An Identification Technique for Damped Distributed Structural Systems Using the Method of Collocation

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ABSTRACT

An identification scheme in the frequency domain, suitable for one-dimensional distributed structural dynamic systems with damping is considered. For this purpose, the form of a model representing the behavior of an Euler-Bernoulli beam is assumed to be known in the frequency domain. Also, the response of the system is assumed to be given at discrete locations along the beam. Quintic B-splines are then used to obtain a continuous representation of the response and its derivatives. The system parameters appearing in the governing differential equation are considered to be spatially varying functions. Cubic B-splines are used to approximate the parameter space, and their derivatives are obtained from such approximations. The method of collocation in conjunction with the equation error approach is then used to estimate the unknown parameters, which are the unknown coefficients of the parameter splines. A numerically simulated response of an Euler-Bernoulli beam in the presence of viscous damping is considered to validate the identification scheme. The estimated values of mass, stiffness and damping are discussed.

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INTRODUCTION

Damping is inherently present in virtually all types of structures encountered in practice. Hence, an adequate representation of damping and suitable methods to accurately quantify it are essential. In the recent past, there has been an increased interest in the study of distributed damping. For structures that can be modeled as continuous systems, discretization reduces modeling accuracy. In such cases, if the form of a model representing the physical system is known along with the initial and boundary conditions, the actual distributed system itself can be considered without resorting to approximations. A distributed representation is likely to yield more accurate predictions of the system behavior. Identification techniques suitable for distributed structural dynamic systems have been reported in the last decade [References 1-8].

At present, there are only very few techniques available to identify the unknown parameters of distributed structural dynamic systems in the presence of damping. Among these, the finite element and spline-based techniques have received considerable attention. The finite element techniques are primarily concerned with systems that include proportional or general viscous damping. A detailed discussion of such methods is presented in Reference 9. In the spline-based technique, time-domain data of the systems are used. The parameters are considered to be either constant or spatially varying functions, and also include different damping mechanisms [Reference. 5]. Also, in most of the available techniques some of the parameters are assumed to be known a priori.

In this vein, an identification technique that employs frequency domain data is discussed in this paper. For this purpose, the form of a model representing a distributed dynamic system within the framework of the Euler-Bernoulli beam theory was assumed to be known. The damping of the system was included using the linear viscous damping model. Also, the acceleration response was assumed to be given at discrete locations along the beam. The parameters appearing in the model were taken to be spatially varying functions. Quintic and cubic B-splines were then used to obtain approximate representations of the response and parameters, respectively. Their higher order derivatives were then obtained from such representations. These approximate functions were then substituted in the original distributed model, and using the collocation method a set of algebraic equations was obtained. The equation error approach was then used to estimate the unknown parameters, which are the coefficients of the parameter splines. The validity of the identification scheme was demonstrated using numerically simulated data, and the estimated values of mass, stiffness and damping are discussed.

For this purpose, none of the parameters was assumed to be known a priori.

PHYSICAL SYSTEM MODEL

The primary objective of the work presented in this paper was to develop an identification scheme suitable for a one-dimensional dynamic system in the presence of damping. The dynamic system was assumed to be modeled within the framework of the Euler-Bernoulli beam theory. The external damping to the system was included using the linear viscous damping model. A form of the equation governing the behavior of such systems was assumed to be known a priori in the frequency domain and can be written as follows:

$$d^2/dx^2 \left[EI(x)d^2a^*(x,\omega)/dx^2\right] + [j\omega Cv(x) - \omega^2 qA(x)]a^*(x,\omega) = -\omega^2 F(x,\omega) \qquad (1)$$

where $a^*(x,\omega)$ is the acceleration response due to the applied forcing function $F(x,\omega)$, x is the axial distance, and ω is the frequency in radians/second. The beam was assumed to be cantilevered at x=0 and free at x=L, where L is the length of the beam. EI, C_v and ρA are the stiffness, damping and mass distributions, respectively, and were assumed to be continuous functions in x. These are the unknown parameters to be estimated. Also, in equation (1), the initial conditions were taken to be equal to zero.

