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Vol. I

THE ALLOCATION OF SYSTEM RELIABILITY

Volume I - Development of Procedures for Reliability Allocation and Testing

(ARINC Research Publication No. 152-2-274, dated 30 November 1961; Air Force Contract AF 33(616)-7468, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio)

Pages 131, 133 - Replace the second paragraph of Section 8.4.3.3, Summary of Exponential Test Plans, with the following paragraph:

> For truncated replacement life tests, one can use this table to completely describe all the test characteristics since the rejection number r_0 and $\frac{\chi^2_{1-\alpha, 2r_0}}{2}$ given in the table, the sample size n, the test termination time T_0, and the termination on total test hours T^* are functionally related to each other as follows:
>
> $$\frac{\chi^2_{1-\alpha, 2r_0}}{2} = \frac{nT_0}{\theta_0} = \frac{T^*}{\theta_0}.$$
>
> Hence, if any one of n, T_0, or T^* are specified (as well as θ_0), the other two parameters can be determined. As an example, assume that a maximum of 75 hours of test time is available for each item on test. If $\theta_0 = 200$ and $\theta_1 = 100$, what is the appropriate plan if $\alpha = .10$, $\beta = .05$? From the table, for $k = 2$, $\alpha = .10$, $\beta = .05$, we have
>
> $\frac{\chi^2_{1-\alpha, 2r_0}}{2} = \frac{nT_0}{\theta_0} = 12.8.$ Since $T_0 = 75$ and $\theta_0 = 200$,
>
> $n = (12.8)(200)/75 = 34$. Hence, 34 items are put on test, each for a maximum of 75 hours. Replacements or repairs are made upon failure. If less than 18 failures occur before $(75)\cdot(34) = 2550$ total test hours are accumulated, the lot is accepted. If 18 failures occur before 2550 test hours are accumulated, the lot is rejected.

Page 132 - Replace Table 8-7 with the attached revised table.

TABLE 8-7

TEST PARAMETERS AND EXPECTED NUMBER OF FAILURES FOR VARIOUS TRUNCATED AND SEQUENTIAL REPLACEMENT LIFE TESTS

$k = \frac{\theta_0}{\theta_1}$	α	β	Truncated Tests - replacement				Sequential Tests - replacement		
			Rejection Number, r_0	$\frac{\chi^2_{1-\alpha, 2r_0}}{2}$	Expected Number of Failures		Truncation Number, r^*	Expected Number of Failures	
					$E_{\theta_0}(r)$	$E_{\theta_1}(r)$		$E_{\theta_0}(r)$	$E_{\theta_1}(r)$
1.5	0.05	0.05	67	54.13	54.0	66.8	201	28.0	36.7
	0.05	0.10	55	43.40	40.5	54.6	165	21.1	32.9
	0.05	0.25	35	25.87	24.0	34.0	105	12.0	23.5
	0.10	0.05	52	43.00	37.6	51.8	156	25.1	27.6
	0.10	0.10	41	33.04	32.8	40.7	123	18.6	24.4
	0.10	0.25	25	18.84	18.7	24.2	75	10.1	16.5
	0.25	0.05	32	28.02	27.3	31.9	96	18.0	15.7
	0.25	0.10	23	19.61	19.0	22.7	69	12.6	13.2
	0.25	0.25	12	9.52	9.1	11.4	36	5.8	7.6
2	0.05	0.05	23	15.72	15.6	22.9	69	8.6	13.7
	0.05	0.10	19	12.44	12.4	18.8	57	6.5	12.3
	0.05	0.25	13	7.69	7.6	12.4	39	3.7	8.8
	0.10	0.05	18	12.82	12.7	17.9	54	7.7	10.3
	0.10	0.10	15	10.30	10.2	14.8	45	5.7	9.1
	0.10	0.15	9	5.43	5.3	8.5	27	3.1	6.2
	0.25	0.05	11	8.62	8.2	10.9	33	5.5	5.9
	0.25	0.10	8	5.96	5.6	7.8	24	3.9	4.9
	0.25	0.25	5	3.37	3.2	4.7	15	1.8	2.8
3	0.05	0.05	10	5.43	5.4	9.9	30	2.9	6.1
	0.05	0.10	8	3.98	3.9	7.8	24	2.2	5.5
	0.05	0.25	6	2.61	2.6	5.6	18	1.3	3.9
	0.10	0.05	8	4.66	4.6	7.9	24	2.6	4.6
	0.10	0.10	6	3.15	3.1	5.9	18	2.0	4.1
	0.10	0.25	4	1.74	1.7	3.6	12	1.1	2.8
	0.25	0.05	5	3.37	3.2	5.0	15	1.9	2.6
	0.25	0.10	4	2.54	2.4	3.9	12	1.3	2.2
	0.25	0.25	2	0.96	0.86	1.7	6	0.61	1.3
5	0.05	0.05	5	1.97	1.9	5.0	15	1.1	3.3
	0.05	0.10	4	1.37	1.4	3.9	12	0.83	2.9
	0.05	0.25	3	0.82	0.81	2.7	9	0.47	2.1
	0.10	0.05	4	1.74	1.7	4.0	12	0.99	2.5
	0.10	0.10	3	1.10	1.1	2.9	9	0.73	2.2
	0.10	0.25	3	1.10	1.1	2.9	9	0.40	1.5
	0.25	0.05	2	0.96	0.86	1.9	6	0.71	1.4
	0.25	0.10	2	0.96	0.86	1.9	6	0.50	1.2
	0.25	0.25	1	0.29	0.26	0.8	3	0.23	0.68

NOTE: If either n, T_s, or T^* is specified, the other two test parameters can be determined from the relationship:

$$\frac{\chi^2_{1-\alpha, 2r_0}}{2} = \frac{nT_s}{\theta_0} = \frac{T^*}{\theta_0}$$

For expected total accumulated test hours: $E_{\theta_0}(T^*) = \theta_0 E_{\theta_0}(r)$; $E_{\theta_1}(T^*) = \theta_1 E_{\theta_1}(r)$

For expected waiting time for n items on test; $E_{\theta_0}(WT) = \frac{1}{n} E_{\theta_0}(T^*)$; $E_{\theta_1}(WT) = \frac{1}{n} E_{\theta_1}(T^*)$

FOREWORD

This report, in two volumes, presents the final results of an ARINC Research Corporation study on methods for allocating weapon-system reliability requirements. The study was conducted under Air Force Contract AF 33(616)7468 for the Engineering Services Division, Directorate of Operational Support Engineering, Aeronautical Systems Division, Air Force Systems Command, U. S. Air Force, Wright-Patterson Air Force Base, Ohio. Mr. A. L. Cleveland, Procurement Data Branch, Engineering Services Division, was Project Engineer. Research started on July 1, 1960 and was completed November 30, 1961

The development of the allocation model, procedural methods, and data inputs required for implementation of the methods are described in detail in Volume I. One section is devoted to various reliability-testing techniques for determining compliance to allocated requirements, and also presents guidelines for the selection of appropriate test plans.

Volume II outlines the step-by-step procedure for implementing the allocation models. Two of the more complicated steps are detailed in appendices of Volume II. The basic data inputs and the procedure for using them are described in Appendix A. Methods for determining the feasibility of the system requirements are described in Appendix B. Detailed examples of the complete allocation procedure for serial, modified serial, redundant and bimodal systems are presented in Appendix C. Several sections of Volume I are duplicated or condensed in Volume II so that the latter may be self-contained and used independently of Volume I.

Principal contributors to the research and report were Mr. H.S. Balaban, Mr. H.R. Jeffers, Mr. D.O. Baechler, and Mr. R.T. Williams, Program Leader, all of ARINC Research Corporation, Washington, D.C.

This is the final report under contract AF 33(616)-7468. Contractor's report number is 152-2-274.

ABSTRACT

Volume 1

Methods for allocating a system reliability requirement to subsystem and lower levels were investigated for four basic system types; serial, modified-serial, redundant and bimodal. An allocation model is presented for determining unit (component, equipment or subsystem) reliability requirements based on such factors as feasibility of the overall system requirement, unit/system failure relationships, unit or functional capability, relative unit state-of-the-art, duty cycles, and gross unit environment. The input data required by the model together with means for determining compliance to the allocated requirements through reliability or life tests are also discussed in detail.

PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or conclusions contained herein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

Harvey R Shute

HARVEY R. SHUTE
Chief, Engineering Services Division
Directorate of Operational Support Engineering

TABLE OF CONTENTS

TABLE OF CONTENTS

1. INTRODUCTION

Military agencies responsible for providing weapon systems for operational use must translate the overall system requirement into quantitative reliability requirements at many system sublevels. Realistic and consistent reliability requirements for units, equipments, or subsystems must be assigned in order to achieve and demonstrate specified operational weapon-system reliability. This assignment is commonly called "reliability allocation."

The purpose of this study was to develop practical and rigorous methods for reliability allocation. Full consideration was given to essentiality, state-of-the-art, mission operating time and other factors which have a direct bearing. A secondary objective was to investigate means for ascertaining the extent of a contractor's compliance with the allocated requirements, and to develop guidelines for choosing appropriate reliability-testing procedures.

This volume presents the development of the allocation model, the required data inputs, and the procedures and guidelines for reliability testing. Volume II presents the step-by-step procedure for implementing the allocation model and the basic data inputs.

Manuscript released by authors 30 November 1961, for publication as an ASD Technical Documentary Report.

2. GENERAL DISCUSSION OF THE ALLOCATION PROBLEM

System design engineers must translate overall weapon system characteristics, including reliability characteristics into many detailed design and development specifications. Reliability allocation is the process by which reliability requirements are assigned to individual units† to achieve a required weapon system reliability. The prime requisite of the assignment of individual reliability requirements is that, when recombined, the total system requirement is met. The allocation process, however, is more than a mathematical equality. The reliability of individual units varies because of the type of function to be performed, the complexity of the unit, and the method of accomplishing the function, to name a few of the more important factors. The role that a unit plays in a particular system is another factor which enters into consideration in allocating reliability.

The various factors influencing unit reliabilities must be considered if the most economic and realistic requirements are to be specified. Today, the designer and his reliability specialists recognize these influences but, generally, they have no quantitative method of relating the various factors. The problem of realistic allocation is further complicated by the fact that detailed information on many of these factors, such as detailed designs and part distributions, is not available early in the system design and analysis phases. Consequently, reliability requirements on units, equipments, and subsystems are often assigned on the basis of past performance data, which have been arbitrarily adjusted to make system reliability equal to the combined reliabilities of these system sub-levels.

Individuals charged with preparing specified requirements have had little guidance in establishing realistic reliability requirements by a practical, nonsubjective method that relates the important factors affecting the system life-characteristic. Development of a practical allocation model including these basic reliability

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† The term "units" as used herein represents that level of the system at which the system requirement is to be allocated.

relationships, which can be applied with the limited information available at the early stages of the system life-cycle, was the primary objective of this study. A secondary objective was the development of guidelines for specifying reliability tests for determining contractor compliance with the allocated requirements.

The problem of allocating system reliability involves solving the basic inequality

$$f(\hat{R}_1, \hat{R}_2, \ldots \hat{R}_n) \geq R^* \tag{2-1}$$

where

$\hat{R}_i$ is the allocated reliability parameter for the i^{th} unit.

R^* is the system reliability requirement parameter.

f is the functional relationship between unit and system reliability.

For a simple series system, in which the R's represent probability of survival for a time period of t hours, equation (2-1) becomes

$$\hat{R}_1(t) \cdot \hat{R}_2(t) \ldots \hat{R}_n(t) \geq R^*(t) \tag{2-2}$$

Theoretically, an infinite number of solutions exist to equation (2-2) assuming no restrictions on the allocation. The problem is to establish a procedure capable of yielding a unique or a limited number of solutions by which consistent and reasonable reliabilities may be allocated. (For example, the allocated reliability for a simple unit of demonstrated high reliability should be greater than for a complex unit whose observed reliability has always been low.)

2.1 The Value of an Allocation Program

Although several methods for attacking the problem have appeared in the literature (some are briefly reviewed in Section 3), the need still exists for a standardized program for reliability allocation. Some of the benefits to be derived from establishing such a program follow:

(a) The well-meaning but ineffectual philosophy on reliability -- "we will do the best we can" -- would be replaced by a contractual obligation in the form of quantitative reliability requirements that force contractors to consider reliability equally with other system parameters such as performance, weight, and cost.

(b) Since an allocation forces contractors to plan on meeting specified reliability goals, improved design, procurement, manufacturing, and testing procedures would result. This would not only ensure a reliable system but, in the long run, should improve the state-of-the-art.

(c) Reliability allocation focuses attention on the relationships between component, equipment, subsystem, and system reliability, leading to a better and more complete understanding of the basic reliability problems inherent in the design.

(d) Requirements determined through an allocation procedure would be more realistic, consistent, and economical than those obtained through subjective or haphazard methods, or those resulting from crash programs initiated after bitter field experiences.

(e) A reliability allocation program can be used to achieve an optimum reliable system because it can provide for handling such factors as essentiality, cost, maintenance, weight, and space.

(f) The overall cost of such a program is negligible when compared to the savings of time and money expended in meeting specified reliability goals in addition to the substantial reductions of operational, maintenance and management costs that would be realized.

Implementation of an allocation program requires a quantitative contractual reliability requirement at the system level. The method by which the overall system requirement is determined was not considered in this study, but its meaning and feasibility were examined. Mutual acceptance by the Air Force and its contractors is also required of success and failure definitions, criticality of various failure modes, along with procedures and criteria for establishing methods for demonstrating conformance to the allocated unit requirements.

Moreover, the program and methods developed from this study can and should apply for suballocating reliability to lower levels within the primary allocation units. The allocation program is necessarily one of continual refinement. Original requirements determined at the design stage should be critically examined and revised as more experience, knowledge, and test data become available during the advance of the system life-cycle through the design, development and production phases.

3. SUMMARY OF PRESENT METHODS

The range of methods presently in use or those available for allocating reliability extends from the extremely simple to the extremely complex. Some of the more commonly used methods are described in this section, but no attempt has been made to evaluate any of them in detail. However, Section 4, which presents the approach developed in this study, also contains a discussion of the advantages and disadvantages of the methods collectively.

3.1 Air Force Specifications on Reliability Requirements

A review of Air Force specifications was made to determine present means for the quantitative assignment of unit, subsystem, and system reliability requirements. The following specifications, exhibits and standards were examined:

MIL-STD-441 - Reliability of Military Electronic Equipment, June 20, 1958.

MIL-R-25717C - Reliability Assurance Program for Electronic Equipment, March 9, 1959.

USAF Spec. Bulletin 506 - Reliability Monitoring Program for Use in the Design, Development and Production of Air Weapon Systems and Support Systems, May 11, 1959.

AFBM Exhibit 58-10 - Reliability Program for Ballistic Missiles and Space Systems, June 1, 1953.

MIL-R-26667A - Reliability and Longevity Requirements for Electronic Equipments, General Specification For, June 2, 1959.

MIL-R-26674 - Reliability Requirements for Weapons System, General Specification, June 18, 1959.

MIL-R-26474 - Reliability Requirements for Production Ground Electronic Equipment, June 10, 1959.

MIL-R-27173 - Reliability Requirements for Electronic Ground Checkout Equipment, July 6, 1959.

MIL-R-26484A - Reliability Requirements for Development of Electronic Subsystems or Equipments, April 18, 1960.

MIL-R-27542 - Reliability Program Requirements for Aerospace Systems, Subsystems and Equipment, June 28, 1961.

A significant result of the review was that only one specification, MIL-R-26474, provided for reliability requirements at system sub-levels, setting forth a firm procedure for determining equipment mean time between failures (MTBF) requirements. The method is based on a part class count (tubes, motors and relays, semi-conductors, and other electrical and mechanical parts are the four classes considered). A simple formula using average failure rates of these part classes is given for determining the minimum MTBF. This approach, of course, is not truly an allocation procedure since no consideration is given to the requirement of the system which contains the equipments.

