CLASSIFICATION IN THE FREQUENCY-TEMPERATURE RANGE OF VISCOELASTIC MATERIALS FOR DAMPING OF FLEXURAL WAVES IN SANDWICH STRUCTURES WITH VARIOUS BOUNDARY CONDITIONS.

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Damping of flexural waves by constrained or unconstrained viscoelastic layers is considered in order to classify the viscoelastic materials according to their efficiency for given ranges of temperature and frequency . The loss factor is computed for structures of various geometries , such as beams, plates and tubes, with various materials of the constraining layers , such as steel, aluminum, fiber glass composite . The influence of boundary conditions is studied . The curves corresponding to particular loss factors are plotted in the frequency - temperature plane for a given structure, so that the efficiency of the damping treatment may be evaluated immediately for each range of temperature or frequency . A classification between different materials can then be made . An experiment giving the modes and the corresponding loss factors of free sandwich plates is presented .

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INTRODUCTION

The damping of flexural vibrations by the mean of viscoelastic materials is a classic method involving different techniques such as extensional damping by unconstrained layers, and shear damping by constrained layers . The resulting loss factor for an elementary structure such as a beam, a plate or a tube, is given by well-known theories (Oberst, Ruzicka and Kerwin). However, one of the user's problems is the choice of the different added layers : the viscoelastic material and, eventually, the material of the constraining layer, and their dimensions.

This paper presents a method of classification of viscoelastic materials, based on their intrinsic loss factor or on the composite loss factor of damped structures in which they are involved. The principal results are curves representing a given loss factor in the temperature-frequency plane, so that the user can immediately evaluate the damping's efficiency in the ranges of temperature and frequency he is interested in. It is also possible to plot the loss factor of a composite structure versus the frequency (or the temperature) for given temperatures (or given frequencies), or versus different thickness ratios for given temperatures and frequencies.

The combination of all these possibilities helps to find the best viscoelastic material, and eventually the constraining material, and to optimize the thickness of each layer.

The utilization of the method will be illustrated with some examples of damping by five different viscoelastic materials .

INTRINSIC DAMPING .

Characterization of a viscoelastic material .

Under linear conditions, the complex modulus is a classic way to characterize the behavior of a viscoelastic material . The stress-strain relation can be written :

$$\sigma = E(f,T)\,(1+i\beta(f,T))\,\varepsilon$$

where f is the frequency, T is the temperature, and E, β are respectively the Young's modulus and the loss factor of the material .

The complex modulus $E(1+i\beta)$ is provided by experimental data giving the variations of E and β with temperature and frequency . Usually, there is an equivalence between temperature and frequency effects, so that the separate variables f and T can be combined in a single variable $f\alpha_T$ called the reduced frequency , where the 'shift factor' α_T is a non-dimensional parameter depending only on temperature . The Young's modulus E (or the shear modulus $G=E/3$) and the loss factor are then given ,in function of the reduced frequency , by the 'master curves' which characterize each viscoelastic material . Figures 1 and 2 show the master curves of two viscoelastic materials : M1 and M4 .

Classification of viscoelastic materials according to their intrinsic loss factor .

One way of comparing the efficiencies of different viscoelastic materials is to look for the frequency intervals where their loss factor is greater than a certain value, for the temperatures one is interested in.Table 1 shows the frequency intervals where $\beta > 0.5$ for five materials: M1 to M5, and for three temperatures . This method gives a first indication about the best materials available for given temperatures and frequencies .

In order to avoid tedious manipulations, the master curves of the viscoelastic materials have been stocked in a library ; the user can then compute E and β for each value of f and T by the mean of a simple program using the following method : it first computes the shift factor α_T,then the reduced frequency $f\alpha_T$, and finally E and β .The user can obtain more global results than the table above by plotting E and β versus frequency (or temperature) for the temperatures (or the frequencies) he is interested in. However, if he wants to have a general view of the efficiency of the material in order to make a first selection, the most appropriate method consists in plotting the curves corresponding to different values of β in the (f,T) plane . These curves are obtained by a program which computes β for several values of f and T, and then plots contour lines corresponding to the desired values of β . In order to have reliable results, one should consider a great number of points in f and T, and make regular subdivisions in log(f) and T . Figures 3 and 4 show the curves obtained for M1 and M4 for 10 Hz < f <10000 Hz and 0°C< T < 60°C . A comparison with the results of Table 1 or the master graphs shows that the curves give quite good results if we take into account the imprecision on the master graphs .

The different types of viscoelastic damping treatments .

There are two types of viscoelastic damping treatments :

- the extensional damping (by unconstrained layer), in which the extensional deformation of the damping layer accounts for the damping
- the shear damping (by constrained layer) , in which the energy losses due to shear motions are dominant .

We will study these two types of treatments with one viscoelastic layer and for elementary structures such as beams, plates, and tubes .

EXTENSIONAL DAMPING.

This method consists in adding a viscoelastic layer of Young's modulus $E_V(1+j\beta)$ to the base structure (Figure 5) .

The loss factor of the composite structure in the case of a damped beam or plate is given by : [1,2,3] :

$$\eta = \frac{\beta eh(3+6h+4h^2+2eh^3+e^2h^4)}{(1+eh)\,(1+4eh+6eh^2+4eh^3+e^2h^4)}$$

H_1 , H_v : Thicknesses of the layers

$e = E_v / E_1$

$h = H_v / H_1$

$eh = E_v H_v / E_1 H_1$: Ratio of the extensional stiffnesses of the two layers

In most practical cases, eh << 1 .

For a damped tube, the loss factor is :

$$\eta = \frac{\beta E_v(R_3^4 - R_2^4)}{E_1(R_2^4 - R_1^4) + E_v(R_3^4 - R_2^4)}$$

with R_1, R_2 : Internal and external radii of the initial tube
R_3 : External radius of the damped tube

For the beam or plate as well as for the tube, the composite loss factor increases with the intrinsic loss factor , the Young's modulus and the thickness of the viscoelastic layer . The best materials for extensional damping are then those which have the greatest loss factor and extensional stiffness . Increasing the thickness of the viscoelastic layer improves the efficiency of the treatment, however there is a limit above which the damping tends to saturate and even to decrease .

For example, the material M4 is better than M1 for extensional damping (Table 1, Figures 3 and 4) . In fact , M4 and M5 are used for extensional damping, whereas M1,M2 and M3 are used for shear damping .

Figures 6 and 7 show the curves η (f,T) for a beam damped by M4 and M5 . By comparing them, one can deduce that :

- M4 is less efficient than M5 for high temperatures and low frequencies, and more efficient for low temperatures and high frequencies
- M4 is more efficient than M5 for intermediate temperatures and frequencies

More precise results can be obtained by superimposing the figures 6 and 7). The best material for the particular case considered is then deduced immediately for each range of temperature and frequency .

The influence of H_v can also be studied by plotting the curves $\eta(H_v/H_1)$ for given values of f and T (Figures 8 and 9) .

SHEAR DAMPING .

This treatment, which has been considered by many authors [1,2,4], consists in applying a constrained viscoelastic layer (Figures 10 and 11) . Ruzicka and Kerwin [4] have provided a simplified theory with the following assumptions :
-The considered modes are sinusoidal (simply supported structure)
-The effects of the boundary constraints are negligible
-Shear and torsional distorsions of the elastic elements are negligible
-The dimensions of the different cross-sections remain constant
-There is contact without slippage at all the interfaces
-The stress-strain relations are linear in all the layers
-The axial inertial forces are negligible
-The elastic elements have zero extensional and shear loss factors
-The elastic elements are considerably stiffer in extension than the viscoelastic material
-The viscoelastic material is thin and of approximately constant thickness

The loss factor of the composite structure is :

$$\eta = \frac{\beta X Y}{1 + X(Y+2) + (1+\beta^2)X^2(Y+1)}$$

with β : Intrinsic loss factor of the viscoelastic material
X: Shear parameter
Y: Geometrical parameter

The intrinsic loss factor is deduced of the master graphs . It depends on the frequency and thetemperature : β (f,T) .

The geometrical parameter Y is defined as

$$Y = \{(EI)_\infty / (EI)_0\} - 1$$

where $(EI)_0$ (resp. $(EI)_\infty$) is the flexural rigidity of the composite structure when the elastic elements are completely uncoupled (resp. coupled) . Another expression for Y is :

$$Y = \frac{M A_1 A_2 d_2}{(A_1 + MA_2)(I_1 + MI_2)}$$

with $M = E_2/E_1$
$A_{1,2}$: Cross sections of the elastic elements
$I_{1,2}$: Moments of inertia of the elastic elements
d : Distance between the neutral planes of the elastic elements

More generally, $Y = Y_0 x (Y/Y_0)$
where Y_0 is a function of dimension ratios and ratios of Young's moduli of the elastic layers
Y/Y_0 is a correction factor representing the influence of the viscoelastic layer
$Y_0 = Y(H_v = 0)$

The expressions of Y for a beam, a plate or a tube are given with figures 10 and 11 ;
The shear parameter for the mode n is given by

$$X_n = \frac{G'_v \; B_v \; d_0^2}{p_n^2 H_v Y_0 (EI)_0}$$

where G'_v, B_v and H_v are respectively the shear modulus, the mean length and the thick ness of the viscoelastic layer
d_0 is the distance between the neutral planes of the elastic elements when $H_v = 0$
p_n is the wave number

The frequency of the mode n for the beam or the tube is :

$$f_n = \frac{a_n^2}{2\pi L^2} \sqrt{\frac{(EI)_n}{m}}$$

where $(EI)_n$ is the flexural rigidity of the composite structure
m is its mass per unit length
L is its length
a_n is a coefficient depending on the boundary conditions

For a simply supported structure, the modes are sinusoidal ; the wave length is related to L by:

$$\lambda_n = \frac{2L}{n}$$

and $a_n = n\pi$

The wave number p_n is then given by :

$$p_n^2 = \left(\frac{2\pi}{\lambda_n}\right)^2 = 2\pi f_n \sqrt{\frac{m}{(EI)_n}}$$

If we suppose that $(EI)_n$ is the real part of the complex rigidity $(EI)_n^*$ [4], then

$$(EI)_n = \mathrm{Re}\,(EI)_n^* = (EI)_0 \,\mathrm{Re}\left(1 + \frac{X_n^*}{1+X_n^*} Y\right)$$

with $X_n^* = X_n (1 - i\beta_n)$

If we introduce the 'coupling parameter' :

$$Z_n = \frac{X_n(1+X_n) + X_n^2\beta_n^2}{(1+X_n^2) + X_n^2\beta_n^2}$$

the flexural rigidity can also be written :

$$(EI)_n = (EI)_0\,(1+Z_nY)$$

Then the shear parameter for the mode n is :

$$X_n = \frac{G'_v\ B_v\ d_0^2}{2\pi f_n \sqrt{\frac{m}{(EI)_n}}\ H_v\ Y_0\ (EI)_0} = \frac{G'_v\ B_v\ d_0^2\ \sqrt{1+Z_nY}}{2\pi f_n\ H_v\ Y_0 \sqrt{m(EI)_0}}$$

(the expressions of $(EI)_0$ and d_0 are given with figures 10 and 11)

For a given frequency, X_n and Z_n are obtained by an iterative method, then the loss factor is deduced.

