ON AN APPLICATION OF COMPLEX DAMPING COEFFICIENTS

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ABSTRACT

Complex damping theory is a useful tool in analysis of energy transformations among modes of a vibration system. Based on this theory, there are many applications and improvements in the areas of system identification, vibration control and damper optimization design.

This paper presents an application of the theory in regard to finite element model corrections. First, a common shortfall of usual correction procedures is analyzed. In order to deal with this problem, a correct correspondence rule is then proposed. With the help of complex damping coefficients, improvements to certain correction procedures are discussed.

INTRODUCTION

The dynamic performance of a structure may be characterized by different mathematical models. Among them, the modal model and the physical model are most frequently seen. A modal model which consists of a set of modal parameters is often used in harmonic-related vibration control, in dynamic behavior analysis, and in physical model modification. Since a modal model is essentially a dynamic representation of the physical model in the modal domain. The transformation of physical coordinates into modal coordinates is always accompanied by a certain loss of information, a modal model is generally considered to be a weak model but relatively easy to obtain.

A physical model consists of three coefficient matrices: The mass, damping and stiffness matrices. If such a model is known, all the modal parameters can be calculated. Conversely, a physical model can not be determined in general from a modal model. In this regard, a physical model is considered superior than a modal model.

In engineering practice, a physical model is not always available because not all the coefficient matrices can be directly measured. The measurable quantities are often the various dynamic responses and modal parameters of the structural system. Based on these data, we can typically generate an approximate model - an analytical model, using the finite element method (FEM). In most cases, the analytical model is inaccurate and requires various adjustments or corrections. In the past decades, many attempts have been made to develop better algorithms to modify the FEM models. At present, the need to develop appropriate algorithms continues to exist.

From the analytical model to the physical model, an important step is to perform model corrections. In a general model correction procedure, the goal is to obtain a set of coefficient matrices, mass **M**, damping **C** and stiffness **K**. What we have at the beginning is the analytical model data $\mathbf{M}^{(a)}$, $\mathbf{C}^{(a)}$ and $\mathbf{K}^{(a)}$, along with some dynamic parameters of the physical model, such as measured response $X^{(m)}$ and/or modal parameters: Undamped natural frequencies $\Omega^{(m)}$, damping ratios $\Xi^{(m)}$ and mode shapes $\mathbf{p}^{(m)}$. In

each step of the corrections, we obtain certain corresponding matrices $\mathbf{M}^{(i)}$, $\mathbf{C}^{(i)}$ and $\mathbf{K}^{(i)}$ as approximations to the real **M**, **C** and **K**. Then we typically compare the measured response $X^{(m)}$ and/or modal parameters $\Omega^{(m)}$, $\Xi^{(m)}$ and $P^{(m)}$, with the calculated response $X^{(i)}$, and/or modal parameters $\Omega^{(i)}$, $\Xi^{(i)}$ and $P^{(i)}$ from the revised analytical model. If the discrepancy between the two sets of data is less than a certain preset level, then the revised analytical model is accepted as the physical model. Otherwise, the correction procedure is continued.

In such a correction procedure described above, a number of factors can influence the final result. There are many existing algorithms that do not converge in general. For those that converge may have problems in targeting the correct **M**, **C** and **K** because the comparison criterion used is not sufficiently comprehensive.

In this paper, we propose an alternative judgment on the effectiveness of model corrections. Our discussions will be restricted to finite element models and their corrections under the assumption that the models are linear, time-invariant and have lumped-masses.

RESPONSE-FITTING

One of the simplest model correction methods is the time domain *response-fitting*. In order to carry out this method, a time history (or transfer function) of the testing structure must first be recorded. The time history can be a free decay response with an initial input such as sine-burst, white noise-burst, impulse, etc. Or it can be a forced response under an excitation such as sinusoidal, sine-sweep, sine-dwell, pseudo white-noise or simulated seismic ground motion. In a carefully conducted experiment, the measured response is considered "noise-free". Thus it is ready to be used as the correction reference. Once the reference is available, corresponding samples are collected from a calculated response of the analytical model with same initial phases and time intervals. Then by using certain mathematical techniques such as the least-square method or the maximum-likelihood method, a cost function is generated to measure the discrepancy between the two responses. Equation (1) gives a least-

square cost function (Natke, 1988).

$$J = \sum_{i=1}^{p} \sum_{j=1}^{n} \alpha_{ij} \left(x_{ij}^{(m)} - x_{ij}^{(a)} \right)^2 \qquad (1)$$

where, $x_{ij}^{(m)}$ and $x_{ij}^{(a)}$ are the i^{th} samples of measured and calculated responses from the j^{th} node respectively. The total number of samples from each source is p and the number of nodes is n. α_{ij} are weighted coefficients. In general, at certain nodes, samples are measured more accurately than at other places, the weighted coefficients of these samples will then be assigned with greater values. Conversely, the tail of a free decay response is thought to have poor signal-to-noise ratio, the weighted coefficients of samples from this portion will be assigned with smaller values.

