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ATMOSPHERIC TURBULENCE FIELD PARAMETERS DETERMINATION

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FOREWORD

This report is the result of an in-house effort under project 1367, "Structural Design Criteria for Military Aerospace Vehicles", Task 136702, "Aerospace Vehicle Airframe Design Criteria". The manuscript was released by the authors in April 1972 for publication as a technical report.

This technical report has been reviewed and is approved.

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ABSTRACT

A Newton-Raphson Least Squares Percentage Error Method is developed for the determination of atmospheric turbulence field parameters. A correction function is proposed to deemphasize the effects of data points with low statistical confidence.

The method is used on various sets of LO-LOCAT (Low Altitude Critical Atmospheric Turbulence) data to demonstrate the excellence of the curve fits obtained. Comparisons are made with other curve fits found in the literature.

It is recommended that this method be adopted as the standard method for the determination of atmospheric turbulence field parameters.

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SECTION I
INTRODUCTION

Current U.S. Air Force specifications (Ref. 1) for aircraft structural design for atmospheric turbulence contain a mission analysis requirement that the "frequency of limit load exceedances shall be determined as a function of load level by the equation:

$$N(y) = \sum t\, N_o \left[P_1 \exp\left\{ \frac{-(y - y_{1g})}{b_1 A} \right\} + P_2 \exp\left\{ \frac{-(y - y_{1g})}{b_2 A} \right\} \right] \qquad (1.1)$$

Where y = net value of load or stress

y_{1g} = value of load or stress in 1g level flight.

$N(y)$ = number of exceedances per second of the level y with a positive slope.

N_o = number of exceedances per second of the level $y = y_{1g}$ with a positive slope.

t = fraction of total mission time spent in each mission segment.

$A = \frac{\sigma_y}{\sigma_w}$ = ratio of r.m.s. incremental load or stress to r.m.s. gust velocity.

P_1, P_2 = percent of mission segment time spent in turbulence of types 1 and 2, respectively.

b_1, b_2 = r.m.s. value of σ_w considering only the time spent in turbulence of types 1 and 2, respectively."

The P and b parameters are referred to as turbulence field parameters and are specified in Ref. 1 as a function of altitude. The maximum allowable frequency of limit load exceedances $N(y_{limit})$ is also specified. The parameters t, N_o, and A are determined from the aircraft response characteristics for all suspected critical locations in the structure for each mission segment. A detailed discussion of the methodology for applying Eqn. (1.1) is given in Ref. 2.

The problem addressed in this report is that of determining the values for the P's and b's to be used to describe the atmospheric turbulence environment. In general, these quantities are determined from measured data in the form of frequency of exceedance as a function of some measure of gust intensity, usually true gust velocity or aircraft acceleration (e.g., center of gravity acceleration). A typical example of such data is given in Figure 1. In Figure 1 and throughout this report, all data points f_i are divided by f_1 (number of exceedances of zero gust velocity) so that a probability of exceeding is obtained. The rationale for describing the gust environment in terms of P and b parameters is based on the fact that the exceedance data generally can be described accurately by the following analytical function, which is similar to and relatable to Eqn. (1.1).

$$F(x) = \frac{N(x)}{N_{ox}} = P_1 e^{-x/b_1} + P_2 e^{-x/b_2} \qquad (1.2)$$

Where x = gust velocity

F(x) = probability of equalling or exceeding any gust velocity x.

$N(x)$ and N_{ox} are defined equivalently to $N(y)$ and N_o, respectively in Eqn. (1.1).

From examination of Eqn. (1.2) and Figure 1 it is obvious that the two terms each plot as straight lines in the semi-log format. Hence, the values of P_1 and P_2 are obtained as the respective intercepts of the two straight lines on the ordinate axis. The values for b_1 and b_2 are determined from the respective slopes (m_1, m_2) of the straight lines as follows:

$$m_1 = -1/b_1$$

$$m_2 = -1/b_2$$

In spite of the straight forward interpretations given to the P's and b's, the determination of specific values to be used for aircraft structural design has been slightly complicated by the lack of a universally accepted method for fitting Eqn. (1.2) to a set of data. Consequently, turbulence field parameters derived from various sets of data by independent investigators are frequently inconsistent with each other because different curve fitting methods are used.

This problem is occasionally demonstrated when the same set of data are analyzed by different individuals. Table 1 presents P's and b's obtained from identical data by two different sources. Example plots of two of the data sets are shown in Figures 2 and 3 along with the analytical fit of Eqn. (1.2) using the respective P and b values of Ref. 3 and 4.

Table 1. Turbulence Field Parameters Derived by Different Investigators from Identical Data Sets

Data Set*	P_1		P_2		b_1		b_2	
	Ref 3	Ref 4	Ref 3	Ref 4	Ref 3	Ref 4	Ref 3	Ref 4
1	.898	.92	.091	.08	2.262	3.472	6.185	6.667
2	.902	.922	.098	.078	2.513	2.898	7.000	7.519
3	.851	.9	.149	.1	2.279	2.825	5.469	5.917
4	.795	.88	.205	.12	2.766	3.763	6.251	6.947
5	.774	.70	.226	.30	2.992	3.995	7.039	7.186
6	.648	.78	.352	.21	2.297	3.908	5.495	6.079

*1 - LO-LOCAT, Phase III, All data, longitudinal component
2 - LO-LOCAT, Phase III, All data, lateral component
3 - LO-LOCAT, Phase III, All data, vertical component
4 - LO-LOCAT, Phase III, High Mountain data, longitudinal component
5 - LO-LOCAT, Phase III, High Mountain data, lateral component
6 - LO-LOCAT, Phase III, High Mountain data, vertical component

Obviously, there is considerable subjectiveness in determining the specific values for P_1, P_2, b_1, and b_2 from any given set of data. Since these quantities, rather than the actual data, are used in Ref. 1 to describe the gust environment for aircraft structural design, it is highly desirable that the values derived for P's and b's are those which give the "closest fit" to the data distribution. However, there is no unique quantitative definition of what is meant by "closest fit". Since the MIL Spec (Ref. 1) P and b values are continually being revised as new data become available or previous data are reanalyzed, reduction of the subjective errors is a desirable goal which can be accomplished by consistently applying a particular data fitting procedure to Eqn. (1.2).

The specific objective of this report is to propose a "standard" procedure for fitting Eqn. (1.2) to gust data for the purpose of deriving P and b values. There are many techniques available for curve fitting and the method presented in this report is not claimed to be an optimum method. The primary goal for this effort was simply to develop a procedure which provides a reasonably good fit of Eqn. (1.2) to the data and can be universally adopted by various investigators to get consistent results.

General Comments

The procedure presented here might be considered to be preliminary in that some subjectiveness in applying this method still exists. At the largest gust velocities in any data set, relatively few sample values are obtained (usually only one sample of the maximum gust is obtained); consequently, statistical confidence is much lower for these large gust velocity samples than for the smaller gust velocity samples. Generally, the extreme gust velocity samples deviate somewhat from the trend of the rest of the data. The subjectiveness arises in determining how much weight should be placed on these less reliable samples. The weighting method described herein is arbitrary and may not give satisfactory results for some data sets. Judgment is still required to develop the most appropriate weighting method for any particular data set.

Subjective judgment is also required in treating the low amplitude (near zero) gust velocity portion of the frequency of exceedance curve. The question might be stated as "what is the dividing gust velocity below which the atmosphere is considered to be 'smooth air' and above which the atmosphere is considered to be turbulent?" Philosophically, any gust velocity greater than zero could be defined as turbulence. Practically, real gust data have some low amplitude limit dictated by the sensors and data handling equipment and procedures. Hence, for every data set there exists some finite range of gust velocities near zero for which essentially there is no data. The most common practice in the past has been to arbitrarily define (for each data set) a threshold (other than zero) below which the atmosphere is considered to be "smooth" or non-turbulent. This also would seem to be acceptable intuitively since the smallest gust velocities would not be expected to significantly affect aircraft structural integrity.

When a non-zero threshold value is established for a set of gust data, $P_1 + P_2$ no longer add up to one. For the LO-LOCAT data used in this report, it is reasonable to assume that $P_1 + P_2 = 1$. The data were collected during terrain following flights at altitudes below 1000 feet above the terrain. However, as noted previously, errors exist in the data near zero gust velocity, and the peak count data were extrapolated to get zero gust velocity peak counts. For the more typical gust velocity data, collected at higher altitudes where gusts of any given magnitude occur much less frequently, extrapolation of the peak counts to zero gust velocity has a much larger influence on the resulting P's and b's. The extrapolation range is a greater proportion of the total data range in terms of the frequency of exceedance.

Consequently, the usual procedure is to fit Eqn. (1.2) to the data above the threshold value and $P_1 + P_2$ then represents the percent of flight time (or distance) spent in turbulence with intensity equal to or greater than the threshold value. Since different data sets in general have different threshold values, effectively we frequently have different definitions of what is turbulence and what is smooth air. Elimination of this source of subjectiveness from P and b determinations would obviously be desirable, but is not treated in this report.