If we consider a motion in one direction, the formulation is the same for a plate,with analogous expressions for the wave number and the frequency (Table 2).

In order to compare the effects of different constraining layers, the curves $\eta(f,T)$ have been plotted for a steel beam damped by M1, and constrained by steel, aluminium or fiber glass layers introducing the same added mass (Figures 12,13,14) . It appears that in this case, the most efficient material is aluminium, which can provide a loss factor of 0.2 . However, this is a global conclusion, and another material can be more efficient for particular values of temperature and frequency .

The influence of the viscoelastic material can be studied by considering the steel beam damped by M1, M2 or M3, with the same constraining layer, for example steel (Figures 12, 15 and 16). It appears that, globally, the most efficient material is M1, which can provide a loss factor of 0.15 .

Influence of boundary conditions (free structure)

In the case of a free structure, the modes are no more sinusoidal, so that the theory is not valid . However, analogous relations for f_n and a_n may be used [5], knowing that the expression for a_n is not valid for the first five modes . If one is interested in the value of the frequency , independantly of the modal analysis, the loss factor is the same as for the simply supported plate.

EXPERIMENT

Modal damping measurements have been made on steel plates damped by constrained viscoelastic layers (with steel constraining layers) . The plates were free and excited by a hammer . The measurements were made in five different points, and a modal analysis has given the modal frequencies as well as the corresponding loss factors . In the frequency ran ge of measurement, a few flexural modes were identified . We plotted he experimental and theoretical values of the loss factors on the same curves (Figures 17 and 18). We can see that the experimental values are a little lower than the theoretical ones. However, the agreement between experiment and theory remains quite acceptable.

CONCLUSION

We have developed a program based on well-known theories and which can be of great help for the designer of damping devices with viscoelastic layers . It allows the user to visualize immediately the efficiency of damping treatments and then to choose the most appropriate . It offers different possibilities such as :

- extensional or shear damping
- beams, plates or tubes
- various viscoelastic layers, which master curves are stocked in a library
- various constraining layers, such as steel, aluminium, fiber glass composite

with different thicknesses of the added layers .

However, one has to make many tries before finding the best damping device . The program needs to be extended to an optimization program which would give the best materials with the appropriate dimensions for a given structure to damp .

	M1	M2	M3	M4	M5
T=0°C	1 1740	1 1000	1 10 000		
T=20°C	6 10 000	20 10 000	20 10 000	1 600	1 40
T=40°C	300 10 000		250 10 000	3 10 000	15 3000

Table 1 - Frequency intervals (between 10 Hz and 10000 Hz) where the intrinsic loss factor is greater than 0,5 for the viscoelastic materials considered (M1 to M5) .

	FREQUENCY OF THE MODE n	SIMPLY SUPPORTED	FREE (n>5)
BEAM/TUBE	$f_n = \frac{a_n^2}{2\pi L^2}\sqrt{\frac{(EI)_n}{m}}$	$a_n = n\pi$ $\lambda_n = \frac{2L}{n}$	$a_n = \frac{(2n+1)\pi}{2}$ $\lambda_n = \frac{4L}{2n+1}$
PLATE (motion along one direction)	$f_n = \frac{a_n^2}{2\pi L^2}\sqrt{\frac{D_n}{m}}$	$a_n = n\pi$ $\lambda_n = \frac{2L}{n}$	$a_n = \frac{(2n+1)\pi}{2}$ $\lambda_n = \frac{4L}{2n+1}$

L : Length
m : Mass per unit length / surface
$(EI)_n$: Flexural rigidity of the beam or the tube
D_n : Flexural rigidity of the plate

