# AN APPLICATION OF FINITE ELEMENT METHODS TO PANEL FLUTTER OPTIMIZATION

Terry A. Weisshaar
University of Maryland*

This paper presents the results of an aeroelastic optimization study. The weight of a panel in high Mach number supersonic flow is minimized subject to the requirement that the critical aerodynamic parameter for flutter be held within prescribed limits. Panel equilibrium is described with finite element techniques. The study shows that finite element techniques, when used together with standard optimization methods, yield accurate results to this difficult problem. The numerical results of the investigation are compared to other converged numerical results to illustrate accuracy. The optimization mechanism itself is studied and discussed to provide qualitative results which may be applied to other aeroelastic optimization problems. The results of the paper show that, with an effective finite element model, weight savings and mass distributions found with finite element methods are comparable to those found by more complicated methods.

## Nomenclature

$A_{ij}, a_{ij}$ = System and element aerodynamic matrix elements, respectively (nondimensional)

$a$ = Plate chordwise dimension

$D(x)$ = Plate chordwise bending stiffness

$g_i$ = Constraint boundary derivatives $(\partial\lambda_{cr}/\partial t_i)$

grad( ) = Gradient of a function with respect to design variables

$J$ = Objective function

$K_{ij}, k_{ij}$ = System and tapered element stiffness matrix elements, respectively (nondimensional)

$m(x)$ = Mass per unit area

MR = Mass ratio (equation 13)

$M_{ij}, m_{ij}$ = System and tapered element mass matrix elements, respectively (nondimensional)

$n$ = Number of equal-length tapered elements used in panel model

$q_o$ = Dynamic pressure (equation 5)

$q_i$ = Nondimensional panel nodal displacements

$S_i$ = Elements of step direction vector

* Assistant Professor, Aerospace Engineering

$t_i$ = Design variable, nondimensional nodal thickness

$T(x)$ = Dimensional thickness

$w_i$ = Elements of the gradient of the unconstrained objective function

$x$ = Nondimensional chordwise coordinate ($x = x/a$)

$\omega$ = Panel oscillation frequency

$Y_i$ = Tapered element nondimensional nodal displacement (Fig. 2)

$\delta_1$ = Ratio of face-sheet mass to total mass, reference panel

$\epsilon$ = Optimization step size

$\lambda_o$ = Aerodynamic parameter (equation 5)

$\tau$ = Time

$\omega_r$ = Reference frequency, $\pi^2(D_o/m_o a^4)^{\frac{1}{2}}$ (equation 4)

$\{ \}$ = Column matrix

$\lfloor \rfloor$ = Row matrix

$[ \ ]$ = Square matrix

## 1.0 Introduction

The aeroelastic optimization of structures involves the combination of two-well-developed, sophisticated scientific disciplines; aeroelasticity and optimization. This paper will study a panel flutter optimization problem and discuss the results together with some of the difficulties encountered. The term "panel flutter optimization", as used in this paper, refers to the search for a least-weight design of a panel in high Mach number supersonic flow. The search for this optimum design is constrained by the requirement that a critical aerodynamic parameter for flutter be held within specified design limits.

Panel flutter optimization studies are of recent origin. Ashley and McIntosh (Ref. 1) presented a differential equation approach to panel flutter optimization. In their analysis, the equilibrium differential equations and the associated eigenvalues were treated as constraints. Variational calculus methods provided the equations necessary for panel optimality. However, they did not present a solution to their equations. The differential equation, with variational calculus, approach used in Ref. 1 has both advantages and difficulties. The primary advantage of this approach is that a great deal of useful information about the mathematical properties of the optimum solution may be found. If a solution can be found, it often provides useful insight into other, more sophisticated problems. The main difficulties of this approach arise from the face that only relatively simple structures may be described with differential equations. Also, although the governing equations are easily found, their solution is usually difficult. Two-point, nonlinear boundary value problems are encountered and often must be solved by numerical techniques. The convergence of these techniques are not always assured.

Turner (Ref. 2) approached the panel flutter optimization in a different manner. His panel model consisted of a series of finite elements, each with a different constant thickness. He then used the resulting matrix equations and variational methods to iteratively solve for a least-weight design. The numerical calculations included only several design variables and the weight savings were insignificant. More importantly, however, Turner studied the governing equations and deduced the presence of a least-weight design whose mass distribution is symmetric about the panel midchord. While this approach sacrifices some solution accuracy, it does eliminate some of the difficulties encountered with the differential equation approach. However, Turner's approach to the optimization portion of the problem is also subject to convergence difficulties.

Another recent study (Ref. 3) presented a converged numerical solution to a panel flutter optimization problem. This study used the differential equation approach of Ref. 1. There are several characteristics of this solution which determined the choice of a finite element model for the present study. First of all, the boundary conditions of the problem show that the panel thickness, in the absence of a minimum thickness requirement, must be zero at both the leading and trailing edges. Also, the resulting least-weight thickness distribution changes drastically from one panel chordwise position to another. A panel which is composed of uniform thickness elements is incapable of satisfying the zero-thickness boundary conditions. Because of this, an element whose thickness varies linearly from one end to the other was chosen to model the system. A preliminary study in Ref. 3 showed good results were possible with this "tapered element" approach. The study was, however, not comprehensive. The present study seeks to incorporate all the best features of the previous work in panel flutter optimization to demonstrate a method whose attributes are ease of application and accuracy of results.

## 2.0 The Panel Flutter Problem

Figure 1 shows a panel with supersonic airflow on one side and "dead" air on the other. The panel rests on simple supports and is of sufficiently large extent in the spanwise direction that its behavior may be considered that of a one-dimensional or semi-infinite panel. If the Mach number is greater than 1.6, then one-dimensional, linearized, quasi-steady, supersonic theory will adequately describe the airloads generated by panel oscillation (Ref. 4). The elastic and inertial behavior of the panel is described by one-dimensional plate equations. Thus, the plate equations are identical to beam equations with $D(x)$ substituted for the bending stiffness and $m(x)$ substituted for mass per unit length. This study will consider a panel composed of two variable thickness face-sheets with a uniform sheet of nonstructural material sandwiched between them.

