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13. ABSTRACT

The Systems Effectiveness Analyzer (SEA) is a weapon system effectiveness measuring system which is based primarily upon the utilization of existing equipment (controllers and peripherals) supplemented by the additional software required to enhance existing AMRL capabilities to evaluate simulated weapon systems.

The SEA System has two major functions. First to checkout, control, monitor and perform statistical analyses associated with tracking simulators. Second, to provide an estimation of weapon system effectiveness. The weapon system includes the human tracker as its principal sensor of target motion. The metrics used to evaluate weapon system performance are the weapon's round-by-round probability of kill and the resulting target's probability of survival. Angular error statistics are used to evaluate tracker performance.

This report describes the SEA system functions and interfaces and discusses the models used for the analysis of system effectiveness.

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PREFACE

A system has been developed for the Aerospace Medical Research Laboratory (AMRL) to enhance its capabilities for evaluation of the human operator in Air Force weapon systems. The enhancement of these capabilities has been achieved by integrating the capabilities of existing AMRL controller facilities, supplementing these facilities with supporting peripheral equipments and developing appropriate software and simulation models.

The system developed to provide the expanded capabilities is called the Systems Effectiveness Analyzer (SEA). The system was developed by Raytheon under Air Force contract F33615-72-C-1442 and was made operational in July of 1973.

This Summary Report describes the SEA System and the mathematical models that were developed as a basis for evaluating the human operator in AF weapon systems.

Dr. Frank M. Holden, AMRL/EME was the contract monitor for this SEA project. The authors wish to thank Dr. Holden for his suggestions on the formalization of this report.

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INTRODUCTION

One of the prime functions of the Aerospace Medical Research Laboratory (AMRL) is to evaluate the performance of the human operator in Air Force weapon systems. This function is performed by simulating the weapon system complex, integrating the operator(s) into the system, conducting simulated tactical missions and analyzing the collected performance data.

The use of weapon system simulators (rather than actual field tests) to determine system effectiveness is necessary for the following reasons:

- Complete data from actual combat missions are often not available.
- The cost of conducting the numerous field test trials under many sets of conditions is prohibitive.
- It is not possible to test systems during concept development and early stages of system optimization because the systems do not exist (this is possible using simulation techniques).
- Foreign systems are not generally available for exploitation.

The Systems Effectiveness Analyzer (SEA) developed by Raytheon Company, is a weapon system effectiveness measuring system which is based primarily upon the utilization of existing equipment (controllers and peripherals) supplemented by the additional software required to enhance existing AMRL capabilities to evaluate simulated weapon systems.

This report describes the SEA System and its interfaces and the models developed as a basis for systems effectiveness analysis.

SEA SYSTEM DESCRIPTION

SYSTEM CONSIDERATIONS

There are several design features that were considered imperative to the development of the SEA measurement system. In addition to the basic functions of system control and measurement, it was necessary to consider system flexibility in terms of accommodating a variety of simulators and weapon/target combinations. It was also necessary to consider methods of minimizing simulation setup times and expanding system capabilities. These features are described in the paragraphs that follow.

Input Flexibility

A basic SEA system requirement is that different weapon-target combinations and different target trajectories be accommodated with a minimum of changeover time. Use of card input for data pertaining to the target and weapon system accomplishes this objective. Individual data points may be adjusted by replacing cards. Full target, target trajectory, and/or weapon systems descriptions can be sorted in card decks and inputted to the system as a batch, without any further changes in the SEA.

System software modifications must also be accommodated easily. This leads to the requirement that the SEA software be structured in modular form so that available modules may be called by the SEA executive routine on an as-required basis. By using operator or card inputted executive state control commands, any or all modules may be used on a particular run. This allows optimum utility of the SEA for its various tasks. For instance, tracking experiments conducted to analyze the tracking subject's performance under stress or to train the tracking subject do not generally require any weapon effectiveness evaluation but do require that tracker statistics be compiled. The operator may command the SEA system to perform and output the necessary statistical routines while suppressing the weapon effectiveness analysis routines.

Operator Input Options

The wide range of SEA system control and analysis options had to be made available to the operator, either by sense switch or TTY commands to permit selection of the following:

- the number of sequential runs and the order in which target trajectories are selected
- the type and depth of analysis to be performed on each run, and the associated output device(s)
- manual override of all automatic procedures.

Simulator Interface

Since the SEA must accommodate a wide variety of simulators, the simulator/SEA interface must be flexible and the changeover time must be minimized. Two specific requirements on the SEA system design results from the above. First, all SEA commands (gain control settings, hit and view window sizes, stage drive levels and slow rates) to the simulator are embodied in the system software in such a way that their numeric values are easily accessible to the operator for adjustment or change. Second, an adequate number of channels on the analog to digital converter, digital to analog converter, and digital I/O trunk are available for future use.

Turn Around Time Minimization

Situations wherein any extended elapsed time between runs is intolerable (e.g., - primate tracking experiments) are common in SEA usage. Thus the SEA is required to perform many of the simulator pre-run checks and run initialization routines automatically. Where the simulated run conditions (e.g., trajectory, countermeasure programs, etc.) are preset, minimum operator participation is a desirable feature. For example the operator should only be required to issue the "start" command, via TTY or sense switch, to initiate a run, with the SEA rapidly performing all "house-keeping" chores without further commands. These features have been incorporated.

Expansion

Since the SEA is basically a system for experimental research, provision for expansion or restructuring must be maintained so that it can continue to meet new requirements for new applications as they arise. To this end, the SEA software has been structured so that intra-module modification or changes can be accomplished without the necessity for extensive inter-module changes. The SEA is configured such that its overall capabilities can be expanded by addition of new or improved controller hardware.

