

## INTRODUCTION

At the present time, most of the works on the control of distributed parameter dynamical systems have been proceeded by first replacing the distributed parameter mathematical models by approximate lumped parameter models. Thus, in the analytical design of control systems for an aerospace vehicle with flexible structures, one usually starts with the derivation of an approximate lumped parameter model by means of spatial discretization or modal truncation, and then proceeds to design the control system via the established techniques for lumped parameter systems. Such an approach is natural from the practical standpoint. However, it does not always lead to satisfactory results. For example, although the design leads to a stable control law for the approximate lumped parameter model, the equilibrium state of the distributed model with the same control law may be unstable.

The main objective of the present investigation is to develop a control theory for distributed parameter dynamical systems with emphasis on flexible vehicles, whose mathematical models are in the form of integral or partial differential equations. Hopefully, the results can shed some light on how and where to make the necessary approximations for obtaining solutions to practical control problems in a rational manner.

In view of the meager existing results in this area, the intrinsic difficulties in the theory of partial differential equations, and the time limitation imposed by the present contract, the investigation has been primarily of an exploratory nature. A major portion of the effort has been spent on the establishment of a mathematical framework for the development of a control theory and in the formulation of control problems associated with distributed parameter systems. Emphasis has been placed on attempting to manifest the nature of the control problems which are intrinsic to this class of systems, viewing them with a broad perspective, rather than attempting to obtain solutions to problems of a specialized nature. It is felt that such an approach at this time can provide some insight for establishing directions for future work in this area. In the work on the control of flexible vehicles, emphasis has been placed on stability problems rather than optimization problems. This choice has been based on the observation that, for distributed parameter systems, approximation in one form or another is imperative in the implementation of any optimum control law; moreover, in aerospace vehicle design, flexibility usually has the effect of a disturbance, and one is willing to increase the margin of reliability or safety at the expense of flight performance.

## SUMMARY

In what follows, we shall summarize the main results of the present investigation in each of the following areas: (1) mathematical description of distributed parameter dynamical systems; (2) optimum control; (3) stability; (4) approximation problems.

### Mathematical Description of Distributed Parameter Dynamical Systems

In "Control of Distributed Parameter Systems, (Advances in Control Systems, Theory and Applications, Academic Press, 1964, pp. 72-172)", the notions of state, state space and state transitions for distributed parameter dynamical systems are clarified. Also, various types of distributed systems and the manner in which the control variables may be introduced into such systems are discussed. In general, due to certain simplifying assumptions made in deriving the mathematical model, the resulting equations may not have the properties of a dynamical system. The establishment of conditions for which the properties of a given partial differential equations are consistent with those of a dynamical system is generally a difficult task. These consistency conditions for linear partial differential equations are discussed in the light of certain known mathematical results in semi-group theory.

In Section 1, the equations of motion for an elastic body entering a planetary atmosphere at hypersonic velocities are derived. The results are applied to the derivation of a mathematical model for a simplified flexible aerodynamic re-entry vehicle. Hopefully, the results can provide some insight for making rational approximations in deriving useful mathematical models for more complex flexible aerodynamic re-entry vehicles.

### Optimum Control

One of the fundamental aspects in control theory involves the identification of those properties intrinsic to a dynamical system which are of importance to control. In "Control of Distributed Parameter Systems, Advances in Control Systems, Theory and Applications, Academic Press, 1964, pp. 75-172", the notions of controllability and observability are extended to distributed parameter dynamical systems. Necessary and sufficient conditions for the controllability and observability of linear distributed parameter dynamical systems are established.

Also, in "Control of Distributed Parameter Systems", various formulations of optimum control problems associated with distributed parameter systems are discussed. Certain functional equations associated with the optimum control of a certain class of distributed parameter systems in the form of a set of partial differential equations are derived via dynamic programming. The results are applied to the problem of optimum control of linear systems with a generalized quadratic performance index.