Because of the sandwich construction, the plate bending stiffness is proportional to the face-sheet thickness. If $D_0$ represents a reference plate stiffness, then

$$\frac{D(x)}{D_o} = \frac{T(x)}{T_o} = t(x) \tag{1}$$

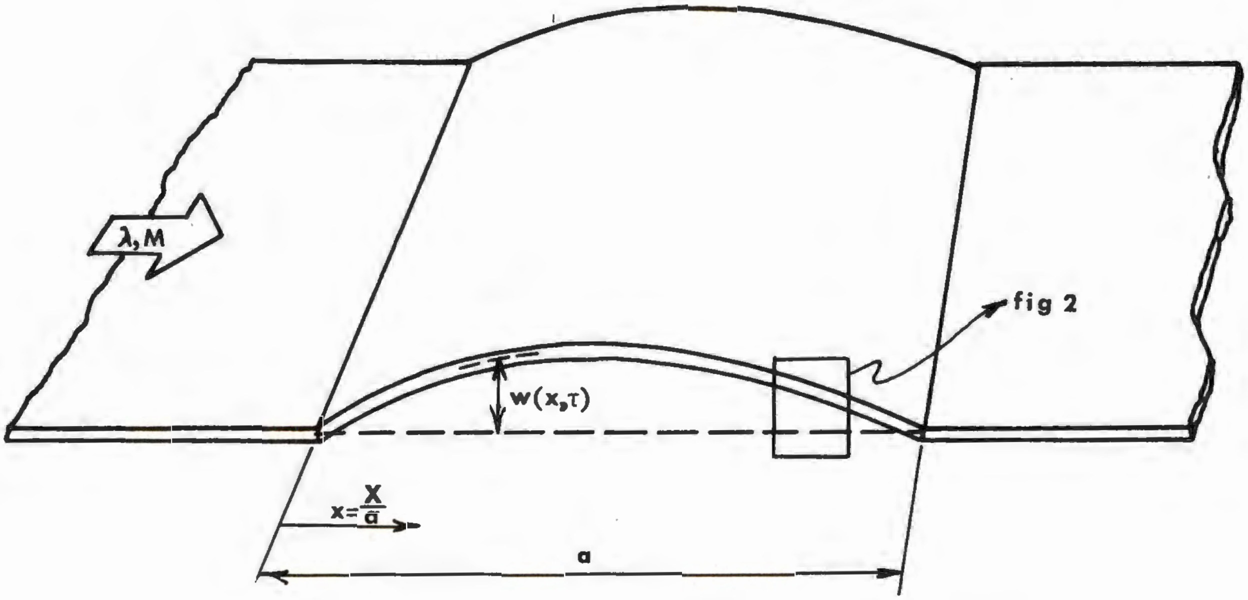


FIG 1 ONE-DIMENSIONAL PANEL FLUTTER MODEL

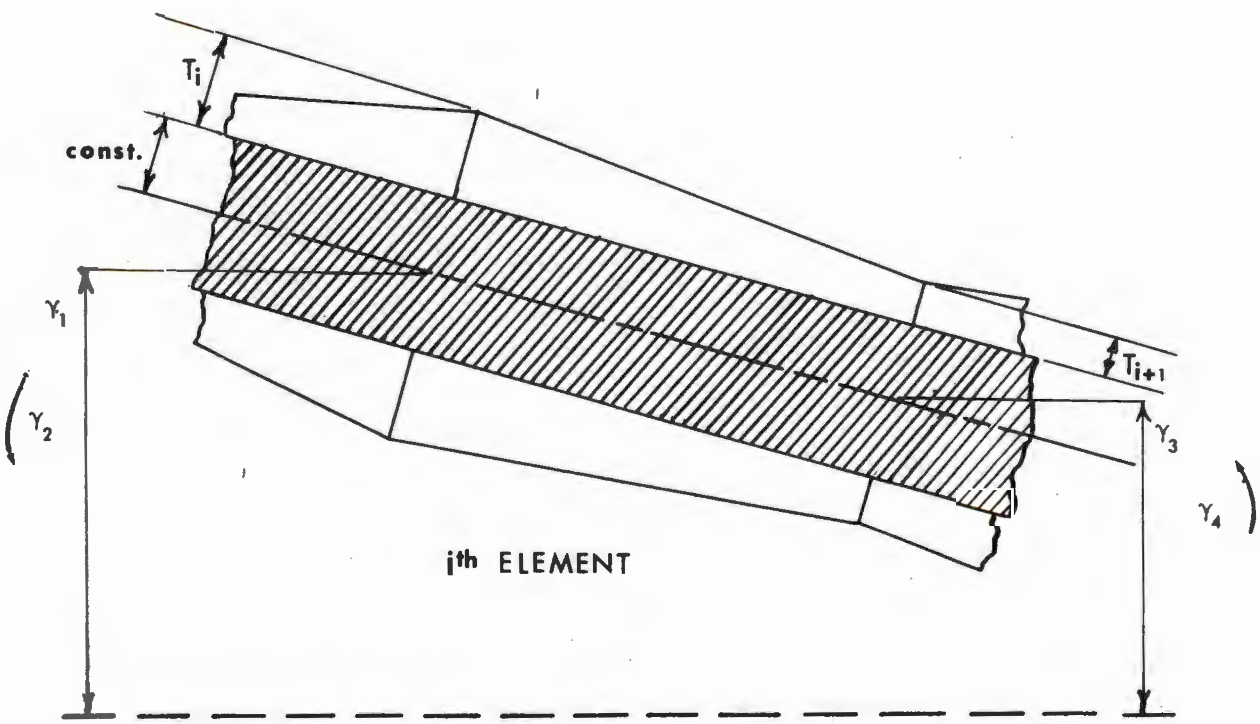


FIG 2 FINITE ELEMENT MODEL

The variable $t(x)$ is a nondimensional face-sheet thickness. Similarly, the mass per unit area may be nondimensionalized.

$$\frac{m(x)}{m_o} = \delta_1 t(x) + (1 - \delta_1) \tag{2}$$

The constant $\delta_1$ is the ratio of the total weight of the face-sheets of a uniform-thickness, reference panel to the total weight of the reference panel.

