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# DYNAMIC SYSTEM STUDIES: ANALOG COMPUTATION

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# FOREWORD

The Advisory Board on Simulation has concluded a three-year research program in air weapon system dynamics sponsored by Wright Air Development Center, with P. W. Nosker/WCRR as project engineer. This volume is one of the following 16 comprising the final report, WADC TR 54-250, entitled Dynamic System Studies:

| Part No. | Subtitle | Editing Agency |
|---|---|---|
| 1 | Conclusion and Recommendations | University of Chicago |
| 2 | The Design of a Facility | University of Chicago |
| 3 | The Mission of a Facility **(Confidential)** | University of Chicago |
| 4 | Technical Staff Requirements | University of Chicago |
| 5 | Analog Computation | Naval Ordnance Laboratory - Corona |
| 6 | Operation and Maintenance Procedures for Analog Computers | University of Chicago |
| 7 | Digital Computers | University of Chicago |
| 8 | Recorders | University of Chicago |
| 9 | Flight Tables **(Confidential)** | University of Chicago |
| 10 | Performance Requirements for Flight Tables | Massachusetts Institute of Technology |
| 11 | Load Simulators **(Confidential)** | Cook Research Lab. |
| 12 | Guidance Simulation **(Secret)** | Naval Ordnance Laboratory - Corona |
| 13 | Error Studies | University of Chicago |
| 14 | Error Analysis for Differential Analyzers (written by F. J. Murray, Columbia U., and K. S. Miller, N.Y.U.) | University of Chicago |
| 15 | Air Vehicle Characteristics **(Secret)** | University of Chicago |
| 16 | Aerodynamic Studies (written by M. Z. Krzywoblocki, U. of Illinois) | University of Chicago |



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FOREWORD

The history of the project and a complete bibliography may be found in Part 1. All reports may be obtained through the project engineer.

This report represents the culmination of the assignment to determine the proper mission, equipmentation, operating procedures, and personnel for an engineering facility in the field of air weapon systems dynamics. The subdivision of the report correspond to these four basic objectives and the subsidiary work in their support, and reflect the role of simulation as a dominant technique. The functions of each part and the relations among them are indicated in the technical summary, Part 2.

The following organizations have participated directly in the program:

| Organization | Contract No. | Time of Performance |
|---|---|---|
| University of Chicago | AF33(038)-15068 Supplements 2 and 11 | 1 Feb '51 - 31 Aug '54 |
| J. B. Rea Company | AF33(038)-15068 Subcontract 2 | 1 Feb '51 - 31 Oct '52 |
| Cook Research Labs | AF33(038)-15068 Subcontracts 3 and 9 | 1 Feb '51 - 31 May '54 |
| RCA Laboratories | AF33(038)-15068 Subcontract 4 | 1 Feb '51 - 1 Mar '53 |
| Armour Res. Foundation of Ill. Inst. of Technology | AF33(038)-15068 Subcontract 5 | 1 Feb '52 - 30 Nov '52 |
| Northwestern University, Aerial Meas. Lab. | AF33(038)-15068 Subcontract 8 | 17 Jul '52 - 22 Aug '52 |
| Mass. Inst. of Technology, Flight Control Lab. | AF33(038)-15068 Purchase Order A2086 | 20 Apr '54 -31 Aug '54 |
| Mass. Inst. of Technology, Dynamic Analysis and Control Laboratory | AF33(038)-15068 Purchase Order A23883 | 22 Jul '53 -30 Nov. '53 |
| Mass. Institute of Technology, D. A. C. L. | AF33(616)-2263 Task Statement 2 | 1 Dec '53 - 30 Sept. '54 |
| Nat'l Bur. of Standards, Corona (which became) | AF33(038)-51-4345-E | 25 Feb '51 - Sep '53 |
| Naval Ordnance Lab., Corona | MIPR(33-616)54-154 | 20 Nov '53 - 31 Dec '55 |

This is a record of formal participation only; the program was aided immeasurably by the splendid cooperation of all governmental, industrial, and educational organizations (particularly the simulation laboratories) contacted. Although it is impractical to mention them all here, the extent



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of their assistance is evident throughout the reports and is hereby gratefully acknowledged. Details of these affiliations, including statements of work, may be found throughout the 21 Bimonthly Progress Reports issued by the University of Chicago during the course of the work. (All formal participation in the program is recorded above; missing supplement and subcontract numbers do not pertain to this project.)

The University of Chicago was assigned prime responsibility for integration of the program. This has been effected by a full-time staff at the University and by periodic meetings of the following advisory committee, selected by the Air Force:

| | | |
|---|---|---|
| Dean Walter Bartky, Chairman | University of Chicago | 1 Feb '51 - 31 Aug '54 |
| Prof. C. S. Draper | Mass. Inst. of Tech. | 1 Feb '51 - 28 Feb '53 |
| Mr. Donald Mc Donald | Cook Research Lab. | 1 Feb '51 - 31 Aug '54 |
| Prof. F. J. Murray | Columbia University | 1 Apr '52 - 31 Aug '54 |
| Dr. J. B. Rea | J. B. Rea Company | 1 Feb '51 - 28 Feb '53 |
| Prof. R. C. Seamans, Jr. | Mass. Inst. of Tech. | 1 Sep '53 - 31 Aug '54 |
| Mr. R. J. Shank | Hughes Aircraft Co. | 1 Jul '51 - 31 Aug '54 |
| Dr. H. K. Skramstad | NBS-NOLC | 1 Feb '51 - 31 Aug '54 |
| Mr. A. W. Vance | RCA Laboratories | 1 Feb '51 - 31 Aug '54 |
| ex officio: | | |
| Mr. P. W. Nosker, Project Engineer | WADC | 1 Feb '51 - 31 Aug '54 |
| Dr. B. E. Howard, Secretary | University of Chicago | 1 Feb '51 - 31 Aug '54 |

The meetings have been recorded in the Bimonthly Progress Reports previously mentioned. Except for Dr. Skramstad, who has participated through direct arrangement between NBS-NOLC and WADC, members of the advisory committee who are not connected directly with the University have participated in the program through consulting agreements with the University of Chicago. In addition, similar consulting agreements with the University have provided for the participation of:

| | | |
|---|---|---|
| Dr. R. R. Bennett | Hughes Aircraft Co. | 1 Jan '52 - 31 Jan '54 |



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|---|---|---|
| Mr. J. P. Corbett | Libertyville, Ill. (formerly with the U. of C.) | 11 May '54 - 31 Aug '54 |
| Dr. Thornton Page | Johns Hopkins U. (formerly with the University, and Sec't to the Board until 1 Aug '51) | 7 Aug '51 - 1 Mar '53 |
| Prof. M. Z. Krzywoblocki | University of Illinois | 15 Jan '52 - 31 Aug '54 |
| Prof. K. S. Miller | New York University | 2 Nov '53 - 31 Aug '54 |
| Dr. J. Winson | Riverside, New York (formerly consultant to Project Cyclone) | 1 Mar '53 - 30 June '54 |
| Mr. G. L. Landsman | Motorola, Inc. | 1 May '54 - 31 Aug '54 |

Many others have contributed significantly to the progress of the work. Among those from other organizations in regular attendance at most of the meetings of the committee have been Mr. Charles F. West, Air Force Missile Test Center; Prof. L. L. Rauch, University of Michigan, representing Arnold Engineering Development Center; Col. A. I. Lingard, WADC; and Dr. F. W. Bubb, WADC.

Coordination of the program and administration of the prime contract at the University of Chicago has been under the charge of Dr. Walter Bartky, Dean of the Division of Physical Sciences and Director of the Institute for Air Weapons Research; Dr. B. E. Howard, Assistant to the Director; and Messrs. William R. Allen and William J. Riordan, Group Leaders. The work at the cooperating institutions has been directed by the appropriate member of the advisory committee and his assistants: Dr. H. K. Skramstad and Mr. Gerald L. Landsman at the National Bureau of Standards-Naval Ordnance Laboratory, Corona; Messrs. Donald McDonald and Jay Warshawsky at Cook Research Laboratories; Messrs. A. W. Vance, J. Lehman, and Dr. E. C. Hutter at RCA Laboratories; Dr. J. B. Rea at J. B. Rea Company; Prof. R. C. Seamans at the Flight Control Laboratory and Dr. W. W. Seifert and Mr. H. E. Blanton at the Dynamic Analysis and Control Laboratory, Massachusetts Institute of Technology. V. H. Disney, S. Hori, and G. F. Warnke at Armour Research Foundation and J. C. MacAnulty and George Goelz at Northwestern University, Aerial Measurements Laboratory have directed the contributory studies at their respective organizations. More explicit credit is found in appropriate places throughout the reports; biographical sketches are in Part 1. Space does not allow full credit that is due to all the workers on the combined project, but special mention is certainly due the project engineer for his conception of the project and for his cooperation during its execution.



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## ABSTRACT

Recently available analog computer components and systems are described and their characteristics discussed. No comparisons or evaluations are made, since their design and utilization is at present in an empirical state of development.


## PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

Aldro Lingard

ALDRO LINGARD
Colonel, USAF
Chief, Aeronautical Research Laboratory
Directorate of Research



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TABLE OF CONTENTS


| | Page |
|---|---|
| 1. Introduction | 1 |
| 2. General purpose analog computers | 2 |
| 3. Analog computer components | 3 |
| 4. Future developments | 8 |
| References | 10 |
| Appendix 1 - Commercially available analog computing equipment | 13 |
| 2 - Operating characteristics of the Reeves analog computing equipment | 32 |
| 3 - Measurement techniques | 56 |
| 4 - Analysis of an operational amplifier | 65 |


## ILLUSTRATIONS


| | |
|---|---|
| Figure 1 - Operational amplifier | 4 |
| 2 - Amplifier amplitude response schematic | 34 |
| 3 - Amplifier phase response schematic | 34 |
| 4 - Average amplitude response of four REAC amplifiers | 35 |
| 5 - Phase response of a REAC amplifier | 36 |
| 6 - Amplifier drift schematic | 38 |
| 7 - Schematic of damping effect (with amplifier in loop) | 38 |
| 8 - REAC oscillator damping effect | 40 |
| 9 - Schematic of servo amplitude and phase response | 41 |




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Contrails

Illustrations (cont.)

Page

Figure 10 - Amplitude response of two low-speed servos . . . . 43
11 - Phase shift characteristic of two low-speed servos . . . . . 44
12 - Amplitude response of two high-speed servos . . . 45
13 - Phase shift characteristic of two high-speed servos . . . . . 46
14 - Servo velocity and acceleration limit schematic . . 47
15 - Servo backlash schematic . . . . . 47
16 - REAC recorder amplitude response schematic . . . 49
17 - REAC recorder amplitude response . . . . . 50
18 - REAC recorder linearity schematic . . . . . 51
19 - REAC recorder linearity . . . . . 52
20 - Plotting board amplitude response schematic . . . . 54
21 - Plotting board amplitude and phase response . . . . 55
22 - Plotting board phase response schematic . . . . . 56
23 - Computer circuit phase shift measurements . . . . 60
24 - Phase shift for one REAC inverter . . . . . 62
25 - Phase lag of one integrator . . . . . 63
26 - Phase shift of GEDA L3 summer . . . . . 64
27 - Operational amplifier . . . . . 65
28 - Equivalent block diagram . . . . . 67
29 - Basic amplifier circuit . . . . . 68

Table 1 - Damping effect data (with amplifier in loop) . . . . 37
2 - Velocity and acceleration limits of low-speed servos . . . . . 42
3 - Velocity and acceleration limits of plotting board . . . . . 57



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# 1. INTRODUCTION

The problems of flight simulation involve solutions of systems of high order linear and nonlinear differential equations that represent the aerodynamics, guidance, and control systems of a missile or aircraft. In physical simulation, where actual physical components of a missile or airplane under study "simulate themselves," it is required that the solution proceed at a real-time rate. A study of the problems of flight simulation indicates that the required speed and accuracy can be achieved with analog computing equipment. Equipment is available or is being developed that has or will have the accuracy, speed of response, and range of parameters required by this basically physical problem.

The analog computers considered in this volume are the modern electronic types sometimes referred to as electronic differential analyzers. Such computers usually may be placed in one of two broad categories: (1) special purpose, or (2) general purpose computers. Both of these classes of computers utilize the same types of computing elements. The differences lie largely in the fact that special purpose computers are permanently or semipermanently wired to solve one particular set of equations; general purpose computers may be set up to solve any of a large class of problems, with a premium placed upon the ease of changing problems. A large computing installation might very well include units of both types.

The variables of problems handled by electronic analog computers are represented by electrical voltages and perhaps by servo shaft positions. The independent variable is always a multiple of real physical time and may or may not be represented explicitly by voltages varying linearly with time. It is considered essential that modern large-scale computers intended for simulation of guided missiles and other high-speed aircraft be capable of solving such problems in real (1 to 1) time. (This allows portions of the physical equipment of the actual aircraft to be included in the simulation.)

The voltages representing problem variables are subjected to various mathematical operations as required for problem solutions. These operations are considered in some detail in Section 3.

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## 2. GENERAL PURPOSE ANALOG COMPUTERS

Modern general purpose analog computers are designed to solve systems of ordinary differential equations of great complexity. In particular, many general purpose computers can solve equations of motion and control equations in real time for guided missiles and other complex physical systems.

Analog computer units often are classified by manufacturers as linear or nonlinear. Linear racks or units usually include operational amplifiers or other elements whose operational characteristics may be expressed in terms of linear transfer functions. In addition, linear units may include limiters, diodes, or other nonlinear elements. Nonlinear units usually comprise multipliers, resolvers, and function generators singly or in combinations. Appendix 1 lists and describes in some detail the racks or other units of equipment made by a number of manufacturers.

General purpose analog computers may conveniently be classified into three types: (1) feedback operational-amplifier type computers, (2) passive network analyzers, and (3) fast-time computers. Each type has advantages and disadvantages depending on the problems considered.

Feedback operational-amplifier type computers are the most important for aircraft simulation problems. Mathematical operations expressible as linear transfer functions are performed by one or a series of amplifiers using resistance-capacitance networks as input and feedback elements. In some computers the operational amplifiers are largely standardized into integrators and summers.

In addition to operational amplifiers, passive-network type computers may use resistance-inductance-capacitance networks to instrument linear transfer functions. Alternating current transformers are sometimes used to provide coupling between networks. Computers of this type are well adapted to simulation of space distributed systems represented by partial differential equations.

Fast-time computers are of the operational-amplifier type with amplifier (and other component) frequency characteristics allowing many problem solutions per second. The curves representing problem solutions commonly are displayed on cathode ray oscilloscopes. Accuracies of existing commercial equipment are materially less than those possible to attain using real-time computers; however, the effects of making changes (such as changing parameters) in problems are rapidly available to the operator.

Interconnection of general purpose analog computer components is almost universally accomplished using plug-in patch cords. Most modern



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computers use removable patch-boards, which allow problems to be preserved after they have once been set up. In this manner large problems may be quickly interchanged on the computer.[1]

For very large computing installations, even the present removable patch-board arrangements become unwieldy, costly, and unreliable. For these reasons other interconnecting arrangements having automatic and semi-automatic features have been investigated.[2]

## 3. ANALOG COMPUTER COMPONENTS

Operational Amplifiers. The mathematical operations of summation, integration, differentiation, and other operations equivalent to the solution of low order linear differential equation with constant coefficients may be performed by the use of operational or computing amplifiers.

An operational amplifier consists essentially of a high-gain amplifier, input networks, and a feedback network connected as shown in Figure 1. $Y_1(S) \ldots Y_n(S)$ and $Y_f(S)$ are the admittances (inverse impedances) of the input and feedback networks, respectively. The triangle represents a high-gain dc amplifier of amplification -A with input grid at g. It is easily shown that the transfer function for such an arrangement is

$$\frac{E_o(S)}{\sum_{i=1}^{n} Y_i(S)\, E_i(S)} = -\frac{1}{Y_f(S)} \left( \frac{1}{1 + \frac{1}{A}\left[1 + \sum_i Y_i(S)/Y_f(S)\right]} \right), \qquad (1)$$

or, since $A \gg 1$ within the operating frequency range of the amplifier, equation 2 is very nearly satisfied:

$$E_o(S) = -\sum_{i=1}^{n} \frac{Y_i(S)\, E_i(S)}{Y_f(S)} = \sum_{i=1}^{n} \frac{E_i(S)}{Z_i(S)}\, Y_f(S) . \qquad (2)$$

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[1] Rogers and Abramis discuss the use of problem boards in a large-scale computing facility in reference 1, pp. 7--8.

[2] See Appendix 1, Section 8, and reference 2, pp. 43--57.



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Equation 2 is the idealized operational amplifier transfer function ordinarily used in setting up computer problems. A more complete analysis of an operational amplifier including error effects not shown here is given in Appendix 4.

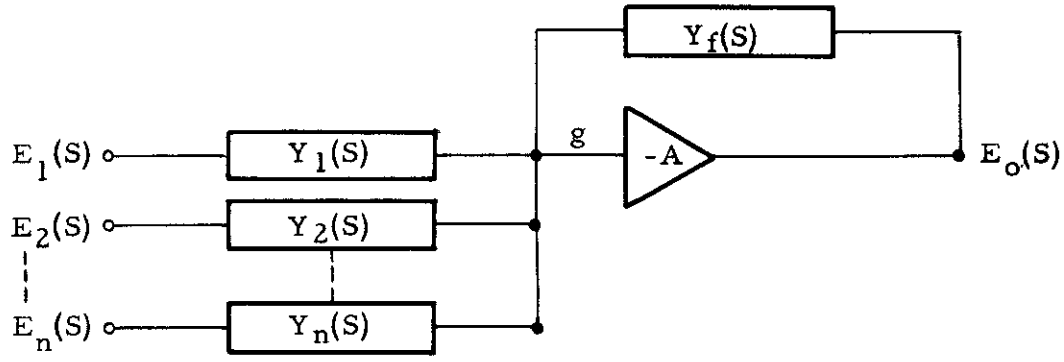


FIGURE 1 - OPERATIONAL AMPLIFIER

It is clear that the accuracy of an operational amplifier depends only upon the accuracy of the (passive) input and feedback elements, provided equation 2 is satisfied.

Multiplication. Multiplication by constant factors ordinarily is performed by a combination of amplifier gains and precision potentiometers. These potentiometers usually are of the 10-turn helical type and may be set to accuracies up to 0.01 percent of full scale. Most computers have a number of these potentiometers mounted in rack panels, with their terminals brought out to connection boards. Arrays of potentiometers arranged to be set remotely from digital keyboards (or possibly completely automatically from punched cards) are commercially available.[3]

Multiplication of two voltages that are varying with time may be done in a great variety of ways.[4] One important method utilizes servo-driven potentiometers: One variable is used to position a servo shaft driving several precision potentiometers, while another is used to excite the potentiometer, giving the product of the two variables at the

[3] Appendix 1, Section 3.10 describes such a system as does reference 1, pp. 96--113. See reference 1, p. 9 for description and use of such devices in existing simulation facilities.

[4] See reference 3 for a comprehensive survey and bibliography on analog multiplication schemes.



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potentiometer arm. By application of a third variable rather than a constant voltage to the position potentiometer of the servo, a product divided by the third variable may be obtained, though usually at a sacrifice of servo performance. While servo-type multipliers perform well in many applications, they suffer from poor high-frequency response.[5] For this reason, intensive efforts have been made to develop electronic multipliers of comparable accuracy, but with good characteristics at high computing frequencies.

Of all the electronic multipliers that have been proposed, the time-division type appears to be best adapted to real-time simulation-type computing. In this class of multipliers, one variable is used to vary the duration of the pulses in a pulse train whose frequency is well above the analog computing range. The other variable controls the amplitudes of the pulses. The average value of the pulse train is proportional to the desired product.[6]

In addition to servo and time-division multipliers, several other all-electronic multiplication devices are in use. The "quarter-squared" method utilizes a squaring device, such as the square-law portion of a vacuum tube characteristic, to obtain products using the identity

$$xy = \frac{1}{4}\left[(x + y)^2 - (x - y)^2\right] .$$

Crossed electric and magnetic fields have been used with a cathode-ray tube having a line mask over the face. A photo-sensing device and feedback circuit maintain the spot behind the mask; one of the required deflecting plate voltages is proportional to the product of the current generating the magnetic field and the other deflection voltage.

