Optimization of Dynamic Vibration Absorber –Case of Cantilever Boring Bar

E. I. Rivin, H. L. Kang
Department of Mechanical Engineering
Wayne State University
Detroit, MI 48202

Abstract

Passive dynamic vibration absorbers (DVAs) are very popular for vibration control/enhancement of effective damping in various structures. This paper describes techniques which allow one to substantially enhance the effectiveness of DVAs, specifically for long overhang cantilever structures (on the example of cantilever boring bar). A so-called combination structure is designed, in which the root segment is made of a high stiffness material, while the overhang segment is made of a light material. Optimization of such a structure results in a stiff but light system with greatly increased dynamic stiffness $K\xi$ Optimal parameters of a DVA for main mass under self-excited vibration and random excitations are discussed. Test results are given for an optimized combination boring bar and DVA parameters with length-to-diameter ratio L/D=15.

1. Introduction

Passive dynamic vibration absorbers (DVAs) are very popular for vibration control/enhancement of effective damping in various structures. In boring bars, like in many other cantilever structures, vibrations are easily developed due to their weakness in both structural stiffness and damping, and thus DVAs are often used. But, due to the intrinsic limitations of the space available for installation of inertia mass, the mass ratio is limited and the DVAs often have curtailed efficiency. Thus, boring bars with length-to-diameter ratios exceeding L/D=9–10 were generally considered not feasible. This paper will describe several techniques which allow one to enhance effectiveness of a DVA for long overhang cantilever structures, specifically for cantilever boring bars.

A so-called combination cantilever bar was designed [1], in which the root segment is made of a high stiffness material, possibly having high specific density, while the overhang segment is made of a light material, possibly having a low Young modulus. Optimization of such a structure results in a rigid but light system, usually with greatly increased natural frequency and mass ratio of DVA, and with reduced usage of expensive materials.

Self-excited vibrations and random excitations are often encountered in structural vibrations, especially in the cutting processes, and they are the main factors of system instability. Classical (DenHartog [2]) optimization for DVAs' parameters is based on a case of harmonic external excitation applied to the main mass. Frequently, however, optimal parameters for this case are considered as a universal approach for DVAs' design. It will be shown that for other practical cases, such as random excitation of the main mass and the case of self-excited vibrations, optimal parameters of a DVA are quite different.

Test results will be given which show that a boring bar designed as an optimized combination structure and furnished with a properly optimized DVA demonstrates substantially lower vibration amplitudes during cutting and can operate with L/D=15. The proposed concept are applicable for a wide range of cantilever structures.

2. DVA attached to the main mass which is under self-excitation conditions

In the self-excited vibrations case the alternating force that sustains the motion is created or controlled by the motion itself; when the motion stops the alternating force disappears. The general expression of the dynamic cutting force can be written as [3]:

$$F_x = -K_{cx}X_1 - C_{cx}\frac{dX_1}{dt} \tag{1}$$

where K_{cx} is stiffness coefficient and C_{cx} is damping coefficient, and K_{cx}, C_{cx} can be defined as "effective cutting stiffness" and "effective cutting damping", respectively. At some combinations of parameters, force (1) can lead to self-excitation of vibrations.

Dynamic vibration absorber can be modeled as a absorber mass M_2 attached by a spring with stiffness K_2 and a damper C_2 to the main system whose mass M_1 is subjected to the excitation force F(t). A model of the main mass with a damped vibration absorber is given in Fig.1 where K_1 and C_1 are stiffness and damping of the main mass.

Equations of motion of this system can be written as:

$$\ddot{X}_1 + 2(\xi_1\omega_1 + \xi_2\omega_2\mu)\dot{X}_1 + (\omega_1^2 + \omega_2^2\mu)X_1 - 2\xi_2\omega_2\mu\dot{X}_2 - \omega_2^2\mu X_2 = \frac{F(t)}{M_1} \tag{2}$$

$$\ddot{X}_2 + 2\xi_2\omega_2\dot{X}_2 + \omega_2^2X_2 - 2\xi_2\omega_2\dot{X}_1 - \omega_2^2X_1 = 0 \tag{3}$$

with

$$\frac{K_1}{M_1} = \omega_1^2; \quad \frac{K_2}{M_2} = \omega_2^2; \quad \frac{M_2}{M_1} = \mu; \quad \frac{C_1}{\sqrt{K_1M_1}} = 2\xi_1; \quad \frac{C_2}{\sqrt{K_2M_2}} = 2\xi_2$$

where X_1, X_2 are vibration amplitudes of the main mass and absorber, ω_1, ω_2 are partial natural frequencies of the main mass and absorber subsystem, μ is the mass ratio of absorber mass to main mass, and ξ_1, ξ_2 are damping ratios of the main mass and absorber subsystems, respectively.