Consider a panel centerline displacement of the form

$$W(x,\tau) = \xi(x)e^{i\alpha\tau} \tag{3}$$

where $\alpha$ is, in general, a complex number $\alpha = \omega + i\beta$. The differential equation of equilibrium for the panel can be written in nondimensional form as (Ref. 4, pp. 418-437)

$$(t\xi'')'' + \lambda_o\xi' + \left(\frac{i\alpha}{\omega_r}g_\alpha\right)\xi(x)\pi^4 - \frac{\alpha^2}{(\omega_r)^2}\pi^4(\delta_1 t + (1 - \delta_1))\xi(x) = 0 \tag{4}$$

where $\omega_r$ is a reference frequency. The symbol $(\;)'$ denotes differentiation with respect to x. The parameter $\lambda_o$ is an aerodynamic parameter given by

$$\lambda_o = 2q_o a^3/D_o(M^2 - 1)^{\frac{1}{2}} \tag{5}$$

while $g_\alpha$, the aerodynamic damping parameter, is given by

$$g_\alpha = (M^2 - 2)\rho U/m_o\omega_r(M^2 - 1)^{3/2} \tag{6}$$

where $\rho$ and $U$ are the air density and speed, respectively.

If $t(x)$ is equal to unity, equation (4) is the equilibrium equation of a reference panel, with constant thickness face-sheets, oscillating in supersonic flow. For a given value of $g_\alpha$, the value of the frequency, $\alpha$, is a function of $\lambda_o$. The stability problem is simplified considerably if $g_\alpha$ is equal to zero. Since the case where $g_\alpha$ equals zero represents a realistic problem (Ref. 4, p. 422), this assumption will be used throughout this paper. For a certain range of values, with zero aerodynamic damping,

$$0 \leq \lambda_o < \lambda_{cr}$$

all values of $\alpha$ are real and distinct. However, for the value $\lambda_o = \lambda_{cr}$ the first two frequencies merge or become equal. For $\lambda_o > \lambda_{cr}$, the lowest two frequencies, $\alpha_1$ and $\alpha_2$, are complex conjugates of one another. Because of equation (3), the motion, for values of $\lambda_o$ above $\lambda_{cr}$, will be divergent with time. Thus $\lambda_{cr}$ corresponds to a situation where the panel motion is neutrally stable. Neutral stability, for a uniform thickness or reference panel, is found to occur (Ref. 4, p. 422) when

$$\lambda_{cr} \cong 3.52\pi^4 \tag{7}$$

The design problem involves the search for a least-weight panel design whose critical value $\lambda_{cr}$ is the same as that of the heavier reference panel.

Olson (Ref. 5) solved the uniform thickness, one-dimensional, panel flutter problem with a finite element description of the panel and the air-loads. His results showed that $\lambda_{cr}$ could be determined accurately with only a few elements. The present study uses Olson's consistently derived airload matrix.

## 2.1 The Finite Element Model

For oscillatory motion of the type given in equation (3), the nondimensional panel equilibrium matrix equation, equivalent to equation (4), is (Ref. 5)

$$\left[ -\left(\frac{\alpha}{\omega_r}\right)^2 \pi^4 \left[M_{ij}\right] + \left[K_{ij}\right] + \lambda_o \left[A_{ij}\right] \right] \left\{q_i\right\} = \left\{0\right\} \tag{8}$$

The variables $q_i$ are nondimensional displacements and rotations at the panel node points. The matrices $[M_{ij}]$, $[K_{ij}]$, and $[A_{ij}]$ are nondimensional mass, stiffness and aerodynamic matrices, respectively. A typical element for the panel is shown in Fig. 2. This element has a face-sheet thickness which varies linearly from one node point to another. The panel model consists of a series of $n$ of these tapered elements, connected to each other at their node points. The modal deflection $\xi(x)$ of an element is assumed to be a third order polynomial, just as was done by Turner in Ref. 2. The element stiffness and mass matrices were rederived in nondimensional form by considering the strain energy and kinetic energy of the tapered element. In one form or another, these tapered element matrices are well-known. The nondimensional element matrices for the tapered element shown in Fig. 2 are given by

$$\left[m_{ij}\right] = \frac{zf_1}{420}\begin{bmatrix} 156 & 22z & 54 & -13z \\ & 4z^2 & 13z & -3z^2 \\ & & 156 & -22z \\ \text{(Symmetric)} & & & 4z^2 \end{bmatrix} + \frac{\delta_1 z(t_{p+1} - t_p)}{840}\begin{bmatrix} 72 & 14z & 54 & -12z \\ & 3z^2 & 14z & -3z^2 \\ & & 240 & -30z \\ \text{(Symmetric)} & & & 5z^2 \end{bmatrix} \tag{9a}$$

$p = 1, 2, \cdots n$

where $f_1 = 1 + \delta_1(t_p - 1)$ and $z = 1/n$.

$$\left[k_{ij}\right] = \frac{2t_p}{z^3}\begin{bmatrix} 6 & 3z & -6 & 3z \\ & 2z^2 & -3z & z^2 \\ & & 6 & -3z \\ \text{(Symmetric)} & & & 2z^2 \end{bmatrix} + \frac{t_{p+1} - t_p}{z^3}\begin{bmatrix} 6 & 2z & -6 & 4z \\ & z^2 & -2z & z^2 \\ & & 6 & -4z \\ \text{(Symmetric)} & & & 3z^2 \end{bmatrix} \tag{9b}$$

The element airload matrix is, from Ref. 5,

$$\left[a_{ij}\right] = \begin{bmatrix} \frac{1}{2} & \frac{z}{10} & \frac{1}{2} & -\frac{z}{10} \\ & 0 & \frac{z}{10} & -\frac{z^2}{60} \\ & & \frac{1}{2} & \frac{z}{10} \\ \text{(anti-symmetric)} & & & 0 \end{bmatrix} \tag{9c}$$

Element equilibrium is, in terms of these matrices, given by

$$\left[-\left(\frac{\alpha}{\omega_r}\right)^2 \pi^4 \left[m_{ij}\right] + \left[k_{ij}\right] + \lambda_o\left[a_{ij}\right]\right]\left\{\gamma_i\right\} = \left\{0\right\} \tag{10}$$

By using a compatibility matrix and the simple support boundary conditions, these matrices may be used to form the system matrices $[M_{ij}]$, $[K_{ij}]$, and $[A_{ij}]$ in equation (8). The nondimensional thickness parameters at each node point, $t_i$, are design variables. The value of $\lambda_{cr}$ is determined in the manner described in Section 2.0. For a given value of $\delta_1$, the value of $\lambda_{cr}$ is a function of the design variables, $t_i$.