The value of J indicates whether the analytical model is close to the physical model. When the response of the analytical model does not fit the response of the structural system, J will assume a large value. Corrections to the analytical model will then be made to reduce the value of J.

Due to several reasons, response-fitting is often considered unsatisfactory in terms of its model correction effect. First, when a given excitation with a nearly straight spectrum, the structural response should theoretically incorporate the influences of all modes of the structural system. However, lower modes are usually associated with large percentage of the total energy involved, these modes have dominate influence to the response. In fact, most engineering applications only require to consider the first mode. Therefore, information from the higher modes may be lost in the response.

Secondly, despite the measured response being assumed noise-free, noise can not be completely eliminated. The commonly used noise-reduction techniques in response-fitting are essentially pre-treatments such as averaging the noise in the frequency domain. Since the participating factors of higher modes are relatively small, these modes give poor signal-to-noise ratios. The existence of these modes can hardly be identified in a response function. Consequently, the *order* of the

reference can not be determined by the response-fitting method unless more sophisticated time domain modal analysis is used. It is conceivable that without prior knowledge of the reference order, response-fitting is more likely to accept a degenerate model with only the first few modes of the real structural system.

Thirdly, the higher modes decay faster than the lower modes in a free decay response. This may also induce the problem of losing information from the higher modes in a recorded response.

EXAMPLE 1: Figure 1 (a) shows a five story structure. This model steel frame is considered to have at least fifteen degrees-of-freedom. A free decay time history measured at a point on the third floor of the frame is shown in Figure 1 (b). Two other responses are also given here. They are calculated responses from two analytical models one with 2 DOF the other with 3 DOF. Although the 15 DOF structure should not be treated as a 2 or 3 DOF system, by using response-fitting, we could have accepted the 2 or 3 DOF analytical model as the real physical model.

FREQUENCY-FITTING

Frequency-fitting is another commonly applicable method for model corrections. The reference in this method is the measured natural frequencies which are either obtained directly from vibration test or extracted through modal analysis. The two ways give damped and undamped natural frequencies respectively. The number of the natural frequencies collected in the reference corresponds to the order of the structural system. So there has no problem in determining the number of modes in the system. The cost function is given by

$$J = \sum_{i=1}^{n} \alpha_i \, (\omega_i^{(m)} - \omega_i^{(a)})^2 \qquad (2)$$

where $\omega_i^{(m)}$ and $\omega_i^{(a)}$ are the measured and calculated undamped natural frequencies of the i^{th} mode respectively. α_i are the corresponding weighted coefficients. Since this method utilizes information from all relevant natural frequencies, which have been accurately measured, it has

better overall performance over the response-fitting method. Especially, frequency-fitting is suitable for correcting models with many higher modes

Equation (2) can be further modified by including both the natural frequencies and the damping ratios in J.

$$J = \sum_{i=1}^{n} \alpha_i(\omega_i^{(m)} - \omega_i^{(a)})^2 + \beta_i(\xi_i^{(m)} - \xi_i^{(a)})^2 \qquad (3)$$

where $\xi_i^{(.)}$ denotes the i^{th} damping ratio of (m) or (a). This improved Equation (3) is suitable for correcting models demanding high accuracy for both natural frequencies and damping ratios.

A useful variation of the above method is the less accurate FRF curve-fitting technique. Similar to Equation (1), the cost function in this case is

$$J = \sum_{i=1}^{p} \alpha_i (f_i^{(m)} - f_i^{(a)})^2 \qquad (4)$$

where $f_i^{(.)}$ is the i^{th} sample taken from the reference FRF (m) or the analytical FRF (a). p is the total number of samples. Selection of weighted coefficients is empirical. If the FRF samples are collected from a forced response with a feedback controlled excitation whose input spectrum has been kept a straight line, then α_i are the same for all i = 1, ..., n.

In using Equation (2), a correspondence between the referential and the analytical frequencies must be established first. One such correspondence is described below.