By letting $F(t)=F_x$ for the exciting force in the equation (1), the system characteristic equation from the equations of motion becomes:

$$S^4+B_3S^3+B_2S^2+B_1S+B_0=0 \tag{4}$$

where

$$B_3=2(\xi_e\omega+\xi_2\omega_2(1+\mu))$$
$$B_2=\omega^2+\omega_2^2(1+\mu)+4\xi_e\xi_2\omega\omega_2$$
$$B_1=2(\xi_e\omega\omega_2^2+\xi_2\omega_2\omega^2)$$
$$B_0=\omega^2\omega_2^2$$
$$\frac{K_1+K_{cx}}{M_1}=\omega^2; \qquad \frac{C_1+C_{cx}}{M_1}=2\xi_e\omega$$

Here ω is the self-excited vibration frequency which is close to but different from the natural frequency of the main mass subsystem (due to addition of K_{cx}) and ξ_e is damping ratio of the main mass subsystem during cutting. Parameter ξ_e combines the damping of the main mass (always positive) and effective damping from the expression for the dynamic cutting force (1). The latter can be positive, thus assuring an unconditional dynamic stability of the system, or negative, which then should be compensated by the positive damping of the main mass and by the stabilizing effects of the absorber in order to achieve stable conditions. Thus, effectiveness of the absorber can be judged by the critical value of ξ_e, which corresponds to the stability boundary of the system. And the maximum effectiveness of the absorber can be characterized by the maximum magnitude of negative critical value of ξ_e, which the absorber can still compensate.

Routh stability criterion states that for a system to be stable, all the coefficients of the characteristic equation must be positive and also must satisfy inequalities [2]:

$$B_1\ B_2\ B_3 > B_1^2+B_3^2\ B_0 \tag{5}$$

From the former requirements, we arrive to conditions:

$$\xi_e > -r\xi_2(1+\mu) \tag{6}$$
$$\xi_e > -\left(\frac{1}{4\xi_2 r}+\frac{r(1+\mu)}{4\xi_2}\right) \tag{7}$$
$$\xi_e > -\frac{\xi_2}{r} \tag{8}$$

where

$$r=\frac{\omega_2}{\omega}$$

is frequency ratio of the partial frequency of the absorber subsystem to the self-excited vibration frequency of the main mass.

The critical value of ξ_e can be obtained by replacing (6)-(8) with equalities, from which the largest value can be determined and then checked with equation (5). If the latter is not satisfied, the critical value of ξ_e can be determined by iterations to satisfy (5).

Fig.2 gives the critical value of ξ_e at various mass and frequency ratios of the absorber. It can be seen that at a given mass ratio and damping ratio of absorber, there exists a optimal (tuning) frequency ratio ω_2/ω_1 at which critical negative value of ξ_e has maximum magnitude (maximum effectiveness of the absorber).

The influence of absorber damping on system stability under optimal frequency ratio conditions can be seen in Fig.3. If the absorber damping is too low, it will result in a poor system stability because of small effectiveness of absorber. If the absorber damping is too high, it also gives poor system stability because the absorber mass is in fact locked together with main mass and low effectiveness will result. There is an optimal damping of the absorber which gives the maximum negative value of ξ_e and results in the main mass remaining stable at higher magnitudes of negative damping induced by the cutting process for a given mass ratio.

Fig.4 gives the influence of mass ratios on system stability under both optimal frequency and optimal absorber damping ratio. It is obvious that the higher mass ratio, the better system stability.

The optimal frequency ratio at the optimal absorber damping condition will be called the global optimal frequency ratio. Since the absorber damping in practical designs may not be optimal, the optimal frequency ratio at this situation can be called a locally optimal, which means that if a value of absorber damping which is not optimum is used, the local optimal frequency ratio has to be chosen for absorber to be the most effective at this damping. Optimal absorber damping and global optimal frequency ratio for a given mass ratio are shown in Fig.5 and Fig.6 together with the results for random (see below) and sinusoidal [2] excitations.

3. DVA attached to the main mass which is under random excitation conditions

Here optimal tuning parameters will be discussed for a case of random excitation having white noise characteristics with a constant spectral density function S_0 and zero memory. In real circumstances random signal is rarely constant over the frequency range $0-\infty$, but it is frequently constant over a wide frequency band. Thus white noise excitation is used in the analysis as an approximation of typical random signals. The frequency response method is used to get the mean square response of the system [4].

In order to get the frequency response functions $H_1(\omega)$ and $H_2(\omega)$, let:

$$\frac{F(t)}{M_1} = e^{i\omega t} \qquad (9)$$

$$X_1 = H_1(\omega) e^{i\omega t} \qquad (10)$$

$$X_2 = H_2(\omega) e^{i\omega t} \qquad (11)$$

Then substituting above expressions and their derivatives into equations of motion (2) and (3), the frequency response functions can be written as follows:

$$H_1(\omega)=\frac{\omega_2^2+2\xi_2\omega_2(i\omega)+(i\omega)^2}{A_0+A_1(i\omega)+A_2(i\omega)^2+A_3(i\omega)^3+(i\omega)^4} \tag{12}$$

where

$A_3=2(\xi_1\omega_1+\xi_2\omega_2(1+\mu))$

$A_2=\omega_1^2+\omega_2^2(1+\mu)+4\xi_1\xi_2\omega_1\omega_2$

$A_1=2(\xi_1\omega_1\omega_2^2+\xi_2\omega_2\omega_1^2)$

$A_0=\omega_1^2\omega_2^2$

and

$$H_2(\omega)=\frac{\omega_2^2+2\xi_2\omega_2(i\omega)}{(i\omega)^2+2\xi_2\omega_2(i\omega)+\omega_2^2}H_1(\omega) \tag{13}$$