MIL-STD-441 gives an allocation equation, which, in very general terms of failure rates, is equivalent to equation (2-2). The standard, however, does not specify how the allocation is to be performed. MIL-R-27073 gives one minimum MTBF for all systems and another for all major subsystems if MTBF's are not otherwise specified. USAF Spec. Bulletin 506, MIL-R-26674 and MIL-R-27542 specifically state that reliability allocations be performed with consideration given to importance (effect of failure), complexities, functions, time of operation, and environmental conditions.

Several of the general specifications either state or imply that reliability requirements shall be established in the detailed equipment or system specification. When contractors are specifically directed to establish requirements, this is accompanied by the mandate that due regard be given to the reliability of government furnished equipments destined for integration in a complete operational system.

From the above cited documents a lack of detailed and rigorous methods for allocating reliability is evident. These are general specifications, however; individual subsystem and equipment detailed specifications may have numerical reliability requirements based on some type of allocation procedure. Several of these are discussed in the following sections.

3.2 Basic Allocation Methods[†]

The methods described in this section are considered basic because little or no consideration is given to factors such as functional differences, unit essentiality, feasibility of the overall requirement and redundancy, or multimodal operation. The procedures are based on part failure (hazard) rates which are assumed to be constant. Two other necessary assumptions are that

(a) Unit failures are independent; and

(b) Failure of any unit will result in system failure, i.e., a serial system.

These assumptions lead to the following equations:

Let

$R_j(t)$ = reliability of the j^{th} unit over t operating hours,

$R(t)$ = reliability of the system over t operating hours.

Then

$$R(t) = R_1(t)\, R_2(t) \cdots R_n(t) \tag{3-1}$$

[†] More detailed descriptions of the methods reviewed in this section can be found in the following two references:

Electronic Industries Association (formerly RETMA), "Determination of Permissible Component Part Failure Rates," Electronic Applications Review, Volume 4, No. 1, September 1956, pp. 10-12.

Frederick, H.E., "A Reliability Allocation Technique," Proceedings of the Fourth National Symposium on Reliability and Quality Control, January 1958, pp. 314-317.

If

λ_j = failure rate of the j^{th} unit

λ_s = failure rate of the system

Equation (3-1) becomes

$$e^{-\lambda_s t} = e^{-\lambda_1 t} e^{-\lambda_2 t} \cdots e^{-\lambda_n t} \qquad (3\text{-}2)$$

Reliability allocations can be performed by essentially identical approaches for overall requirements expressed in terms of R(t) or in terms of λ. Allocations based on a system failure rate requirement is discussed first. Since a series system with constant failure rates is assumed, the method is identical for requirements stated in terms of mean life or MTBF which is the reciprocal of failure rate.

The method can be outlined by the following steps:

(i) Given a series system with n units, the system failure rate is equal to the sum of the unit failure rates. If λ^* is the system failure rate requirement, allocated unit failure rates $\hat{\lambda}_j$ must be chosen so that

$$\hat{\lambda}_1 + \hat{\lambda}_2 + \cdots + \hat{\lambda}_n \leq \lambda^*.$$

(ii) Obtain observed or estimated unit failure rates,

$$\lambda_1, \lambda_2, \cdots, \lambda_n.$$

(iii) Compute relative unit weights from the equation

$$w_j = \frac{\lambda_j}{\sum_{k=1}^{n} \lambda_k}$$

(iv) Since w_j represents the relative failure vulnerability of the j^{th} unit and $\sum_{j=1}^{n} w_j = 1.0$, the system failure rate requirement λ^* can be apportioned over the units by the formula

$$\hat{\lambda}_j \leq w_j \lambda^* \tag{3-3}$$

(If the equality sign holds, maximum allocated failure rates result which satisfy the requirement $\sum_{j=1}^{n} \hat{\lambda}_j = \lambda^*$).

For a system requirement expressed in terms of probability of survival over t hours, i.e., $R^*(t)$, the same formulas can be used since it is required that

$$R^*(t) = e^{-\lambda^* t} \leq e^{-\hat{\lambda}_1 t} e^{-\hat{\lambda}_2 t} \cdots e^{-\hat{\lambda}_n t}$$

and by equation (3-3)

$$R^*(t) \leq e^{-w_1 \lambda^* t} e^{-w_2 \lambda^* t} \cdots e^{-w_n \lambda^* t}$$

leading to the basic allocation formula

$$\hat{R}_j(t) \geq [R^*(t)]^{w_j}. \tag{3-4}$$

3.3 The AGREE Allocation Method

The reliability allocation method described in the AGREE Report† is somewhat more sophisticated than the methods discussed in the previous sections. The principal difference is that unit-complexity rather than unit-failure rates are used as the basis for the allocation. Also, unit importance or essentiality is considered explicitly in the allocation formula, by considering the relationship between unit and system failure.

The allocation formula is used to determine a minimum acceptable mean life for each unit (AGREE uses the term equipment) to satisfy a minimum acceptable system reliability. The assumption is made that units within the system operate independently and in series in their effect on mission success.

Unit complexity is defined in terms of modules, where a module is an electron tube, a transistor, or a magnetic amplifier, and its associated circuitry. Diodes represent half a module. AGREE states that for digital computers, where the module count is high, reductions should be made to the module count to allow for the fact that failure rates for digital parts are generally far lower than for radio-radar types.

The importance factor for the j^{th} unit is defined in terms of the probability of system failure if the j^{th} unit fails. If the importance factor of a unit equals one, the unit must operate satisfactorily for successful system operation; if it equals zero, then failure of the unit has no affect on the system operation with respect to the system-failure definition.

The specific basis of the allocation is to require that each module make an equal contribution to system success. An equivalent requirement is that each module have the same mean life or failure rate. By using the approximating formula $e^{-x} \approx 1 - x$ for small x, the allocated failure rate of the j^{th} unit is shown in the AGREE

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† Advisory Group on Reliability of Electronic Equipment, Office of the Assistant Secretary of Defense, Reliability of Military Electronic Equipment, June 4, 1957, pp. 52-57.

report to be

$$\hat{\lambda}_j = \frac{n_j\,[-\log R^*(T)]}{E_j t_j N} \qquad (3\text{-}5)$$

where

n_j is the number of modules in the j^{th} unit

E_j is the importance factor of the j^{th} unit

t_j is the number of hours the j^{th} unit will be required to operate in T system hours ($0 < t_j \leq T$)

N is total number of modules in the system.

The AGREE report cautions against use of the allocation formula for units of very low importance which, if included, will distort the allocation. The report also briefly discusses allocation when redundancy exists, but the formulas given are very poor approximations, at best.

3.4 Allocation Models Based on Cost Considerations

Papers have appeared in publications which present models for allocation based primarily on cost considerations.† The basic problem in using this approach is summarized below:

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† Breipohl, A.M., "A Unique Allocation of Required Component Reliability," Proceedings of the Seventh National Symposium on Reliability and Quality Control January 1961, pp. 189-202.

Truelove, A.J., "Mathematical Models for Optimizing Strategic Reliability and for Minimizing Cost," Proceedings of the Sixth Joint Military Industry Guided Missile Reliability Symposium, February 1960, Volume 2, pp. 87-108.

A system reliability requirement of R^* exists. Unit reliabilities $\hat{R}_j$ are to be assigned so that $R^* = \hat{R}_1 \hat{R}_2 \cdots \hat{R}_n$. The cost of achieving a unit reliability of $\hat{R}_j$ is $C(\hat{R}_j)$. The allocation method must solve for an optimum set of $\hat{R}_j$ so that the total cost

$$C = C(\hat{R}_1) + C(\hat{R}_2) + \cdots + C(\hat{R}_n) \qquad (3\text{-}6)$$

is a minimum and the system reliability goal is met.

One of the basic problems of this approach is to derive cost functions which are realistic as well as mathematically tractable. The cost functions should satisfy the logical requirements that

(a) $C(\hat{R}_j) > 0$

(b) $C(\hat{R}_j)$ is monotonically increasing with $\hat{R}_j$.

(c) $C(\hat{R}_j)$ increases rapidly as $\hat{R}_j$ approaches one and, in fact, becomes infinitely large as $\hat{R}_j$ approaches one.

Cost functions proposed in the papers by Breipohl and Truelove[†] are, respectively:

$$C(\hat{R}_j) = \frac{A_j}{1 - \hat{R}_j} e^{-B_j(1-\hat{R}_j)}$$

$$C(\hat{R}_j) = \frac{A_j}{(1 - \hat{R}_j)^k}$$

[†] Ibid.

where A, B and k are suitable constants. Both of these functions satisfy the logical requirements. Lagrange multipliers are necessary for minimizing equation (3-6) subject to the constraint of R*. Because of the mathematical complexity involved in using exact formulas, approximations are employed. One common to both cost functions is the formula

$$R^* \approx 1 - \sum_{j=1}^{n} (1 - R_j)$$

which is valid only for R* close to one.

4. BASIS OF THE ALLOCATION MODEL

4.1 Criteria for Effective Allocation

In order to realize the benefits that can be derived from a reliability allocation program, the following criteria were used as guidelines in developing the allocation model:

(a) The model must be generally applicable. It should not be restricted to a limited number of system configurations or classes of equipment. It should be capable of accommodating the various levels of system complexity, i.e., assemblies, equipments, and subsystems, and it should be appropriate for suballocations within these levels. The model must also be applicable at the various phases of the system life-cycle starting from the early design stage.

(b) The model should be based on the ultimate use of standard input data. This will provide a common basis for comparisons and, to a great extent, will eliminate variance in allocated requirements due to subjective influences. The standardized data should be amenable to adjustment for stress or environmental factors pertaining to the particular application.

(c) The methods provided in the model must be economically feasible. The implementation of a reliability allocation program depends on its costs and time requirements in relation to other design and development tasks, as well as upon the increased degree of assurance that system reliability goals will be achieved. The methods, therefore, should not require highly specialized personnel and should not take an undue amount of time for application.

(d) The model must yield realistic and attainable requirements. The model should, in the course of application, provide for determining if the overall system requirement is feasible in order to assure attainable allocated unit requirements. Realistic requirements can only be assured if the model provides means for including the important factors influencing the unit/system reliability relationships.

Examination of these criteria indicates that a balance must be sought between model sophistication and the degree of success in meeting each of the criteria. The first three criteria require that the allocation model should be relatively simple and easy to apply; an overly simplified model, however, may not satisfy the fourth criterion.

4.2 Factors to Consider

In order to develop a suitable reliability allocation model, all possible factors and influences that may be necessary for inclusion must be considered. The following list contains many of the factors which, to various degrees, are important in reliability and allocation:

- Basic Objective
  - System reliability requirement
  - Feasibility of the requirement
- Unit Capability
  - State-of-the-art
  - Complexity
- Failure Characteristics
  - Failure definitions
  - Failure relationships
  - Failure modes
  - Time-to-failure distributions
  - Environmental and stress relationships
- System Design
  - Unit or functional importance
  - Redundancy
  - Duty cycles
  - Maintenance factors

Cost

Cost of achieving reliability goals
Cost of non-conformance
Cost of proving conformance.

A mathematical allocation model that includes all of the above factors in a rigorous manner would be extremely complex. Also, that each of the above elements can be stated in numerical or mathematical terms is highly unlikely. On the other hand, to allocate reliability on a subjective, non-scientific basis is obviously unwise. Therefore this list must be pared down to those elements absolutely essential; provision for adjustments based on good engineering judgment for factors not explicitly included must be made in the allocation program.

4.3 Factors Included in the Allocation Model

This section describes those factors and influences which are considered to be of sufficient importance for inclusion, explicitly, in the mathematical allocation model and which are believed to satisfy the criteria listed in Section 4.1. These factors, therefore, will have to be translated into quantitative terms or be capable of mathematical representation and analysis. The method for handling the described factors in the allocation is discussed in Section 5, which derives the basic allocation model. Section 6 contains a detailed discussion of the data inputs developed in this study that relate to these factors.

4.3.1 System and Failure Definitions

The system under consideration must be clearly defined in terms of its functions and boundaries. The conditions that constitute failure or unsatisfactory performance can be determined from a study of the operational demands and the functional requirements of the system. These conditions can then be translated into measurable unit characteristics. The boundaries surrounding the system and each unit must be clearly defined to insure that important items are neither neglected nor considered more than once.

4.3.2 System Reliability Requirement

The primary element in a reliability allocation model is the system reliability requirement. It is usually determined on the basis of ultimate user requirements and feasibility, but it may derive from an allocation performed at a higher echelon. The requirement may be stated in any appropriate measure such as mean life, system failure rate, or, preferably, a reliability over a fixed period of time.

The success probability requirement on a weapon system may be based on the desires of field personnel who, naturally, think in terms of the probability that the system can successfully complete some specific mission, probably under wartime conditions. The supplier of the system cannot, however, design or test the system under these same conditions. The translation, therefore, must be made in the writing or interpretation of a specification, which requires certain measurable system and equipment parameters to be within specified limits under specified environmental conditions, with the hope that hardware meeting these requirements will also fulfill the military mission. This leads to the concept of system effectiveness which is a function of at least two factors -- reliability and design adequacy.

As an example, a procurement specification may require that a rifle eject a bullet of specified weight with a specified muzzle velocity within a specified dispersion cone or C.E.P. The rifle's reliability is the probability that it will accomplish this task under given environmental conditions. The design adequacy of the same rifle, however, may vary from nearly unity if it is used as an antipersonnel weapon at close range to practically zero if it is being used to fire at high-altitude jet aircraft, even though its reliability is constant under the two conditions.

If the possibility exists that a system which is performing all its designed functions satisfactorily can still fail to accomplish the mission, the system requirement may be subject to misinterpretation. The following factors must be considered:

> System Effectiveness $S^*(T)$ = probability that the system can successfully meet a stated operational demand for T hours of operation under stated conditions.

System Reliability $R^*(T)$ = probability that the system will satisfactorily perform its designed functions for T hours of operation under stated conditions.

System Design Adequacy D_s = probability that satisfactory performance of designed functions will lead to accomplishment of the mission.

Probability-of-success requirements on systems which have design adequacies less than one (1.0) shall be considered to be system effectiveness requirements unless otherwise stated. The system reliability requirement is related to the system effectiveness requirement by the formula †

$$R^*(T) = \frac{S^*(T)}{D_s} .$$

Design adequacy must be determined before allocation in order to obtain $R^*(T)$. Theoretical investigations, Monte Carlo simulations, or experimentations may be necessary to estimate D_s. Since design adequacy is usually a function of many variables such as system accuracy, environmental conditions, and system inputs, an average value for D_s may be used by considering the relative frequency distribution of these parameters. It is probable that, at the design stage, system design adequacy will have to be assigned on an intuitive basis after careful consideration of the operational demands on the system and the abilities of various units to meet these demands.

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† For the general case, system effectiveness is related to reliability by the formula, $S^*(T) = R^*(T) \cdot D_s \cdot P_{or}$, where P_{or} is the operational readiness defined as the probability that, at any point in time, a system is either operating satisfactorily or is ready to be placed in operation, on demand, when used under stated conditions. For the purpose of allocation, P_{or} is assumed to be one. A thorough discussion of these concepts will be found in the following report: ARINC Research Monograph No. 9, "Concepts Associated With System Effectiveness," E. L. Welker, R. C. Horne, Jr., ARINC Research Corporation, July 15, 1960, Publication No. 123-4-163.

4.3.2.1 Feasibility of the System Reliability Requirement

An inherent part of an allocation procedure is the assessment of the feasibility of the overall reliability requirement. The allocation model provides for such assessment through comparison with past reliability experience on systems of similar complexity and gross environment. A mathematical model for incorporating the reliability improvement to be expected from redundancy and multimodal operation is also developed.

