

Contrails

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**THE THEORETICAL BEHAVIOR OF KNITTED FABRIC
SUBJECTED TO BIAXIAL STRESSES**

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FOREWORD

This report was prepared by the Fibrous Materials Branch and was initiated under Project No. 7320, "Fibrous Materials for Decelerators and Structures," Task No. 73203 "Fibrous Structural Materials." The work was administered under the direction of the Nonmetallic Materials Laboratory, Materials Central, Directorate of Advanced Systems Technology, Wright Air Development Division, with Lt. Peter Popper acting as Project Engineer.

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ABSTRACT

In this report the theoretical mechanical behavior of a plain knitted fabric subjected to biaxial stresses is derived. An approximate mathematical model of the fabric structure has been established from which the stress vs. fabric-geometry and stress vs. strain relationships have been determined. The work was done by considering only the properties of the fabric structure, completely independent of the fiber properties. The results of this report may be used for such applications as predicting the performance of a plain knitted fabric in situations where it will be stressed biaxially.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:



C. A. WILLIS, Chief
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Contrails

INTRODUCTION

The stress strain properties of fabric structures are of great importance in the utilization of textiles as engineering materials. Perhaps the greatest single factor that determines these properties is the geometrical arrangement of the yarns in the structure. Fabrics can be made in a wide range of each of the following types of constructions: interlaced (woven or braided), interlooped (knitted), bonded, knotted, or intertwisted (as lace). In general, the nature of the individual fibers is only of secondary importance. For example, the differences between a woven fabric and a knitted fabric are more pronounced than the differences between a specific woven fabric made of two different fiber types.

Of course, the fiber properties are extremely important in establishing the chemical, thermal, ultimate strength, and other properties of a fabric, but the stress strain properties prior to rupture are dependent primarily on the geometry of the fabric.

In order to get an understanding of the mechanical properties of textiles it is of great importance to study individually the various fabric geometries, independent of fiber type, and determine the characteristics of each one. This is especially true at the present time when textiles are being used in more and more sophisticated applications such as pressure suits, g-seats, and inflatable antennas. In these uses, trial and error techniques are certainly inadequate for design purposes.

Of the basic properties of fabric structures, the relationships between biaxial stress and strain are of great importance. They will be determined in this report for a particular type of fabric, - the "plain knit" (commonly called knitted jersey). This fabric is not usually thought of as an engineering material, but because of its low cost and unusual behavior it may eventually find more widespread applications. Presently it is used in the reinforcement of garden hoses; and, the restraining layer of one type of full pressure suit uses a lace fabric that has an almost identical structure.

Previous studies done on the subject of fabric structures under stress include such work as Haas (1) on woven fabrics and Petterson's (2) on non-woven or bonded fabrics. The geometry of the plain knit was studied by several authors, most notably Munden (3), who includes a review of previous work in his paper. However, the previous investigations of knitted fabrics did not involve fabrics under stress, and therefore, their results are not directly comparable with those in this report.

ANALYSIS

A. Objective

There are two major objectives of this analysis:

1. To determine the relationships between stresses and fabric geometry for a knitted fabric under biaxial stress.
2. To determine the relationships between stresses and strains for this fabric. More completely stated, the objectives are as follows:

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a. Consider a plain knitted fabric that is placed in a situation where it will be subjected to biaxial stresses. (See Fig. 1) The fabric is oriented so that the stresses are parallel to the courses and wales. The fabric will then assume a certain geometry so that it can be in equilibrium with the imposed stresses, and this will depend on the magnitude of the stresses and on certain properties of the fabric. As the stresses are changed, the fabric may extend - by means of its loops slipping through one another - until it again comes into equilibrium. The object of this analysis is to determine the particular geometry that the fabric will assume for a given set of stresses and for a given set of initial conditions.

b. The second objective of this analysis is to determine the relationship between stresses and strains. This is done by considering the fabric to start at a particular state of biaxial stress and then relating stresses to changes in geometry instead of geometry itself. By expressing the changes of geometry as strains, the desired results are obtained.

In both parts of this analysis consideration is given to the fact that at some fabric extension the yarns will be jammed and the fabric structure cannot deform any further. (See Figs. 5 and 7). These "end points" will be established in terms of both the geometry and the strains. In an actual fabric, it is possible to extend the fabric beyond these calculated points because the yarns themselves may extend. However, that extension is a function of the yarn properties and not of the fabric structure and consequently will not be considered in this report.

B. Assumptions and Approximations

In order to accomplish the objectives, it is necessary to make several assumptions:

1. The fabric when under biaxial stress will always maintain a geometry similar to that shown on Fig. 2. This geometry consists of straight lines connected by almost circular arcs.
2. The yarns are frictionless, inextensible at the loads encountered, and flexible.
3. The yarns diameter is small compared to the length of yarn in each loop. (Less than 10%).

C. Fabric Geometry

To obtain any theoretical relations for the mechanical behavior of this fabric structure it is necessary to have as a means for numerically describing the geometry at all stress conditions. This can be done by selecting a mathematical model which closely represents the actual fabric. This mathematical model is shown on Fig. 2, and is very similar to actually stressed fabrics which were observed under a microscope.

Referring to Fig. 2, it is seen that the fabric geometry can be described by knowing the values of certain geometric parameters. Thus, if it is possible to determine the values of each of these parameters for a given set of stresses, the relation between stress and fabric geometry is known. This is done in the following sections by using a

somewhat different set of parameters. It is important to note that the geometry on Fig. 2 is assumed to be representative of the fabric at all conditions of deformation. Only the values of the geometric parameters will change.

D. Relations Between Geometric Parameters

The object of this section is to determine the relations between the geometric parameters, and also to define some new parameters. The results will be used in the following sections.

In order to obtain the relations desired, an element of structure has been isolated from the fabric. (See Fig. 3). The entire fabric is made up solely of these elements and their mirror images and therefore the overall behavior of the fabric will be directly related to the behavior of each element. It is assumed that each element behaves exactly as every other.

Referring to Fig. 3:

L = length of yarn in one element

l = length between two numbers

r = yarn radius

a, b, c, d, θ as defined in Fig. 3

$$L = l_{12} + l_{23} + l_{34} + l_{56} + l_{67} + l_{78}$$

$$l_{12} = a$$

$$l_{34} = l_{56} = \frac{c}{2 \sin \theta}$$

$$l_{78} = b$$

$$l_{23} = l_{67} \approx \frac{\pi r}{2}$$

$$L = a + b + \frac{c}{\sin \theta} + \pi r$$

$$L = d + \frac{c}{\sin \theta} + \pi r$$

Note: This is only approximately true, the error involved is a very small part of the total length "L". (1)

E. Forces

Before deriving the equations for stresses, it is necessary to obtain some information about the forces acting on each element.