Arbitrary function generators. The arbitrary function generators commonly used in analog computation may be classified into four general categories: (1) mechanical servo-type generators, (2) straight-line segment type generators, (3) photoelectric function generators, and (4) magnetic tape recorder and playback units.[7]

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[5] See Appendix 1, Section 7.7 for typical performance figures.

[6] Manufacturers' specifications for time-division type multipliers are given in Appendix 1, Sections 4.2 and 7.6.

[7] Reference 4, Chapter 6, discusses types 1, 2, and 3 extensively. See reference 5 for example of type 4.



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Mechanical servo arbitrary function generators may take several forms. The usual schemes involve forming a conducting wire (or a line of conducting ink or soft pencil) in the shape of the desired function. This conducting curve is mounted on a roll-table[8] or a plotting board[9] and caused to move the contact arm on a linear potentiometer. In some cases the contact arm is mechanically connected to a movable arm. A servo enables the movable arm to follow the curve. In other cases, the conducting curve contacts the linear potentiometer strip directly. Another servo method of arbitrary function generation is to drive a multitapped linear potentiometer with appropriate voltages applied to the taps. The tap voltages may be set accurately and the potentiometer winding will interpolate between the settings in an approximately linear manner.

These mechanical function generators suffer from the poor high-frequency response inherent in such devices. However, particularly in the plotting board type, very high accuracy is possible at low frequencies.

Straight-line segment type function generators[10] find wide use due to their inherent simplicity with attendant ease of operation and reliability. In this type of generator, biased diode switches are used with resistance networks to fit a desired function with a series of straight-line segments. In most of these generators, both the beginning and end points and the slope of each segment are adjustable. As many as 22 or as few as 5 segments may be provided to fit a desired function in commercially available generators of this type. Some straight-line segment function generators give either positive or negative slopes for any given segment; others yield only monotonic functions. Diode limiters are a special type of straight-line function generators having, usually, three segments.

Photoelectric type function generators[11] use an optical mask in front of a cathode-ray tube with photoelectric cell error detectors. The mask is formed in the shape of the desired function and the light spot on the CRT screen is forced to follow the edge of the mask by a closed-loop system through the photoelectric cell. The deflecting plate voltage required to make the spot follow the mask edge is an electrical replica of the desired function.

Magnetic tape recorders of the fm type[12] are capable of recording and playing back functions in the range of frequencies of interest in analog

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[8] See Appendix 1, Section 4.3.

[9] See Appendix 1, Section 3.11.

[10] See Appendix 1, Sections 4.2 and 6.2.

[11] See Appendix 1, Sections 4.3 and 7.8.

[12] Reference 5 gives details of such a recorder.



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computation. Such recorder-playback combinations are of particular utility in problems where one computation produces results to be used in a succeeding computation.[13]

Trigonometric resolvers. Resolvers are used to produce trigonometric functions and to convert from polar to rectangular or from rectangular to polar coordinates. Perhaps the most commonly used resolvers are servo-driven sine-cosine potentiometers. These are capable of good accuracy but suffer from the poor frequency response inherent in electromechanical devices.[14]

Any function generator of the type discussed in the previous section may be used (with a multiplier) as a resolver, when set up, to generate sines and cosines of the independent variable. The use of implicit sine-cosine loops has been suggested for generating sines and cosines when rectangular components are available.[15]

Recorders. Recorders, plotting boards, and other output devices are treated in detail in the volume on recorders.

Special purpose equipment. In addition to the usual analog computing elements, various special-purpose units are necessary in analog computation. Noise generators[16] are used in missile simulation problems to introduce randomness, often by simulating actual noise sources in the missile guidance loop. Relays,[17] with and without associated amplifiers, are often useful to perform switching operations, such as introducing discontinuities in functions. A whole class of specialized equipment comes under the heading of physical simulation equipment and is considered in detail in the volumes of this series on flight tables, target simulation, and load simulation.

Programmers. Many simulation facilities have found it worth while to construct special programming devices to allow automatic or semi-automatic operation of the facility. Such programming can be used

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[13] See reference 1, pp. 219-233, for description and use of a "time-ratio" modulated tape-recording system in analog computation.

[14] See Appendix 1, Sections 3.5, 4.3, and 7.7.

[15] See reference 1, p.47 and reference 6, p. 7, as well as Appendix 1, Section 8.

[16] The design and use of such a generator are discussed in reference 7, pp. 83-91.

[17] Reference 8, pp. 187-195, gives applications of differential relays to analog computer problems.



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to run the computer through a predetermined sequence of operations in an automatic manner. Such operation might be desirable when conducting studies involving the effects of a random variable or of systematic variation of one or more problem parameters.[18]

## 4. FUTURE DEVELOPMENTS

Analog computers. Currently operating large-scale analog computing facilities suffer from several operating difficulties. These difficulties arise largely from (1) equipment and maintenance troubles and (2) troubles involving the setting up, changing, and checking of problems.

Experience has shown that much equipment failure occurs in electromechanical elements, such as the electromagnetic choppers used in operational amplifier stabilization. Also, electromechanical elements ordinarily are the limiting factors in the upper frequency limit of operation. For these reasons, the tendency in new equipment development is to eliminate electromechanical units in favor of all-electronic devices. Examples of recent developments in this direction are the time-division multiplier and the photoelectric function generator described in Section 3. Recent work has shown the feasibility of eliminating electromagnetic choppers in favor of photoelectric chopping devices for amplifier stabilization.[19] This development, when completed, should materially decrease the maintenance required by the electromagnetic type of choppers.

In addition to equipment failures, it is found that much time is consumed in large simulation facilities by human errors. This fact indicates that the setting-up and programming of problems should be made as automatic as is feasible in future facilities. Particularly, such automatic operation would insure that rerunning problems after they once have been set up would be as fast and trouble-free as possible, under equipment limitations.

Reliability of computer results (assuming no equipment malfunctioning) is a field of investigation only recently attacked from a theoretical point of view.[20] Past efforts in this field have been confined to the development of checking procedures, such as comparison of a sample computer

[18] Programming arrangements for automatic analog computer operation are described in reference 8, pp. 129–139 and pp. 173–183, and reference 1, pp. 65–82.

[19] See Appendix 1, Section 8, and reference 1, pp. 141–155.

[20] See volume on "Error Studies" and references 9 and 4, pp. 139–147.



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solution with a check solution obtained by independent means, usually on a digital computer.[21] The perfection of individual computing components undoubtedly contributes to the accuracy and reliability of overall results, but the relationship is not clear except in the limited case considered in detail by Winson in reference 9. As a result of Winson's work, some of the important factors to be considered in computing amplifier design have been clarified and put on a sound theoretical basis.[22]

Computers with digital components. Problems arise in dynamic systems analysis where the speed and accuracy requirements exceed those which can be obtained with available analog equipment. It appears possible to develop a computer which combines the advantages of both analog and digital computers by using a system design based on the electronic analog computer with components which are digital in nature, having the inherent basic speed of an analog computer and yet allowing computation with digital accuracy. A report outlining the concept of such a computer is described in reference 10. A prototype computer of this nature has been built to demonstrate the feasibility of the system, and a technical report (NOL-Corona Report 172) on this work will be issued shortly by the Naval Ordnance Laboratory, Corona.

[21] See volume on "Procedures" and reference 8, pp. 113-116.

[22] Reference 9, Section 5.3.



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# APPENDIX 1
# COMMERCIALLY AVAILABLE ANALOG COMPUTING EQUIPMENT

This survey is intended to provide a representative cross-section of the commercial analog computer field. While it is by no means exhaustive, this survey does include some of the most widely used analog computing equipment.[23]

The A. B. S. publications "Survey of Simulation Facilities" and "Summary Progress Report for the Year Ending 1 February 1953, Volume IV, Computers" contain descriptions of older models of some of the equipment included here as well as of some of the units described here. This survey includes the most recent models made by the various manufacturers, except where otherwise noted. The manufacturers included here are:

1. Beckman Instruments Inc. (ref. 11)
2. Boeing Airplane Company (refs. 6 and 12)
3. Electronic Associates, Inc. (ref. 13)
4. Goodyear Aircraft Corporation (ref. 14)
5. William Miller Corporation (refs. 15 and 16)
6. George A. Philbrick Researches, Inc. (ref. 17)
7. Reeves Instrument Corporation (refs. 18 through 22)
8. R. C. A. Laboratories (refs. 1 and 23)

## 1.1 BECKMAN INSTRUMENTS INCORPORATED, BERKELEY DIVISION

The main units of the EASE computer are discussed below.

Operational amplifier unit. This unit contains ten identical high-gain dc amplifiers that may be used as operational amplifiers by connections to resistances and capacitors mounted in the control unit. This amplifier uses positive feedback to obtain an infinite gain at zero frequency. The amplifiers may be used with or without a stabilizing unit. The unstabilized drift is 0.6 millivolt referred to the grid. The stabilized drift is less than 100

[23] The information presented here is from data provided by manufacturers or other equipment sources. NOL-Corona takes no responsibility for the validity of this data.



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microvolts referred to the grid. The phase shift at a gain of unity is about 1 degree at 4000 cps.

Function multiplier unit. This multiplier is of the time-division type and provides outputs in the ±100 volts range. The carrier frequency is 10 kc and a filter removes all except 0.1 percent of the 10 kc in the output. The maximum error from all sources is ±2.5 percent. The phase shift is less than 1 degree at 20 cps for an amplitude of 75 volts.

Function generator unit. This arbitrary function generator approximates a desired curve by 22 straight-line segments. Two models are available: one with fixed lengths for the line segments and one in which the beginning and end points of each line segment are adjustable.

Control unit. The control unit is a rack consisting of a removable setup panel, a receptacle unit, and all RC elements. The standard setup panel provides for the following: 24 helipots, 18 decade resistors, 6 double diodes, 6 operational relay contacts, 24 function relay contacts, 4 function generators, 10 multipliers, 10 dc initial condition sources, 30 amplifiers, 30 capacitors, 60 fixed resistances, 20 external connections, and 36 auxiliary connections for extra components and special adaptations. RC elements are of 1 percent tolerance on the standard computer; however, 0.1-percent components may be specified for extra cost.

## 1.2 BOEING AIRPLANE COMPANY

The basic computer unit consists of 12 amplifiers with associated power supplies, control panel, and patch panel mounted in a cabinet 23 3/4 by 19 by 81 3/4 inches. Six panel spaces are available in each standard rack for installation of additional amplifiers, limiters, potentiometers, operational servos, or function generators. A prepatch panel is available so that problems may be set up away from the computer. An automatic balance unit that can be connected to the operational amplifiers is available as optional equipment.

Operational amplifier. The operational amplifier is a three-stage amplifier with positive internal feedback resulting in infinite dc gain and zero dc output impedance. For unity gain, the amplitude response is flat to 2000 cps; the 3-db down point occurs above 8 kc, and the phase shift is less than 0.5 degree at 100 cps. Forty integrating capacitors ranging in value from 0.05 to 5 microfarads are provided.



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Operational servo. The Boeing operational servo consists of a dc motor tachometer, a high-gain amplifier, and five single-turn ganged linear potentiometers. Additional mounting parts, gears, sine-cosine units, and nonlinear elements can be attached to the standard servo unit. This velocity servo with its accessories serves as multiplier, divider, and resolver unit. Its amplitude response, within the torque and acceleration limits of the motor, is flat to 10 cps, and the phase shift is negligible at 10 cps.

Function multiplier. The Type 7081 function multiplier is a time-division unit. This multiplier accepts two inputs in the ±50 volt range and provides an output in the same range. The output is linear to within 1 percent of full scale and, after a warmup period of 1 hour, the drift in 8 hours will not be over 1 percent of full scale. A Type 7082 function multiplier keying source is required, but one keying source may be used for up to twelve multipliers.

## 1.3 ELECTRONIC ASSOCIATES INCORPORATED

Computer control console type 16-24E( ). This control console is a double-width rack with an inclined attenuator panel across the top, a single patch panel assembly on the left, and a control panel on the right. A work shelf is provided just below the control panel and patch board assembly. There is ample space for expansion of the operational controls or for the installation of accessory equipment. Each control console is equipped with one prepatch panel. Details of the patch panels and other features are given below. (Type No. 16-24E(20) would indicate that 20 operational amplifiers are provided for in that console.)

Computer control console type 16-24D( ). When the control of more than three amplifier groups and associated nonlinear elements is necessary, control console Type 16-24D( ) may be used. This console is a triple-width rack with inclined attenuator panel across the top, two patch board assemblies, and a double control panel. As in type 16-24E( ) there is ample extra space available.

The control panel contains a vacuum tube voltmeter with several scales, which can be switched to read the output of any element of the computer, such as an amplifier, during its operation. The control switch has five positions: standby, ready, initial condition, hold, and operate. The control console components and passive elements for 48 operational



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amplifiers are listed below.

(a) 72 attenuators (potentiometers)
(b) 1 set of the necessary resistors, capacitors, and relays to provide the following modes of amplifier operation: 20 amplifiers, summing or integrating; 20 amplifiers, summing only; 8 amplifiers, inverting only.
(c) 6 diode-type limiters
(d) 1 set of passive elements: 6 1-megohm resistors, 2 1-megohm resistors, and 2 1-microfarad capacitors.
(e) Switches, dials, patch cords, etc.

Patch panel system type 16-5A. Patch panel system type 16-5A is a prepatch system with a 15- by 20-inch metal work board containing 1800 terminations. Construction of the equipment confines all leakages to the ground paths; i.e., no terminal-to-terminal leakage is possible. The tips of the shielded patch cords make direct contact with springs behind the patch board. It is not necessary to remove the work board to insert or remove patch cords or plugs. Patch cords are available in multiples of 6 inches in length.

Amplifier group type 16-31C. This unit is the basic analog computer component. The rack-type cabinet contains 14 dual dc computing amplifiers which, when used in conjunction with a suitable control console, can be used to solve high-order linear differential equations. All required power supplies are furnished with the equipment. Reference supplies can be included at the sacrifice of two dual computing-amplifiers. The performance characteristics of the dual dc amplifier type 16-6H used in this amplifier group are:

| | |
|---|---|
| Gain of unstabilized portion: | 100,000 |
| Gain of stabilizer: | 3,000 |
| Grid current: | 90 μμamperes, average |
| Integrator drift: | 0.005% per minute average, for $R = 1/C = 10^6$ |
| Noise output | |
| High frequency: | 9 mv |
| Low frequency: | 1 mv |
| Output load: | ±100 v at 7500 ohms with no boost |
| | ±100 v at 5000 ohms with "low boost" |
| | ±100 v at 3000 ohms with "high boost" |

Resolver assembly type 16-31D. This assembly provides four resolving channels, each capable of converting polar data into rectangular



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data or vice versa. It is also capable of multiplying three variables by a fourth variable. The unit contains all necessary power supplies, inverting amplifiers, summing amplifiers, summing networks, operating controls, and provisions for switching the dc amplifiers out of the circuit if they are required elsewhere as operational amplifiers. The resolver rack type 16-31D consists of the following components:

(a) 4 resolver chassis type 16-8A, each including: 1 type 16-A8AA dual sine-cosine potentiometer, 1 automatic gain servo amplifier, 2 drive motors and associated gearing, 1 linear followup potentiometer, and 3 linear multiplying potentiometers
(b) 6 dual amplifiers type 16-6H (12 amplifiers); 8 are used for inverting and 4 for summing. They may be switched for use elsewhere as operational amplifiers.
(c) All additional power supplies and networks required.

The slewing rate for the resolvers is approximately 640 degrees per second.

Multiplier group type 16-31L. This is a 20-channel servo multiplier system in which the following multiplier units are housed: 12 channels with type 16-7E servo multipliers, 4 channels with type 16-7F servo multipliers, and 4 channels with type 16-7G servo multipliers. The type 16-7E servo multiplier is constructed to accept up to 5 variable dc voltages, multiply each by another variable dc voltage, and furnish 5 individual output voltages proportional to any of the products. The multiplying and feedback potentiometers are a single-turn 6-gang unit with all the cups center-tapped. The type 16-7F servo multiplier is similar to the 16-7E, but the 6-gang potentiometer consists of 2 linear center-tapped cups and 4 linear cups each with 17 taps. The multitapped cups may be used either as multiplying potentiometers or as function generators. The type 16-7G servo multiplier uses a 6-gang unit made up of 4 linear center-tapped cups and two linear cups each with 17 taps. Other specifications are the same as for the 16-7E and the 16-7F.

The errors in multiplying by the units are of three major types:

(1) Potentiometer inaccuracies--The potentiometers used are single-turn 0.1-percent linear units. Since two windings are associated with each multiplication, potentiometer errors can be ±0.2 percent. In general, however, this error will be somewhat smaller.
(2) Servo nulling errors--The static nulling error does not exceed 0.06 percent.
(3) Dynamic errors--The multiplying circuit produces a dynamic error of less than 0.5 percent for full excursion of the potentiometer at one-half cps.



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Multiplier group type 16-31N. Multiplier group type 16-31N is a 20-channel servo multiplier. Each channel is capable of multiplying 5 variables by a sixth. This unit contains 20 servo multipliers type 16-7E. A switch panel contains switches for grounding the inputs to the individual servo amplifiers and for selecting either internal or external reference voltages.

Multiplier group type 16-31M. Multiplier group type 16-31M differs from type 16-31N only in the types of servo multiplier units that it houses. These are: 15 channels with type 16-7E multipliers and 5 channels with type 16-7F servo multipliers.

Servo group 16-31P. This unit contains 2 resolving channels, the means to multiply 2 sets of 3 variables by a fourth variable, and the means to multiply 4 sets of 5 variables by a sixth variable. This assembly includes all necessary power supplies required to perform the described computations when it is used with an associated dc amplifier group, such as EA type 16-31C. This equipment contains 2 resolver chassis type 16-8A which include: 1 type 16-A8AA dual sine-cosine potentiometer, 1 automatic gain servo amplifier, 2 drive motors and associated gearing, 1 linear followup potentiometer, 3 linear multiplying potentiometers, 4 multipliers type 16-7E, a test panel, a control panel, and all the necessary power supplies.

Digitalized attenuator system. This keyboard controlled digitalized attenuator system provides a rapid means of precisely setting large quantities of computer potentiometers under load conditions. The system will handle up to 1000 potentiometers, thereby eliminating from a given program the time involved in making manual adjustments of the potentiometers and the errors introduced by manual setting. A complete system consists of: 1 keyboard type 16-9C, 1 dc servo amplifier type 16-7C, 1 control chassis type 16-4A, sufficient attenuator assemblies type 16-9A for the number of potentiometers to be set, and 1 matrix power supply type 16-10M. Attenuator assembly type 16-9A houses a servo motor that drives a gear train which in turn serves 18 precision potentiometers. Each potentiometer is serviced by its individual potentiometer relay and a magnetic clutch that provides drive to the potentiometer from the servo amplifier through the gear train. The keyboard centralizes all potentiometer setting operations once the proper gain setting has been made on the servo amplifier. By using the keyboard, the particular potentiometer and the voltage it is to deliver to the computing system may be selected. Bar type switches control the operate, clear, and readout functions for the system.



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The control chassis provides circuits that automatically time the functions of the keyboard and give visual indications at the keyboard of the operate and clear conditions of the system. The red indicator light shows that potentiometer setting is in progress while the green indicator light shows that setting is complete and the keyboard is ready for the next operation. With this system, the potentiometers may be set to 0.01 percent of the desired value.

Function generator type 17-18A. The function generator type 17-18A is used to convert the Variplotter[24] plotting board from a recorder to a curve following unit. The head assembly is substituted for the pen on the plotting board and with minor changes to the Variplotter, the motion of the head along the curve generates a signal proportional to the amplitude of the curve. A small resistance mandrel which rides along the curve to be followed is mounted in a plastic block at the leading edge of the head.

The curve to be traced is laid out on the plotting board in the form of a soft copper wire. A servo enables the head to follow the curve, giving an electrical replica of the desired function at the moving arm of a linear potentiometer.