The mean square response of the mass M_1 under the white noise excitation is then given as:

$$E(X_1^2)=\int_{-\infty}^{\infty}|H_1(\omega)|^2S_0d\omega \tag{14}$$

which represent the total energy of the main system after attachment of a dynamic vibration absorber. For the main system without dynamic vibration absorber, the frequency response function is:

$$H_{10}(\omega)=\frac{1}{(i\omega)^2+2\xi_1\omega_1(i\omega)+\omega_1^2} \tag{15}$$

The mean square response can also be given by equation (14) in which $H_{10}(\omega)$ is used. A non-dimensional normalized mean square response of the main subsystem, i.e. the ratio of mean square response of M_1 with absorber to mean square response of M_1 without absorber, which reflects effect of the absorber on the main mass, is then defined as:

$$\frac{E(X_1^2)}{E(X_1^2)_0}=2\xi_1\omega_1^3\frac{A}{B} \tag{16}$$

where

$A=-A_0A_1-A_0A_3(4\xi_2^2\omega_2^2-2\omega_2^2)+\omega_2^4(A_1-A_2A_3)$

$B=A_0(A_0A_3^2+A_1^2-A_1A_2A_3)$

The calculation results of the normalized mean square response of the main mass are given in Fig.7 for a main system damping ratio ξ_1 =0.02. Since the mean square response represents the total energy of the system over the entire frequency range, the normalized mean square response gives the total energy ratio of the response of the main mass M_1 but not the response itself. The larger the value of the normalized mean square response, the larger is response of the main mass and the lesser effect of the absorber on the main system behavior. Influence of mass ratio on the local optimal frequency ratio is similar to the case of self-excited vibration, but the different values of the optimal frequency ratios. Computed optimal global frequency ratio and optimal absorber damping ratio are plotted in Fig.5 and Fig.6.

A mass with dynamic vibration absorber under sinusoidal excitation has been analyzed by J. P. DenHartog [2] where he considered the case with zero main system damping which is a good approximation for the system: cantilever bar with damped vibration absorber.

Comparing optimal tuning and damping conditions under various excitations in Fig.5 and Fig.6, it can be seen that at the same mass ratios, the required optimal absorber damping values are the lowest for the case of white noise excitation, and the highest for the case of sinusoidal excitation. The required optimal frequency ratios are the lowest for the case of sinusoidal excitation and the highest for the case of self-excited vibrations. For the case of cantilever boring bar, since both self-excitation and random excitation exist during cutting process [5], the optimal tuning values for the frequency ratio and damping could be chosen in between of the optimal values shown in Figs.5,6 for cases of self-excitation and random white noise excitation.

4. Optimization of combination cantilever bar

A combination bar of length L with sintered carbide in the root segment (length L_1) and aluminum in the free end segment (solid part of length L_2 and hollow part of length L_3), shown in Fig.8, was analyzed. The absorber made of heavy machinable tungsten alloy is located in the hollow part of the free end as shown in Fig.9. An optimization procedure was applied to choose parameters L_1, L_2, and L_3 in order to have the highest dynamic stiffness $K\xi$ of the cantilever bar, where K is static stiffness and ξ is effective damping ratio of the main mass subsystem since both damping and stiffness are important for the system stability.

The Rayleigh expression for fundamental natural frequency of the system without absorber was used [1],

$$\omega_1^2 = \frac{\int_0^L E(z)I(z)\left(\frac{d^2x}{dz^2}\right)^2 dz}{\int_0^L m(z)x^2 dz + M_t x_t^2} \tag{17}$$

where E(z) is Young's modulus, I(z) is moment of inertia of the cross section, m(z) is mass per unit length, and all these parameters are considered as function of z (coordinate along the axis of the cantilever bar). M_t is mass at the free end, x is vibration amplitude of cantilever bar (a function of z), and x_l is vibration amplitude at the free end of cantilever bar.

The effective mass at the length $L_0=L-L_3/2$ which is the midpoint of the absorber cavity L_3 is [1]:

$$M_1 = \frac{\int_0^l E(z)I(z)\left(\frac{d^2x}{dz^2}\right)^2 dz}{x_0^2\omega_1^2} \tag{18}$$

The effective stiffness at L_0 then is:

$$K_1 = \omega_1^2 M_1 \tag{19}$$

To determine the effective stiffness at the tool end, the effective mass at the length L should be determined. The approximate fundamental mode shape of the cantilever bar to be used in the Rayleigh formula (17) is assumed to be:

$$x(z) = 1 - \cos\left(\frac{\pi z}{2L}\right) \tag{20}$$

which satisfies the boundary conditions for this mode.