Referring to Fig. 3, it can be seen that for zero friction:

$$T_A = T_B$$

$$T_C = T_D$$

From equilibrium requirements:

$$T_A = T_C$$

$$T_D = T_B$$

Thus, all the tensions are equal and will be designated by "T".

It should be noted that by taking moments about point 9 the same result could be obtained without the zero friction assumption. If friction was significant, the geometry would have to be different.

From Fig. 3 it can be seen that:

$$F_1 = T \sin \theta \quad (2)$$

$$F_2 = T \quad (3)$$

F. Relations Between Stresses and Geometry

Let:

S_1 = Fabric Stress parallel to wales (vertical stress)

S_2 = Fabric Stress parallel to courses (horizontal stress)

From the definition of fabric stress:

$$S_1 = \frac{F_1}{d}$$

$$S_2 = \frac{F_2}{c}$$

Substituting from (2) and (3),

$$S_1 = \frac{T \sin \theta}{d}$$

$$S_2 = \frac{T}{c}$$

and

$$\frac{S_1}{S_2} = \frac{c \sin \theta}{d} \quad (4)$$

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It is of importance to note that the magnitude of stress does not enter into the equations, only the stress ratio. The same thing was shown by Haas (1) on his study of the deformation of a woven structure made of inextensible, incompressible, flexible yarns.

Equation (4) gives the relationship between stresses and fabric geometry. The problem remaining is merely to put it in terms of more convenient variables.

Substituting for d from (1) into (4) gives

$$\frac{S_1}{S_2} = \frac{c \sin \theta}{L - \pi r - c/\sin \theta}$$

Since L and r are constants for a given fabric, it is convenient to normalize in terms of them. This will be done by using "primes" on the variables, as in

$$d' = \frac{d}{L - \pi r} \quad (5)$$

$$c' = \frac{c}{L - \pi r} \quad (6)$$

$$r' = \frac{r}{L - \pi r} \quad (7)$$

Thus, the following relation between stresses and geometry is obtained.

It is plotted on Fig. (4). The approximation of setting $\sin \theta = 1$ has been shown to have virtually no effect on the results of this equation.

$$\frac{S_1}{S_2} = \frac{c' \sin \theta}{1 - c'/\sin \theta} \approx \frac{c'}{1 - c'} \quad (8)$$

Equation (8) is one of the results which was to be determined since from it, the stress ratio for a given c' can be calculated.

The corresponding values for d' can be found from triangle 5, 10, 9, (Fig. 3)

$$\cos \theta = \frac{2r}{c} \quad (9)$$

or, since

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin \theta = \sqrt{1 - \left(\frac{2r}{c}\right)^2}$$

Substituting into eq. (1) yields

$$d = L - \pi r - \frac{c}{\sqrt{1 - \left(\frac{2r}{c}\right)^2}}$$

$$d' = l - \frac{c'}{\sqrt{1 - \left(\frac{2r'}{c'}\right)^2}} \quad (10)$$

Equation (10) giving the relation between c' and d' is plotted on Fig. (4) for several values of r' .

G. Limiting Geometries due to Yarn Jamming

Referring to Figs. (5 and 7), it can be seen that there are limits to the extensions and contractions that the fabric can undergo because of yarns jamming against one another. The object of this section is to determine when this jamming occurs in terms of the geometric parameters used in the previous sections. The results will be fairly rough approximations.

Fig. (5) shows the approximate geometry of a fabric that has extended horizontally until the yarns are jammed. The same element of structure that has been used is isolated on Fig. (6) and the relation between vertical spacing and radius is as follows:

$$\left(\frac{c}{2}\right)_{\min} = 2r$$

$$c_{\min} = 4r$$

$$c'_{\min} = 4r' \quad (11)$$

Fig. 7 shows the approximate geometry of a fabric that has extended vertically until the yarns are jammed. The element of structure appears on Fig. (8). In order to simplify the mathematics, it appears reasonable at this point to assume that $\sin \theta = 1$. This will give a slightly larger value of c_{\max} ; and since some additional extension will undoubtedly take place due to yarn extension or compaction, the error will be in the right direction. The following equations are derived from Fig. 8 and eq. (1) for the value of maximum "c".

$$d \text{ (at } c_{\max} \text{)} = \frac{4r}{\sin \theta} \approx 4r$$

$$L = d + \frac{c}{\sin \theta} + \pi r \approx d + c + \pi r$$

$$c_{\max} = L - \pi r - 4r$$

$$c'_{\max} = l - 4r' \quad (12)$$

These limiting values are indicated on Fig. (4) for different values of r' . The corresponding limiting values of d' can also be found from Fig. (4) using the relationship between c' and d' .

H. Relations between Stresses and Strains

The relations between stress and strain can be obtained from the results of the first section which relates stress and fabric geometry. This is done by considering the fabric to start at some arbitrary geometry designated by "o" subscripts. Any other geometry that the fabric takes on will mean that a certain percent elongation or contraction from the original has occurred. The new geometry will require a different set of stresses to hold it in equilibrium. The equations in this section will relate these stresses to the changes from an initial setting, rather than to the geometry itself which was previously done.

The strains are denoted as follows:

e_1 = vertical strain (in direction of wales)

e_2 = horizontal strain (in direction of courses)

c_o, d_o, c'_o, d'_o initial values of geometric parameters

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}}$$

$$e_1 = \frac{c - c_o}{c_o}$$

$$e_1 = \frac{c}{c_o} - 1$$

$$e_2 = \frac{d}{d_o} - 1$$

or,

$$c = c_o (1 + e_1)$$

$$d = d_o (1 + e_2)$$

$$c' = c'_o (1 + e_1) \tag{13}$$

$$d' = d'_o (1 + e_2) \tag{14}$$

From eq. (8),

$$\frac{S_1}{S_2} = \frac{c'}{1 - c'}$$

and,

$$\frac{S_1}{S_2} = \frac{c'_o (1 + e_1)}{1 - c'_o (1 + e_1)} \tag{15}$$

Equation (15) gives the desired relation between stresses and vertical strain. The relation involving horizontal strains is derived by assuming the relation between d' and c' to be:

$$d' = 1 - c' \tag{16}$$

This is a reasonably good approximation as can be seen on Fig. 4. Substituting eq. (16), in (14), and (13), gives:

$$\begin{aligned} 1 - c' &= d'_0 (1 + e_2) \\ c'_0 (1 + e_1) &= 1 - d'_0 (1 + e_2) \end{aligned} \tag{17}$$

$$\frac{S_1}{S_2} = \frac{1 - d'_0 (1 + e_2)}{1 - [1 - d'_0 (1 + e_2)]}$$

$$\frac{S_2}{S_1} = \frac{d'_0 (1 + e_2)}{1 - d'_0 (1 + e_2)} \tag{18}$$

Equation (18) gives the relation between the stresses and horizontal strain.