FUNCTIONAL DESCRIPTION

General

The SEA System has two major functions. First, to checkout, control, monitor and perform statistical analyses associated with tracking simulators. Second, to provide an estimation of weapon system effectiveness. The weapon system includes the human tracker as its principal sensor of target motion. The metrics used to evaluate weapon system performance are the weapon's round-by-round probability of kill and the resulting target's probability of survival. Angular error statistics are used to evaluate tracker performance. Figure 1 is a functional diagram of one SEA simulation that has been implemented. The simulated weapon system is an AAA battery using manned optical sights. The target being engaged is an aircraft deploying optical countermeasures. The SEA commands the tracking simulator sights picture displays, both with regard to target position and countermeasure stress level. Outputs from the simulator include the subject's control movements and the gun trigger state. These outputs are used by the SEA, in conjunction with the target trajectory, to update the sights picture, to accumulate the statistical measures of the tracker's performance, and to compile the data on which to base the effectiveness analysis of the AAA system. The collected data is essentially that which would have been determined from an actual AAA system. The Lead Angle Computer directs the gun to the predicted position of the aircraft. The mean miss distance of each shell fired is estimated and error sources appropriate to the gun system are used to find the distribution of shot relative to the target aircraft. Weapon lethality and target vulnerability data are then used in conjunction with the shell mean miss distance and scatter area to determine the kill probability of each shell. The probability that the aircraft survives the mission is derived after each round is fired.

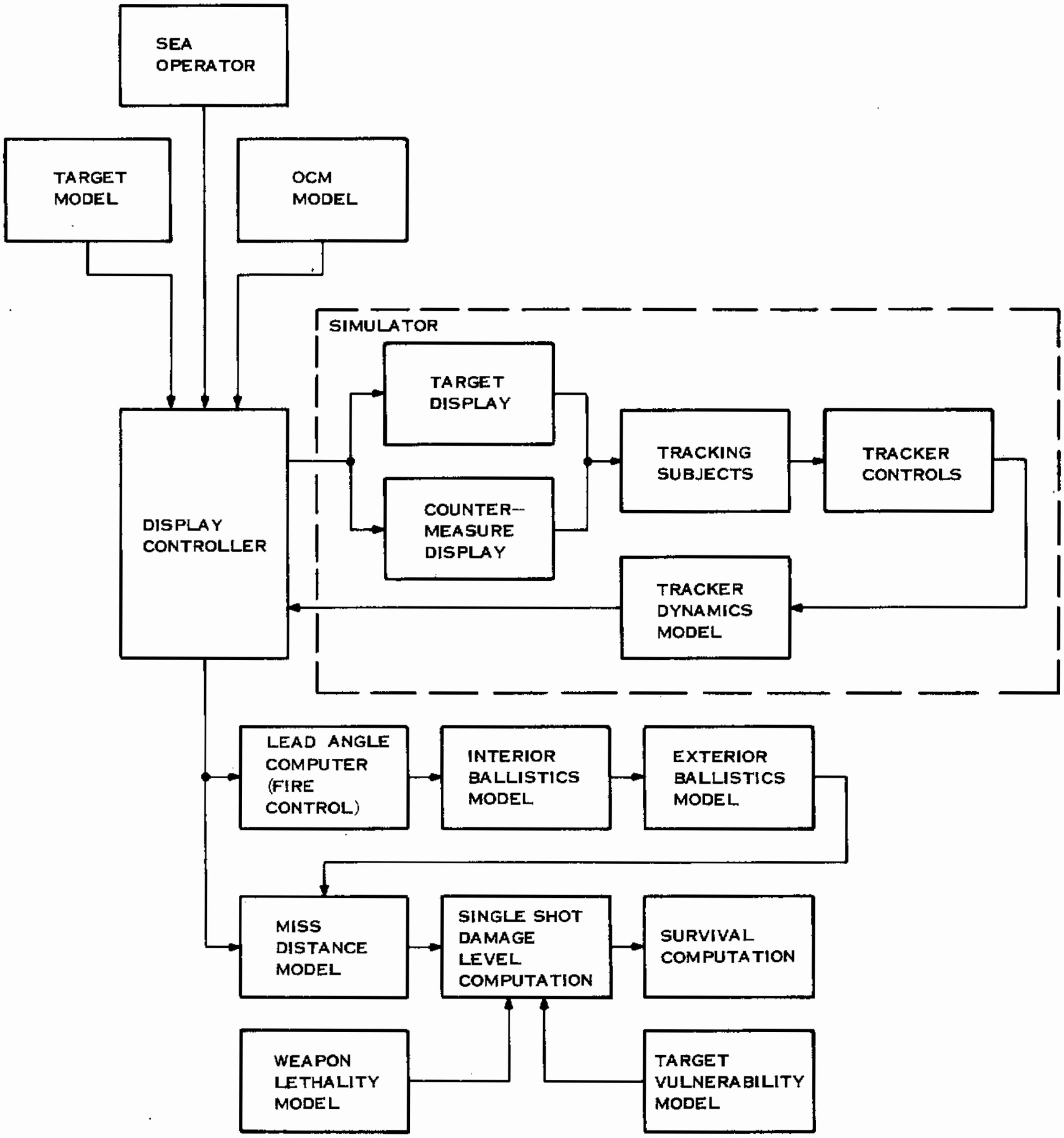

Figure 1. SEA AAA Simulation

SEA Controller

A principal SEA requirement is to control and monitor various tracking simulators. Control of a tracking simulator includes providing the various drive signals that determine the display of target/sight information to the tracking subject and that determine the simulated deployment of various types of countermeasures by the target. Monitoring functions include the measurement of the tracking subject's actions (e.g., handcrank or stick positions), presentation of auxiliary displays to the personnel conducting the simulation, real time simulator status and system checks to assure that simulator operation is proceeding safely and within specific performance tolerances.

Simulator Monitor and Control - Real Time

Figure 2 shows the SEA control loop - a first order sampled feedback system - used during simulator operation. A 50 Hz clock is used to coordinate and phase the various sampling procedures in process. During the 20 ms interval following each clock pulse, the tracker's control movements are sensed (azimuth, azimuth rate, elevation, elevation rate, and trigger up/down). The tracker's observed target position is compared with the stored target position at that time and error signals (azimuth error, elevation, and elevation error) defining the displacement of the target from the center of the gunsight are computed and outputted to the display control system. Multiplexed sixteen channel A/D and D/A converters are used to provide the interface between the digital controller and the analog display system.

Clock

The 50 Hz clock pulse train is input to the SEA controller. Upon arrival of the clock pulse, the CLOCK functional element first checks that various simulator initialization procedures have been completed. If the simulator is "up" for the run, the pulse train is permitted to actuate the SEA real time functions. Each pulse triggers the entire sampling and control process and results in updated display signals. This 50 Hz rate is accepted as sufficient to assure flicker free display to human and primate trackers.

The run is normally concluded when the prestored target trajectory data has been expended. The clock must sense this condition and cease actuation of the sampling process at that time. Additional conditions that may cause termination of a run are excessive elapsed time in the run and operator override.