4.3.3 Unit State-of-the-Art

State-of-the-art measures are required in order to determine the relative reliabilities of the allocation units within the system. These are the basic data inputs in a typical reliability allocation procedure and are usually stated in terms relatable to the measure used for the system reliability requirement. Relative average failure rates are the state-of-the-art measures adopted in this study and, as shown in the next section, they will give exact answers for units with constant failure rates; furthermore, they represent a reasonable approach for most other typical failure densities. A detailed discussion of the actual data inputs developed, in addition to their use in the allocation model, is set forth in Section 6.

4.3.4 Relationships Between Unit and System Failure

The relationships between unit failure and system failure must be determined before the allocation is made. Four types of basic relationships, for which allocation methods are presented, are as follows:

(1) Serial system: no functional duplicates exist and each unit must operate successfully for system success.

(2) Modified serial system: no functional duplicates exist but units can fail without necessarily causing system failure.

(3) Redundant system: components of the system are duplicated for increased reliability but each redundant path or mode of operation is equally effective in performing its function.

(4) Multimodal system: redundant paths or modes of operation are not equally effective in performing their function.

These unit/system failure relationships require consideration of the two associated factors, namely, unit essentiality and modal design adequacy.

4.3.4.1 Unit Essentiality

The concept of essentiality, used to describe the effect of unit failure on mission success, is considered only if a failed unit has no functional duplicate. It is defined as follows:

> The essentiality of a unit is the probability that the system will fail to accomplish its mission if the unit fails while all other units perform satisfactorily.

An example of a unit which might have an essentiality less than one is a radar beacon transmitter on a satellite used for tracking purposes. If the beacon fails after the orbit has been firmly established, the orbital position may possibly be obtained through mathematical analysis.

Unit essentiality must be considered in the allocation of reliability of modified serial systems; it may also be involved in redundant and multimodal systems. At the design stage of system development, the likelihood is that the essentiality of various units within the system will have to be assigned intuitively on the basis of experience gained with similar systems. If appropriate system failure data is available, essentiality can be estimated by the ratio,

$$E_j = \frac{\text{Number of Mission Failures due only to } j^{th} \text{ Unit Failure}}{\text{Number of } j^{th} \text{ Unit Failures}}$$

4.3.4.2 Modal Design Adequacy

A mode of operation is defined as a unique combination of components which are required to perform the system function. Modal design adequacy is defined as follows:

> Modal design adequacy is the probability that, given satisfactory operation in the mode, the system will accomplish its mission.

A redundant system is defined to be one having more than one mode of operation (because of functional duplicates) but with equal design adequacies for each mode (e.g., the components in each mode are identical). For allocation purposes, the groups of components which have functional duplicates are termed redundant units.

A multimodal system is defined as one incorporating more than one mode of operation (because of functional duplicates), each mode having a different design adequacy (e.g., secondary modes result in some degradation in performance). For allocation purposes, the groups of components which have functional duplicates are termed modal units. In determining values for modal design adequacy, the discussion of system design adequacy in Section 4.3.2 is applicable.

4.3.5 Unit Duty Cycles

Duty cycles must be included in an allocation model to reflect any variance in unit operational time requirements with respect to systems operation-time. Units which have a limited operational period because of a low duty cycle (e.g., the hydraulic system of a airplane) should have a relatively high allocation over the system operating period.

4.3.6 Other Factors

Section 7 presents a discussion of the analysis and interpretation of the allocation. Factors such as cost, state of design, and type of research and development effort, which are not explicitly included in the allocation model, are considered. These factors are the basis of trade-offs among the mutual allocated unit reliabilities to yield an optimum set of requirements.

5. MATHEMATICAL RELIABILITY ALLOCATION MODEL

5.1 Requirements on the Model

The basic allocation model developed in this study is predicated on the factors discussed in Section 4.3,[†] namely,

System and Failure Definitions
System Reliability Requirement
Unit State-of-the-Art
Unit/System Failure Relationships
Unit Essentiality
Unit Duty Cycles

In general, the following requirements exist for any reliability allocation model:

(a) Allocated unit reliability increases as unit state-of-the-art decreases.

(b) Allocated unit reliability increases as essentiality increases.

(c) Allocated unit reliability increases as duty cycle or required time of operation decreases.

(d) Units in a system with equal essentiality, duty cycle and state-of-the-art should have the same allocated reliability whether in series or in a redundant configuration within the same system.

5.2 Basic Assumptions

The following two basic assumptions are made in developing the allocation model:

[†] The model for determining feasibility of the system requirement is discussed in Section 6.8

(1) Allocation units can be so chosen that failure probabilities are independent.

(2) Unit state can be described in discrete terms of success and failure.

These two assumptions greatly simplify the mathematics of allocation and are believed to be reasonable for the purposes of a design-stage reliability allocation procedure. With regard to the first assumption, if components within the system are known to be dependent, they may possibly be grouped into one allocation unit, making the failure probability of this unit independent of the state of other units. The state-of-the-art of this unit can then be adjusted for the dependence that exists, thereby minimizing error.

The second assumption is reasonable in the sense that reliability, by definition, requires that satisfactory performance be uniquely defined. In practice, this is often a most difficult problem, and success/failure definitions sometimes are necessarily somewhat arbitrary. If allocation is primarily regarded as a procedure for defining parameters of reliability acceptance tests, such tests usually require that this second assumption be satisfied in order to determine if a unit has passed. In this case, the same success/failure definitions for such tests should hold for allocation. The allocation model does not require explicit success/failure definitions, but since the input data is based on success/failure appraisals of field personnel, it is implicitly assumed that similar appraisals can be made for the units under consideration.

5.3 Derivation of the Model for Serial or Modified Systems

The basic allocation equations for serial and modified serial systems are developed in this section. The model is a modification and extension of that presented in the AGREE report which was described in Section 3.3. Many of the AGREE recommendations for further study have been followed and solutions obtained. The major modification is that because of the provided input data, the AGREE requirement that "...each module make an equal contribution to mission success...." is unnecessary. Other modifications include the distinction between functions, allowance for active element type and environment differences, and the inclusion of design adequacy. It is also believed that the models developed for simple redundant and bimodal systems represent a significant improvement over those presented in the AGREE report.

5.3.1 General Notation

$S^*(T)$ - the system effectiveness requirement for T hours of operation

$R^*(T)$ - the system reliability requirement for T hours of operation

D - the design adequacy of the system

t_j - required operating time of the j^{th} unit over T system hours ($j = 1,2,\ldots, n$).

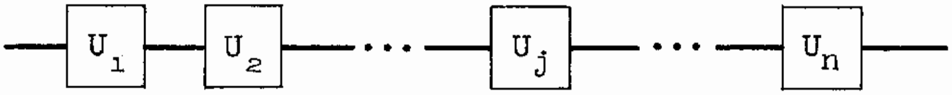
E_j -- essentiality of the j^{th} unit

K_j - failure index of the j^{th} unit (the measure of unit state-of-the-art derived from the basic data inputs described in Section 6)

$R(t_j)$ - reliability of the j^{th} unit for t_j hours of operation.

5.3.2 Serial Systems

In this section the basic allocation equation for serial systems is derived and shown to be equal to the allocation equation given by Equation (3-4). A reliability block diagram of a serial system with n units is shown below. The symbol U_j represents the j^{th} unit.



The assumption is that an overall requirement exists on the system. If the system has a design adequacy(D) less than one and the requirement pertains to mission or system effectiveness, the system reliability requirement can be estimated by the equation

$$R^*(T) = \frac{S^*(T)}{D} \qquad (5\text{-}1)$$

From the input data provided by this study, a measure of unit state-of-the-art in terms of expected total relative failure rate can be obtained. This measure is called the unit failure index. It is obtained through consideration of complexity, type of function, part types, environment, etc.† If the j^{th} unit has a failure index of K_j, the total system failure index, K, is defined by

$$K = \sum_{j=1}^{n} K_j \qquad (5\text{-}2)$$

and the failure index ratio or relative weight of each unit is

$$w_j = \frac{K_j}{K} \qquad \left(\sum_{j=1}^{n} w_j = 1.0\right) \qquad (5\text{-}3)$$

The basis for the allocation is that each unit of 1/K has an equal effect on system reliability in the same sense that each unit of failure rate has an equal effect on reliability. Since w_j is the number of (1/K) units for the j^{th} equipment, the contribution of the j^{th} unit to system unreliability is, in some way, proportional to the failure index ratio w_j. It will now be shown that the

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† See Section 6 for explanation.

use of w_j as exponent weighting factors is a reasonable approach for determining allocated unit reliabilities, given a serial system reliability requirement.

Under the assumption of unit independence, the system reliability function can be expressed by

$$R(T) = \prod_{j=1}^{n} R(t_j)$$

where $R(t_j)$ is the reliability function of the j^{th} unit which is required to operate t_j hours during T system hours of operation.

The density function of system failure-times is, by definition of the reliability function,

$$f(t) = \frac{-d[R(t)]}{dt} .$$

The hazard rate which is the instantaneous rate of failure is generally defined by

$$z(t) = \lim_{h \to 0} \frac{R(t) - R(t+h)}{h\, R(t)}$$

$$= \frac{f(t)}{R(t)}$$

$$= - \frac{d\,[\log R(t)]}{dt}$$

Since $\log R(T) = \sum_{j=1}^{n} \log R(t_j)$ for a serial system, the system hazard rate is

$$z(T) = -\frac{d\,[\log R(T)]}{dT} = -\frac{d\left[\sum_{j=1}^{n} \log R(t_j)\right]}{dt_j}$$

or

$$z(T) = \sum_{j=1}^{n} z(t_j) \qquad (5\text{-}4)$$

where $z(t_j)$ is the hazard rate function of the j^{th} unit.

Equation (5-4) says that the system hazard rate is equal to the sum of the unit hazard rates under the assumption of independence of unit failures.

Since

$$-\frac{d\,[\log R(T)]}{d(T)} = z(T),$$

we have, by integrating both sides,

$$\log R(T) = -\int_0^T z(\tau)\, d\tau$$

or

$$R(T) = e^{-\int_0^T z(\tau)\, d\tau} \qquad (5\text{-}5)$$

Substituting for $z(\tau)$ by equation (5-4), we have the basic relationship,

$$R(T) = e^{-\sum_{j=1}^{n} \int_0^{t_j} z(\tau_j)\, d\tau_j} \qquad (5\text{-}6)$$

For the exponential distribution, $Z(\tau_j)$ is constant over time and is equal to what is commonly called the failure rate, say λ_j. For other failure densities, if the hazard rate function is approximately constant over t_j hours, e.g., the effects of wearout in time period $(0, t_j)$ are negligible, the $Z(\tau_j)$ in equation (5-6) can be replaced by average hazard rates estimated from appropriate failure data. A common formula for obtaining such an estimate is

$$\lambda_j = \frac{\text{Number of failures in } (0,\ t_j)}{\text{Total accumulated operating time}}$$

The substitution for $Z(\tau_j)$ by λ_j in equation (5-6) yields the relationship

$$R(T) = e^{-\lambda_s T} = e^{-\sum_{j=1}^{n} \lambda_j t_j} \qquad (5\text{-}7)$$

where λ_s is the single estimate for $z(\tau)$ $(0 \leq \tau \leq T)$

λ_j is the single estimate for $z(\tau_j)$ $(0 \leq \tau_j \leq t_j)$

If a system requirement of $R^*(T)$ exists, one can find an equivalent λ^*_s by Equation (5-7). Since $\lambda_s T = \sum_{j=1}^{n} \lambda_j t_j$ allocated average failure rates of $\hat{\lambda}_j$ must be determined so that

$$\sum_{j=1}^{n} \hat{\lambda}_j t_j = \lambda^*_s T$$

A reasonable approach is to replace each $\lambda_j t_j$ in Equation (5-7) by $w_j \lambda_s{}^* T$ since the reliability contribution of the j^{th} unit is proportional to w_j. Hence, allocated average unit failure rates can be determined from the equation

$$\hat{\lambda} t_j = w_j \lambda^*_s T \tag{5-8}$$

or

$$R^*(T) = e^{-\lambda^*_s T} = e^{-w_1 \lambda^*_s T} \, e^{-w_2 \lambda^*_s T} \ldots \, e^{-w_n \lambda^*_s T}$$

Thus the allocated reliability of the j^{th} unit for t_j operating hours is

$$\hat{R}(t_j) = e^{-w_j \lambda^* T} \tag{5-9}$$

or

$$\hat{R}(t_j) = [R^*(T)]^{w_j}$$

$$\left[\text{Since } \sum_{j=1}^{n} w_j = 1.0, \right.$$

$$\left. \prod_{j=1}^{n} \hat{R}(t_j) = \prod_{j=1}^{n} [R^*(T)]^{w_j} = R^*(T) \right]$$

Under the assumption of constant failure rates, $R(t) = e^{-\lambda t}$, and therefore allocated failure rate or mean life requirements are obtained as follows:

$$\hat{\lambda}_j = - \frac{\log \hat{R}(t_j)}{t_j} \tag{5-10}$$

$$\hat{\theta}_j = - \frac{t_j}{\log \hat{R}(t_j)} \tag{5-11}$$

By using the approximation $\hat{R}(t_j) = e^{-w_j \lambda^* T} \approx 1 - w_j \lambda^* T$ which is quite good for $R(t_j) \geq 0.9$, we have

$$\hat{\lambda}_j \approx - \frac{\log [1 - w_j \lambda^* T]}{t_j}$$

Upon applying the equivalent approximation to the numerator in the opposite direction to partially cancel errors,

$$\begin{aligned} \hat{\lambda}_j &= \frac{w_j \lambda^* T}{t_j} \\ &= - \frac{w_j \log R^*(T)}{t_j} \end{aligned} \tag{5-12}$$

For units known to have a failure rate which is not constant over its operating time, an average failure can be allocated by

$$\hat{\lambda}_j = \frac{1 - \hat{R}(t_j)}{t_j} \tag{5-13}$$

5.3.3 Modified Serial Systems

For modified serial systems, one or more units have essentialities less than one and therefore these units may fail without necessarily causing system failure. The probability that the system will not fail due to failure of the j^{th} unit is

$$1 - E_j \, [1 - R(t_j)] \qquad (5\text{-}14)$$

Under the assumption of independent unit failures and serial operation, a good approximate formula for system reliability is

$$R(T) = \prod_{j=1}^{n} \left\{ 1 - E_j \, [1 - R(t_j)] \right\} \qquad (5\text{-}15)$$

This formula is approximate in the sense that it implies independence of unit essentialities, e.g., the probability of system failure given failure of units A and B is $E_a \, E_b$. Since E will most likely be one for the majority of units, the above equation is reasonable.

If $R^*(T)$ is the system reliability requirement, the allocated contribution of the j^{th} unit to system reliability as given by Equation (5-9) is

$$[R^*(T)]^{w_j}$$

Hence, by Equation (5-14), $\hat{R}(t_j)$ must be chosen so that

$$1 - E_j \, [1 - \hat{R}(t_j)] = [R^*(T)]^{w_j}$$

or

$$\hat{R}(t_j) = 1 - \frac{1 - R^*(T)^{w_j}}{E_j} \qquad (5\text{-}16)$$

This is the formula derived in the AGREE report. It is important to note that E_j must be greater than $1 - R^*(T)^{w_j}$ in order to avoid negative reliability allocations. In most practical situations, especially where weight and space is at a premium, units with low essentiality and high failure indices are not common. If a unit does exist for which $E_j > 1 - R^*(T)^{w_j}$, it is recommended that this unit be eliminated from the allocation and the failure index ratios, w, of the remaining units be recomputed.

Failure rate and mean life allocation equations can be derived in the same manner as for serial systems. By computing $\hat{R}(t_j)$ from Equation (5-16), equations (5-10), (5-11), and (5-13) remain unchanged. The approximate formulas for failure rate and mean life allocations become

$$\hat{\theta}_j = - \frac{w_j \log R^*(T)}{E_j t_j}$$

$$\hat{\lambda}_j = - \frac{E_j t_j}{w_j \log R^*(T)} \qquad (5\text{-}17)$$

5.4 Redundant Systems

A redundant system is defined in this section to be one where some (or possibly all) of the elements have functional duplicates for purposes of increasing system reliability. Each redundant path or mode of operation is assumed to be equally effective in performing its function, i.e., the design adequacies of all modes of operation are equal. Equation (5-1) applies for translating a system effectiveness requirement to a system reliability requirement.