Both eq. (15) and (18) are plotted on Fig. 9 for various values of d'_0 and c'_0 .

I. Relations Between Strains

When the fabric is extended in one direction it will contract in the other. The relation between these strains can be determined by rewriting eq. (17) and substituting eq. (16).

$$\begin{aligned} 1 - c'_0 (1 + e_1) &= d'_0 (1 + e_2) \\ 1 - c'_0 - c'_0 e_1 &= d'_0 - d'_0 e_2 \\ c'_0 e_1 &= 1 - c'_0 - d'_0 - d'_0 e_2 \\ 1 - c'_0 &= d'_0 \\ e_1 &= - \frac{d'_0}{c'_0} e_2 \end{aligned} \tag{19}$$

This fraction d'_0/c'_0 can be considered the Poisson's ratio of the fabric, since it is the ratio of extension in one direction to contraction in the other. It is dependent on the initial geometry and may be numerically greater than one.

J. Limiting Values of Strain

Referring to eqs. (11) and (12) it is seen that:

$$\begin{aligned} c'_{\min} &= 4r' \\ c'_{\max} &= 1 - 4r' \end{aligned}$$

If:

$$d' = 1 - c'$$

$$d'_{\min} = 4r'$$

$$d'_{\max} = 1 - 4r'$$

(Noting that d'_{\min} occurs at c'_{\max})

Substituting into eqs. (13) and (14)

$$e_{1\min} = \frac{4r'}{c'_0} - 1 \quad (20)$$

$$e_{1\max} = \frac{1 - 4r'}{c'_0} - 1 \quad (21)$$

$$e_{2\min} = \frac{4r'}{d'_0} - 1 \quad (22)$$

$$e_{2\max} = \frac{1 - 4r'}{d'_0} - 1 \quad (23)$$

SUMMARY OF RESULTS AND SAMPLE SOLUTIONS

A. Relationship between Stresses and Fabric Geometry

The knitted fabric under consideration is assumed to have a characteristic geometry when under stress. (See Figs. 2 and 3). In addition to the geometric parameters which are defined on these diagrams, the following notations will be used.

L = length of yarn in one element (Fig. 3) (Same as half the "stitch length").

S_1 = stress (lbs/in-width) applied vertically (walewise).

S_2 = stress (lbs/in-width) applied horizontally (coursewise).

$$d' = \frac{d}{L - \pi r}$$

$$c' = \frac{c}{L - \pi r}$$

$$r' = \frac{r}{L - \pi r}$$

By considering the requirements of equilibrium, and assuming the yarns to be flexible, inextensible, incompressible, frictionless, and fine compared to the spaces between them, the following relations have been established:

$$\frac{S_1}{S_2} = \frac{c \sin \theta}{d} \quad (4)$$

$$d = L - \pi r - \frac{c}{\sin \theta} \quad (1)$$

$$\cos \theta = \frac{2r}{c} \quad (9)$$

For ease of calculation in most problems, and for simple graphical representation, these equations may be rewritten as: (See Fig. 4).

$$\frac{S_1}{S_2} = \frac{c'}{1 - c'} \quad (8)$$

$$d' = 1 - \frac{c}{\sqrt{1 - \left(\frac{2r'}{c'}\right)^2}} \quad (10)$$

The limiting geometries at which the yarns will jam against one another can be determined from:

$$4r' \leq c' \leq 1 - 4r'^2 \quad (11)$$

$$(12)$$

Sample Solution #1.

Perhaps the best way to explain the graph on Fig. 4 is to demonstrate a sample solution. Consider a fabric placed into a situation where it is stressed 10 lbs/in vertically (parallel to the wales) and 5 lbs/in horizontally. The length L (actually half of the commonly used "stitch length") is 0.1 inch and the yarn radius is 0.005 inches. The vertical and horizontal spacing of the fabric elements c and d are to be determined. The following calculations can be made:

Given: $S_1 = 10$ lb/in.

$S_2 = 5$ lb/in.

$L = 0.1$ in.

$r = 0.005$ in.

Find: c,d

Calculations:

$$\frac{S_1}{S_2} = \frac{10}{5} = 2.0$$

$$r' = \frac{r}{L - \pi r}$$

$$r' = \frac{.005}{.1 - (\pi)(.005)}$$

$$r' = .059$$

From Fig. (4)

$$\text{at } \frac{S_1}{S_2} = 2.0, \quad c' = .665$$

$$\text{at } r' = .059 \\ c' = .665, \quad d' = .31$$

$$d = d'(L - \pi r) \\ = (.31)(.1 - .005\pi) \\ = \underline{\underline{.026 \text{ in}}}$$

$$c = c'(L - \pi r) \\ = (.665)(.1 - .005\pi) \\ = \underline{\underline{.0561 \text{ in}}}$$

These values of c and d are the solution to this problem. If the total width and length of a fabric subjected to these conditions was to be found, it would be a simple matter of multiplying by the total number of elements in each direction. Note, however, that there are two elements (as shown on Fig. 2) in each full loop or wale, and therefore the number of elements horizontally is twice the number of wales. Also, the number of elements per inch is twice the number of wales per inch.

Sample Solution #2.

Using the numbers of the above problem, suppose it were required to determine how far this fabric could be extended horizontally, and also, how much additional horizontal stress would have to be applied.

Given: At Original Condition

$$S_{O_1} = 10 \text{ lb/in.}$$

$$S_{O_2} = 5 \text{ lb/in.}$$

$$L = 0.1 \text{ in.}$$

$$r = 0.005 \text{ in.}$$

Find: d_{\max}' , S_{x_2} (Stress which must be added to S_{o_2} to achieve d_{\max}')

Solution:

a) As in solution no. 1, $r' = .059$

b) From figure 4

$$\text{at } r' = .05, d_{\max}' = .77$$

$$\text{at } r' = .10, d_{\max}' = .54$$

Interpolating gives the approximate result of:

$$\text{at } r' = .059, d_{\max}' = .73$$

$$\begin{aligned} \text{c) } d_{\max} &= d_{\max}' (L - \pi r) \\ &= (.73)(.1 - .005\pi) = \underline{\underline{.060 \text{ in}}} \end{aligned}$$

d) at $d' = .73$, $c' = .24$

$$\frac{S_1}{S_2} = 0.3$$

$$S_1 = S_{o_1}$$

$$S_2 = S_{o_2} + S_{x_2}$$

$$\frac{S_{o_1}}{S_{o_2} + S_{x_2}} = .3$$

$$S_{x_2} = \frac{10}{.3} - 5 = \underline{\underline{28 \text{ lb/in.}}}$$

B. Relationship Between Stresses and Strains

By taking the results of the previous section, it is possible to determine the relations between stresses and strains. Since strains are defined as a percent increase of length, it is necessary to have some means for describing the initial lengths of the elements in the fabric. This has been done by using the same notation and adding the subscript "o".