Figure 1
LO-LOCAT, Phase III
All Data, Vertical Component

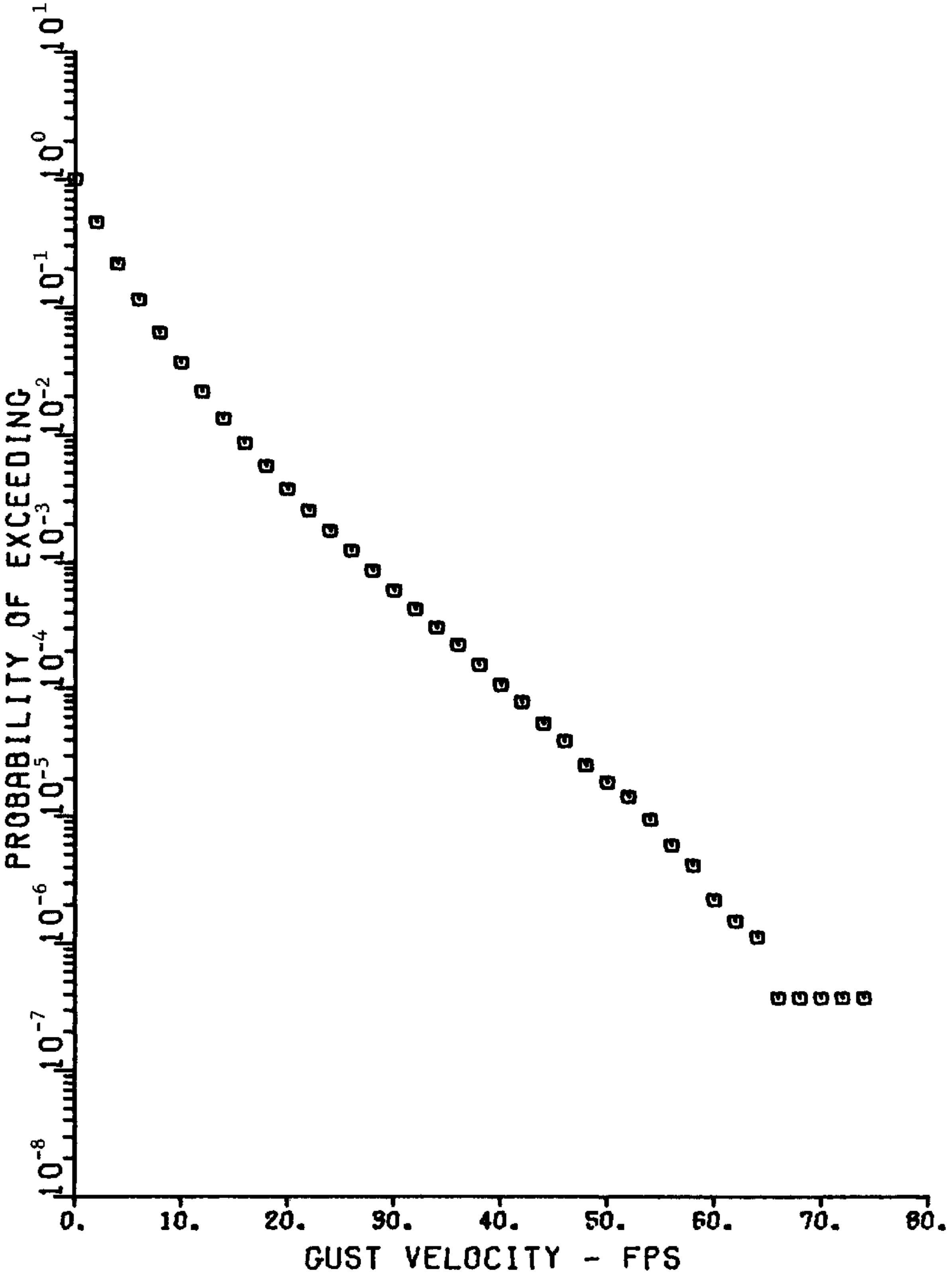


Figure 2
LO-LOCAT, Phase III
All Data, Vertical Component

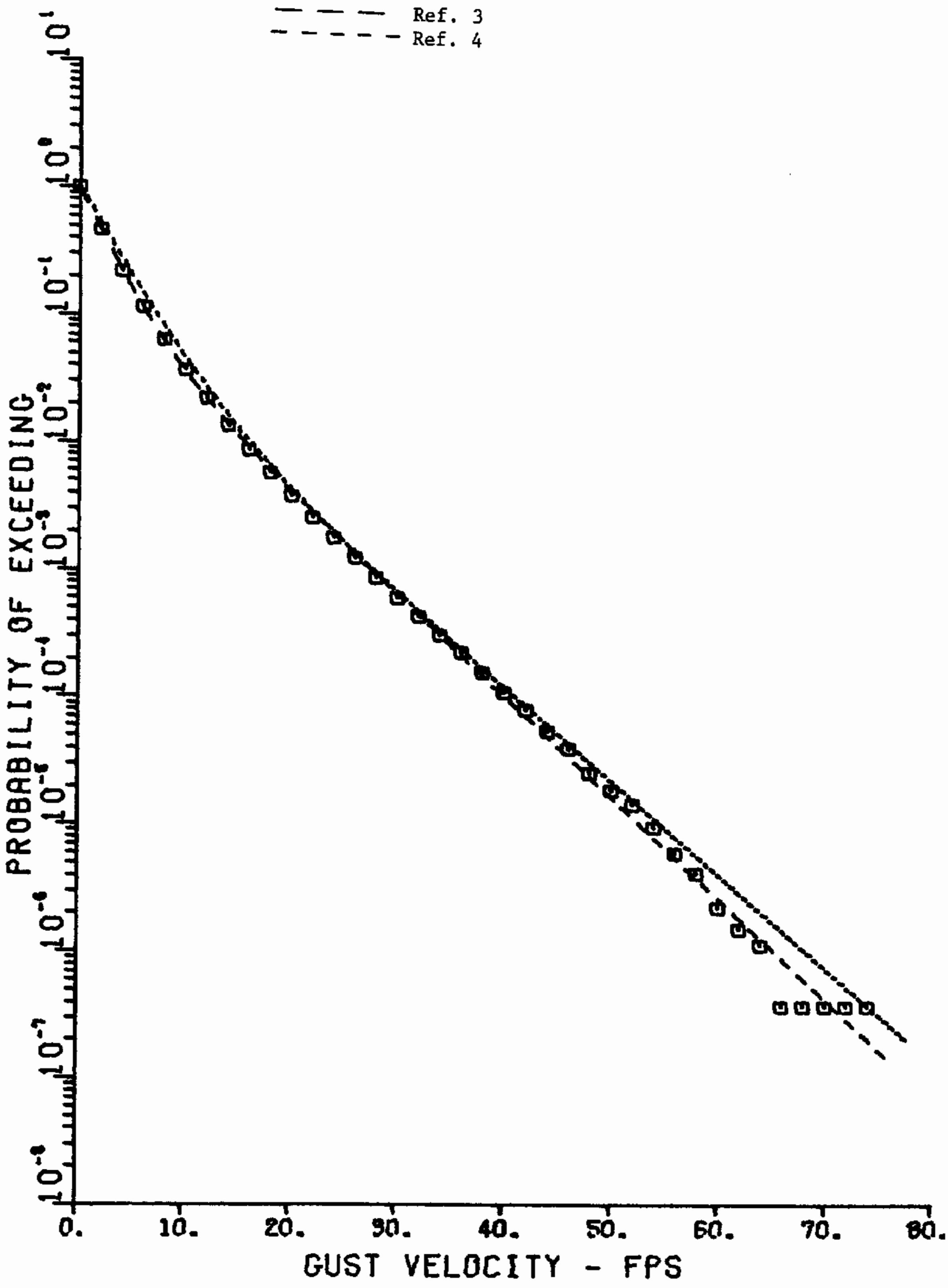
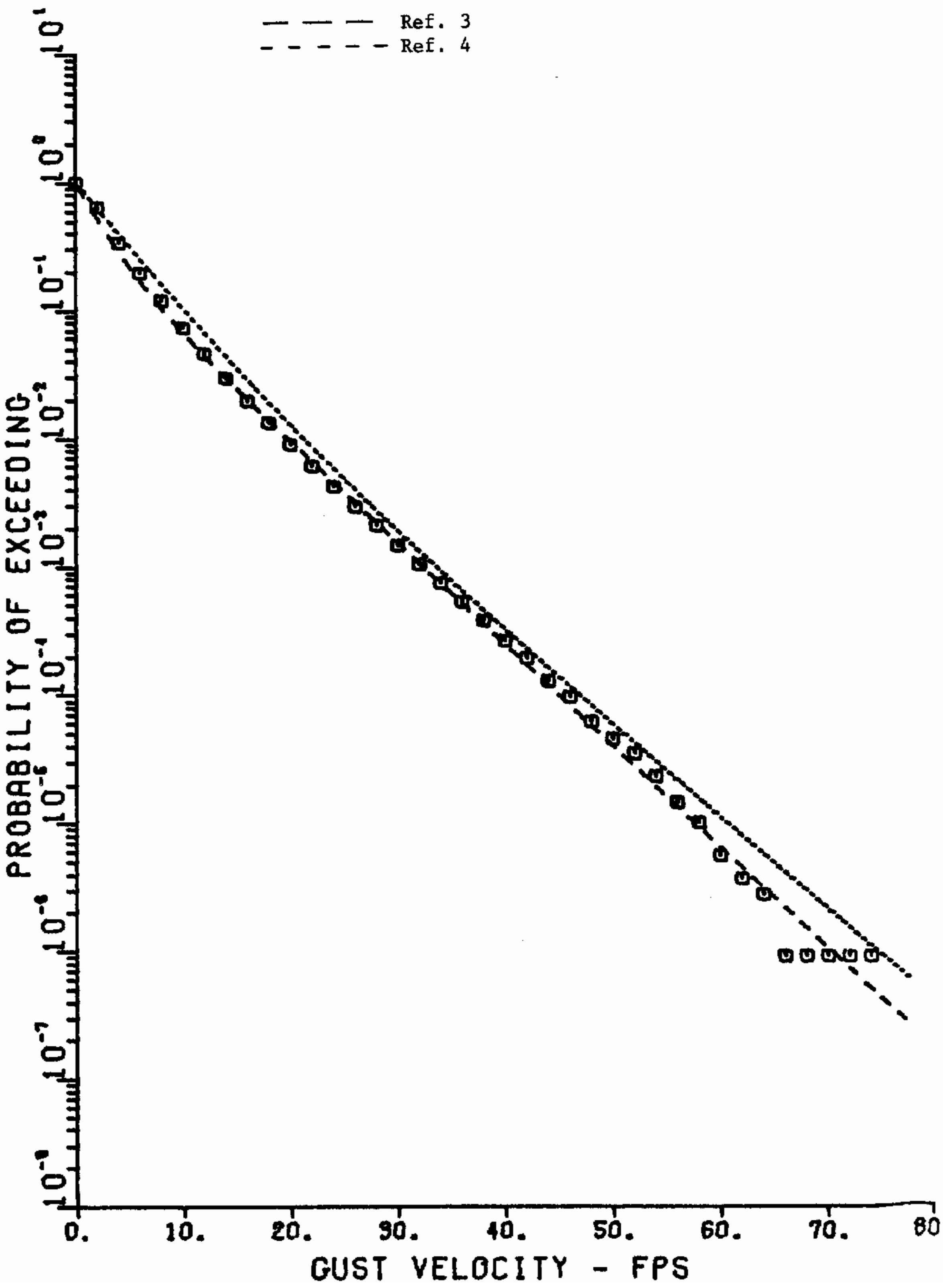


Figure 3
LO-LOCAT, Phase III
High Mountain, Vertical Component



SECTION II
DESCRIPTION OF PROCEDURE

2.1 Mathematical Formulation

The standard method of least squares (Ref. 5) cannot be applied directly to Eqn. (1.2) for the best fit of N data points with respect to parameters P_1, P_2, b_1, and b_2 because F is nonlinear in b_1 and b_2. Furthermore, the wide range of values taken on by F (10^{-7} to 1, for the data of interest here) doesn't permit putting equal weight on all data points. Another consideration, discussed in Section I, is the weighting of data points upon which little statistical confidence exists. Thus, a different procedure for curve fitting is required. The procedure chosen here might be described as a Newton-Raphson Least Square Percentage Error Method. It is outlined in the following paragraphs (a similar procedure is described in Ref. 6).

To determine the best fit, the following approach is used. Let $F(x;P_1,P_2,b_1,b_2)$ be expanded in a Taylor Series about P_{10}, P_{20}, b_{10}, and b_{20} as in Eqn. (2.2). The notation $F(x;P_1,P_2,b_1,b_2)$ indicates the dependence of the function F(x) on the parameters P_1,P_2,b_1,b_2. That is,

$$F(x;\, P_1,\, P_2,\, b_1,\, b_2) = P_1\, e^{-x/b_1} + P_2\, e^{-x/b_2} \qquad (2.1)$$

$$\begin{aligned} F(x;\, P_1,\, P_2,\, b_1,\, b_2) = {} & F(x;\, P_1,\, P_2,\, b_{10},\, b_{20}) + \\ & \frac{\partial F}{\partial b_1}(x;\, P_{10},\, P_{20},\, b_{10},\, b_{20})(b_1 - b_{10}) + \qquad (2.2) \\ & \frac{\partial F}{\partial b_2}(x;\, P_{10},\, P_{20},\, b_{10},\, b_{20})(b_2 - b_{20}) + \dots \end{aligned}$$

Let $F_a(x;P_1,P_2,b_1,b_2,P_{10},P_{20},b_{10},b_{20})$ be defined as the linear terms in Eqn. (2.2).

$$F_a\,(x;\, P_1,\, P_2,\, b_1,\, b_2,\, P_{10},\, P_{20},\, b_{10},\, b_{20}) = \qquad (2.3)$$

$$\exp\left[-\frac{x}{b_{10}}\right](P_1 + P_{10}\frac{x}{b_{10}^2}(b_1 - b_{10})) + \exp\left[-\frac{x}{b_{20}}\right](P_2 + P_{20}\frac{x}{b_{20}^2}(b_2 - b_{20}))$$

The parameters b_1 and b_2 appear linearly in Eqn. (2.3) and the least squares error method may be applied.

An iterative procedure is now used. First, choose b_{10} and b_{20}. This may be an educated guess or the values derived by "eyeballing" the data as in the method described in Section 1. Next, find a best estimate of P_1 and P_2 by minimizing the corrected square percentage error S given by

$$S = \sum_{i=1}^{N} \left\{ \left[\frac{F(x; P_1, P_2, b_{10}, b_{20}) - f_i}{f_i} \right]^2 \right\} \frac{1}{CR(x_i)} \qquad (2.4)$$

by requiring that

$$\frac{\partial S}{\partial P_1} = \frac{\partial S}{\partial P_2} = 0 \qquad (2.5)$$

where f_i are the measured probabilities of exceeding gust values x_i and $CR(x_i)$ is the correction to the percentage error for points with low statistical confidence. The function $CR(x_i)$ weights the residual error (see Ref. 7 for a discussion of similar weighting techniques).

Then, with an estimate for P_1 and P_2, find a better estimate for b_1, b_2, P_1 and P_2 by minimizing the corrected squares percentage error S_a given by

$$S_a = \sum_{i=1}^{N} \left\{ \left[\frac{F_a(x; P_1, \ldots, b_2, P_{20}, \ldots, b_{20}) - f_i}{f_i} \right] \right\}^2 \frac{1}{CR(x_i)} \qquad (2.6)$$

requiring that

$$\frac{\partial S_a}{\partial P_1} = \frac{\partial S_a}{\partial P_2} = \frac{\partial S_a}{\partial b_1} = \frac{\partial S_a}{\partial b_2} = 0 \qquad (2.7)$$

The derived values of b_1 and b_2 are then substituted for b_{10} and b_{20}. Parameters b_{10} and b_{20} along with the calculated values of P_1 and P_2 are used in Eqns. (2.6) and (2.7) and the cycle repeated until the desired degree of accuracy is obtained. Since $F_a(x) = F(x)$ when $b_{10} = b_1$ and $b_{20} = b_2$, the condition for convergence used was

$$\left| \frac{b_{10} - b_1}{b_{10}} \right| < \varepsilon \tag{2.8}$$

and

$$\left| \frac{b_{20} - b_2}{b_{20}} \right| < \varepsilon \tag{2.9}$$

where ε is selected as desired.

2.2 Programming Considerations

The iterative evaluation of parameters P_1, P_2, b_1 and b_2 as described in Section 2.1 is well suited to computer programming. The calculation of the parameters requires the simultaneous solution of the two equations (Eqn. 2.5) for the initial values of P_1 and P_2, and the solution of the four simultaneous equations (Eqns. 2.7) for P_1, P_2, b_1 and b_2. Since Eqns. (2.5) and (2.7) are linear in the desired parameters, standard matrix inversion routines can be used.

