ϕ^2_{opt} must be increased in proportion to k_r/Eb. Alternatively the ratio ϕ^2_{opt} : k_r/Eb must be kept constant. This fact assists in the explanation of the effect of plate flexibility on the value of ϕ_{opt}, dealt with below.

The effect of plate flexibility when $k_r/Eb > 0.003$ is to increase the relative displacements of the plates over and above that associated with the rivet deflection. This in turn results in greater energy dissipation than in the rigid plate joint. With increasing rivet stiffness, the energy dissipation associated with rivet deflection becomes relatively smaller, whereas that associated with unequal stretching of the plates becomes relatively larger. The net effect of increasing the rivet stiffness is, of course, to decrease the energy dissipation at all values of k_r/Eb.

Now tables I to IV show that the values of ϕ_{opt} for the elastic plate joint ($\phi_{opt.e}$, taken from computed results) become less than the values of ϕ_{opt} corresponding to the rigid plate theory ($\phi_{opt.r}$). This may be accounted for in the following manner:

The effect of unequal plate stretching is to increase the shear strain in the interface layer above that corresponding to the rigid plate theory. With a given applied load, therefore, more load will be transferred through the interfacial layer when the plates are flexible than when they are rigid. This is tantamount to a relative reduction of the rivet stiffness compared with the interface stiffness. Now according to the rigid plate theory, optimum conditions obtained when the ratio, interface stiffness (ϕ^2): Rivet Stiffness (k_r/Eb), is kept constant. If the effect of plate flexibility is equivalent to a reduction of rivet stiffness, then the interface stiffness ϕ_{opt} must be reduced to keep the ratio the same. The value of ϕ_{opt} for the flexible plate condition will therefore be less than that for the rigid plate condition. This effect is more marked as ℓ/h increases.

The curves of Figs. 9 and 11 demonstrate the asymptotic behaviour of the maximized energy dissipation function as β gets very large. (The extremely high values of β that have been considered - i.e. up to 64 - are not representative of those which are practically obtainable. They have been considered only in order to determine the ultimate trends and asymptotes). When $k_r/Eb < 0.003$ and $\ell/h < 50$, the asymptotic value of the energy dissipation is virtually the same as the "absolute maximum" amount of energy that can be dissipated in a rigid plate joint (Δ_r). For larger values of k_r/Eb the effect of plate flexibility is to increase this asymptotic value. The reason for the asymptotic behaviour is outlined in the Appendix. It should be noted in Figs. 10 and 12 that 80% of the theoretical maximum is obtained with β = 4.0, using the optimum configuration.

5. The Potentialities of Interfacial Layers for Increasing Structural Damping

The foregoing theory, computations and discussion relate entirely to a joint in which the insert is subjected only to shear strain, i.e. the joint is subjected to an 'ideal' longitudinal load. Such idealization is approached, for example, in the chordwise skin joints of an aeroplane wing undergoing flexural vibration, or in the circumferential skin joints of a fuselage in flexural vibration. Considerable increase in the mode damping should be obtainable using visco-elastic inserts at such joints.

When a fuselage is subjected to acoustic excitation, it is possible for modes of vibration to be excited which involve distortion of the whole cross-section of the fuselage (ref. 4). These modes will cause the circumferential and longitudinal skin joints, and joints in the stringer and frame flanges to be loaded longitudinally. Again, the damping of the modes should be considerably increased by visco-elastic inserts at the joints. These modes will also cause shear loads to be transferred through the rivets attaching stringers to skin and frames to skin, by virtue of the transverse shear forces in the stringer-skin and frame-skin beam combinations vibrating in flexural modes. The loads on these rivets, however, will be much less than the loads on skin-joint rivets since they are proportional to the local shear force, and not to the local bending moment. The wave-lengths of the flexural modes that are likely to be excited are such that stringer skin joints will be lightly loaded compared with the joints transmitting bending moment loads. Furthermore, unlike the rivets at skin-skin joints the pitch of these rivets is not determined in the first place from considerations of strength, but from consideration of surface smoothness and/or inter-rivet buckling. There are, therefore, many more rivets that are required to carry any static load. It follows that the load received by these rivets in flexural vibrations is very small compared with the load on the skin-skin joints.

Now the energy dissipated at a joint (or rivet) is proportional to the square of the joint load. The total energy that could be dissipated at the stringer and frame-skin joints is therefore likely to be very much less than that dissipated at skin-skin joints. For the same reason, the energy dissipated at spar boom to web joints in spar boom-skin surface joints in a wing undergoing flexural vibrations, is likely to be much less than that dissipated at the chordwise joints.

Consider now the vibrations of skin panels riveted to stringer or frame boundary members. Longitudinal loads on these joints can only exist if large amplitude, non-linear flexural vibrations take place, in which stretching of the plate middle surface is important. Such stretching is only likely to be important if the boundaries of the panel are very stiff, and the average stringer or frame would be unlikely to provide the necessary stiffness. That non-linear vibrations of such panels do occur is due to the stiffness of adjacent panels, and not to the stringer or frames. It follows, therefore, that no 'longitudinal' load is likely to pass through the joint at the boundary member. If, however, a skin to skin joint exists at the boundary then the longitudinal load transferred from one panel to the next may dissipate energy at the joint, and it would be advantageous to insert a visco-elastic interface.

So far, the possibility of bending of either of the joint plates has been ignored. Such bending is likely to be important at the boundaries of a vibrating panel, as above. The effect will be to cause the two plates to 'open' and 'close' relative to one another, and energy will be dissipated by the alternate tension and compression of the interfacial layer. A separate analysis is required to deal with this mechanism, but its potentiality has already been recognized in work by Mentel (ref. 5) and theoretical work is already proceeding.

The overall usefulness of the addition of interfacial layers at joints can only be assessed by comparing the modal damping increment achieved thereby with the damping of the modes without the layers. This initial damping derives from three or more principal sources:

(a) The hysteresis of the material of the structure

(b) The damping deriving from the joints before treatment

(c) The acoustic radiation damping

(b) depends upon the size and disposition of the rivets, and also upon the modal stress distribution. (c) depends upon the panel shape, frequency and mode of vibration. It is evident that a comparison of the interface damping with the initial damping can only be made for specific configurations, and no broad generalizations may be made. However, some typical values of the damping due to (b) and (c), in relation to the modes of vibration of a fuselage, are given in Ref. 4. For comparison with these values, the interface damping must be expressed as a damping ratio of the appropriate mode of vibration. This is obtained by calculating the total interface energy dissipation per cycle of displacement of the configuration in the relevant mode, and dividing by 4π x the maximum strain energy stored during the cycle (see Ref. 3).

The maximum potentiality of interface damping at riveted joints can only be achieved if interface materials with high loss factors can be developed. In particular, if loss factors of 4 can be achieved, 80% of the maximum possible damping will be obtained. It must be pointed out, however, that at the time of writing loss factors as high as this have not been achieved. The value of 4 therefore sets a target for high-polymer research work. There is no stipulation with regard to the real part of the shear modulus of the material, as the geometry of the joint may be adjusted to correspond with any value of this.

This maximum damping obtainable is also dependent upon the stiffness of the rivet. Experimental work is in progress to measure this stiffness, and with this knowledge, a better idea will be obtained of the potentiality of interfaces at riveted joints as a means of increasing structural damping.

6. Conclusions

This paper develops a theory for the energy dissipated in a singly-riveted lap joint having a visco-elastic interfacial layer and subjected to longitudinal harmonic loading. The effects of elastic deformation of the plates are included.

It has been shown that the energy dissipated per cycle depends upon a number of dimensionless parameters.

(a) The rivet stiffness parameter (rivet shear stiffness ÷ plate stiffness)

(b) The interface shear parameter (interface shear stiffness ÷ plate stiffness)

(c) The interfacial layer loss factor (normalized imaginary part of the complex shear modulus)

(d) The overlap parameter (length of plate overlap ÷ plate thickness)

In general, the energy dissipated passes through a true maximum when any one of the parameters (b), (c), (d) is varied.

When the rivet stiffness parameter is low ($< .003$) an approximate expression for the energy dissipation, derived assuming the joints plates to be inextensible, has been shown to be sufficiently accurate for engineering purposes. This permits the formulation of a simple rule to design a joint with an interfacial layer which dissipates the maximum amount of energy as follows:

If the rivet stiffness and the interface material shear modulus are known, then provided the rivet stiffness parameter is less than .003, the modulus of the complex shear stiffness of the layer should be equal to the rivet elastic stiffness.