Mathematically,

$$e_1 = \text{vertical strain}$$

$$e_2 = \text{horizontal strain}$$

$$e_1 = \frac{c}{c_o} - 1$$

$$e_2 = \frac{d}{d_o} - 1$$

The results of the stress strain analysis are as follows:

Relation between stresses and vertical strain

$$\frac{S_1}{S_2} = \frac{c'_0 (1 + e_1)}{1 - c'_0 (1 + e_1)} \quad (15)$$

Relation between stresses and horizontal strain

$$\frac{S_2}{S_1} = \frac{d'_0 (1 + e_2)}{1 - d'_0 (1 + e_2)} \quad (18)$$

Relation between strains

$$e_1 = - \frac{d'_0}{c'_0} e_2 \quad (19)$$

Limiting Values of Strain

$$\frac{4r'}{c'_0} - 1 \leq e_1 \leq \frac{1 - 4r'}{c'_0} - 1 \quad (20)$$

$$\frac{4r'}{d'_0} - 1 \leq e_2 \leq \frac{1 - 4r'}{d'_0} - 1 \quad (22)$$

$$(23)$$

Sample Solution #3.

A fabric is available with a yarn radius of 0.1 inch and an "L" of 2.0 inches. The fabric is initially subjected to vertical stress of 15 lbs/in. and a horizontal stress of 30 lb/in. (which will remain constant). Additional vertical stress is added to the initial stress until the fabric stretches 25%. Calculate the additional stress required to do this, and check to see whether the fabric can stretch 25% without jamming. (A problem of this type comes up in predicting the behavior of pressurized joints for full pressure suits.) (See Ref. 4).

Given: $r = 0.1$ in.

$L = 2.0$ in.

$S_{O1} = 15$ lb/in. (Original Stress)

$S_{O2} = 30$ lb/in. (Original Stress)

$e_1 = 0.25$

Find: S_{x1} (Vertical stress which must be added to the original to produce a 25% strain).

Calculations:

$$a) \quad r' = \frac{r}{L - \pi r}$$

$$r' = \frac{.1}{2 - (.1)\pi} = \underline{0.060}$$

$$b) \quad \frac{S_{01}}{S_{02}} = \frac{15}{30} = \underline{0.5}$$

$$c) \quad c'_0 = .33 \quad (\text{From figure 4})$$

$$d) \quad \frac{S_1}{S_2} = \frac{c'_0 (1 + e_1)}{1 - c'_0 (1 + e_1)} \quad (\text{Equation 15})$$

$$S_1 = \frac{(30)(.33)(1.25)}{1 - (.33)(1.25)}$$

$$S = 21.1 \text{ lb/in} \quad (\text{Total Vertical Stress})$$

$$e) \quad S_{x_1} = 21.1 - 15$$

$$S_{x_1} = \underline{\underline{6.1 \text{ lb/in}}}$$

$$f) \quad e_{1\text{max}} = \frac{1 - 4r'}{c'_0} - 1 \quad (\text{From equation 21})$$

$$= \frac{1 - (4)(.06)}{(.33)} - 1$$

$$e_{1\text{max}} = \underline{\underline{130\%}}$$

This maximum allowable strain is well in excess of the required 25% and therefore the yarns will not jam.

PLANS FOR FUTURE WORK

The work presented in this report is based on several assumptions and approximations, some of which may not be valid in all cases. In particular, the assumption that the fabric will maintain a particular type of geometry and the assumption that friction is negligible. It is therefore quite important to verify or modify the theoretical results which have been found by experimental evidence. This is the first step which will be taken in the future.

Other work which will be done on this topic includes:

1. Determination of the effects of shear stresses on the fabrics. This will permit an evaluation of the fabric under biaxial stresses of any orientation.
2. Presentation of the results in simplest form, - possibly in a nomograph.
3. Application of the results to specific engineering design problems such as the full pressure suit and fabric-reinforced materials.

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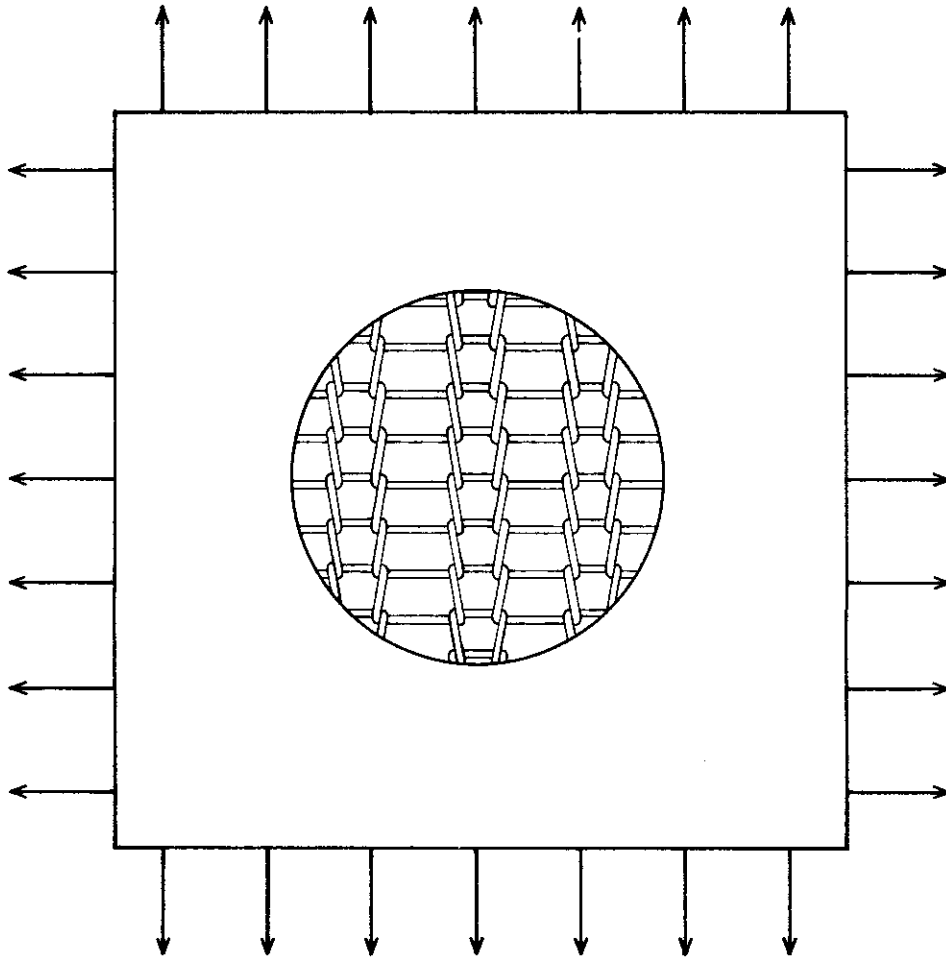