A correction factor, CR(x), is used in Eqns. (2.4) and (2.6) to reduce the percentage error due to data points having low statistical confidence. One way to do this is to deemphasize data points with M or fewer cumulative observations. A correction function of the following form was selected.

$$CR(x_i) = 1 \qquad \text{for all } x_i \text{ such that } f_i > Mf_N \tag{2.10}$$

and

$$CR(x_i) = C_1 + C_2(i - j + 1) \quad \text{for all } x_i \text{ such that } f_i < Mf_N \text{ and } j \text{ is the largest } k \text{ for which } f_k > Mf_N$$

NOTE: In Eqns. (2.10) the points x_i are assumed to be ordered. That is,

$$x_1 < x_2 < \cdots < x_N$$

where N is the index of the data point for the highest gust velocity observed. This formulation of the correction function assumes that f_N represents one cumulative observation. In the data examined this usually was the case. If it is known that f_N represents more than one observation, the expression Mf_N can be replaced by the appropriate value. The constants C_1 and C_2 are arbitrarily chosen to produce an appropriate CR for the data being fitted.

Convergence has been achieved for every set of data considered. In some instances, more than one set of initial values for b_1 and b_2 were tried before convergence was achieved. In application, ε from Eqns. (2.8) and (2.9) was selected to be 10^{-7}. Two hundred iterations were considered to imply no convergence. In no data set considered did the number of iterations reach two hundred. Failure to converge was detected by error messages from the matrix inversion routines when the values of the parameters were unrealistic.

SECTION III
APPLICATION OF THE PROCEDURE

The Newton-Raphson Least Squares Percentage Error Method described in Section 2.1 was applied to various sets of Phases I, II, and III LO-LOCAT data (Refs. 3 and 8). As turbulence is encountered virtually at all times and the probability of encountering gusts of velocity zero or greater is exactly one, it was decided to define

$$P_1 + P_2 = 1 \tag{3.1}$$

thus

$$F(x) = P_1 e^{-x/b_1} + (1 - P_1) e^{-x/b_2} \tag{3.2}$$

The assumption in Eqn. (3.1) is discussed in Section I and Ref. 4. Its validity for low altitude gusts is demonstrated by the excellent data fit in the low gust velocity range (see, for example, Figs. 4-9). For turbulence data from higher altitude ranges, such an assumption would be inappropriate.

Presented in Tables 3 through 8 and Figures 4 through 15 are various fits of data using the Newton-Raphson Least Squares Percentage Error Method and comparisons with fits found in the literature, as indicated. The following values were selected for C_1, C_2, and M, defining the correction function, $CR(x_i)$, given by Eqn. (2.10):

$$\begin{aligned} C_1 &= 5 \\ C_2 &= 2.5 \\ M &= 6 \end{aligned} \tag{3.3}$$

As stated previously, the choice of a correction function, $CR(x_i)$, is made somewhat subjectively. Only one form of $CR(x_i)$ was considered in this report, but two different values of C_1 were tried. That is, for four data sets, the values of C_1, C_2, and M given in Eqn. (3.4) were used as well as those given in Eqn. (3.3).

$$
\begin{aligned}
C_1 &= 10 \\
C_2 &= 2.5 \\
M &= 6
\end{aligned}
\qquad (3.4)
$$

Table 2 presents a comparison between the values of P_1, b_1, and b_2 for the two different C_1 values on four data sets. The only significant difference is in the b_2 for the data set "LO-LOCAT, Phase III, Desert, Vertical Component." This data set warrants further discussion.

The desert data (see Table 3 and Fig. 4) exhibits strange behavior at the higher gust velocities. Only three observations or measurements are represented in the range of gust velocities from 18 to 30 feet per second. All of these data points were deemphasized through the use of $CR(x_i)$. Since the value of b_2 is largely determined by the higher gust velocities, b_2 is much more sensitive to changes in $CR(x_i)$ for the desert data set.

The procedure described in Section II and applied in this Section permits rapid determination of P's and b's. For example, the program GUSTP (see Appendix) required no more than 3.6 seconds central processor time (on a CDC 6600 computer) for compilation for any of the data sets considered in this report. The total time for compilation and execution never exceeded 6.5 seconds of central processor time.

Table 2. Turbulence Field Parameters for Two Values of C_1

	P_1		b_1		b_2	
Data Set*	$C_1 = 5$	$C_1 = 10$	$C_1 = 5$	$C_1 = 10$	$C_1 = 5$	$C_1 = 10$
1	.9999	.9998	1.529	1.525	9.142	7.795
2	.8498	.8432	1.823	1.809	3.137	3.122
3	.9992	.9992	2.345	2.346	5.975	5.976
4	.8531	.8575	2.287	2.307	5.483	5.512

*1 - LO-LOCAT, Phase III, Desert, Vertical Component
2 - LO-LOCAT, Phase I & II, All Data, Longitudinal Component
3 - LO-LOCAT, Phase I & II, All Data, Vertical Component
4 - LO-LOCAT, Phase III, All Data, Vertical Component

NOTE: Here and throughout this report, the data set "LO-LOCAT, Phase I & II, All Data" doesn't include data from non-contour flights at Peterson.

Table 3. LO-LOCAT, Phase III, Desert, Vertical Component

X	F	FCALC
0.0	.10000E+01	.10000E+01
2.0	.23863E+00	.27050E+00
4.0	.69238E-01	.73203E-01
6.0	.20129E-01	.19836E-01
8.0	.52928E-02	.53963E-02
10.0	.13891E-02	.14849E-02
12.0	.38918E-03	.42205E-03
14.0	.17363E-03	.13060E-03
16.0	.53885E-04	.48549E-04
18.0	.17962E-04	.23759E-04
20.0	.11975E-04	.14966E-04
22.0	.11975E-04	.10910E-04
24.0	.11975E-04	.84642E-05
26.0	.59873E-05	.67194E-05
28.0	.59873E-05	.53770E-05
30.0	.59873E-05	.43145E-05

B1 = 1.5294 B2 = 9.1417 P1 = .9999 P2 = .0001

X = Gust velocity in feet per second
F = Normalized data points (probability of exceeding)
FCALC = Calculated probability of exceeding using
derived turbulence field parameters

Table 4. LO-LOCAT, Phase III, High Mountain, Vertical Component

X	F	FCALC
0.0	.10000E+01	.10000E+01
2.0	.64112E+00	.55264E+00
4.0	.33983E+00	.31616E+00
6.0	.19612E+00	.18733E+00
8.0	.11819E+00	.11475E+00
10.0	.73098E-01	.72397E-01
12.0	.46198E-01	.46817E-01
14.0	.29784E-01	.30876E-01
16.0	.19612E-01	.20672E-01
18.0	.13127E-01	.13996E-01
20.0	.88383E-02	.95535E-02
22.0	.60257E-02	.65589E-02
24.0	.41888E-02	.45215E-02
26.0	.29501E-02	.31258E-02
28.0	.20762E-02	.21653E-02
30.0	.14544E-02	.15020E-02
32.0	.10362E-02	.10429E-02
34.0	.74645E-03	.72459E-03
36.0	.53647E-03	.50367E-03
38.0	.37965E-03	.35021E-03
40.0	.26320E-03	.24356E-03
42.0	.19166E-03	.16941E-03
44.0	.12930E-03	.11785E-03
46.0	.96287E-04	.81987E-04
48.0	.61440E-04	.57040E-04
50.0	.45851E-04	.39685E-04
52.0	.34847E-04	.27611E-04
54.0	.22926E-04	.19211E-04
56.0	.14673E-04	.13367E-04
58.0	.10087E-04	.93002E-05
60.0	.55022E-05	.64710E-05
62.0	.36681E-05	.45024E-05
64.0	.27510E-05	.31327E-05
66.0	.91703E-06	.21797E-05
68.0	.91703E-06	.15166E-05
70.0	.91703E-06	.10553E-05
72.0	.91703E-06	.73425E-06
74.0	.91703E-06	.51088E-06

B1 = 2.7063   B2 = 5.5142   P1 = .6560   P2 = .3440

Table 5. LO-LOCAT, Phase III, All Data, Vertical Component

X	F	FCALC
0.0	.10000E+01	.10000E+01
2.0	.45412E+00	.45780E+00
4.0	.21449E+00	.21921E+00
6.0	.11244E+00	.11106E+00
8.0	.62175E-01	.59955E-01
10.0	.35828E-01	.34472E-01
12.0	.21378E-01	.20950E-01
14.0	.13234E-01	.13302E-01
16.0	.84725E-02	.87167E-02
18.0	.55608E-02	.58358E-02
20.0	.36925E-02	.39618E-02
22.0	.24963E-02	.27132E-02
24.0	.17293E-02	.18682E-02
26.0	.12094E-02	.12906E-02
28.0	.85080E-03	.89343E-03
30.0	.59400E-03	.61922E-03
32.0	.42059E-03	.42948E-03
34.0	.30254E-03	.29801E-03
36.0	.21805E-03	.20684E-03
38.0	.15459E-03	.14359E-03
40.0	.10736E-03	.99686E-04
42.0	.78217E-04	.69212E-04
44.0	.52760E-04	.48055E-04
46.0	.39108E-04	.33366E-04
48.0	.25089E-04	.23167E-04
50.0	.18447E-04	.16086E-04
52.0	.14020E-04	.11169E-04
54.0	.92238E-05	.77555E-05
56.0	.59032E-05	.53850E-05
58.0	.40583E-05	.37391E-05
60.0	.22137E-05	.25963E-05
62.0	.14758E-05	.18027E-05
64.0	.11068E-05	.12517E-05
66.0	.36895E-06	.86914E-06
68.0	.36895E-06	.60349E-06
70.0	.36895E-06	.41904E-06
72.0	.36895E-06	.29096E-06
74.0	.36895E-06	.20203E-06

B1 = 2.2871 B2 = 5.4828 P1 = .8531 P2 = .1469

Table 6. LO-LOCAT, Phase I & II, All Data, Vertical Component

X	F	FCALC
0.0	.10000E+01	.10000E+01
2.0	.39483E+00	.42647E+00
4.0	.15293E+00	.18194E+00
6.0	.66724E-01	.77663E-01
8.0	.29828E-01	.33184E-01
10.0	.13259E-01	.14202E-01
12.0	.58276E-02	.60947E-02
14.0	.25690E-02	.26272E-02
16.0	.11431E-02	.11409E-02
18.0	.50862E-03	.50133E-03
20.0	.23276E-03	.22445E-03
22.0	.11431E-03	.10337E-03
24.0	.56379E-04	.49573E-04
26.0	.29138E-04	.25073E-04
28.0	.14052E-04	.13508E-04
30.0	.75345E-05	.77766E-05
32.0	.43448E-05	.47592E-05
34.0	.28103E-05	.30621E-05
36.0	.19138E-05	.20448E-05
38.0	.14052E-05	.14008E-05
40.0	.10207E-05	.97574E-06
42.0	.76552E-06	.68686E-06
44.0	.51034E-06	.48665E-06
46.0	.38276E-06	.34616E-06
48.0	.38276E-06	.24682E-06
50.0	.12759E-06	.17624E-06

B1 = 2.3454   B2 = 5.9754   P1 = .9992   P2 = .0008

Table 7. LO-LOCAT, Phase I & II, All Data. Lateral Component

X	F	FCALC
0.0	.10000E+01	.10000E+01
2.0	.39516E+00	.38196E+00
4.0	.15032E+00	.15382E+00
6.0	.65645E-01	.65573E-01
8.0	.30000E-01	.29526E-01
10.0	.14032E-01	.13939E-01
12.0	.68226E-02	.68308E-02
14.0	.34194E-02	.34399E-02
16.0	.17419E-02	.17652E-02
18.0	.89677E-03	.91727E-03
20.0	.47581E-03	.48055E-03
22.0	.25000E-03	.25306E-03
24.0	.13355E-03	.13370E-03
26.0	.69032E-04	.70782E-04
28.0	.36290E-04	.37521E-04
30.0	.20645E-04	.19905E-04
32.0	.11290E-04	.10565E-04
34.0	.70645E-05	.56093E-05
36.0	.38387E-05	.29788E-05
38.0	.13645E-05	.15820E-05
40.0	.74355E-06	.84026E-06
42.0	.37258E-06	.44631E-06
44.0	.24839E-06	.23707E-06
46.0	.24839E-06	.12593E-06
48.0	.24839E-06	.66890E-07
50.0	.12403E-06	.35532E-07

B1 = 1.7982 B2 = 3.1614 P1 = .7374 P2 = .2626

Table 8. LO-LOCAT, Phase I & II, All Data, Longitudinal Component

X	F	FCALC
0.0	.10000E+01	.10000E+01
2.0	.35000E+00	.36304E+00
4.0	.12458E+00	.13665E+00
6.0	.51458E-01	.53789E-01
8.0	.22500E-01	.22277E-01
10.0	.10146E-01	.97214E-02
12.0	.47292E-02	.44530E-02
14.0	.22292E-02	.21250E-02
16.0	.10937E-02	.10469E-02
18.0	.53958E-03	.52791E-03
20.0	.27292E-03	.27055E-03
22.0	.13854E-03	.14017E-03
24.0	.69375E-04	.73150E-04
26.0	.35833E-04	.38352E-04
28.0	.19625E-04	.20168E-04
30.0	.11042E-04	.10626E-04
32.0	.58333E-05	.56051E-05
34.0	.27708E-05	.29590E-05
36.0	.13792E-05	.15628E-05
38.0	.92083E-06	.82570E-06
40.0	.61250E-06	.43633E-06
42.0	.30625E-06	.23060E-06
44.0	.15312E-06	.12188E-06
46.0	.15312E-06	.64424E-07