## 1.4 THE GOODYEAR AIRCRAFT CORPORATION

GEDA L3 linear rack. The size of the unit is 72 by 34 by 30 inches and it weighs 875 pounds. It is enclosed in a castered cabinet with 30 by 6 by 12 inches available for auxiliary equipment. Each of the 24 dc operational amplifiers in this unit may be used to perform linear operations by the use of suitable passive elements. Twelve built-in 1-μfd integrating capacitors and a number of built-in summing and feedback resistors are provided. Fixed and adjustable-decade plug-in resistors may be used in conjunction with the built-in components. The built-in power supplies provide ±200 volts at 1 ampere, with an output impedance of 0.05 ohm and -450 volts at 200 milliamperes with an output impedance of 0.15 ohm. The power consumption is approximately 1200 watts. Eighteen 10-turn precision potentiometers are provided along with 3 limiters and 10 thermionic diodes. The connections to these and to the other computing elements are brought out to a patch bay in front of the rack. The removable problem boards are made of a glass-fiber reinforced plastic and provide high leakage resistance as well as lightness and strength. A precision calibrator and null-voltmeter on the front panel may be used to measure voltages in

[24] See the volume on recorders for a detailed description of the Variplotter.



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the range ±100 volts and to set potentiometers or variable plug-in resistors to within 0.1 percent of the full-scale setting of the calibrator.

GEDA N3 nonlinear rack. The N3 nonlinear rack consists of a control unit, power supplies, junction box, stabilizers, calibrator, all the necessary interconnecting cables, and spaces for 10 auxiliary plug-in units. The plug-in units may at present be selected from seven units available at this time: A-multiplier (GN215-N3A), B-multiplier (GN215-N3B), reference power supply (GN215-N3C), servomultiplier (GN215-N3D), function generator (GN215-N3E), potentiometer (GN215-N3M), and summer (GN215-N3Q).

The GEDA N3A unit is a time-division type of stabilized electronic multiplier. The N3A accepts three inputs (X, Y, and Z) in the ±100 volt range with input impedances of 1 megohm for X and 160K ohms for Y and Z. There are two outputs (XY and XZ) in the ±100 volt range. The dc output impedances are less than 20 ohms. The static accuracy is within 0.2 percent of full scale. The amplitude response is essentially flat to 200 cps with less than 18 degrees phase-shift at that frequency. There is less than 0.1-volt noise at the switching frequency of 15 kc, and the specified accuracy can be maintained with monthly calibration.

The GEDA N3B unit also is a stabilized time-division multiplier and is identical with the N3A except that it accepts three inputs (U, V, and W) and produces three products (UX, VX, and WX) where the X-information is received from one or more N3A units.

The GEDA N3C is a reference power supply providing either a plus or minus 100-volt reference signal for analog computers. It is capable of independent operation, using a self-contained reference voltage, or using a laboratory reference, L3, or N3 regulator, or another N3C. Two N3C units are required to supply both plus and minus 100 volts.

The GEDA N3D is a servomultiplier that produces the product of each of five variables multiplied by a sixth variable. This unit consists of a 60-cps chopper-modulated servo amplifier, a servo actuator with a 6-section linear potentiometer, and 5 sign-changing amplifiers. Input signals of either sign can be used. The amplitude frequency response is essentially flat to 10 cps with a phase shift of less than 3 degrees at 1 cps within the velocity and acceleration limits. The velocity limit is 400 volts per second; the acceleration limit is 40,000 volts per second; and the static accuracy is within 0.25 percent of full scale.

The GEDA N3E unit is a straight-line segment type of function generator. It accepts input voltage ±X and ±Y in the ±100 volt range and generates f(X) and g(Y) in this range. Each function can be approximated by a maximum of 5 straight-line segments; if more than 5 segments are required, the 2 halves of the unit may be ganged to approximate a single



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function with a maximum of 10 line segments. For Y = f(X) = X, the output is essentially flat to 1000 cps with a phase shift of less than 11 degrees at 400 cps.

The GEDA N3M potentiometer unit provides 9 supplementary potentiometers of the 10-turn precision type. One side of each potentiometer is grounded. By means of switches, the potentiometers may be connected to an external calibrating circuit to increase the accuracy of the settings.

The GEDA N3Q is a summer unit with 6 summing amplifiers, each having 3 inputs. Panel jacks allow the operator to select input and feedback impedances. Scale factors may be selected by the operator to 0.1 percent. The amplifiers are automatically stabilized and dc plug-in units having a dc gain of more than $2(10^7)$ and a gain of more than $5(10^4)$ below 20 cps. The output capabilities are ±100 volts at 3 milliamperes. The amplifier drift is equivalent to less than 1 millivolt per month at the input grid.

The N3 rack also contains built-in regulated power supplies giving ±200 volts at 1.0 ampere with 0.05 ohm output impedance and -450 volts at 200 milliamperes with 0.1 ohm impedance. The built-in ac power supplies give 115 volts, 60 cps, ±1 percent, 1000 volt-amperes; 6.3 volts, center-tapped at ground, 100 amperes; and 6.3 volts, center-tapped at -200 volts dc, 50 amperes.

T1 curve follower. The model GN215-T1 GEDA curve follower combines a function generator and a function plotter into a single compact instrument. The T1 generates functions from continuous penciled graphs, including curves having small step discontinuities. The T1 consists basically of X- and Y-servos. The X-servo controls the angular displacement of a drum and the Y-servo controls the lateral displacement of a carriage holding either a pickup element or a pen. The function Y = f(X) is plotted with soft pencil on 8 1/2 by 11-inch graph paper which is placed on the drum. The pickup element provides the Y-servo with a voltage proportional to the error of the pickup element in following the curve. A precision output potentiometer, mechanically linked to the carriage, provides a voltage proportional to the lateral displacement of the pickup. The error voltage is added to the displacement voltage to produce an output that is truly proportional to Y. The maximum static error in function generation is less than 0.3 percent of full scale; the maximum error in the Y-channel while tracking is ±1 percent of full scale.

Other GEDA equipment. The GEDA GN215-R5 6-channel recorder and the GN215-R3 2-channel recorder amplifier are discussed in detail in the volume on recorders. Goodyear also manufactures the GN215-M1



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impedance bridge for high-accuracy resistance and capacitance measurements. This bridge measures resistance from 100 ohms to 10 megohms and capacitance from 0.0001 to 1 μfd with accuracies of 0.1 percent or 100 ohms, whichever is greater for resistance, and 0.1 percent for 0.0001 μfd, whichever is greater, for capacitance.

## 1.5 WILLIAM MILLER CORPORATION[25]

The computer manufactured by the William Miller Corporation is of the "direct" or "physical" analog type developed by Professor G. D. McCann at the California Institute of Technology. In this type of computer considerable advantage is taken of the similarity in form of a set of ordinary differential equations and the equations of an electrical network (Kirchhoff's laws). The elements of the physical system are represented by electrical units of inductance, capacitance, and resistance. Amplifiers, limiters, and arbitrary function generators are available for representing active elements and nonlinearities. "Distributed" systems are represented by finite difference methods.

A large-scale setup might consist of 50 sets of passive elements, 60 amplifiers, 4 multipliers, 4 arbitrary function generators, 2 power amplifiers, 1 control desk, and 1 power supply cabinet. This apparatus would be housed in 21 cabinets, each 2 by 3 by 7 feet. The control desk is contained in 3 of these cabinets and is set up to handle 100 sets of passive elements and any required number of amplifiers or other nonpassive units. It is relatively easy to add units to increase the capacity.

The frequency range is approximately 50 to 2000 cps. However, all the elements other than the transformers and inductors are useful over a wider band extending to 0 cps.

This type of equipment has a certain advantage over differential analyzers when studies are made of such problems as bending beams, temperature distribution in turbine rotors, stresses on plates, etc., in that the system under investigation and its electrical analogy have striking similarity in form; hence the physical problem is easier to visualize.

Problems are set up using plug-in cords and dial settings. The elements are in general accurate to 1 percent, but the feedback amplifiers are somewhat more accurate.

Two types of amplifiers are available, positive and negative gain, or phase inverter. Phase shift is less than 1 1/2 percent at 1000 cps. Amplifier response is linear to 3 db at 90,000 cps. The amplifiers are

[25] This material was adapted from the references given by H. P. Messinger of the Advisory Board on Simulation, Chicago.



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all chopper stabilized and are provided with plug-in panels at the front that allow the input, output, or feedback networks to be set up. Floating operation is also possible when the floating power supplies are utilized.

Inductors cover 3 decades with maximum values of 1.11 or 11.1 henries. Capacitors are of the 3-decade type with maximum values of 1.11, 11.1, or 111.0 μfd. Resistor boxes cover 4 decades with maximum values of 11,110, 111,100, or 1,111,000 ohms. Transformers are widely variable as to ratio and have a variable tap on one winding. The passive elements come in sets of 10. The multipliers are electronic and have almost the same phase and amplitude responses as the amplifiers. The function generators are of the photo-tube cathode-ray type, with mask type functions. The manufacturer states that any function may be used as the dependent or independent variable.

Trigonometric resolution is accomplished by the use of the multipliers and function generators. Potentiometers and limiters may be mounted in amplifier panels or in racks, depending on the problem requirements. A specially made precision 5-inch dual-beam cathode-ray oscilloscope with built-in photographic equipment is supplied. If required, pen-type recorders may be connected.

Remote control measuring of current and voltage going to elements may be achieved from the control desk. Electronic voltmeters are supplied to measure sinusoidal voltages, and the cathode ray oscilloscope is used for recording transients.

## 1.6 GEORGE A. PHILBRICK RESEARCHES, INCORPORATED

The GAP/R computers are assembled from standard components, as needed, although various complete computing assortments are suggested. These computers are primarily of the fast-time variety, described in Section 2, although the components may be operated at real time if desired. The K3 and K4 components combine differentiation, integration, summation, constant-factor multiplication, and inversion in various combinations.

K3 series of analog computer components. The K3 components have the following characteristics in common:

Input
- Range: ±50 volts
- Impedance: 1/2 megohm or over

Output
- Range: ±50 volts
- Impedance: under 1 ohm to 1 megohm

Amplifiers: dc with various gains
rise-time: 2 μsec at unity gain



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Accuracy
- Calibrations: 1 percent
- Drift: negligible
- Fidelity to dynamic form and resolution to 0.1 percent
- Precision: 0.01 percent

Size: 5 1/4 by 7 1/4 by 4 1/4 inches
Weight: 5 pounds
Power requirements:
±300 volts dc at 10 to 16 ma
0.1 to 0.13 amp at 115 volts ac

The characteristics of the K3 components are summarized below. All units provide both positive and inverted outputs. In the notation used here, p represents the differential operator d/dt, and 1/p represents the operation of integration with respect to time. The inputs are represented by x and the outputs by y.

K3-A adding component.
$\pm y = x_1 + x_2 + x_3 + x_4 \pm X$, X = constant < 10 volts

K3-C coefficient component.
$y = \pm Cx$, $0 < C < 100$

K3-J integrating component.
$\pm y = x/T_o p$, $T_o$ = multiple of 0.4 millisecond or 1 second

K3-D differentiating component.
$\pm y = T_o p x$, $T_o$ = 0.4 millisecond

K3-K augmenting integrator.
$\pm y = (1 + 1/Tp)x$, T adjustable up to 0.4 millisecond

K3-E augmenting differentiator.
$\pm y = (1 + Tp)x = \pm x$, T adjustable up to 0.4 millisecond

K3-L unit-lag component.
$(1 + Tp)y = \pm x$, T adjustable up to 0.4 millisecond

K3-B bounding component.
$\pm 2y = (x + B) - (x - B)$, B adjustable from 0 to 100 percent of standard range

K3-H backlash component.
±y = (x with backlash H), H adjustable up to 10 percent of the full excursion

K3-Z inert-zone component.
$$\pm y = \begin{cases} 0, & |x| < D/2 \\ x - D/2, & x > D/2 \\ x + D/2, & x < -D/2 \end{cases},$$



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D adjustable up to 10 percent of range

K3-S squaring component.

$\pm y = Ax^2$, A adjustable from 0 to 1

K3-T square-root component.

$\pm y = A\sqrt{x}$, $x \geq 0$, scale adjustment hyperbolic and covers a 10 to 1 range

K3-V absolute-value component.

$\pm y/A = x$, or $\pm y/A = 2x - 50$, A adjustable from 0 to 1

K4 series of analog computer components. The K4 components have the following characteristics in common:

| | |
|---|---|
| Inputs (2-4) | |
| Range: | ±50 volts dc |
| Impedance: | above 1/2 megohm |
| Outputs (2-5) | |
| Range: | ±50 volts dc |
| Impedance: | under 1 ohm |
| Accuracy: | same as K3 components |
| Size: | 17 by 7 by 5 1/4 inches |
| Weight: | 20 pounds |
| Power requirements: | |
| | ±300 volts dc at 20 to 60 ma |
| | 115 volts ac at 0.13 to 0.38 ma |

The K4-DY dynamic component embodies the following second-order linear differential equation:

$$(A_2T_o^2p^2 + A_1T_op + A_o)\,y = x(t)\ ,$$

where $T_o$ is 0.4 millisecond and $A_o$, $A_1$, and $A_2$ are directly calibrated on coefficient-type 0 to 100 dials. The input is x and the outputs are $\pm y$, $\pm \dot{y}$, and $\pm \ddot{y}$.

The K4-MU multiplier component consists of a pair of independent multiplying units. For development of the product, the so-called quarter-square principle is applied (see section 3).

The K4-FF functional component is designed to cause the output voltage to follow an assignable function of the input voltage. It utilizes an approximation based on ten connected line segments, each of which is independently adjustable as to slope and length. (See section 3.)



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Model CU central unit. The central unit consists of a central signal component model CS, a power supply, and a standard 6-foot open rack with three component shelves. The central signal component serves to initiate and automatically to monitor the repetitive solutions that are carried out with the standard computing components. This component provides several pulse-type outputs having a period of 40 milliseconds, which is the normal computing interval for fast-time computing with the GAP/R equipment. The outputs are used to introduce initial conditions, to clamp integrators, and are also used with output and calibrating equipment.

Complete computing assortments. Philbrick offers several collections of components of varying degrees of complexity and cost. Four examples of complete computing assortments are the CA, DA, FA, and the GA assortments, which are capable of solving third, seventh, twelfth, and eighteenth order linear and nonlinear equations, respectively.

## 1.7 REEVES INSTRUMENT CORPORATION

REAC model C-301 Mod 0 computer. The model C-301 computer is a portable, desk-top type electronic analog computer designed for use in the office. The components are housed in a cabinet measuring 20 by 25 by 20 inches. All operating controls are mounted on the front panel. The computer houses 12 computing amplifiers on 6 dual chassis. Amplifier input grids and outputs are connected to the patch bay. Passive elements necessary for forming inverting, summing, and integrating loops are also connected to the patch bay as are the contacts of relays controlled by an operate-reset switch. A portable prepatch plug-board is provided for remote problem setup and problem storage. The component connections available at the plug-board include the input grids and outputs of the 12 computing amplifiers, both ends of forty-eight 0.1-percent wire-wound temperature compensated resistors, both sides of six 1-μfd 0.1-percent polystyrene capacitors, one end and arm of 9 precision potentiometers, both ends and one arm of 9 precision potentiometers, connections for relay controlled by operate-reset switch, connections to 8 diodes, and various voltage and switch connections. All the necessary power supplies, mounted on a single chassis, are in the computer. These include regulated ± 200 volt and ± 400 volt plate supplies, the 26-volt relay supply, and the ± 105 volt computing reference supply.

REAC model C-302 Mod 0 computer. This model is identical to the model 301 with the following exceptions:



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(1) Six of the 12 computing amplifiers have permanently wired computing circuits containing a normal-special switch. When the switch is set to normal, the amplifier is wired as an integrator; when it is on special, the amplifier becomes a summer. Each amplifier has a permanently wired unity-gain input as well as initial condition, grid, and output connections.

(2) The other 6 amplifiers are permanently wired as summers with one unity-gain input, grid, and output connections.

(3) Relays controlled by a hold-operate-reset switch are permanently wired for control of the integrators.

(4) Fewer passive elements are provided, since the amplifiers have permanently wired computing circuits.

C-203 Mod 0 computer. The components of computer C-203 Mod 0 are contained in a 7-foot-high relay rack cabinet. The computer is wired so that problems can be set up on patch boards that plug into a patch bay on the front. Computer C-203 Mod 0 contains 24 computing amplifiers, 7 integrators, 7 summers, and 10 inverters. Four of the integrators and 8 of the inverters are instantly convertible to high-gain amplifiers by means of switches on the front panels. Provision is made for introducing 23 constant parameters and 7 initial conditions by 29 precision 10-turn helical potentiometers with vernier dials. The initial-condition potentiometers may be excited with ±100 volts or zero volt by 3-position switches on the front panel. When the initial-condition potentiometers are not being used for providing initial conditions, they may be used in any other way desired. Four diode limiters permit 4 variables to be limited at any positive or negative values within the normal operating range. Regulated ±200 volt and ±400 volt power supplies are built in to operate the computing amplifiers. External 26-volt relay power and ±100 volt computing reference power are required.

The computing amplifiers in the C-203 Mod 0 computer are of the A200 Mod 0 type. The A200 Mod 0 amplifier chassis consists of 2 identical amplifier channels. Each amplifier has one dc amplifier section and one ac balancing amplifier section. The dual construction permits the dual tubes and the vibrator between the amplifiers to be shared. The open loop dc gain of the amplifiers is at least $30(10^6)$, of which $15(10^3)$ is in the dc section and $2(10^3)$ is in the ac balancing amplifier section. As a unity-gain summing amplifier, the amplitude-frequency response is flat from 0 to over 1000 cps. The phase-shift for a single unity-gain input is less than 1 degree below 100 cps. The output of a summing amplifier with a +1.000 volt signal into a unity gain input is -1.000 ± 0.005 volt. For input signals of 10 volts or higher, within the operating range of the summing



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amplifier, the output signal is within 0.1 percent of the theoretical value. The phase-shift of an integrating amplifier for a unity-gain input signal is 90 degrees minus less than 0.1 degree, over the range 0.01 to 100 cps. For an input of +1 volt into a unity-gain input (time constant of one second), the output rate of the integrating amplifier is -1.000 ± 0.005 volt per second. For input signals of 10 volts or higher, within the operating range of the amplifier, the output rate is within 0.1 percent of the theoretical value.

REAC model C-202 Mod 0 computer. The Reeves electronic analog computer C-202 Mod 0 is identical to the C-203 Mod 0, with the following exceptions: 4 of the dc amplifiers in the C-203 are replaced by 4 servo amplifiers and 4 servomechanical units are included. Each unit drives 2 dual 10-turn precision helical potentiometers. Each of the four 20-kilohm potentiometers is linear to 0.1 percent. One potentiometer of each pair is center-tapped. The potentiometer used as the followup potentiometer is not center-tapped, so that of the three potentiometers remaining for multiplication two are center-tapped. The servos have a maximum velocity of 70 volts per second and a maximum acceleration of 1500 volts per second per second.

REAC model C-201 Mod 0 computer. The C-201 Mod 0 computer is identical to the model C-202 Mod 0 except for differences in the servomechanical units and the servo switches on the front. Each of the 4 servomechanical units has 6 ganged 1-turn L-type helipots: 4 are center-tapped, 30-kilohm, 0.1 percent linear; 2 are 9-tapped, 100-kilohm, 0.5 percent linear. The last helipots provide ten segments each for function generation. Panel switches permit the followup potentiometers to be connected across 0 to 100 volts, 0 to -100 volts, and -100 to +100 volts. The servos in the C-201 Mod 0 have a maximum velocity of 20 radians per second.

REAC model EM-101 electronic multiplier. The EM-101 electronic multiplier is a variation of the time-division type described in Section 3. Each channel of this multiplier will simultaneously perform the operations $W_1 = K_1(XY/U)$ and $W_2 = K_2(XZ/U)$, where $K_1$ and $K_2$ are constants and X, Y, Z, and U are single-ended variable inputs from a REAC system. By keeping U constant, multiplication may be performed; by keeping Y constant, division may be performed, subject only to the restriction that X/U does not exceed ± 1.5. The accuracy and noise will be within the values specified only for values of X/U up to unity; at this ratio the multiplier outputs are limited. The inputs X, Y, and Z may be of either sign, but the U-input must always be positive. The multiplier preserves the proper



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signs with these inputs. The amplitude response is uniform from zero to one cps within 0.3 percent, and from one to 10 cps within one percent. The phase shift at one cps is less than 0.5 degree and at 20 cps is less than 5 degrees. The outputs are within 0.25 volt of the correct values over the entire range of ±100 volts. The probable error within this range is of the order of 0.1 volt. Over any 8-hour period, after suitable warmup, the output drifts less than 0.1 volt for zero Y-input and less than 0.2 volt for Y-inputs other than zero. For values of X, Y, and Z between ±100 volts, for values of U between 0 and +100 volts, and for values of X/U not exceeding ±1.0, the maximum output noise is about 0.3 volt. In normal use, a weekly balance check may be required; readjustment should normally not be required more often than once a month, or when tubes are replaced.