A combination cantilever bar and a steel bar were analyzed and compared. The parameters used for combination bar are: outside diameter D=1.25 in (31.75 mm), for carbide segment: E=80,000,000lb/in (55 N/cm), specific gravity ρ_1=0.516 lb/in (0.01428 Kg/cm), for aluminum segment: inside diameter d_3=1.0 in (25.4 mm), E=10,000,000lb/in (7 N/cm), specific gravity ρ_2=0.09384 lb/in (0.0028 Kg/cm); for steel bar: E=28,600,000lb/in (19 N/cm), specific gravity ρ=0.28 lb/in (0.008 Kg/cm). Mass at the free end is M_t=0.0004857$lb-sec^2$/in (0.085 N-sec^2/m), material for absorber mass is machinable tungsten with specific gravity ρ_a=0.6497 lb/in (0.01798 Kg/cm)and lengths L_3=4,5,6 in (0.1,0.13,0.15 m) were chosen for the cantilever bar with the overall length L=18 in (0.457 m).

By calculating stiffness values at tool end K_t and critical value of ξ_r of the combination boring bar with damped vibration absorber under optimal tuning and damping conditions, the performance index $K\xi_r$ vs L_1/L ratios can be obtained as shown in Fig.10. Since both higher stiffness K_t and more negative critical value of ξ_r give better cutting process stability, higher magnitudes of absolute value of the performance index (dynamic stiffness) are corresponding to a better stability of the system. It can be seen that L_1/L in the range 0.45–0.6 corresponds to the best stability of the boring bar. It has been shown [6] that for L_1/L=0.45, it corresponds to the highest natural frequency of the combination bar, and for L_1/L=0.6, the combination bar with damped vibration absorber has the minimum vibration amplitude under harmonic excitation.

The results of natural frequency f, stiffness at free end K_t, and mass ratio of absorber μ for the combination cantilever bar at L_1/L=0.45 and for the steel bar with the same dimension are given in Table-1. It can be seen that values of all this three parameters for the combination bar are about double of the values for the steel bar. Such increase in natural frequency and stiffness should improve dynamic performance of the cantilever structure, and the increase in absorber mass ratio makes the absorber more effective for a given limited absorber mass.

5. Cutting test using combination boring bar

A combination boring bar of 1.25 in (31.75 mm) diameter and 18.75 in (476 mm) long (L/D=15) was designed with parameters L_3=6 in (152 mm) and L_1/L=0.6. Rubber resilient elements were used which provide both necessary compliance and damping for the absorber. Absorber frequency tuning can be done by adjusting the screw which results in preloading of the rubber elements. The absorber mass ratio is μ=1.07. The damping ratio of the boring bar is ξ_1=0.02 and damping ratios of absorber ξ_2 are 0.07, 0.18, and 0.45 for three rubberlike materials used. Natural frequency of the boring bar is f_1=173 Hz. The recommended tuning frequencies for absorber are about 84 Hz in case of sinusoidal excitation, 105 Hz in case of random excitation, and 117 Hz in case of self-excited vibrations.

Vibration displacements of the boring bar in horizontal (x) directions were measured by LVDT at the distance 13.5 in (342 mm) from the clamp (since the measurement of the tool end vibration is impossible during the cutting). Deformations both at LVDT and at the tool end under static load were measured and it was shown that displacement at the tool end is about 1.91 times of displacement at the LVDT position. This factor was used as an approximation to get vibration displacements at the tool end from measured values from LVDT.

Table-2 gives maximum vibration peak-to-valley (p-v) values at the tool end for the boring bar without absorber and for the boring bar with damped vibration absorber having various absorber damping and tuning adjustments. The results show substantial improvements of cutting conditions while using boring bar with dynamic vibration absorber as compared with the original boring bar. The results show that if tuning at the local optimal frequency ratio at a given damping of rubber was realized according to the self-excitation and random white noise excitation case, smaller vibrations were observed (10-30% lower p-v values), as compared with cases of tuning as recommended for sinusoidal excitation [2]. It was also observed that when damping of the absorber is closer to the optimal damping values, the vibration amplitudes are smaller.

6. Conclusions

1. A combination cantilever bar with high rigidity material in the root segment and light weight material in the free end segment has much higher natural frequency, stiffness, and mass ratio of DVA compared with steel bar, which results in a higher dynamic stiffness (better dynamic performance) and higher effectiveness of dynamic vibration absorber.

2. The optimal tuning conditions and damping values of DVA are different for cases when the main mass is under self-excitation and under random excitation , than the classical case of sinusoidal excitation. Optimal tuning/damping parameters for actual cutting thus should be chosen accordingly. When absorber damping deviates from the obtained optimal values, the local optimal frequency ratios should be used for maximum effectiveness of absorber.

3. The cutting test results confirmed that the best results are obtained when absorber is tuned in accordance with the self-excitation/random excitation cases (smaller vibration amplitudes were recorded). The reasonably good results were obtained for a combination boring bar with length-to-diameter ratio L/D=15.