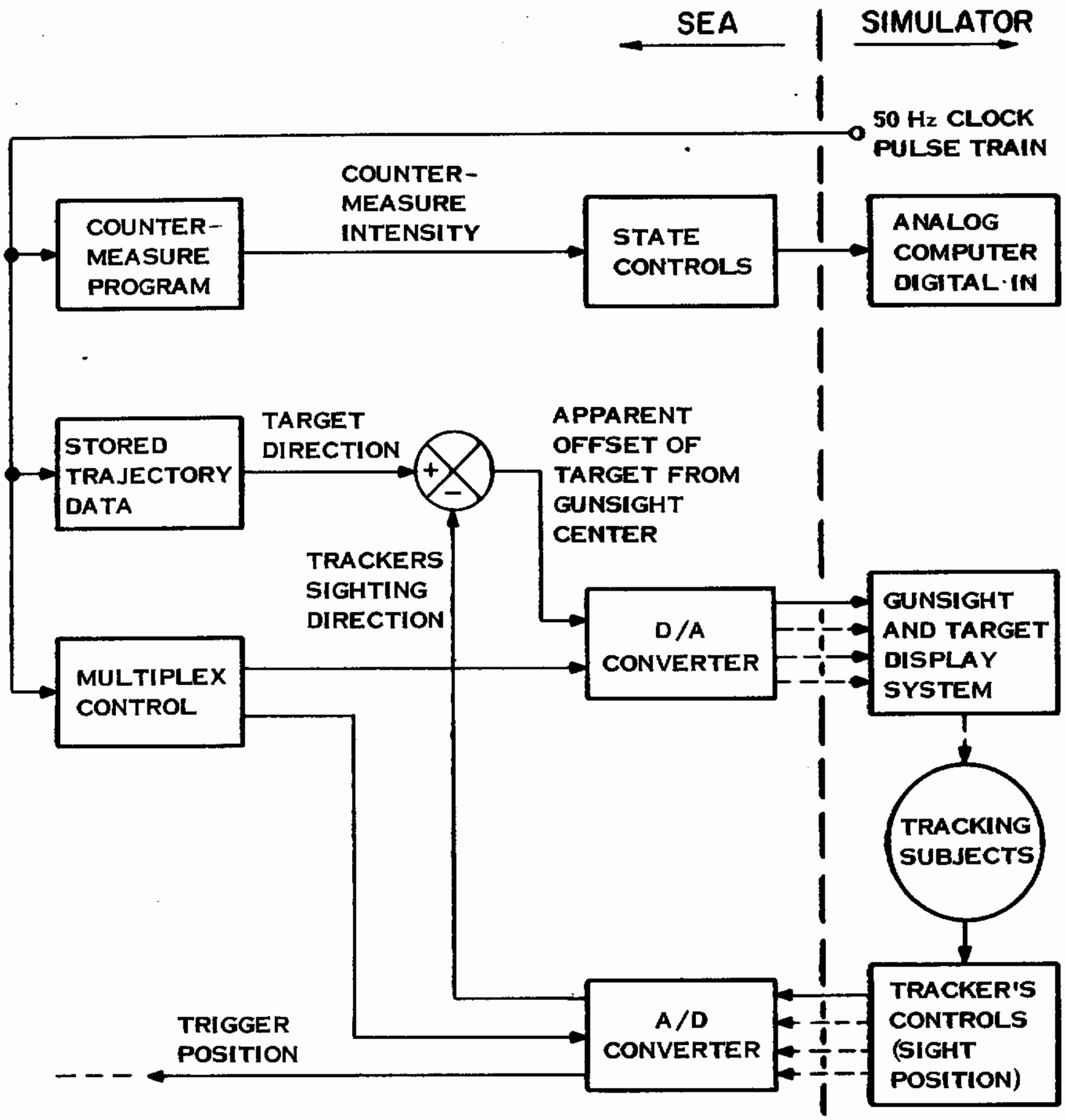

Figure 2. Basic SEA Control Loop

. Trajectory Data

Trajectory data consists of the target spatial coordinates at 20 ms intervals and is to be stored in the SEA prior to the run. These coordinates (range, azimuth, and elevation, as seen from the tracking site) are prescaled in such a way that their values are identical with the output of the A/D converter when the sights are operated by an ideal (zero-error) tracker. This is done to reduce comparator computational requirements during the real-time portion of the run and allow the high speed (50 Hz) update rate required for flicker-free display.

The trajectory triplets are organized in records of appropriate length so that records may be read sequentially during the run. This is done to minimize core storage space for use by other elements of the controller software.

Each trajectory is identified with an appropriate name designator, and this title is stored at the beginning of the record.

Countermeasure Program

Various types of countermeasure simulations can be controlled by the SEA system. The countermeasure program controls the duty cycle (0 to 100 percent) timing (variable and fixed repetition rates), and intensity (discrete), depending on the particular simulator in use. It also provides real time fail-safe provision to shut down the countermeasure so that subject stress levels do not exceed preset limits.

Multiplex Control of A/D and D/A Converters

The SEA digital controller must sample/command several analog devices associated with the simulator. Sixteen channel A/D and D/A converters are used for this purpose. Multiplexing the converters, to drive or sample analog channels, is performed by inserting the address word into the appropriate converter and then actuating (strobing) the device.

A/D conversion requirements typically include:

- azimuth potentiometer setting (calibration - zero set)
- elevation potentiometer setting (calibration zero set)
- azimuth sights output voltage

- elevation sights output voltage
- azimuth rate output voltage
- elevation rate output voltage

and ten spare channels for auxiliary inputs or future expansion.

D/A Conversion requirements include:

- target trajectory elevation (display multiplier constant)
- elevation error (display drive input voltage)
- azimuth error (display drive input voltage)
- target range
- hit score (used for subject motivation)
- monitor scope x input
- monitor scope y input
- monitor scope z input (intensity axis)

and eight spare channels for auxiliary outputs or future expansion.

The multiplexer control generates the appropriate address words and strobe commands to operate the converters in a sequential fashion during each clocking interval.

Simulator State Controls

In addition to the 50 Hz data flow, the controller also maintains and monitors the simulator system state. This is done by transmitting digital data to the simulator's digital interface unit. Values/commands to be transmitted include:

Manually Set

- zero elevation potentiometer setting command
- zero azimuth potentiometer setting command
- azimuth control gain setting
- slew control gain setting
- background intensity setting

Automatically Set

- run indicator
- target in hit circle indication
- I/O time determination (time check)
- countermeasure shutter control
- timing commands

Four spare channels to be reserved for future expansion or auxiliary outputs.