REAC model S-101 Mod 4 servo unit. The Reeves computer servo unit S-101 Mod 4 extends the capability of the Reeves electronic analog computers to include the following operations: multiplication and division of variables, rotation of axes, resolution of vectors into rectangular components, addition of vectors, solution of other trigonometric problems and ordinary nonlinear differential equations, and the introduction of arbitrary or empirically determined functions into problems. The components of servo unit S-101 Mod 4 are housed in a standard 7-foot-high REAC cabinet. The unit contains two multiplying and two resolving servos. The multiplying servos drive a followup potentiometer and three 10-turn multiplying potentiometers, and have provisions for mounting 2 functional potentiometers. The resolving servos differ from the multiplying servos in the following ways: A large sine-cosine potentiometer driven by the servo which, together with the special inverter amplifiers provided with the unit, allows transformation of two vectors from polar to rectangular coordinates or vice versa. The special inverter amplifiers (4 total) may be used for ordinary computing purposes when the resolving servos are not being used. Using standard 0.05-percent linear potentiometers, the static accuracy of the servo is better than 0.05 volt and the dynamic accuracy better than 0.1 volt for the normal operating frequency ranges (i.e., from 0.02 to 0.5 cps). Somewhat better accuracy may be obtained by using 0.025-percent linear potentiometers. The accuracy of the resolving potentiometer is within $\frac{1}{8}$ percent (of full scale) of the theoretical value.

REAC function generator EFG-101 Mod 1. The Reeves electronic function generator EFG-101 Mod 1 enables the Reeves electronic analog computers to perform computations involving single-valued arbitrary



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functions. Trigonometric operations, multiplication, division, the solution of linear differential equations with variable coefficients, and the solution of nonlinear differential equations can thus be performed electronically by inserting the appropriate functions. The function generator consists of 6 identical channels, associated power supplies, and calibration, metering, and control circuits. These are housed in a standard 7-foot-high REAC cabinet. The system of connections is designed in such a manner that the inputs and outputs of all of the channels are terminated at the prepatch panel of a REAC computer for convenient interconnection with the computing elements of the REAC system. This function generator, which is of the phototube-mask type described in Section 3, is designed to operate with inputs up to ±100 volts and to deliver outputs up to the same limits. The slides used to generate arbitrary functions are 3 1/4 by 4 1/4 inches and may be prepared by the CU-101 Mod 1 camera unit.

The amplitude response is uniform from zero to 10 cps within 0.25 percent and from 10 cps to 30 cps within 1.0 percent. The phase shift at 10 cps is less than 1 degree and at 30 cps is less than 5 degrees. The output is within 0.5 volt of the correct value over the entire range of ±100 volts for slopes up to and including 80 degrees. The output will drift less than 0.25 volt over any 15-minute period after suitable warmup. In normal use, readjustment should not be required more than once a week, or when tubes are replaced. Calibration means are built in for adjusting the function generator and for calibrating function slides.

REAC model 10-101A input-output function generator. The use of this unit as an output device is discussed in the volume on recorders. This function generator is of the mechanical servo type discussed in Section 3. It consists of a rotating servo-controlled drum on which a contact wire in the shape of the desired function is mounted. The drum is controlled by the independent variable to rotate past a fixed linear resistance card with the contact wire making contact at a variable point determined by the desired function. The linear resistance card usually has a fixed voltage across its ends. Functions also may be generated manually by using an aided-tracking handwheel to position a cross-hair to follow a plot of the function.

## 1.8 RADIO CORPORATION OF AMERICA, R. C. A. LABORATORIES DIVISION

The dynamic systems synthesizer which R. C. A. Laboratories is currently developing for Wright Air Development Center is not a



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commercially available, general-purpose type computer but is included here for the sake of completeness. This synthesizer is a large-scale analog computer to be used for dynamic systems engineering problems, and the design makes use of the experience gained in the development of the Typhoon computer.

The proposed specifications for the D. S. S. have been published in the R.C.A. Laboratories report "Proposed Specifications for a Dynamic Systems Synthesizer" and are reproduced in reference 24. Several unique developments proposed by R.C.A. deserve special mention. The need for electromechanical resolvers in the D. S. S. will be eliminated by the use of electronic curve followers, and electronic sine-cosine loops. These loops will utilize the high-accuracy time-division multipliers being developed to provide accurate multiplication by sin $\alpha$ and cos $\alpha$ when x and y are given, where $\tan \alpha = y/x$.

Zero stabilization of computing amplifiers commonly is accomplished by the use of an auxiliary ac amplifier and vibrator-type chopping and rectifying elements. R.C.A. Laboratories has developed a successful chopper-stabilizer using photoelectric chopping and rectifying elements, thus eliminating one computer component that requires extensive maintenance in a large analog computing facility.

Considerable work has been done by R.C.A. toward the development of an automatic or semi-automatic problem setup arrangement. Two systems currently are under consideration. One of these systems would utilize sandwich-type problem boards with a large number of spring-loaded contact plugs in the cover. Punched cards would be inserted and the cover locked down, causing the contact plugs to make contact wherever a hole in the cards appears. Problem storage would consist merely of filing the appropriate cards. The other system under consideration would use latching relays in place of the contact plugs. The relays would be remotely actuated from a simultaneous punched card reader, thus making problem-connection entirely automatic. Using as many as 10,000 relays in a single problem is under consideration, which would make a very reliable relay design mandatory.



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# APPENDIX 2

## OPERATING CHARACTERISTICS OF THE REEVES ANALOG COMPUTING EQUIPMENT

The following material is reproduced from reference 25. Sections of this report are included to indicate the methods of measurement and the type of data which are of use in an analog computer facility. The data given here is not necessarily applicable to the latest models of Reeves analog computing equipment. All tabulated data is omitted where both graphical and tabular data are given.

### 2.1 INTRODUCTION

This report is intended as a guide for the evaluation of the accuracy and reliability of the results of computation on the Reeves Electronic Analog Computer. Response characteristics such as amplitude response, velocity, and acceleration limits, drift, backlash, and linearity are considered. Since quadratic terms of the form of simple harmonic motion with damping are common to many problems, an evaluation of the error in the damping term due to the computing equipment is included.

The methods of measurement used to obtain the data in this report are indicated with schematic diagrams in conventional REAC notation. Mathematical derivations are presented where required.

The data is presented in tabular form and graphs are provided for suitable characteristics. The equipment is considered in the following sections for convenience of presentation: Computer, consisting of diode limiters and summing, inverting, and integrating amplifiers; servos, high and low speed; REAC recorder, modified Sanborn; plotting board; and input-output table.

### 2.2 COMPUTER

#### 2.2.1 Amplifier Characteristics

The inverters, summers, and integrators of the REAC all utilize a standard plug-in amplifier unit (standard Reeves notation for amplifiers is used throughout this report). The summing resistors and a feedback resistor or capacitor are connected externally to each unit.



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(a) Amplitude response. The amplitude response (Fig. 2) was obtained by comparing the difference between input and output peak amplitude with the peak amplitude of the input (maintained at 10 volts) over the frequency range from 1 to 4000 cps. The results are plotted in Figure 4.

(b) Phase response. The phase response (Fig. 3) was obtained by measuring the difference between a sin $\omega t$ and b sin $(\omega t + \phi)$ over the frequency range from 2 to 1000 cps where a sin $\omega t$ was the input to the test amplifier and -b sin $(\omega t + \phi)$ was the output. The potentiometer was set to the reciprocal of $e_o/e_i$, thus maintaining a equal to b. From the maximum amplitude of the resulting y, and the known value of a, $\phi$ was evaluated as indicated by the following equations:

$$y = a \sin \omega t - a \sin (\omega t + \phi)$$

$$y = a \left[\sin \omega t (1 - \cos \phi) + \cos \omega t (-\sin \phi)\right]$$

$$y = a \sqrt{2(1 - \cos \phi)} \cos (\omega t + \psi)$$

where

$$\tan \psi = \frac{(1 - \cos \phi)}{-\sin \phi}$$

$$y = a \sqrt{2(1 - 1 + \phi^2/2! - \phi^4/4! + \phi^6/6! - -)} \left[\cos (\omega t + \psi)\right]$$

$$y \cong a \phi \cos (\omega t), \text{ for small } \phi$$

$$y_{max} \cong a \phi$$

The results are plotted in Figure 5.

(c) Amplifier drift. To measure amplifier drift (Fig. 6), summers and inverters were first balanced to 0.0 millivolt, and integrators to 0.4 millivolt. (The tolerance for these measurements was ±0.05 millivolt.) In general, a properly operating amplifier had a drift of less than 0.2 millivolt over an eight-hour period. It was found that 0.4 millivolt was the setting producing minimum drift for the integrators (as integrators) in the computer rack tested. Readings were made on the one-volt scale in the presence of an average apparent noise level of approximately 0.5 millivolt peak-to-peak. Amplifiers No. 11 and 18 were apparently defective with a sporadic, high noise level and poor drift characteristics.



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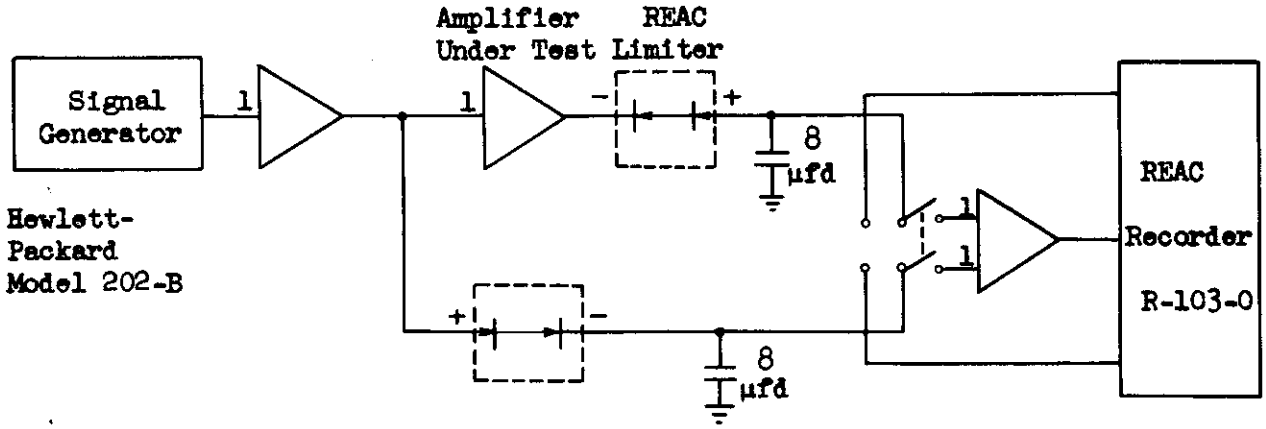


FIGURE 2 - AMPLIFIER AMPLITUDE RESPONSE SCHEMATIC

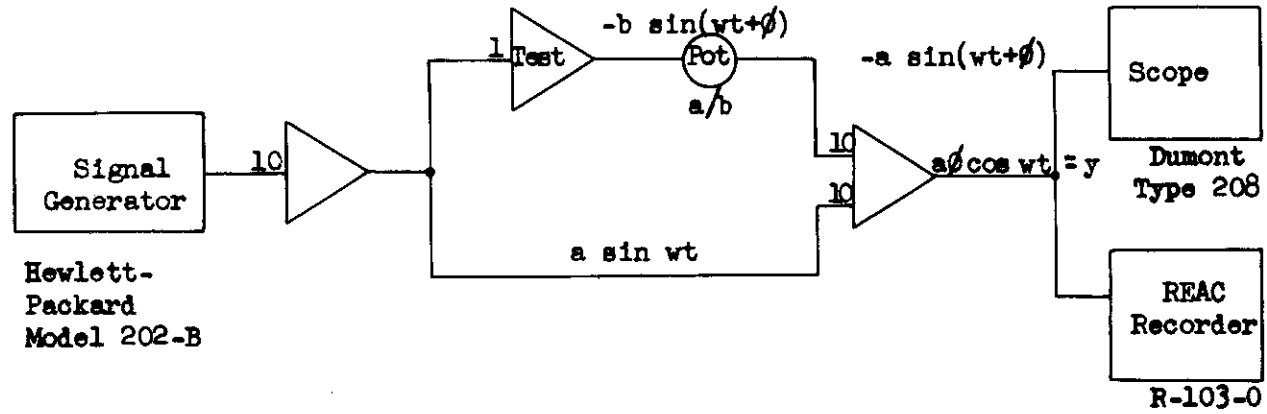


FIGURE 3 - AMPLIFIER PHASE RESPONSE SCHEMATIC



SA-313

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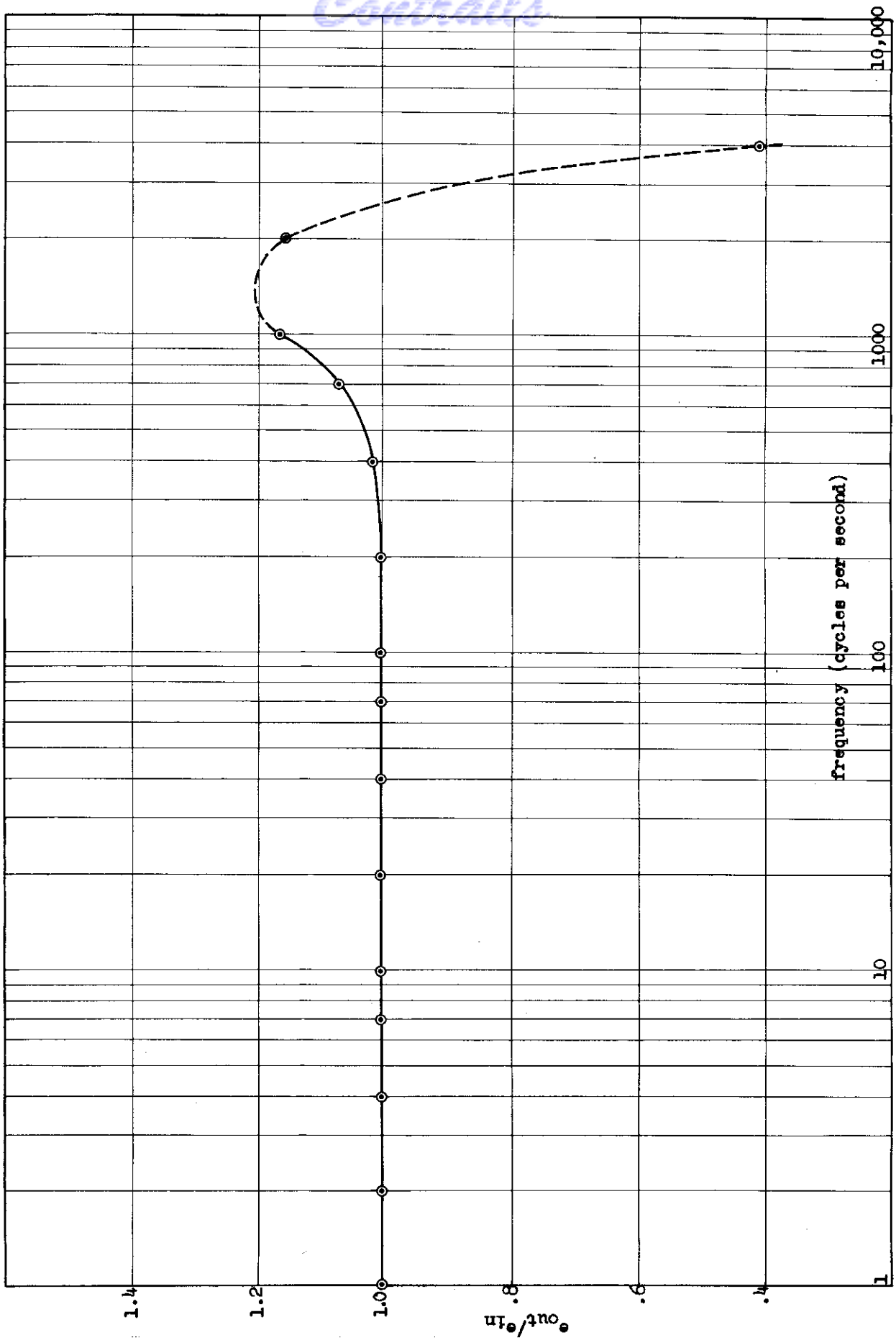


FIGURE 4 - AVERAGE AMPLITUDE RESPONSE OF FOUR REAC AMPLIFIERS



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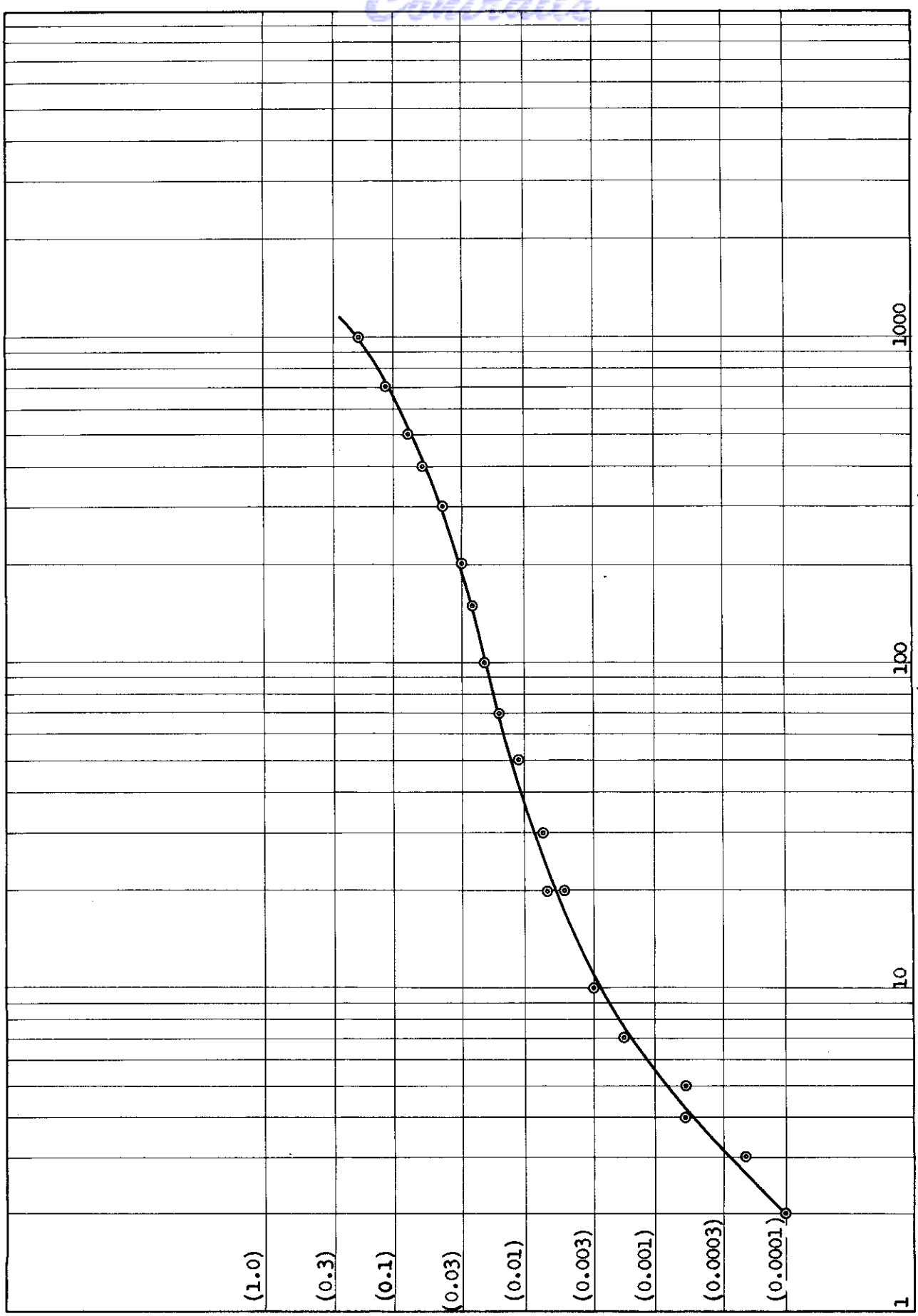


FIGURE 5 - PHASE RESPONSE OF A REAC AMPLIFIER

SA-304



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### 2.2.2 Drift of Integrating Amplifier

It was found that the integrator drift characteristic in the OPERATE position improved when the initial balancing took place with two "10" inputs connected to ground and the integrator adjusted for minimum drift on OPERATE. The output was observed as described for amplifier drift measurement.