$$\lambda_{cr} = \lambda(\delta_1, t_i) \tag{11}$$

## 2.2 The Objective Function

The object of this study is to reduce panel weight by varying only the face-sheet thickness. A measure of the panel face-sheet weight is given by

$$J = \int_0^1 t(x)\,dx \tag{12}$$

The integral $J$ represents the ratio of the face-sheet weight of a variable thickness panel to the face-sheet weight of the reference panel. The ratio of the total weight of a variable thickness panel to that of the reference panel is called the mass ratio, MR.

$$MR = \delta_1 \int_0^1 t(x)dx + (1 - \delta_1) \tag{13}$$

Since the nonstructural mass is not disturbed, the only way to reduce the mass ratio is to reduce J. With a tapered element approach, the objective function is given by

$$J = \frac{1}{n}\left( (t_1 + t_{n+1})/2 + \sum_{i=2}^{n} t_i \right) \tag{14}$$

For the reference panel, J is equal to unity. The objective of this study will be to find a minimum value of J, as defined by equation (14), subject to the flutter parameter constraint.

## 2.3 The Flutter Parameter Constraint

The search for a minimum to the objective function defined in equation (14) is constrained by the requirement that the flutter parameter be held within design limits. Theoretically, the design requirements are that the least weight panel have a value $\lambda_{cr}$ equal to that of a similar, uniform thickness, reference panel. But, because of linear approximations used in the optimization procedure, the flutter constraint will, in practice, be an inequality constraint.

$$\begin{aligned} &\lambda_1 < \lambda_{cr} < \lambda_2 \\ &\lambda_1 = (0.995)\lambda_{ref} \\ &\lambda_2 = (1.005)\lambda_{ref} \end{aligned} \tag{15}$$

The value $\lambda_{ref}$ is the flutter parameter for the reference panel. Thus, a 0.5% variance on either side of the reference value is permissible. If, at any time, the value of $\lambda_{cr}$ for the variable thickness panel falls outside these limits, the design must be modified to bring it back within design specifications of equation (15). $\lambda_{ref}$ will be determined by finite element analysis.

The derivatives of the theoretical design constraint, $\lambda_{cr}$ = constant, with respect to changes in design variables, are necessary to the optimization procedure. These derivatives are calculated by perturbing, one at a time, each value of $t_i$ by an amount $\Delta t_i$. The value of $\lambda_{cr}$ due to this perturbation is then used to calculate the derivative.

$$\frac{\partial \lambda_{cr}}{\partial t_i} \cong \frac{\Delta \lambda_{cr}}{\Delta t_i} \tag{16}$$

## 2.4 The Optimization Problem

Two possible approaches to the optimization portion of the problem are available. The first approach is to convert the constrained minimization problem to an unconstrained minimization problem. This method is called elimination (Ref. 6). The constraint that $\lambda_{cr}$ be fixed at a certain value during optimization may be expressed mathematically as

$$d\lambda_{cr} = \sum_{i=1}^{n+1} \frac{\partial\lambda_{cr}}{\partial t_i} dt_i = 0 \qquad (17)$$

In terms of finite, small, design variable changes $\Delta t_i$, a truncated Taylor series gives

$$\Delta\lambda_{cr} = \sum_{i=1}^{n+1} \frac{\partial\lambda_{cr}}{\partial t_i}\Delta t_i = \sum_{i=1}^{n+1} g_i \Delta t_i = 0 \qquad (18)$$

Equation (18) may be used to eliminate one of the design variable changes.

Since the objective function is given by equation (14), changes in $J$ are given by

$$\Delta J = (\Delta t_1 + \Delta t_{n+1})/2n + \sum_{i=2}^{n} \Delta t_i/n \qquad (19a)$$

For simplification let us define new coefficients such that

$$\Delta J = \sum_{i=1}^{n+1} w_i \Delta t_i \qquad (19b)$$

These coefficients $w_i$ are elements of the gradient of $J$. Equation (18) may be used to eliminate a design variable change, call it $\Delta t_k$, from equation (19b). Equation (19b) then becomes

$$\Delta J = \sum_{i=1}^{n+1} G_i \; t_i = \lfloor G_i \rfloor \{\Delta t_i\} \qquad i \neq k \qquad (20a)$$

where

$$G_i = (w_i g_k - w_k g_i)/g_k \qquad (20b)$$

The matrix elements $G_i$ are also those of a gradient of $J$, but with the constraints included.

A combination of design variable changes must be determined which reduces the value of the objective function. That is, a vector of design variable changes $\Delta t_i$ must be formed such that $\Delta J$, given in equation (20a) is negative. If one thinks of searching through a multi-dimensional design space, then a "direction vector" is found to guide the search from one design to another, lesser weight design. The step direction vector given by $\{S\}$ can be formed in many different ways, each with its own advantages and disadvantages. In terms of $\{S\}$, the design variables changes are given by

$$\left\{\Delta t_i\right\} = \epsilon \left\{S_i\right\} \tag{21}$$

where $\epsilon$ is a constant which determines the step size. Note that in the unconstrained problem, $\{S\}$ is an n-dimensional vector. The change $\Delta t_k$ can then be calculated from equation (18).

Rubin (Ref. 7) presented an automated method for solving constrained natural frequency problems. He chose his step direction vector as

$$\left\{S_i\right\} = \left\{\frac{1}{|G_i|_{max}} t_i G_i\right\} \qquad i \neq k \tag{22}$$

This method was used with good results in Ref. 3 for a panel flutter optimization problem with a large minimum thickness constraint.

Many different optimization techniques are discussed in a recent book by Fox (Ref. 8). One method which was found effective in this study was the Fletcher-Reeves or conjugate gradient method (Ref. 8, pp. 87-89). This method eliminates some difficulties which are encountered with the steepest descent method. The Fletcher-Reeves method chooses $\{S\}$ as follows.

$$\left\{S_i\right\}_{new} = -\left\{G_i\right\}_{new} + \beta \left\{S_i\right\}_{old} \tag{23}$$

The constant $\beta$ is found from the relation

$$\beta = \sum_{i=1}^{n+1} (G_i)^2_{new} \Big/ \sum_{i=1}^{n+1} (G_i)^2_{old} \qquad i \neq k \tag{24}$$

In addition to the elimination method, a second method of approaching the optimization problem exists. This approach does not directly eliminate the constraints. Instead, the constraints are incorporated into the direction finding problem directly. With this approach, the vector $\{S\}$ is an $n + 1$ dimensional vector. One such method of finding a step direction is the method of feasible directions (Ref. 9). This technique has been used with excellent results on other flutter optimization problems (Ref. 10). An easily readable explanation of this technique is also given in Ref. 8. The choice of a step direction is shown to be an optimization problem in itself.