Real-Time Statistics

For purposes of subject motivation and SEA operator evaluation, the SEA is required to provide real-time "quick look" evaluation measures of the tracking subject's performance. The following values are required during the run:

- Sights elevation and azimuth
- Sights elevation and azimuth rates
- Target elevation and azimuth
- Tracker elevation and azimuth error
- Hits (defined as the number of 200-millisecond intervals during which the tracker maintains track within the window corresponding to "good" tracking performance. "Good" tracking performance criteria, in milliradians, is preset prior to the run based on subject proficiency and degree of difficulty of the track).

In addition to the above values, which are outputted real time to the strip chart recorder, statistical measures are accumulated for listing on the line printer at the end of the run. These include:

- Mean elevation and azimuth error for the run
- Variance of elevation and azimuth error for the run
- Total number of hits during the run
- Possible number of hits during the run

Non Real Time Simulator Control

The SEA simulator controller is required to perform a number of non-real time functions concerned with the simulator set up and post-run record-keeping. These requirements are discussed in the following subsections.

Simulator Set-Up

Prior to the beginning of a simulator run, the SEA must configure both itself and the simulator for the run. Storage units must be checked to assure that the data - trajectory and countermeasure - are available for the upcoming run. The real-time statistical routines described previously must be initialized. A message is sent to the system operator that the SEA is ready. Upon receipt of the operator command, the simulator settings (azimuth, elevation and countermeasure controls) are checked automatically to see that they are properly adjusted before the run can commence.

End of Run

Upon completion of the run, several operations are required before commencement of the next run. The real-time statistics are scaled and outputted to the line printer. These statistics relate to the performance of the tracking subject and include the mean tracking errors and variance in azimuth and elevation. Additional metrics include the number of hits based on the time on target, and the maximum number of hits. Outputted records include identification of the tracking team, the trajectory, the date, and time of the run. These records are then written on magnetic tape for storage and returned as required.

Upon completion of the above tasks, the SEA proceeds to analyze the performance of the weapon system of which the tracker is a part. This is discussed separately below. Upon completion of all functions related to the simulation and its analysis, the SEA can reconfigure automatically both itself and the simulator in preparation for the next run.

EFFECTIVENESS ANALYSIS

The SEA must determine the performance of the weapon system. The task requirements for this function are presented through a discussion of the mathematical models of the various weapon system functional elements, as shown in Figure 3.

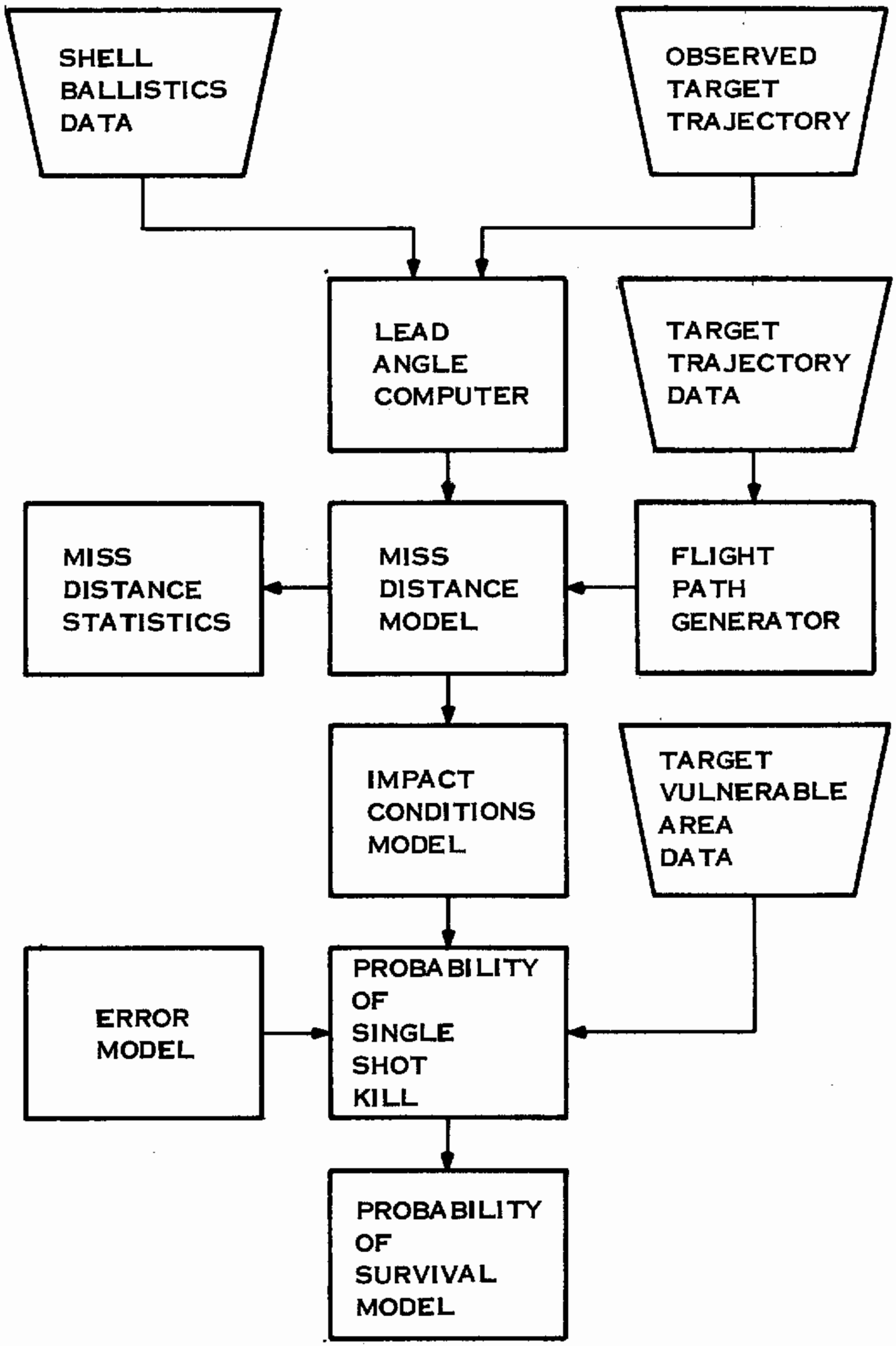

Figure 3. Weapon Effectiveness Analysis

Flight Path Generator Model[1]

As discussed previously, the SEA must perform comparison between simulator's sight positions and the target coordinates in order to generate the various commands used to control the simulator. This, in turn, requires that the target trajectory be available to the controller during the run.