The energy dissipated per load cycle will then be given by the expression

$$\frac{\pi P^2}{2k_r} \times \frac{\beta}{(\beta^2 + 1)^{\frac{1}{2}} + 1}$$

The effect of the plate flexibility is to increase the relative plate displacements above those associated with rivet deflection alone. Only when the rivet stiffness parameter exceeds .003 does this begin to affect the energy dissipation significantly. The energy dissipated per cycle is then given by

$$\frac{\pi P^2}{Eb} \cdot f(\phi, \ell/h, \beta, k_r/Eb)$$

in which f is a non-dimensional function of the non-dimensional parameters. The value of the shear parameter at which f passes through a maximum has been found to be less than that predicted by the inextensible plate theory in corresponding conditions. This effect becomes more marked with increasing values of the rivet stiffness and overlap parameters.

The potentialities of interfacial layers for increasing aircraft structural damping have been discussed, with particular reference to the choice of the most suitable joints for their inclusion. It is concluded that circumferential and longitudinal skin-to-skin joints in fuselages, and the chordwise skin joints in wings are best suited to this treatment.

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APPENDIX

The reasons for the occurrence of the observed maximum values of energy dissipation for particular joint configurations and material properties can readily be seen from the rigid plate theory by constructing vector diagrams of the forces and displacements involved in the harmonic deformation of the joints.

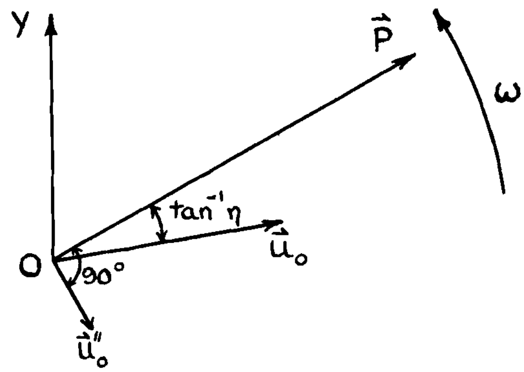


Fig. a.

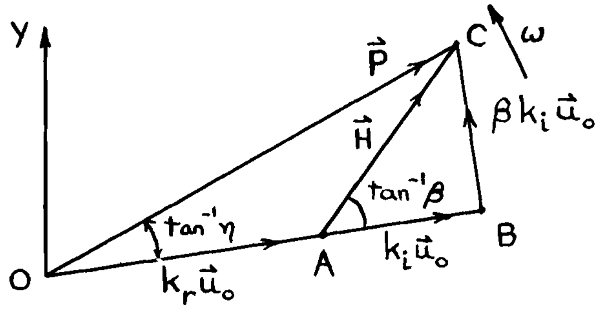


Fig. b.

In fig.(a), $\vec{P}$ represents the harmonic exciting force. This vector can be considered to rotate anticlockwise with an angular velocity corresponding to the frequency of the exciting force. The rotating vector $\vec{u}_0$ represents the corresponding joint displacement and lags behind the exciting force by an angle $(\tan^{-1}\eta)$. The vector $\vec{u}''_0$ is the component of the displacement in quadrature with P. The magnitudes of the actual force and displacement at any instant can be considered to be represented by the projections of these vectors on the line OY.

Equation (13) represents the equilibrium of the force P with the elastic restoring force of the rivet, $k_r\vec{u}_0$, and the elastic restoring and damping forces of the insert $k_i\vec{u}_0$ and $i.\beta k_i.\vec{u}_0$, respectively. The elastic restoring forces are both in phase with $\vec{u}_0$ and may be represented by a vector along $\vec{u}_0$ as shown in fig.(b). The damping force is in quadrature with $\vec{u}_0$ and is therefore drawn at right angles to $\vec{u}_0$ to complete the triangle of forces (fig. (b)). $\vec{H}$, is the resultant complex 'restoring' force from the insert, acting at angle $(\tan^{-1}\beta)$ to the direction of the displacement vector, u_0, and of magnitude $k_i(1+\beta^2)^{\frac{1}{2}}|\vec{u}_0|$, where $|u_0|$ indicates the modulus of the vector.

Now the energy dissipated per cycle is given by $\pi|\vec{P}||\vec{u}''_0|$. When examining the effect on the energy dissipation of varying the joint component properties it is sufficient, therefore, to to examine the variation of $|\vec{u}_o''|$, $|\vec{P}|$ remaining constant. When $|\vec{u}_o''|$ passes through a maximum so also does the energy dissipation.

The Effect of varying β, k_r and k_i remaining constant

Here, an explanation is sought for the existence of an optimum value of the material loss factor, when the other joint properties are specified.

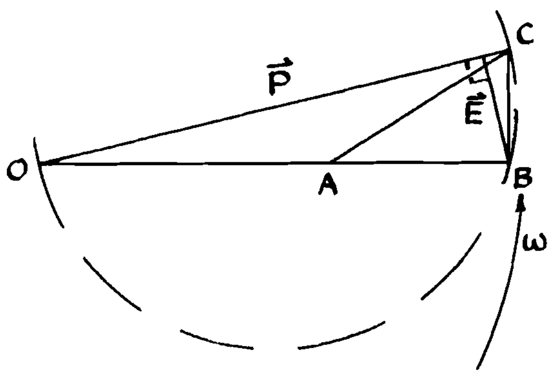


Fig. c.

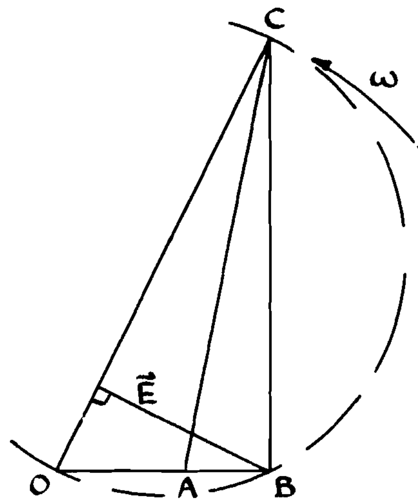


Fig. d.

In figs, (c and d) vector diagrams are shown for low and high values of β, respectively. Since k_r and k_i are fixed, the ratio OA : OB remains constant as β varies. $\vec{u}_o$ is obviously proportional to the total vector $(k_r + k_i)\,\vec{u}_o$; $\vec{u}_o''$ is then proportional to $\vec{E}$ which is drawn perpendicularly to P. The effect of increasing β is to increase the angle $C\hat{A}B$ whilst $A\hat{B}C$ remains a right angle. B, therefore, always lies on a circle of diameter $|\vec{P}|$.

When β is small:

$$\vec{E} \doteqdot CB = k_i \beta \vec{u}_o$$

and $$\vec{P} \doteqdot OB = (k_r + k_i)\,\vec{u}_o.$$

Hence $$\vec{E} \doteqdot \vec{P}\,\frac{k_i \beta}{k_r + k_i}.$$

i.e. for small values of β , $\vec{E}$ is approximately proportional to β .

When β is large:

$$OB \ll BC \quad ,$$

hence $$AC \doteq BC \doteq \vec{P}$$

and $$\vec{P} \doteq k_i \beta \vec{u}_o \; .$$

Also $$\vec{E} \doteq OB = (k_r + k_i)\vec{u}_o$$

therefore $$\vec{E} \doteq \vec{P} \frac{k_r + k_i}{k_i \beta}$$

i.e. for large values of β , $|\vec{E}|$ is approximately inversely proportional to β .

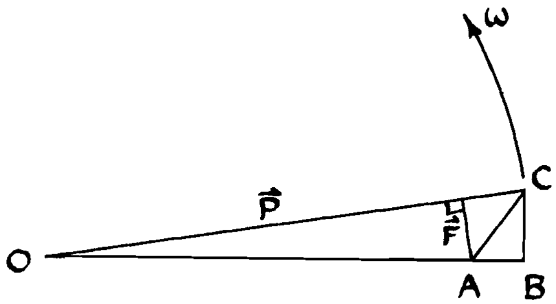
It is evident therefore that a maximum value of $|\vec{E}|$ occurs at some intermediate value of β . It is readily seen that this maximum occurs when OB = BC. We therefore have

$$(k_r + k_i)\vec{u}_o = \beta_{opt} \, k_i x \, \vec{u}_o$$

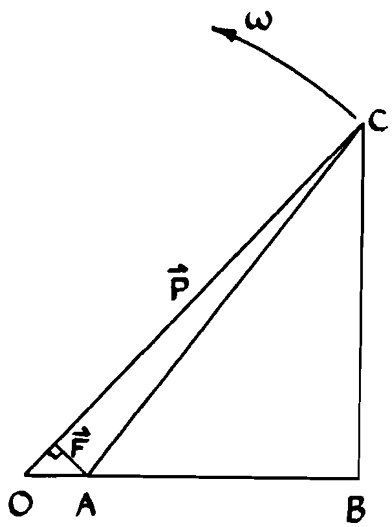
i.e. $$\beta_{opt} = \frac{k_r + k_i}{k_i} \; .$$

<u>The effect of varying k_i, keeping k_r and β constant</u>

This is equivalent to specifying the material properties, $G(1 + i\beta)$, and then varying the thickness of the interfacial layer. k_i is then inversely proportional to the thickness.



<u>Fig. e</u>.