Consider the two sets of natural frequencies

$$\{ \omega_i^{(m)} \mid i = 1, \ldots, n \} \quad \text{and} \quad \{ \omega_j^{(a)} \mid j = 1, \ldots, n \}.$$

First, arrange them by a linear ordering

$$\begin{aligned} &\omega_{i_1}^{(m)} \le \omega_{i_2}^{(m)} \le \omega_{i_3}^{(m)} \le \ldots. \le \omega_{i_n}^{(m)} \\ &\omega_{j_1}^{(a)} \le \omega_{j_2}^{(a)} \le \omega_{j_3}^{(a)} \le \ldots. \le \omega_{j_n}^{(a)} \end{aligned} \qquad (5)$$

where the subscripts are some permutations of 1, 2, ..., n. Then the frequencies are paired according to the ordering. With this one-one

correspondence, Equation (2) can be restated as

$$J = \alpha_1 (\omega_{i_1}^{(m)} - \omega_{j_1}^{(a)})^2 + \alpha_2 (\omega_{i_2}^{(m)} - \omega_{j_2}^{(a)})^2 + \ldots + \alpha_n (\omega_{i_n}^{(m)} - \omega_{j_n}^{(a)})^2 .$$

Such a correspondence has the nice mathematical property that it gives the cost function J the smallest value when the weighted coefficients in Equation (2) are the same.

There are some problems with response-fitting method too. This can be seen from the following example.

EXAMPLE 2: A 4 DOF structure is shown in Figure 2 (a). Its physical model and an analytical model have the generalized damping and stiffness matrices as given in Table 1 (a), (b) respectively. Figure 2 (b) gives two FRF's, in which the dotted curve is from the physical model and the dashed curve is from the analytical model.

Table 1 (a) Generalized Damping Matrices $M^{-1}C$

Physical				Analytical			
27.3598	-19.2436	-22.8993	24.2555	6.3384	-1.3278	-1.2976	0.6308
	32.2713	1.7445	-23.1446		3.1574	-0.7730	-1.1894
		43.2177	-32.4571			3.1408	-1.6335
			49.1511				2.5634

Table 1 (b) Generalized Stiffness Matrices $M^{-1}K$ 1,000 ×

Physical				Analytical			
8.3284	-2.7139	-2.6032	2.3111	8.3130	-2.7085	-2.5973	2.3056
	1.8560	0.1289	-0.8413		1.8549	0.1266	-0.8398
		2.3377	-1.5225			2.3361	-1.5209
			1.4619				1.4607

It is clear that the generalized damping matrices of these two models are quite different. In fact, the physical model is non-proportionally damped whereas the analytical model is proportionally damped. (most finite element algorithms only generate proportionally damped models). Consequently, the mode shapes of these two models are different. The first

model has complex-valued mode shapes but the second has only real-valued ones. Such differences can not be detected in a single pair of frequency response functions. Therefore, curve-fitting FRF or frequency-fitting is inadequate for correcting errors in such category. This is seen in Figure 2 (b), where, in spite of the aforementioned differences between the two models, the two FRF still appear to be close.

COMPLETE MODAL-FITTING

A more sophisticated fitting method is developed by include the mode shape influences into the cost function, namely

$$J = \sum_{i=1}^{n} \alpha_i (\omega_i^{(m)} - \omega_i^{(a)})^2 + \beta_i (\xi_i^{(m)} - \xi_i^{(a)})^2$$

$$+ (p_i^{(m)} - p_i^{(a)})^H \Gamma_i (p_i^{(m)} - p_i^{(a)}) \qquad (6)$$

where $p_i^{(.)}$ is the i^{th} mode shape, and Γ_i is a diagonal matrix which consists of weighted coefficients. A simplified version of Equation (6) is

$$J = \sum_{i=1}^{n} \alpha_i (\omega_i^{(m)} - \omega_i^{(a)})^2$$

$$+ (p_i^{(m)} - p_i^{(a)})^H \Gamma_i (p_i^{(m)} - p_i^{(a)}) . \qquad (7)$$

Since the complete set of modal parameters is employed in Equation (6), it is called the *complete modal-fitting*. However, complete modal-fitting does not always give a satisfactory correction to an analytical model. One problem is related to the mode shapes. For example, the error in a measured mode shape could reach as high as 500%. (Liang & Inman 1988). Under this circumstance, the weighted coefficients Γ_i must be assigned with very small values. Therefore, the modification effect from mode shapes is limited.

WEAKNESS OF AVAILABLE CORRECTION METHODS

In the preceding sections, we briefly reviewed some commonly used model correction procedures. None of these methods is sufficient in terms of the correction effectiveness. There are certain types of errors in the analytical model that may not be eliminated through the model corrections.