Table 2

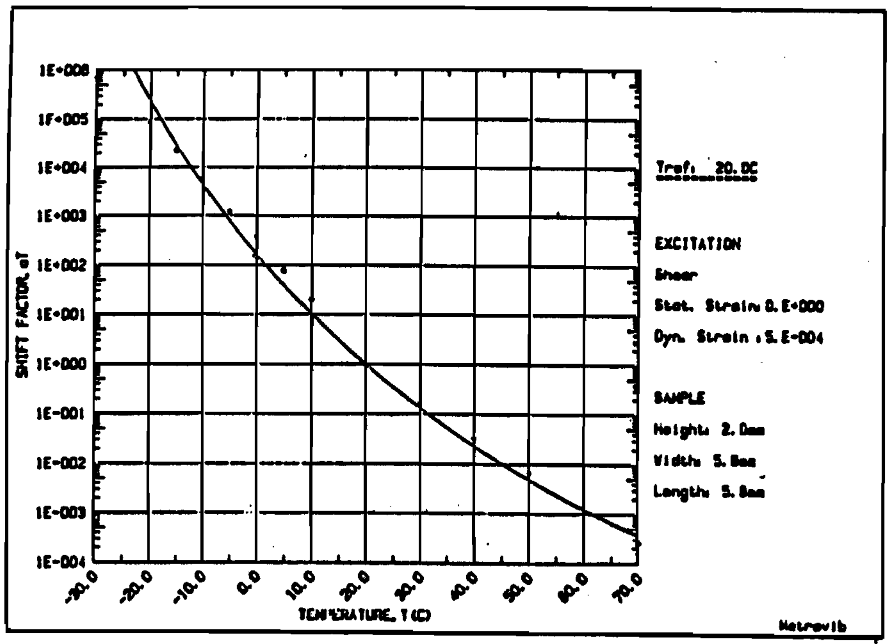


Figure 1a - M1: Shift factor

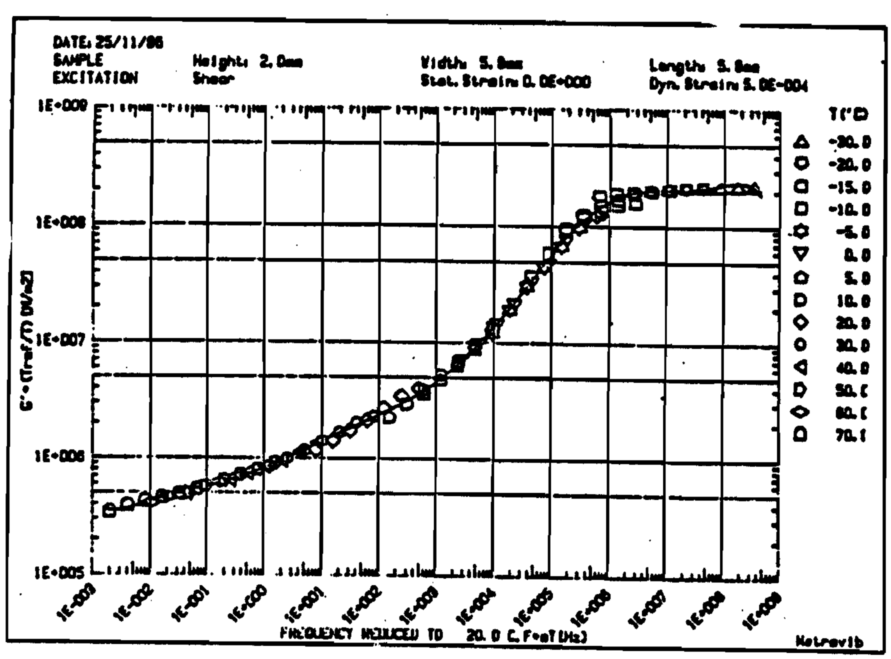


Figure 1b - M1: Shear modulus

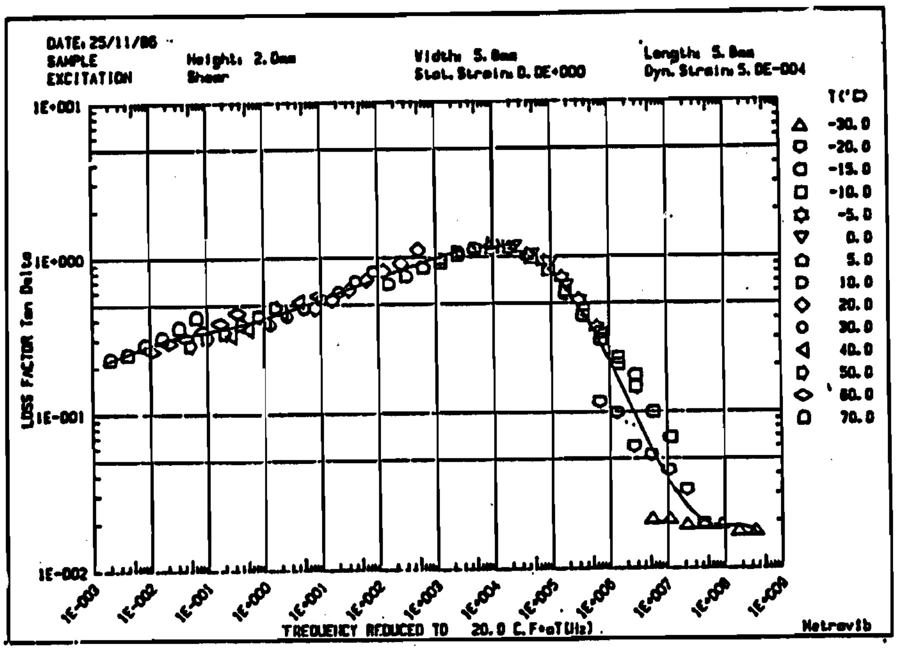


Figure 1c - M1: Loss factor

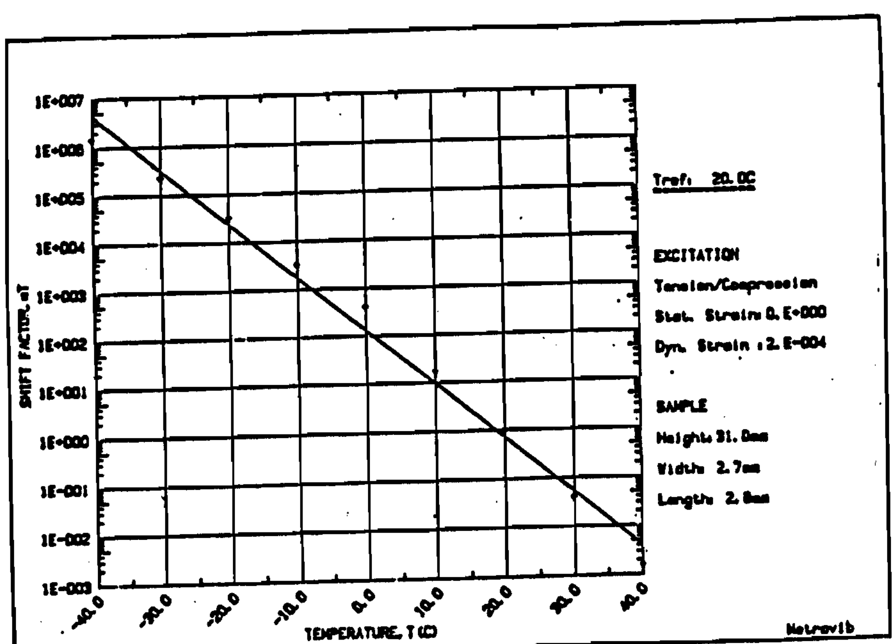


Figure 2a - M4: Shift factor

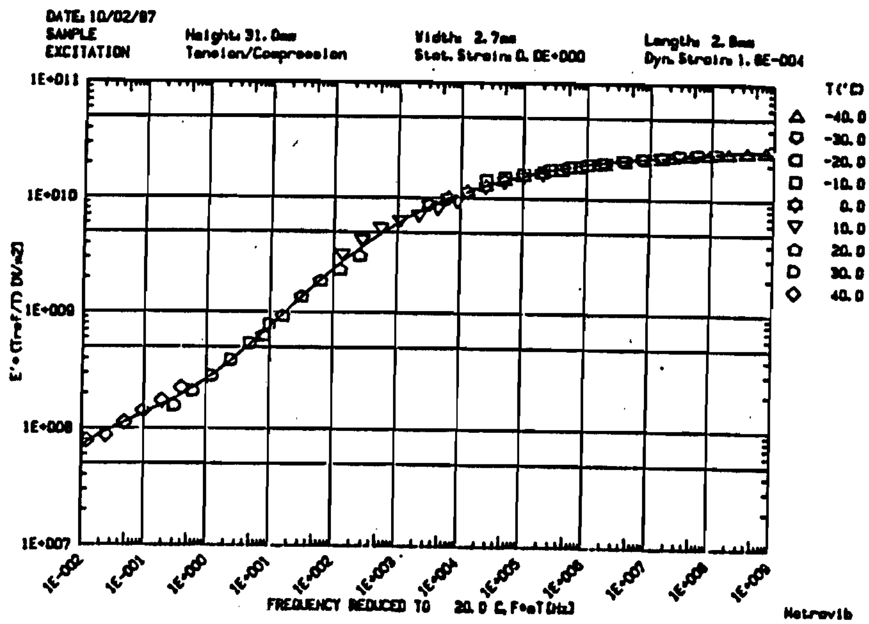


Figure 2b - M4: Shear modulus

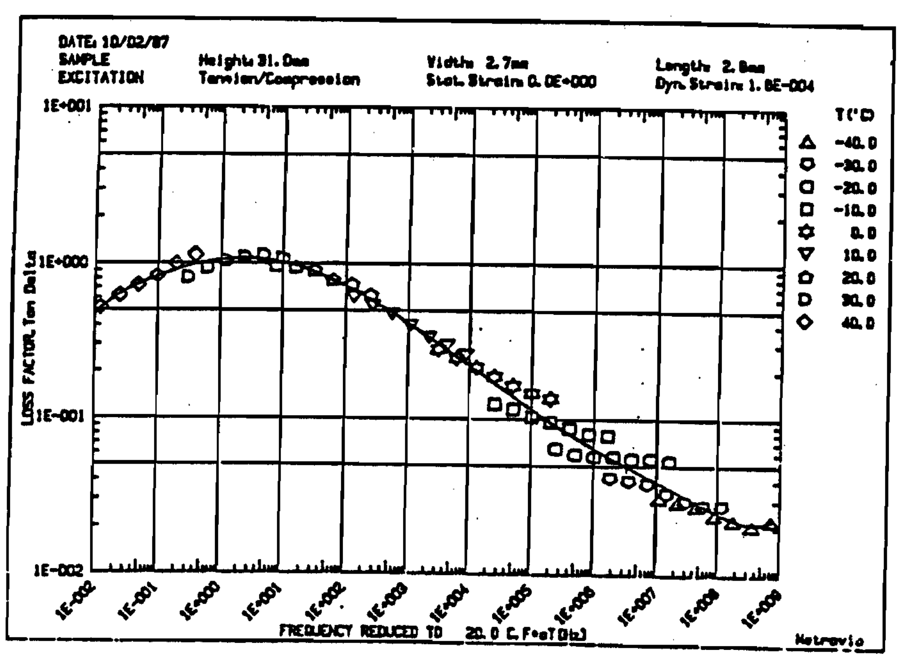


Figure 2c - M4: Loss factor

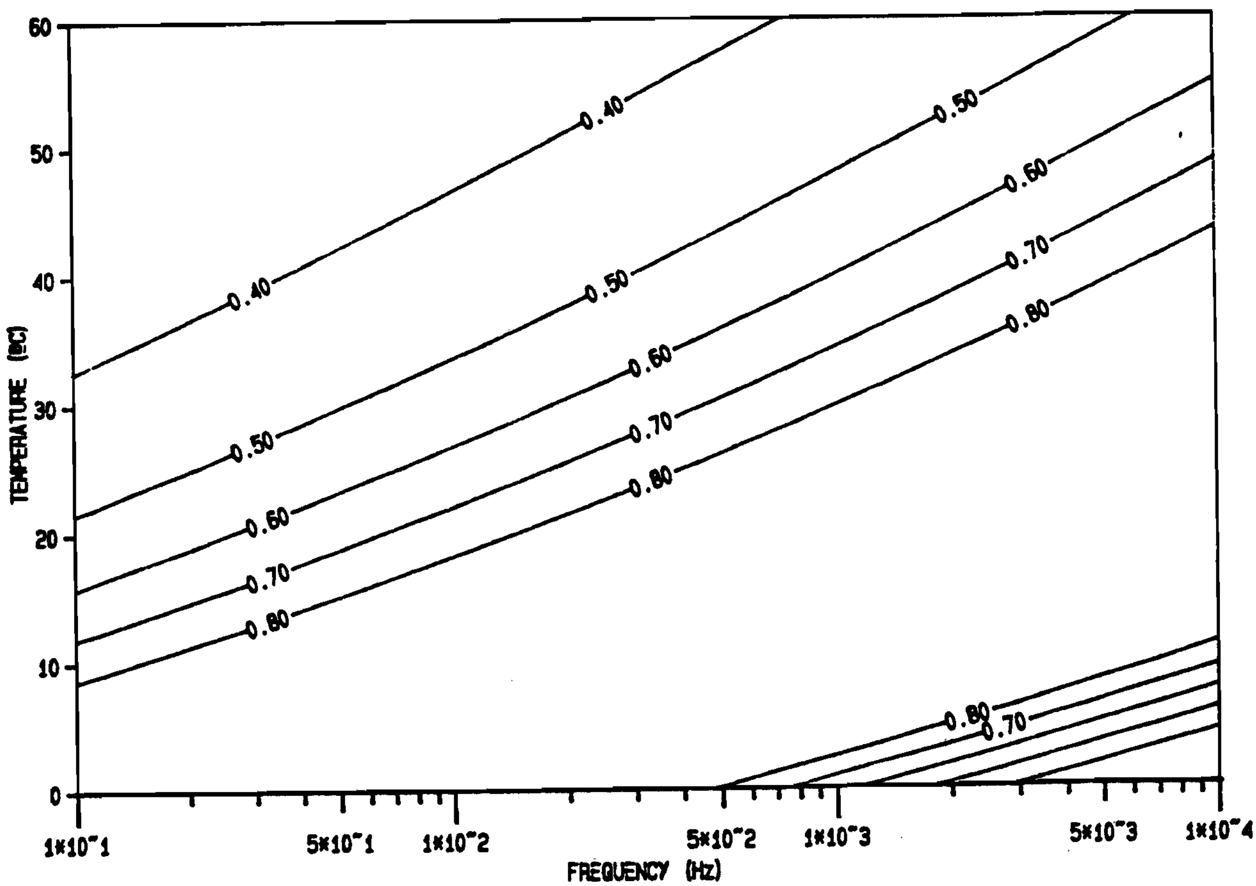