B1 = 1.8227   B2 = 3.1373   P1 = .8498   P2 = .1502

Figure 4
LO-LOCAT, Phase III
Desert, Vertical Component

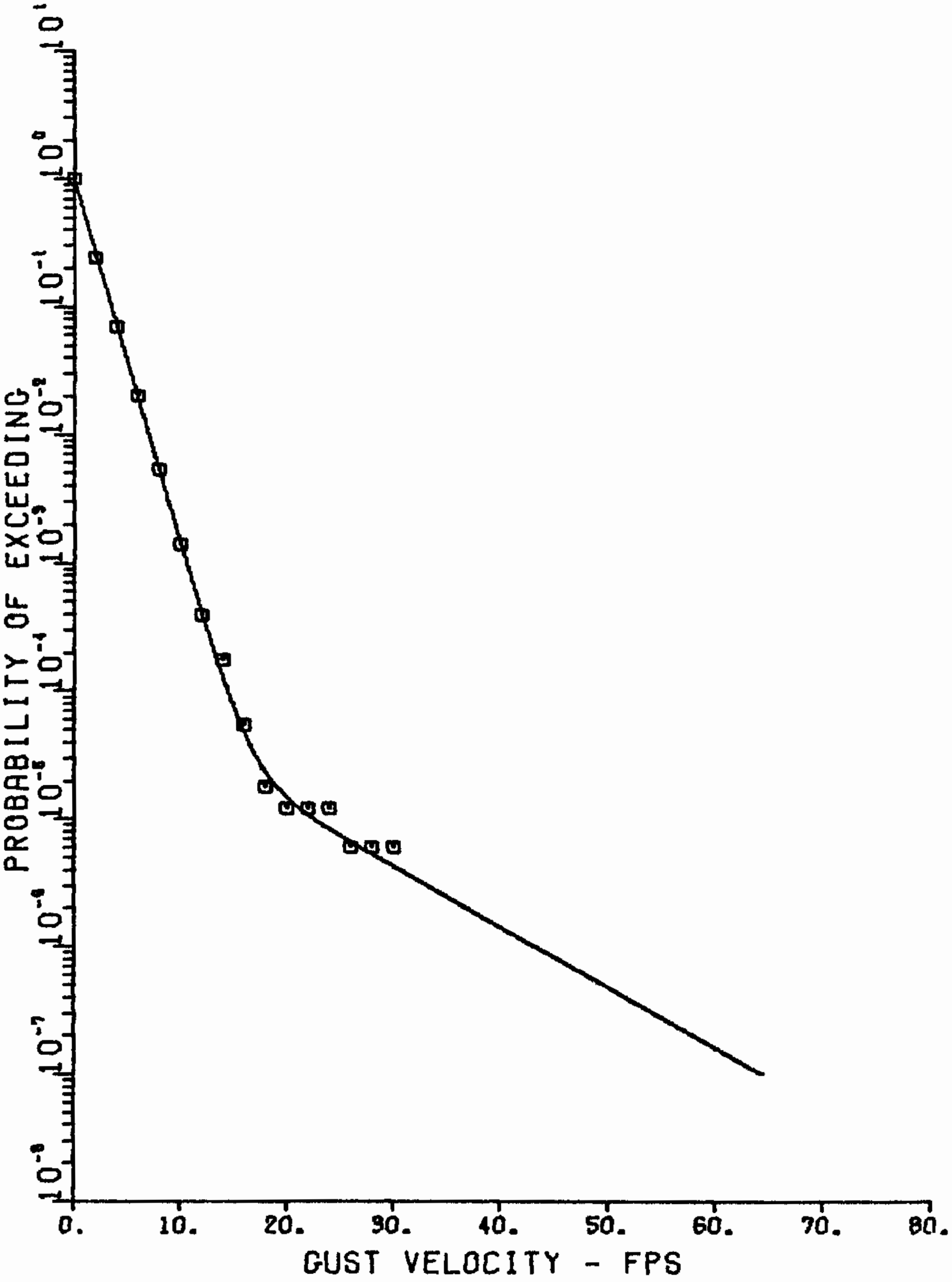


Figure 5
LO-LOCAT, Phase III
High Mountain, Vertical Component

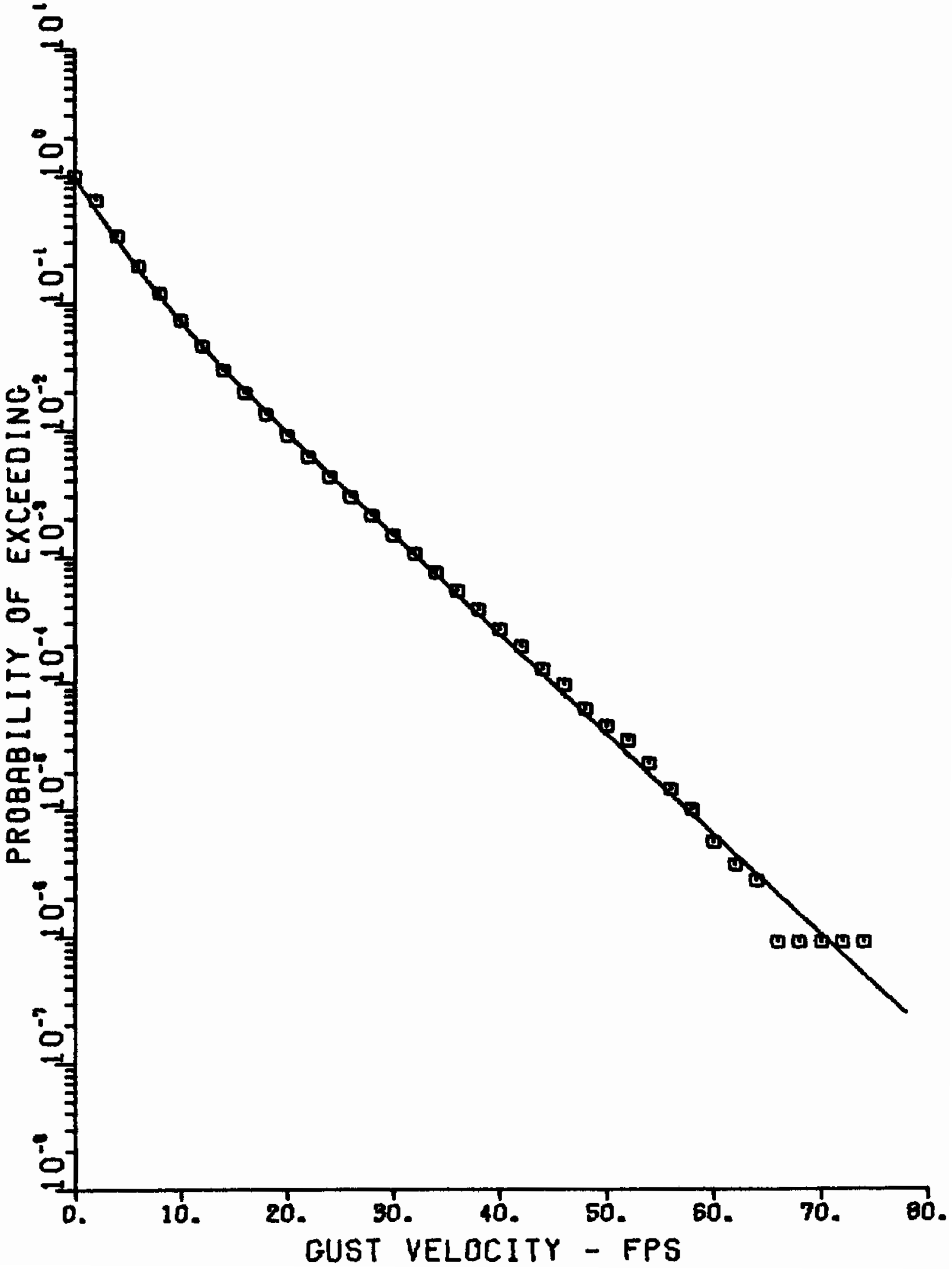


Figure 6
LO-LOCAT, Phase III
All Data, Vertical Component

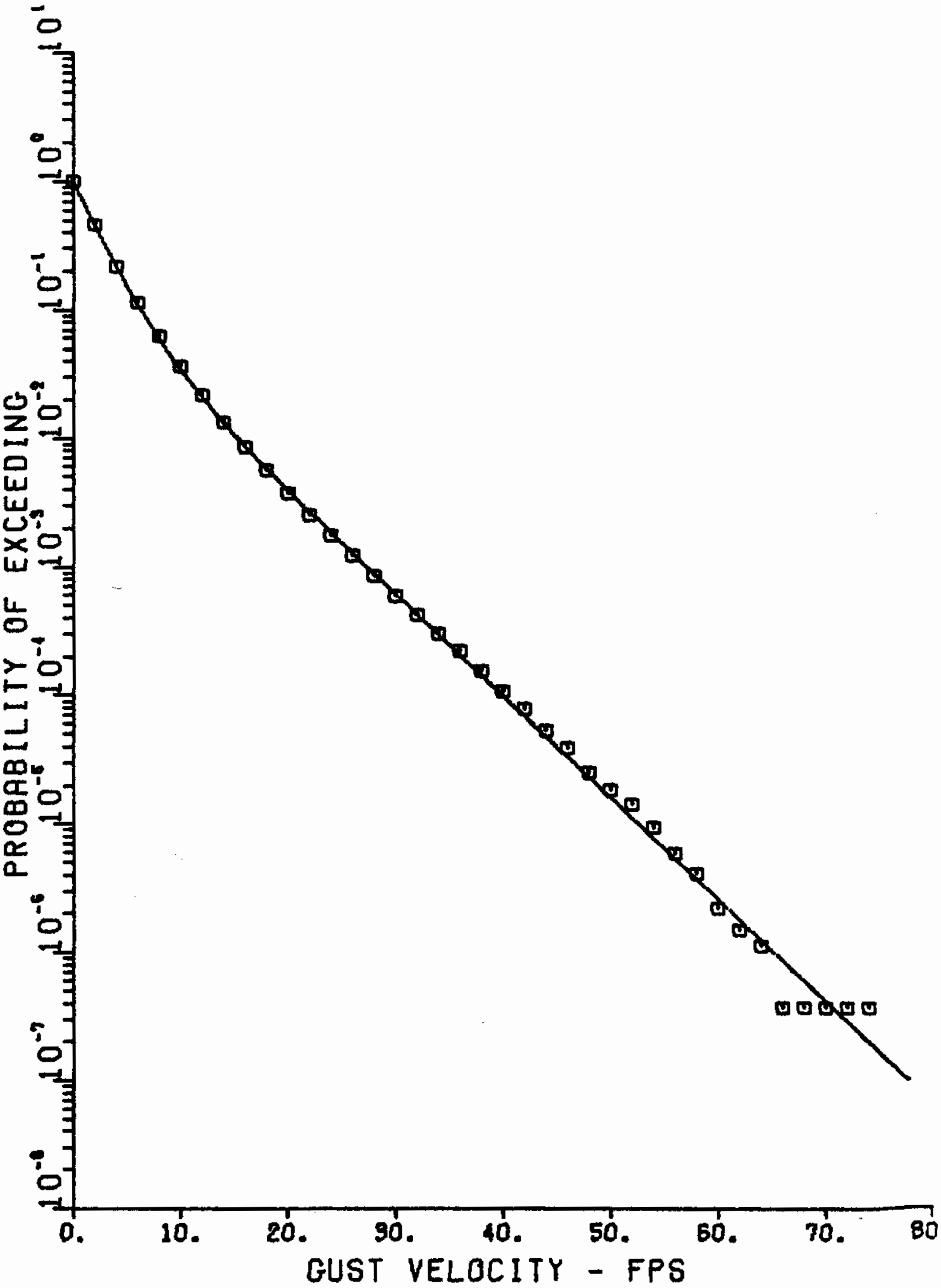


Figure 7
LO-LOCAT, Phase I & II
All Data, Vertical Component

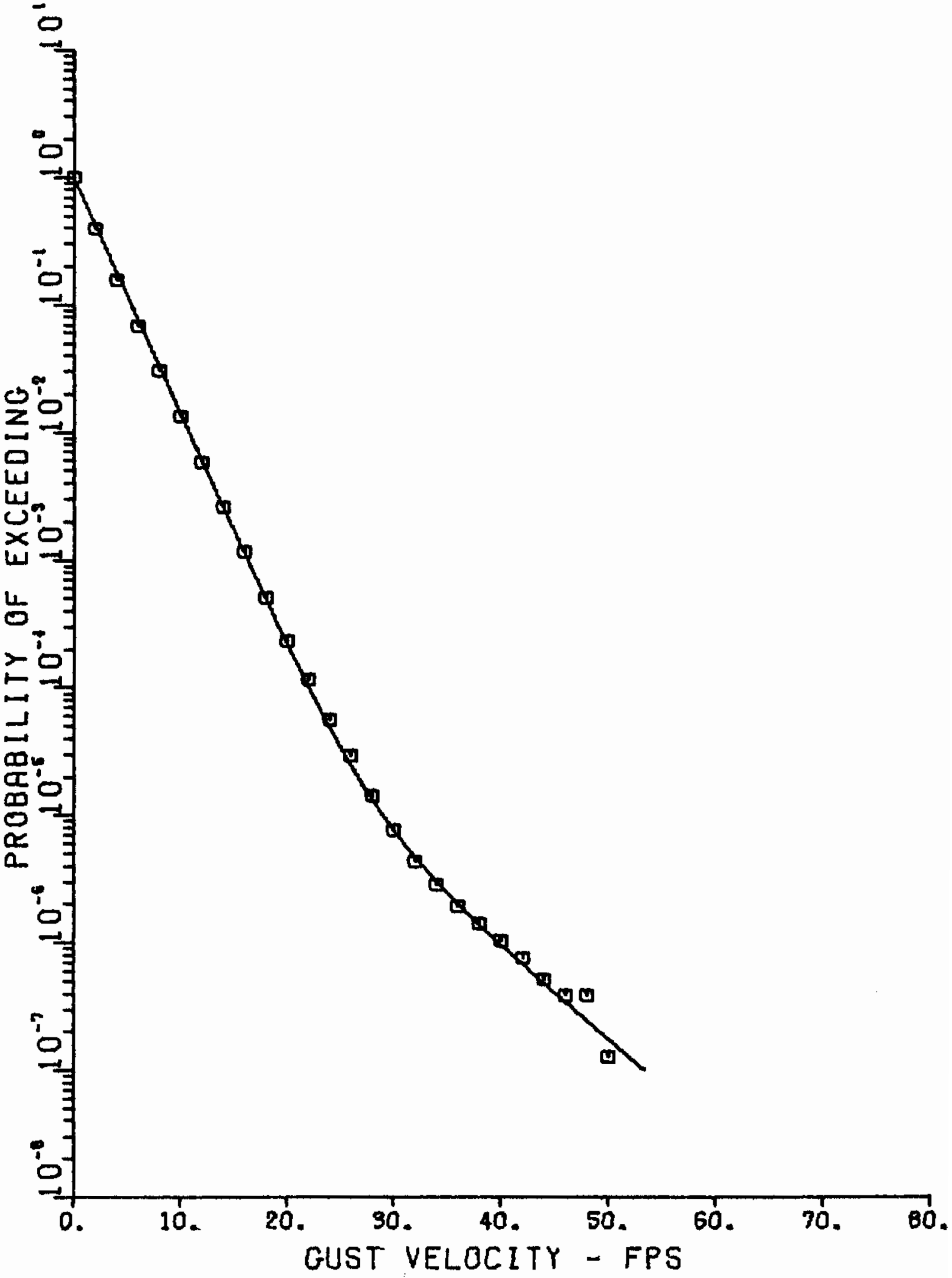


Figure 8
LO-LOCAT, Phase I & II
All Data, Lateral Component

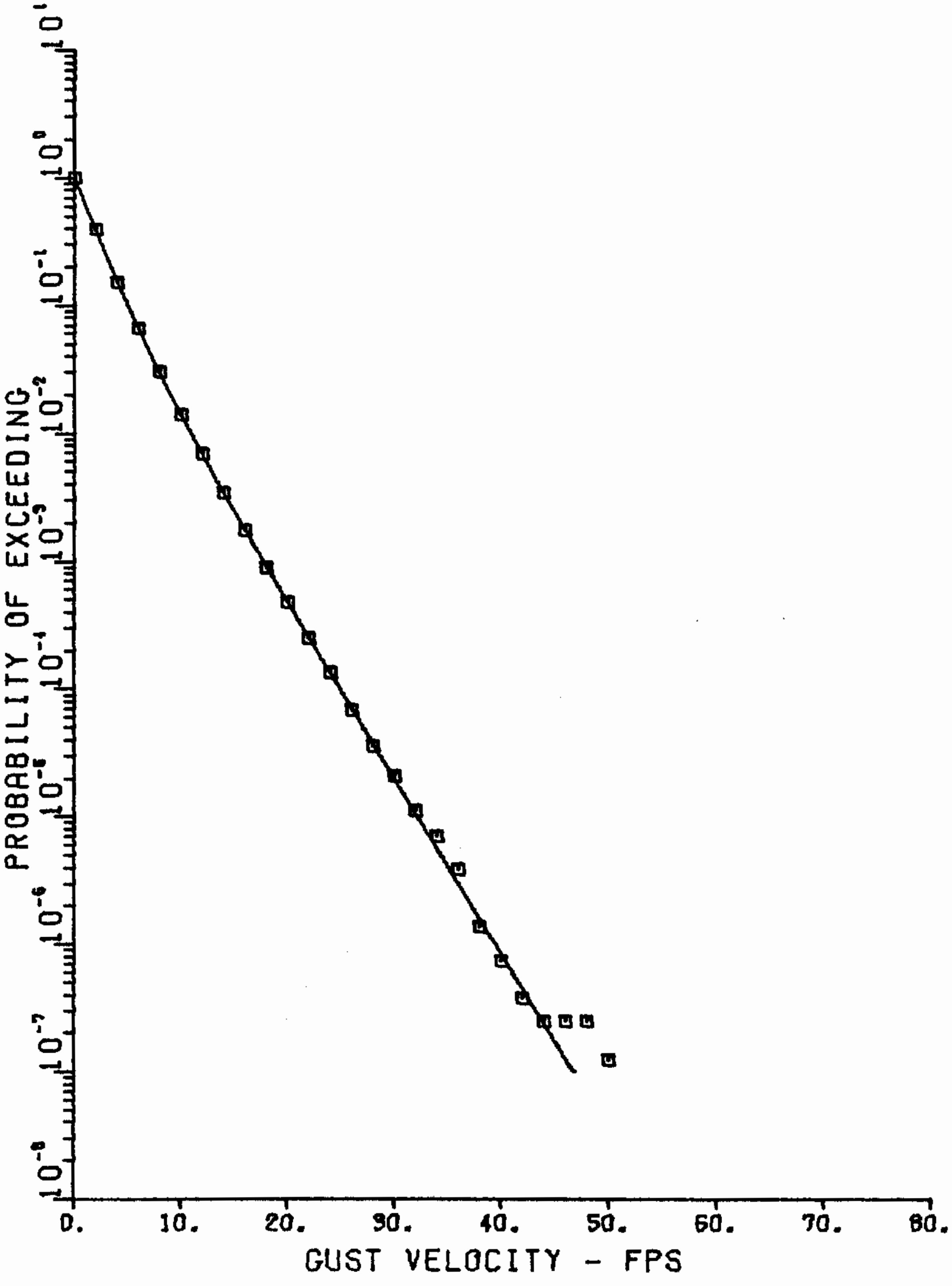


Figure 9
LO-LOCAT, Phase I & II
All Data, Longitudinal Component

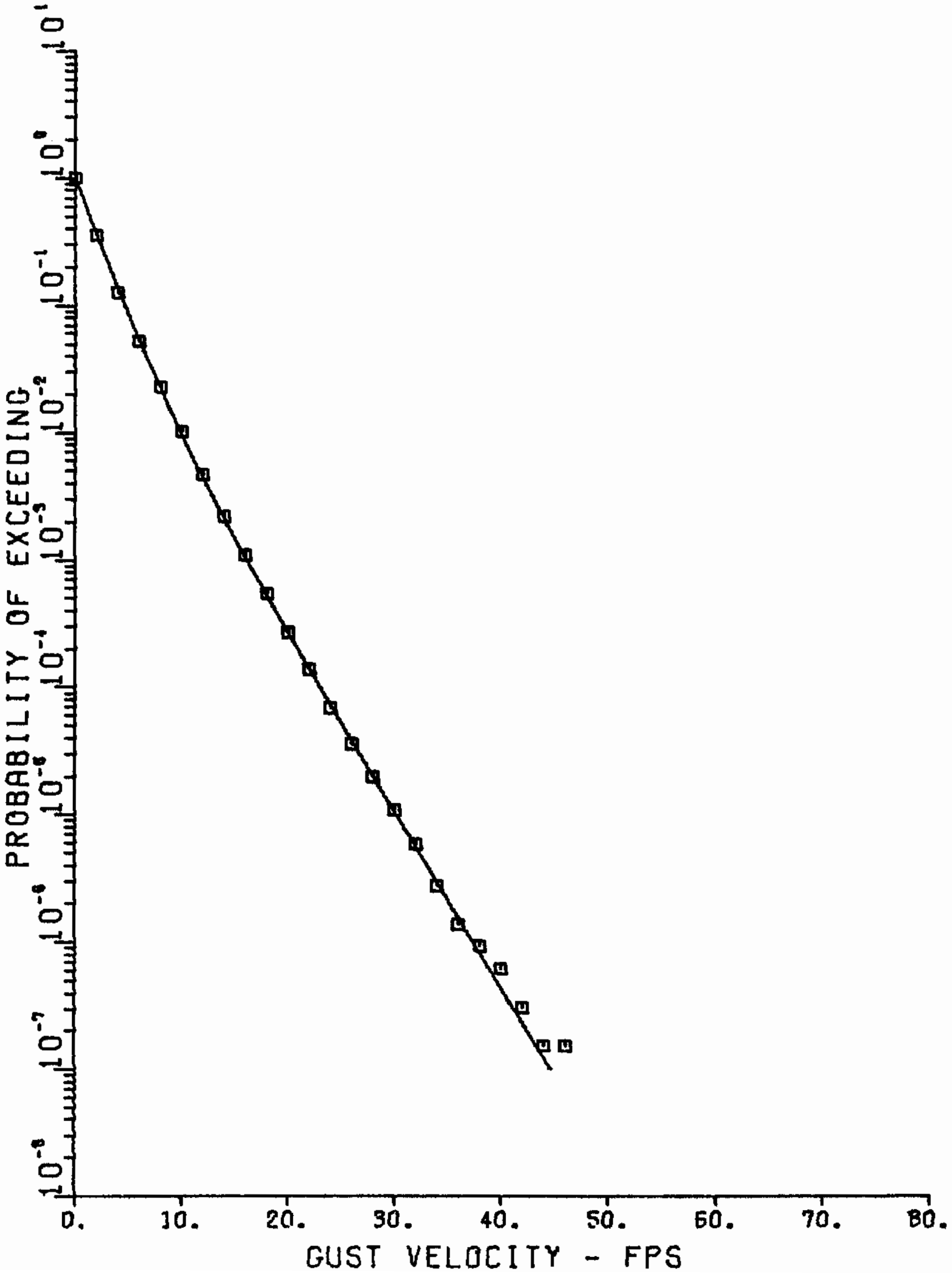


Figure 10
LO-LOCAT, Phase III
Desert, Vertical Component

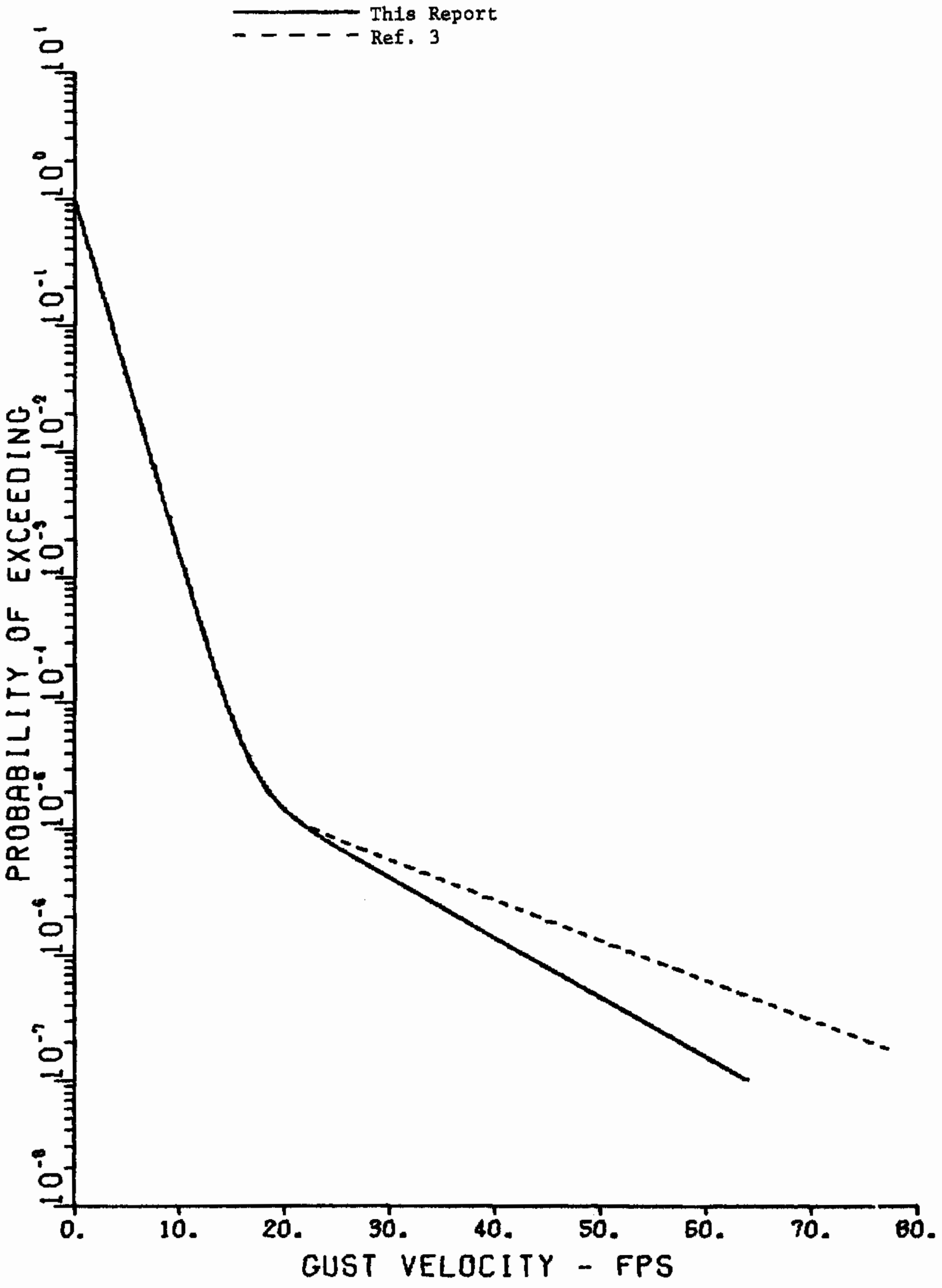


Figure 11
LO-LOCAT, Phase III
High Mountain, Vertical Component

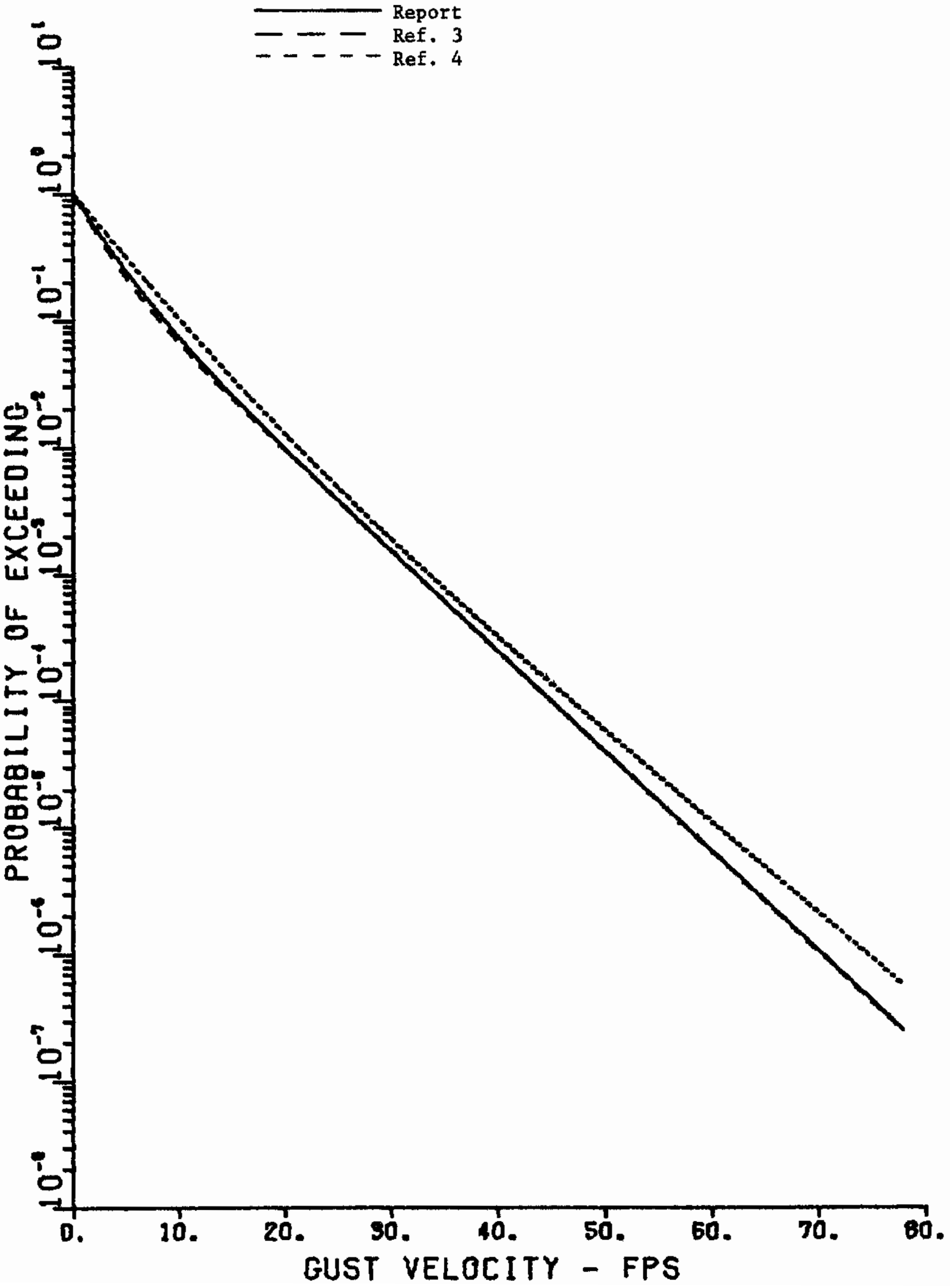


Figure 12
LO-LOCAT, Phase III
All Data, Vertical Component

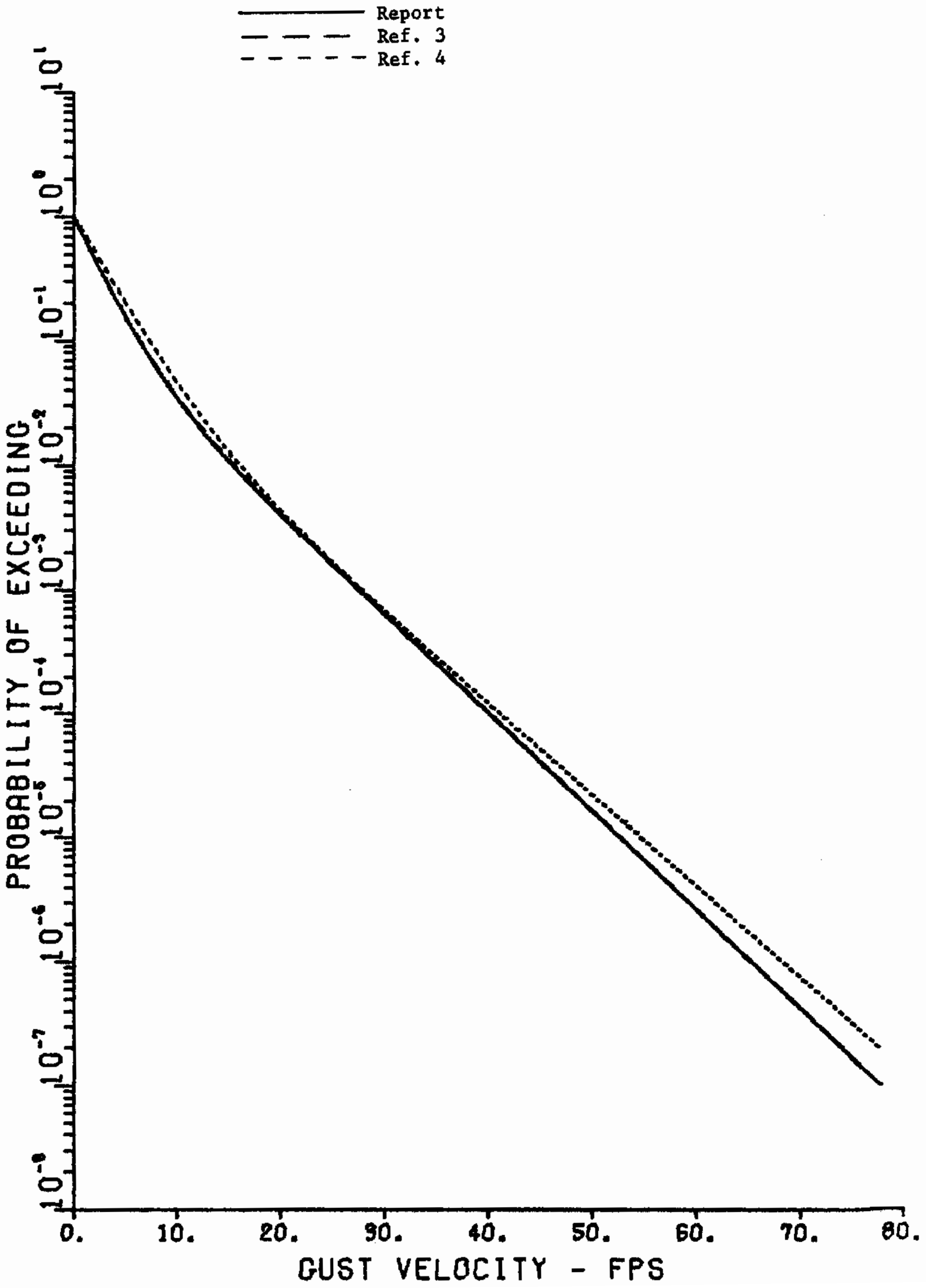


Figure 13
LO-LOÇAT, Phase I & II
All Data, Vertical Component

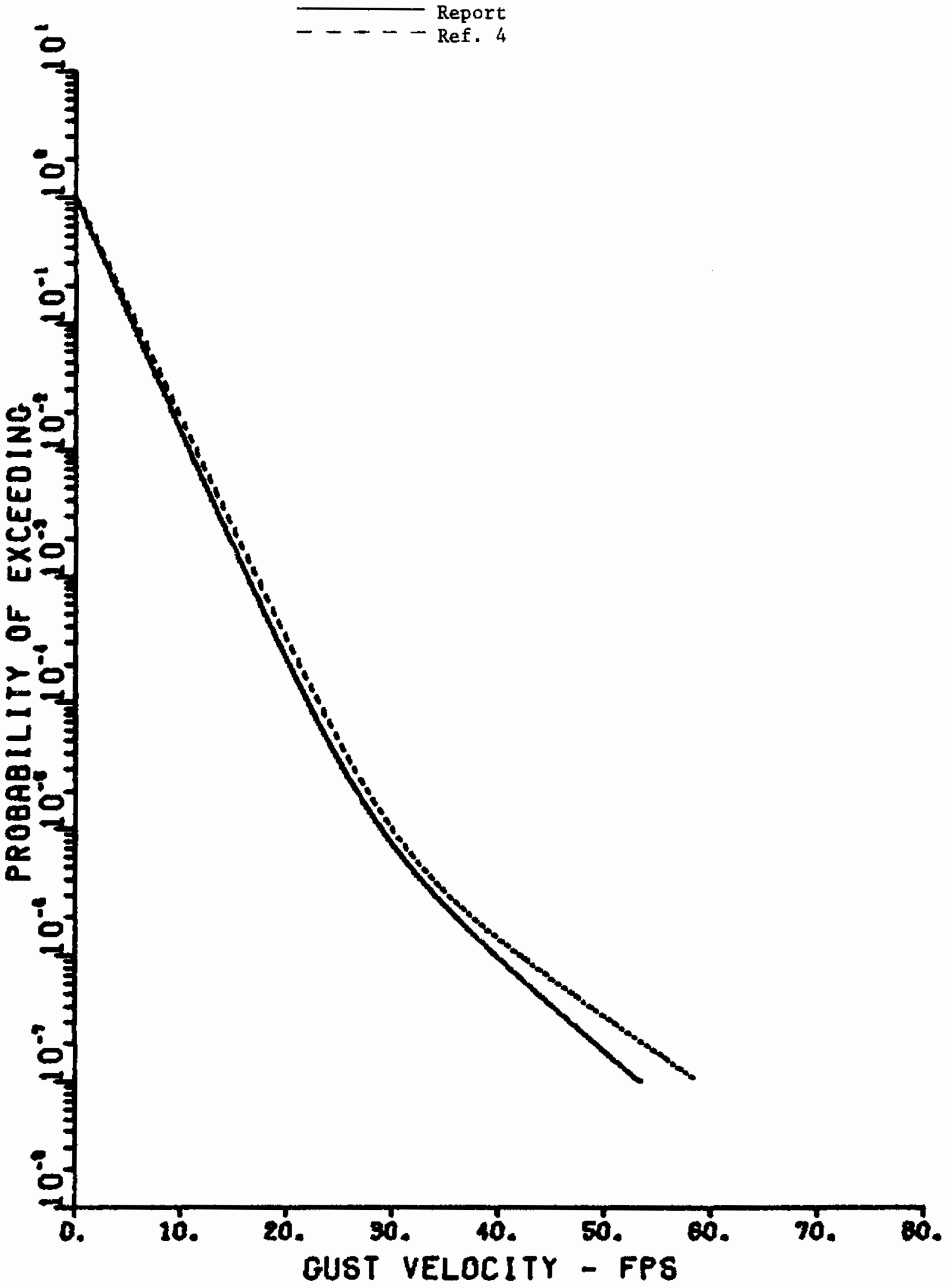


Figure 14
LO-LOCAT, Phase I & II
All Data, Lateral Component

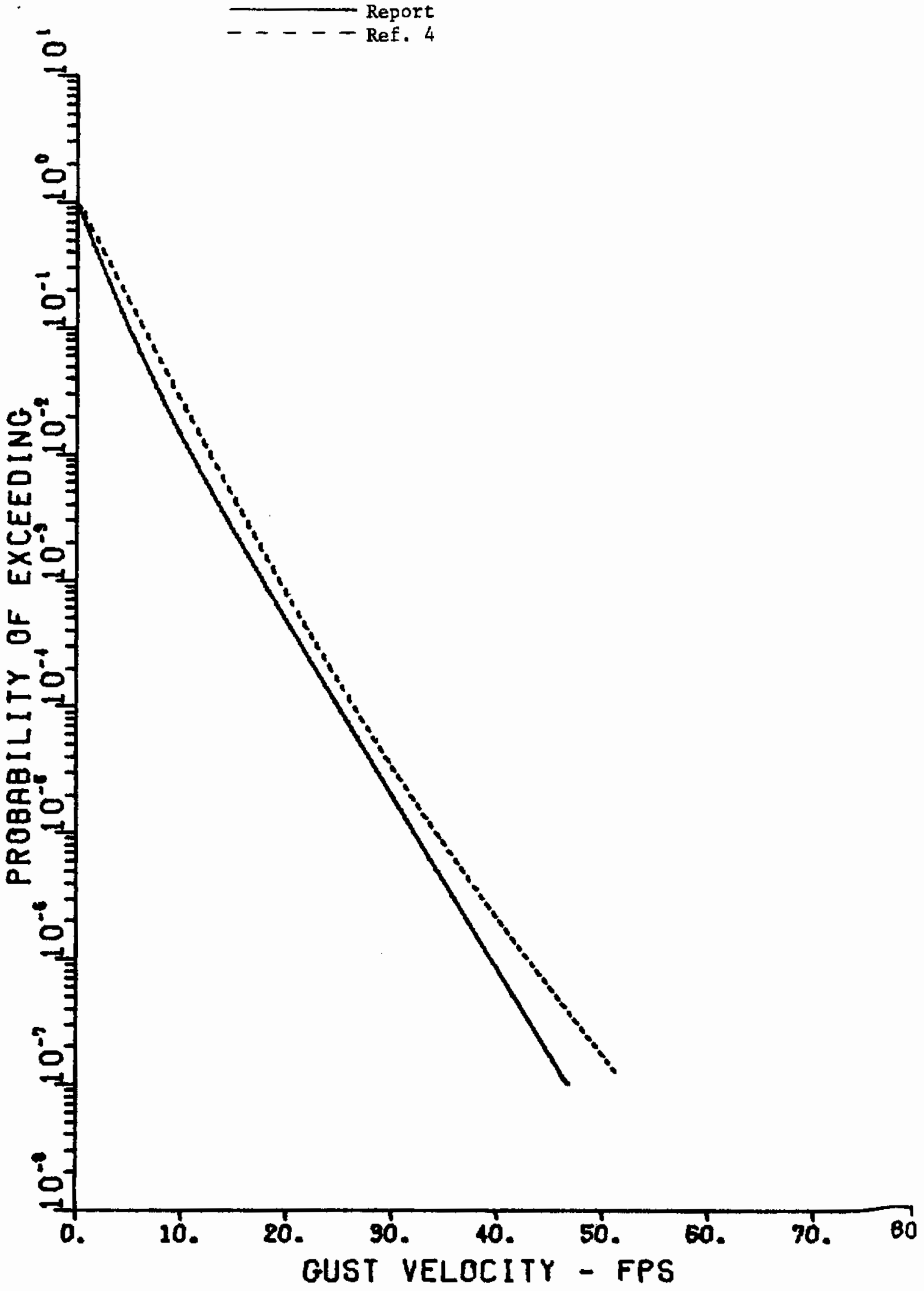
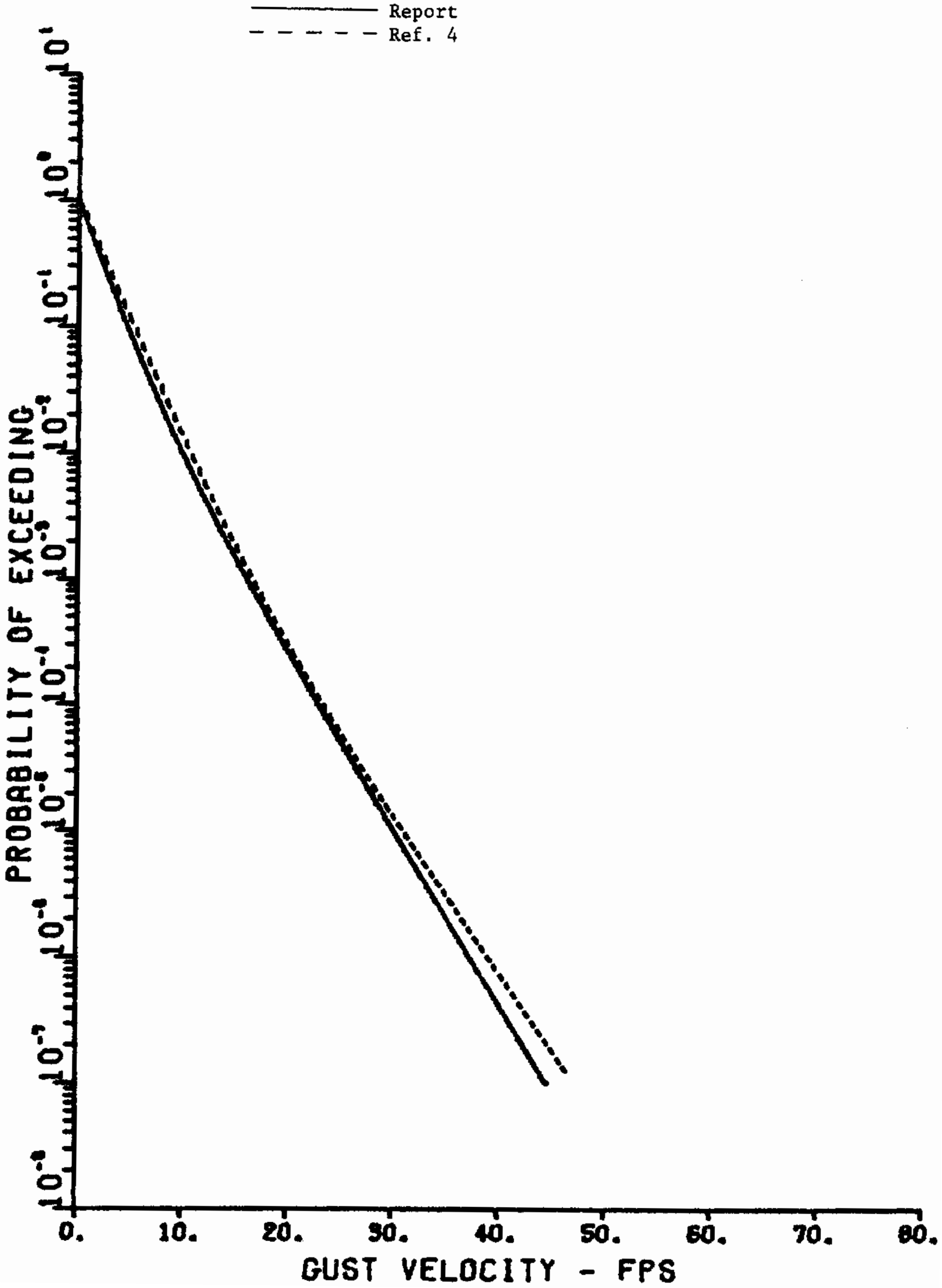


Figure 15
LO-LOCAT, Phase I & II
All Data, Longitudinal Component



SECTION IV
CONCLUSIONS AND RECOMMENDATIONS