A simplification of the feasible directions technique reduces it to a gradient projection method. A complete discussion of the mathematical aspects of this technique are given by Fox (Ref. 8, pp. 179-196). The step direction is determined by the relation

$$\left\{S_i\right\} = -\text{grad}\ (J) - c(\text{grad}\ (\lambda_{cr})) \tag{25}$$

where the elements of grad J are $w_i$ and the elements of grad $\lambda_{cr}$ are the $n + 1$ partial derivatives in equation (16). With equation (25), all components of grad J parallel to the normal to the constraint boundary $\lambda_{cr}$ = constant are subtracted from that vector. The constant, c, in equation (25) is then determined by the relation

$$\lfloor S_i \rfloor\ \text{grad}\ (\lambda_{cr}) = 0 \tag{26}$$

It should be noted that if grad $\lambda_{cr}$ is a linear function of grad J, then no progress can be made because $\left\{S\right\} = 0$.. The relation

$$\text{grad}\ (J) + c(\text{grad}\ (\lambda_{cr})) = 0 \tag{27}$$

is related to the Kuhn-Tucker condition for a local minimum. Geometrically, equation (27) expresses the fact that the gradient of J is expressible as a linear function of the gradient of the constraint, which is normal to the boundary $\lambda_{cr}$ = constant. Thus, there is no possible way to reduce the objective function without violating the constraint (Ref. 8, pp. 167-171). This Kuhn-Tucker condition is helpful in determining how close one is to an optimum design.

## 3.0 Computational Approach

The previous discussion and the number of references cited shows that there is a wealth of information on all aspects of the problem. The aeroelastic analysis techniques with finite element methods, the finite element analysis itself and the optimization logic are all readily available. The combination of all these techniques into a computational design algorithm is the next step in the analysis.

The techniques necessary to analyze the panel flutter problem, to form the direction vector and to calculate the step size $\epsilon$ were programmed for the computer. The program has these basic features

(a) $\lambda_{cr}$ is determined from the analysis of a uniform thickness panel with a number of elements, n, equal to that of the panel being optimized.

(b) The variables $g_i = \partial\lambda_{cr}/\partial t_i$ are calculated for the design.

(c) One of the three methods outlined in the previous section is used to find a step direction.

(d) A step size $\epsilon$ is calculated such that the objective function is reduced by a given percentage.

(e) The design variables are modified by an amount $\Delta t_i$.

(f) $\lambda_{cr}$ for the new design is calculated.

(g) If $\lambda_{cr}$ falls outside the design constraint limits, it is modified by a gradient method. If the desired change in $\lambda_{cr}$ is

$$\Delta \lambda_{cr} = \sum_{i=1}^{n+1} g_i \; t_i \tag{28a}$$

and

$$\Delta t_i = \mu g_i \tag{28b}$$

then

$$\mu = \Delta\lambda_{cr} / \sum_{i=1}^{n+1} (g_i)^2 \tag{28c}$$

(h) If the design constraint is satisfied then the operation begins with step (b) and a new set of design variables.

Steps (f) and (g) are necessary because the functional relationship between and the design variables $\Delta t_i$ is nonlinear. The design modifications $\Delta t_i$ represent linear steps along a nonlinear constraint boundary. If these steps are two large, the design variables will result in a value $\lambda_{cr}$ which grossly violates the design specifications. Therefore, the decrease in the objective function cannot be too rapid.

## 3.1 Results

The initial study of a uniform thickness panel and other symmetric panels showed that the elements $g_i$ are symmetric about the midchord, $x = (1/2)$. That is

$$g_i(x) = g_i(1 - x) \tag{29}$$

Since the uniform thickness panel is symmetric and is the initial design, the optimization procedures outlined can never yield a nonsymmetric design. This fact agrees with Turner's comments in Ref. 2. This symmetry property can be used to reduce the amount of computation for an $n$ element panel. If $n$ is an even number, only $1 + n/2$ values of $g_i$ are necessary.

The value of $g_i$ at the leading edge is, for the reference panel, much smaller than other values $g_i$ at different panel locations. This relative insensitivity of $\lambda_{cr}$ to design variable changes at the leading and trailing edges can be a source of error. If the design variable is not perturbed sufficiently, the value of $g_i$ may be subject to numerical roundoff errors. An examination of the optimization methods outlined shows qualitatively that the design variables associated with the smallest values $g_i$ will decrease in magnitude while those associated with the largest values $g_i$ will increase in magnitude.

The majority of the investigation was done with four and six element models. In all cases the end thicknesses tended to decrease from one design step to another. The Fletcher-Reeves method and the feasible direction method decreased the end thicknesses much faster than the gradient method suggested by Rubin. Because of the possibility that inaccuracy in determining the values of $g_i$ at the ends might lead to inaccuracy in determining the optimum design, the author wanted to eliminate these variables from the design process as quickly as possible. For this reason, Rubin's gradient method was not used except in the early stages of the investigation. Also, because the value of the design variables at the ends approached zero, as predicted by the analytic analysis, a minimum thickness constraint is necessary to guard against meaningless results. This constraint is expressed mathematically as

$$t_i \geq t_{min} \tag{30}$$

In this investigation, $t_{min}$ was taken as 0.10.

Using a six-element model and identical starting conditions, the Fletcher-Reeves method and the feasible directions method were compared. Figure 3 shows the results of the first few, completely automated, design cycles. The values of mass ratio and $\lambda_{cr}$ are plotted versus design cycle. Since $\delta_1$ is equal to unity, the objective function and the mass ratio are identical. When $\lambda_{cr}$ falls outside the allowable range, the design is modified. This results in a slight weight increase. From Fig. 3, it is seen that both methods are of equivalent accuracy. A 1.5% improvement or decrease in the objective function is requested for each design cycle.