Figure 3 - M1: Contour lines for $\beta = 0.4$; 0.5 ; 0.6 ; 0.7 ; 0.8

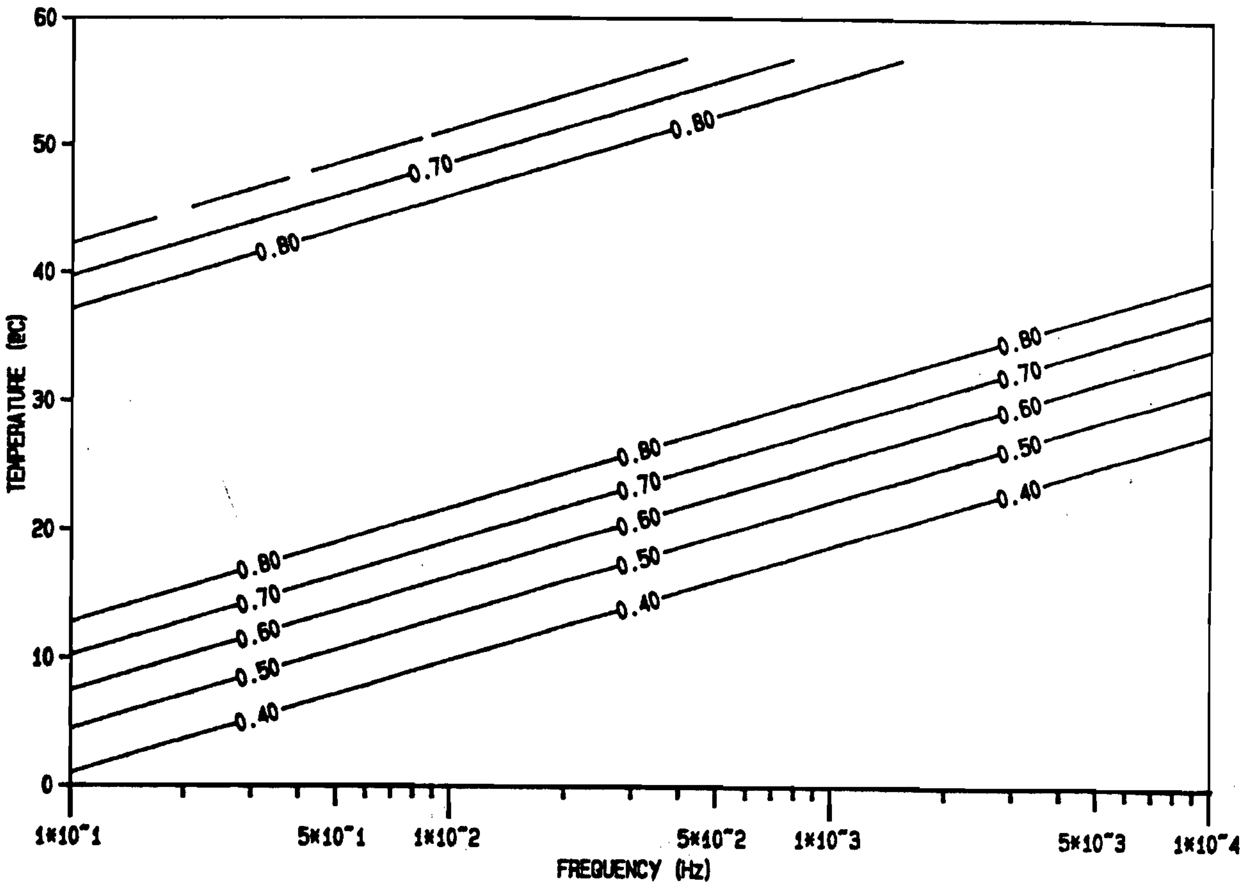


Figure 4 - M4: Contour lines for $\beta = 0.4 ; 0.5 ; 0.6 ; 0.7 ; 0.8$

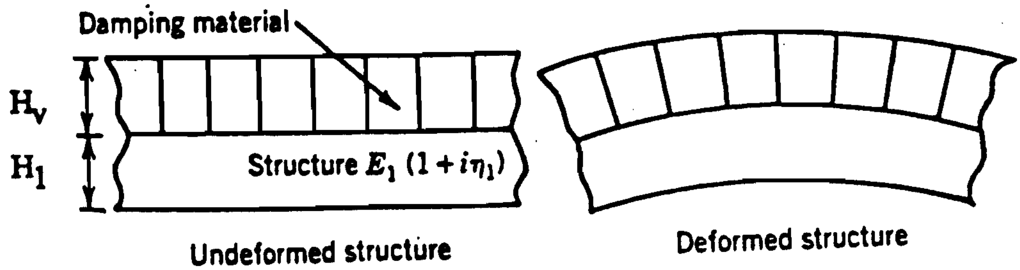


Figure 5 - Unconstrained damping treatment
Structure : $E_v(1+j\beta)$
Viscoelastic layer : $E_1(1+j\eta_1)$, $\eta_1 << 1$

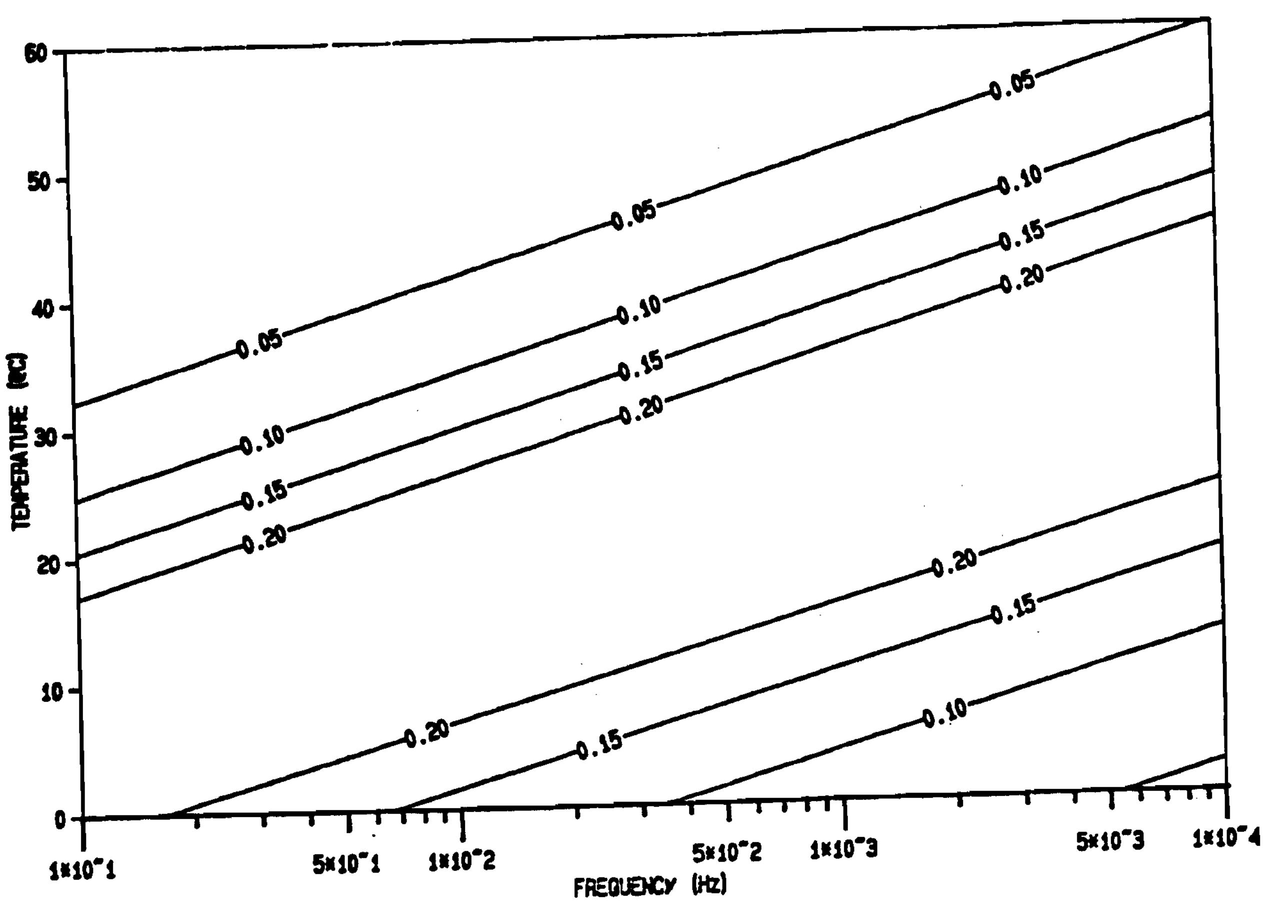


Figure 6 - Extensional damping . Contour lines for $\eta = 0.05$; 0.1 ; 0.15 ; 0.2
DAMPED BEAM
STEEL $H_1 = 0.01$ m
M4 $H_v = 0.02$ m