Acknowledgments

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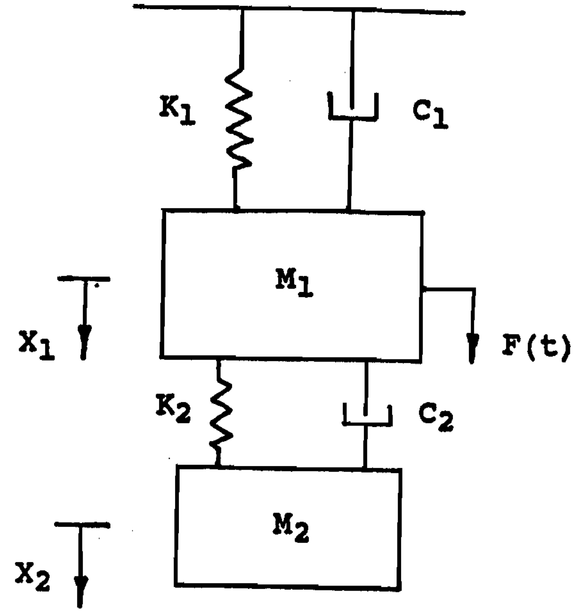

Fig.1 Model of boring bar with dynamic vibration absorber under an external excitation

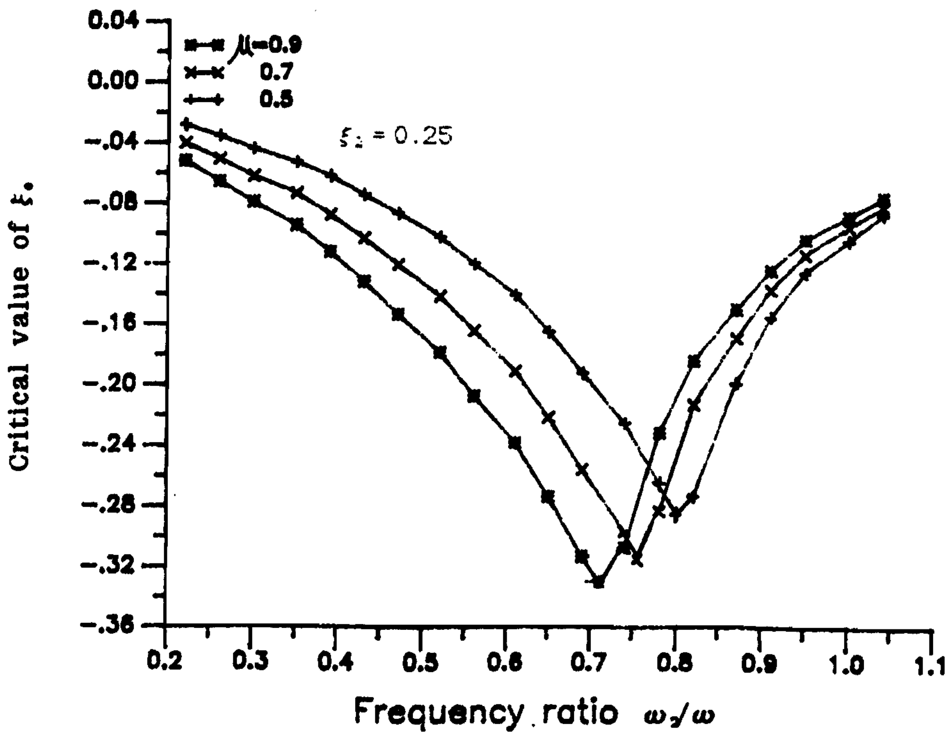

Fig.2 Critical value of ξ_0 vs frequency ratio of absorber at various mass ratios

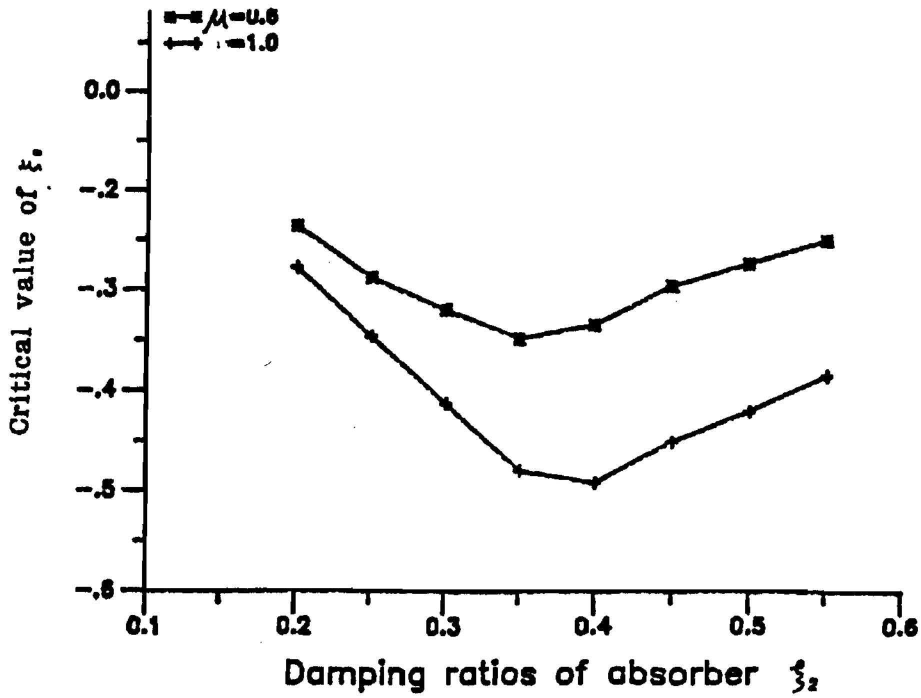

Fig.3 Critical value of ξ_0 vs absorber damping ratio (optimal tuning condition)