The formulation and logic used in the feasible directions technique has a slight advantage over the Fletcher-Reeves method in that the gradient of $\lambda_{cr}$ not only provides a measure of how the design process in progressing, but it provides qualitative information on the next design step. This information is useful when interaction with the computer is desired. The designer can intervene with the design process at any step and modify the design logic.

After the objective function had been reduced by about 10%, the author used the feasible direction method to achieve better designs. The amount of objective function improvement requested at this point was of the order of 0.25 to 0.50%. Larger requests for improvement were found to be inefficient because of unacceptable $\lambda_{cr}$ fluctuations.

Typically, on all models analyzed, the design variables fluctuated as the optimum value of $J$ was approached. This could occur if the value of the objective function is not sensitive to the constraint near the optimum while the design variables themselves are rather sensitive. This sensitivity was more pronounced with larger numbers of elements.

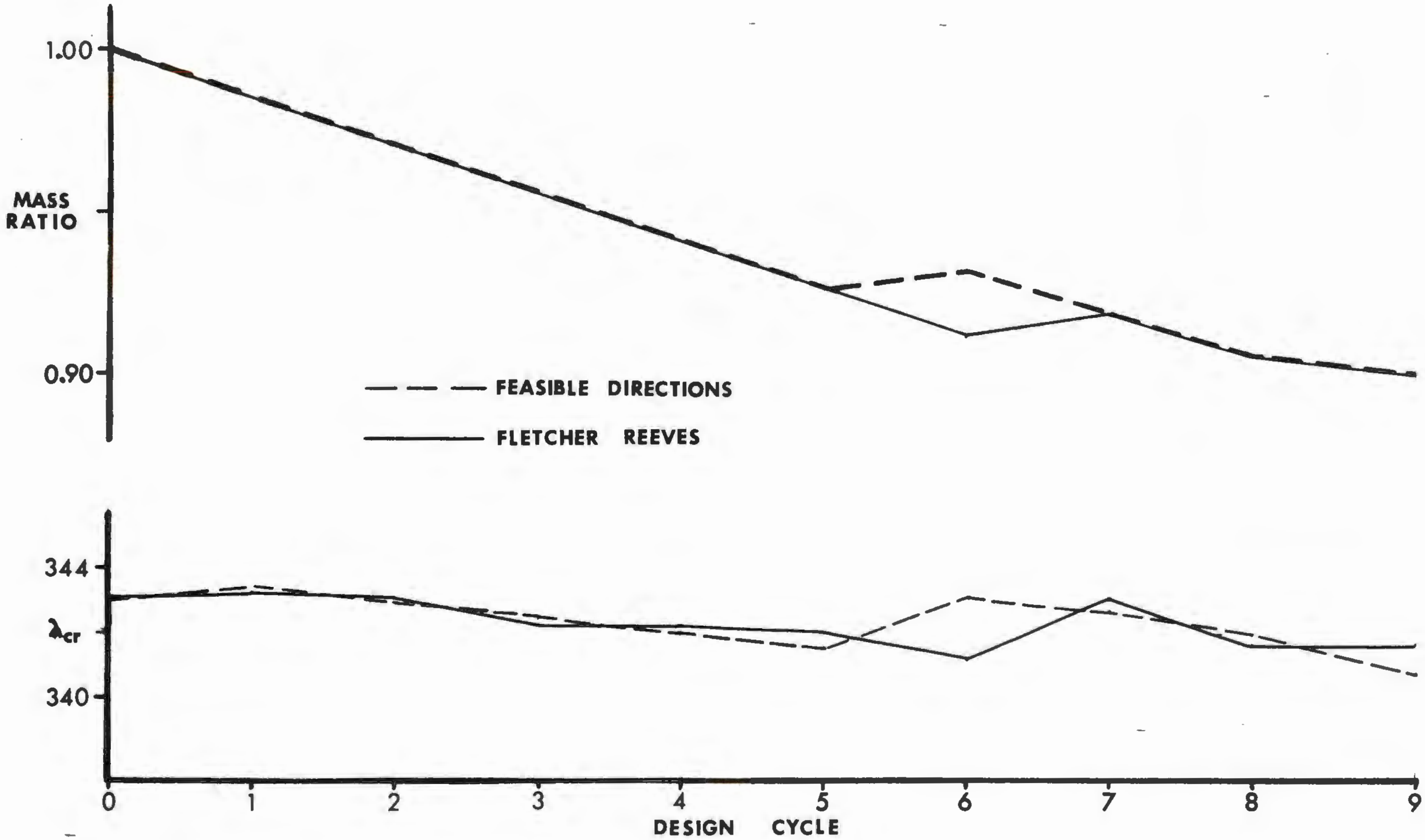


FIG 3 PROGRESSIVE COMPARISON OF TWO SIX ELEMENT OPTIMIZATION METHODS ($\delta_1$= 1.0)

Figure 4 shows how a typical design progresses from a uniform panel to a lesser weight panel. Figure 5 shows the final designs found with 4, 6 and 10 elements. Some further improvement in these designs is possible but, with the techniques used, was thought to be uneconomical. Table 1 below shows, for a six-element model, the initial values of $g_i$ and the final values of $g_i$ compared to the gradient of J.

<u>TABLE 1</u>

A Comparison of $g_i$ Values Before and After Panel Optimization
6-element Model, $\delta_1 = 1$, $t_{min} = 0.10$

| n x grad J ($n \times w_i$) | $g_i$ (Reference panel) | $g_i$ (Final Design) | Design Variable |
|---|---|---|---|
| 0.5 | 5.62 | 29.8 | 1 |
| 1.0 | 56.9 | 69.8 | 2 |
| 1.0 | 85.0 | 62.8 | 3 |
| 1.0 | 63.1 | 65.9 | 4 |
| 1.0 | 85.0 | 62.4 | 5 |
| 1.0 | 56.9 | 69.8 | 6 |
| 0.5 | 5.62 | 29.8 | 7 |

A measure of the accuracy of this method is provided by the comparison in Fig. 6. The converged solution for a panel obtained from Ref. 3 is compared to the best finite element solution obtained for a panel with identical parameters. The mass ratio for the converged solution is MR = 0.885 while the mass ratio for the finite element solution is slightly greater than 0.910. This is a difference of 2.5%.