In Section 5, the problem of optimum control of a class of mixed distributed and lumped parameter systems are considered. The mathematical model for the system is in the form of a coupled system of equations consisting of a scalar linear partial differential equation and a first-order vector linear ordinary differential equation. An example of such a system is an aerodynamic vehicle with a flexible body. The rigid body mode of vehicle's motion can be described by a set of ordinary differential equations, whereas the deforming motion of the flexible body can be described by a partial differential equation. Classical variational techniques are used to derive the optimum control equations. Particular attention is focused on the case where the distributed and lumped parameter sub-systems are weakly coupled to each other. A sufficient condition for asymptotic stability of such a system with a near-optimum control law is derived. The meaning of some of the results is interpreted in the framework of two physical problems pertaining to the control of temperature in a solid and the pitching motion of a simplified flexible aerodynamic vehicle.

## Stability

In "Control of Distributed Parameter Systems, Advances in Control Systems, Theory and Applications, Academic Press, 1964, pp. 75-172", Lyapunov's definitions for the stability of equilibrium of finite-dimensional systems are extended to distributed parameter systems. General stability theorems due to Zubov and Massera are discussed and their applications are illustrated by specific examples.

The main effort in this area has been concentrated on the formulation of stability problems in elastic and aeroelastic systems in the framework of Lyapunov stability theory, and the exploration of the range of applicability of Lyapunov's direct method to this class of problems. Almost all the existing works on the stability analysis of aeroelastic systems have been based on the modal truncation approach. From the mathematical standpoint, except in certain special cases, such an approach generally provide neither necessary nor sufficient conditions for stability. On the other hand, Lyapunov's direct method provides at least sufficient conditions for stability. In Section 2, the physical meaning of stability of equilibrium in the sense of Lyapunov is discussed for elastic and aeroelastic systems. Next, the stability problem for a general elastic system is formulated; some applications of Lyapunov's direct method are demonstrated by examples which cannot be readily analyzed using conventional methods.

In Sections 3 and 4, the application of Lyapunov's direct method to stability analysis of aeroelastic systems is further explored. The particular systems considered consist of a simplified aerodynamic vehicle with flexible tail and concentrated aerodynamic loading, and a similar vehicle with a pitch autopilot. Sufficient conditions for asymptotic stability of equilibrium are obtained for both systems.

Finally, in IBM Research Note No. NJ-56, "On The Stability of Equilibrium of a Diffusion System with Feedback Control" a sufficient condition for the stability of equilibrium of a linear diffusion system with feedback control is derived using Nirenburg's maximum principle for parabolic equations.



## Approximation Problems

From the practical standpoint, the problems of approximation in the optimum control of distributed parameter systems are of primary importance, since approximation in one form or another is imperative in the implementation of any optimum control law.

In "Control of Distributed Parameter Systems, Advances in Control Systems, Theory and Applications, Academic Press, 1964, pp. 75-172", the approximation problems are discussed in general terms. Various approaches to the approximation problems are outlined. Also, a method of incorporating the stability requirement into the formulation of the approximation problem in optimum control is discussed.

Generally speaking, the problem of determining the relation between the discretization level and the errors induced by the approximation associated with partial differential equations is extremely difficult. In the present work, we have restricted ourselves to the class of distributed parameter systems whose time domain behavior is describable by a denumerably infinite system of linear ordinary differential equations. In Section 6, explicit estimates for the errors induced by truncating infinite systems of first and second order linear ordinary differential equations are derived. The truncation approach to approximation is widely used in the engineering literature - usually without justification. An improved error estimate for first order systems is given in Section 7. The sharpness of the estimates is checked by considering specific numerical examples. The results obtained here represent at least a preliminary step toward establishing the relation between discretization level and the error induced by the approximation, and they can be used to derive approximate mathematical models for certain class of distributed parameter systems.

## Background Material

During the period covered by the present contract, a series of lecture on the theory of partial differential equations was presented by W. E. Langlois. The lecture notes are published as IBM Reports (see Foreward). They are primarily for tutorial purposes.

Finally, an up-to-date annotated bibliography on the theory of stability and control of distributed parameter systems is included as Section 8 of the report. It includes only those works which deal directly with distributed parameter systems governed by integral or partial differential equations or by infinite systems of ordinary differential equations.