The target flight path generator model (FPGM) must convert any operator prescribed target trajectory into a trajectory described at the 20-millisecond intervals required to control the simulator. The prescribed trajectories are operator-inputted as specifying a series of linear segments. Each segment is defined by the spatial and time coordinates relating to its beginning and end points. The FPGM then proceeds to interpolate linearly between the prescribed points at 20-millisecond intervals. Up to 100 seconds of flight path are allowed as input with as many or few individual segments as are desired.

The output of the FPGM are the coordinates of the target (range, azimuth, and elevation) relative to the tracker at 20-millisecond intervals.

The mathematical model of the FPGM is described below.

The complete trajectory is derived from operator inputted segments. The following describes the procedures required to convert any segment of the trajectory from operator input to 20 ms coordinates.

The end point conditions for a particular segment are (x_0, y_0, z_0, and t_0) at the beginning of the interval, and (x_1, y_1, z_1, and t_1) at the end of the interval. The number of points, NBPT, corresponding to the 20-millisecond hacks on the segment is given by:

$$\text{NBPT} = \text{largest integer less than } 50\,(t_1 - t_0) + 1 \qquad (1)$$

The index (i) and time (t_i) corresponding to the initial trajectory hack are given by:

$$i = \text{largest integer less than } (50t_0 + 1) \qquad (2)$$

and

$$t_i = i/50 \qquad (3)$$

---

[1] The target flight path generator (FPGM) is part of the pre-run routine and not properly part of the SEA weapon performance analysis. However, since the target trajectory enters intimately into the weapon system performance, the FPGM model discussion is included for the sake of clarity and completeness.

The rectangular coordinates of the target velocity are:

$$v_x = \left(\frac{x_1 - x_0}{t_1 - t_0}\right)$$

$$v_y = \left(\frac{y_1 - y_0}{t_1 - t_0}\right)$$

$$v_z = \left(\frac{z_1 - z_0}{t_1 - t_0}\right) \qquad (4)$$

The target rectangular coordinates are then computed for each of the 20-millisecond time hacks on the segment.

$$x_j = x_0 + v_x \cdot (t_j - t_0)$$

$$y_j = y_0 + v_y \cdot (t_j - t_0)$$

$$z_j = z_0 + v_z \cdot (t_j - t_0) \qquad (5)$$

where t_j is the current time given by

$$t_j = t_i + \left(\frac{k}{50}\right) \text{ where } k = 1 \text{ to NBPT and } j = i + k \qquad (6)$$

These values for the target's rectangular position at time t_j must then be transformed to the azimuth-elevation-range coordinate system (convention: +x direction is north +y direction is west, and +z direction is up).

$$R_j = \text{range} = \sqrt{x^2 + y^2 + z^2} \qquad (7)$$

$$\theta_j = \text{azimuth} = \tan^{-1} (x/y) \qquad (8)$$

$$\varphi_j = \text{elevation} = \sin^{-1} (z/\text{range}) \qquad (9)$$

For those degenerate cases where y = 0 or both x and y are zero, appropriate tests must be made on neighboring points to determine the proper quadrant for the azimuth and elevation.

Shell Ballistics Model

The representation of AAA shell flyout curves is done by lockup and interpolation operations on tabulated data. The SEA system requires as input the following data:

a. Maximum flight time of the shell (not to exceed 14 seconds).

b. Shell range in feet and velocity in ft/second, at 0.2-second intervals of elapsed time after firing.

This method of inputting shell data is used rather than the closed form approximation method so that real data on shell trajectories may be employed.

(The 57 mm ballistics data in current use was derived from data taken under the AFWET Program. Shell ballistics data appropriate to other gun systems may be entered into the SEA by changing the input data card deck to reflect the new data.)

AAA Lead Angle Computer Model (LACM)

The purpose of an AAA lead angle computer is to determine the gun pointing direction on the basis of the observed target velocity and position relative to the gun site. The mathematical model used in the SEA system to simulate a real world lead angle computer is based on the assumption that the target flight path continues along a straight line at constant velocity from the time last observed by the tracker. This model parallels lead angle computers used by real AAA systems.

The lead angle computer can thus be modeled by the following vector equation:

$$|\overrightarrow{AIM}| = \left| \begin{Bmatrix} x(t_0+\tau) \\ y(t_0+\tau) \\ z(t_0+\tau) \end{Bmatrix} \right| = \left| \begin{Bmatrix} x'(t_0) + v'_x(t_0)\tau \\ y'(t_0) + v'_y(t_0)\tau \\ z'(t_0) + v'_z(t_0)\tau \end{Bmatrix} \right| = R_{SHELL}(\tau) \qquad (10)$$

Where $\overrightarrow{AIM}$ is the gun aiming point (the theoretical point of impact), (x, y, z) is the predicted target position in the earth coordinate system at the time of predicted shell impact, t_0 is the time at which the firing observation is made and τ is the predicted time of flight of the projectile to the AIM. The position and velocity of the target as observed by and relative to the tracker are (x', y', z') and (v'_x, v'_y, v'_z). $R_{SHELL}(\tau)$ is the range of the shell from the gun tracking site when time τ has elapsed since firing the gun. The gun pointing angles commanded by the lead angle computer are then defined as:

$$\theta_G = \tan^{-1} \frac{x(t_0+\tau)}{y(t_0+\tau)} \tag{11}$$

$$\varphi_G = \tan^{-1} \frac{z(t_0+\tau)}{R(t_0+\tau)} \tag{12}$$

where φ_G and θ_G are the aiming azimuth and elevation angles of the gun, respectively, and R is the range of AIM[(1)].

In general, the shell ballistics are highly nonlinear and nonconductive to the closed form solution of Equation (10). Therefore, an iterative procedure is required.