Some difficulty was experienced when working with large numbers of elements. More difficulty was experienced with the calculation of the $g_i$ elements when using ten elements than with four elements. This can be caused by a number of numerical difficulties. The important aspect of this problem is that it does exist and probably prevents the generation of more accurate results. This $g_i$ calculation is an area where improvement possibly can be made. Rubin (Ref. 7) used analytical expressions for frequency gradients in his work. The use of a similar technique for the panel flutter problem to calculate $g_i$ is hampered by the fact that flutter occurs when two frequencies merge. This factor considerably complicates the mathematical aspects of the problem.

## 4.0 Conclusions

The results of this study and the comparison with a converged numerical result shows that finite element techniques provide accurate models for the

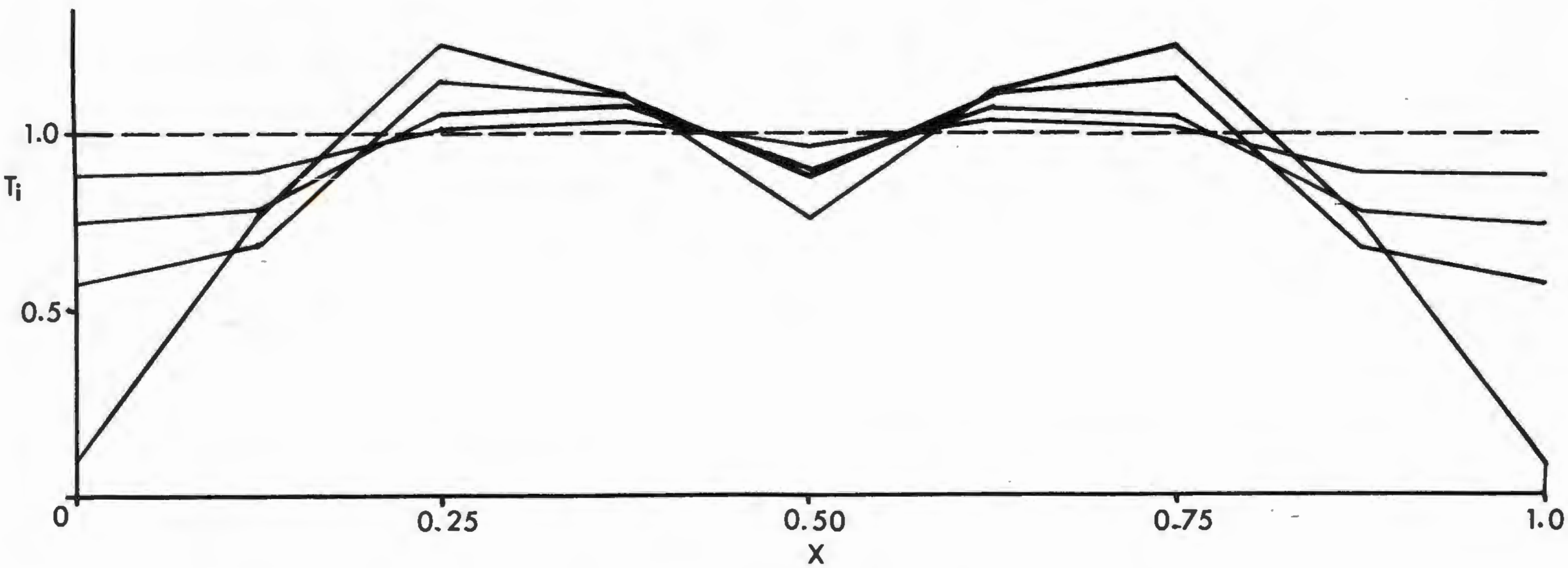


FIG 4 SEVERAL SUCCESSIVE EIGHT ELEMENT DESIGNS ( $\delta_1 = 1.0$ )