Due to the popularity of acceleration as the most often measured quantity, it was chosen as the response variable in the model. For identification purposes, it was assumed to be known at as many frequencies as required. In general, it is not possible to have a continuous measurement of the response, hence it was assumed to be known at only a discrete number of locations. From this information, an approximate continuous representation was obtained and used in the identification scheme. To this effect, quintic B-splines were used to obtain a continuous response from the discrete data at each frequency.

Also, each of the parameters appearing in equation (1) was approximated using cubic B-splines due to their continuous nature. The task of parameter identification then reduces to merely estimating the unknown coefficients of the cubic spline functions. The method of collocation in conjunction with an equation error approach was used for this purpose. Frequency response functions at discrete locations along the length of the beam for an impulse load applied at a known location were used as the data in the identification scheme.

B-SPLINE APPROXIMATION

It is evident from equation (1) that the highest order derivative of the parameters and response are two and four, respectively. Hence, their approximating functions were sought to be at least twice and four times differentiable. In this regard, cubic and quintic B-splines were chosen as their respective approximating functions.

RESPONSE REPRESENTATION

The infinite dimensional response space was approximated by an (N+1) dimensional function as follows:

$$a^*(x,\omega) = \sum_{i=0}^{N} a_i(\omega)\phi_i(x) \tag{2}$$

or in matrix notation

$$a^*(x,\omega) = \{\phi\}^T \{a(\omega)\} \tag{3}$$

where $\phi_i(x)$ are the approximating functions and should satisfy a given set of boundary conditions, and $a_i(\omega)$ are constants to be determined at each frequency. The functions $\phi_i(x)$ were constructed using the fundamental quintic B-spline basis functions, which are described below [Reference 10].

$$B_i^5(x) = 1/h^5 \Big[(x-x_{i-3})_+^5 - 6(x-x_{i-2})_+^5 + 15(x-x_{i-1})_+^5 - 20(x-x_i)_+^5 + 15(x-x_{i+1})_+^5 - 6(x-x_{i+2})_+^5 + (x-x_{i+3})_+^5\Big] \tag{4}$$

where

$\{x_i\}^N_{i=0}$ are the spline knots; $x_0 = 0$; $x_N = L$; and $h = L/N$.

Also,

$$x_{-3} < x_{-2} < x_{-1} < x_0; \quad x_{N+3} > x_{N+2} > x_{N+1} > x_N$$

and

$$(x-x_i)^5_+ = (x-x_i)^5, \text{ if } x \ge x_i$$
$$= 0, \text{ if } x < x_i$$

The basis functions given in equation (4) do not necessarily satisfy the required boundary conditions at hand. However, a combination of these basis functions was used to construct the expressions for $\phi_i(x)$ satisfying the given boundary conditions. Though many such expressions are possible, the following expressions satisfying the cantilevered boundary conditions were used in this paper.

$$\phi_1(x) = B_1^5(x) - (9/4)B_{-1}^5(x) + (65/2)B_{-2}^5(x)$$

$$\phi_2(x) = B_2^5(x) - (1/8)B_{-1}^5(x) + (9/4)B_{-2}^5(x)$$

$$\phi_N(x) = B_N^5(x) + (3/2)B_{N+1}^5(x) + 3B_{N+2}^5(x)$$

$$\phi_{N-1}(x) = B_{N-1}^5(x) - 2B_{N+2}^5(x)$$

$$\phi_{N-2}(x) = B_{N-2}^5(x) - (1/2)B_{N+1}^5(x)$$

$$\phi_i(x) = B_i^5(x), \qquad I = 3,4,\ldots\ldots,N-3 \tag{5}$$

Substituting the expressions for $\phi_i(x)$ from equation (5) in equation (2), for a given set of measured responses at the knot locations $i=0,1,2,\ldots,N$ at a given frequency, the coefficients $a_i(\omega)$ in equation (2) can be uniquely obtained. The higher order derivatives of the response involved in equation (1) could then be simply obtained by differentiating equation (2) as many times as required.