<u>Fig. f</u>.

In Figs. (e) and (f), vector diagrams are shown for small and large values of k_i, respectively. The angle $C\hat{A}B$ (and hence $C\hat{A}O$) remains constant whilst k_i is varied. 'A' therefore moves on the circumference of a circle with chord OC. Since k_i is different in each figure, the relative values of $\vec{u}_o$ in the two cases are no longer given by the relative lengths of the vector OA. $k_r.\vec{u}_o$ is now proportional to $\vec{u}_o$ in each case. $\vec{u}_o''$ is therefore proportional to $\vec{F}$, which is drawn perpendicularly to $\vec{P}$.

When k_i is small:

B is close to C

AB and BC are $\ll$ OA

and $$\vec{P} \doteqdot OA = k_r\, \vec{u}_o$$

Also $$\vec{F} \doteqdot BC = \beta k_i\, \vec{u}_o\ ,$$

hence $$\vec{F} \doteqdot \vec{P}\, k_i/k_r\beta\ .$$

i.e. for small values of k_i, $|\vec{F}|$ is directly proportional to k_i.

When k_i is large:

'A' is now close to O and therefore OC and AC nearly coincide, i.e.

$$\vec{P} \doteqdot k_i.\vec{u}_o\,(1 + \beta^2)^{\frac{1}{2}}.$$

Also, since $C\hat{O}B \doteqdot C\hat{A}B$,

$$\vec{F} \doteqdot OA \text{ Sin } C\hat{A}B$$
$$= k_r\, \vec{u}_o \cdot \frac{\beta}{(1 + \beta^2)^{\frac{1}{2}}}\ ,$$

hence, $$\vec{F} \doteqdot \vec{P}\, \frac{k_r}{k_i} \cdot \frac{\beta}{1 + \beta^2}$$

i.e. for large values of k_i, $|\vec{F}|$ is inversely proportional to k_i.

It is evident that a maximum value of $|\vec{F}|$ will occur at some intermediate value of k_i. It can be seen that, since A lies on a circumference of a circle of which $\vec{P}$ is a chord, the maximum value of $|\vec{F}|$, and hence of $|\vec{u}_o''|$, and the energy dissipation, occurs when OA = AC. This gives

$$k_{i_{opt}}\,(1 + \beta^2)^{\frac{1}{2}}\, \vec{u}_o = k_r\, \vec{u}_o\ .$$

i.e. $$k_{i_{opt}} = \frac{k_r}{(1 + \beta^2)^{\frac{1}{2}}}$$

The limit of the energy dissipation as β gets very large

This may be examined by considering the diagram (g) with k_i at the optimum value, i.e. with OA = AC.

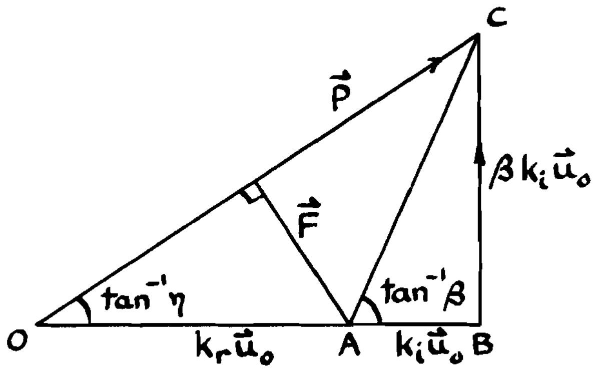


Fig. g.

As β increases, $C\hat{A}B$ increases and approaches $\pi/2$, i.e. point B approaches point A, and $AB \rightarrow 0$, or $k_{i\,opt} \rightarrow 0$. The vector $\vec{F}$ cannot exceed $OA/\sqrt{2}$, however large β becomes. OA ($=k_r.\vec{u}_o$) approaches the value $\frac{OC}{\sqrt{2}} = \frac{\vec{P}}{\sqrt{2}}$

i.e. $$|\vec{u}_o''| \rightarrow \frac{|\vec{P}|}{2k_r}.$$

There is therefore a limit on the magnitude of energy that can be dissipated (for rigid plate theory), and this is set by the rivet stiffness. This limit is approached as $\beta \rightarrow \infty$ and $k_i \rightarrow 0$ such that the product $k_i \beta \rightarrow k_r$. It is evident from Fig. (g) that beyond a value of $\beta \doteq 4$, $\vec{F}$ will not increase very much for a further increase of β.

TABLES I to IV: Corresponding optimum values of the shear parameter obtained from

(1) Elastic Plate Theory ($\phi_{opt.e}$)

and (2) Rigid Plate Theory ($\phi_{opt.r}$)

for particular values of ℓ/h and β, and the range of $\frac{k_r}{Eb}$: 0.01 to 0.3

TABLE I $k_r/Eb = 0.01$

ℓ/h \ β	0.25	1.0	4.0	16.0	64.0	
	ϕ_{opt}					$\frac{\phi_{opt.e}}{\phi_{opt.r}}$
6.25	0.0384	0.0336	0.0197	0.00999	0.0050	1.0
7.5	0.036	0.0307	0.0179	0.00912	0.00456	1.0
10.0	0.0311	0.0266	0.0156	0.0079	0.00395	1.0
15.0	0.0254	0.0217	0.0127	0.00645	0.00323	1.0
25.0	0.0197	0.0168	0.00985	0.0050	0.00250	1.0

TABLE II $k_r/Eb = 0.03$

ℓ/h \ β	0.25	1.0	4.0	16.0	64.0	
	ϕ_{opt}					$\frac{\phi_{opt.e}}{\phi_{opt.r}}$
6.25	0.0682	0.0583	0.0314	0.0173	0.00866	1.0
7.5	0.0623	0.0532	0.0296	0.0158	0.0079	0.999
10.0	0.0539	0.0461	0.027	0.0137	0.00685	0.997
15.0	0.044	0.0376	0.022	0.0112	0.00559	0.995
25.0	0.0341	0.0291	0.017	0.0087	0.00433	0.993

TABLE III $k_r/Eb = 0.1$

ℓ/h \ β	0.25	1.0	4.0	16.0	64.0	
	ϕ_{opt}					$\frac{\phi_{opt.e}}{\phi_{opt.r}}$
6.25	0.1246	0.01064	0.0623	0.0316	0.0158	1.0
7.5	0.1137	0.0971	0.0569	0.02884	0.01443	0.99
10.0	0.0985	0.0841	0.0492	0.025	0.0125	0.98
15.0	0.0804	0.0687	0.0421	0.0204	0.0102	0.9
25.0	0.0623	0.0532	0.0311	0.0158	0.0079	0.82

TABLE IV $k_r/Eb = 0.3$

ℓ/h \ β	0.25	1.0	4.0	16.0	64.0	
	ϕ_{opt}					$\frac{\phi_{opt.e}}{\phi_{opt.r}}$
6.25	0.2158	0.01842	0.1079	0.547	0.0274	0.88
7.5	0.197	0.1682	0.0985	0.05	0.25	0.85
10.0	0.1706	0.1456	0.0853	0.0433	0.216	0.78
15.0	0.9393	0.1189	0.0696	0.03353	0.177	0.68
25.0	0.1079	0.0921	0.0539	0.0274	0.0137	0.55

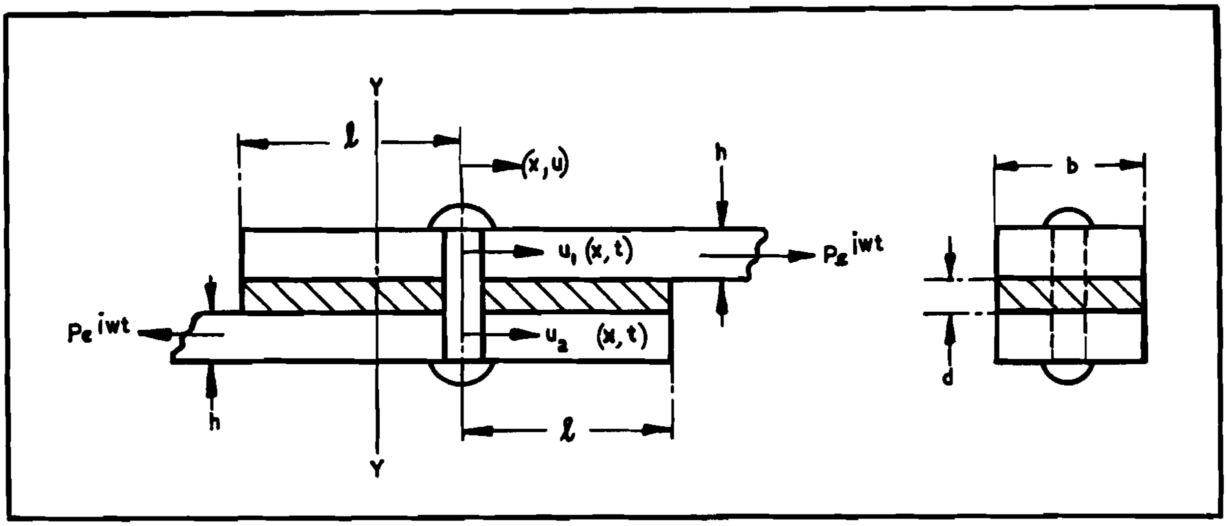


FIG.1 LAP JOINT CONFIGURATION AND CO-ORDINATE SYSTEM.