Figure 1. Knitted Structure Under Biaxial Stress

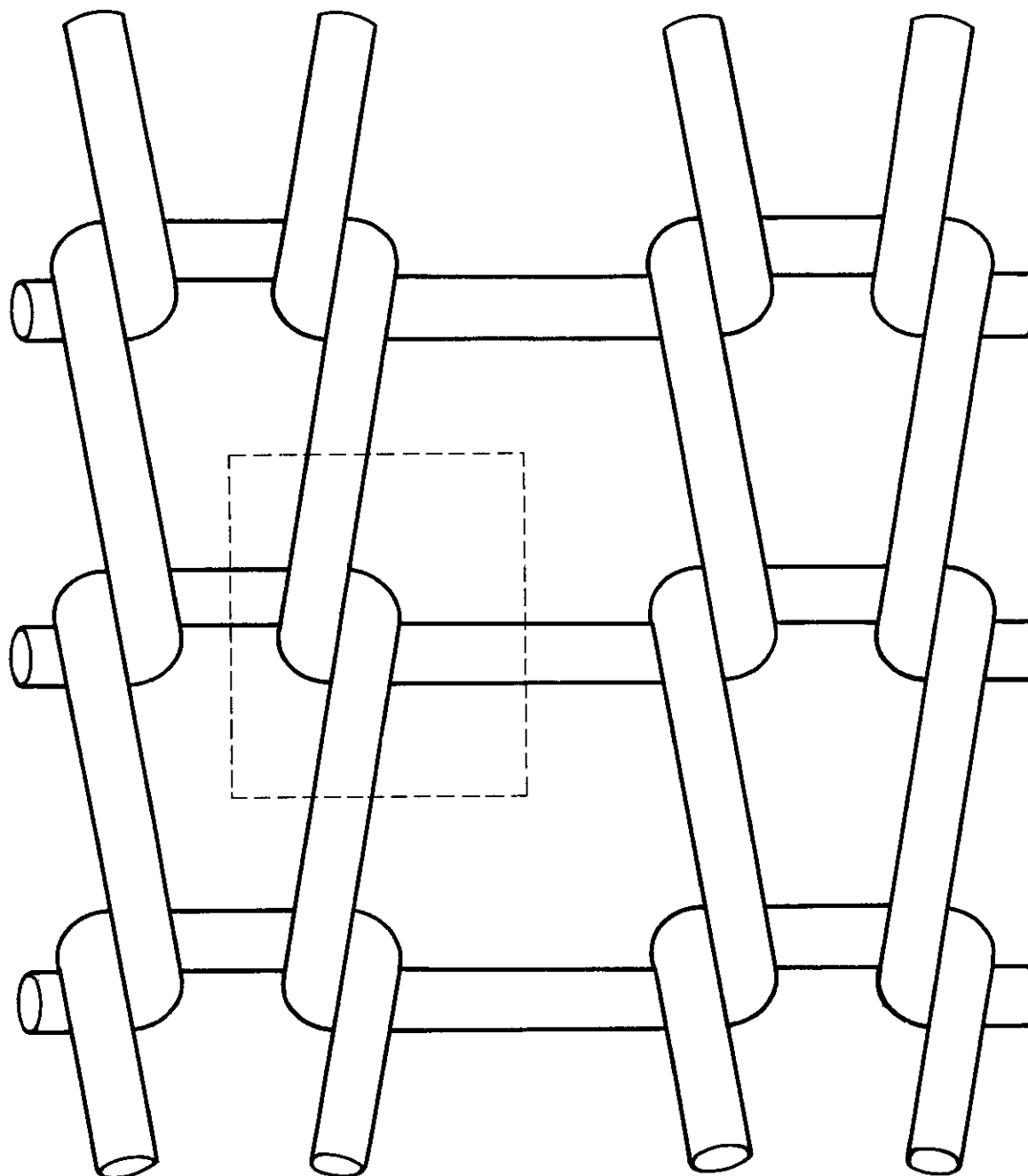


Figure 2. Mathematical Model of Knitted Structure Under Stress