The Newton-Raphson Least Squares Percentage Error Method described herein provides a fast consistent method for the determination of "turbulence field parameters". Based on the excellent results obtained in treating the LO-LOCAT data, it is recommended that the procedure be adopted as the standard method for determining P's and b's.

While this procedure is much less subjective than graphically "eyeballing" the data, some subjective judgment is still required. The threshold value of gust velocity, which is defined as turbulence, must be selected. The procedure for treating data with low statistical confidence must be decided, especially for data sets similar to LO-LOCAT, Phase III, Desert, Vertical Component.

Note that the general procedure can be extended to fit data to an equation such as:

$$F(x) = \sum_{i=1}^{N} P_i \, e^{-x/b_i} \qquad (4.1)$$

In fact, the method can be generalized to fit data to any nonlinear function.

SECTION V

REFERENCES

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3. Jones, J. W., et al, "Low Altitude Atmospheric Turbulence, LO-LOCAT Phase III", AFFDL-TR-70-10, W-PAFB, Ohio, November 1970.

4. McCloskey, J. W., et al, "Statistical Analysis of LO-LOCAT Turbulence Data for use in the Development of Revised Gust Criteria", AFFDL-TR-71-29, W-PAFB, Ohio, April 1971.

5. Kuo, S. S., Numerical Methods and Computers, Addison-Wesley Publishing Company, Inc.; Reading, Massachusetts, 1965, pp 90-95, pp 219-227.

6. McCalla, T. R., Introduction to Numerical Methods and FORTRAN Programming, John Wiley & Sons, Inc., New York, 1967, pp 255-260.