The measurements of drift were taken with no input and in the OPERATE condition. The readings were obtained directly on the integrator output from the REAC panel meter on the appropriate scale.

### 2.2.3 Damping Effect

There is an observed change of amplitude with respect to time in the oscillator shown in Figure 7. To measure small amplitude changes conveniently, the sine and cosine components were applied to the X and Y axis inputs of the REAC plotting board. The change in radius of the circle formed in this manner was observed during a time t. For higher frequencies the change in amplitude was measured on the REAC recorder.

It was found that the gain of the amplifier in the oscillator loop affected the values obtained from this measurement. In general, optimum results are obtained for a given loop gain when the maximum possible gain is obtained in the integrators and a minimum gain obtained from the amplifier. In the present REAC, the maximum gain in an integrator is 31. For the high frequencies the oscillator was measured with three fixed values of amplifier gain. A single amplifier with a gain of 1, a single amplifier with a gain of 10, and three amplifiers in series with a gain of 4 each, were used. The measurements were essentially independent of amplitude. However, the initial condition of the amplifier output was maintained at 10 volts.

The change in amplitude was assumed to be exponential and of the form found in an oscillator with damping; that is,

$$\ddot{X} + 2\zeta \omega_n \dot{X} + \omega_n^2 = 0$$

$$X = e^{-\zeta \omega_n t} X_o \sin (\omega_n \sqrt{1 - \zeta^2})t$$

$100\zeta$ = percent of critical damping

$\omega_n \cong \omega$, for small $\zeta$

In terms of peak amplitude of sin $\omega t$



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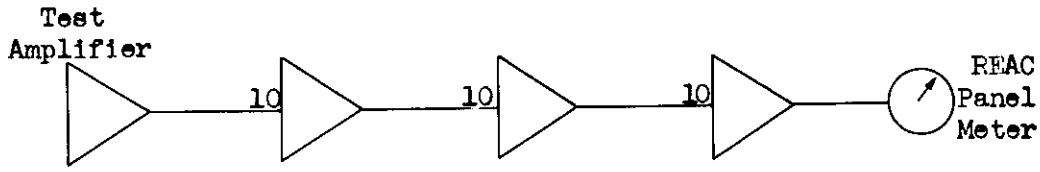


FIGURE 6 - AMPLIFIER DRIFT SCHEMATIC

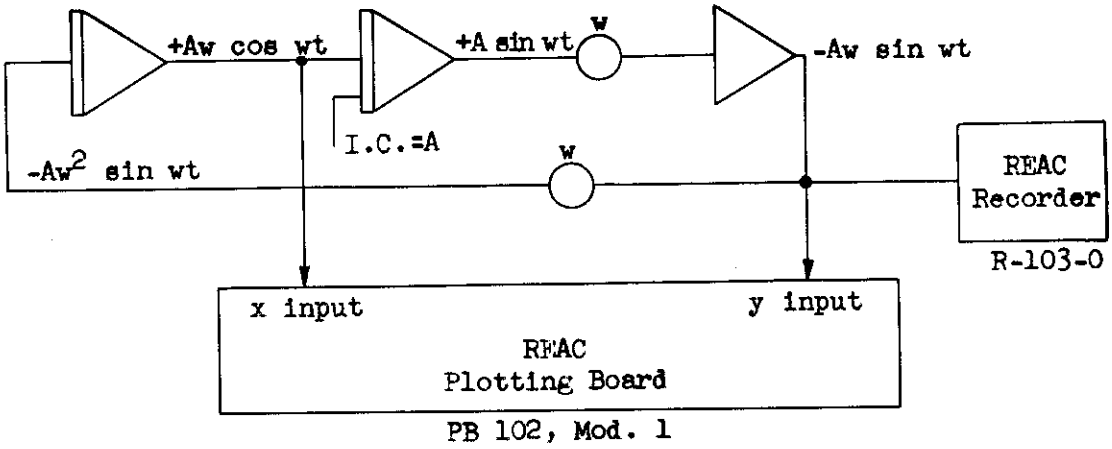


FIGURE 7 - SCHEMATIC OF DAMPING EFFECT (WITH AMPLIFIER IN LOOP)



SA-314

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$$X_{max} = X_o e^{-\zeta \omega t}$$

$$\zeta = \frac{-\ln X_{max}/X_o}{\omega t} = \frac{-\ln X_{max}/X_o}{2\pi n}$$

n = number of cycles observed during time t.

The percent damping obtained in this test represents an error in the damping term at a particular frequency. The results are tabulated in Table 1 and plotted in Figure 8. Additional data, not presented here, indicate that the particular integrators and amplifiers used in this measurement are representative of the REAC equipment.

| $x_{max}/x_o$ | n (cycles) | $\omega^2$ (radians/sec)$^2$ | f (cps) | $100\zeta$ (percent) |
|---|---|---|---|---|
| | | Amplifier Gain = 1 | | |
| 0.940 | 50 | 0.001 | 0.00504 | .012 |
| 0.932 | 93 | 0.01 | 0.0159 | .012 |
| 0.928 | 103 | 0.1 | 0.0504 | .012 |
| 0.605 | 668 | 1.0 | 0.159 | .012 |
| 0.492 | 813 | 10.0 | 0.504 | .014 |
| 0.305 | 2860 | 100.0 | 1.54 | .007 |
| 1.84 | 280 | 400.0 | 3.18 | -.035 |
| 2.25 | 129 | 783.0 | 4.45 | -.1 |

TABLE 1 - DAMPING EFFECT DATA (WITH AMPLIFIER IN LOOP)

## 2.3 SERVOS

The principal nonlinear errors in the servo output result from backlash and friction in the gear train, nonlinearity of follow-up potentiometers and servo motor, and velocity and acceleration limits. These errors define a region of useful operation within the velocity and acceleration limits for amplitudes large compared with amplitude errors caused by backlash, friction, and potentiometer nonlinearity. In this region the nonlinear errors that still exist introduce an amplitude and phase error in the principal frequency of any given periodic input as well as in other frequencies. A linear analysis based on this principal frequency is then a useful approximation, although for some frequencies and amplitudes a large part of the error present may be of nonlinear origin.



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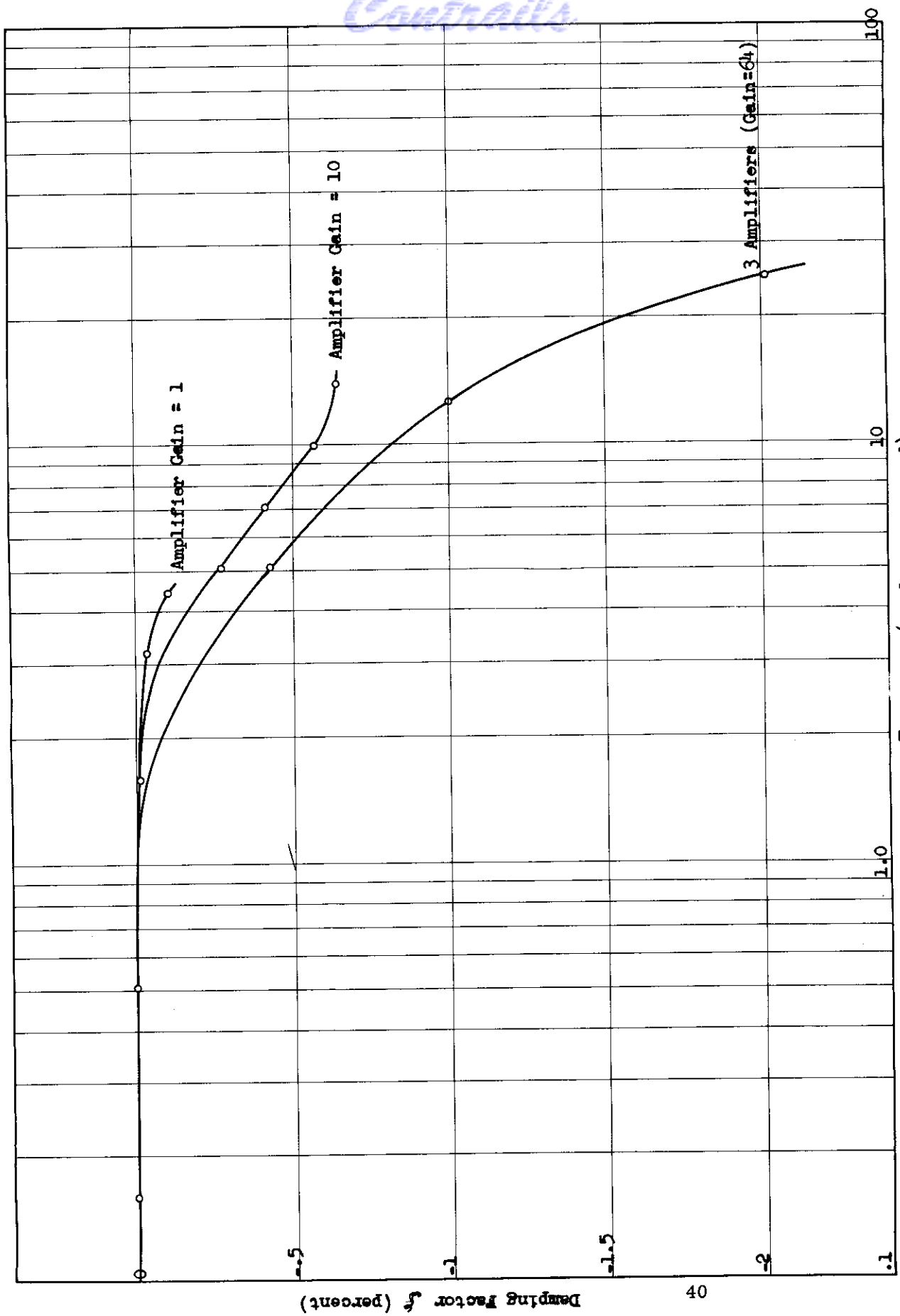


FIGURE 8 - REAC OSCILLATOR DAMPING EFFECT

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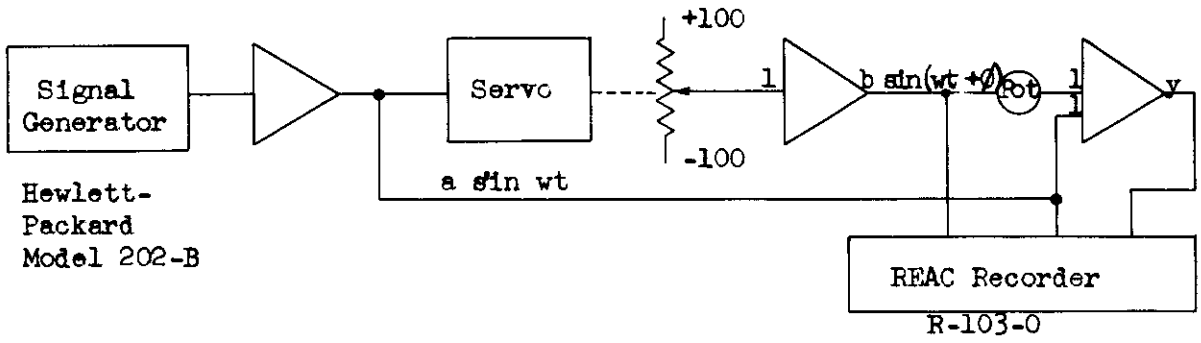


FIGURE 9 - SCHEMATIC OF SERVO AMPLITUDE AND PHASE RESPONSE

SA316



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The operations indicated in the following section were performed on both high and low speed servos.

2.3.1 Amplitude and Phase Response (Fig. 9)

$$y = a \sin \omega t - b \sin (\omega t + \phi)$$

$$= (a - b) \sin \omega t + b \sin \omega t - b \sin (\omega t + \phi)$$

$$= (a - b) \sin \omega t + b \phi (\cos \omega t), \text{ for small } \phi$$

For each point the potentiometer was varied until y was a minimum. The potentiometer reading was then equal to $e_{in}/e_{out}$ with a = b. The phase shift was evaluated directly from the amplitude of y. Refer to Figures 10 through 13.

2.3.2 Velocity and Acceleration Limit (Fig. 14)

A differentiator was formed as indicated. The output was calibrated on the recorder in terms of displacement equivalent to volts per second. Potentiometer K was adjusted so that the differentiated output of the servo was the same order of magnitude as the calibrating voltage. The differentiated displacement output was then interpreted as velocity and the differentiated output slope as acceleration. It was found that the high-speed servo did not reach a velocity limit with the maximum step input of 95 volts. The results of these measurements are shown in Table 2. See also Figures 11 and 13.

| | Low-Speed Servo No. | | | |
|---|---|---|---|---|
| | I | II | III | Average |
| Velocity limit (volts/sec) | 96 | 95 | 91 | 94 |
| Acceleration limit (volts/sec$^2$) | 519 | 503 | 505 | 509 |

TABLE 2 - VELOCITY AND ACCELERATION LIMITS OF LOW-SPEED SERVOS



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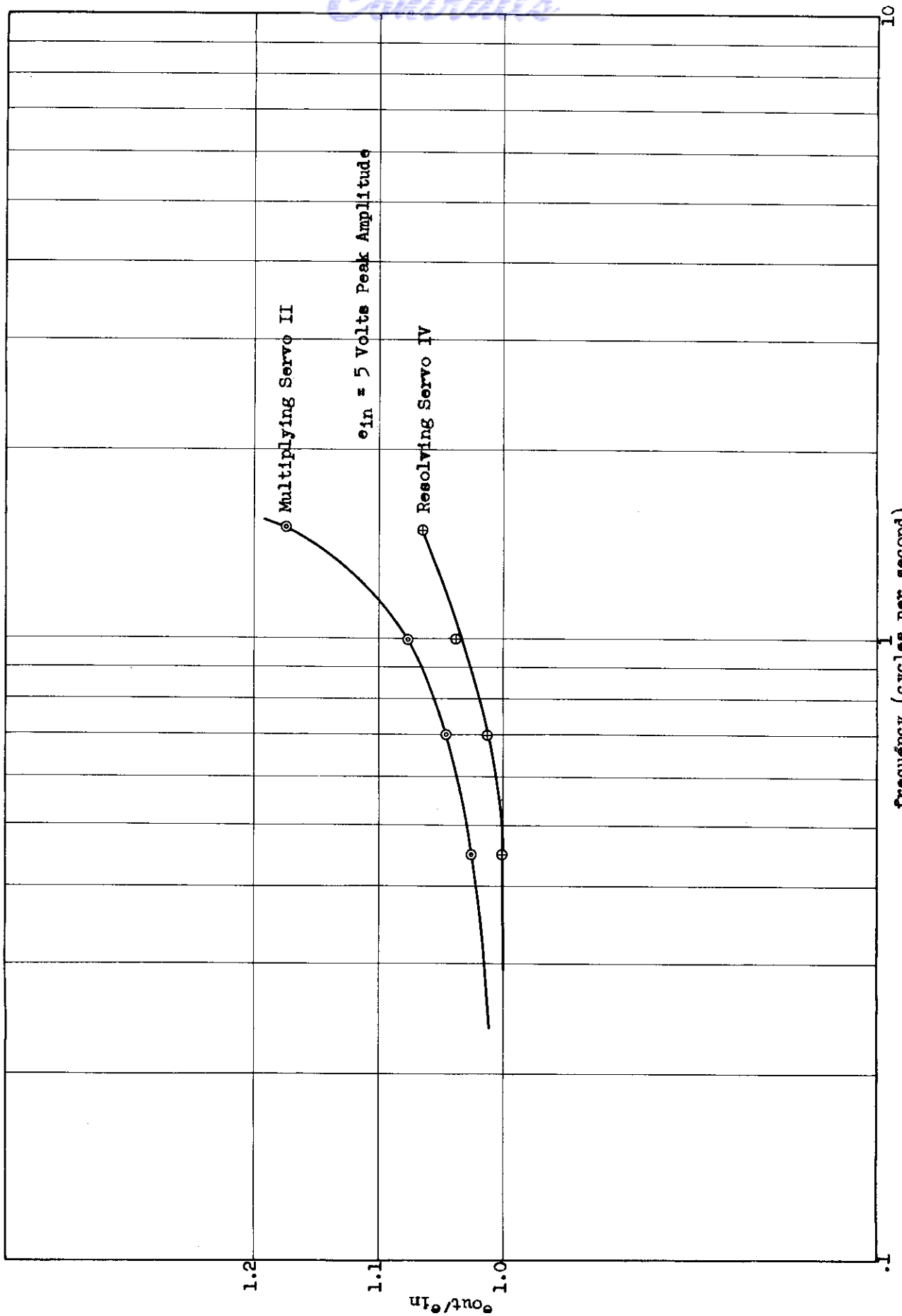


FIGURE 10 - AMPLITUDE RESPONSE OF TWO LOW-SPEED SERVOS

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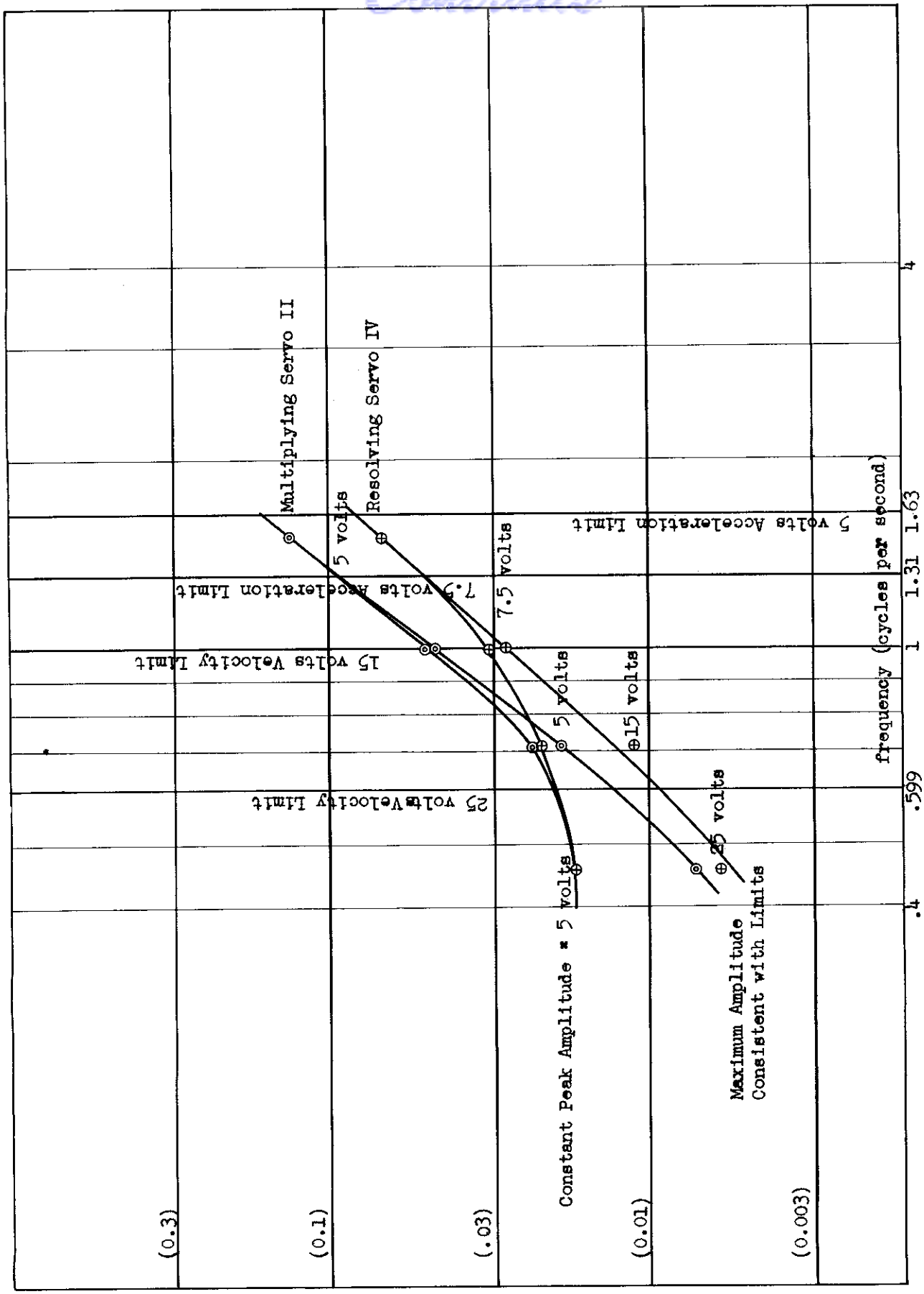