Two specific redundancy types are considered:

(a) Active-parallel or continuous redundancy where all redundant units are continuously energized.

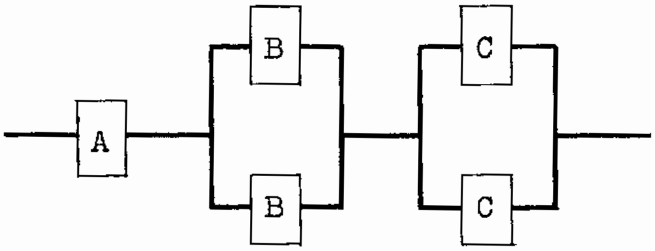
(b) Standby or sequential redundancy where only one of the redundant units is energized at any one time.

If switching is involved (as it always is for standby redundancy), the probability of premature switching (switching when not required) shall be assumed to be relatively small as compared to the probability of failure to switch when required. The switching mechanism, if it is subject to failure, can therefore be considered as a series unit.

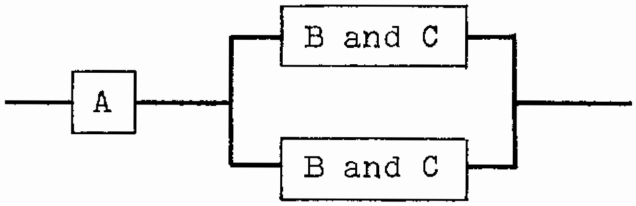
The following model applies only to redundant systems which contain a single redundant configuration, i.e., only one unit or one group of units is duplicated. The degree of redundancy is fixed at two, i.e., there are only two paths of operation for the particular function which is duplicated. The latter restriction was made primarily because of the belief that, at the design stage, redundancy is not and should not be used extensively since the technique can be employed much more effectively after allocations are made and predictions or laboratory tests performed to determine possible trouble areas. The extension of the model to degrees greater than two is easily made and briefly discussed in Section 5.4.1.

The restriction on the number of redundant configurations is also justified by the above argument and, in addition, the complexity of the allocation model is greatly increased for more than one configuration. If the system has two or more redundant configurations, an approximation that will yield conservative allocations can be made.

Assume two units are duplicated in a redundant design. The reliability block diagram of the system will therefore be as shown below.



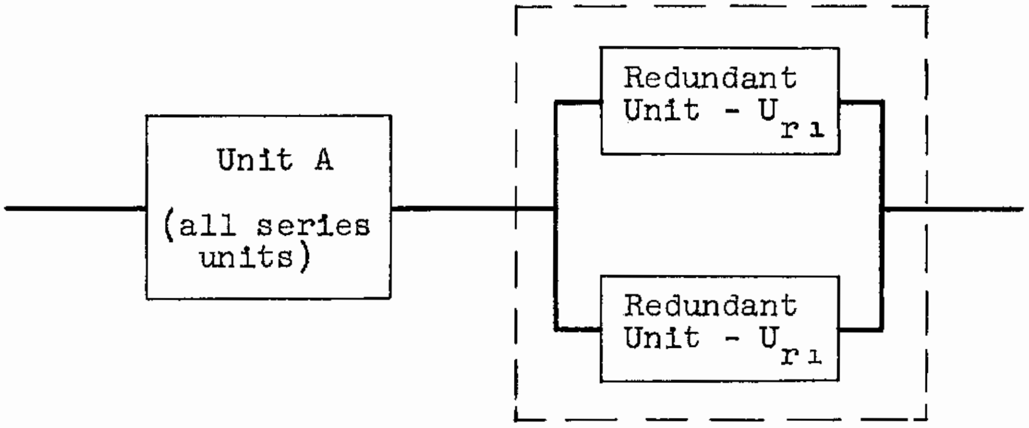
"A" represents all series units. By eliminating the cross-connects between the B and C configurations, the block diagram reduces to the following:



This is a single redundant configuration for which the model applies. The model also permits allocation to the individual B and C units as well as to the redundant configuration and to the redundant units composed of B and C. Since the reliability of the second system is generally lower than that of the first, the reliabilities allocated will be somewhat higher than actually required.

5.4.1 Identical Redundant Paths

The reliability block diagram of a system with a single redundant configuration consisting of identical redundant paths is shown in the following figure:



Redundant Configuration - U_r

Unit A represents the combination of all series units. Unit U_{r_1} is a redundant unit (possibly including more than one allocation unit) which is duplicated to form the redundant configuration. K_a shall be used to designate the total failure index of the series units, and K_{r_1} the total failure index of each redundant unit.

The approach used to allocate the system reliability requirement to Unit A, to the redundant configuration, and to each redundant unit is to determine an equivalent complexity for the redundant configuration K_r which will justify use of the basic allocation formulas for serial or modified serial systems. The derivation for determining K_r is given below.

Equation (5-9) gives the basic allocation equation for serial systems

$$\hat{R}_j = R^{*w_j} \text{ or } w_j = \frac{\log \hat{R}_j}{\log R^*}$$

(operating times can be neglected for the present.)

Since redundant configuration U_r is in series with Unit A, allocations based on Equation (5-9) can be performed if values can be found for w_a, the failure index ratio of U_a, and for w_r, the failure index ratio of U_r. By definition,

$$w_a = \frac{K_a}{K_a + K_r}$$

$$w_r = \frac{K_r}{K_a + K_r} \qquad (5\text{-}18)$$

where K_r is as yet undetermined.

If some combination of the units that make up Unit A (the series unit) had a total failure index of K_{r_1} (the failure index of each redundant element), the failure index ratio of this combination is

$$w_{r_1} = \frac{K_{r_1}}{K_a + K_r} \qquad (5\text{-}19)$$

Since units with the same failure index ratio are required to have the same allocated reliability (assuming equal essentiality and duty cycle) whether in series, parallel, or both, w_{r_1} above is also the failure index ratio of U_{r_1} and U_{r_2}. By equations (5-9), (5-18), and (5-19)

$$\frac{w_r}{w_{r_1}} = \frac{\log \hat{R}_r}{\log \hat{R}_{r_1}} = \frac{K_r}{K_{r_1}}$$

Hence

$$K_r = \frac{K_{r_1} \log \hat{R}_r}{\log \hat{R}_{r_1}}$$

Substituting for K_r in equation (5-18) yields

$$w_r = \frac{K_{r_1} \log \hat{R}_r}{K_a \log \hat{R}_{r_1} + K_{r_1} \log \hat{R}_r} \qquad (5\text{-}20)$$

Since we also have

$$w_r = \frac{\log \hat{R}_r}{\log R^*}$$

equation (5-20) can be rewritten and simplified to

$$\log \hat{R}_r = \frac{K_{r_1} \log R^* - K_a \log \hat{R}_{r_1}}{K_{r_1}}$$

or

$$\log \hat{R}_r = \log R^* - \alpha \log \hat{R}_{r1} \tag{5-21}$$

where
$$\alpha = \frac{K_a}{K_{r1}} \tag{5-22}$$

In general, $\hat{R}_r$ is some function of $\hat{R}_{r1}$, the allocated reliability of the redundant units, e.g., for active-parallel redundancy

$$\hat{R}_r = 2\hat{R}_{r1} - (\hat{R}_{r1})^2$$

Hence, by the inverse relationship,

$$\hat{R}_{r1} = 1 - (1 - \hat{R}_r)^{1/2}$$

(Note: Since relationships of this type exist for any number of redundant units, the model applies to all degrees of redundancy.)

Writing $\hat{R}_{r1}$ as some function, $\hat{R}_{r1} = f(\hat{R}_r)$, we have from equation (5-21)

$$\log \hat{R}_r = \log R^* - \alpha \log [f(\hat{R}_r)] \tag{5-23}$$

For a given α and R^*, equation (5-23) can be used to determine $\hat{R}_r$ for a specific type of redundancy. The remainder of the system (Unit A) is then allocated a reliability of $R^*/\hat{R}_r$. It is possible, however, to use equation (5-23) to determine K_r directly as shown below.

From equation (5-18)

$$K_r = \left(\frac{w_r}{1 - w_r}\right) K_a \tag{5-24}$$

By equation (5-9)

$$w_r = \frac{\log \hat{R}_r}{\log R^*}$$

Hence, for a given α and R^*, equation (5-23) can be used to obtain $w_r(\alpha, R^*)$. This enables us to obtain the ratio

$$Z(\alpha, R^*) = \frac{w_r(\alpha, R^*)}{1 - w_r(\alpha, R^*)} \qquad (5\text{-}25)$$

Then from equation (5-24)

$$K_r = Z(\alpha, R^*)\, K_a \qquad (5\text{-}26)$$

Nomographs have been constructed giving values of $Z(\alpha, R^*)$ for wide ranges of α and R^* for both active-parallel and standby redundancy. (These nomographs are presented in Volume II as Figures 9 to 12.) Once K_r is determined, the total failure index of the system can be found by

$$K = K_1 + K_2 + \cdots + K_m + K_r$$

where K_1 to K_m are the failure indices of the units in series (represented by K_a in the above derivation).

Failure index ratios are then found by

$$w_j = K_j/K$$

for each series unit, the redundant configuration and for each redundant unit as well. The allocation equations for serial or modified serial systems then apply.

5.4.1.1 Duplicate Systems

For designs where the complete system is duplicated, allocation is relatively simple. For active-parallel operation, the reliability requirement for each system is

$$\hat{R}_{r_1}(T) = 1 - [(1 - R^*(T)]^{1/2}$$

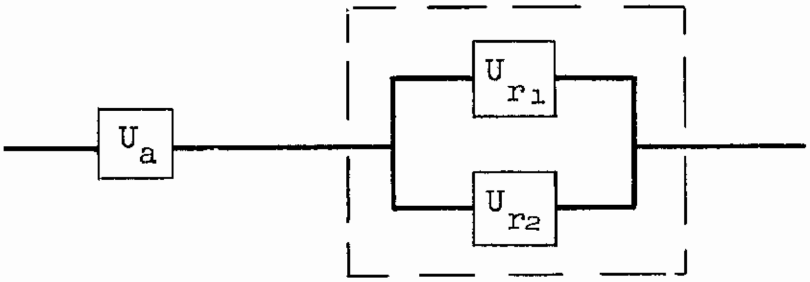
$\hat{R}_{r_1}(T)$ can then be considered to define $R_{r_1}{}^*(T)$, the reliability requirement of each system, which then can be suballocated among the units of the system by methods previously described. For standby redundancy (under the assumption of constant failure rates),

$$R^*(T) = \hat{R}_{r_1}(T)\ [1 - \log \hat{R}_{r_1}(T)]$$

For a given $R^*(T)$, $\hat{R}_{r_1}(T)$ can be graphically determined and suballocations within the system for $R_{r_1}{}^*(T) = \hat{R}_{r_1}(T)$ can be performed.

5.4.2 Dissimilar Redundant Paths

Assume the block diagram of the system is as shown below.



Redundant Configuration - U_r

U_a represents all units which are in series. The redundant configuration, U_r, is composed of two dissimilar redundant units, U_{r_1} and U_{r_2} which are equally effective in performing the required function. K_a will be used to designate the failure index of U_a, and K_{r_1} and K_{r_2} the failure indices of U_{r_1} and U_{r_2}, respectively. The approach used to allocate reliability is to find a failure index for each redundant unit, K'_r, so that for a given time period

$$R_r\,(K'_r,\ K'_r) = R_r\,(K_{r_1},\ K_{r_2})$$

where $R_r\ (K_i,\ K_j)$ represents system reliability for a given time period, given redundant unit failure indices of K_i and K_j.

Given an equivalent failure index of K'_r, equation (5-26) can be used to obtain K_r, the failure index of the redundant configuration, and the basic allocation equations then obtain. The following discussion is limited to redundant units which have approximately constant failure rates.

5.4.2.1 Active-Parallel Redundancy

Let $\bar{\lambda}$ represent the average failure rate of the normalizing function. The $K_j\bar{\lambda}$ represents the absolute failure rate of the j^{th} unit since K_j is obtained as the sum of component failure rates in the unit relative to the normalizing function. The reliability function for two units in an active-parallel redundant configuration is

$$R(t) = e^{-\lambda_{r_1}t} + e^{-\lambda_{r_2}t} - e^{-(\lambda_{r_1}+\lambda_{r_2})t}$$

where λ_{r_1} and λ_{r_2} are the failure rates of the redundant units. The problem then is to find a value of K'_r so that

$$e^{-K_{r_1}\bar{\lambda}t} + e^{-K_{r_2}\bar{\lambda}t} - e^{-(K_{r_1}+K_{r_2})\bar{\lambda}t} = 2e^{-K'_r\bar{\lambda}t} - e^{-2K'_r\bar{\lambda}t}$$

If we use the approximation $e^{-x} = 1 - x + x^2/2$, the above equation reduces to

$$K'_r = (K_{r_1} \cdot K_{r_2})^{1/2} \tag{5-27}$$

This approximate formula for K'_r is generally quite satisfactory. Equation (5-26) can then be used to obtain K_r and the allocations for the units in series, for the redundant units, and for the redundant configurations are obtained by the allocation equations given for serial or modified serial systems.

5.4.2.2 Standby Redundancy

From the general reliability function of a standby redundant configuration, K'_r must be determined so that

$$\frac{K_{r_1}}{K_{r_1}-K_{r_2}}\, e^{-K_{r_2}\overline{\lambda} t} - \frac{K_{r_2}}{K_{r_1}-K_{r_2}}\, e^{-K_{r_1}\overline{\lambda} t} = e^{-K'_r\overline{\lambda} t}\,(1 + K'_r\overline{\lambda} t)$$

The same approximation for e^{-x} as was used for active-parallel redundancy can also be employed to obtain an estimate for K'_r. This expression, however, will yield, for the right hand side, a term that involves $\overline{\lambda}^3 t^3$. Since $\overline{\lambda}$ will be quite small (say on the order of 20×10^{-6}), $\overline{\lambda}^3 t^3$ is negligible for the range of t usually involved. On dropping the term involving $\overline{\lambda}^3$, the approximate formula for K'_r is identical to equation (5-27). Tests of fifteen pairs of K_{r_1} and K_{r_2} showed an average error of 2% for K'_r. The maximum error was about 10% which occurred for the extremely unlikely ratio of $K_{r_1}/K_{r_2} = 100$.

Equation (5-26) can be used to obtain K_r and then basic allocation equations obtain.

5.5 Multimodal Systems

A multimodal system is defined in this study to be a system with redundant paths or modes of operation which are not equally effective in performing their function Modal design adequacy, D_i, shall be used to represent the probability that, given satisfactory operation in the i^{th} mode, the system will accomplish its mission. Therefore, a multimodal system is one with differing modal design adequacies. (Redundant systems have identical modal design adequacies.) For allocation purposes, the groups of units which have functional duplicates are termed modal units.

The allocation model described applies specifically to bimodal systems (two modes of operation); this limitation was made for the same reasons given in Section 5.4 in the discussion pertaining to the similar limitation on the amount of redundancy.

Because different modal design adequacies prohibit an exact determination of the system reliability requirement, R*(T), from the system effectiveness requirement, an approximate method is presented for estimating R*(T) by first estimating an average design adequacy for the system.

5.5.1 Types of Bimodal Operation

To evaluate the relationships between system effectiveness, modal design adequacy, and unit reliability for all types of bimodal operation would be impossible. This is apparent if one considers a complex weapons system such as a fighter plane, which may have several mission objectives, since the type of bimodal operation is dependent on the mission and modal function. Two types of operation for which an allocation model is developed are described below. These types are necessarily very general; the bimodal operation of a specific system will most likely deviate from the types discussed. However, for allocation purposes, such deviations, unless very large, probably will not invalidate the results.