7. Deming, W. E., Statistical Adjustment of Data, John Wiley & Sons, Inc., New York, 1943.

8. Gunter, D. E., et al, "Low Altitude Atmospheric Turbulence LO-LOCAT Phase I & II", AFFDL-TR-69-12, W-PAFB, Ohio, February 1969.

APPENDIX
PROGRAM GUSTP

Program GUSTP was written in Extended Fortran for the CDC 6600 at W-PAFB, Ohio. This program evaluates parameters P_1, b_1, and b_2 from the following equation:

$$F(x) = P_1 \, e^{-x/b_1} + (1 - P_1) \, e^{-x/b_2}$$

by the Newton-Raphson Least Square Percentage Error Method and plots F(x) and the data point~~s~~ on a semi-logarithmic plot. Up to three sets of data can be plotted distinctively on one plot. A solid line with data points marked with squares and two different dashed lines with triangular and octagonal markings of data points are available.

Program GUSTP normalizes data so that the first data point is (1,0). It is assumed that Probabilities of Exceeding, f_i, correspond to gust velocities, x_i, starting with zero and occurring in intervals of two feet per second.

Input

The following input cards are required:

CARD 1 CONTROL CARD (I3, 2F5.3, 2I2)

Col. 1-3	N	Number of data points.
4-8	PO(1)	Initial guess for b_1.
9-13	PO(2)	Initial guess for b_2.
14-15	IDASHP	Selects type of line and data point designation - 0 gives solid line and squares, 1 gives fine dash line and triangles, 2 gives large dashes for line and octagons.