PARAMETER SPLINES

The unknown structural parameters present in equation (1) were represented as follows.

$$\theta^{(p)}(x) = \sum_{i=0}^{M} \theta_i^{(p)} C_i(x), \qquad p = 1,2 \text{ and } 3. \tag{6}$$

where $\theta^{(1)}$, $\theta^{(2)}$ and $\theta^{(3)}$ represent EI, C_v and ρA, and $\theta_i^{(1)}$, $\theta_i^{(2)}$ and $\theta_i^{(3)}$ are their corresponding coefficients. The value of M in equation (6) depends on the number of knot locations at which the parameters were identified. For a rapidly varying cross-sectional beam, a large value of M is required for an accurate identification. In the present case, the number of locations at which the parameters were to be identified was taken to be equal to the number of locations at which the response was known. The approximating basis functions Ci(x) were taken to be cubic B-splines and are defined as follows [Reference 10].

$$C_i^3(x) = 1/h^3 \Big[(x-x_{i-2})_+^3 - 4(x-x_{i-1})_+^3 + 6(x-x_i)_+^3 - 4(x-x_{i+1})_+^3 + (x-x_{i+2})_+^3\Big] \tag{7}$$

where

$$(x-x_i)_+^3 = \begin{cases} (x-x_i)^3, & \text{if } x \ge x_i \\ 0, & \text{otherwise} \end{cases}$$

Since the parameter knots were assumed to coincide with that of the response knots, equation (6) could be rewritten as

$$\theta^{(p)}(x) = \sum_{i=-1}^{N+1} d_i^{(p)} C_i(x) \tag{8}$$

To obtain a unique solution for the d_i's in equation (8), the following interpolatory conditions were to be satisfied.

$$\theta'^{(p)}(x) = d_{-1}^{(p)} C'_{-1}(x) + d_0^{(p)} C'_0(x) + \ldots + d_{N+1}^{(p)} C'_{N+1}(x) \qquad \text{at } x = x_0$$

$$\theta^{(p)}(x) = d_{-1}^{(p)} C_{-1}(x) + d_0^{(p)} C_0(x) + \ldots + d_{N+1}^{(p)} C_{N+1}(x) \qquad \text{at } x = x_0$$

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$$\theta^{(p)}(x) = d_{-1}^{(p)} C_{-1}(x) + d_0^{(p)} C_0(x) + \ldots + d_{N+1}^{(p)} C_{N+1}(x) \qquad \text{at } x = x_{N+1}$$

$$\theta'^{(p)}(x) = d_{-1}^{(p)} C'_{-1}(x) + d_0^{(p)} C'_0(x) + \ldots + d_{N+1}^{(p)} C'_{N+1}(x) \qquad \text{at } x = x_{N+1}$$

(9)

where a " ′ " denotes the firest derivative with respect to the axial coordinate. Now, define a vector $\{\theta^{(p)}\}$ as

$$\{\theta^{(p)}\} = \left[\theta'^{(p)}(x_0) \quad \theta^{(p)}(x_0) \quad \theta^{(p)}(x_1) \ldots \theta^{(p)}(x_N) \quad \theta'^{(p)}(x_N) \right] \tag{10}$$

The above vector, which also includes the first derivatives of the parameters at x=0 and x=L, is the unknown quantity to be identified.