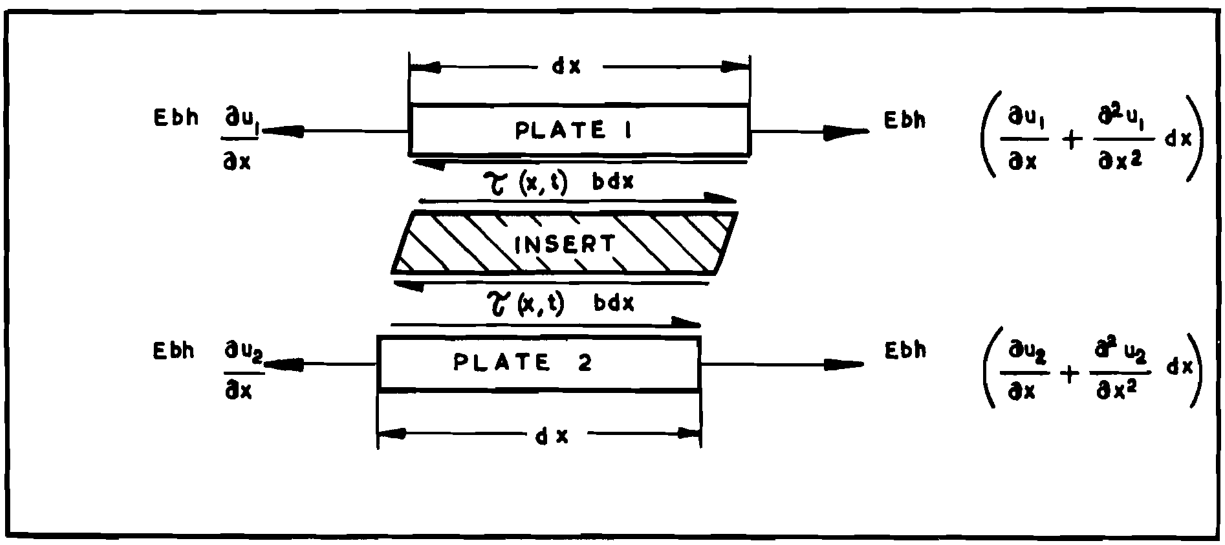


FIG.2 LOADING ACTIONS ON THE SECTION OF AN ELEMENTAL LENGTH OF THE JOINT.

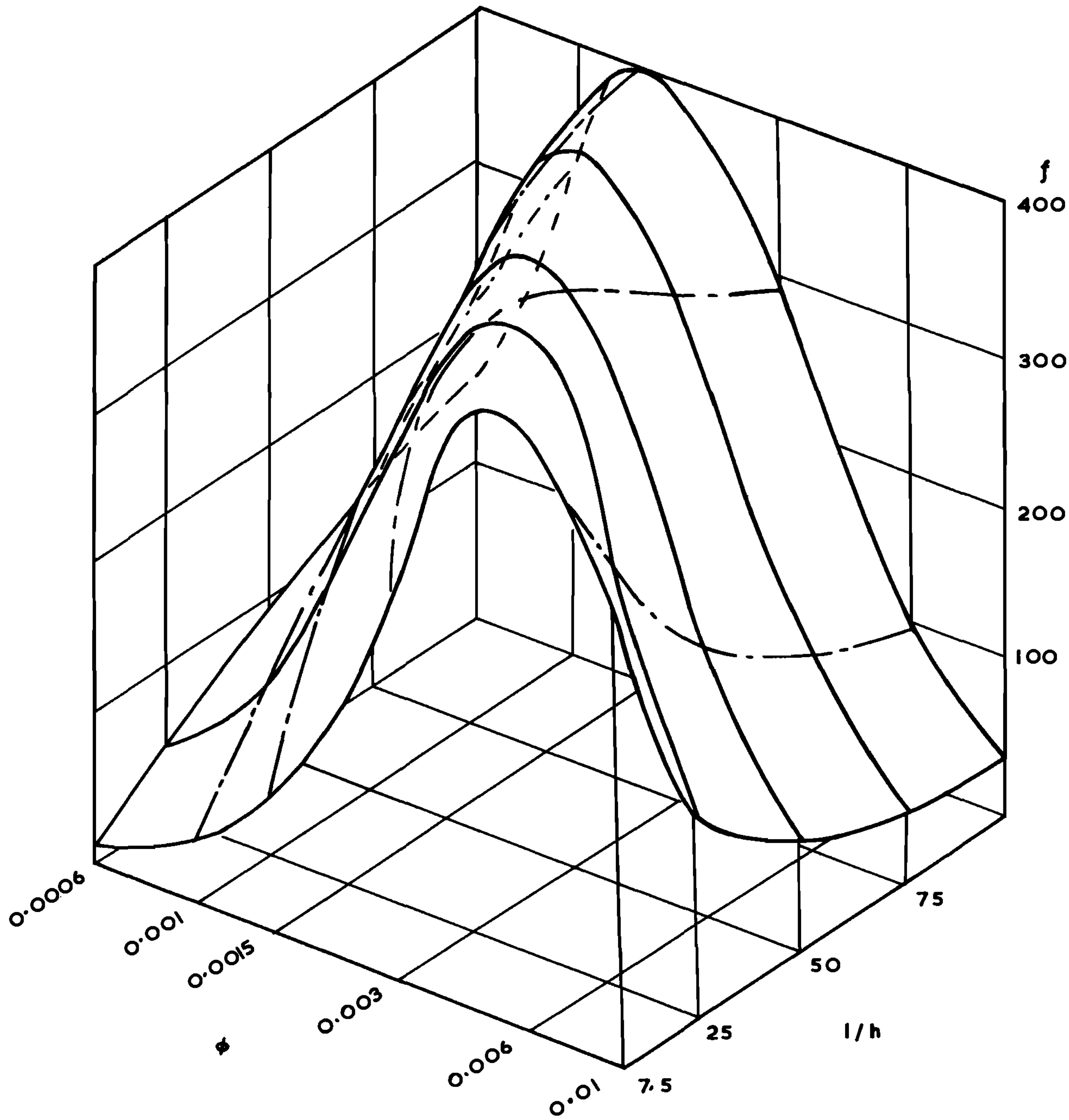


FIG. 3 ENERGY DISSIPATION FUNCTION, f, AS A FUNCTION OF THE SHEAR AND OVERLAP PARAMETERS
RIVET STIFFNESS PARAMETER, Kr/Eb = 0·0003
MATERIAL LOSS FACTOR, $\beta = 0{\cdot}5$