FIGURE 11 - PHASE SHIFT CHARACTERISTIC OF TWO LOW-SPEED SERVOS

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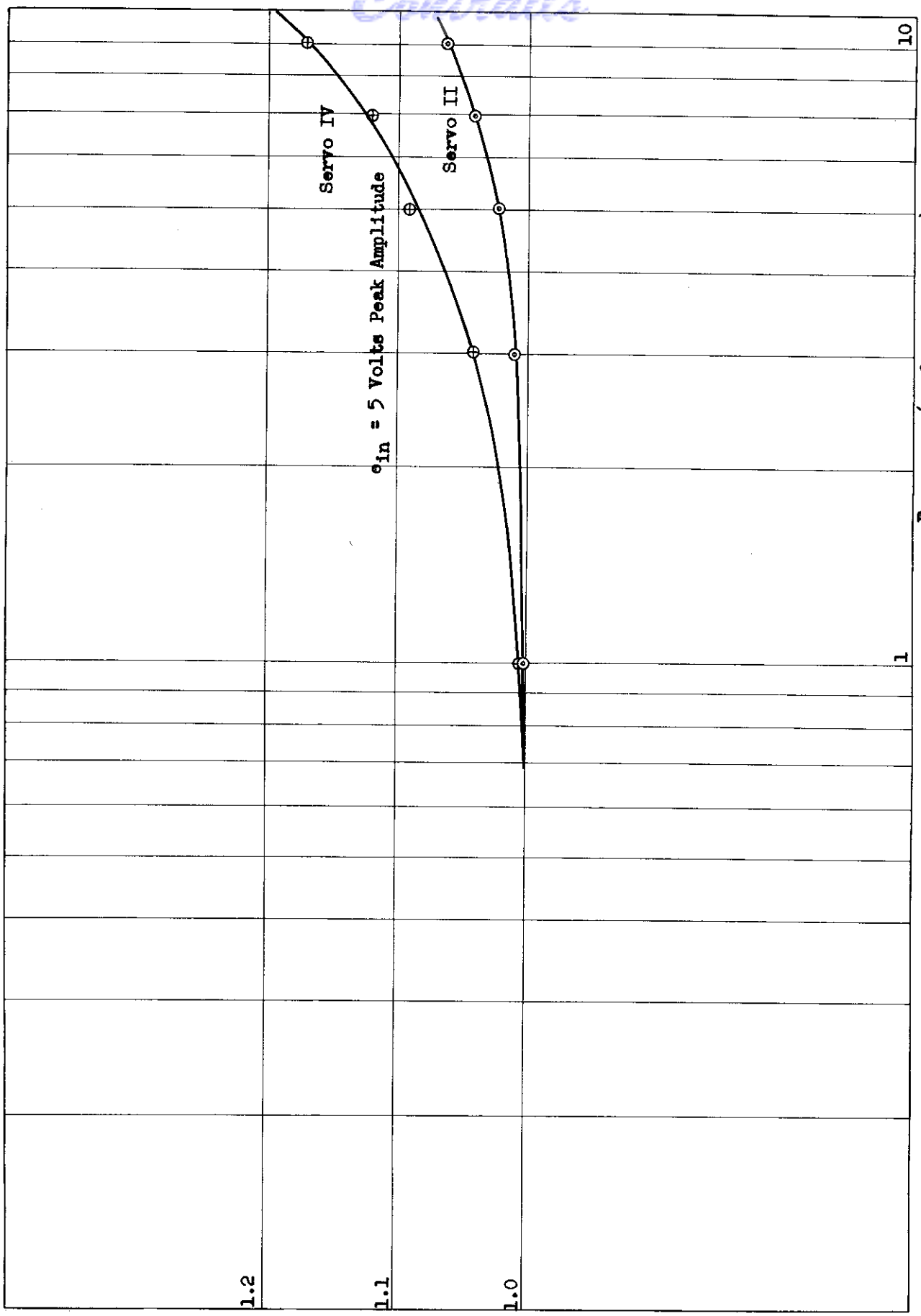


FIGURE 12 - AMPLITUDE RESPONSE OF TWO HIGH-SPEED SERVOS





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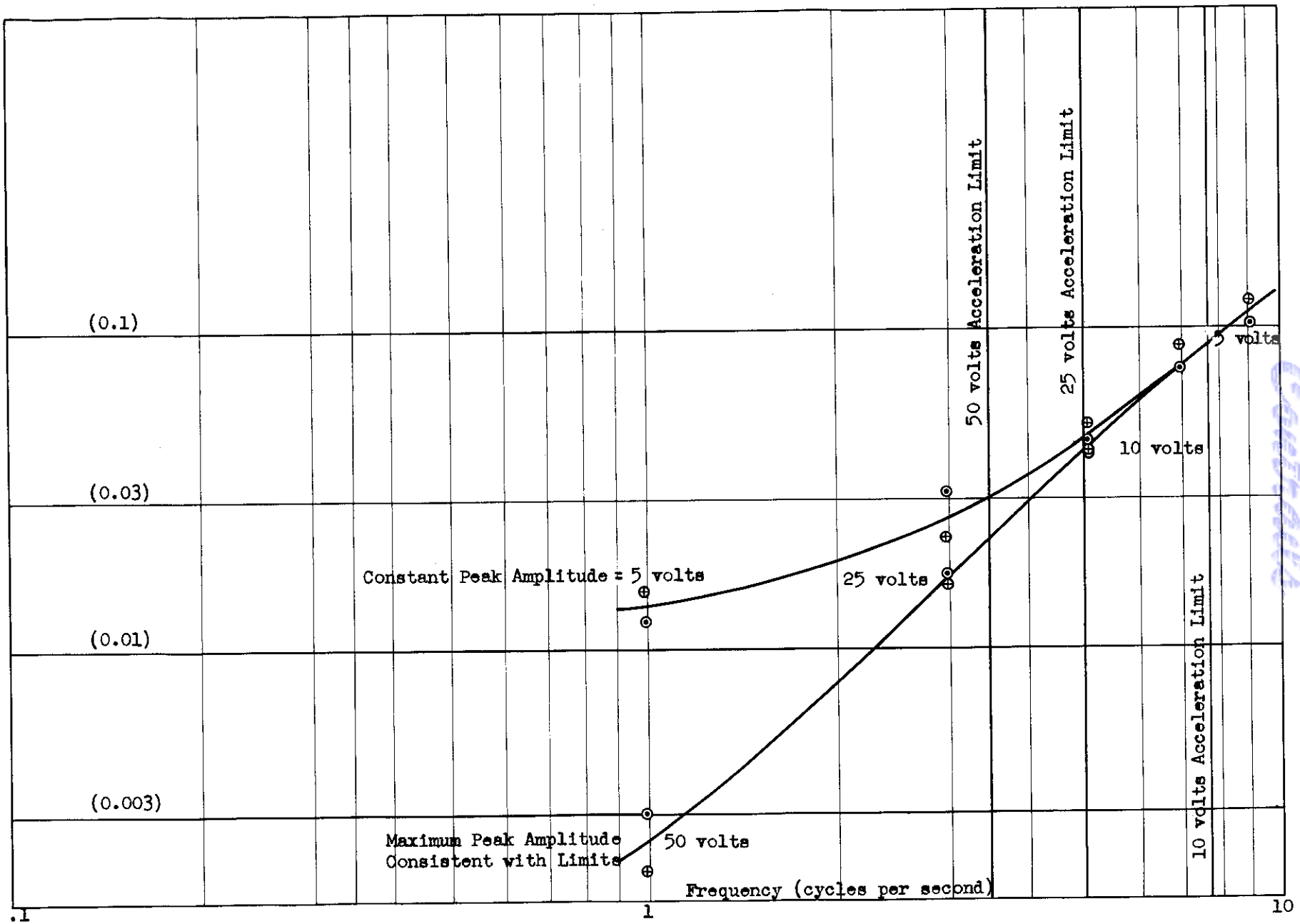


FIGURE 13 - PHASE SHIFT CHARACTERISTIC OF TWO HIGH-SPEED SERVOS



SA-310

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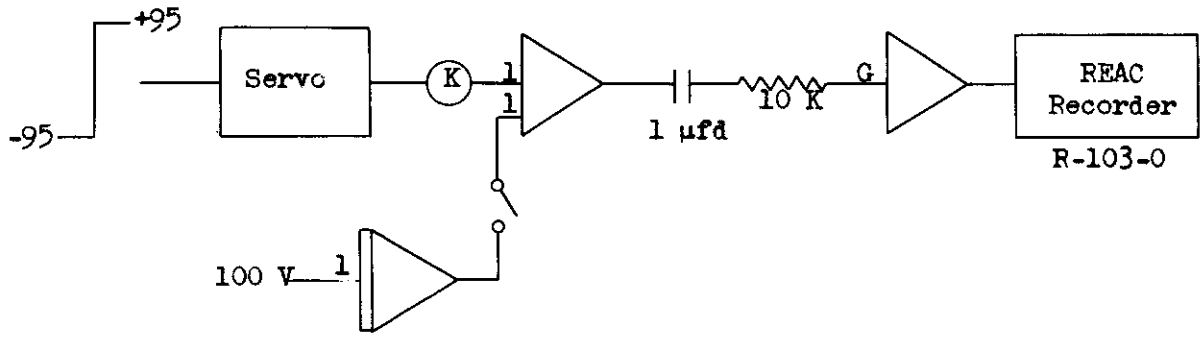
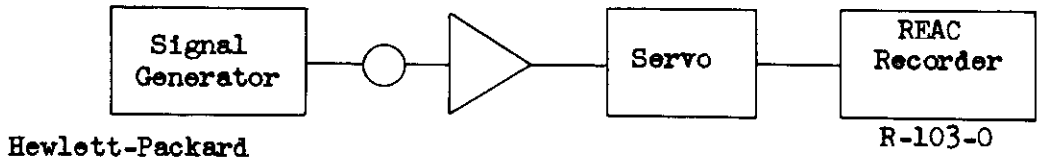



FIGURE 14 - SERVO VELCCITY AND ACCELERATION LIMIT SCHEMATIC




Hewlett-Packard
Model 202-B

FIGURE 15 - SERVO BACKLASH SCHEMATIC



SA-317

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2.3.3 Backlash (Fig. 15)

The amplitude was reduced to a value for which the output approached zero. The peak-to-peak amplitude was then used as a measure of the backlash. The values are as follows:

| Low-speed servos | | High-speed servos | |
|---|---|---|---|
| I | 0.05 volt | II | 0.015 volt |
| II | 0.14 volt | IV | 0.04 volt |
| III | 0.08 volt | | |
| IV | 0.06 volt | | |

## 2.4 REAC RECORDER

### 2.4.1 Amplitude Response (Fig. 16)

To study the amplitude response of the REAC recorder (R-103-0, Modified Sanborn), input voltage was obtained from a signal generator, Krohn-Hite, Wide Range Ultra-Low Frequency Oscillator, Model 410-A. According to the manufacturer, the amplitude varies less than ±3 percent over the frequency range from 0.02 to 20,000 cycles per second. The output amplitude on three channels was measured directly as the frequency of the oscillator was varied. The response was obtained for three initial values of amplitude at 0.1 cycle per second. The damping adjustment of the recorder was set for approximately 4 percent overshoot on a 20-millimeter square wave. The average output of the three channels is plotted for the 20-millimeter data in Figure 17.

### 2.4.2 Linearity and Backlash (Fig. 18)

The amplitude was varied with REAC, a scale-factor potentiometer, linear to 0.1 percent with corrections applied for the loading effect of the amplifier input. The amplitude of the recorder output was set to 50 millimeters peak-to-peak for full scale of the potentiometer. The input, at 1 cycle per second, was reduced in steps of 0.1 and the average output amplitude of three channels was measured directly.

An attenuation of 100:1 was then set on the oscillator, and the potentiometer setting was reduced until there was essentially zero output. The range of values of backlash for the three channels was approximately 0.1 or 0.2 millimeters. The results over the entire range are plotted in Figure 19.



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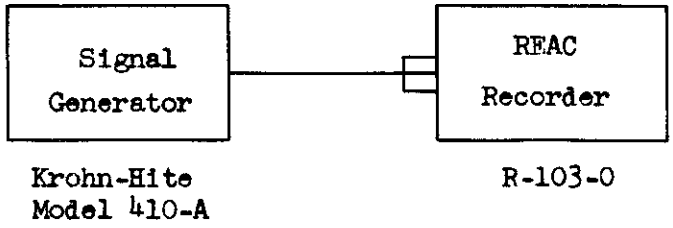


FIGURE 16 - REAC RECORDER AMPLITUDE RESPONSE SCHEMATIC



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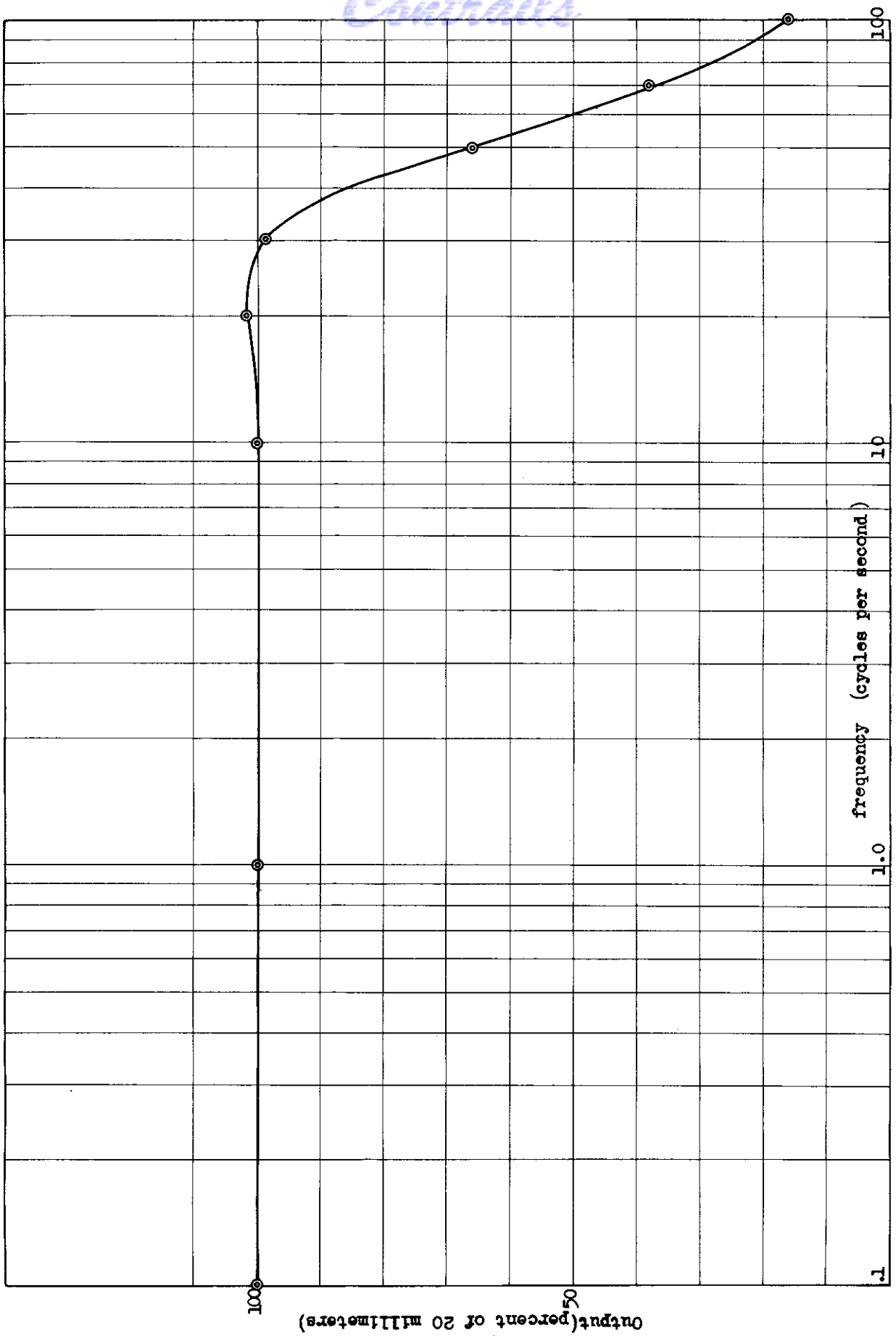


FIGURE 17 - REAC RECORDER AMPLITUDE RESPONSE

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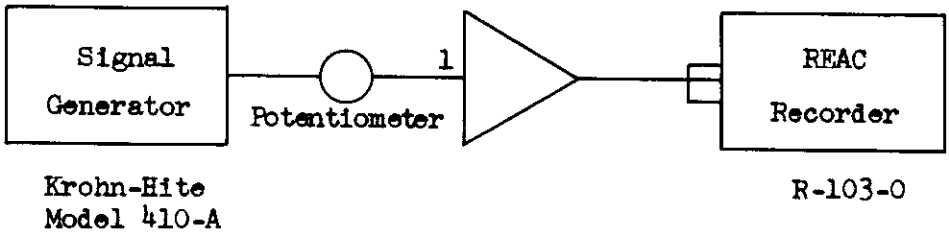


FIGURE 18 - REAC RECORDER LINEARITY SCHEMATIC

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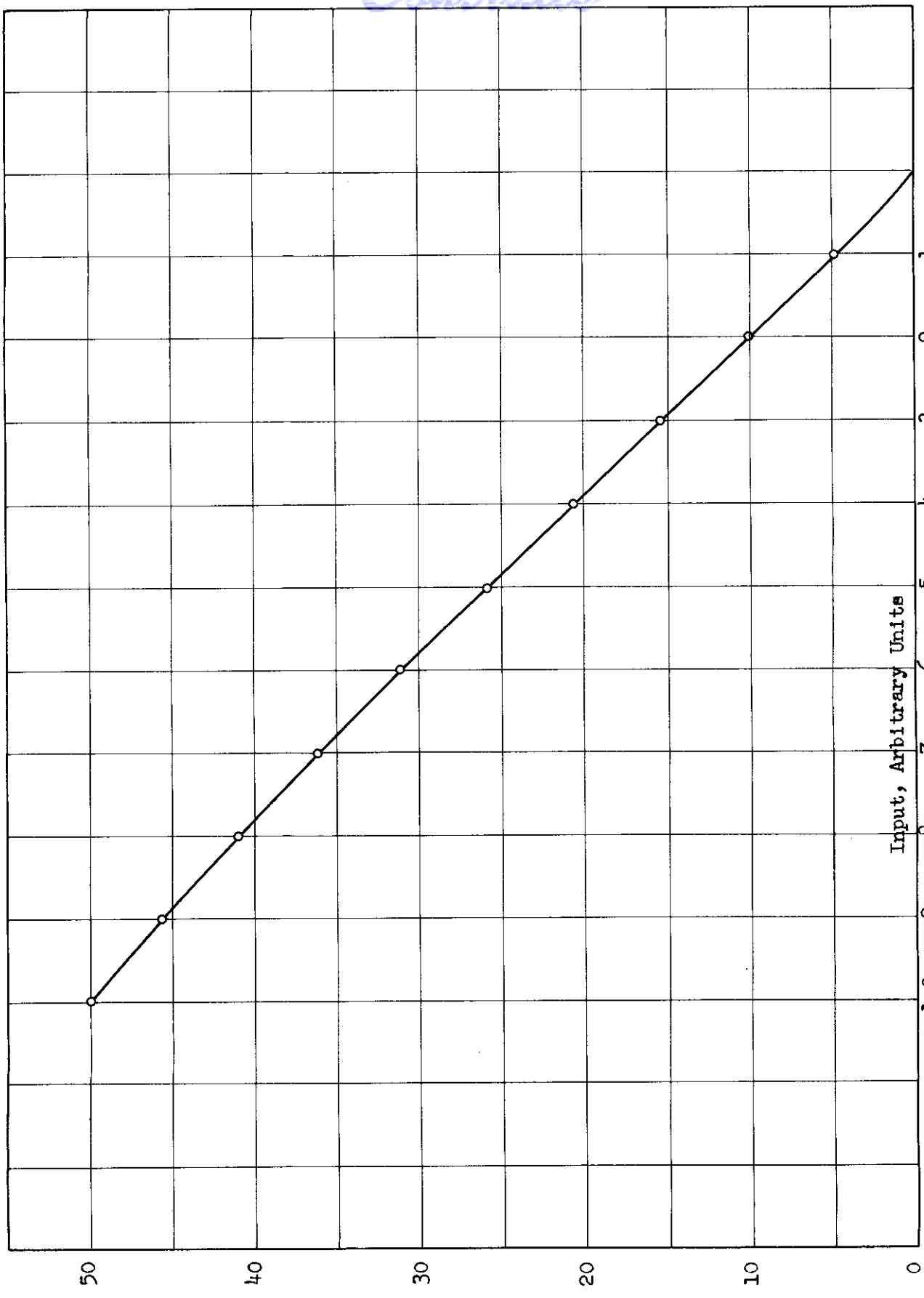


FIGURE 19 - REAC RECORDER LINEARITY

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## 2.5 PLOTTING BOARD

### 2.5.1 Amplitude Response (Fig. 20)

The amplitude response of the plotting board (PB 102, Mod 1) was studied with an input obtained from a Krohn-Hite, Model 410-A, Ultra-Low Frequency Oscillator applied to the Y and Z axis inputs. A time base, from an integrator with a constant input, was applied to the X axis. The frequency of the input was varied and the output was measured directly. The response was obtained for an initial value of one-half-inch peak amplitude at 0.1 cycle per second. For the measurement of the X axis response, the X and Y inputs were interchanged.