IDENTIFICATION SCHEME

Equation (9) can be written in the matrix form as

$$\{\theta^{(p)}\} = [C^*]\{d^{(p)}\}$$

or

$$\{d^{(p)}\} = [C^*]^{-1}\{\theta^{(p)}\} \tag{11}$$

Substituting from equation (11) for $\{d^{(p)}\}$, equation (8) and its derivatives were written in the matrix form as

$$\theta^{(p)}(x) = \{C(x)\}^T[C^*]^{-1}\{\theta^{(p)}\}$$

$$\theta'^{(p)}(x) = \{C'(x)\}^T[C^*]^{-1}\{\theta^{(p)}\}$$

$$\theta''^{(p)}(x) = \{C''(x)\}^T[C^*]^{-1}\{\theta^{(p)}\} \tag{12}$$

Equation (12) was evaluated at the knot locations $\{x_k\}^N{}_{k=0}$, and the following equations were obtained.

$$\theta^{(p)}(x) = \{P_k\}^T\{\theta^{(p)}\} \quad \text{at } x=x_k$$

$$\theta'^{(p)}(x) = \{Q_k\}^T\{\theta^{(p)}\} \quad \text{at } x=x_k$$

$$\theta''^{(p)}(x) = \{Q_k\}^T\{\theta^{(p)}\} \quad \text{at } x=x_k \tag{13}$$

where

$$\{P_k\}^T = \{C(x)\}^T[C^*]^{-1} \quad \text{at } x=x_k$$

$$\{Q_k\}^T = \{C'(x)\}^T[C^*]^{-1} \quad \text{at } x=x_k$$

$$\{R_k\}^T = \{C''(x)\}^T[C^*]^{-1} \quad \text{at } x=x_k \tag{14}$$

The dimensions of each of the above vectors is $1 \times (N+3)$.

A term $a^*{}_{kl}$ was defined as the quantity $a^*(x,\omega)$ in equation (3) evaluated at a given location x_k and frequency ω_l. Equation (13) was combined with this definition, and equation (1) was rewritten as follows.

$$\{P_k\}^T\{\theta^{*(1)}\}(a^*{}_{kl})'''' + 2\{Q_k\}^T\{\theta^{*(1)}\}(a^*{}_{kl})''' + \{R_k\}^T\{\theta^{*(1)}\}(a^*{}_{kl})'' +$$

$$\{P_k\}^T[-\omega_l{}^2\{\theta^{*(3)}\} + j\omega_l\{\theta^{*(2)}\}]\, a^*{}_{kl} = F_{kl} \tag{15}$$

where F_{kl} is the force applied at location x_k and at frequency ω_l. In equation (15) both $a^*{}_{kl}$ and F_{kl} are complex quantities. Hence, they could be separated into real and imaginary parts as

$$a^*{}_{kl} = a^R{}_{kl} + ja^I{}_{kl}$$

and

$$F_{kl} = F^R{}_{kl} + jF^I{}_{kl} \tag{16}$$

Using the above definition, equation (15) could be finally written as

$$\begin{bmatrix} \{R_1\}^T{}_{kl} & -\omega_1\{R_4\}^T{}_{kl} & -\omega_1{}^2\{R_3\}^T{}_{kl} \\ \\ \{R_2\}^T{}_{kl} & \omega_1\{R_3\}^T{}_{kl} & -\omega_1{}^2\{R_4\}^T{}_{kl} \end{bmatrix} \begin{Bmatrix} \theta^{(1)} \\ \theta^{(2)} \\ \theta^{(3)} \end{Bmatrix} = \begin{Bmatrix} F^R{}_{kl} \\ F^I{}_{kl} \end{Bmatrix} \tag{17}$$

where

$$\{R_1\}^T{}_{kl} = \{P_k\}^T(a^R{}_{kl})'''' + 2\{Q_k\}^T(a^R{}_{kl})''' + \{R_k\}^T(a^R{}_{kl})''$$

$$\{R_2\}^T{}_{kl} = \{P_k\}^T(a^I{}_{kl})'''' + 2\{Q_k\}^T(a^I{}_{kl})''' + \{R_k\}^T(a^I{}_{kl})''$$

$$\{R_3\}^T{}_{kl} = \{P_k\}^T(a^R{}_{kl})$$

$$\{R_4\}^T{}_{kl} = \{P_k\}^T(a^I{}_{kl}) \tag{18}$$

Equation (17) was obtained for a single frequency. Similar sets of equations could be written at other frequencies. Combining the different sets at various frequencies, the resulting equations could be written as follows.