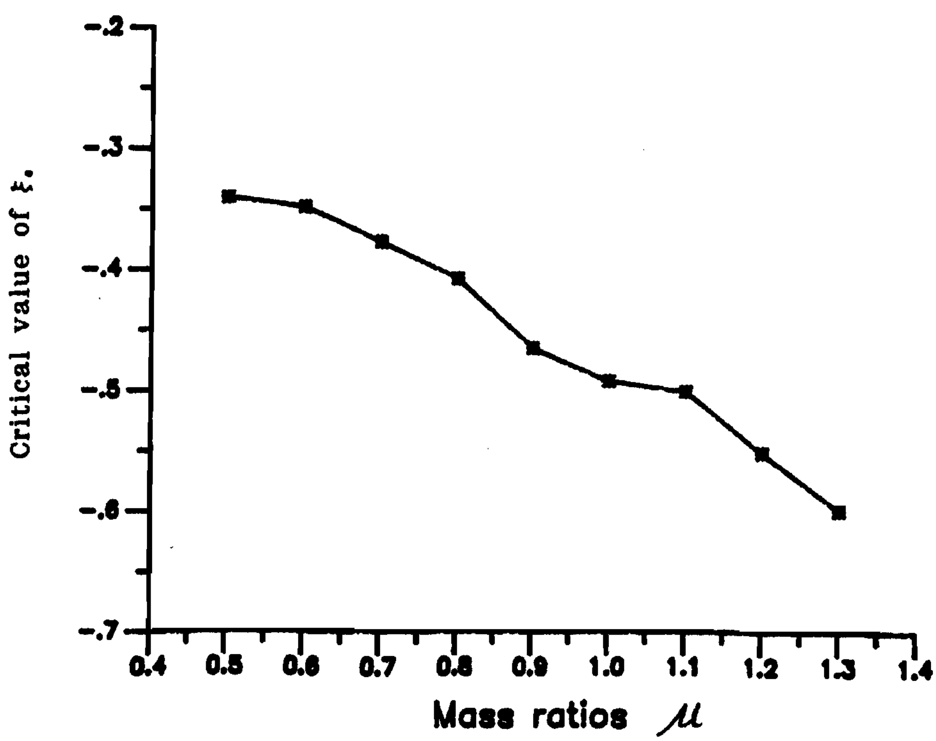

Fig.4 Critical value of ξ_0 vs mass ratio of absorber (optimal tuning and damping condition)

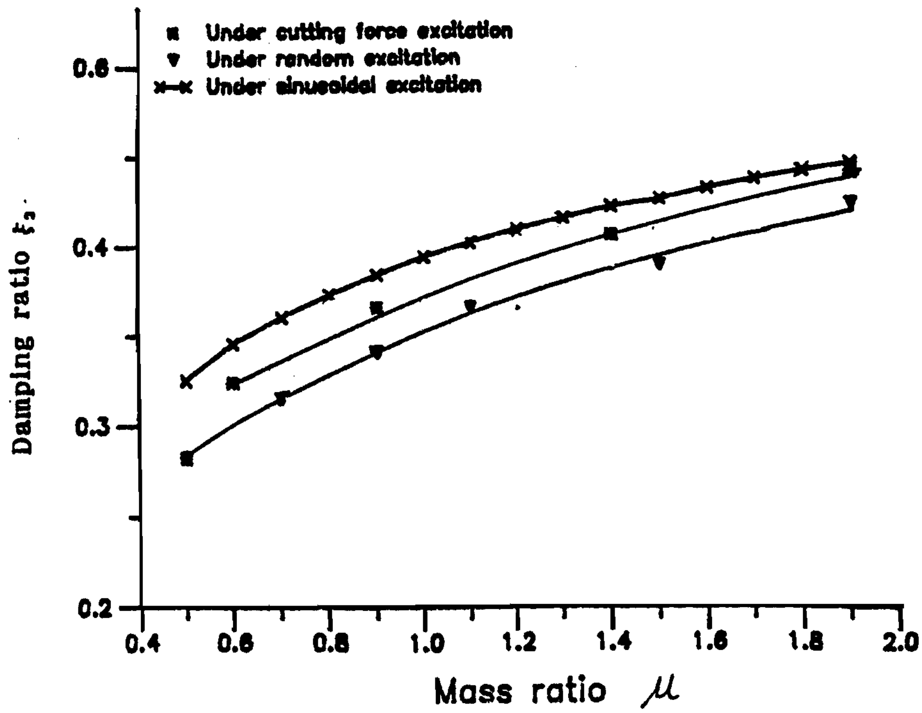

Fig.5 Optimal damping ratio of absorber vs mass ratio (optimal tuning condition)