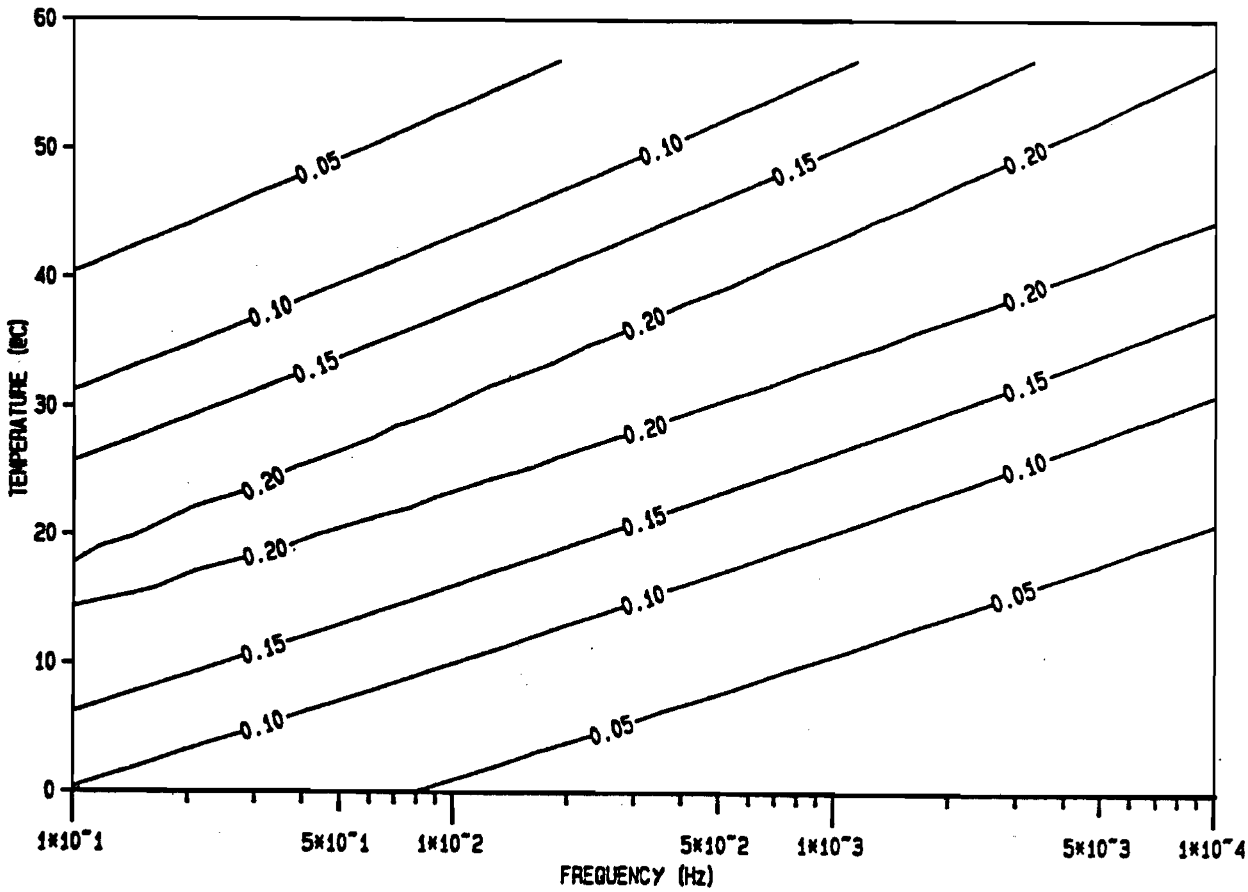


Figure 7 - Extensional damping . Contour lines for $\eta = 0.05$; 0.1 ; 0.15 ; 0.2

DAMPED BEAM

STEEL $H_1 = 0.01$ m

M5 $H_v = 0.02$ m

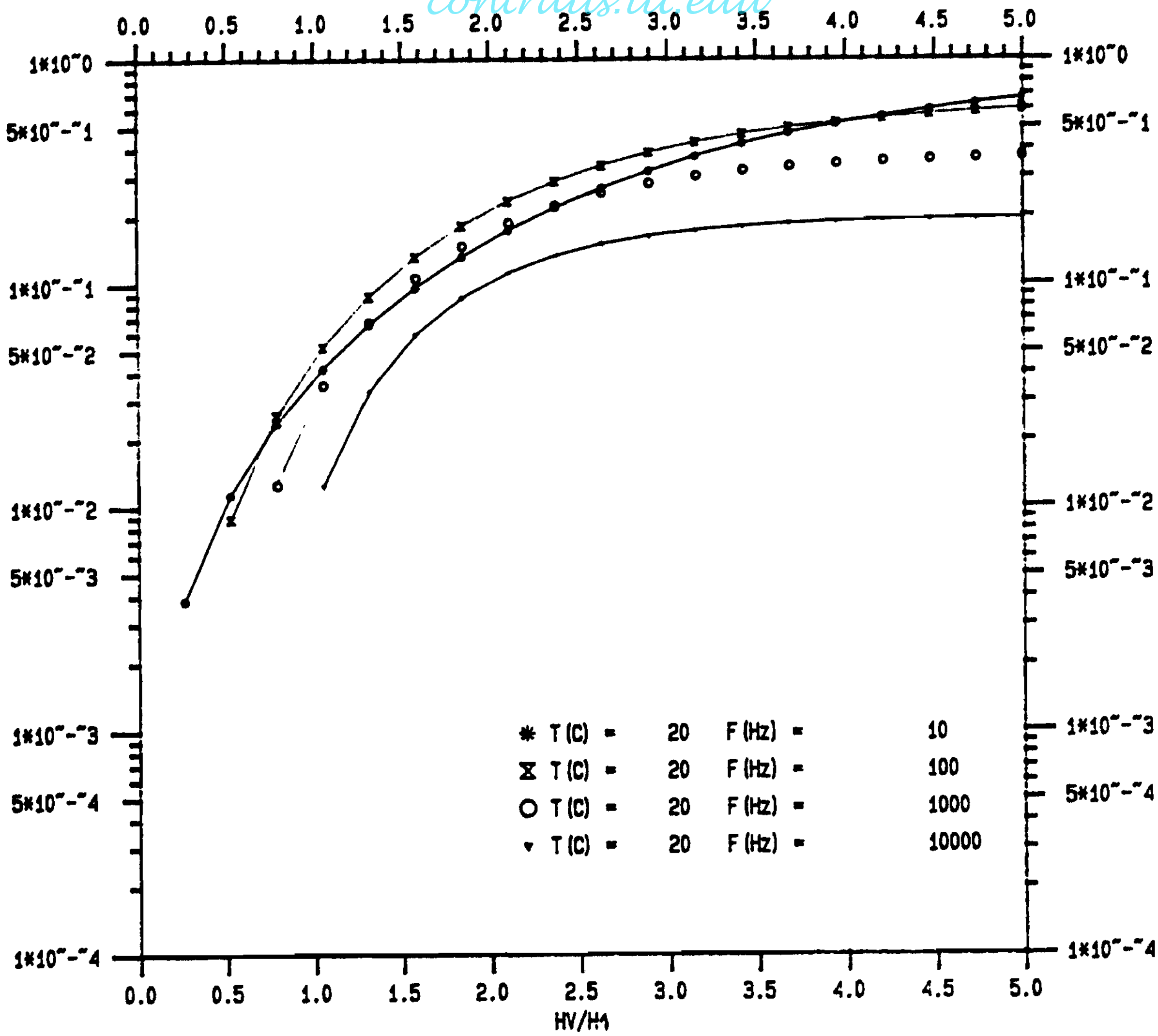


Figure 8 - Extensional damping - Composite loss factor
DAMPED BEAM
STEEL $H_1 = 0.01$ m
M4

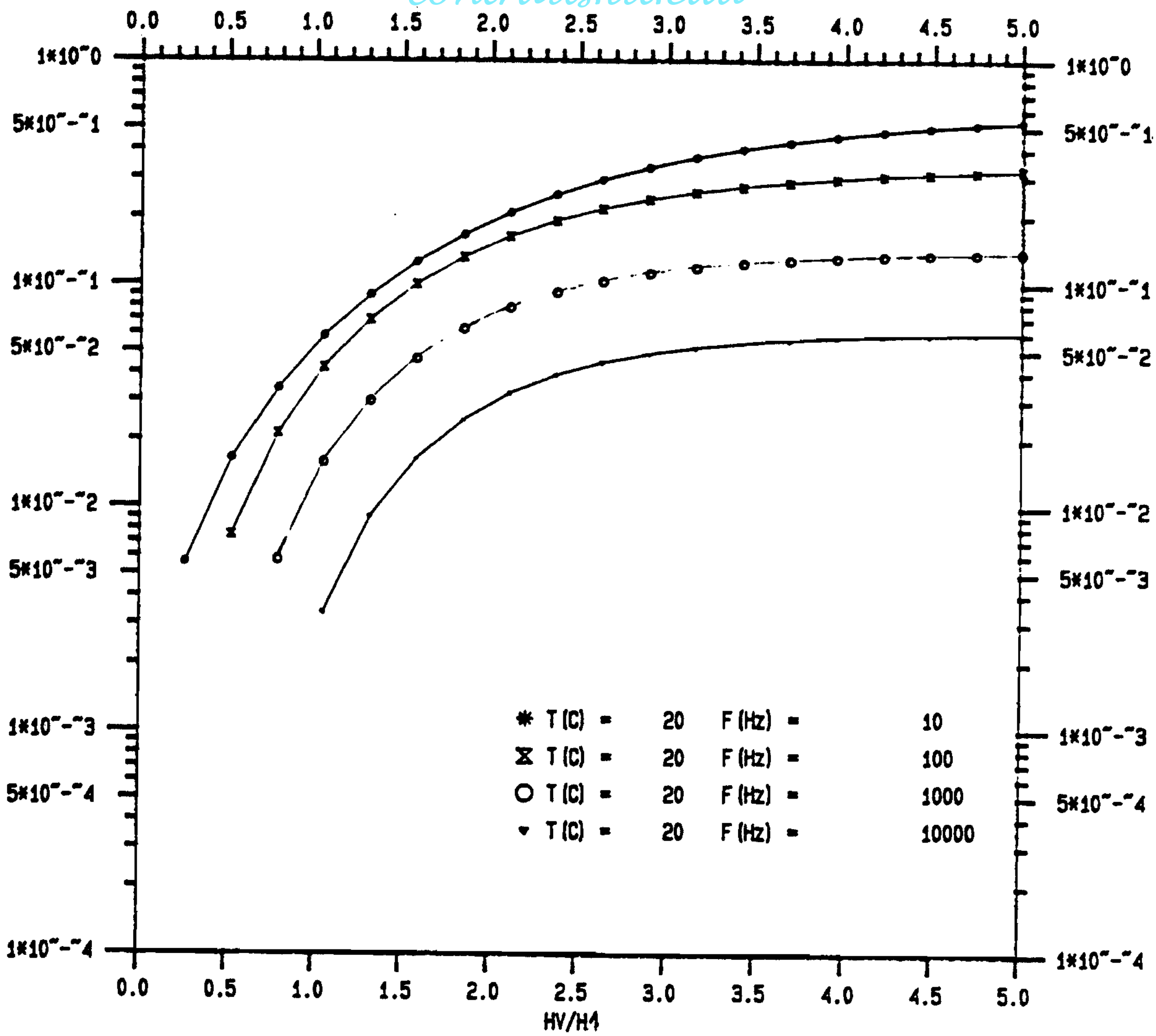
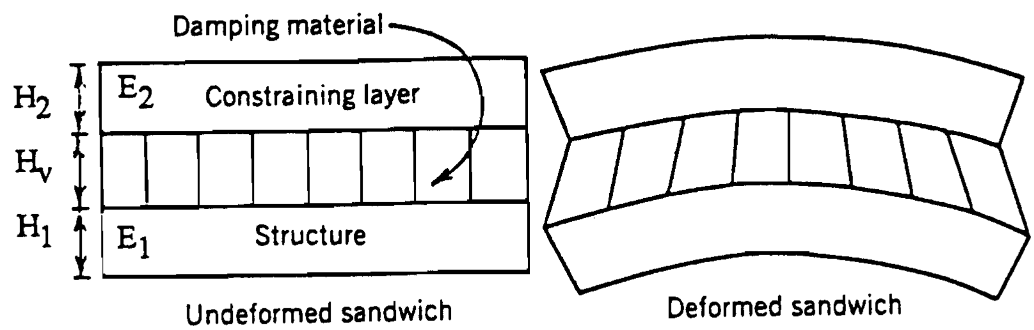