Type I - Uncommitted Case

In the uncommitted case, the operator or decision maker can determine if a reliable mode (all modal design functions are satisfactory) will result in mission success. Where an indication of mission failure in a selected mode appears, sequential switching to alternate modes is possible until the desired objective is attained. Failure occurs only when the objective is not attained after all modes are exhausted. Note that this case also includes the situation in which all modes are used simultaneously to perform the function. An example of a Type I case is a communication system in which the amount of time permitted to get a message through is long enough to allow trial of all possible transmitting modes.

(The term "continuous operation" will be used to denote the situation in which both modes are continuously energized. If the secondary mode is not activated until required, the term "sequential operation" will apply.)

Type II - Committed Case

The committed case occurs (1) when the operator or decision maker has no way of assessing whether reliable operation in a given mode will lead to mission success, or (2) even if assessment is possible, it is too late to switch to an alternate mode. (This situation does not preclude modal switching if a component in the modal unit fails to perform as specified.) An example would be a reconnaissance satellite with a mission to obtain information over a particular area. If an optical mode is selected and cloud cover exists over the area, the mission might fail even though no modal failure was experienced.

NOTE: For both the uncommitted and committed cases, modal design adequacy may depend on when a mode is initially activated, e.g., a dead-reckoning mode in an aircraft navigation system. These cases are not considered.

5.5.2 Assumptions and Conditions

The assumptions and conditions by which the effectiveness equations for bimodal systems were derived are listed below. Most systems follow these restrictions to a great extent and, for the purpose of design stage allocation, they are reasonable. In any case, the methodology used in deriving formulas for bimodal systems is general enough to serve as a framework for developing an allocation model that is applicable to a specific system.

(a) If the system has more than one possible mission, a primary mission can be selected.

(b) A system effectiveness requirement exists for the selected primary mission.

(c) Average modal design adequacies can be estimated for the primary mission.

(d) The mode with the greater design adequacy is selected initially; the alternate mode is not activated unless

(1) the primary mode fails, or

(2) it can be determined that the primary mode will not yield satisfactory results (Type I case), or

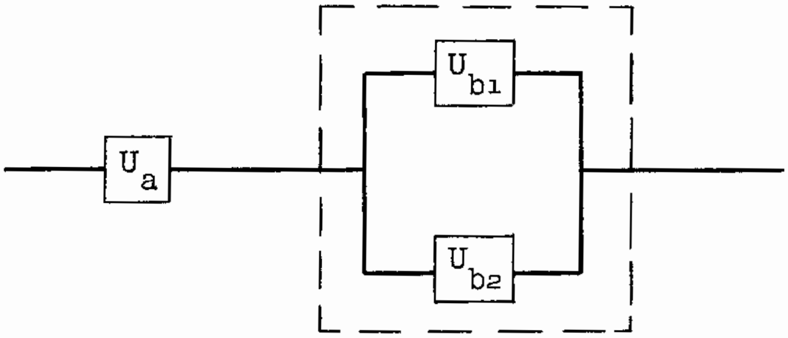
(3) both modes operate continuously.

(e) Modal switching is in one direction only -- from the primary to the alternate mode.

(f) Modal switching is failure-free, or the switching mechanism is considered to be another series unit (see discussion of switching failure in Section 5.4).

5.5.3 Derivation of Allocation Formulas

This section derives the formulas for reliability allocation for the two types of bimodal systems discussed. The following reliability block diagram and unit identification will be used throughout:



Bimodal Configuration - U_b

U_a represents all units in series

U_{b1} and U_{b2} are the modal units

U_b is the bimodal configuration

The design adequacy of the primary mode (U_a, U_{b1}) shall be designated by D_1; the design adequacy of the alternate mode (U_a, U_{b2}) shall be designated by D_2.

The approach used is essentially the same as that for redundancy. A failure index K_b is assigned to each modal unit that yields the same contribution to system effectiveness as the true modal unit failure indices of K_{b1} and K_{b2}. Knowing K_b', the equivalent failure index of the bimodal configuration K_b can be obtained from equation (5-26). Total system complexity is then calculated and basic allocation formulas can then be used to allocate the reliability requirement over the units.

5.5.3.1 Estimating the System Reliability Requirement

The first requisite of the model is to estimate the system reliability requirement $R*(T)$ given the system effectiveness requirement of $S*(T)$. This can be accomplished by first obtaining a preliminary estimate of the allocated reliability of the primary modal unit. Without alternate mode capabilities, the system would be of the serial type and have a total failure index of $K_s = K_a + K_{b1}$. The reliability requirement of this equivalent series system is, by equation (5-1).

$$R_s*(T) = S*(T)/D_1 .$$

Hence, the preliminary estimate of the allocated reliability of the primary modal unit U_{b1} is

$$\hat{r}_{b1} = \left[\frac{S*(T)}{D}\right]^{K_{b1}/K_s} \qquad (5\text{-}28)$$

The average design adequacy of the bimodal system is, therefore,

$$\bar{D} = \hat{r}_{b1} D_1 + (1 - \hat{r}_{b1})\, D_2 \qquad (5\text{-}29)$$

and the system reliability requirement of the bimodal system can be estimated by

$$R*(T) = \frac{S*(T)}{\bar{D}} \qquad (5\text{-}30)$$

5.5.3.2 Estimating K_b'

The procedure for estimating K_b' is similar to that described in Section 5.4.2, namely to find a K_b' that satisfies the equation

$$S_b\ (K_b', K_b') = S_b\ (K_{b_1}, K_{b_2})$$

where $S_b\ (K_i, K_j)$ represents system effectiveness for a given time period, given modal unit failure indices of K_i and K_j. Continuous operation of the two modes shall be assumed since, as in the redundancy case, the formula for K_b' assuming continuous operation will not usually be significantly different than that for sequential operation.

In order to obtain the approximation formula, it is desirable to express the modal design adequacies as an exponential relationship, i.e., the value for D_i can be expressed as

$$D_i = e^{-d_i \lambda T} \qquad (5\text{-}31)$$

where $d_i \lambda T$ is a constant. λ in this case is the average failure rate of the normalizing function required to meet the system requirement and can be estimated by

$$\lambda = \frac{-\log R^*(T)}{K_s T} \qquad (5\text{-}32)$$

Given D_1 and D_2, the modal design adequacies of the two modes, we have

$$d_1 = \frac{-\log D_1}{T}, \quad d_2 = \frac{-\log D_2}{T} \qquad (5\text{-}33)$$

For the uncommitted case with continuous operation, both modes are operating and the system effectiveness formula is given by

$$S(T) = D_1 R_{b1}(T) + D_2 R_{b2}(T) - D_1 D_2 R_{b1}(T)\, R_{b2}(T)^{\dagger}$$

By assuming the exponential failure law and replacing D_i by equivalent expression given in equation (5-31), we have

$$S(T) = e^{-(d_1+K_{b1})\lambda T} + e^{-(d_2+K_{b2})\lambda T} - e^{-(d_1+d_2+K_{b1}+K_{b2})\lambda T} \tag{5-34}$$

The system effectiveness equation if both modal failure indices were equal to K_b' is

$$S(T) = e^{-(d_1+K_b')\lambda T} + e^{-(d_2+K_b')\lambda T} - e^{-(d_1+d_2+2K_b')\lambda T} \tag{5-35}$$

On equation the right hand sides of equations (5-34) and (5-35) and solving for K_b' by using the approximation $e^{-x} = 1 - x + x^2/2$ we find

$$K_b' = \frac{1}{2}\left[-(d_1+d_2) + \sqrt{(d_1+d_2) + 4(K_{b1}K_{b2}+d_1K_{b2}+d_2K_{b1}}\right] \tag{5-36}$$

$\dagger$ The right hand side should actually be multiplied by $R_a(T)$, the reliability of the series portion of the system. For this discussion, however, $R_a(T)$ can be ignored.

For the committed case with continuous operation, the second mode does not perform the system function unless the primary mode fails. Hence, assuming that the design adequacy concept applies only after a particular mode is committed,

$$S(T) = D_1 R_{b1}(T) + [1 - R_{b1}(T)]\, R_{b2}(T)\, D_2$$

Using the same approach as for the uncommitted case, the approximate formula for K_b' is

$$K_b' = \frac{1}{2}\left[(d_1-d_2) + \sqrt{(d_1-d_2)^2 - 4K_{b1}(d_1-d_2-K_{b2})}\right] \quad (5\text{-}37)$$

Two points should be noted

(a) If K_{b1} and K_{b2} are nearly equal, K_b' can be approximated very satisfactorily by

$$K_b' = (K_{b1} \cdot K_{b2})^{1/2}$$

(b) If $D_1 = 1.0$, equations (5-36) and (5-37) are identical since $d_1 = 0$; hence an uncommitted case can always be assumed if the primary mode has a design adequacy nearly 1.0. This is intuitively acceptable since, for a committed case, the alternate mode is activated only if the primary mode fails. If $D_1 = 1.0$, the alternate mode in an uncommitted case is also activated only upon primary modal failure since reliable operation of the primary mode will always lead to mission success.

Once K_b' is determined, the procedure for determining the allocated unit reliabilities is identical to that for redundant systems, except for the bimodal configuration allocation. Because of different modal design adequacies, the reliability allocation for the bimodal configuration (if one is required) is best computed from the reliabilities allocated to the modal units. Therefore, for continuous operation

$$\hat{R}_b(t_b) = \hat{R}_{b1}(t_b) + \hat{R}_{b2}(t_b) - \hat{R}_{b1}(t_b)\,\hat{R}_{b2}(t_b) \qquad (5\text{-}38)$$

and for sequential operation

$$\hat{R}_b(t_b) = \frac{L_2}{L_2 - L_1}\,\hat{R}_{b1}(t_b) - \frac{L_1}{L_2 - L_1}\,\hat{R}_{b2}(t_b) \qquad (5\text{-}39)$$

where

$$L_1 = \log \hat{R}_{b1}(t_b) \quad \text{and} \quad L_2 = \log \hat{R}_{b2}(t_b)$$

6. INPUT DATA

The methods of analysis used in the preparation of the data required for the allocation procedure are described in this section. The data and the preliminary steps for their use are presented in Appendix A, Volume II of this report.

6.1 Unit State-of-the-Art Measure

The unit failure indices (K_1) are the basic failure measures required by the mathematical allocation model. These indices are relative measures of the reliability state-of-the-art of the individual units to which reliability requirements are being allocated. Although the unit failure indices are not available directly, they can be derived from relative functional failure rates of individual active element groups as part of the allocation procedure.

An active element group (AEG) is defined as consisting of an active element (a part capable of a single valving or controlling action) and its associated group of passive parts. Tubes, transistors, diodes, and magnetic cores are all examples of electronic active elements, while resistors, capacitors, and transformers are examples of passive parts. From the above definition, it may be seen that dual stage tubes are considered as two active elements each, which, with the associated group of passive parts, constitute two active element groups. In the preparation of the data for allocation procedures, two exceptions were made to the above definition: (1) solid state diodes of less than one watt dissipation were considered as active elements only in the special case of digital computers; and (2) dual stage tubes were considered as one active element if the two stages were connected entirely in parallel. The unique problems associated with the AEG definition in non-electronic areas will be discussed in Section 6.3.1.

Consideration of the information available on the system and its units at the early design stage led to the choice of the AEG as the basis of analysis. Ordinarily, this information is limited to functional descriptions of the system and its units. From the description of the functional requirements of a unit (each function corresponding to one performed by an AEG), the numbers and types of functions that will be required on the AEG level can be estimated.

The data are presented as relative functional failure rates (relative to a standard function) for the various types of functions performed by AEG's. Essentially, the unit failureindex used in the allocation procedure is the sum of the relative failure rates of the individual AEG's of the unit. The data required to compute this sum (Procedural Steps of Appendix A, Volume II) consists of the basic relative functional failure rates;the adjustments in the relative rates for the various types of active elements or AEG's used to perform the function; and the changes in the relative rates for various special characteristics of the unit, such as gross environment. Accordingly, these three types of data are presented and discussed separately in the sections that follow.

6.2 Standard Electronic Functions

6.2.1 Identification of Functional Categories

A preliminary step in the analysis of electronic failure data was the establishment of functional categories of AEG's. The categories are presented in Table 6-1 and each includes various specific electronic functions. The relative failure rate, assigned to each of the nine categories, is expected to be characteristic of any function or combination of functions within the category.

Functions falling within any given category are assumed to have a relatively common set of stress conditions and a fairly uniform distribution of parts. On this premise, they also may be expected to have a fairly common failure rate. The differences between the categories generally reflect variations in application stress, part distribution, or tolerance requirements. In all but one case, the functional category is defined in terms of the performance criteria rather than the part types employed to accomplish the function. The special, or exotic, category, however, is defined in terms of the type of active element employed. (The identification of a special function is somewhat arbitrary, as the definition may overlap

TABLE 6-I

DEFINITIONS OF FUNCTIONAL CATEGORIES OF STANDARD ELECTRONIC FUNCTIONS

Functional Categories[1]	Definition	Examples
Audio	Active element groups acting on or supplying signals of an audio range used as audio output without further detection	Detectors Audio amplifiers
Primary Power	Active element groups acting to supply, modify, or control electrical power (as opposed to signals) in a form suitable to act as a power input for other active element groups	Rectifiers Rectifier bridges Voltage regulator tubes
Pulse, High Power (> 1 watt)	Active element groups acting on or developing signals of a pulse nature greater than one watt	Trigger circuits
Pulse, Low Power[2] (≤ 1 watt)	Active element groups acting on or supplying signals of a pulse nature equal to or less than one watt on the average	Trigger circuits Blocking oscillators
RF, High Power (> 1 watt)	Active element groups acting on or supplying high-frequency signals greater than one watt -- i.e., those not presentable as audio, servo, or video without detection or an equivalent operation	RF output stages
RF, Low Power (≤ 1 watt)	Active element groups acting on or supplying high-frequency signals equal to or less than one watt -- i.e., those not presentable as audio, servo, or video without detection or an equivalent operation	IF amplifiers RF amplifiers Local oscillators
Servo	Active element groups acting on or supplying signals used to perform a servo (electromechanical) function, drive a servo element, or transmit electromechanical information. (Low frequency servo carrier AEG's are classed as servo.)	Servo amplifiers
Special[3]	Non-normal AEG types containing klystrons, magnetrons, or hydrogen thyratrons, etc. Sometimes classed as exotic tube types	
Video	Active element groups acting on or supplying signals of a form to be presented as video output without further detection or equivalent operation, and active element groups which serve to present video information	Video amplifiers Cathode ray tubes

[1] No separation according to power has been made for categories other than RF and Pulse. In any case, however, it is advisable to assign a higher relative failure rate to AEG's handling extremely high power.

[2] The 'Pulse, Low Power' category is not considered to include computer digital applications. The functional categories and relative rates of digital computers are discussed in Section 6.4.1

[3] The 'Special' category is quite general with respect to the nature of the AEG's involved, and greater variation in relative failure rates of AEG's can be expected in this category than in any of the other standard categories. When possible, the data for the 'Special' category should be supplemented by experimental observation of relative failure rates for the specific AEG type.

other functional categories. In the data analysis, active element groups containing magnetrons, klystrons, and hydrogen thyratrons were included in the exotic category.)

The justification for a separate category in this case lies primarily in the extremely high failure rates that have been observed for these devices in the past. The various types in the exotic category do not, however, appear to have the same high degree of uniformity of influencing factors found for the other categories (e.g., the power levels, bandwidth requirements, and associated part distributions for a magnetron are frequently quite different than for a hydrogen thyratron).

Separate categories have been established reflecting differences in power dissipation in only two cases, RF and pulse. While such a division is probably appropriate for the other categories as well, the incidence of extremely high power applications is quite rare, and such cases are to be treated as exceptions with higher relative failure rates. In only one such case was any data available -- a system contained two audio AEG's each with approximately 100 watts power dissipation. The observed failure rates were 27.5 times greater than the failure rates of lower power audio AEG's in the same system.