$$[C_\theta]\{\theta\} = \{F_\theta\} \tag{19}$$

$[C_\theta]$ is the coefficient matrix of dimension $2(N+3)n \times 3(N+3)$, where n is the number of frequencies used in the estimation. $\{\theta\}$ and $\{F_\theta\}$ are respectively the parameter and force vector of order $3(N+3)$. A least-square solution $\{\theta^*\}$ for equation (19) could be written as follows.

$$\{\theta*\} = C_\theta{}^T \; C_\theta{}^{-1} \; C_\theta{}^T \; F_\theta \tag{20}$$

NUMERICAL RESULTS

The identification scheme discussed in the previous section was demonstrated using simulated data for a cantilever beam with the following properties.

$$L = 0.61\text{m}$$

$$EI(x) = 18.01*10^2\;[1-(x/2L)]^4\;\text{N-m}^2$$

$$\rho A(x) = 4.22\;[1-(x/2L)]^2\;\text{N/m}$$

$$C_v(x) = 17.3\;[1-(x/2L)]^2\;\text{N-sec/m}^2$$

The above parameter distributions correspond to a beam of linearly varying cross section from tip to root, with the dimensions at the tip being half of those at the root. The beam was subdivided into 12 regions (N=12), and an independent finite element program was used to calculate the response at the resulting 13 knots. The assumed impulse was applied at the eighth interior knot x_8 (Figure 1). The first three natural frequencies of the beam determined from the finite element program were found to be 36.9Hz, 155.9Hz and 387.4Hz, respectively. In the identification, the frequency response data in the following frequency bandwidths at 1Hz intervals were used:

25-34Hz and 39-48Hz (regions surrounding the first mode)
144-153Hz and 158-167Hz (regions surrounding the second mode)
376-385Hz and 390-399Hz (regions surrounding the third mode)

Including the data in the immediate vicinity of the modal peaks resulted in less accurate estimates of damping, hence they were omitted. The probable cause for this phenomenon is the fact that the frequency response function tends to vary rapidly around the modal peaks for lightly damped structures, increasing the error in the response close to the peak regions. This in turn could significantly reduce the parameter estimates.

It can be seen from Figures 2-4, that the estimated values are in excellent agreement with the actual values used in generating the frequency response functions at the interior knots. Unacceptable mass and damping estimates at the root location were obtained and are not shown in the figures. This phenomenon may be due to the little or no contribution of these parameter values at the root to the error in satisfying the beam differential equation. Since the parameters are calculated by the subsequent minimization of this error, the procedure yields highly inaccurate estimates at these locations.

SUMMARY AND CONCLUSIONS

A spline based identification technique in the frequency domain that is suitable for damped distributed structural dynamic systems was developed. A beam whose behavior can be modeled within the framework of the Euler-Bernoulli beam theory was considered for the identification scheme. The parameters were allowed to vary linearly along the length of the beam. The infinite-dimensional response and parameter spaces were approximated by quintic and cubic B-splines, respectively. A Galerkin type weighted residual procedure was used to estimate the unknown parameters. Simulated frequency response data for an impulse applied at a known location were used to validate the technique. Acceleration response data around the first three modes of the beam were employed to estimate the mass, stiffness and damping properties. None of the parameters was assumed to be known a priori. The estimated results showed excellent agreement with the actual values at all the interior locations of the beam.