16-17	ICON	If ICON = 0 program terminates with current data set, ICON = 1 program will look for another data set.

CARD 2 DATA CARD (2E10.5, 12)

Col. 1-10 C1 - Correction function parameter C_1 from Eqn. (2.10)
11-20 C2 - Correction function parameter C_2 from Eqn. (2.10)
21-22 M - Correction function parameter M from Eqn. (2.10)

CARD 3 DATA CARDS (8E10.5)

Col. 1-10 etc. - Data points f_i from Eqn. (2.6). Points f_i need not be normalized.

If more than one set of data is to be used, repeat cards as given above such that CARD 4 would be in the format of CARD 1, etc.

Output

For each iteration, the following information is printed on line: (1) iteration number, K; (2) b_{10}, b_{20}, P_1 as VECTOR PO; (3) b_1, b_2, P_1 derived in iteration as VECTOR P; (4) a table of (a) gust velocities, X, (b) normalized data points (probabilities of exceeding), F, and (c) calculated values of F using current b_1, b_2, P_1 as FCALC. The final values of b_1, b_2, P_1, P_2 are printed on line as B1, B2, P1, and P2 respectively. After convergence is obtained the data points and calculated F(x) are plotted using CALCOMP routines. Plots of F(x) are terminated at $F(x) = 10^{-7}$ or x = 78 feet per second. The graphs are the size reproduced in this report.

Special Subroutines

Subroutines SPAXIS, LGAXIS, and LGLINE have been attached from the auxiliary file. These subroutines are used in obtaining the CALCOMP plots.