The time of projectile flight, τ, is determined as follows, using the target range and range rates of the target and the shell. The shell range and range rate data are inputs to the model. The target range and speed data are derived from man-operated tracker measured data at the gun firing time. At the ith iteration, the range of the target, R_T is

$$R_T(t_0+\tau_i) = \sqrt{x^2(t_o+\tau) + y^2(t_o+\tau) + z^2(t_o+\tau)}$$

and the range of the shell, R_{SHELL} is

$$R_{SHELL}(\tau_i)$$

If R_T $(t_o+\tau_i)$ and R_{SHELL} (τ_i) are different, the new time of bullet flight, τ_{i+1}, is defined by the following recursion formula:

$$\tau_{i+1} = \tau_i + \frac{(R_T-R_{SHELL})}{v_{SHELL}-v_T} \tag{13}$$

---

(1) This model neglects gravity droop in the shell trajectory. A real-world lead angle computer would increase the actual elevation (the "super-elevation" correction) to compensate for gravity droop.

where v_{SHELL} and v_T are the range rate of the shell and the target respectively. This iterative process is started from $\tau_o = 0$ terminated when

$$\left| R_T(t_0+\tau_j) - R_{SHELL}(\tau_i) \right| < \epsilon \quad (14)$$

where ϵ is a present small value. The value of ϵ selected (Equation (14)) is related to the accuracy of the lead angle computer being modeled. A typical lead angle computer (57 mm PUAZO) has been exploited and typically has an error of a few milliradians. For ranges of interest (R > 300 yards), this error amounts to several feet or more. Therefore, since the mathematical model is intended to correspond to the "ideal" lead angle computer, ϵ is chosen as 1 foot, better than the actual system being modeled. For other systems, ϵ can be chosen to fit corresponding conditions. The value τ_j which satisfies the inequality (Equation (14)), is then the predicted projectile flight time used in the determination of the gun aiming angles in Equations (11) and (12).

End Game Model

At the instant of interaction (time of impact) between the weapon projectile and the target, several factors are required to assess the outcome of the ensuing engagement. These are (1) miss-distance, (2) striking speed, and (3) the striking direction (in terms of striking angles) in the target referenced coordinate system. The miss-distance is used in conjunction with the AAA errors to define the projectile density function in the neighborhood of the aircraft. The striking speed and the striking direction are used in the estimation of target vulnerable area to the shell.

Miss-distance is determined by analysis of the actual mean flight paths of the target and shell. Striking speed is determined by the vector relation between target and shell velocity at the impact time. The striking direction is defined as that from which the shell approaches the target, in the target reference frame.

Each of these models is discussed in the following subsections.

Miss-Distance Model

The AAA projectile range and the actual target aircraft range are computed iteratively in much the same manner as described for the lead angle computer model and for the same reasons. The equation to be solved is:

$$R_{SHELL}(\tau) = R_{AC}(\tau + T) \tag{15}$$

where

T is the time at which the gun is fired,

τ is the time elapsed since the gun was fired,

R_{SHELL} is the range of the shell from the gun,

and

R_{AC} is the true range of the target from the gun.

This equation is similar to Equation (10), except that the true, rather than predicted, target trajectory appears.

At the instant when these two ranges are approximately equal (again chosen as within one foot), the distance between AAA projectile and the target is computed. This distance is defined as the miss-distance. The expression for the miss-distance is:

$$d_{miss} = R_T \sin^{-1} (\sin^2 \Delta\varphi + \sin^2 (\Delta\theta \cos \varphi_T) - \sin^2 \Delta\varphi \sin^2 (\Delta\theta \cos \varphi_T))^{1/2} \tag{16}$$

where

R_T = range to target (at $T+\tau$)

$\Delta\varphi$ = $\varphi_T - \varphi_G$ = difference between target and shell azimuth angles

$\Delta\theta$ = $\theta_T - \theta_G$ = difference between target and shell azimuth angles

φ_T = elevation of target (at $T+\tau$)

θ_T = azimuth of target (at $T+\tau$)

φ_G = shell elevation (at $T+\tau$)

θ_G = shell azimuth (at $T+\tau$)

For cases of interest to the SEA, the miss angles, $\Delta\varphi$ and $\Delta\theta$, can be expected to be less than 10 degrees. (This value corresponds to an approximate miss of 140 feet at 100 yards, or 1400 feet at 1000 yards.) When these conditions are satisfied ($\Delta\varphi \le 10^o$, $\Delta\varphi \le 10^o$) the following approximate expression for d_{miss} is correct to within 1 percent of the true value.

$$d_{miss} = R_T \sqrt{\Delta\varphi^2 + \Delta\theta^2 \cos^2\varphi_T} \tag{17}$$

This approximation leads to the following identification of the elevation and azimuth components of the miss-distance:

$$\begin{aligned} \text{elevation component} &= R_T\Delta\varphi \equiv D_\varphi \text{ (in feet)} \\ \text{azimuth component} &= R_T\Delta\theta \cos\varphi \equiv D_\theta \text{ (in feet)} \end{aligned} \tag{18}$$

Striking Speed Model

The striking speed, V_S, is defined as the magnitude of the relative velocity between the AAA projectile and the target aircraft. In the earth reference coordinate systems:

$$\begin{pmatrix} V_{SE}(1) \\ V_{SE}(2) \\ V_{SE}(3) \end{pmatrix} = \begin{pmatrix} V_{BE}(1) \\ V_{BE}(2) \\ V_{BE}(3) \end{pmatrix} - \begin{pmatrix} V_{TE}(1) \\ V_{TE}(2) \\ V_{TE}(3) \end{pmatrix} \tag{19}$$

where V_{SE}, V_{BE}, and V_{TE} are the striking velocity, shell velocity and target velocity, respectively. The numbers 1, 2, and 3 represent the three components of each velocity vector. The velocity V_{BE} can be obtained from the shell range rate (V_B), shell azimuth (θ_G) and shell elevation (φ_G) angles, under the assumption that the bullet flies in a straight line.

$$\begin{pmatrix} V_{BE}(1) \\ V_{BE}(2) \\ V_{BE}(3) \end{pmatrix} = \begin{pmatrix} \cos\varphi_G \cos\theta_G \\ -\cos\varphi_G \sin\theta_G \\ \sin\varphi_E \end{pmatrix} V_B \tag{20}$$