Figure 9 - Extensional damping - Composite loss factor
DAMPED BEAM
STEEL $H_1 = 0.01$ m
M5



$$M = E_2/E_1$$

$$R = H_2/H_1$$

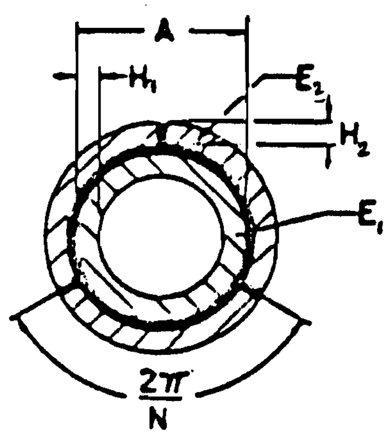
$$Y_0 = \frac{3MR(R+1)^2}{(MR+1)(MR^3+1)}$$

$$\frac{Y}{Y_0} = 1 + 2\,\frac{H_v}{H_1 + H_2}$$

$$(EI)_0 = \frac{B_v}{12}\left(\frac{E_1H_1^3}{1-\nu_1^2} + \frac{E_2H_2^3}{1-\nu_2^2}\right) \qquad \text{(with } \nu_1 = \nu_2 = 0 \text{ for a beam)}$$

$$d_0 = \frac{H_1 + H_2}{2}$$

Figure 10 - Shear damping for a plate or a beam



$M = E_2/E_1$

$R = H_2/H_1$

$S = H_1/A$

$T = 1+2RS$

$$Y_0 = \frac{8MN^2(T^3-1)^2 \sin^2(\pi/N)}{9\pi^2(T^2-1)\{(1-(1-2S)^4) + M(T^4-1)\} - 8MN^2(T^3-1)^2 \sin^2(\pi/N)}$$

$Y = Y_0$

$$(EI)_0 = E_1A^4 \left\{ \frac{\pi(1-(1-2S)^4)}{64} + \frac{\pi M(T^4-1)}{64} - \frac{MN^2(T^3-1)^2 \sin^2(\pi/N)}{72\pi(T^2-1)} \right\}$$