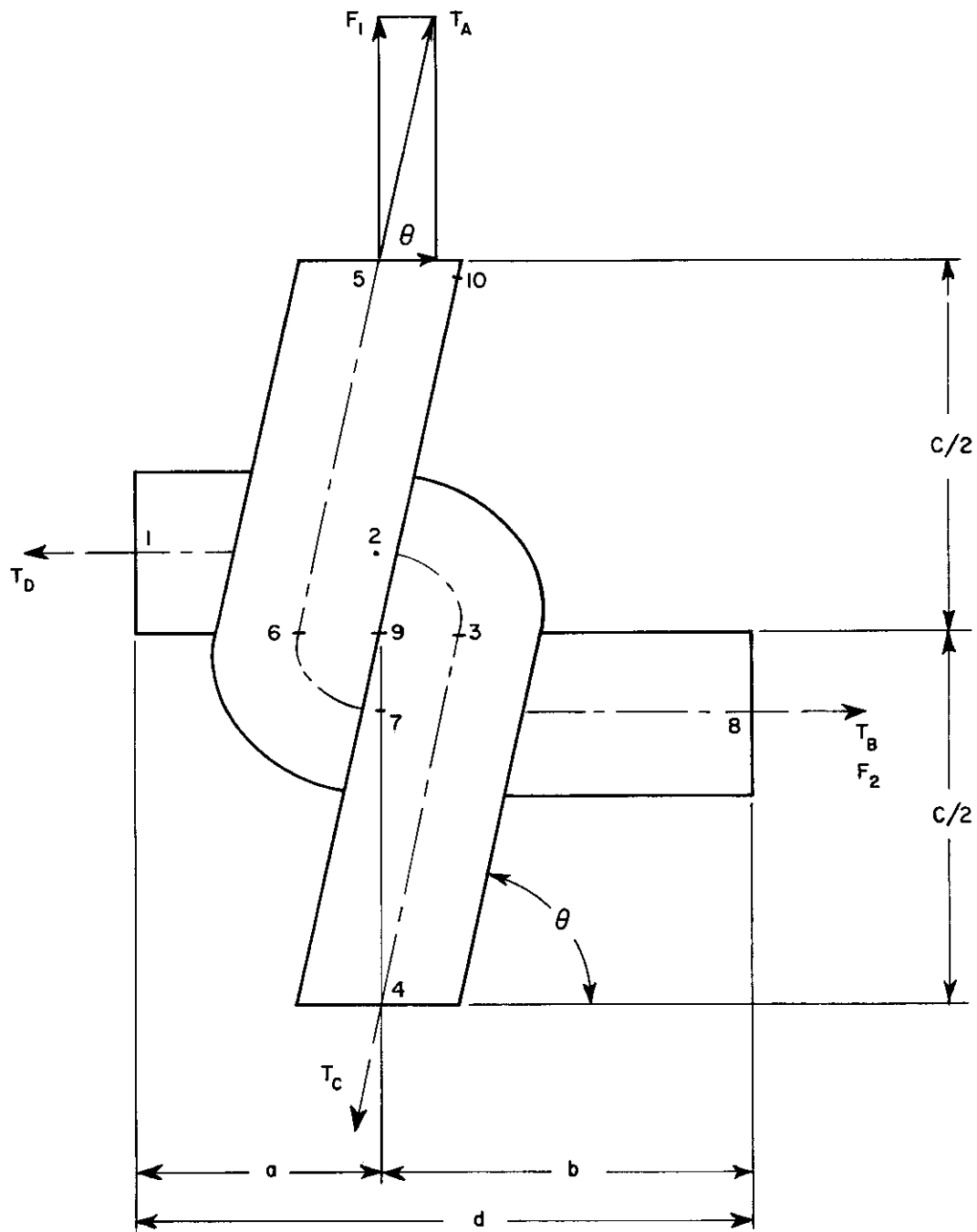


Figure 3. Basic Element of Structure

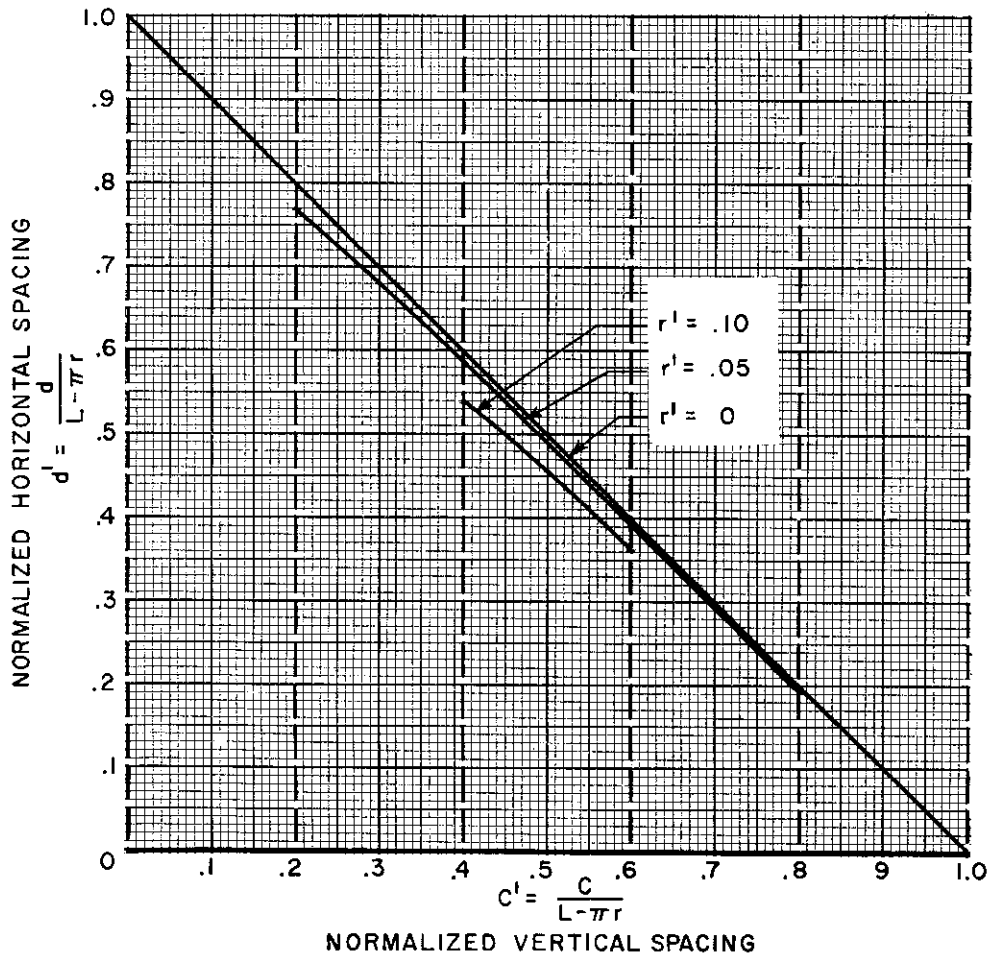
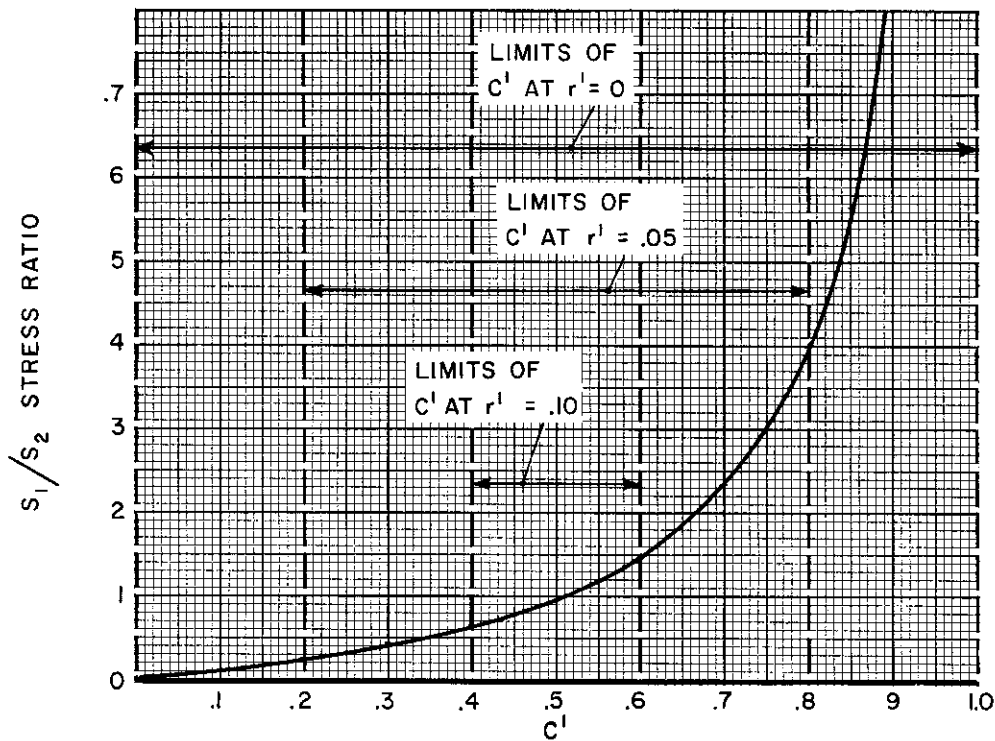