Therefore, the striking velocity, in the earth referenced coordinate system is

$$\begin{pmatrix} V_{SE}(1) \\ V_{SE}(2) \\ V_{SE}(3) \end{pmatrix} = \begin{pmatrix} V_B \cos\varphi_G \cos\theta_G - V_{TE}(1) \\ -V_G \cos\varphi_G \sin\theta_G - V_{TE}(2) \\ V_B \sin\varphi_G - V_{TE}(3) \end{pmatrix} \tag{21}$$

The striking speed is the magnitude of the striking velocity vector.

$$V_S = \sqrt{V_{SE}^2(1) + V_{SE}^2(2) + V_{SE}^2(3)} \tag{22}$$

Striking Direction Model

The striking direction is defined by the polar angle, α_s. These angles are defined below:

$$\alpha_p = \tan^{-1} \frac{\sqrt{V_{SA}^2(2) + V_{SA}^2(3)}}{V_{SA}(1)} \text{ (in radians)} \tag{23}$$

$$\alpha_s = \tan^{-1} \left(\frac{V_{SA}(2)}{V_{SA}(3)}\right) \text{(in radians)} \tag{24}$$

where V_{SA} is the striking velocity in the airframe reference coordinate system, $V_{SA}(1)$, $V_{SA}(2)$, and $V_{SA}(3)$ are the three components of V_{SA} in nose, right wing, and downward directions, respectively. This vector, V_{SA}, can be obtained from V_{SE} with a linear transformation

$$\left[V_{SA}\right] = \left[A\right]\left[V_{SE}\right] \tag{25}$$

where A is the matrix that transforms the striking velocity in the earth coordinate system (V_{SE}) into the airframe reference coordinate system.

The derivation of the rotation matrix A is described below.

$$[A] = \begin{pmatrix} A(1,1), & A(1,2), & A(1,3) \\ A(2,1), & A(2,2), & A(2,3) \\ A(3,1), & A(3,2), & A(3,3) \end{pmatrix} \tag{26}$$

Assuming the angle of attack of the target aircraft is small during the engagement, the first row of the A matrix (A(1, 1)) is the unit vector, $\hat{V}_{TEN}$ (normalized target velocity in earth reference coordinate system). Therefore,

$$\begin{aligned} A(1,1) &= V_{TEN}(1) \\ A(1,2) &= V_{TEN}(2) \\ A(1,3) &= V_{TEN}(3) \end{aligned} \qquad (27)$$

where

$$V_{TEN} = \frac{V_{TE}(i)}{\sqrt{\sum_i V_{TE}^2(i)}}, \quad i = 1, 2, 3 \qquad (28)$$

and $\vec{V}_{TE}$ is the target velocity in earth reference coordinate system.

The second row of matrix A, A(2,.) represents the components of the unit vector parallel to and in the same sense as the right wing of the target aircraft. Assuming the wings are leveled during engagement, the component

$$A(2,3) = 0.$$

In addition, the vector A(2,) is orthogonal to the vector A(1,.). Therefore,

$$\begin{cases} A(2,1) = \dfrac{V_{TEN}(2)}{\sqrt{V_{TEN}^2(1) + V_{TEN}^2(2)}} \\[2ex] A(2,2) = \dfrac{-V_{TEN}(1)}{\sqrt{V_{TEN}^2(1) + V_{TEN}^2(2)}} \\[2ex] A(2,3) = 0 \end{cases} \qquad (29)$$

Since the three vectors, $\hat{A}(1,.)$, $\hat{A}(2,1)$, and $\hat{A}(3,.)$ form an orthonormal set, the third row of the A matrix, A(3,.), is the vector cross product of $\hat{A}(1,.)$ and $\hat{A}(2,.)$,

$$\hat{A}(3,\cdot) = \hat{A}(1,\cdot) \times \hat{A}(2,\cdot) \qquad (30)$$

Therefore,

$$A(3,1) = A(1,2)A(2,3) - A(1,3)A(2,2)$$
$$A(3,2) = A(1,3)A(2,1) - A(1,1)A(2,3)$$
$$A(3,3) = A(1,1)A(2,2) - A(1,2)A(2,1) \quad (31)$$

Substituting Equations (25), (27), (29), and (31) into Equations (23) and (24) yields the striking angles α_p and α_s.

Analysis of the Effects of Random Errors

In the models discussed so far, the random errors from various sources are not included. Therefore, the results obtained from these models are mean values in the statistical sense. In a real AAA system, these random errors are inevitable; therefore, the performance of each shell fired will deviate from the mean on a round-to-round basis. This section delineates the error sources and describes the SEA's approach to handling these random errors in computing the single-shot kill probability.

The weapon system errors represented in the SEA system are listed as follows:

a. Lead angle computer error (Two types of computers are modeled by the SEA, the electronic computer and the mechanical computer.)

b. Error due to gun jitter (servo errors and misalignment).

c. Variation in muzzle velocity (due to barrel wear and shell miss variations).

d. Variation due to exterior ballistics dispersive effects.

e. Variation due to atmospheric perturbation (wind, atmospheric turbulence, etc.).

The composite distribution of shell trajectories, formed by combining all the error sources listed above, is three dimensional with components in azimuth, elevation and range, centered about the average trajectory (i.e., that which passes through the theoretical aiming point, AIM). The range type error, which causes the interception at a greater or lesser range than the average shell, has been treated successfully in the P001 AAA model as an enlarging of the distribution which is obtained from combining all azimuth and elevation type errors at mean intercept (mean point of impact, or aiming point, AIM). The SEA system adopted this approach of processing the range type error and, thus, all of the effects of the three-dimensional shell trajectory error distribution can be "projected" onto the mean intercept plane creating a final distribution that is planar in nature and upon which the integration over the target's vulnerable area is to take place.

The variances of the random errors are first segregated into two directions of influence, azimuth and elevation components. These variance components are then summed in the rms (root mean square) sense; that is,

$$\left\{ \begin{aligned} \sigma_{\theta}^2 &= \sigma_{1\theta}^2 + \sigma_{2\theta}^2 + \sigma_{3\theta}^2 + \sigma_{4\theta}^2 + \sigma_{5\theta}^2 \\ \sigma_{\varphi}^2 &= \sigma_{1\varphi}^2 + \sigma_{2\varphi}^2 + \sigma_{3\varphi}^2 + \sigma_{4\varphi}^2 + \sigma_{5\varphi}^2 \end{aligned} \right. \quad (\text{ft}^2) \tag{32}$$

57 MM Error Model

At the present time, the SEA system is using an error budget appropriate to the 57 MM AAA system. The following describes the numeric data associated with that system.