One of the shortfalls is that the cost function J is based on numerical judgments of some necessary but not sufficient properties of the model. Therefore, no matter how small the value of J could be reduced to, the correction effect still may not be greatly improved. In addition, there exist possible experimental errors as well. So it is necessary to establish more suitable criteria for evaluations of the correction effects.

The goal of model correction is to obtain the correct **M**, **C**, and **K** matrices. However, in many engineering applications, it is the properties of the structural system that are of our interests. As described at the introduction section, using the M-C-K model we can calculate these system's properties. On the other hand, when some of the properties the systems are known such as the order of the system, they may be used in model corrections. Following this line of thought, we can consider and treat model correction on the basis of its ability of preserving system properties in addition to its ability to satisfy the prescribed numerical criteria such as cost function J. Since there is no single property of the system that is strong enough to guarantee the correctness of the physical model (at least it is the case at present), the best analytical model is the one that preserves most properties of the system.

CORRECT CORRESPONDENCE AND ITS INTERPRETATION IN MODEL CORRECTIONS

Consider again the 5-story structure shown in Figure 1 (a). A diagrammatic finite element representation generated according to the real measurements is shown in Figure 3 (a). In Figure 3 (b) and (c), the modal deformations of the first and second modes of the structural system are illustrated. Figure 3 (b) shows a simple translational mode and Figure 3 (c) shows a simple torsional mode. In more complicated situations, modes of the structural system may not be as simple as the ones given in these figures. Nevertheless, they possess distinct modal deformations, which are the most basic dynamic performances of the structural system. Since the structural system for testing is also the object for finite element modeling, the modal deformations of the modes obtained from the two approaches should be essentially the same, despite of numerical disparities due to the errors of measurements and calculations. Based on

this observation, we investigate in the following some possible model correction methods that make the revised analytical model preserve similar modal deformations as the real structural system.

The invariance of modal deformations for the modes in both physical and analytical models can be characterized by the correct correspondence between the modes (system eigenvalues and system eigenvectors) of the two models. Conceptually, it is easy to understand that such a correspondence should relate the modes with similar deformations to each other. To establish such correspondence however, we have to define the correct correspondence in terms of model elements. In usual, the stiffness **K** and the mass **M** of the analytical model are obtained with more accuracy than damping matrix **C**, a correct correspondence can be obtained easily between the stiffness eigenvalues of the physical and analytical models. Since each individual mode is dominated by an unique stiffness eigenvalue, we can achieve the correct correspondence of modes of the physical and analytical models by first numbering the modes in each model with respect to the given subscripts of the stiffness eigenvalues in that model, and then relate the modes according to the correct correspondence between the stiffness eigenvalues of the two models.

Examine the governing equation

$$M\ddot{X} + C\dot{X} + KX = F \tag{8}$$

where **M**, **C** and **K** are mass, damping and stiffness matrices respectively. Vectors $\ddot{X}$, $\dot{X}$, X and F denote the acceleration, velocity, displacement and forcing function respectively. In free vibration, F is zero. Equation (8) becomes

$$M\ddot{X} + C\dot{X} + KX = 0 \tag{9}$$

Applying some matrix operations to Equation (9), we obtain the following $D\text{-}A_k$ model

$$I\ddot{Y} + D\dot{Y} + A_k Y = 0 \tag{10}$$

where I is an identity matrix, and $Y = Q^T M^{1/2} X$

$$D = Q^T M^{-1/2} C\, M^{-1/2} Q \quad \text{and} \quad A_k^T = QM^{-1/2} K M^{-1/2} Q = \begin{bmatrix} \omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ddots & \\ & & & \omega_n^2 \end{bmatrix},$$

Q is an orthogonal matrix. This is called the canonical vibration model (Liang et al 1990). Its advantage over the general vibration model (9) lies in the simplification of **M** and K matrices. With Λ_k diagonal, the stiffness eigenvectors in this model are always e_i (the unit vectors) i = 1, 2, .. n. Now we discuss how to number the modes with respect to subscript i.

In the case of a proportionally damped system, it is known that Caughey's criterion (Caughey, 1976)

$$D\,\Lambda_k = \Lambda_k D \qquad (11)$$

is satisfied. Using Equation (11), we can find an orthogonal matrix **R**. By applying **R** from the left and the Hermitian transpose of **R** from the right to Equation (10), we have a canonical model with both RDR^H and $R\Lambda_k R^H$ diagonal. Such a system is completely decoupled. There are n separate single DOF equations each of which corresponds to a mode of the system. The numbering is easily determined in the way that the eigenvalue of the i^{th} equation,

$$\ddot{y}_i + d_i\dot{y}_i + \omega_i y_i = 0$$

is assigned with subscript i.