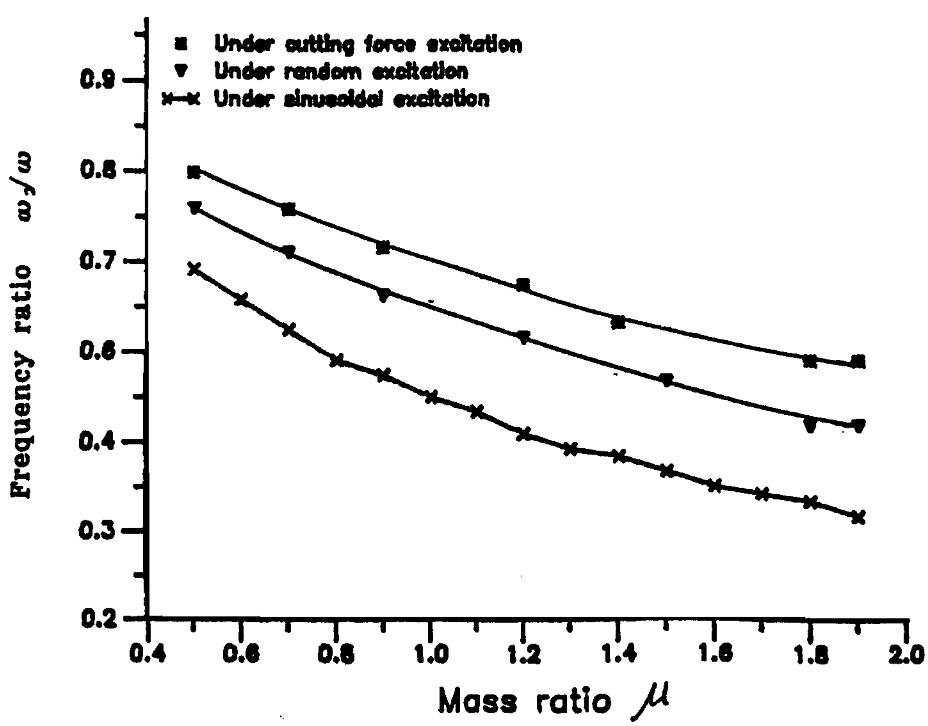

Fig.6 Optimal frequency ratio vs mass ratio (optimal absorber damping condition)

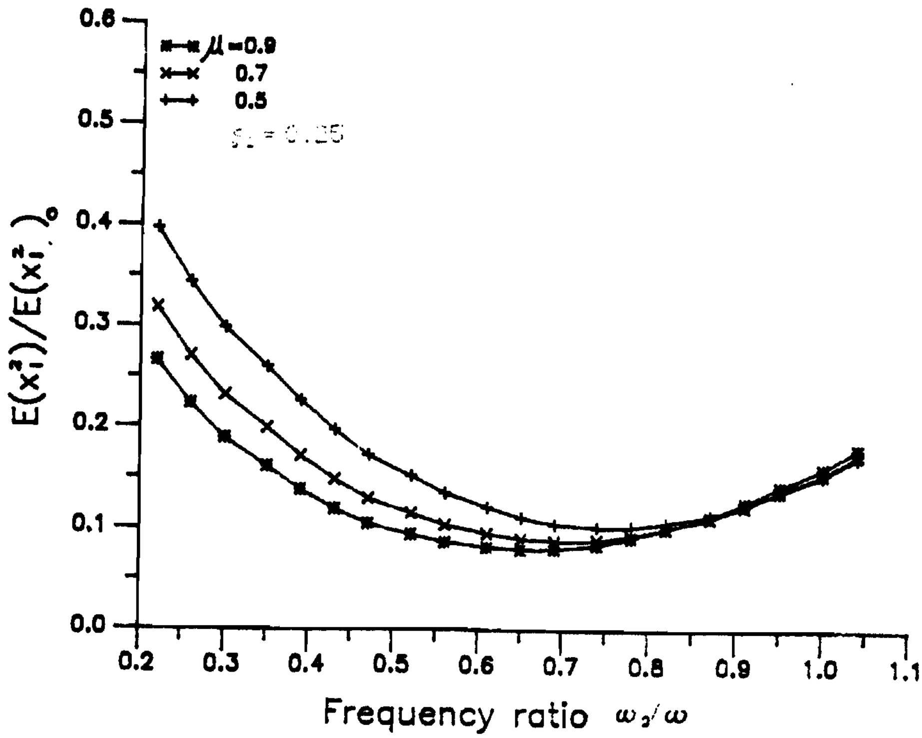

Fig.7 Normalized mean square response vs frequency ratio of absorber at various mass ratio