ACKNOWLEDGEMENT

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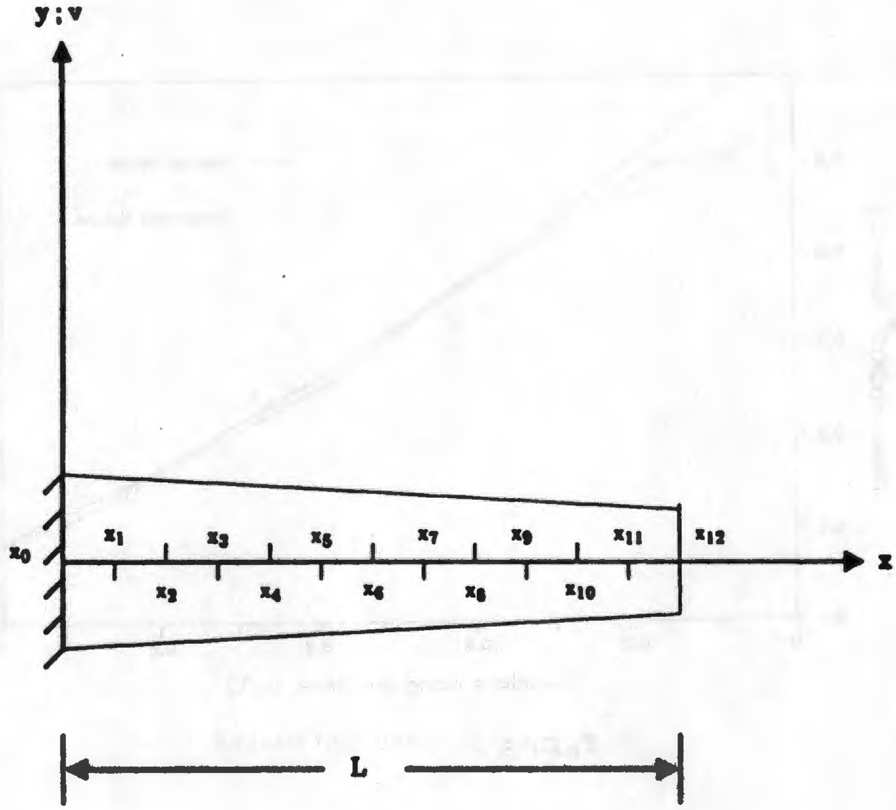