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ denote the system eigenvalues and $p_1, p_2, \ldots, p_n$ denote the mode shapes of an canonical model. A recent result by Liang et al (1990) offers another convenient way to obtain the numbering. The result unfolds the following property of a proportionally damped system

$$\lambda_i\bar{\lambda}_i = \omega_i^2 \quad i = 1, 2, \ldots, n \qquad (12)$$

where ω_i^2 is the i^{th} eigenvalue of the stiffness matrix. The subscript i in (12) enumerates the system eigenvalues such that it gives an one-one correspondence between the system eigenvalues λ and eigenvalues ω^2 of the stiffness matrix. Since the inverse of this result is also valid, a system satisfying (12) is automatically proportionally damped and possesses the desired numbering.

Using complex-damping coefficients, the above numbering can be justified in terms of the system energy relations. By definition, a complex-damping coefficient is a generalized Rayleigh quotient

$$\alpha_i = a_i + b_i j = \frac{q_i^T D p_{r_i}}{q_i^T p_{r_i}} \qquad i = 1, 2, \ldots, n$$

where q_i is an eigenvector of stiffness matrix, p_{r_i} is a system eigenvector. a_i and b_i are the real and imaginary parts of α_i. For a different numbering of the system's modes, we usually get a different set of corresponding complex-damping coefficients. Otherwise the generalized Rayleigh quotients are not defined at all. As reported by Liang and Lee (1990), there is one correct set of complex-damping coefficients which can be used to describe the energy transformations among the modes of a vibration system. In their report a complex damping ratio μ_i is defined as

$$\mu_i = \alpha_i / 2\omega_i \simeq \xi_i + j\zeta_i$$

where ω_i is the i^{th} undamped natural frequency and ξ_i is the i^{th} damping ratio. The ζ_i is a ratio of the energy transformed in a cycle over the total energy stored before the cycle in the i^{th} mode. If ζ_i is zero, μ_i has no imaginary part. Thus there is no energy transformed into or out of the i^{th} mode. Consequently, such a mode can be decoupled from the system. By this theory, the set of correct complex-damping coefficients for a proportionally damped canonical vibration model is a set of real-valued scalars, because in such model every mode can be decoupled. This condition is satisfied by the complex-damping coefficients calculated with the numbering described earlier. In fact, this numbering is the only one that satisfies the requirement $q_i^T p_{r_i} \neq 0$, for complex-damping coefficients.

For non-proportionally damped systems, the correct numbering is also associated with the correct set of complex-damping coefficients, which describe the energy transformations among the coupled modes. Although a natural generalization of the numbering discussed for proportionally damped systems, namely relating a stiffness eigenvalue ω^2 with a closest $\lambda\bar{\lambda}$, is not correct in general for non-proportionally damped systems, (see Tong et al), given the correct set of complex-damping coefficients, the correct numbering is shown to be unique (see Tong et al). Therefore, we can search the correct numbering from the complex-damping coefficients.

By using the correct correspondence of modes in model corrections, individual modes in the corrected model preserve their deformations. In most cases, we may produce a model having similar energy transformation pattern to the physical model. Due to the limit of space, we omit the examples.

COMPLEX DAMPING FITTING AND EIGEN-MATRIX FITTING

The first way to improve the correction procedures is to use the complex-damping coefficients to determine the correct correspondence. The imaginary part of a complex-damping ratio satisfies

$$\omega_i = \omega_{n_i} \exp(\zeta_i), \quad i = 1, 2, \ldots, n \tag{13}$$

when the system is lightly damped, i.e.

$$|\mu_i| \leq 0.3, \quad i = 1, 2, \ldots, n. \tag{14}$$

Equation (14) is satisfied with most engineering structures.

In Equation (14), ω_{r_i} is the square root of the r_i^{th} eigenvalue of the generalized stiffness matrix, where r_i is a designated permutation of 1, 2, ..., n. Thus, by using equation (14) and μ_i, we can determine the correct correspondence quantitatively.

We propose a improved model correction criterion as follows.

$$J = \sum_{i=1}^{n} \alpha_i (\omega_i^{(m)} - \omega_i^{(a)})^2 + \beta_i (\xi_i^{(m)} - \xi_i^{(a)})^2 +$$

$$\gamma_i (\zeta_i^{(m)} - \zeta_i^{(a)})^2 + (p_i^{(m)} - p_i^{(a)})^H \Gamma_i (p_i^{(m)} - p_i^{(a)}) \tag{15}$$

where γ_i are weighted coefficients for least square approximation of ratio $\zeta^{(.)}$. The term $(\zeta_i^{(m)} - \zeta_i^{(a)})^2$ is a good monitor of non-proportionality. With the complex-damping ratios available and the systems considered being lightly damped, The correct correspondence can be solved from Equation (13).