Program Listing

The listing of program GUSTP begins on the next page.

```
      PROGRAM GUSTP (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PLOT)
C
C     PROGRAM GUSTP EVALUATES PARAMETERS P1, B1, AND B2 FROM EQUATION
C                       F(X)=P1*EXP(-X/B1)+(1.-P1)*EXP(-X/B2)
C     BY THE NEWTON-RAPHSON LEAST SQUARE PERCENTAGE ERROR METHOD WITH
C     CORRECTIONS
C
      DIMENSION IBUF(1024),FNN(2605),Y(2605),FNNT(27),YT(27)
      DIMENSION X(80),D1(80),D2(80),D3(80),T(3,3),A(3,3),R(3,3),C(3),
     1P0(3),P(3),B(80),F(80)
      DIMENSION CR(80),FNP(80)
      P1(X,Y)=(EXP(-X/P(2))-Y)
      P2(X)=EXP(-X/P(2))-EXP(-X/P(1))
C
C     DERIVED FUNCTION FOR PROBABILITY OF EXCEEDING
C
      FN(X)=P(3)*EXP(-X/P(1))+(1.-P(3))*EXP(-X/P(2))
      CALL PLOT (0.0,0.0,-3)
      NC=0
C
C     READ NUMBER OF DATA POINTS, N, INITIAL B1, B2 AS P0(1) AND P0(2)
C     RESPECTIVELY, DASH LINE PARAMETER, IDASHP, AND CONTINUATION
C     PARAMETER, ICON
C
  222 READ(5,1) N,P0(1),P0(2),IDASHP,ICON
C
C     READ CORRECTION PARAMETERS C1, C2, AND M
C
      READ(5,77) C1,C2,M
      NC=NC+1
      IF (N.EQ.0) GO TO 10
C
C     READ DATA F(I)
C
      READ (5,2) (F(I),I=1,N)
      F1=F(1)
      DO 3 I=1,N
    3 F(I)=F(I)/F1
      X(1)=0.
      DO 4 I=2,N
    4 X(I)=X(I-1)+2.
      DO 20 I=1,N
      IF(F(N-I).GT.((M-1)+.1)*F(N)) GO TO 21
   20 CONTINUE
   21 J1=N-I+1
C
C     COMPUTE CORRECTION FACTORS, CR(I)
C
      DO 22 I=1,J1
   22 CR(I)=1.
      DO 23 I=J1,N
   23 CR(I)=C1+C2*(I-J1)
C
C     COMPUTE INITIAL P(3) USING LEAST SQUARE PERCENTAGE ERROR WITH
C     COMPUTED CORRECTION FACTORS
```

```
C
      DO 16 I=1,3
   16 P(I)=P0(I)
      XNUM=0.
      XDEN=0.
      DO 19 I=1,N
      XNUM=P1(X(I),F(I))*P2(X(I))/(F(I)*F(I)*CR(I))+XNUM
   19 XDEN=P2(X(I))*P2(X(I))/(F(I)*F(I)*CR(I))+XDEN
      P(3)=XNUM/XDEN
      P0(3)=P(3)
      DO 17 I=1,N
   17 FNP(I)=FN(X(I))
C
C     PRINT INITIAL P(1), P(2), P(3), DATA POINTS, AND CORRESPONDING
C     VALUES OF DERIVED FUNCTION
C
      K=0
      WRITE (6,15) K
      WRITE(6,53)
      WRITE(6,72)(P0(I),I=1,3)
      WRITE (6,41)
      WRITE(6,42)(X(I),F(I),FNP(I),I=1,N)
      K=1
   13 DO 5 I=1,N
      D1(I)=X(I)*EXP(-X(I)/P0(1))
      D2(I)=X(I)*EXP(-X(I)/P0(2))
      D3(I)=EXP(-X(I)/P0(1))-EXP(-X(I)/P0(2))
    5 B(I)=EXP(-X(I)/P0(2))-F(I)-P0(3)/P0(1)*D1(I)-(1.-P0(3))/P0(2)*
     1D2(I)
C
C     PRINT K-1 ITERATIVE VALUES OF P(1), P(2), P(3)
C
      WRITE (6,15) K
      WRITE (6,53)
      WRITE (6,72) (P0(I),I=1,3)
      DO 6 I=1,3
      C(I)=0.
      DO 6 J=1,3
      T(I,J)=0.
    6 A(I,J)=0.
      DO 7 I=1,N
      C(1)=C(1)-D1(I)*B(I)/(F(I)*F(I)*CR(I))
      C(2)=C(2)-D2(I)*B(I)/(F(I)*F(I)*CR(I))
      C(3)=C(3)-D3(I)*B(I)/(F(I)*F(I)*CR(I))
      A(1,1)=A(1,1)+D1(I)*D1(I)/(F(I)*F(I)*CR(I))
      A(1,2)=A(1,2)+D1(I)*D2(I)/(F(I)*F(I)*CR(I))
      A(1,3)=A(1,3)+D1(I)*D3(I)/(F(I)*F(I)*CR(I))
      A(2,2)=A(2,2)+D2(I)*D2(I)/(F(I)*F(I)*CR(I))
      A(2,3)=A(2,3)+D2(I)*D3(I)/(F(I)*F(I)*CR(I))
    7 A(3,3)=A(3,3)+D3(I)*D3(I)/(F(I)*F(I)*CR(I))
      A(2,1)=A(1,2)
      A(3,1)=A(1,3)
      A(3,2)=A(2,3)
      T(1,1)=P0(3)/(P0(1)*P0(1))
      T(2,2)=(1.-P0(3))/(P0(2)*P0(2))
```

```
      T(3,3)=1.
      CALL MMPY (A,T,R,3,3,3)
      CALL MINV (R,R,3)
      CALL MMPY (R,C,P,3,3,1)
C
C     PRINT KTH ITERATIVE VALUES OF P(1), P(2), P(3), DATA POINTS,
C     AND CORRESPONDING VALUES OF THE DERIVED FUNCTION
C
      WRITE (6,76)
      WRITE (6,72) (P(I),I=1,3)
      DO 18 I=1,N
   18 FNP(I)=FN(X(I))
      WRITE (6,41)
      WRITE(6,42)(X(I),F(I),FNP(I),I=1,N)
C
C     CONVERGENCE TEST
C
      E1=(P0(1)-P(1))/P0(1)
      E2=(P0(2)-P(2))/P0(2)
      E3=(P0(3)-P(3))/P0(3)
      IF(ABS(E1).GT.1.E-7)GO TO 8
      IF(ABS(E2).GT.1.E-7)GO TO 8
      IF(ABS(E3).GT.1.E-7)GO TO 8
      P3=1.-P(3)
C
C     PRINT FINAL VALUES OF P(1), P(2), P(3), 1.-P(3)
C
      WRITE (6,9) (P(I),I=1,3),P3
C
C     CALCOMP PLOT ROUTINES
C
      X(N+1)=0.
      X(N+2)=13.333334
      F(N+1)=1.E-08
      F(N+2)=1.125
      IF(NC.GT.1) GO TO 24
      CALL PLOT (1.0,-11.0,-3)
      CALL PLOT (.5,.75,-3)
      CALL SPAXIS (0.0,0.0,19HGUST VELOCITY - FPS,-19,6.,0.0,X(N+1),X(N+
     12),1.65,-.5,.14,0.,.75,0,0.0)
      CALL LGAXIS (0.0,0.0,24HPROBABILITY OF EXCEEDING,24,8.00,90.0,F(N
     1+1),F(N+2))
   24 CALL LGLINE(X,F,N,-1,IDASHP,1)
      DY=.03
      DO 101 I=1,2600
      Y(I)=DY*(I-1)
      FNN(I)=FN(Y(I))
      IF(FNN(I).LE.1.E-7) GO TO 177
  101 CONTINUE
      I=2600
  177 II=I
      Y(II+1)=X(N+1)
      Y(II+2)=X(N+2)
      FNN(II+1)=F(N+1)
      FNN(II+2)=F(N+2)
```

```
      IF (IDASHP.EQ.0) GO TO 888
      JJ=10
      IF(IDASHP.EQ.2) JJ=25
      J2=2*JJ
      FNNT(JJ+1)=FNN(II+1)
      FNNT(JJ+2)=FNN(II+2)
      YT(JJ+1)=Y(II+1)
      YT(JJ+2)=Y(II+2)
      NN=INT(FLOAT(II)/FLOAT(J2))
      DO 901 L=1,NN
      DO 902 I=1,JJ
      YT(I)=Y(I+J2*(L-1))
  902 FNNT(I)=FNN(I+J2*(L-1))
  901 CALL LGLINE(YT,FNNT,JJ,0,0,1)
      GO TO 90
  888 CALL LGLINE(Y,FNN,II,0,0,1)
   90 IF(ICON.EQ.1) GO TO 222
      CALL PLOT (0.0,0.0,-3)
      CALL PLOTE
      GO TO 10
C
C     FAILURE TO CONVERGE CRITERIA
C
    8 IF (K.GE.200) GO TO 11
      DO 12 I=1,3
   12 P0(I)=P(I)
      K=K+1
      GO TO 13
   11 WRITE (6,14) K,(P0(I),I=1,3),(P(I),I=1,3)
   10 STOP
    1 FORMAT(I3,2F5.3,2I2)
    2 FORMAT (8E10.5)
    9 FORMAT(/3X,4HB1 =F7.4,5X,4HB2 =F7.4,5X,4HP1 =F7.4,5X,4HP2 =F7.4)
   14 FORMAT(/1X,17HDOES NOT CONVERGE,3HK =I5,5X,4HP0 =3F7.4,5X,3HP =3F7
     1.4)
   15 FORMAT (///3X,3HK =I5/)
   41 FORMAT (/5X,1HX,16X,1HF,19X,5HFCALC/)
   42 FORMAT (3X,F4.1,9X,E12.5,9XE12.5)
   53 FORMAT (/3X,9HVECTOR P0/)
   72 FORMAT (3(3X,E12.5))
   76 FORMAT (/3X,8HVECTOR P/)
   77 FORMAT(2E10.5,I2)
      END
```

```
      SUBROUTINE MINV(A,B,N)
C
C            MATRIX INVERSION SUBROUTINE
C
C     CALLING SEQUENCE...
C
C     CALL MINV(A,B,N)
C
C        A IS THE INPUT MATRIX (DIMENSIONED N X N)
C        B WILL BE THE INVERSE MATRIX (DIMENSIONED N X N)
C        N IS THE DIMENSION OF A AND B
C
C     NOTE...
C     CALL MINV(A,A,N)
C        CAUSES THE MATRIX A TO BE REPLACED BY ITS INVERSE.
C
C     209 CELLS OF BLANK COMMON ARE USED.
C
      DIMENSION    A(N,N), B(N,N)
      COMMON   I, I1, I2, IP, J, J1, NN, PE, TPE
      COMMON   K(100), P(100)
      DATA    NMAX/ 100/
C
      NN=N
      IF (NN-NMAX) 10,10,500
   10 IF (NN) 500,500,20
   20 DO 30 I=1,NN
      K(I)=I
      DO 30 J=1,NN
   30 B(I,J)=A(I,J)
      DO 330 I=1,NN
      I2=NN-I+1
      PE=0.
      DO 120 I1=1,I2
      TPE=B(I1,1)
      IF (ABS(PE)-ABS(TPE)) 100,100,120
  100 PE=TPE
      IP=I1
  120 CONTINUE
      IF (PE) 160,510,160
  160 DO 170 J=2,NN
  170 P(J-1)=B(IP,J)/PE
      P(NN)=1.0/PE
      IP=K(IP)
      I2=0
      DO 310 J=1,NN
      I1=J-I2
      K(I1)=K(J)
      IF (K(J)-IP) 260,250,260
  250 I2=1
      GO TO 310
  260 TPE=-B(J,1)
      DO 300 J1=2,NN
  300 B(I1,J1-1)=B(J,J1)+TPE*P(J1-1)
      B(I1,NN)=TPE*P(NN)
```

```
 310   CONTINUE
       DO 320  J=1,NN
 320   B(NN,J)=P(J)
 330   K(NN)=IP
       DO 410  I=1,NN
       DO 400  J=1,NN
       I1=K(J)
 400   P(I1)=B(I,J)
       DO 410  J=1,NN
 410   B(I,J)=P(J)
       RETURN
 500   PRINT 1000, NN
       CALL SYSTEM(200,1L )
       RETURN
 510   PRINT 1001
       CALL SYSTEM(200,1L )
       RETURN
1000   FORMAT  (3H0N=,I12,33H IS INCORRECT FOR SUBROUTINE MINV)
1001   FORMAT  (40H0SUBROUTINE MINV FINDS MATRIX A SINGULAR)
       END
```

```
      SUBROUTINE MMPY(A,B,C,M,K,N)
C     MMPY     MATRIX MULTIPLICATION
C
C     MMPY WILL COMPUTE THE MATRIX PRODUCT C=A*B
C
C     USAGE...
C
C     DIMENSION A(M,K), B(K,N), C(M,N)
C     CALL MMPY(A,B,C,M,K,N)
C
C     MMPY USES 3 CELLS OF BLANK COMMON
C
      DIMENSION   A(M,K), B(K,N), C(M,N)
      COMMON          I, J, L
      DO 10 J=1,N
      DO 10 I=1,M
      C(I,J)=0.0
      DO 10 L=1,K
   10 C(I,J)=C(I,J)+A(I,L)*B(L,J)
      RETURN
      END
```

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13. ABSTRACT

A Newton-Raphson Least Squares Percentage Error Method is developed for the determination of atmospheric turbulence field parameters. A correction function is proposed to deemphasize the effects of data points with low statistical confidence.

The method is used on various sets of LO-LOCAT data to demonstrate the excellence of the curve fits obtained. Comparisons are made with other curve fits found in the literature.

It is recommended that this method be adopted as the standard method for the determination of atmospheric turbulence field parameters.

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