Figure 1 Beam Geometry

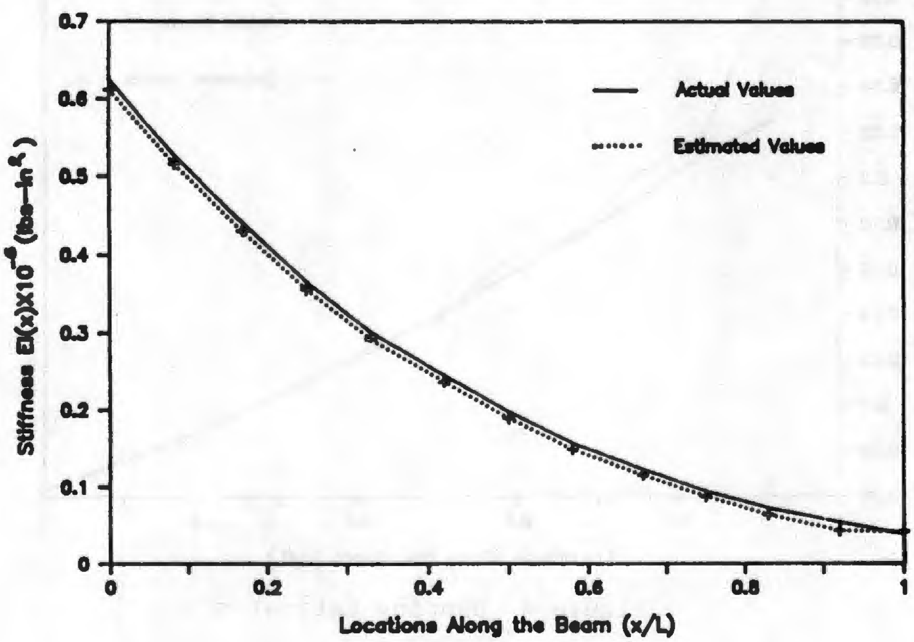

Figure 2 Stiffness Estimates

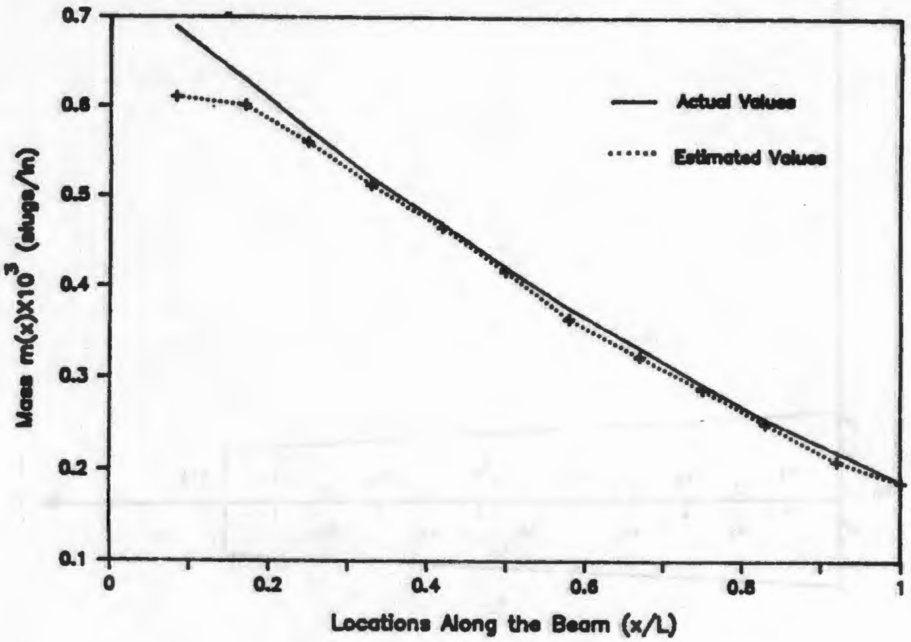

Figure 3 Mass Estimates

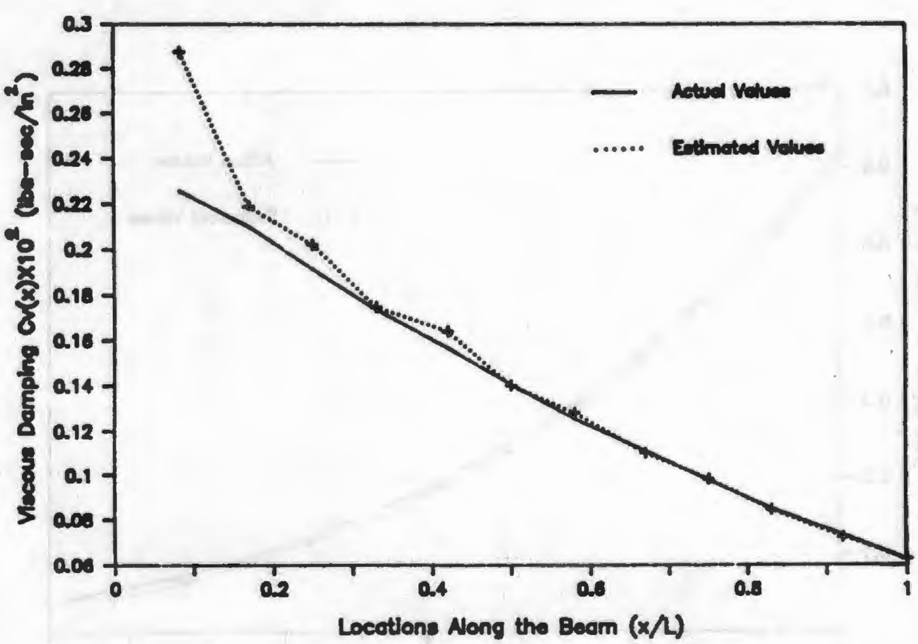

Figure 4 Damping Estimates