A second approach to deal with correct correspondence is to avoid using the modal parameters mode by mode. Instead we can use a more general convergence pattern so that the correct correspondence is assured through the convergence. In this regard we have a choice of either using the state matrix or using the eigen-matrix. Because the size of the state matrix is 2nx2n, (supposing the order of the system is n), we consider the eigen-

matrix whose size is only nxn.

An eigen-matrix **A** is defined by

$$\mathbf{A} = \mathbf{P}\,\Lambda\,\mathbf{P}^{-1} \tag{16}$$

where Λ is a diagonal matrix consists of all eigenvalues λ_i , i = 1,...n, of the system, and

$$\lambda_i = -\xi_i\,\omega_i \pm j(1-\xi_i)^{1/2}\,\omega_i \tag{17}$$

A matrix **A** has the eigen-decomposition (16), if and only if **A** satisfies the following quadratic matrix equation:

$$\mathbf{M}\,\mathbf{A}^2 + \mathbf{C}\,\mathbf{A} + \mathbf{K} = \mathbf{0} \tag{18}$$

where the coefficient matrices **M**, **C** and **K** are defined as in Equation (8). From Equation (16), we can see intuitively, that convergence of an eigen-matrix **A** involves global adjustments of all modal parameters Therefore, the problem of correspondence will not occur here. The cost function can be established by

$$J = \| \mathbf{A}^{(m)} - \mathbf{A}^{(a)} \| \tag{19}$$

where $\| \cdot \|$ stands for a norm of matrix $\mathbf{A}^{(m)} - \mathbf{A}^{(a)}$. For example, it can be the Frobenius norm,

$$J = \| \mathbf{A}^{(m)} - \mathbf{A}^{(a)} \|_F = \{ \sum_{i=1}^{n} \sum_{j=1}^{n} | a_{ij}^{(m)} - a_{ij}^{(a)} |^2 \}^{1/2} \tag{20}$$

where $a_{ij}^{(.)}$ is the ij^{th} entry of matrix $\mathbf{A}^{(.)}$. Or it can be a p-norm, such as the 2-norm,

$$J = \|\mathbf{A}^{(m)} - \mathbf{A}^{(a)}\|_2 = \{ \lambda_{max}[(\mathbf{A}^{(m)} - \mathbf{A}^{(a)})^H(\mathbf{A}^{(m)} - \mathbf{A}^{(a)})] \}^{1/2} \tag{21}$$

where $\lambda_{max}[.]$ is the maximum eigenvalue of matrix [.] .

CONCLUDING REMARKS

In this paper, we first examined several model correction procedures and their common weaknesses. Most available methods emphasize the speed of numerical convergence. In this study we pay attention to the validity of the corrections. We suggested methods to improve some of correction procedures by using the correct correspondence between the modes of physical and analytical models. This study results in the improvement of finite element modeling. It is also shown that a strong connection exist between the theoretical studies such as the complex damping theory and the various practical applications.

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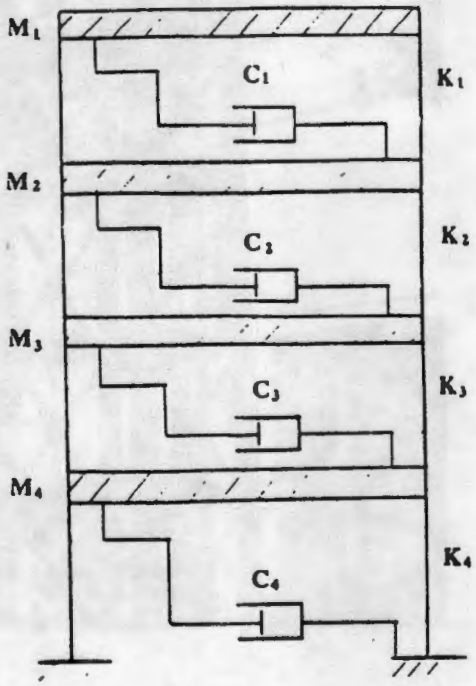