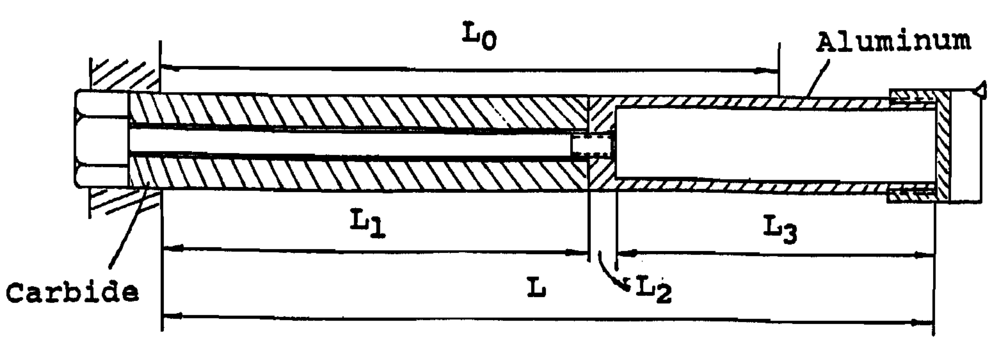

Fig.8 Combination boring bar design

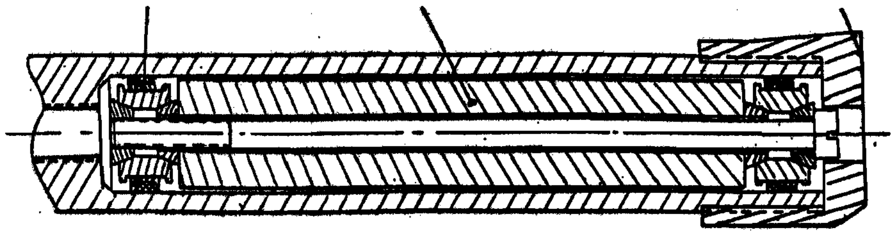

Fig.9 Dynamic vibration absorber design

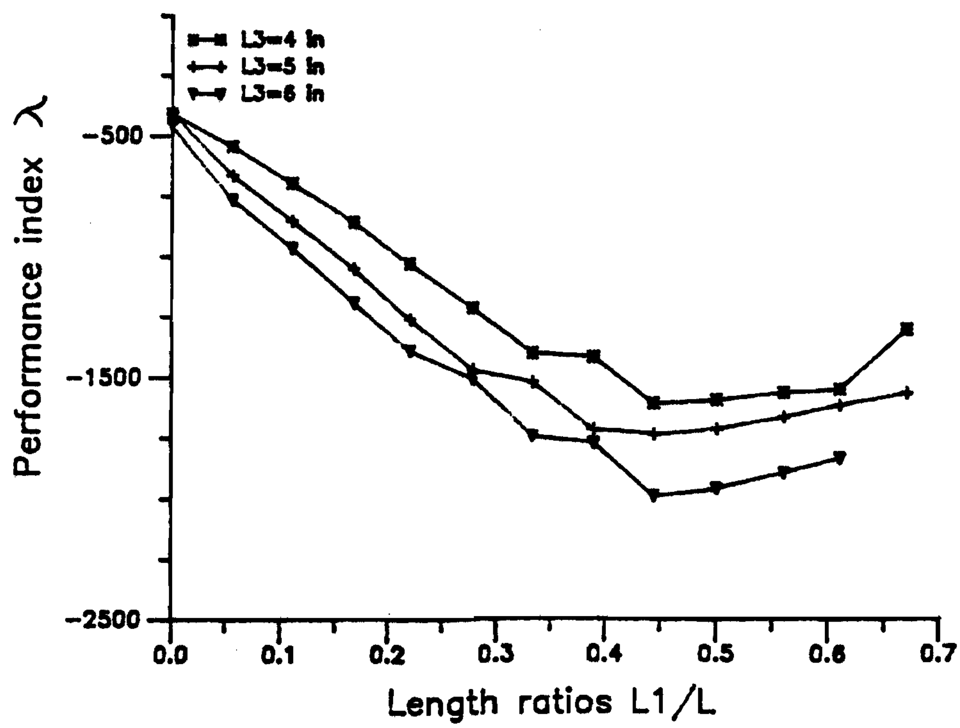

Fig.10 Performance index vs length ratio L_1/L (combination cantilever structure)

L3 (in)	Bars	Frequency (Hz)	Stiffness (lb/in)	Mass ratio
4	Combination	275	3939	1.41
	Steel	133	1776	0.73
5	Combination	283	3935	1.77
	Steel	138	1764	0.94
6	Combination	289	3929	2.07
	Steel	143	1746	1.14

Table-1 Calculated natural frequency, stiffness, and mass ratio for combination bar (L1/L=0.45) and steel bar

Absorber Damping Ratio	Spindle Speed (rpm)	Max. Peak -to-Vally Value (X- Direction)			
		Tuned under Se	Tuned under Ra	Tuned under Si	No Absorber
0.07	80	0.00177	0.00156	0.00192	0.00324
0.07	130	0.00184	0.00137	0.00241	0.00561
0.07	210	0.00265	0.00228	0.00371	0.00721
0.18	42	0.00079	0.00085	0.00114	
0.18	80	0.00104	0.0012	0.00121	
0.18	130	0.00118	0.00144	0.00167	

Se: Self-excited Vibration; Ra: Random Excitation;
Si: Sinusoidal Excitation

Table-2 Maximum p-v values (inches) of boring bar under various cutting speed, absorber damping, and tuning conditions

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