$$m = (\pi/4)\{ \rho_2((A+2H_2)^2-A^2) + \rho_1(A^2-(A-2H_1)^2) \}$$

Figure 11 - Shear damping for a tube

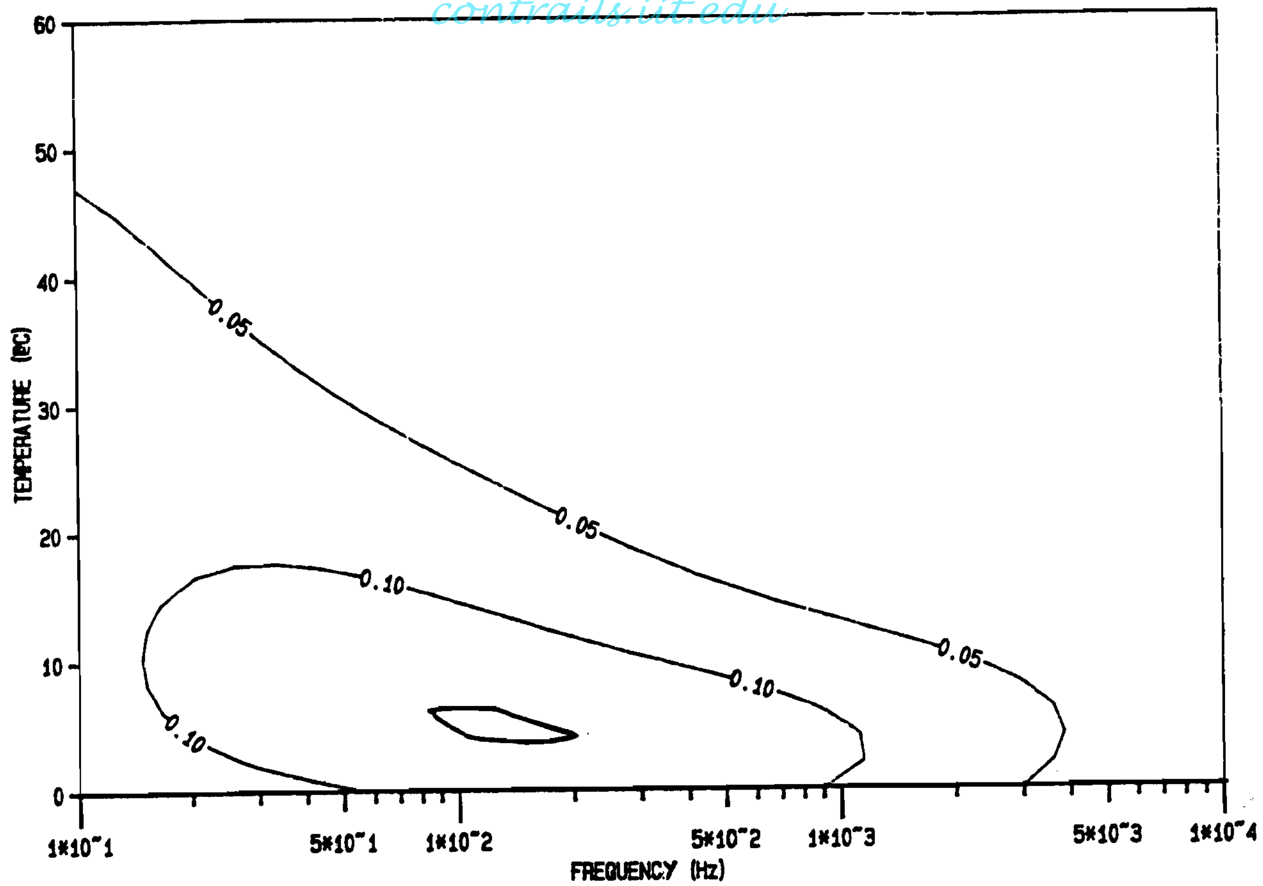


Figure 12 - Shear damping . Contour lines for $\eta = 0.05$; 0.1 ; 0.15 ; 0.2

DAMPED BEAM

STEEL $H_1 = 0.01$ m

M1 $H_v = 0.001$ m

STEEL $H_2 = 0.002$ m

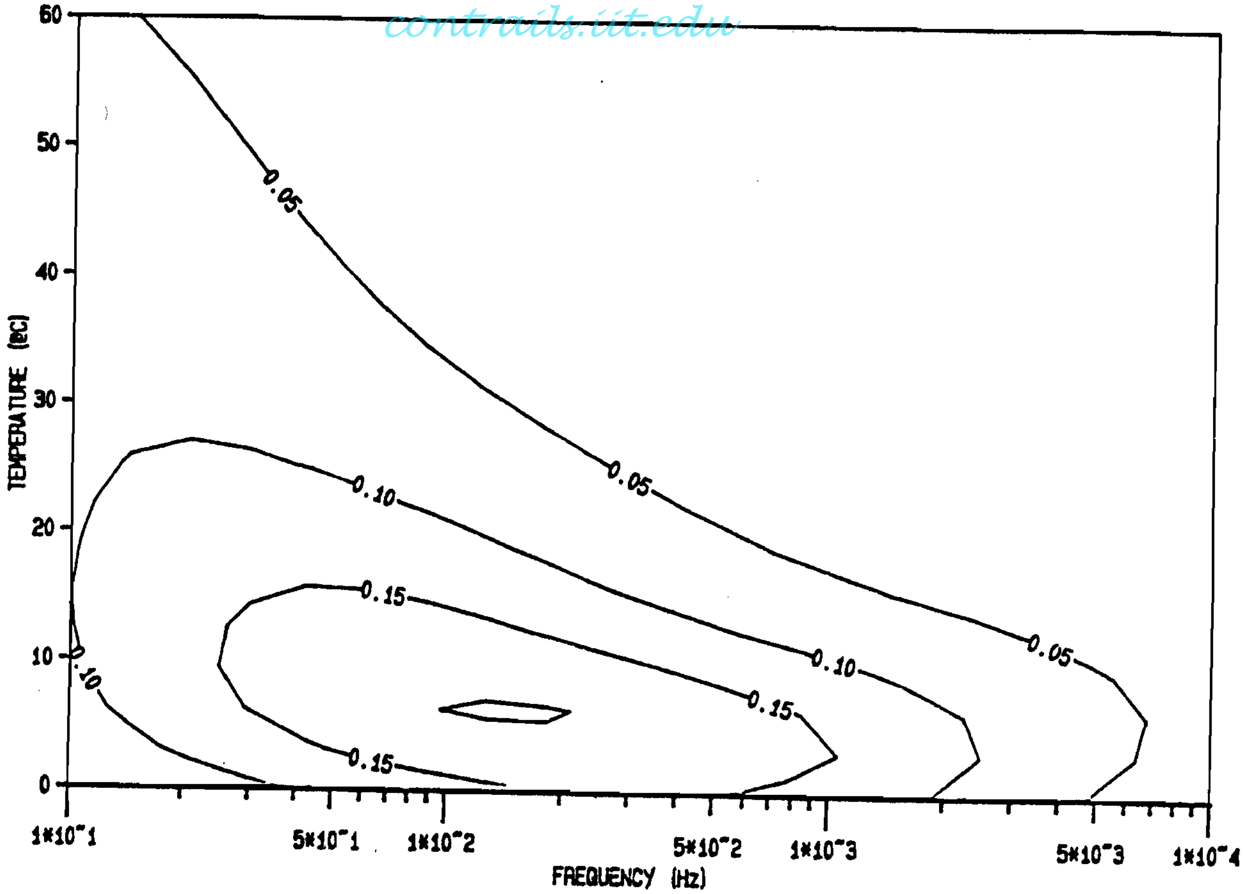


Figure 13 - Shear damping . Contour lines for $\eta = 0.05$; 0.1 ; 0.15 ; 0.2

DAMPED BEAM

STEEL $H_1 = 0.01$ m

M1 $H_v = 0.001$ m

Al $H_2 = 0.006$ m

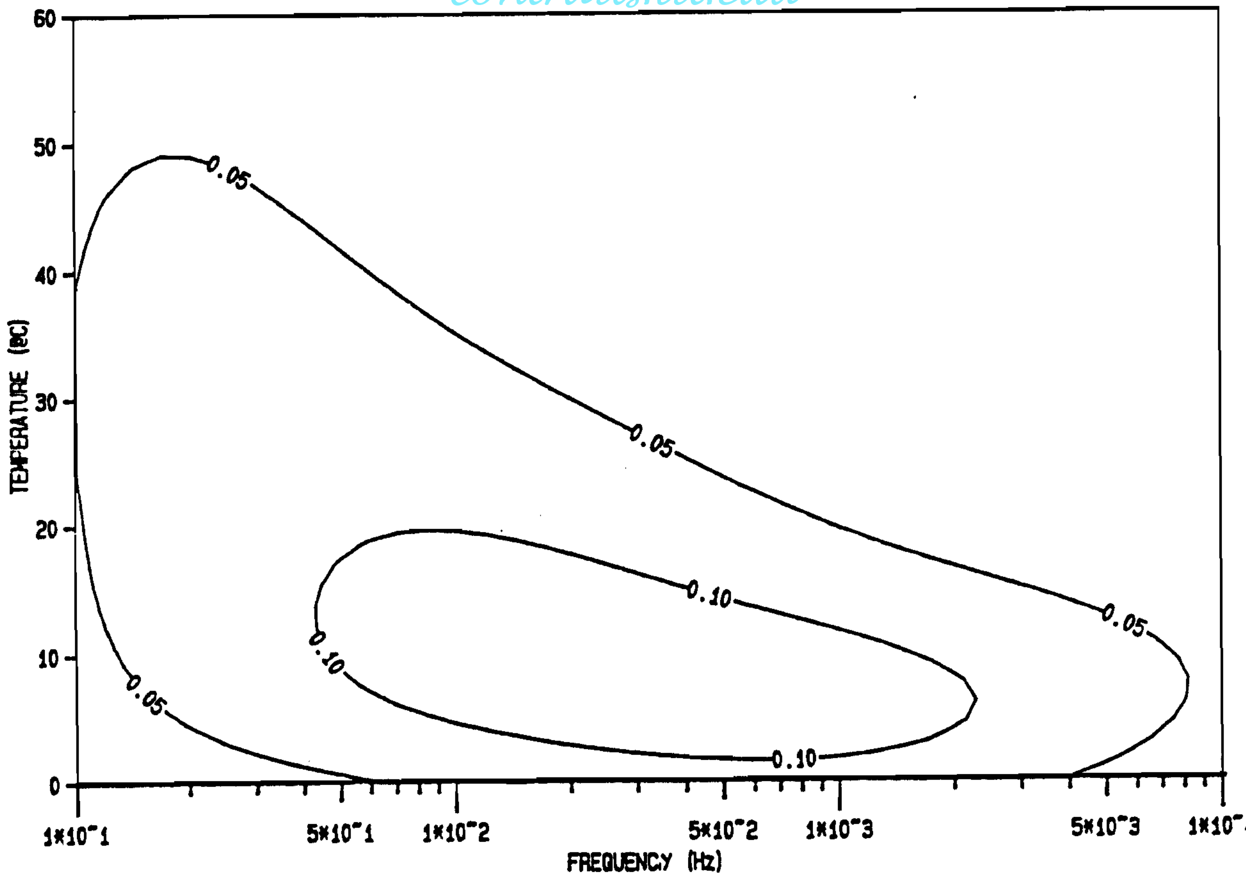


Figure 14 - Shear damping . Contour lines for $\eta = 0.05$; 0.1 ; 0.15 ; 0.2

DAMPED BEAM

STEEL $H_1 = 0.01$ m

M1 $H_v = 0.001$ m

CVR $H_2 = 0.009$ m

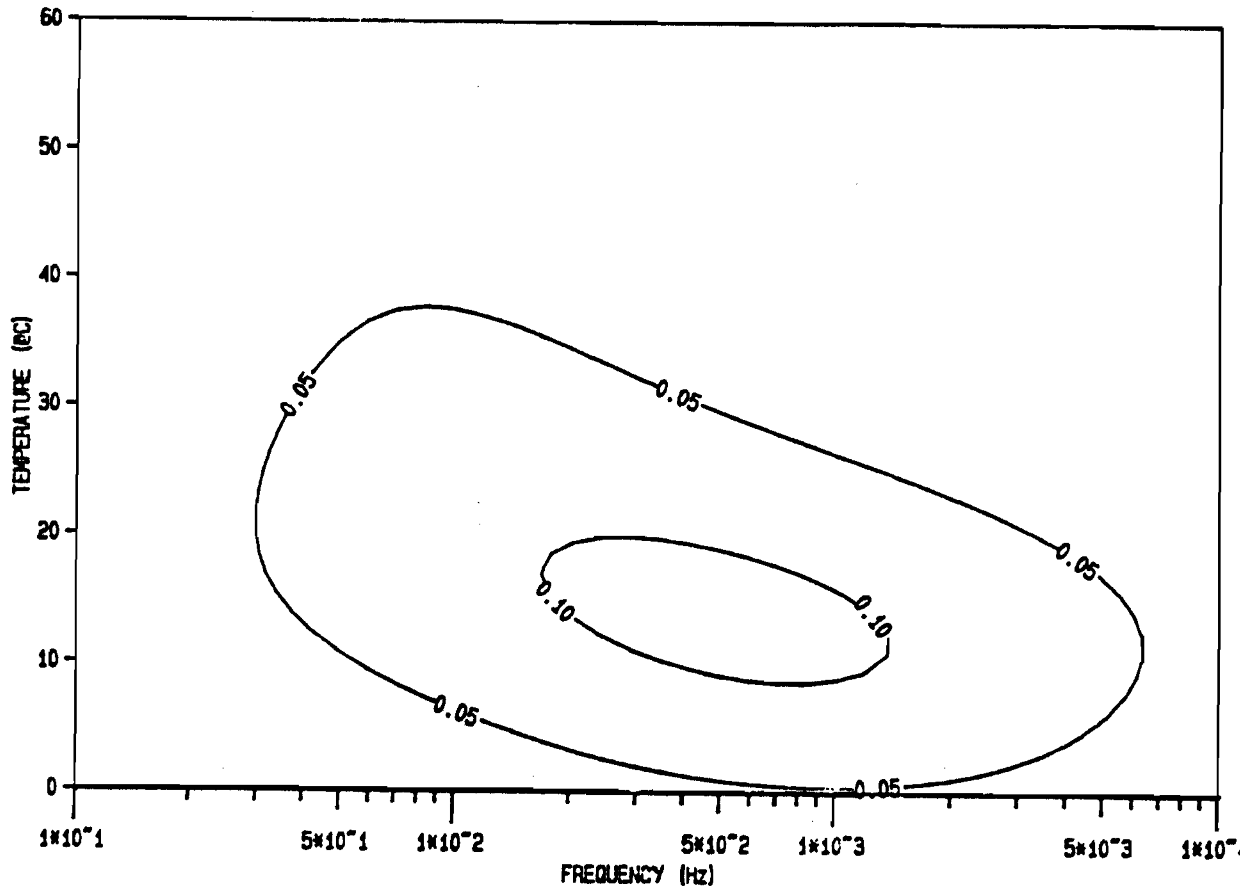


Figure 15 - Shear damping . Contour lines for $\eta = 0.05$; 0.1 ; 0.15 ; 0.2