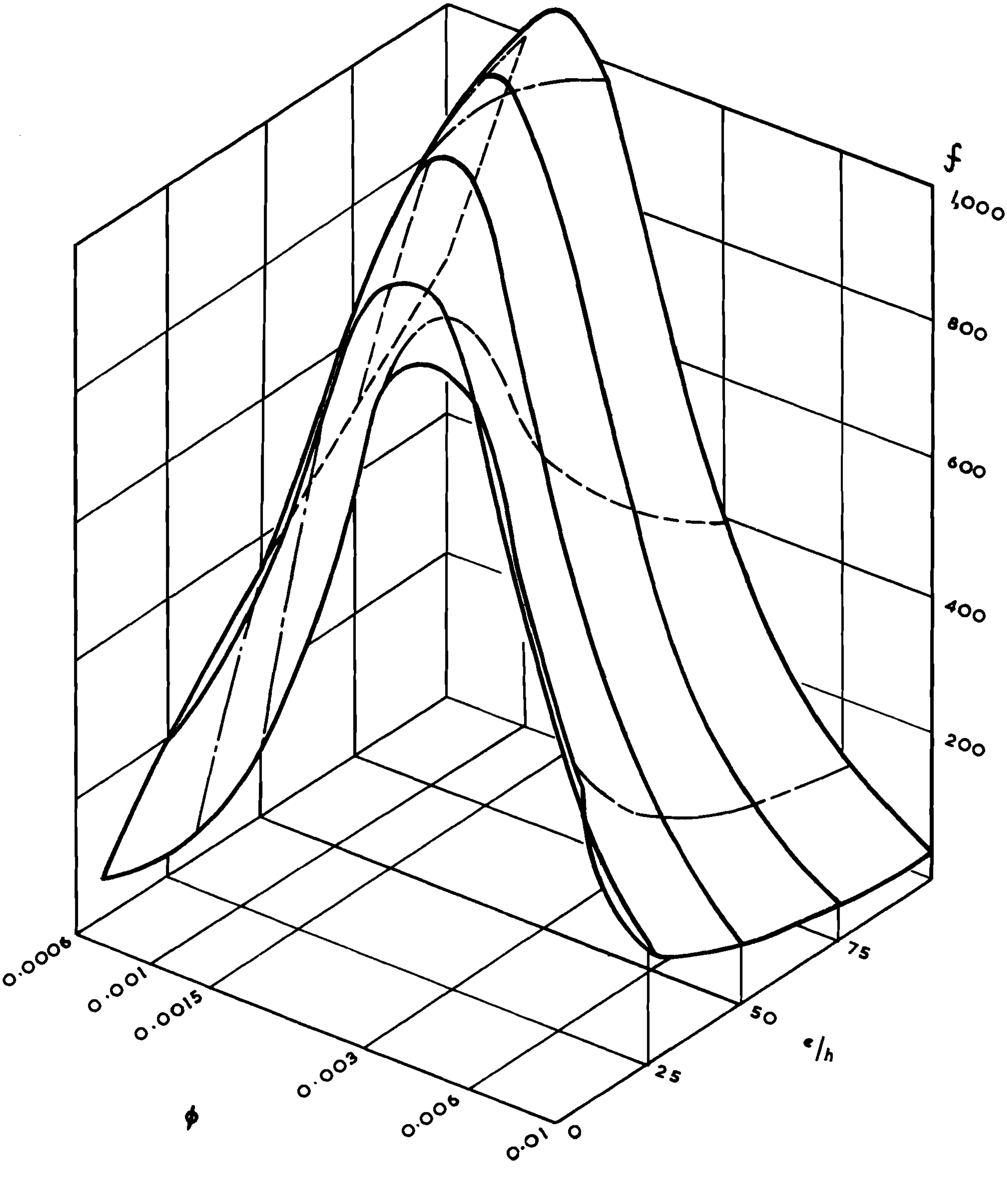


FIG. 4. ENERGY DISSIPATION FUNCTION, f, AS A FUNCTION OF THE SHEAR AND OVERLAP PARAMETERS. RIVET STIFFNESS PARAMETER, $K\gamma/Eb = 0{\cdot}0003$. MATERIAL LOSS FACTOR, $\beta = 2 \cdot 0$

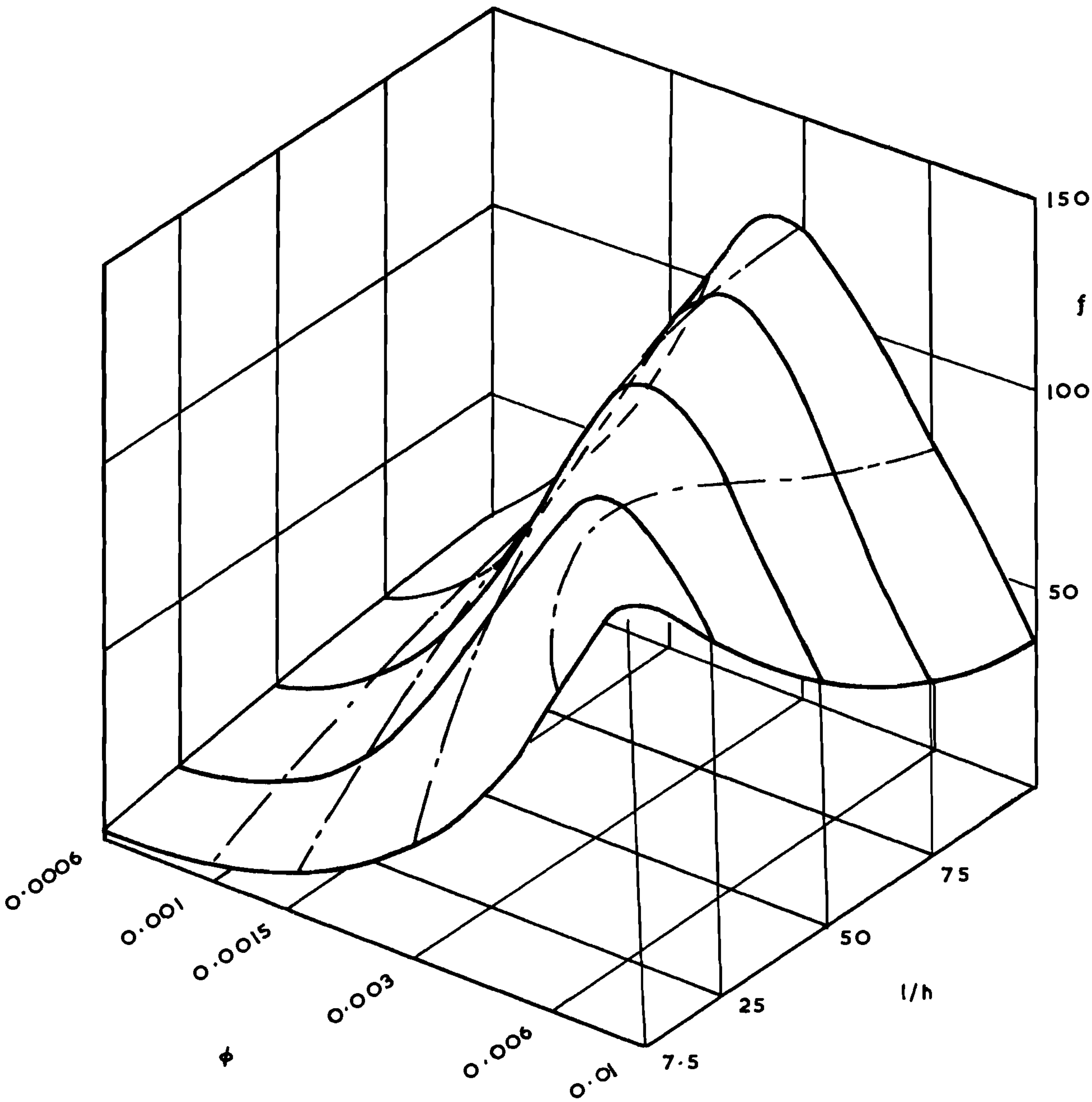


FIG. 5 ENERGY DISSIPATION FUNCTION, f, AS A FUNCTION OF THE SHEAR AND OVERLAP PARAMETERS
RIVET STIFFNESS PARAMETER, $K_p/Eb = 0{\cdot}001$
MATERIAL LOSS FACTOR, $\beta = 0{\cdot}5$