The above functional categories will doubtless be sufficient for the great bulk of electronic applications. Most of the functional groups found in digital computers, however, are specifically excluded both from the above groupings and whenever the expression "standard electronic functions" appears in the text. This is primarily due to the feeling that part population and stress levels in digital computers are not sufficiently similar to those found in any of the above categories to allow common consideration. A more thorough discussion of digital computers is given in Section 6.4.1.

6.2.2 Source of Data

The relative failure rates for standard electronic functions (relative to the audio function) are derived from the failure data accumulated by ARINC Research Corporation during the course of several military contracts. Table 6-2, which lists the types of electronic systems involved, indicates that the coverage is quite broad -- it involves seventeen systems which cover many areas of military electronics applications. The electronic data represent approximately two million hours of equipment operation and over 1.3 billion AEG hours.

TABLE 6-2

SYSTEMS FROM WHICH ELECTRONIC FAILURE DATA WERE ACCUMULATED

Equipment Type	Number of Types Studied	Gross Environment	Reference
Communication Receivers	6	Shipboard	1
Communication Transmitters	4	Shipboard	1
Radar Repeaters	2	Shipboard	1
Fire Control Radar	1	Shipboard	1
Search Radar	1	Airborne	2
Communications Transceivers	2	Airborne	3
Bomb/Nav System	1	Airborne	4

1 "Effects of Cycling on Reliability of Electronic Tubes and Equipments," Volumes 1 and 2, ARINC Research Corporation Publication No. 101-26-160, 30 June 1960.

2a "Reliability of the AN/APS-20E Radar System," ARINC Research Corporation Publication No. 101-11-139, 15 May 1959.

2b "Maintainability and Reliability of the AN/APS-20E Radar System", ARINC Research Corporation Publication No. 101-33-180, 1 September 1960.

3a "Effects of Maintenance Procedures on the Reliability and Maintainability of an Airborne Communication Equipment," ARINC Research Corporation Publication No. 101-32-179, 1 September 1960.

3b "Reliability and Maintainability of the AN/ARC-34 UHF Communications Equipment," ARINC Research Corporation Publication No. 137-1-251, 31 July 1961.

4 Air Force Reliability Assurance Program Progress Report No. 1, ARINC Research Corporation Publication No. 81, 15 February 1956.

6.2.3 Data Analysis Required for Computation of Relative Failure Rates

The required analysis for computing relative failure rates was performed in a series of steps set forth in the following pages. The analysis assumes that the AEG's of any one system have a common operating time. In cases where duty cycles varied for different parts of the system, a separate analysis was performed for each part. Other than this, there is no requirement for a knowledge of operating time.

(1) The first step of the analysis for each equipment is the identification of each active element group according to its functional category. Only the active element involved rather than the entire group needs to be defined for this step. Considerable care is required for the identification in the case of dual section tubes; for this analysis such a tube is counted as two separate active elements except in those cases where the corresponding elements of the sections were connected in parallel for such reasons as greater power capacity, etc. Unused tube sections were not counted. The number of active element groups of each category for each equipment were then tabulated as in Table 6-3.

(2) The second step of analysis for the individual equipment is to assign each part failure to its proper functional category. Thus only the failed parts have to be identified with their corresponding AEG's. The data available provide the exact identification of each failed part in terms of its schematic symbol, part number, etc., and a study of the circuit diagram allows the failure to be associated with the proper active element in order to assign the failure to the appropriate functional category. In some cases of dual section tubes where the two sections were not of the same functional category, the failures were divided evenly between the functional categories of the two sections. Considerable care was required in the case of cluster removals to insure that any given AEG was never recorded as having more than one failure for any one maintenance action. The results of this

TABLE 6-3

FAILURE DATA FOR INDIVIDUAL ELECTRONIC SYSTEMS

Equipment Type †		A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Equipment Hours		44,257	7,219	16,000	3,472	326,196	86,851	71,988	24,072	200,858	53,879	29,661	60,122	112,868	462,681	445,272
No. of Sets in Test		24	12	54	14	26	5	5	2	16	4	2	4	9	33	29
Functional Categories																
Audio	No. of AEG's	1	26	25		14	14	12	14			8	124	12	12	8
	No. of Failures	0	28	55		64	5	24	3			1	174	5	26	33
Primary Power	No. of AEG's	55		6	48	4	4	7	4	42	26	3	3	4	4	3
	No. of Failures	133		9	34	42	8	24	0	65	24	5	20	44	149	51
Pulse High Power	No. of AEG's				3								74			
	No. of Failures				20½								107			
Pulse Low Power	No. of AEG's	81			107					65	59		3			
	No. of Failures	85			28					50	91		22			
RF High Power	No. of AEG's		3	3		3		5	3				9			
	No. of Failures		19	29		443		118	31				58			
RF Low Power	No. of AEG's	21	43	42	16	3	20	21	6			16	38	13	14	16
	No. of Failures	17	188	246	1	101	72	66	1			3	59	20	123	146
Servo	No. of AEG's	38	2	6	309	1	1	6	1	6	6		11			
	No. of Failures	55	2	4	63	5	1	15	0	2	1		186			
Special	No. of AEG's	15			3					4			86			
	No. of Failures	484			42½					97			232			
Video	No. of AEG's	121			119					85	18					
	No. of Failures	214			61					260	38					

† Further identification of systems is not possible due to security restrictions.

step are the number of failures observed for each equipment for each functional category, which, together with the results of Step (1), are the data presented in Table 6-3.

(3) The data given in Table 6-3 form the basis for estimating average functional failure rates relative to a normalizing function. A maximum likelihood approach was used to obtain these estimates. If we let A represent the normalizing function and B some other function, the general representation of Table 6-3 would be as follows:

Function	Equipment			
	1	2	. . .	n
A	T_{a1}	T_{a2}	. . .	T_{an}
	r_{a1}	r_{a2}	. . .	r_{an}
B	T_{b1}	T_{b2}	. . .	T_{bn}
	r_{b1}	r_{b2}	. . .	r_{bn}

where T_{aj} and T_{bj} represent the total AEG hours for the A and B functions of the j^{th} equipment (T = number of AEG's times the number of equipment hours); r_{aj} and r_{bj} represent the number of failures for the A and B functions in the j^{th} equipment.

The likelihood function (L) assuming a Poisson process is

$$\prod_{j=1}^{n} \frac{e^{-\lambda_{aj}T_{aj}} (\lambda_{aj}T_{aj})^{r_{aj}}}{(r_{aj})!} \quad \frac{e^{-\lambda_{bj}T_{bj}} (\lambda_{bj}T_{bj})^{r_{bj}}}{(r_{bj})!}$$

Taking the logarithm of L and dropping all terms not involving the parameters λ_a and λ_b, we have

$$\text{Log } L = \sum_{j=1}^{n} \left[r_{aj} \log \lambda_{aj} + r_{bj} \log \lambda_{bj} \right] - \sum_{j=1}^{n} \left[\lambda_{aj} T_{aj} + \lambda_{bj} T_{bj} \right]$$

Assuming that differences in the relative failure rates λ_b/λ_a over the equipments are due only to sampling variation, we can let $k_b = \lambda_{bj}/\lambda_{aj}$ for all j. Then $\lambda_{bj} = k_b\lambda_{aj}$ and

$$\text{Log } L = \sum_{j=1}^{n} \left[r_{aj} \log \lambda_{aj} + r_{bj} \log (k_b \lambda_{aj}) \right] - \sum_{j=1}^{n} \left[\lambda_{aj} T_{aj} - k_b \lambda_{aj} T_{bj} \right]$$

Using the straightforward maximum likelihood method would lead to a complex set of equations for which solution would be most difficult. Instead, the maximum likelihood estimate for λ_{aj} (denoted by $\hat{\lambda}_{aj}$) can be substituted in the equation

$$\frac{\partial [\text{Log } L]}{\partial k_b} = 0$$

yielding the equation

$$\frac{\sum_{j=1}^{n} r_{bj}}{k_b} - \sum_{j=1}^{n} \hat{\lambda}_{aj} T_{bj} = 0$$

from which one obtains the estimate

$$\hat{k}_b = \frac{\sum_{j=1}^{n} r_{bj}}{\sum_{j=1}^{n} \hat{\lambda}_{aj} T_{bj}}$$

Replacing $\hat{\lambda}_{aj}$ by r_{aj}/T_{aj}, we have

$$\hat{k}_b = \frac{\sum_{j=1}^{n} r_{bj}}{\sum_{j=1}^{n} \frac{n_{bj}}{n_{aj}} r_{aj}} \qquad (6\text{-}1)$$

where n_{aj} and n_{bj} are the number of AEG's of type A and B in the j^{th} equipment.

The denominator of equation (6-1) is an adjustment of the actual number of function A failures to the number of A failures that would have been accumulated if T_{aj} were equal to T_{bj} for $j = 1, 2, \ldots n$.

Since there are many missing entries in Table 6-3, it was expedient to use primary power as the normalizing function and assume one AEG and zero failures for equipment B. This yielded a set of functional failure rates relative to prime power which were then converted to a set of failure rates relative to the audio function. The results of this analysis are presented in Table II of Volume II, Appendix A.

6.3 Relative Failure Rates of Non-Electronic AEG's

6.3.1 Identification of AEG Equivalents

The active element group concept has not as yet been sufficiently developed for rigorous application to any field other than electronics. The data available for analysis consists of failure rate data for various parts of mechanical, electromechanical, hydraulic, pneumatic, and other non-electronic fields. Each of these parts has been considered as representing an AEG equivalent in performing the relative failure rate analysis. (For ease in presentation, the term AEG is also used to represent non-electronic functions which are utilized in the allocation.)

6.3.2 Description of Data

The analysis in the case of non-electronic AEG's is somewhat similar to that for the electronic cases. However, data limitations and necessary approximations in deriving the relative failure rates yield results that are not as consistent or reliable as those for the standard electronic functions.

The relative failure rates are derived from the failure data on seventy-two systems tabulated in the following report.

WADD Technical Report 60-330, "A Compilation of Component Field-Reliability Data Useful in Systems Preliminary Design," SECRET, D. E. Johnston, T. S. Durand, Aeronautical Systems Division (formerly Wright Air Development Division), Wright-Patterson Air Force Base, Ohio.

In some cases, direct reference to this report for appropriate data on specific AEG types may be desirable. The list of parts for which relative failure rates are available is, unfortunately, small, but it represents those items for which sufficient information was available at this writing.

The reference above provides failure information by part type on seventy-two systems. The data consists of information on one or more of the following:

Part hours of operation,
Part failures, and
Part failure rate.

As the discussion below will demonstrate, only those systems can be used for which sufficient data is available to determine both the number of failures and the total operating time. As failure rates only are reported for a number of the systems, these are eliminated from the analysis.

Due to the lack of detailed information on the electronic AEG's of the systems involved, it was not possible to derive relative failure rates with respect to any specified functional category as was done in the case of the standard electronic functions. Instead, relative rates were derived with respect to the average of the electronic AEG's, and provisions are made (see Section 6.5) for converting these to relative failure rates with respect to audio as part of each allocation.

6.3.3 Determination of Average Relative Failure Rates for Non-Electronic Functions

The method of computing the average relative failure rate for each type of non-electronic function involves the same maximum likelihood estimation used for the standard electronic functions in Section 6.2.3. The formula as applied to non-electronic functions is:

$$k_i' = \frac{\sum_j r_{ij}}{\sum_j \frac{r_{ej}\, T_{ij}}{T_{ej}}} \qquad (6\text{-}2)$$

where

k_i' = average relative failure rate of the i^{th} non-electronic type of AEG with respect to the relative failure rate of an average electronic AEG.

r_{ij} = number of failures of the i^{th} AEG type in the j^{th} equipment.

T_{ij} = total accumulated hours of i^{th} AEG type in the j^{th} equipment.

T_{ej} = total accumulated hours of electronic AEG's in the j^{th} equipment.

r_{ej} = number of failures of the electronic portion of the j^{th} equipment.

The values of r_{ij}, T_{ij} and T_{ej} are available directly from the tables of the source document. The values of r_{ej}, however, were computed for each system as the sum of

The number of tube failures

The failures reported for electronic parts except batteries.

The failures of variable capacitors, variable resistors, switches, connectors, and potentiometers, which were included in the lists of electromechanical parts in the source document.

The above calculation necessarily introduces systematic errors into the data system due to the approximations required. Three approximations, noted below, each lead to bias in the same direction, namely, a tendency toward higher apparent failure rates of average electronic AEG's, with a corresponding reduction of the apparent relative failure rate of the non-electronic AEG's.

(a) The number of active element groups in each system was assumed to be equal to the number of tubes. Neglecting the existence of dual section tubes reduces the number of AEG's counted, raising the apparent relative failure rate.

(b) Including all the part failures listed above, possibly covers many part failures that were actually associated with non-electronic AEG's (e.g., motor capacitors, starting switches, and electrical connectors) resulting in a decrease of the apparent relative failure rate of the non-electronic AEG's.

(c) The possibility of individual electronic part failures being reported more than once leads to a third reduction of the non-electronic relative failure rates. This occurred for some parts listed under a major part-type heading and one of the subdivisions (e.g., a particular failure may have been listed under resistors (general) and under resistors, carbon composition.)

Due to the limited information available on each system, the above approximations were required in order to perform the necessary analysis. To account for the bias introduced, a second calculation was performed on three systems listed in the reference which were also included in the ARINC Research Corporation in-house data used to derive the relative failure rates of the standard electronic function. For each of the three systems, relative failure rates with respect to average electronic AEG's were derived for non-electronic AEG's which were in the systems, and the results were compared with the relative failure rates derived from the reference source. For each of these systems, proper accounting was possible for the dual section tubes, and the part failures associated with non-electronic AEG's; and, due to the nature of the available in-house data, there was no concern with multiple reporting of any one failure. The results of this analysis indicated that the apparent relative failure rates previously calculated were low by a factor of one half which was used as a general correction factor. The corrected relative failure rates resulting from this analysis are presented in Tables III through V of Volume II, Appendix A.

6.4 Relative Failure Rates for Unlisted Functions

In many cases, implementation of the allocation procedure will require relative functional failure rates not presently available because of lack of suitable data. To a reduced extent, this deficiency may be expected to continue indefinitely, due to the rapid expansion of technology and accompanying increase in the number of types of systems, components and part categories. Therefore, estimation procedures for filling existing data gaps are required, and form part of the allocation procedure.

This section discusses two methods for estimating relative functional failure rates. Because digital computers are becoming of increasing importance in today's weapon systems, and represent a special class of electronic functions, they are discussed separately in Section 6.4.1.

(1) Estimates Based on Nearest Similar Function

The relative failure rate can be estimated if the comparable rate for a similar function is available. For example, if a relative failure rate is required for a cam, the listed relative rate for a gear drive mechanism may be appropriate. The judgment of similarity should be verified by analysis of available failure data, or, if possible, by observation of actual failures.

(2) Estimates Based on AEG Failure Rates

A relative failure rate for non-electronic AEG's can be defined by

$$k_i' = \frac{\text{Failure Rate of an } i^{th} \text{ Type AEG or } i^{th} \text{ Function}}{\text{Failure Rate of an Average Standard Electronic Tubed AEG}} .$$

The relative rate can be estimated by using part failure rate information to estimate the numerator and denominator of the above ratio. The average part class distribution of the AEG or function being analyzed is estimated, as well as the average part class distribution of the average standard electronic AEG. Using a standard set of part failure rates, the following computations yield an estimate of k_i'.

Let

n_{ix} represent the average number of parts of type x in the i^{th} function being analyzed.

λ_x represent the failure rate of the part class x.

X represent the total number of part classes involved.