Figure 2 (a) 4 Degree-of-Freedom Structure

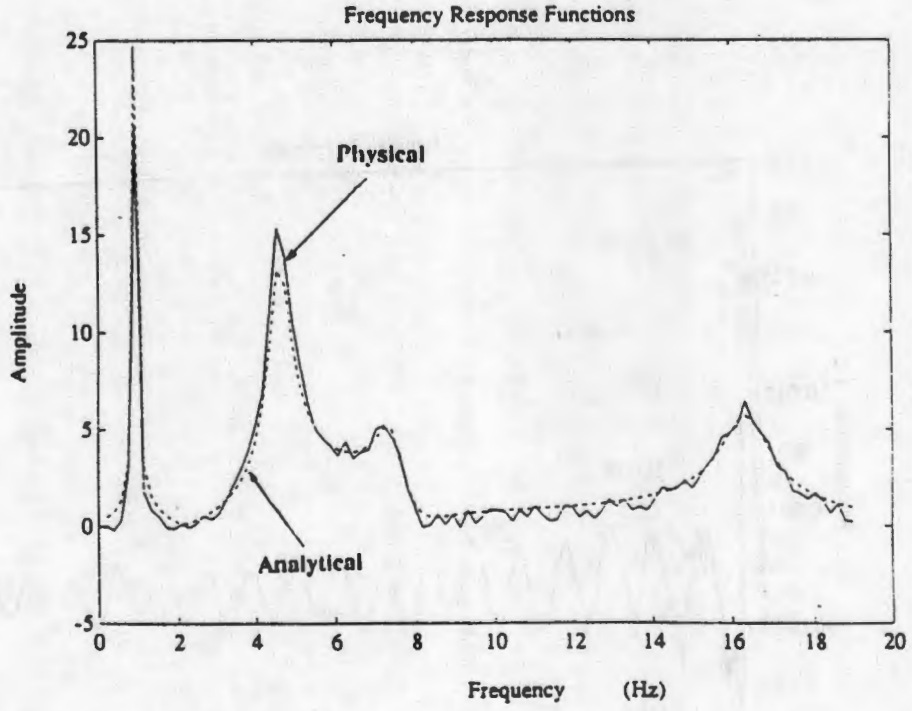


Figure 2 (b) FRF Curves

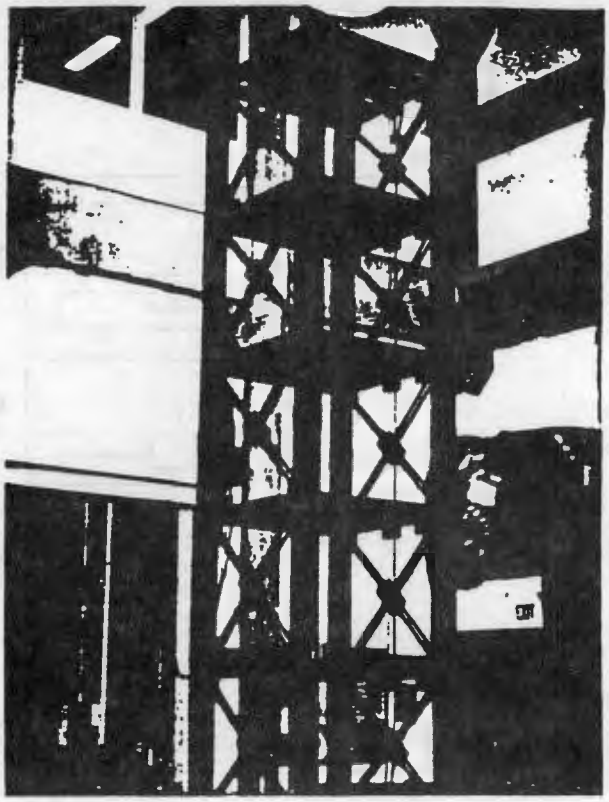


Figure 1 (a) Five Floor Structure

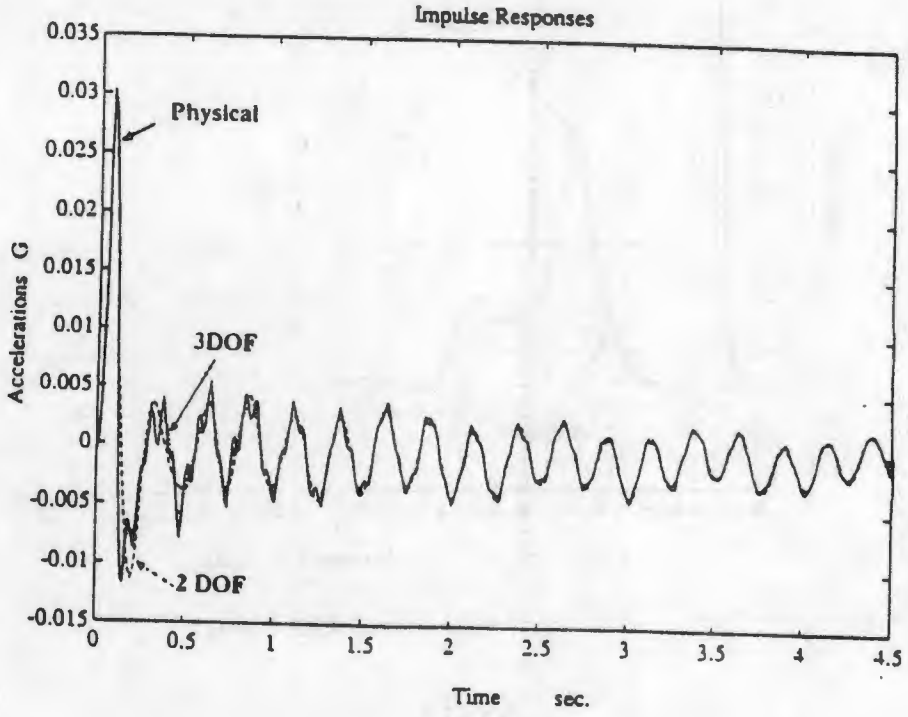
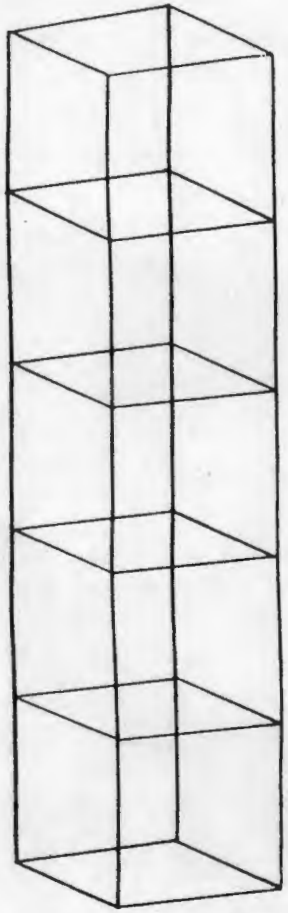