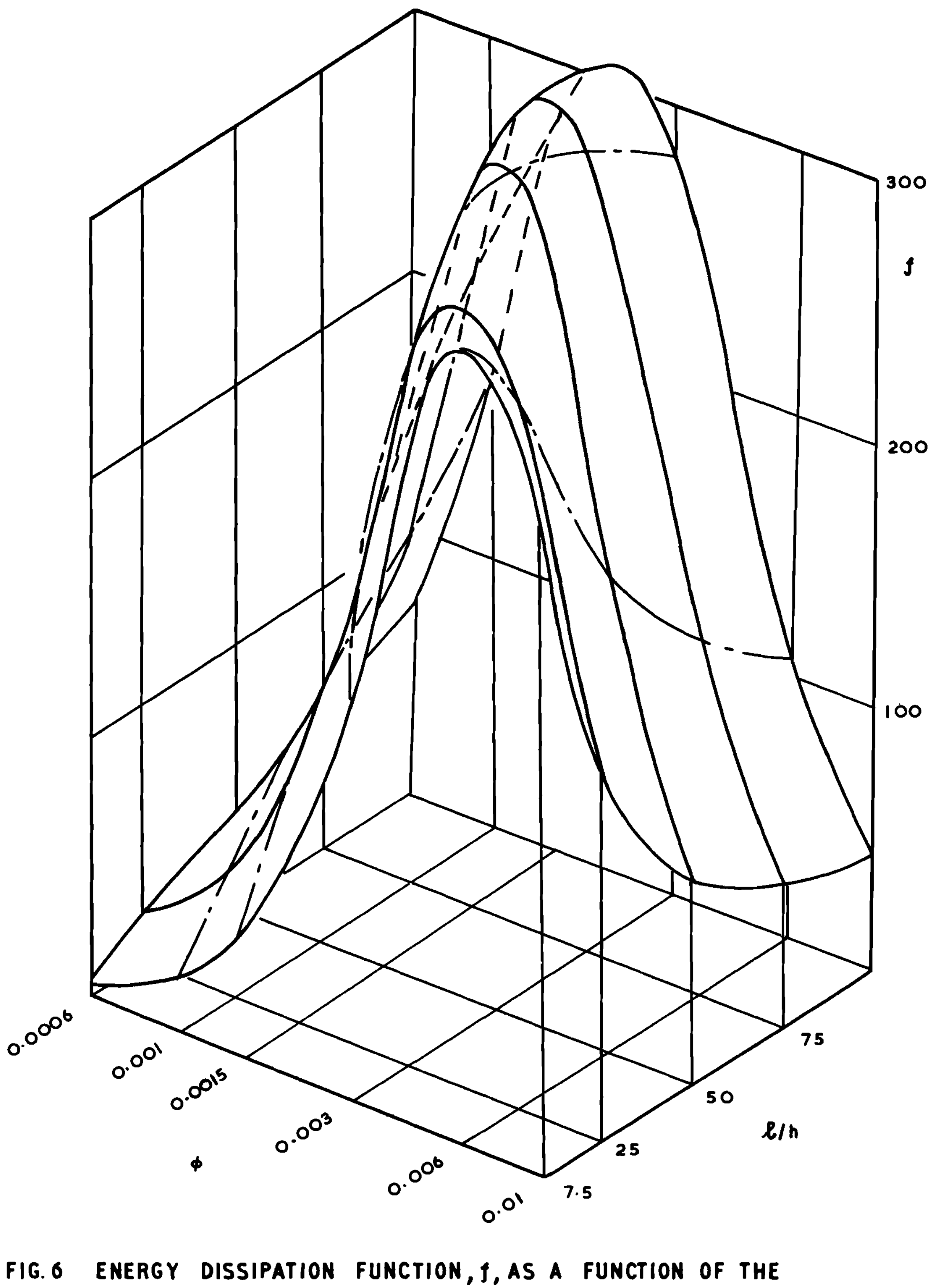


FIG. 6 ENERGY DISSIPATION FUNCTION, f, AS A FUNCTION OF THE SHEAR AND OVERLAP PARAMETERS
RIVET STIFFNESS PARAMETER $K_p/Eb = 0.001$
MATERIAL LOSS FACTOR, β 2.0

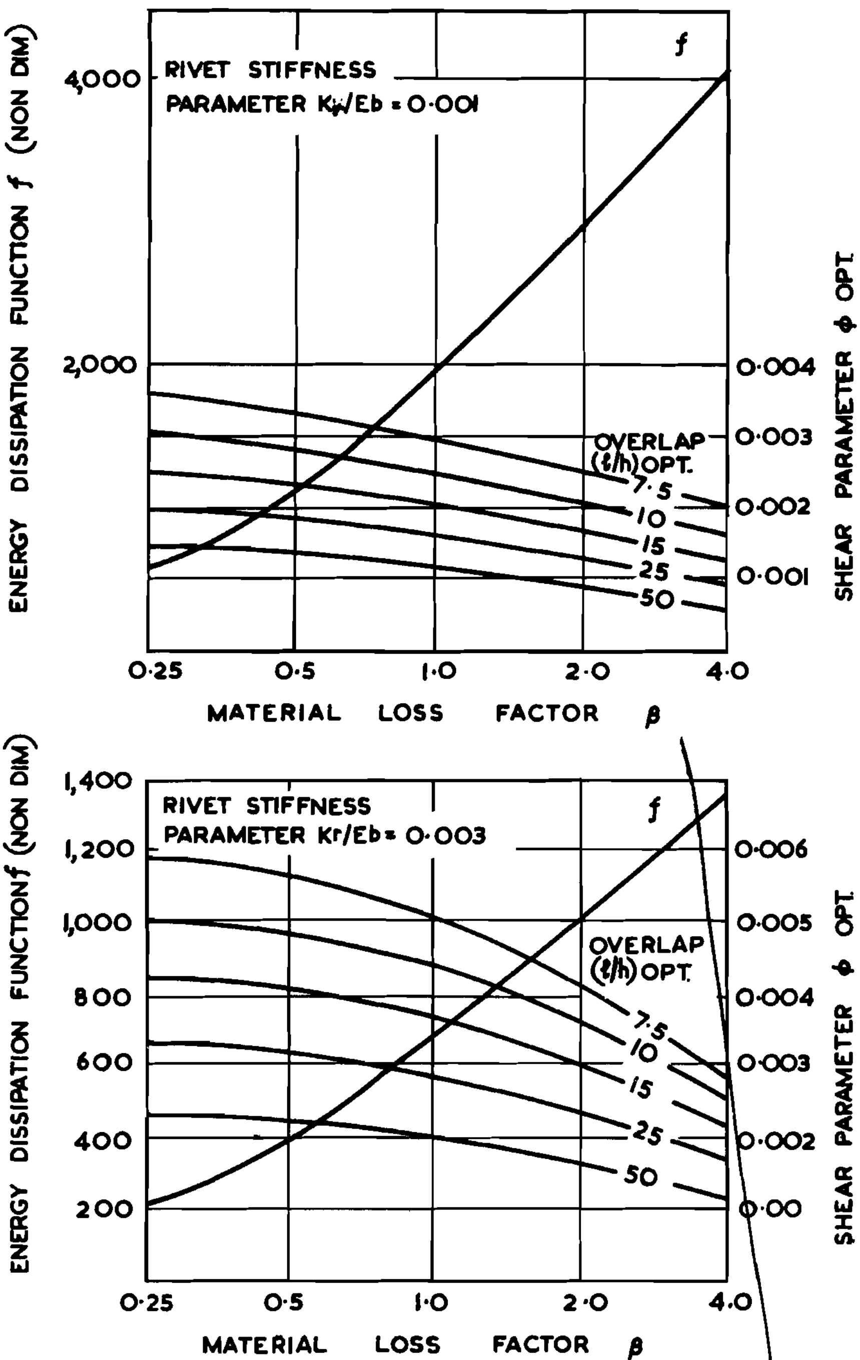


FIG. 7. THE VARIATION (WITH MATERIAL LOSS FACTOR) OF THE ENERGY DISSIPATION FUNCTION, f, CORRESPONDING TO THE OPTIMUM VALUES OF THE SHEAR AND OVERLAP PARAMETERS.

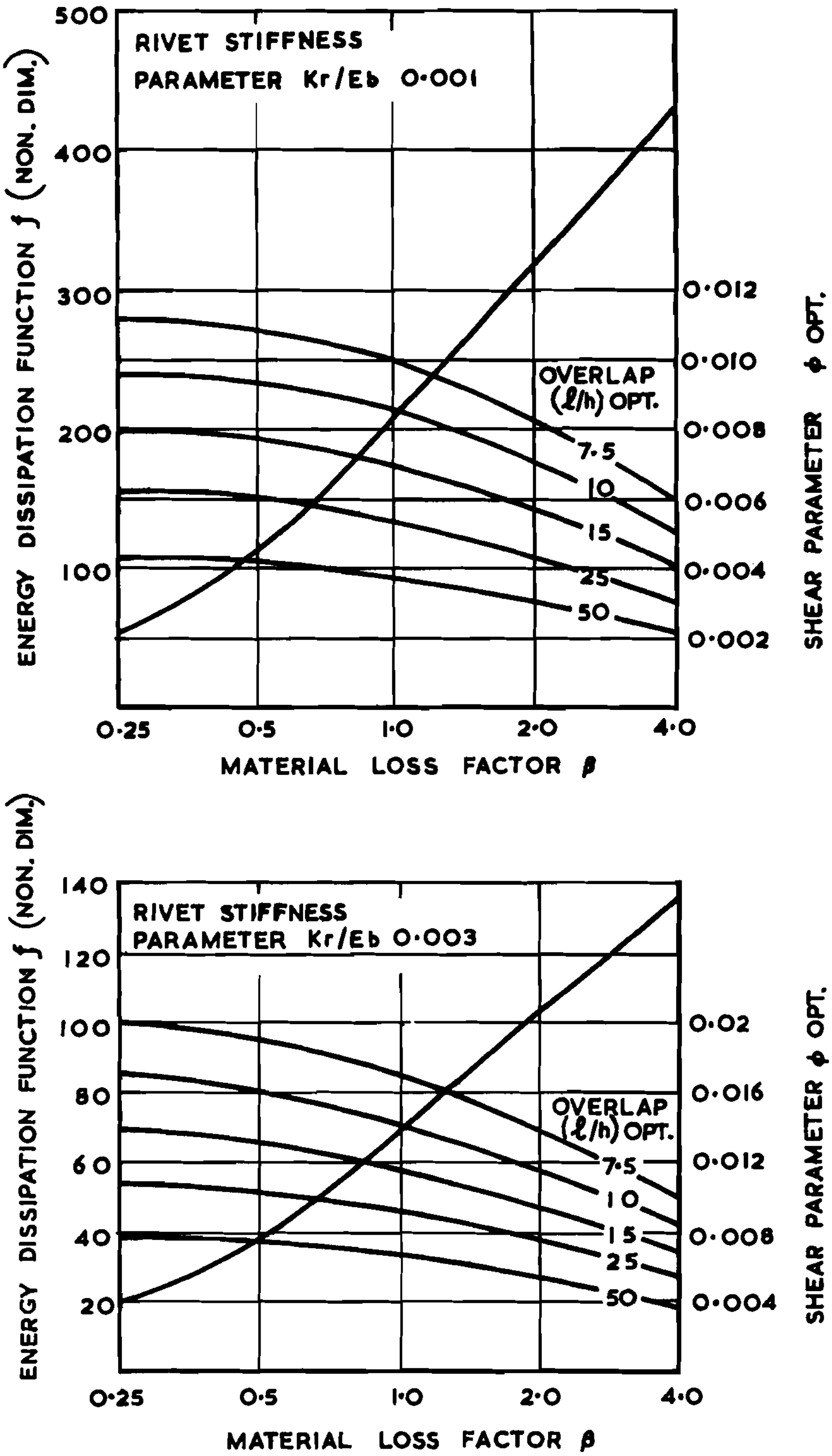


FIG.8. THE VARIATION (WITH MATERIAL LOSS FACTOR) OF THE ENERGY DISSIPATION FUNCTION, f, CORRESPONDING TO THE OPTIMUM VALUES OF THE SHEAR AND OVERLAP PARAMETERS