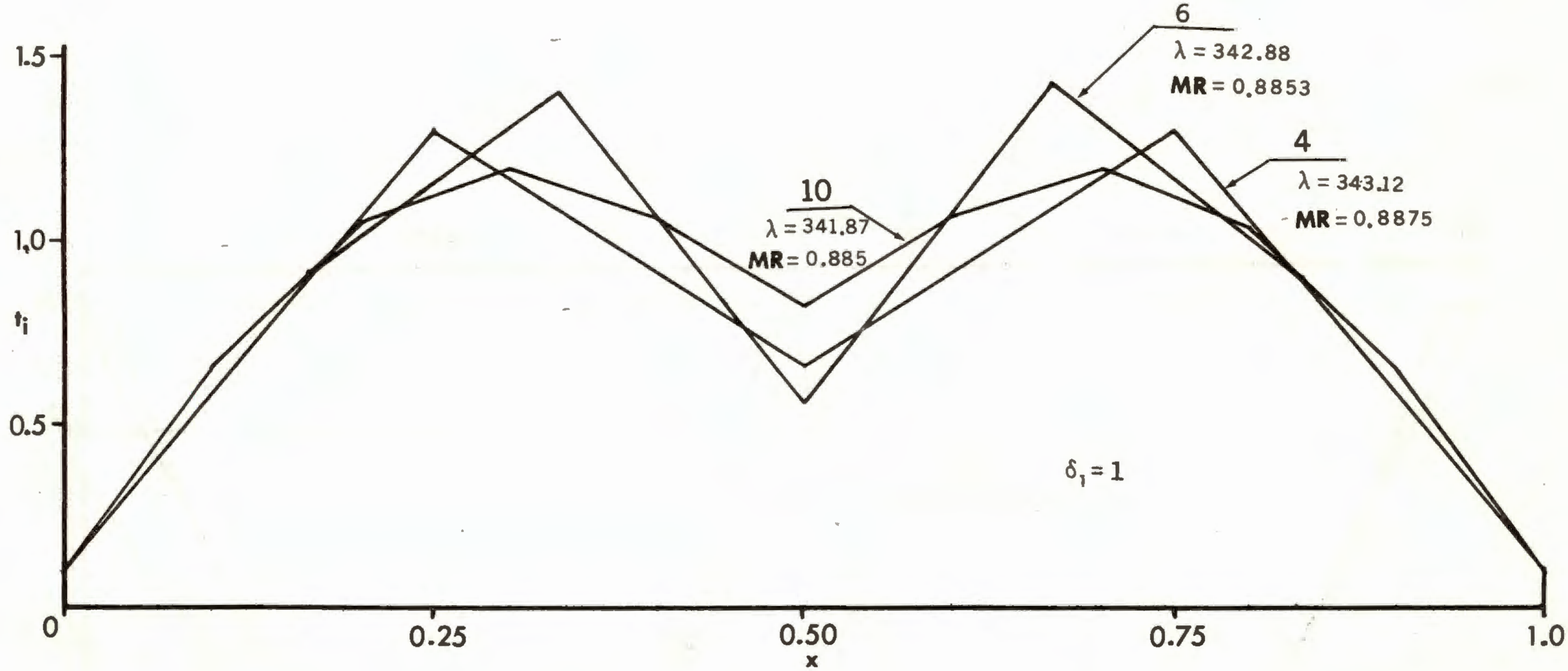


FIG 5 COMPARISON OF FINAL DESIGNS WITH FOUR, SIX AND TEN ELEMENT MODELS

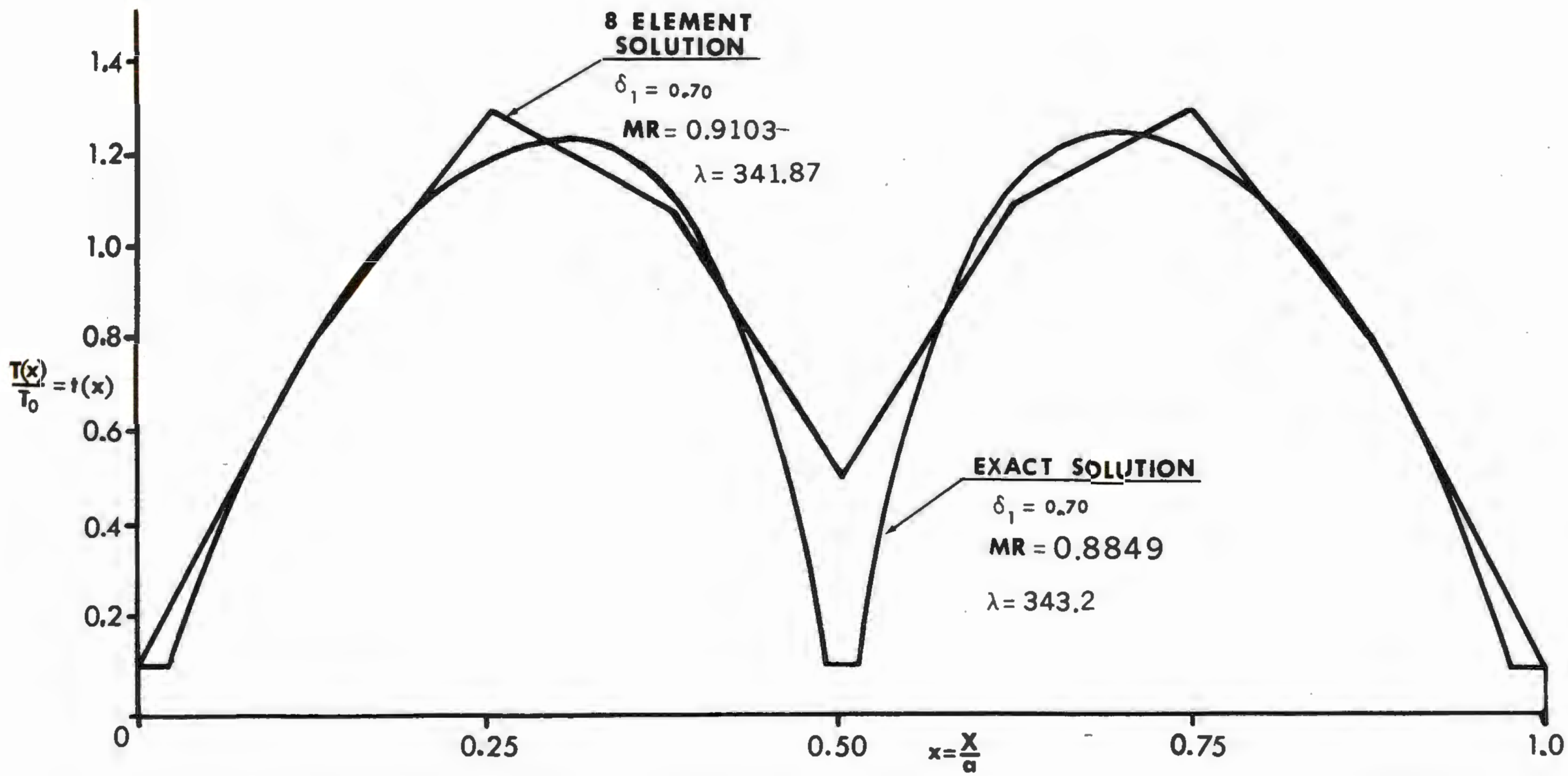


FIG 6 COMPARISON BETWEEN EXACT THICKNESS DISTRIBUTION AND EIGHT FINITE ELEMENT OPTIMIZATION

flutter optimization problem. Weight savings up to 12% have been shown. The ultimate accuracy of the method is dependent on the ability of the finite element model to adequately describe the actual optimal design. A recent flutter optimization study by Craig (Ref. 11) used a series of uniform thickness elements to model the panel. This approach is similar to Turner's except that Craig used a gradient projection technique in the optimization portion of the problem. Craig shows weight savings of around 4%. The present study uses an optimization technique similar to Craig's but, the use of the tapered element results in weight savings of more than twice that shown by Craig for a comparable problem. Thus, the main difference appears to be in the choice of the finite element model.

The great advantage of the finite element approach over the differential equation approach is that very little insight is necessary to generate good solutions. Convergence difficulties do not occur with finite element panel flutter optimization until near the optimal design. Some theoretical aspects of the problem are sacrificed with the finite element approach, but this is far overshadowed by the fact that a reasonable solution can be found. In addition, the model of the panel may be easily made more sophisticated and realistic by the addition of such effects as shear stiffness or the inclusion of different boundary conditions. This can be done without significant alterations in the computer program logic.

Additional improvements in optimization logic and the calculation of constraint gradients can undoubtedly be made. This should improve the solution and reduce the number of design cycles required. More importantly, this study shows that, given a suitable element, finite element techniques can be competitive with other, more complicated optimal structural design techniques.

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