The results of this measurement and of the following phase response measurement depend on the setting of the sensitivity and damping controls for each axis. The criteria for these adjustments were maximum sensitivity consistent with smooth operation when the damping was set for approximately 5 percent overshoot for a one-inch step input. For specific recording requirements, it may be desirable to use different criteria for these adjustments.

The amplitude and phase response also depend on amplitude. The values of amplitude used were the smallest practical ones for the methods of measurement. Larger amplitudes will, in general, result in smaller errors if the velocity or acceleration limit is not reached.

The results of these measurements are plotted in Figure 21.

### 2.5.2 Phase Response (Fig. 22)

The input from the signal generator was applied to the Y and Z axis inputs and to two limiting amplifiers in series. The limiting amplifiers used were obtained from a REAC Auxiliary Computer Mod 0, A 105. The output of the limiters was a square wave with limiting points within 0.5 percent of the sine wave zero. The square wave was summed with the time base as a timing indication of the zero cross over. The phase shift was evaluated by measurement of the position of the break points in the resulting trace relative to the peaks of the sine wave. For the measurement of the X axis phase response the X and Y inputs were reversed.

The results are plotted in Figure 21.

### 2.5.3 Backlash and Drift (Fig. 20)

A sine wave was applied to each axis against a time base, and the output of the signal generator was reduced until the observed amplitude was essentially zero. The step attentuator on the signal generator was



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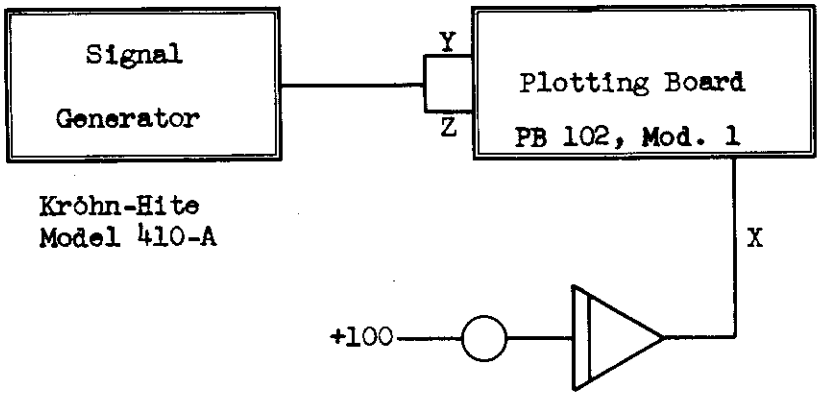


FIGURE 20 - PLOTTING BOARD AMPLITUDE RESPONSE SCHEMATIC



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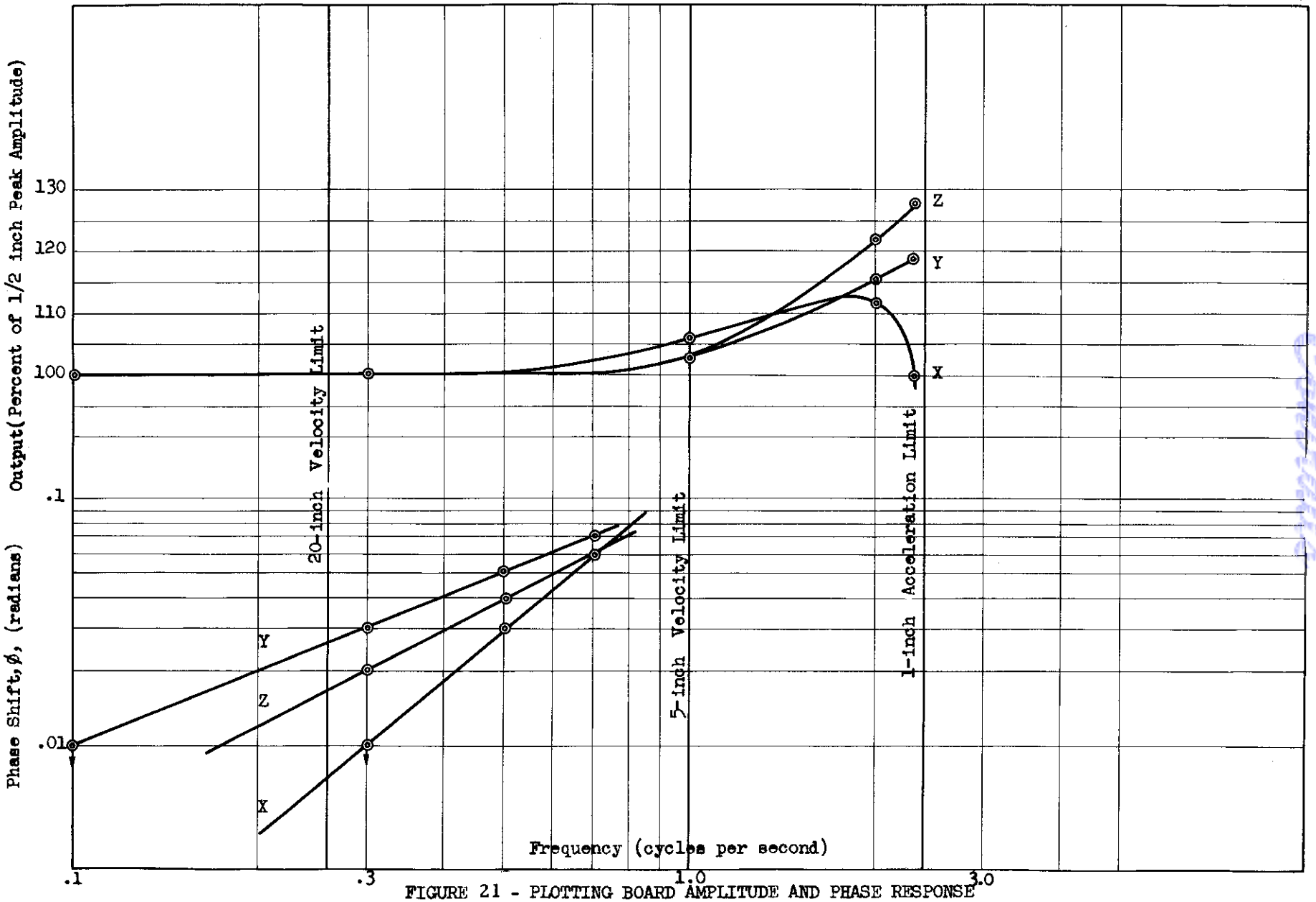


FIGURE 21 - PLOTTING BOARD AMPLITUDE AND PHASE RESPONSE

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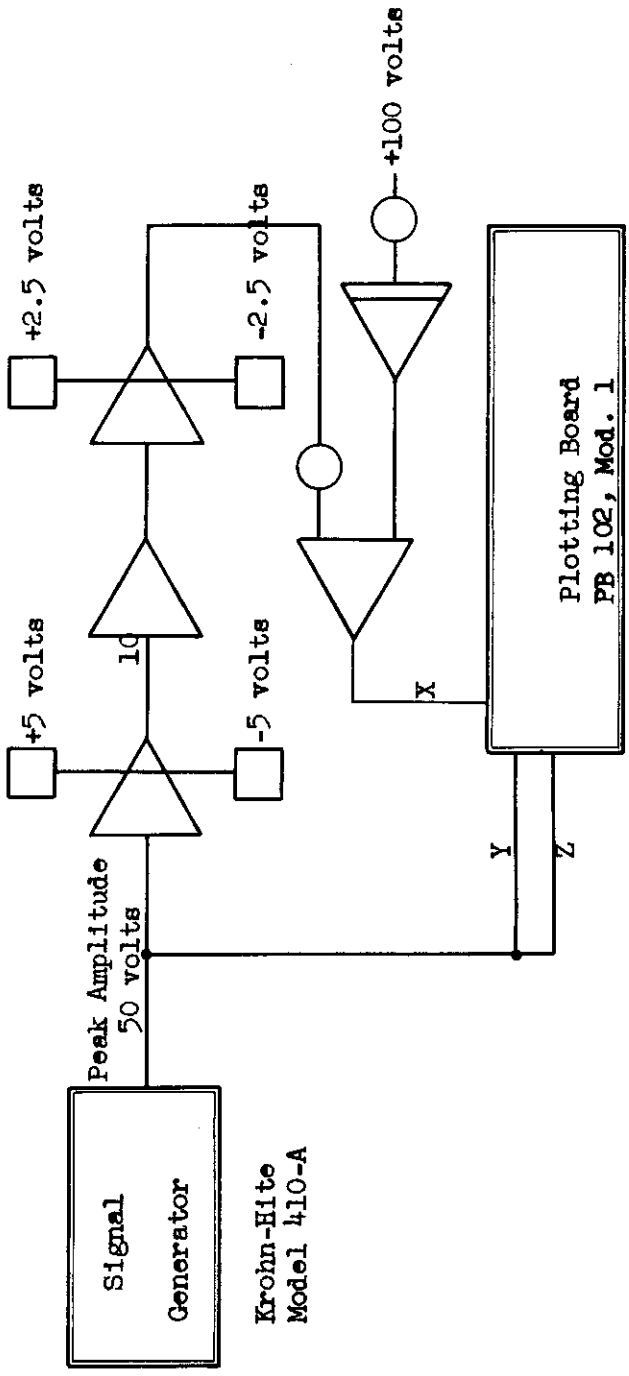


FIGURE 22 - PLOTTING BOARD PHASE RESPONSE SCHEMATIC



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then increased until a measurable amplitude resulted. The backlash for each axis was then evaluated from the measured amplitude and the amount of attenuation. The results were as follows: X axis, 0.03 inches; Y axis, 0.01 inches; Z axis, 0.01 inches.

The input of all channels was connected to ground and over a period of one hour the amount of drift was unmeasurable.

### 2.5.4 Velocity and Acceleration Limit

A saturating step input of voltage was applied to the Y and Z axis inputs with a time base applied to the X axis. From the plot so produced, the velocity limit was measured directly over the region of constant slope. The acceleration limit was obtained by measuring the slope $V_o$ and the displacement $X_o$ at a time $t_o$, and the displacement X at a later time $t_1$ prior to the time at which the displacement curve reached its region of constant slope. The acceleration limit was computed from the following relationship:

$$t_1 - t_o = t, \quad X = \frac{at^2}{2} + V_o t + X_o$$

$$a = 2\frac{X - X_o - V_o t}{t^2}$$

The X and Y inputs were reversed for the measurement of the X axis velocity and acceleration limit. The results are shown in Table 3. Representative limits at 20 inches, 5 inches, and 1 inch, are plotted in Figure 21.

| | Velocity Limit | Acceleration Limit |
|---|---|---|
| X axis | 33 in./sec. | 222 in./sec.$^2$ |
| Y axis | 34 in./sec. | 235 in./sec.$^2$ |
| Z axis | 32 in./sec. | 221 in./sec.$^2$ |

TABLE 3 - VELOCITY AND ACCELERATION LIMITS OF PLOTTING BOARD



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# APPENDIX 3 - MEASUREMENT TECHNIQUES

## 3.1 INTRODUCTION

The mathematical analysis of the dynamic errors that occur in the analog computer solution of differential equations requires a precise knowledge of the operation of each computing component.[26] The idealized operational characteristics assumed for the components when setting up problems are only approximations to the actual performances. These approximations, in general, become less accurate as the frequencies involved become higher (and in the case of electronic integrators, lower).

Operational amplifiers, linear passive networks, and linear servos can be characterized concisely by expressing their operational properties as linear transfer functions.[27] Other components, such as limiters and function generators, are essentially nonlinear and can be described only by families of performance curves, precision figures, and other specifications.[28]

In Appendix 4 a linear analysis of an operational amplifier is given in which effects ordinarily neglected are considered. After an analysis of this type is completed, the coefficients or parameters for use in the derived formulas must be evaluated experimentally. The usefulness of such an analysis depends on the fidelity of the description employed within the frequency band and under the other conditions of actual use. Therefore, measurement techniques capable of providing data under such conditions must be used.

## 3.2 MEASUREMENT TECHNIQUES

Conventional methods are not well adapted to the measurement of extremely small phase-shifts, such as those occurring in summing amplifiers at frequencies below one or two cps (say 0.005 degree).

The methods for measuring the frequency-response characteristics of a summing amplifier at a gain of unity are given in Appendix 2. The method shown for phase-shift measurements is good down to quite low

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[26] See the volume on error studies, references 8 (p. 139), 9, and 26.

[27] For discussion of transfer function of REAC multiplying servo, see reference 8, p. 183.

[28] See Appendix 2.



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frequencies, but requires the determination of the amplitudes of very small ac voltages at such frequencies.

Winson[29] has suggested a method whereby phase-shift measurements in summing and integrating amplifiers may be carried out to a high degree of accuracy for very small values of phase-shift. This method allows the cumulative effects of small phase-shifts to be observed rather than to attempt to measure phase-shifts per cycle. Winson's method utilizes the effects obtained when attempting to solve

$$\frac{d^2x}{dt^2} + \omega^2 x = 0; \quad \frac{dx(0)}{dt} = A, \quad x(0) = B \tag{1}$$

on the analog computer.

The well-known analytical solutions of equation 1 are undamped sinusoidal oscillations; however, the typical analog computer solutions to this equation show positive or negative damping effects that depend on the frequency of oscillation. Since the damping observed is of an exponential form, it may be assumed that the computer actually is solving an equation of the form of

$$\frac{d^2x}{dt^2} + 2\zeta\omega\frac{dx}{dt} + \omega^2 x = 0. \tag{2}$$

The "damping ratio" $\zeta$ may easily be determined from recordings of the analog computer solution and may, in turn, be used to find the excess phase shifts in the integrators and summing amplifiers used to solve equation 1. Two integrators and n summers (n odd) may be used to solve equation 1 in the manner shown in Figure 23. The excess phase-shift around the computing loop is related to the damping ratio by the equation[30]

$$\phi = 2\zeta.$$

As Winson points out, under certain simplified assumptions for the transfer functions of the amplifiers, the overall phase-shift versus frequency curve for the loop can yield information sufficient to evaluate the transfer function coefficients.

The phase-shift curves for the individual amplifiers can be obtained by substitution procedures. For instance, at each frequency the loop phase-shift can be measured first with one and then with three summing

[29] See reference 9, Chapter 5.

[30] For derivations see references 8 (p. 63) and 9.



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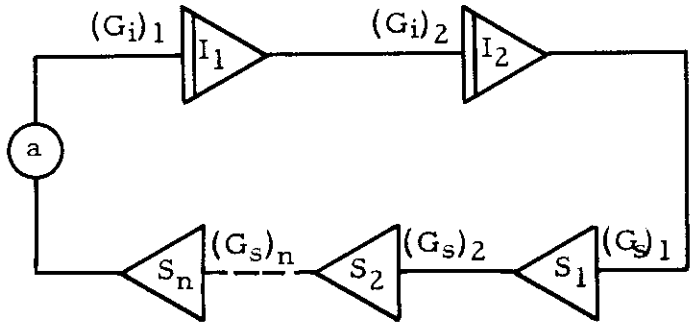


$$\ddot{x} + \omega^2 x = 0$$

$$\omega^2 = a(G_i)_1(G_i)_2 \prod_{j=1}^{n} (G_s)_j$$

$G_i$ = gain of integrator

$G_s$ = gain of summer

FIGURE 23 - COMPUTER CIRCUIT PHASE SHIFT MEASUREMENTS

amplifiers in the loop. Since the phase-shifts are additive, the phase-shifts due to the two inserted amplifiers can be obtained. If the two amplifiers are identical, the phase-shift for each will be half the total. If the two amplifiers are not identical, or if it is desired to ascertain whether they are identical, three amplifiers may be combined in pairs and the phase-shifts of the pairs measured. Simple calculations would then yield the phase-shifts of each. Similarly, by substitution and subtraction, the phase-shift versus frequency characteristics of the integrators can be obtained. The phase-shift characteristics shown in Figures 24, 25, and 26 for REAC and GEDA amplifiers were obtained by these procedures. The circuitry for making these measurements can be quite elaborate or can consist merely of some type of magnetic pen recorder for recording the loop oscillations. The results given here were obtained using a peak voltmeter assembled from computer components with a preset counter to count oscillations and to close a relay throwing the computer to "HOLD" at the end of each run. The peak voltages were recorded on the x-axis of a plotting board with index marks made in the y-direction by pulsing the y-servo at the end of each run (usually after 100 counts on the preset-counter). The runs were started manually by turning the counter on as the changing amplitude of the oscillations crossed a predetermined starting value ($A_1$). This value, together with the final amplitude ($A_2$) and the number of cycles between (m) are used to calculate the loop phase-shift from the formula[31]

$$\phi = 2\zeta = \frac{\ln A_1/A_2}{\pi m} \quad . \tag{3}$$

[31] For derivation see Appendix 2.



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It is found both theoretically and experimentally that computing amplifier phase-shift characteristics are quite sensitive to factors such as gains and loading.[32] For these reasons, care must be taken to maintain all circuit conditions constant when making substitution or other type tests. For instance, in Figure 23, connecting a low-impedance recorder to one of the amplifier outputs could change the amplifier characteristics considerably.

[32] See Appendix 4 for details.