DAMPED BEAM

STEEL $H_1 = 0.01$ m

M2 $H_v = 0.001$ m

STEEL $H_2 = 0.002$ m

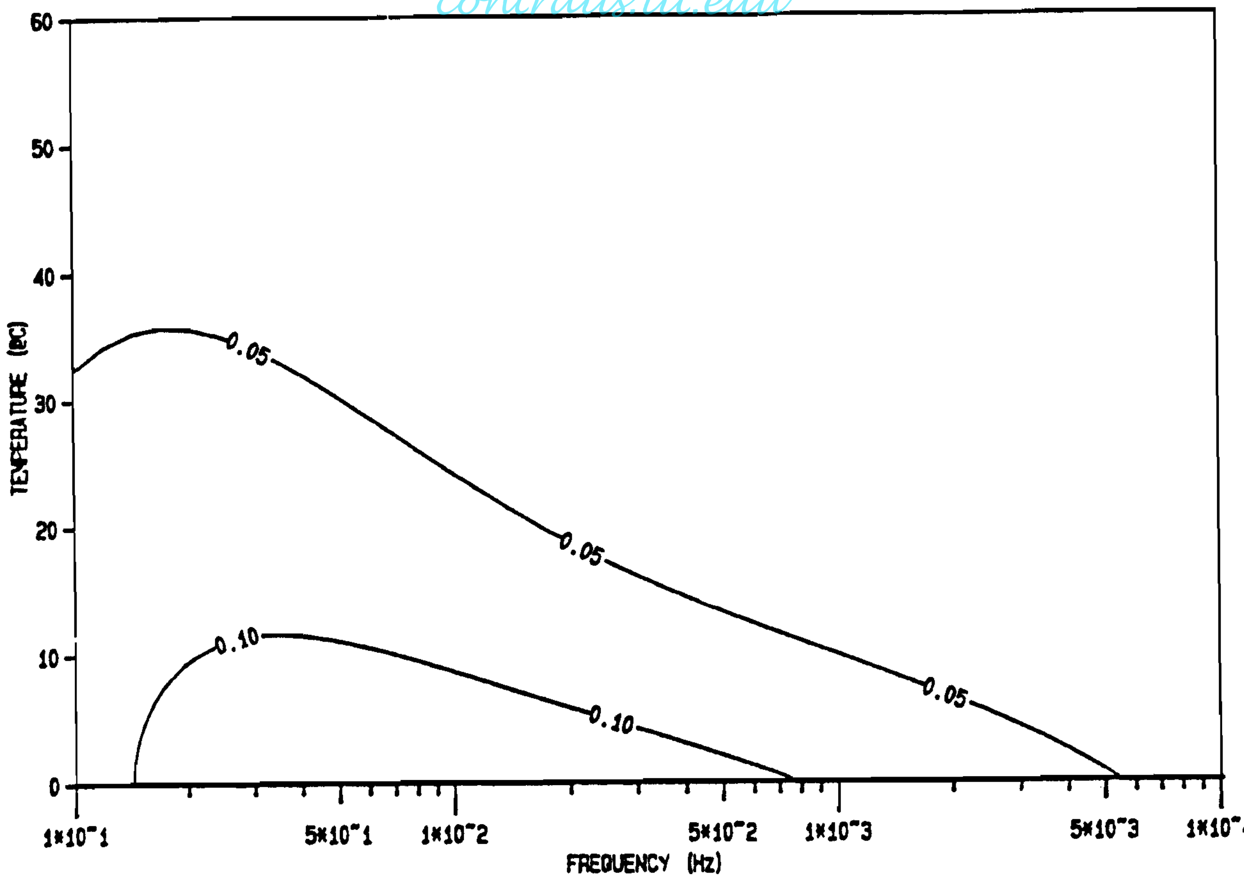


Figure 16 - Shear damping . Contour lines for $\eta = 0.05 ; 0.1 ; 0.15 ; 0.2$

DAMPED BEAM

STEEL $H_1 = 0.01$ m

M3 $H_v = 0.001$ m

STEEL $H_2 = 0.002$ m

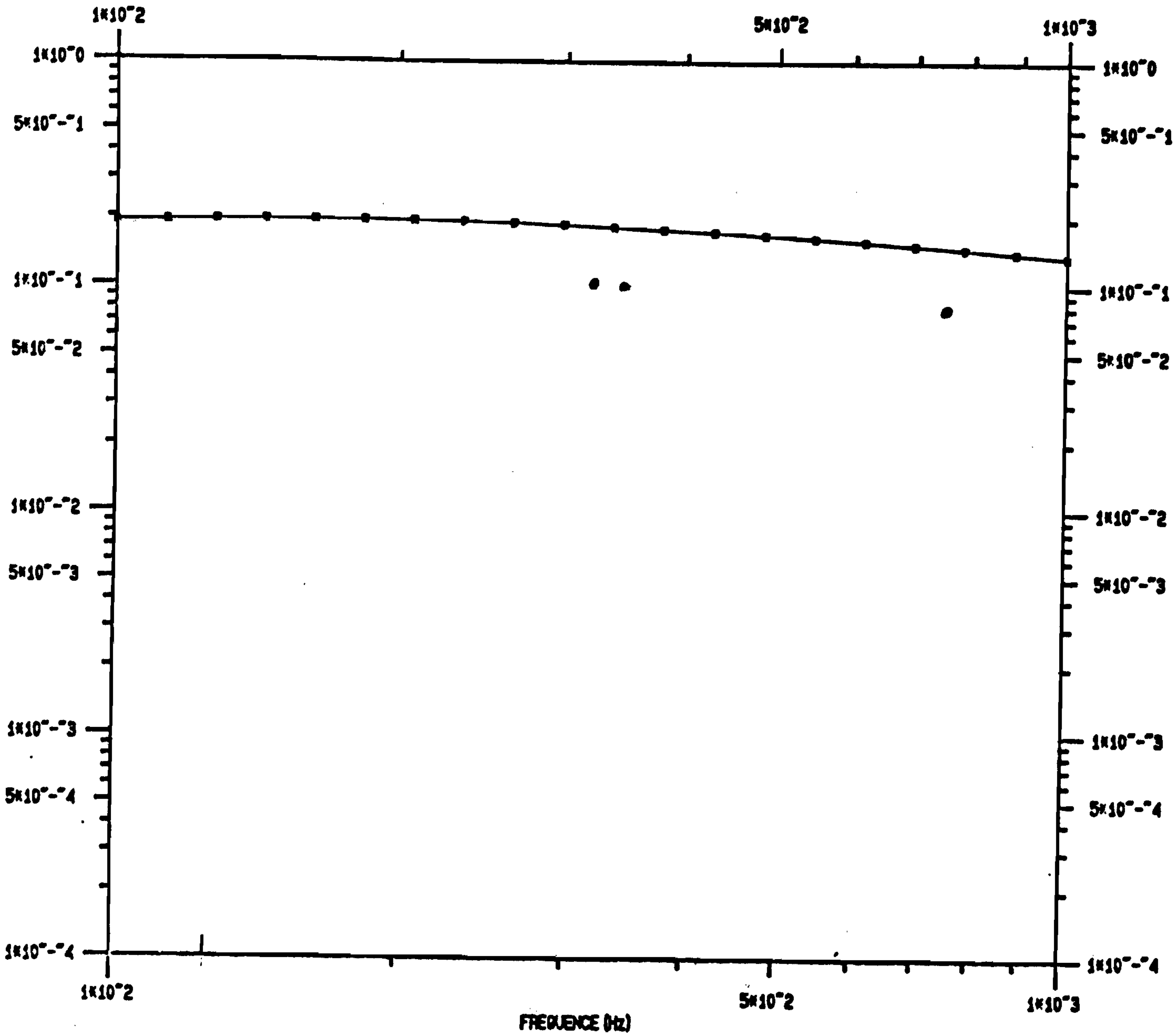


Figure 17 - Shear damping .Loss factor versus frequency for T = 27 °C
Theoretical curve and experimental points (x)
DAMPED BEAM
STEEL $H_1 = 0.008$ m
VISCO $H_v = 0.0017$ m
STEEL $H_2 = 0.0025$ m

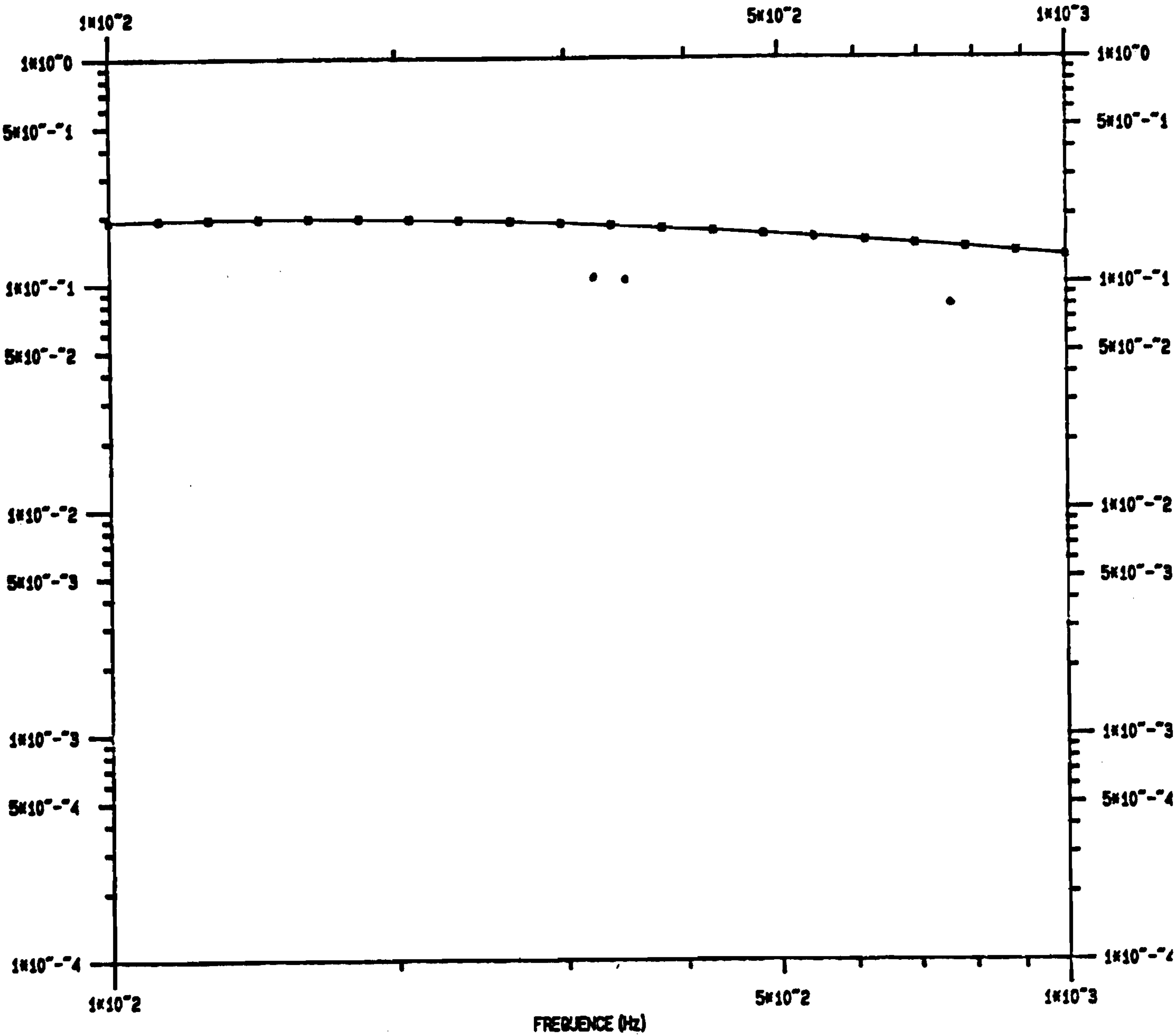


Figure 18 - Shear damping .Loss factor versus frequency for T = 27 °C
Theoretical curve and experimental points (x)
DAMPED BEAM
STEEL $H_1 = 0.008$ m
VISCO $H_v = 0.0025$ m
STEEL $H_2 = 0.002$ m

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