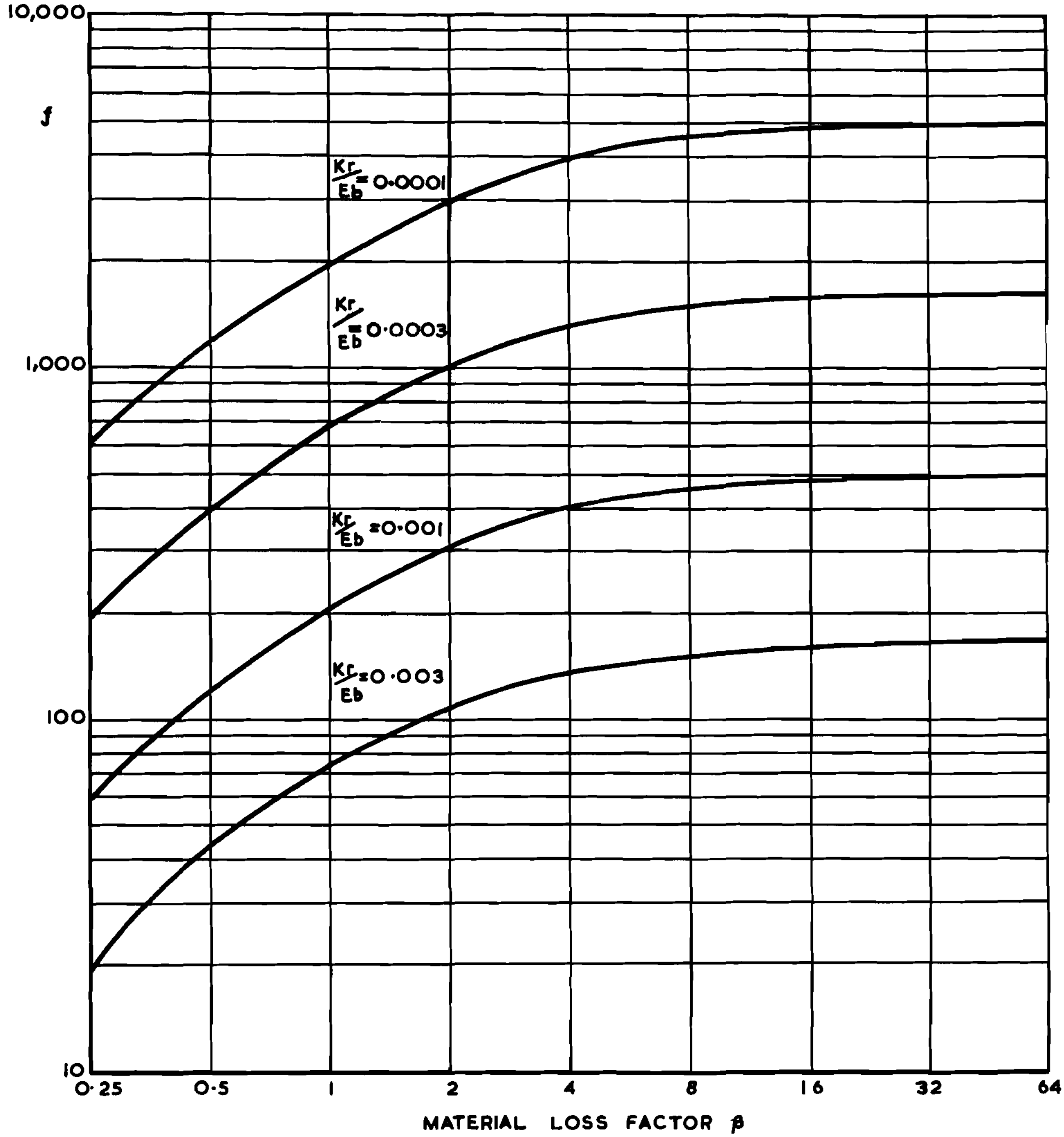


FIG.9 VARIATION (WITH MATERIAL LOSS FACTOR) OF THE MAXIMIZED ENERGY DISSIPATION FUNCTION, f, FOR VALUES OF THE RIVET STIFFNESS PARAMETER, K_r/Eb, 0·0001 TO 0·003

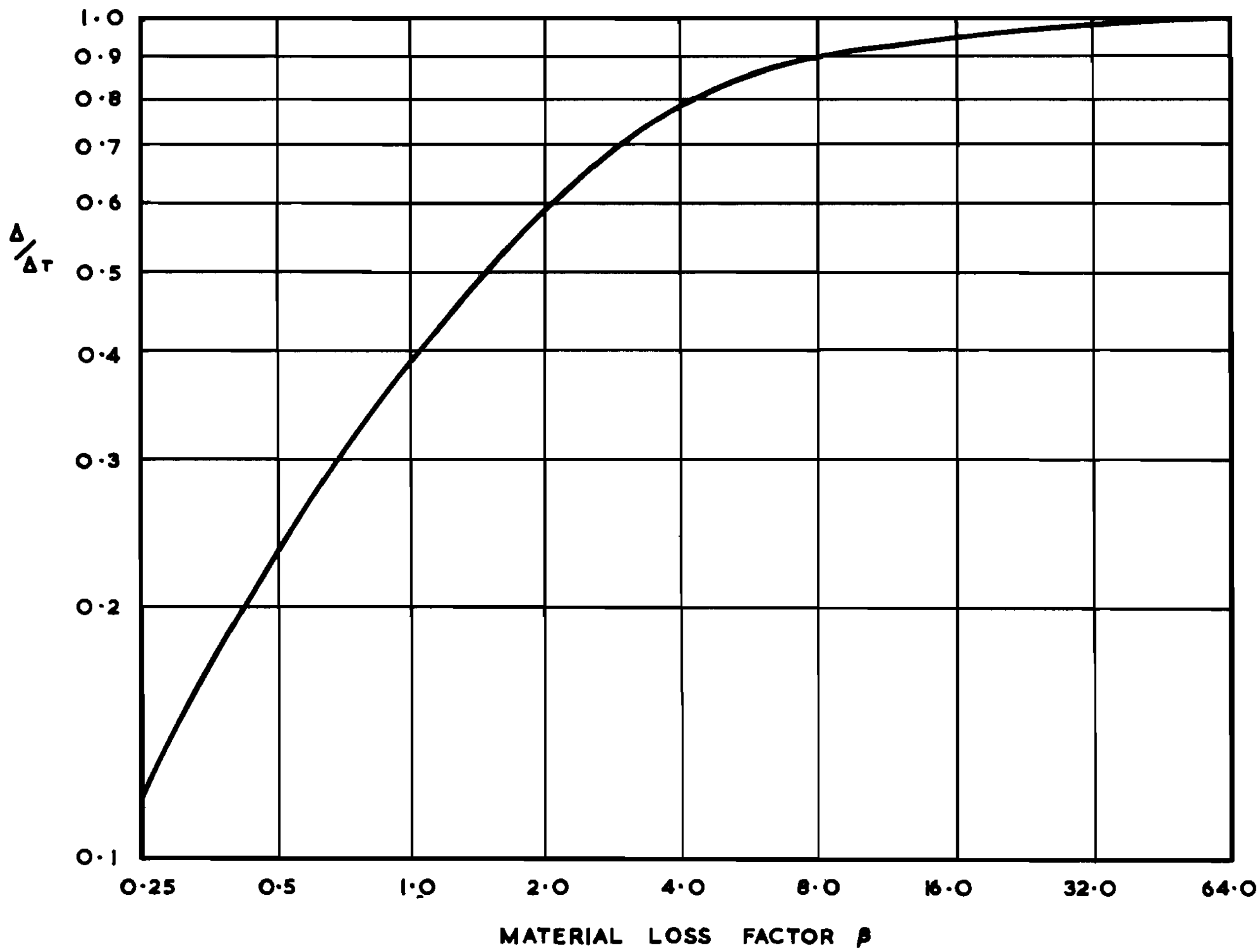


FIG. 10 VARIATION (WITH MATERIAL LOSS FACTOR) OF: THE MAXIMIZED ENERGY DISSIPATION (ELASTIC PLATES) ÷ THE ABSOLUTE MAXIMUM ENERGY DISSIPATION (RIGID PLATES) FOR $K_r / Eb \leq 0{\cdot}003$