Figure 4. Relations Between Stress Ratio and Geometric Parameters

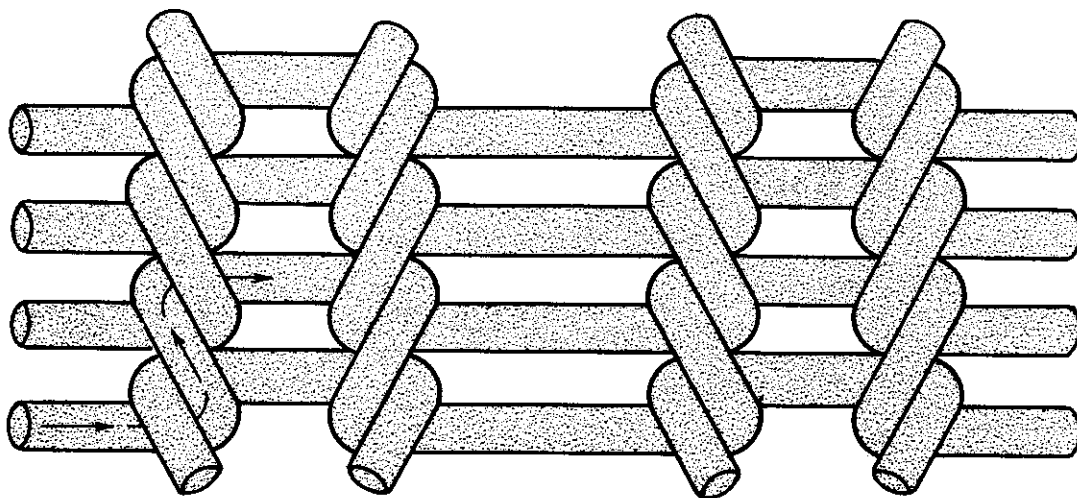


Figure 5. Fabric Under Maximum Horizontal Extension

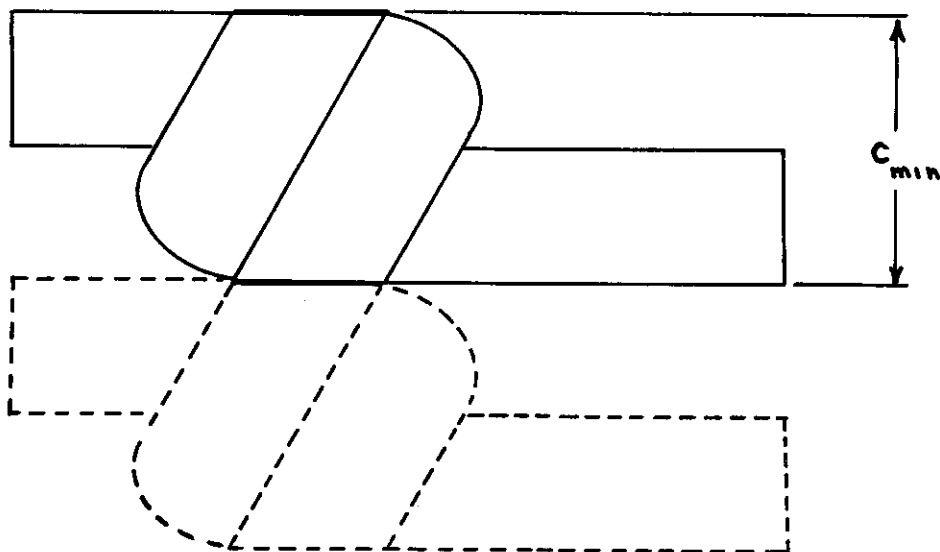


Figure 6. Basic Element Under Maximum Horizontal Extension