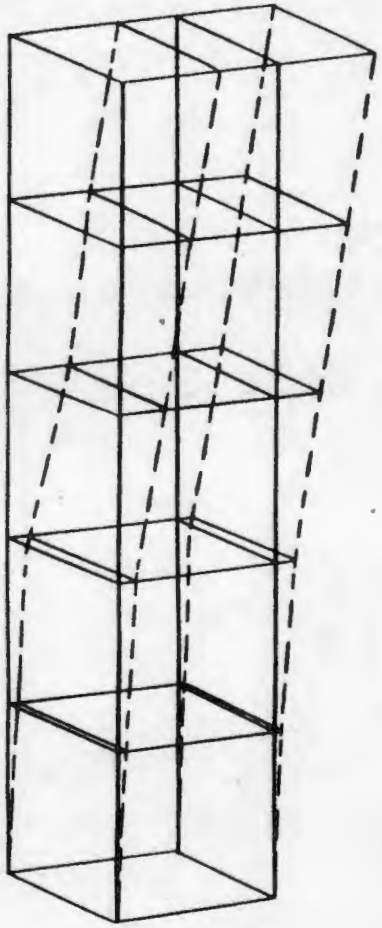


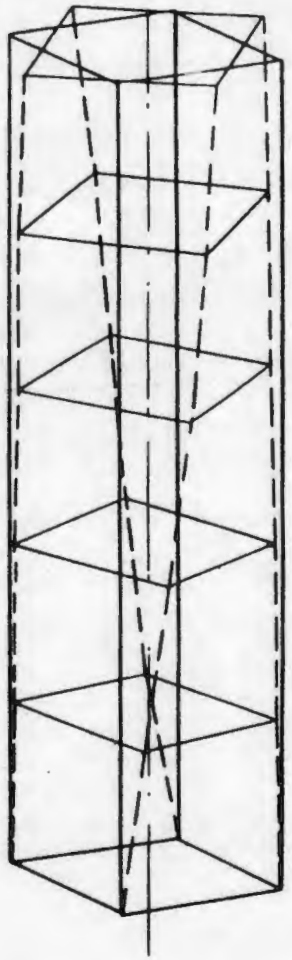
Figure 1 (b) Three Responses



Original Structure (a)



Translational Mode Shape (b)



Torsional Mode Shape (c)

Figure 3 Structure and Mode Shapes

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