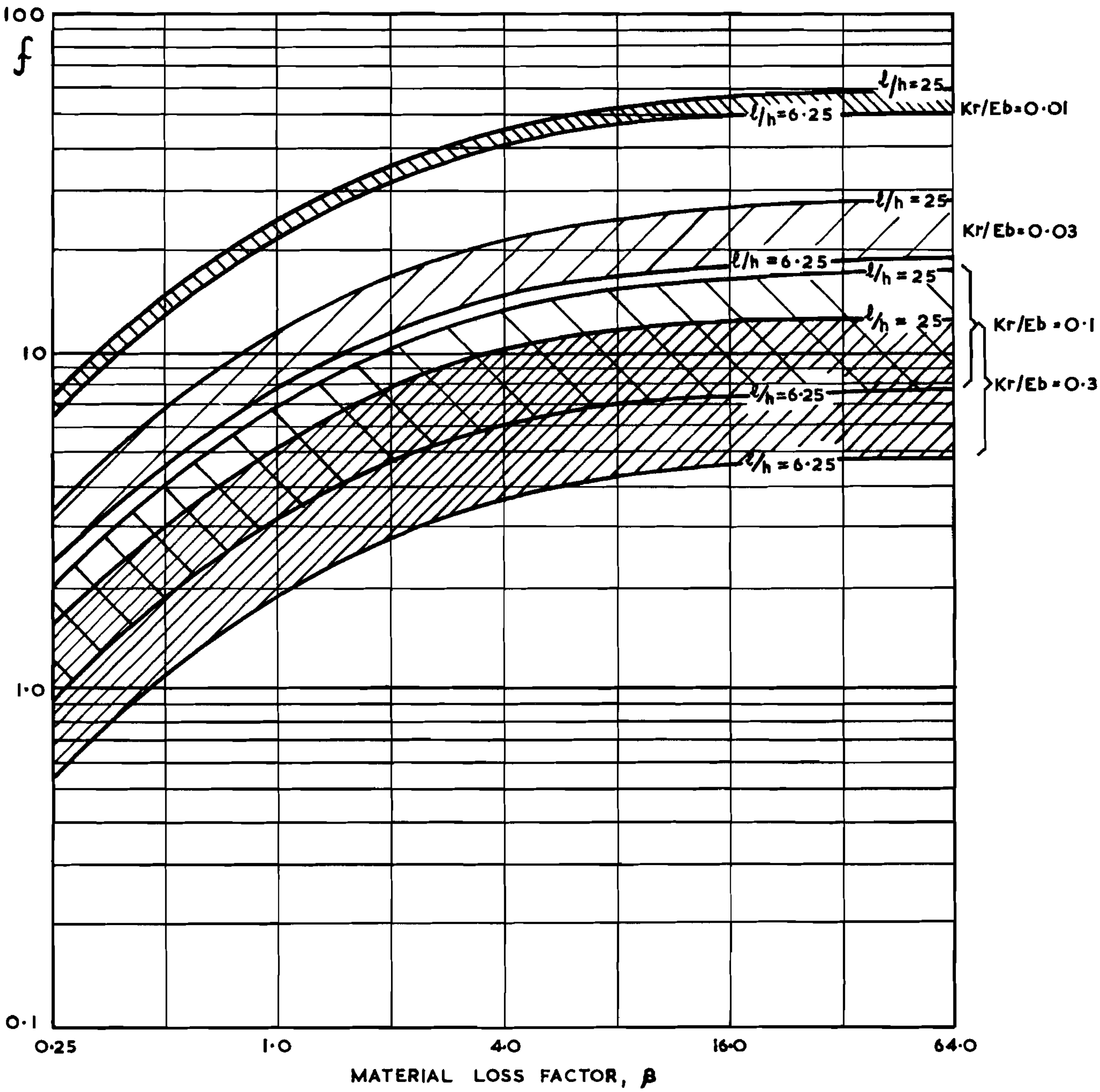


FIG. 11. VARIATION (WITH MATERIAL LOSS FACTOR) OF THE MAXIMIZED ENERGY DISSIPATION FUNCTION, f, FOR VALUES OF THE RIVET STIFFNESS PARAMETER, Kr/Eb, 0·01 TO 0·03

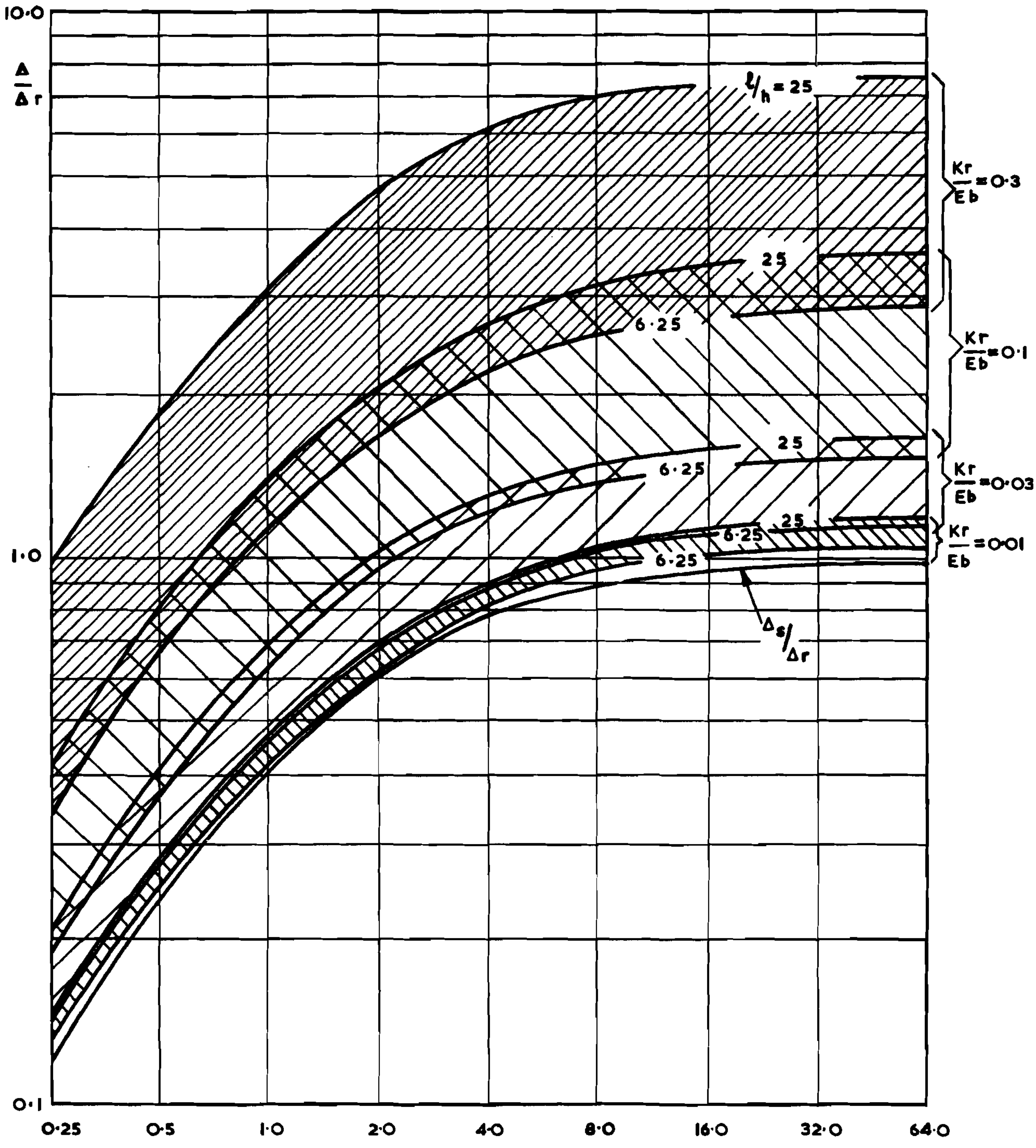


FIG. 12. THE VARIATION (WITH MATERIAL LOSS FACTOR) OF: THE MAXIMIZED ENERGY DISSIPATION (ELASTIC PLATES) ÷ THE ABSOLUTE MAXIMUM ENERGY DISSIPATION (RIGID PLATES) AND: THE MAXIMIZED ENERGY DISSIPATION (RIGID PLATES) ÷ THE ABSOLUTE MAXIMUM ENERGY DISSIPATION (RIGID PLATES)