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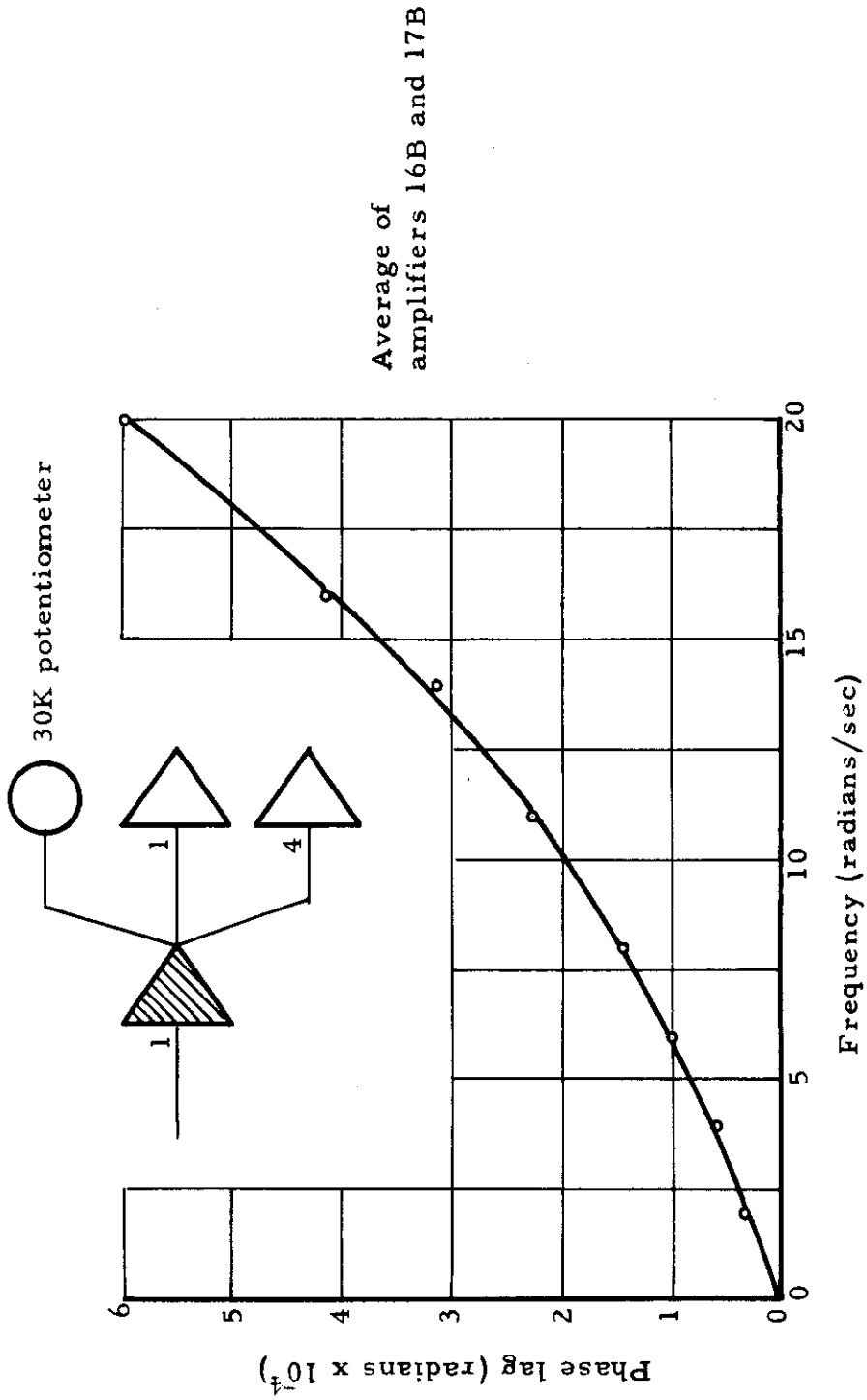


FIGURE 24 - PHASE SHIFT FOR ONE REAC INVERTER



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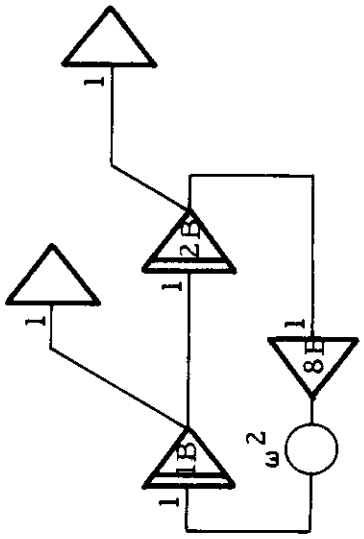


o - measured lag in loop minus lag of the summer divided by 2

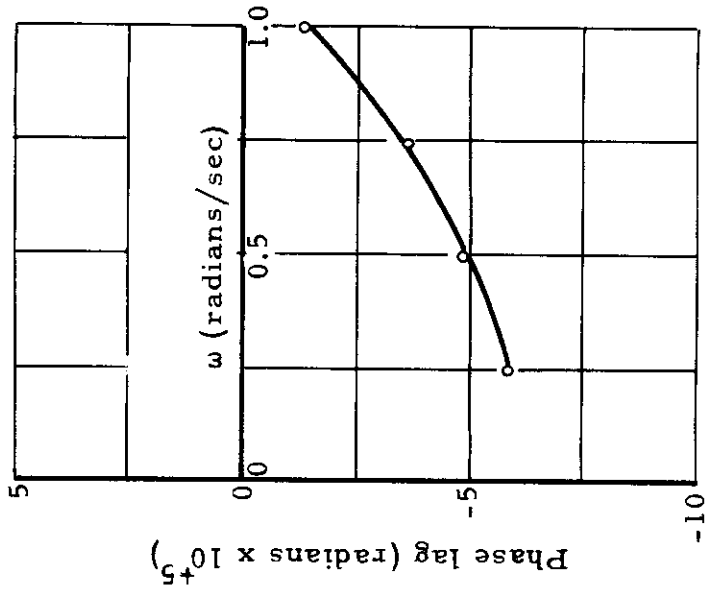


FIGURE 25 - PHASE LAG OF ONE INTEGRATOR



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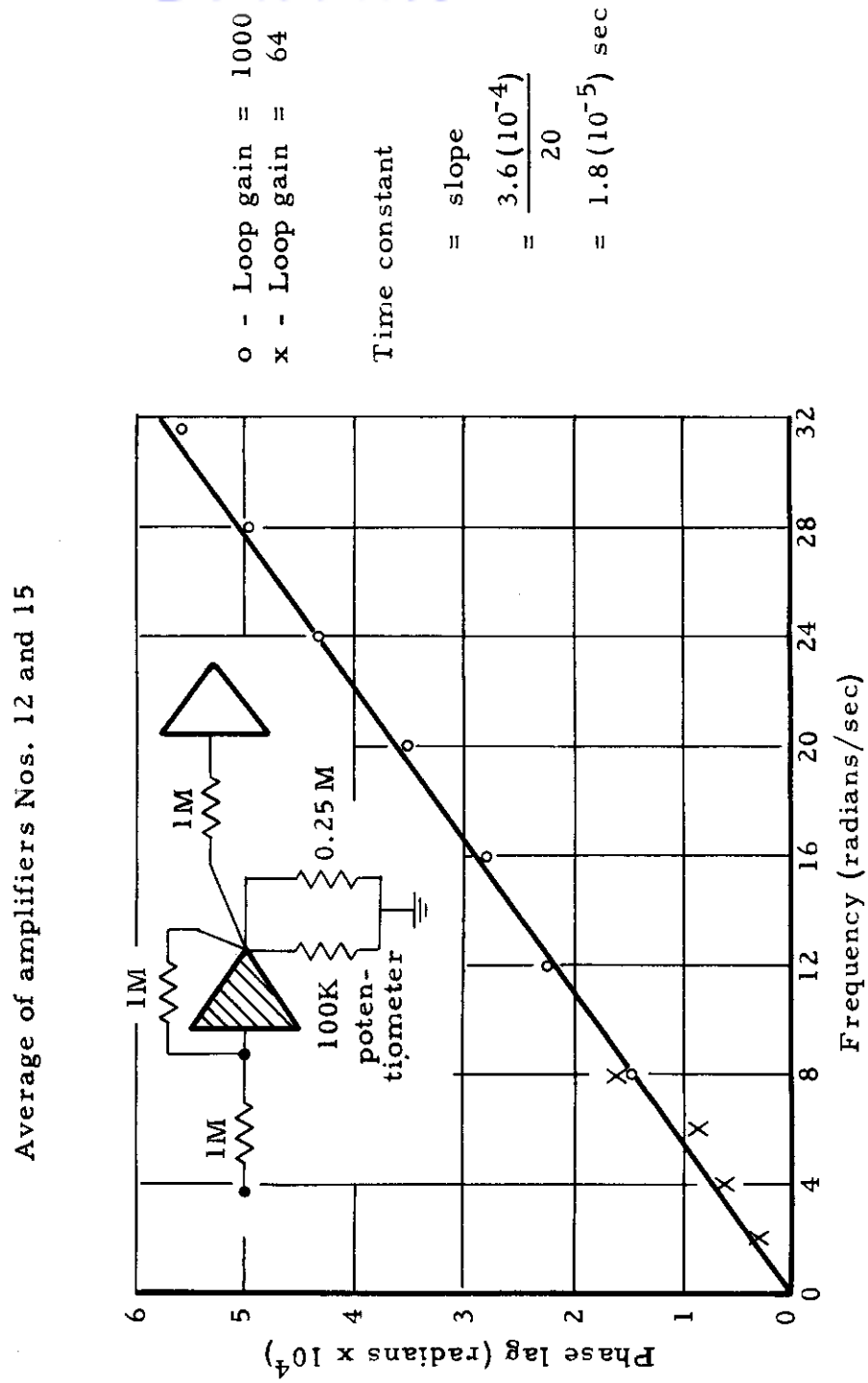


FIGURE 26 - PHASE SHIFT OF GEDA L3 SUMMER



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## 4.1 TRANSFER FUNCTION[33]

This discussion will utilize the notation of the Laplace transformation; particularly the concepts of admittance and transfer function will be used.

The admittance of a two-terminal network is defined as the ratio $I(s)/E(s)$, where $E(s)$ is the Laplace transform of a function of time $e(t)$ that represents a voltage applied to the network terminals. Similarly, $I(s)$ is the Laplace transform of a function $i(t)$ representing the current flow at the network terminals as a function of time. The energy stored in the network at $t = 0$ is assumed to be zero.

The transfer function of a device, in this case an amplifier, is the ratio $E_o(s)/E_i(s)$, where $E_o(s)$ and $E_i(s)$ are Laplace transforms of functions of time representing output and input voltages, respectively.

The schematic diagram used here to represent an operational amplifier is shown in Figure 27. The input admittances $Y_1 \ldots Y_n$ are all connected to a common point (s) called the summing point. In some amplifier designs a compensating network ($Y_c$) may be provided. $Y_s$ and $Y_g$ usually represent undesirable stray leakages and capacitances to ground, and $Y_f$ is the feedback element. The high-gain dc amplifier is represented by $A(s)$ and the output conductance is $g_o$, with an amplifier load $Y_L$. Various simplifications of Figure 27 can, of course, be made by omitting the appropriate elements and making corresponding changes in the mathematical expressions that follow.

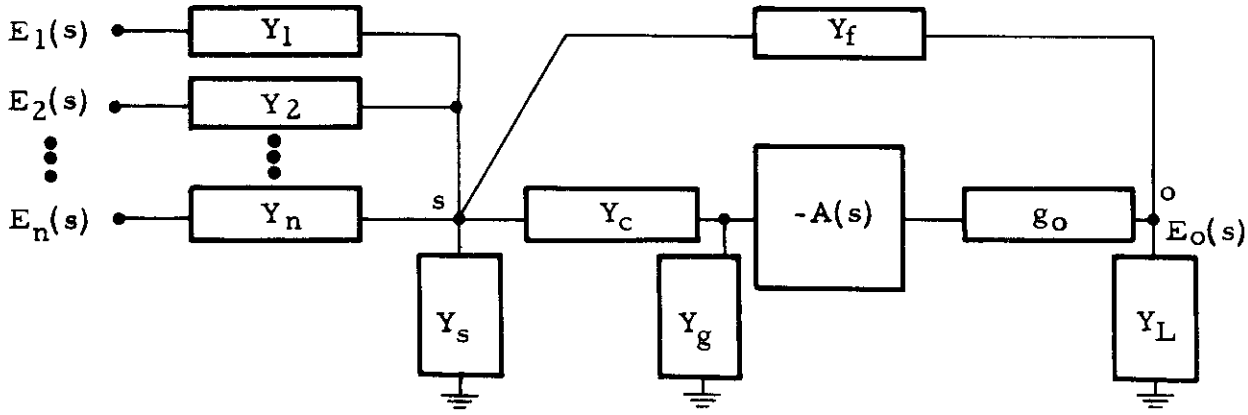


FIGURE 27 - OPERATIONAL AMPLIFIER

[33] References 4, 8 (pp. 61–63), 9, and 27 contain similar, though less general, analyses.



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Equating the sums of the currents to zero at the points s and o yields

$$\sum_{i=1}^{n} (E_i - E_s)Y_i + (E_o - E_s)Y_f - E_s\left(Y_s + \frac{Y_c Y_g}{Y_c + Y_g}\right) = 0 \qquad (1)$$

and $$(E_s - E_o)Y_f - E_o Y_L + \left(\frac{-AY_c}{Y_g + Y_c} E_s - E_o\right) g_o = 0 . \qquad (2)$$

Eliminating $E_s$ between equations 1 and 2 and rearranging the remaining terms gives the transfer function

$$\frac{E_o}{\sum_i E_i Y_i} = -\frac{1}{Y_f} \cdot \frac{1}{1 + \dfrac{(Y_L + Y_f + g_o)\left[Y_I + Y_f + Y_s + Y_c Y_g/(Y_c + Y_g)\right]}{Y_f\left[Ag_o Y_c/(Y_c + Y_g) - Y_f\right]}} . \qquad (3)$$

Here $Y_I = \sum_{i=1}^{n} Y_i$, where it is understood that this sum includes only those input admittances to which some voltage is applied from a low impedance source. In particular, those inputs with zero voltage applied should not be included unless they are effectively grounded.

The usual assumption, that the amplification **A** is for all practical purposes infinite, gives the transfer function

$$\frac{E_o}{\sum_i E_i Y_i} = -\frac{1}{Y_f} , \qquad (4)$$

Equation 4 is the expression ordinarily used in setting up problems on the analog computer.

The following definitions and explanations are useful in gaining some insight into the significances of the terms appearing in equation 3: Note that the quantity $Y_L + Y_f + g_o$ is the sum of all the admittances connected to the output point o in Figure 1. Let this quantity equal $Y_0$. Similarly, the quantity $Y_I + Y_f + Y_s + Y_c Y_g/(Y_c + Y_g)$ is the sum of all the admittances tied to the summing point s. Let this quantity be $Y_S$. Equation 3 now may be written as

$$\frac{E_o}{\sum_i E_i Y_i} = -\frac{1}{Y_f} \cdot \frac{1}{1 + \dfrac{Y_0 Y_S}{Y_f\left[Ag_o Y_c/(Y_g + Y_c) - Y_f\right]}} \qquad (5)$$



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Rearranging the terms slightly gives

$$\frac{E_o}{\sum_i E_i Y_i} = -\frac{1}{Y_f}\,\frac{1}{1+\frac{1}{G(s)}}\,, \qquad (6)$$

where $G(s) = (Y_f/Y_S)\left[(Ag_o/Y_0)(Y_c/Y_g + Y_c) - (Y_f/Y_0)\right]$.

The terms in G(s) may be interpreted in the following way: The first term inside the brackets is the overall gain of the amplifier and associated circuitry, from the summing point s to the output terminal o with no feedback, but with the input (s) end of $Y_f$ grounded. The second term inside the brackets is the direct effect of a voltage at s on the output voltage, through the passive network consisting of the feedback admittance and the sum of all the output admittances. This second term will, in general, be negligible compared to the first term which contains the amplifier gain. The factor outside the brackets, $Y_f/Y_S$, is the complex gain of the feedback loop from the output (o) through $Y_f$ back to the summing point (s). Thus the quantity G(s) may be seen to be the generalized gain of the amplifier loop from s through the amplifier and feedback circuits back to s, considering all factors, such as amplifier loading, grid-to-ground impedance, etc.

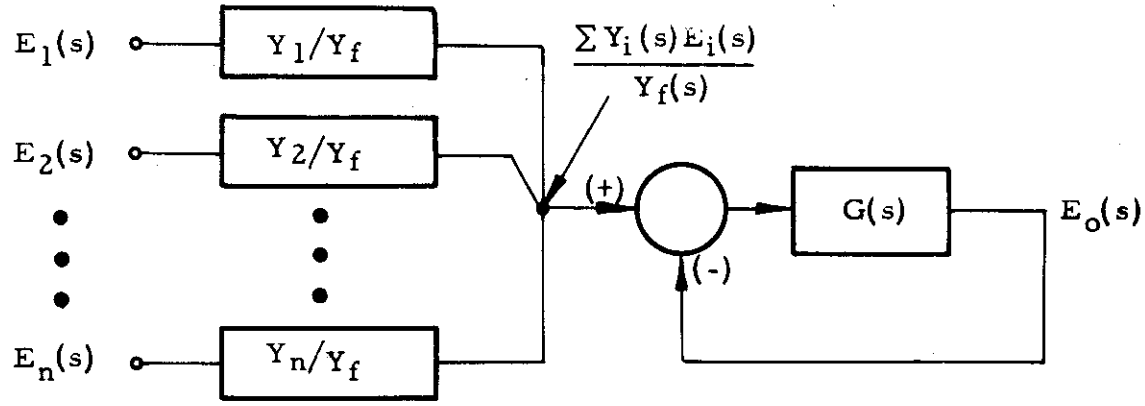


FIGURE 28 - EQUIVALENT BLOCK DIAGRAM

The form of equation 6 suggests the equivalent block diagram shown in Figure 28. The departure of a computing amplifier from ideal performance is seen to be identical to the error occurring in a simple feedback control loop having a forward element with transfer function G(s). The



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problems involved in the design and analysis of such loops have been exhaustively treated in numerous books and articles.[34]

## 4.2 DRIFT

One aim of computing amplifier design is to keep amplifier drift or offset to a negligible level. The relationship of overall feedback-amplifier drift to the drift or offset of the high-gain amplifier is easily derived. A basic amplifier circuit is assumed as in Figure 29.

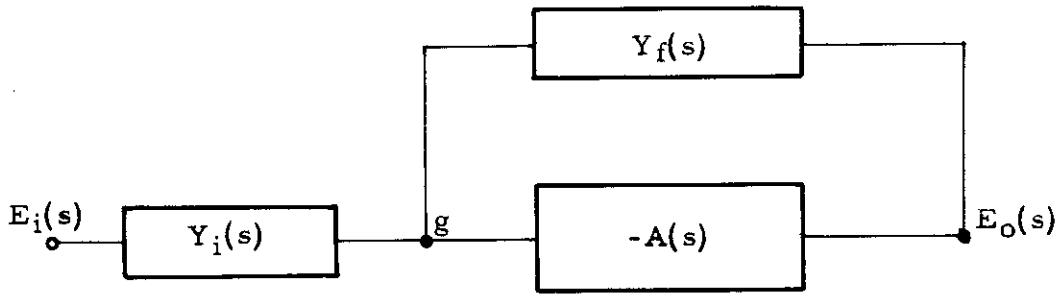


FIGURE 29 - BASIC AMPLIFIER CIRCUIT

Equating the input and feedback currents at g gives

$$(E_i - E_g)\,Y_i + (E_o - E_g)\,Y_f = 0\ . \tag{7}$$

If the output voltage is related to the grid voltage by

$$E_o = -E_g A + e\ , \tag{8}$$

where e represents the output of the high-gain amplifier with zero input, the resulting overall transfer relationship

$$E_o = -\frac{E_i Y_i - \frac{e}{A}(Y_i + Y_f)}{Y_f\left[1 + \frac{1}{A}\left(1 + \frac{Y_i}{Y_f}\right)\right]}\ , \tag{9}$$

[34] See W. R. Ahrendt and J. F. Taplin, Automatic Feedback Control, McGraw-Hill, New York, 1951, for a representative treatment and bibliography.



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or, to a close approximation,

$$\text{offset } E_o = \frac{e}{A}\left(1 + Y_i / Y_f\right) . \qquad (10)$$

From equation 10 it can be seen that the presence of a pole at $s = 0$ (as in an integrator) in the amplifier transfer function $(Y_i / Y_f)$ could cause large errors to accumulate due to amplifier unbalance, even though the value of $e/A$ were quite small. The use of a chopper balancing amplifier is equivalent to the use of a very large value for A at low frequencies, thus making the dc component of the offset $E_o$ very small.[35]

[35] See reference 28 for a discussion of a chopper balancing arrangement.



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