The failure rate of the i^{th} function is therefore

$$\bar{\lambda}_i = \sum_{x=1}^{X} n_{ix} \lambda_x \qquad (6\text{-}3)$$

If a function has parts which do not have a constant failure rate, $\bar{\lambda}_i$ can be estimated by

$$\bar{\lambda}_i = \frac{1 - R_i(t)}{t}$$

where

t is the average number of hours the function will be required to operate over T system hours.

$R_i(t)$ is the reliability estimate of the function, based on an average part class distribution obtained by a standard reliability prediction method.

The average failure rate of a standard electronic AEG, assuming the active elements are primarily tubes and transistors, is obtained in a manner similar to that for the non-electronic AEG's.

Let

n_v represent the average number of parts of the v^{th} part class in the standard electronic AEG's, excluding tubes and transistors.

λ_v represent the failure rate of part class v.

V represent the total number of part classes involved (passive parts)

w_1 represent the proportion of active elements that are tubes (average tube failure rate = λ_{w1}).

w_2 represent the proportion of active elements that are transistors (average transistor failure rate = λ_{w2}).

(w_1 and w_2 should be adjusted so that $w_1 + w_2 = 1.0$.)

The estimated average failure rate of a standard electronic AEG corrected to an all-tubed system is then

$$\bar{\lambda}_e = w_1 \left[\lambda_{w1} + \sum_{v=1}^{V} n_v \lambda_v\right] + 3.3 w_2 \left[\lambda_{w2} + \sum_{v=1}^{V} n_v \lambda_v\right] \qquad (6\text{-}4)$$

(See Section 6.6.2 for discussion of the correction factor, $\frac{1}{0.3} = 3.3$)

The relative failure rate for the i^{th} function can then be estimated by

$$k_i' = \frac{\bar{\lambda}_i}{\bar{\lambda}_e} \qquad (6\text{-}5)$$

6.4.1 Digital Computers

Digital computer AEG's represent a special class of electronic functions not included in the categories listed in Table 6-1. Since parts in digital applications are usually subject to much lower electrical stresses and less severe tolerance factors than similar parts in analog applications, one can expect much lower failure rates for digital computer parts. The pulse-low power standard electronic category represents functions of an analog nature, including such functions as gating or blanking of a video amplifier for a specific length of time, and are dependent on pulse shape and width. In digital computers, however, there are many go-no go circuits very tolerant of pulse width or amplitude variations.

The significant digital functions performed in a computer are:

(1) Gating-logic.

(2) Pulse-shaping, inverting, restoring.

(3) Registering, counting.

(4) Pulse storage, memory.

(5) Analog functions such as a basic computer clock.

The analog functions are consistent with the pulse-low power category and can be treated as such. For the digital categories, relative failure rates can be estimated by the procedure described in the previous section, except that the part class distribution used to derive $\overline{\lambda}_1$ will be composed primarily of electronic components. If the data used to estimate individual part failure rates does not list failure rates for digital transistors or diodes, the following correction factors, obtained from data presented in an ARINC Research Corporation report†, can be used:

For digital transistors, multiply the given analog failure rate by 0.06

For digital diodes, multiply the given analog failure rate by 0.008.

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† ARINC Research Corporation Fifth Quarterly Report, "Reliability of Semiconductor Devices,"(Publication No. 144-5-256, dated 31 August 1961).

6.5 Conversion of Relative Rates to Standard Form

The relative rates of non-electronic AEG's computed according to Section 6.3, and many of the relative rates derived from the estimating procedures discussed in Section 6.4, are not in the standard form required for use with the allocation procedure; they must be converted prior to their use in the model. For simplicity and clarity of discussion, the following notations and definitions are adopted to distinguish between the two types of relative failure rates:

(k_i') Relative failure-rate with respect to average electronic AEG failure-rate, usually shortened to "Relative Failure Rate (to average AEG)".

(k_i) Relative failure rate with respect to audio AEG failure rate, usually shortened to "Relative Failure Rate (to audio)".

Each of the relative functional failure-rates used in the allocation procedure must be related to the same standard function. The audio functional category has been selected as the common function to which all others are to be related. As part of the allocation procedure, therefore, all the relative failure rates (to average AEG) must be adjusted to become relative failure rates (to audio). The adjustment is based on the system failure index of the allocation model. A system electronic failure index (K_e) is computed for only the standard electronic AEG's of the system based on the listed failure rates. When divided by the number of electronic AEG's in the system, **the ratio** $\left(\frac{K_e}{F_e}\right)$, represents the relative failure rate (to audio) of the average electronic AEG of the system. If this ratio is assumed to be numerically equal to the value of the same ratio in the systems used to compute the relative failure rates (to average AEG), the adjustment becomes

$$k_i = \frac{K_e}{F_e} \cdot k_i' \qquad (6\text{-}6)$$

which is the equivalent to

$$k_i = \frac{\text{Average AEG Failure Rate}}{\text{Audio AEG Failure Rate}} \cdot \frac{i^{th}\ \text{Functional Failure Rate}}{\text{Average AEG Failure Rate}}$$

$$= \frac{i^{th}\ \text{Functional Failure Rate}}{\text{Audio AEG Failure Rate}}$$

Alternate possibilities for an adjustment of this type would either require data that is not presently available or additional approximations and assumptions, thereby increasing the possibilities of bias.

6.6 Part Population Factors

6.6.1 General Discussion

In general, the relative failure rates are assumed to be independent of the characteristics of the parts used in a system so long as the part choice is uniform within a system. The differences in passive part distributions between AEG's of the different categories are sufficiently reflected in the differences in the relative failure rates. However, differences in the types of active elements employed, and certain other part population characteristics, are not reflected in these data and must be treated separately.

The relative rates of the functional categories are derived entirely from tubed systems, and that they would be appropriate for entirely transistorized systems, is assumed. However, complete conversion away from tubed systems is not possible in many cases such as those requiring high powered stages, and the complexity of the active element mix is likely to increase rapidly in the future. Provisions are made, therefore, for accounting for differences in part population that can be recognized early in the design life of a system, specifically, differences in the types of active elements employed.

Adjustment factors have been derived for three types of active elements (the factor for tube active elements is equal to 1.0) and are discussed separately in the following sections. These factors, which are presented in Table 6-4 represent the ratio:

$$\frac{\text{Failure Rate of AEG, Non-tube Active Element}}{\text{Failure Rate of AEG, Tube Active Element}}$$

and are used to multiply the listed standard electronic relative functional failure rates, k_i for non-tube electronic active elements in the system.

TABLE 6-4

ADJUSTMENT FACTORS FOR DIFFERENT TYPES OF ACTIVE ELEMENTS

Active Element Type	Adjustment Factor
Tube	1.0
Transistor	0.3
Solid State Power Rectifier	0.4
Tubed Modular Assembly	0.6

6.6.2 Transistorized AEG's

The factor of 0.3 for analog transistorized AEG's represents an engineering judgment based on data and information from several sources which are described below.

In the ARINC Research Corporation report "Relationship of Field Reliability to AGREE Bench-Test Reliability," (Publication No. 162-1-220, dated 30 April 1961), a comparison was presented of tubed and transistorized airborne equipments, which yielded factors of .15 and

.20 on an AEG basis. Since the equipments also contain non-electronic AEG's, these factors are somewhat low. In a paper "On Reliability Prediction in Satellite Systems," ARINC Research Corporation Publication No. 4226-1-201, dated May 1960, failure data analysis from a number of equipments indicated that the ratio of transistor to tube analog AEG failure rates is approximately 0.5. In a paper by J. Naresky and J. Klion, "What Price Reliability," IRE National Convention Record, Part 6, March 1959, the ratio of the failure rate of an average transistorized circuit to an average tubed circuit, based on part failure rates, was computed to be equal to 0.33. From an extensive set of part failure rates presented in a report by D.R. Earles, "Reliability Application and Analysis Guide," The Martin Company Report M1-60-54, dated July 1961, a similar calculation yielded a ratio of 0.65.

No attempt is made to evaluate each of these ratios individually. It is generally accepted that transistorized analog AEG's will, in most applications, exhibit lower failure rates than equivalent tubed AEG's, with the factor possibly ranging from 0.05 to less than one. The factor of 0.3 which is adopted for this study is believed to be a reasonable average based on the limited data available.

6.6.3 Solid-State Power Rectifier

A previous study by ARINC Research Corporation† reported on an experiment involving the substitution of solid-state power rectifiers directly into tube diode sockets without further modifications. The referenced report indicates only the number of crystal failures versus the number of tubed failures. However, a comparison was possible of these results with the studies referenced in Section 6.2 for which all failures were reported. The assumption was that the result of replacing a rectifier tube with a silicon rectifier would be a change in the failure rate of only the active element involved and no change in the number of failures of the associated parts

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† "Evaluation of X-89 and TI-680 Silicon Rectifiers by Comparison with 6X4W and 6X4WA Vacuum-Tube Rectifiers," ARINC Research Corporation Publication No. 101-13-140, May 8, 1959.

of the AEG's. The reported failure rate of the solid-state diodes was used to modify the observed failures of tubes. The estimated failure rate of the AEG using solid-state diodes was computed on this basis and compared with the failure rates of the tubed AEG's. An average failure rate reduction to 0.4 of the tubed rate was derived as the adjustment factor.

6.6.4 Tubed Modular Assemblies

The information on relative reliabilities of tubed modular assemblies and ordinary-tubed AEG's was derived from the report, "Reliability of Modular Assemblies," WADD Technical Report No. 60-515, dated September 30, 1960, prepared by the Light Military Electronics Department, Advanced Electronics Center, General Electric Company.

The cited value of 0.6 is the ratio of percentage of failures of modular assemblies to percentage of failures of standard assemblies made by the same company. The specific equipment involved was the AN/SSQ-23 Sonobuoy, evaluated under operating conditions which included a drop into water from an aircraft. (Shock loadings of 100g's or greater were encountered in the drop-test.) The value of 0.6 represents the best available estimate; however, the equipment was only partially modularized, and the test conditions were rather severe. Similar data for other applications would allow greater confidence in the use of the adjustment factor in reliability allocation.

6.7 Special-Situation Data

6.7.1 Description and Use

The third type of data required as input to the allocation procedure consists of factors which account for various identifiable differences (particularly environmental) between units of a system. In the procedure, the correction factor is applied as a multiplier at the unit level, although it would be equivalent to multiply the relative failure rate of each AEG of the unit by the same correction factor. If only a portion of the AEG's require a modification factor (e.g., a particular AEG type may have a very low duty cycle), the adjustment would apply at the AEG level rather than at the unit level.

In addition to the environmental factors, special unit characteristics such as design maturity, exceptional developmental effort, etc., may be accounted for in this fashion if an estimate can be made of the influence of these characteristics on unit reliability. The general equation used in estimating a correction factor (c_i) is:

$$c_i = \frac{\text{Failure rate of unit under special condition}}{\text{Failure rate of equivalent unit under conditions typical of the system}}$$

It can be seen from the allocation model that to multiply the failure index of each unit of a system by the same constant factor has no influence on the resulting allocated unit requirements. It is therefore not necessary to account for any condition that applies uniformly to <u>each</u> unit of the system.

6.7.2 Ratio of Airborne to Ground Failure Rates

The relationship between ground and airborne failure rates has been obtained from information given in the ARINC Research Corporation paper, "Reliability Predictions in Satellite Systems," by George T. Bird (Publication No. 4226-1-205). This paper presents the relationship between electronic system mean life and complexity (measured by an AEG count) for both ground and airborne environments. The overall results are plotted on a log-log scale, resulting in a linear relationship for each environment. (The plot is reproduced as Figure 6-1 in Section 6.8.2 of this report.) Since the relationships are approximately parallel, the airborne to ground failure rate can be estimated by the ratio

$$C = \frac{\lambda \text{ Airborne}}{\lambda \text{ Ground}} = \frac{\theta \text{ Ground}}{\theta \text{ Airborne}}$$

where θ is the intercept at unit complexity of the appropriate lines. From Figure 6-1, this ratio is equal to

$$C = \frac{187{,}000}{21{,}700} = 8.6.$$

6.7.3 Ratio of Satellite to Ground Failure Rates

At present, little is known concerning the reliability of electronic systems in satellite environments. Limited data on few systems indicate that the upper boundary of the ground band of Figure 6-1 can be used as a tentative midline for the satellite band.† This is equivalent to an adjustment factor of approximately 0.5.

6.8 Feasibility Prediction

The feasibility of the system reliability requirement is determined as part of the allocation procedure. This is accomplished through the following series of steps:

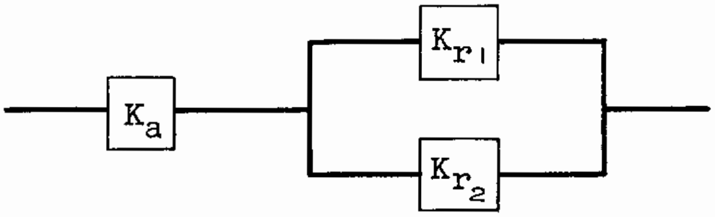
A. Prediction of the feasible mean life of the electronic portion of the system, excluding elements in redundant or alternate modes of operation.

B. Prediction of the feasible mean life of the non-electronic portion, excluding elements in redundant or alternate modes of operation.

C. Combination of the two estimates of A and B to account for redundancy or bimodal operation, to yield an estimate of feasible system reliability.

6.8.1 The Feasibility Prediction Model

Assume that the system under consideration can be represented by a block diagram of the following form:



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† See ARINC Research Corporation Interim Report, "Satellite Reliability Spectrum", 27 July 1961 (Publication No. 173-3-255).

where

K_a represents the total failure index of all units in series, and

K_{r_1} and K_{r_2} represent the failure indices of the two redundant (or bimodal) units.

The basic series system is defined as that design for which no redundant or alternate modes of operation exist (e.g., K_a and K_{r_1} make up the series equivalent). The feasible mean life of the electronic portion of this system (θ_e) can be estimated through use of a reliability-feasibility chart, as discussed in Section 6.8.2. The feasible mean life of non-electronic portion $\tilde{\theta}_{ne}$ can be estimated from experience with similar non-electronic elements or with similar systems. An alternative method described in Section 6.8.3 is to use the allocation input data provided for non-electronic AEG's.

Given $\tilde{\theta}_e$ and $\tilde{\theta}_{ne}$, the feasible mean life of the equivalent series system, under the exponential assumption, is

$$\tilde{\theta}_s = \frac{\tilde{\theta}_e \, \tilde{\theta}_{ne}}{\tilde{\theta}_e + \tilde{\theta}_{ne}} \qquad (6\text{-}7)$$

If the system were actually series, $\tilde{\theta}_s$ would then be compared with the required mean life θ^*, or, in terms of reliability, $\tilde{R}(T) = e^{-T/\theta_s}$ would be compared with $R^*(T)$, to determine the feasibility of the system requirement.

For redundant or bimodal systems, the percent of the system made redundant can be estimated from the equation

$$\gamma = \frac{K_r'}{K_a + K_r'} \qquad (6\text{-}8)$$

where

K_r' is the equivalent failure index of each redundant or bimodal unit which actually have failure indices of K_{r_1} and K_{r_2}.