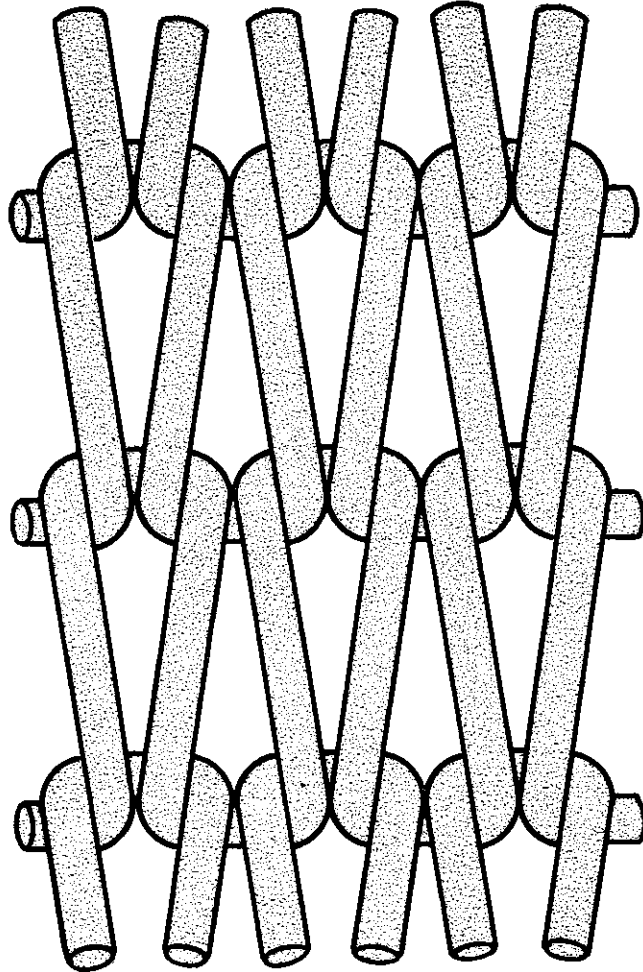


Figure 7. Fabric Under Maximum Vertical Extension

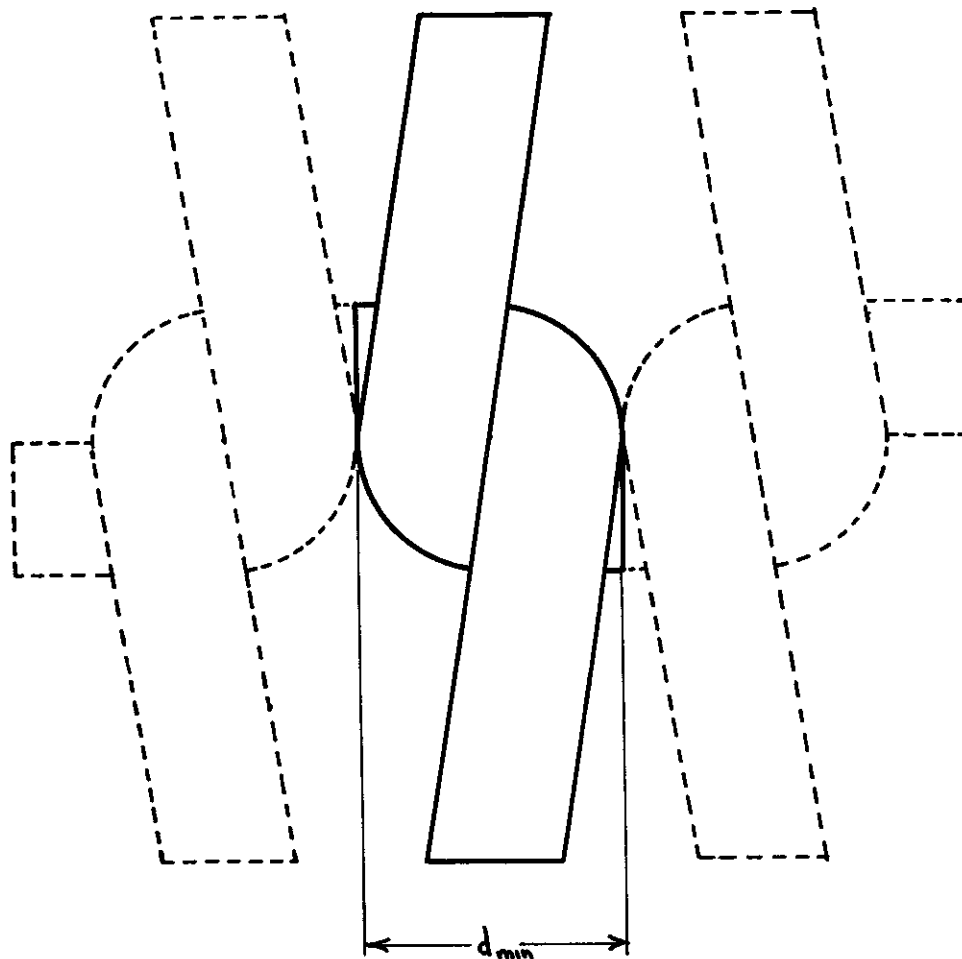


Figure 8. Basic Element Under Maximum Vertical Extension

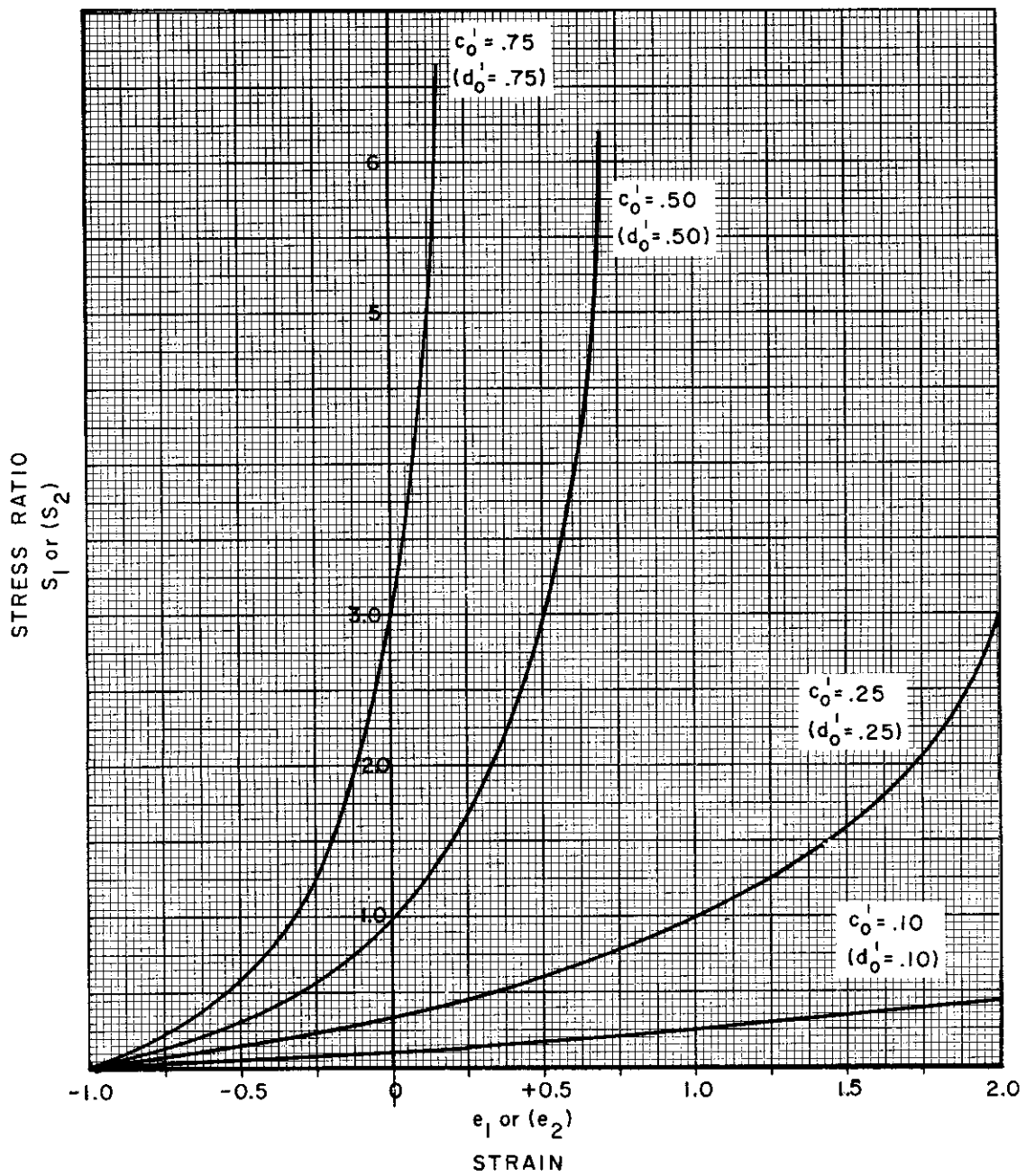


Figure 9. Stress Ratio vs Strain