The AAA model for muzzle speed variation used in the SEA system assumes that the muzzle speeds are normally distributed, their mean for a given projectile type is the muzzle speed quoted for that projectile, and their standard deviation is one percent of that value. Based on this assumption, the influence of muzzle speed error on azimuth and elevation variance components are computed as follows:

$$\begin{aligned} \sigma_{3\theta}^2 &= \left(\frac{0.01\ V_{\theta}}{V\ \text{muzzle}}\right)^2, \ (\text{milliradians}^2) \\ \sigma_{3\varphi}^2 &= \left(\frac{0.01\ V_{\varphi}}{V\ \text{muzzle}}\right)^2, \ (\text{milliradians}^2) \end{aligned} \tag{33}$$

where

$$\begin{aligned} V_{\theta} &= V_T - (\sin \theta_T)\, V_{TE}(1) + (\cos \theta_T)\, V_{TE}(2) \\ V_{\varphi} &= V_T - (\cos \theta_T)(\sin \varphi_T)\, V_{TE}(1) \\ &\quad - (\sin \theta_T)(\sin \varphi_T)\, V_{TE}(2) \\ &\quad + (\cos \varphi_T)\, V_{TE}(3). \end{aligned} \tag{34}$$

V muzzle is the AAA shell mean muzzle velocity and V_T is the target speed at the time of impact[1]. Other 57 mm error variances, based on exploited data as reported in the P001 model are as listed.

$$\sigma_{1\theta}^2 = \sigma_{1\varphi}^2 = 2.89, \text{ (milliradians}^2\text{)}$$

$$\sigma_{2\theta}^2 = \sigma_{2\varphi}^2 = 12.5, \text{ (milliradians}^2\text{)}$$

$$\sigma_{4\theta}^2 = \sigma_{4\varphi}^2 = 8.0, \text{ (milliradians}^2\text{)}$$

and

$$\sigma_{5\theta}^2 = \sigma_{5\varphi}^2 = 4.5. \text{ (milliradians}^2\text{)}$$

To convert these values of $\sigma_{i\theta}$ and $\sigma_{i\varphi}$ into feet, they must be multiplied by the range, R_T, expressed in K-feet.

Vulnerability Model

The purpose of this model is to estimate the vulnerable area of the target. The vulnerable area of the target aircraft is a function of the engagement variables - striking speed and striking angles -, the type of target aircraft, and the type of shell.

In the SEA system, vulnerable area data is input in the form of a three-dimensional array with 240 elements.

$$AV_{i,j,k} = A_v(V_s(i), \alpha_p(j), \alpha_s(k)) \quad i = 1 \text{ to } 10, \quad j = 1 \text{ to } 6; \text{ and } k = 1 \text{ to } 4$$

A_v is the value of the vulnerable area (in square feet) corresponding to the given striking speed ($V_s(i)$), polar angle ($\alpha_p(j)$) and sideswipe angle ($\alpha_s(k)$). In the SEA system, the striking speed, V_s, is divided into ten discrete steps with each step equivalent to 500 feet per second, the polar angle, α_p, is divided into six sectors with each sector 30 degrees wide, and the sideswipe angle, α_s, is divided into four sectors with each sector 45 degrees wide. (Note that target right-left symmetry is assumed.)

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(1) Variables θ_T, $V_{TE}(i)$, and φ_T have been described previously.

At the time of the impact, the striking speed and the striking angles are computed. These computed values are used for selecting the corresponding vulnerable area using the array A_v.

Provision exists to use more finely or coarsely graded A_v models within the SEA system.

Single-Shot Kill Model

The probability that a particular shell will destroy the target aircraft is the summation of the probabilities of the shell being located anywhere within the exposed vulnerable area of the target aircraft at the time of impact. Thus the probability of a single-shot kill is the double integral of the function that expresses shell mean intercept location probability over limits encompassing the target aircraft's exposed vulnerable area. The single-shot kill model is used to estimate this probability for each round fired.

In the SEA system, the probability density function of the shell intercept location is assumed to be a normal bivariate distribution function with zero mean and standard deviations given by σ_θ and σ_φ. (Equation (32)).

The shell density function (in number of shells per square foot) in the vicinity of target is given by:

$$\rho_{SHELL}(\varphi, \theta) = \frac{1}{2\pi\sigma_\theta\sigma_\varphi} \exp\left[-\frac{{R_T}^2}{2}\left(\left(\frac{\varphi-\varphi_G}{\sigma_\varphi}\right)^2 + \left(\frac{(\theta-\theta_G)\cos\varphi_G}{\sigma_\theta}\right)^2\right)\right] \quad (35)$$

The probability that the shell impact the target's vulnerable area (TVA) is the probability of single-shot kill. TVA is a function of the azimuth and elevation coordinate for a particular target location. P_{SSK} is shown below.

$$P_{SSK} = \int_0^{\pi/2} \cos\varphi d\varphi \int_{-\pi}^{+\pi} TVA\ (\varphi, \theta)\, \rho_{SHELL}(\varphi, \theta) d\theta \quad (36)$$

When the target vulnerable area is contained within a small portion of the shell distribution and the ${R_T}^2$ is greater than $10\ \sigma_\theta\sigma_\varphi$, the following approximation is valid. [1]

[1] For example; at 1000 foot range, with $\sigma_\varphi = \sigma_\theta = 50$ feet, and $A_v = 200$ square feet, a typical case, $A_v \ll 2500\ ft^2$ = area of shell distribution, and ${R_T}^2 = 10^6 \gg 25{,}000$. Both conditions are more than adequately fulfilled.

$$P_{SSK} = \frac{1}{2\pi\sigma_\theta\sigma_\varphi} \exp\left[-\frac{1}{2}\left(\frac{D_\theta^{\,2}}{\sigma_\theta^{\,2}} + \frac{D_\varphi^{\,2}}{\sigma_\varphi^{\,2}}\right)\right] \tag{37}$$

Where A_v is as described on page 26, σ_θ and σ_φ are derived as shown on page 25 and D_θ and D_φ are derived as described on page 20.

Target Survival Probability

Equation (37) is used to compute the single-shot kill probability for each round of shells. The target aircraft probability of survival after the j^{th} round has been fired is then:

$$P_{surv}(j) = \prod_{i=1}^{j} (1 - P_{SSK}(i)) \tag{38}$$

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