If we let $\widetilde{\lambda}$ be the feasible audio AEG rate, for active parallel or continuous operation, the feasible system reliability for T hours is

$$\widetilde{R}(T) = e^{-K_a\widetilde{\lambda}T}\left[2e^{-K_r'\widetilde{\lambda}T} - e^{-2K_r'\widetilde{\lambda}T}\right]$$

$$= e^{-(K_a + K_r')\widetilde{\lambda}T} - e^{-(K_a + 2K_r')\widetilde{\lambda}T} .$$

Since $(K_a + K_r')$ represents the total failure index of an equivalent series system, the feasible system failure rate is

$$\widetilde{\lambda}_s = (K_a + K_r')\widetilde{\lambda} = 1/\widetilde{\theta}_s$$

Hence,

$$\widetilde{R}(T) = 2e^{-T/\widetilde{\theta}_s} - e^{-T/\widetilde{\theta}_s}\, e^{-K_r'\widetilde{\lambda}T}$$

But,

$$K_r'\widetilde{\lambda}T = \frac{K_r'}{(K_a + K_r')}(K_a + K_r')\widetilde{\lambda}T$$

$$= \gamma T/\widetilde{\theta}_s$$

Therefore,

$$\tilde{R}(T) = 2e^{-T/\tilde{\theta}_s} - e^{-[(1+\gamma)\,T/\tilde{\theta}_s]} \qquad (6\text{-}9)$$

For standby or sequential operation,

$$R(T) = e^{-K_a\tilde{\lambda}T}\left[e^{-K'_r\tilde{\lambda}T}\,(1 + K'_r\tilde{\lambda}T)\right]$$

$$= e^{-T/\tilde{\theta}_s}\left[1 + \gamma T/\tilde{\theta}_s\right] \qquad (6\text{-}10)$$

6.8.2 Electronic Portion - Estimate of $\tilde{\theta}_e$

Figure 6-1, a log-log plot of system mean life versus non-redundant system complexity, can be used to estimate θ_e. Complexity is measured in terms of the number of active elements making up the system. The data from which the figure was derived were accumulated by various companies over the past several years. The systems shown range in design age from two to ten years, and in functional complexity from ground-based communication receivers to airborne bombing/navigation systems. Failure is defined as a malfunction during the system operating cycle, necessitating a maintenance action, part replacement, or adjustment. Operator knob-adjustments or "fine tuning" during the operating cycle were not classified as system failures.

Available data on airborne and ground-based (or shipborne) systems were sufficient to permit computation of least-squares regression lines through the points. The least-squares regression line for airborne systems was computed to be

$$\tilde{\theta} = 21{,}700\,(N_e)^{-1.33}$$

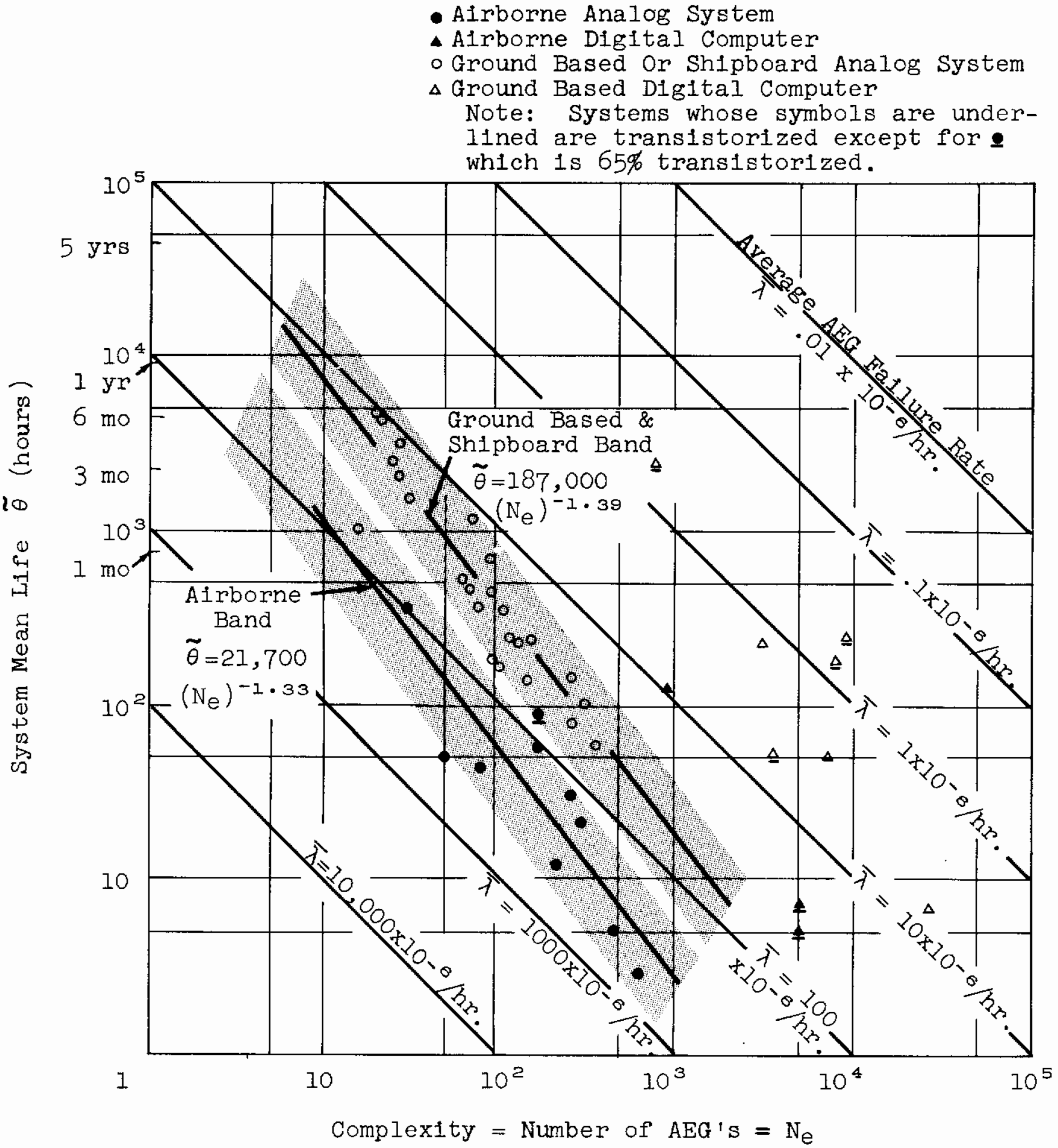

FIGURE 6-1

EFFECTS OF SYSTEM COMPLEXITY ON RELIABILITY where $\tilde{\theta}$ represents the estimated mean time in hours between system failures for a system of N_e non-redundant electronic AEG's; the 21,700 value is the theoretical mean life in hours of a system with one AEG, and the value -1.33 is the slope of the regression line.

Similarly, the least-squares fit through the ground/shipboard scattergram can be expressed by the equation

$$\tilde{\theta} = 187{,}000\ (N_e)^{-1.39} .$$

The most significant feature of the graph is that system mean life falls off more sharply as complexity increases than would be expected if exponentiality were assumed. Because mean life is not directly proportional to complexity, it cannot be stated that System A, with 50 AEG's, has a mean life twice that of System B, with 100 AEG's. If this were true, then the slopes of the regression lines (the exponents in the regression equations) would be equal to 1.0. The 45° lines show how mean life and complexity would be related for various average AEG failure rates, if the hypothesis of direct proportionality were true.

Various hypotheses can be formulated to account for the observed departure from the "norm". Most of the hypotheses could probably be classified into a general category labelled "interaction effects". For example, it is reasonable to assume that as system complexity increases, so must the number of knobs by which the operator can compensate for degradation and instability within individual subsystems. Eventually, the level of complexity and the corresponding number of required knob adjustments would exceed human capacity for effective and timely control during the operating cycle.

While this chart needs much refinement, it can be employed advantageously for determining design feasibility and for making "ball park" predictions of electronic system reliability. Several of the refinements developed as a result of this study are discussed below.

Figure 6-1 is based primarily on tubed systems. If the system under consideration has active elements other than tubes, the mean life given by the Figure should be adjusted. Adjustment is accomplished through use of multiplicative correction factors B_i, listed in the table below, which will yield a reduced electronic AEG failure rate for active elements other than tubes.

i	Type of Active Element	B_i
1	Transistor, analog	0.3
2	Transistor, digital	0.02
3	Diode, digital (AEG)	0.06†
4	Tube	1.0

† Based on ten digital diodes.

B_1 corresponds to the correction factor discussed in Section 6.6.2. The digital transistor factor is derived from data presented in the ARINC Research Corporation report, "Reliability of Semiconductor Devices," (Publication No. 144-5-256, dated 31 August 1961). From the data therein, the ratio of digital transistor field failure rate to analog transistor field failure rate is estimated as approximately 0.06. Multiplying this factor by the B_1 factor of 0.3 yields

$$\frac{\lambda\ (\text{Digital Transistor})}{\lambda\ (\text{Analog Transistor})} \times \frac{\lambda\ (\text{Analog Transistor AEG})}{\lambda\ (\text{Tube AEG})}$$

$$= (0.06)\ (0.3) \approx .02.$$

This factor 0.02 is assumed to represent

$$B_2 = \frac{\lambda\ (\text{Digital Transistor AEG})}{\lambda\ (\text{Tube AEG})}$$

under the assumption that the passive parts of a digital transistorized AEG would have a failure rate approximately 0.06 times the failure rate of passive parts in transistorized analog AEG's. The justification for a factor of this order of magnitude is the lower electrical stresses and lower sensitivity to tolerances for the passive elements in digital applications.

The factor $B_3 = 0.06$ for digital diode AEG's is based on two considerations. Since many digital computers have extremely high diode populations, ten digital diodes are taken to represent one complexity unit or one digital diode AEG. The complexity counts of the digital computers shown in Figure 6-1 are based on this ten-to-one ratio. From the data presented in the referenced report, the digital diode failure rate to the digital transistor failure rate is approximately 0.36: The product

$$\frac{\lambda\ (\text{Digital Diode})}{\lambda\ (\text{Digital Transistor})} \cdot \frac{\lambda\ (\text{Digital Transistor})}{\lambda\ (\text{Analog Transistor})} \cdot B_1$$

is equal to $(0.36)\ (0.06)\ (0.03) \approx .006$.

This factor of .006 is taken to represent

$$\frac{\lambda\ (\text{Digital Diode})}{\lambda\ (\text{Tube AEG})}$$

Since ten digital diodes represent one complexity unit or one equivalent electronic AEG,

$$B_3 = \frac{\lambda\ (\text{Digital Diode, AEG})}{\lambda\ (\text{Tube AEG})} = 10 \times \frac{\lambda\ (\text{Digital Diode})}{\lambda\ (\text{Tube AEG})}$$

$$= 10 \times .006 = .06.$$

Four of the systems in Figure 6-1 can be used to check on this estimate, and give close agreement: the average equivalent B_3 ratio for the four systems is .064.

Although it is realized that the factors derived above are based on assumptions not yet rigorously justified, it is felt that for purposes of a gross feasibility prediction the factors can be used with reasonable confidence until more appropriate data become available.

From a count of all electronic active elements in the basic series system, Figure 6-1 can be used to estimate feasible mean life for an assumed tubed system ($\tilde{\theta}_{et}$) by the formula

$$\tilde{\theta}_{et} = K \, (N_e)^{-p}$$

where K and p are constants which depend on the environment, and N_e is the number of electronic active elements (ten digital diodes are assumed to represent one electronic active element).

To account for non-tube active elements, consideration was given to adjusting N_e by the B_1 factors, but this is valid only if p were equal to one. A more accurate procedure is to adjust $\tilde{\theta}_{et}$, as discussed below.

The feasible system failure rate for a tubed system with N_e tubed active elements is

$$\tilde{\lambda}_{et} = \frac{1}{\tilde{\theta}_{et}}$$

If $\tilde{\lambda}_1$ denotes feasible tube failure rate per active element,

$$\tilde{\lambda}_1 = \frac{\tilde{\lambda}_{et}}{N_e} \, .$$

To account for active element types other than tubes, one can consider the feasible failure rate of a non-tubed active element to be equal to $B_i\tilde{\lambda}_1$; hence, total feasible system failure rate is

$$\tilde{\lambda}_e = \sum_{i=1}^{4} N_i B_i \tilde{\lambda}_1$$

$$= \sum_{i=1}^{4} \frac{N_i B_i \tilde{\lambda}_{et}}{N_e}$$

where N_i is the number of active elements of type i (N_3 is equal to $\frac{1}{10}$ x number of digital diodes).

The feasible mean life of the electronic portion of the system is, therefore

$$\tilde{\theta}_e = \frac{N_e \tilde{\theta}_{et}}{\sum_{i=1}^{4} N_i B_i} \tag{6-11}$$

It is important to note that this model gives only a gross indication of feasibility, because it is based on rough historical data and simplifying assumptions. Further improvements in reliability can be anticipated, and this should be taken into consideration in the appraisal of the feasibility of the system requirement.

6.8.3 Non-electronic Portion - Estimate of $\tilde{\theta}_{ne}$

Feasible mean-life estimates for non-electronic elements in the basic series system can be estimated through experience with similar non-electronic parts or equipments. An alternate method has been developed, based on data inputs of this study. The method is described in this section.

The total failure index of all non-electronic AEG's in the basic series system (K_{ne}) can be obtained by summing the product of relative failure unadjusted rates (k_i') and the number of existing AEG's of the i^{th} type. Hence,

$$K_{ne} = \sum_i f_i' k_i' \qquad (6\text{-}12)$$

where k_i' is the relative unadjusted failure rate of non-electronic active element type i, and

f_i' is the total number of AEG's of the i^{th} type for non-electronic elements in the basic series system

Since k_i' is equivalent to the ratio of the failure rate of a type i AEG to the average electronic failure rate in the system.

$$K_{ne} = \frac{\lambda_{ne}}{\bar{\lambda}_e}$$

where λ_{ne} is the total non-electronic failure rate, and

$\bar{\lambda}_e$ is the average electronic failure rate of one AEG.

From Figure 6-1, the feasible electronic system mean life based on N_e tubed active elements can be obtained and, in the notation of Section 6.8.2, is equal to $\tilde{\theta}_{et}$.

Therefore, one can obtain

$$\tilde{\lambda}_{et} = \frac{1}{\tilde{\theta}_{et}}$$

where $\tilde{\lambda}_{et}$ is the feasible failure rate of a system with N_e tubed electronic elements.

Hence the average feasible failure rate of one electronic active element is

$$\tilde{\lambda}_e = \frac{\tilde{\lambda}_{et}}{N_e} = \frac{1}{N_e \tilde{\theta}_{et}}$$

Replacing $\bar{\lambda}_e$ by $\tilde{\lambda}_e$, and λ_{ne} by $\tilde{\lambda}_{ne}$ in the equation for K_{ne}, yields

$$K_{ne} = \tilde{\lambda}_{ne} N_e \tilde{\theta}_{et}$$

or

$$\frac{1}{\tilde{\lambda}_{ne}} = \frac{N_e \tilde{\theta}_{et}}{K_{ne}}$$

But $\frac{1}{\tilde{\lambda}_{ne}}$ is equal to the feasible mean life of the non-electronic portion of the basic series system or

$$\tilde{\theta}_{ne} = \frac{N_e \tilde{\theta}_{et}}{K_{ne}} \qquad (6\text{-}13)$$

7. ANALYSIS AND INTERPRETATION OF THE ALLOCATION

7.1 Introduction

To establish reliability as a design parameter, reliability requirements and methods of demonstrating compliance with them must be specified in each contract. The allocation of reliability during the initial stages of system evaluation permits reliability to be specified and provides a basis upon which demonstration and acceptance tests can be prepared and costed.

The methods described in Sections 5 and 6 of this document permit a numerical allocation of the over-all reliability requirement. Within the restrictions of time and funding, the numbers so derived can be used in solving management problems associated with development of an effective system for the ultimate user. The extent to which they will serve this purpose depends upon the manner in which they are interpreted and applied.

An allocation obtained by the above-described procedures is based on the following fundamental assumptions:

(1) The allocation process by itself gives no assurance or guarantee that the reliability so assigned will materialize in service operation of the units or system. The allocation procedure takes an assigned over-all reliability and apportions the allowable unreliability to the various units of the system. If the system reliability requirement exceeds the state-of-the-art (see Section 6.8), each unit's allocation will reflect its appropriate share of the required increase in the state-of-the-art.

(2) The allocation process assumes that the system development program is a uniform effort, i.e., that each unit will receive its equivalent share of the development funds and calendar time. The relative weighting of functional units is based on past experience in a wide variety of development and production programs; thus, the relative weighting is based on an "average" development and production background.

(3) The allocations arrived at by the above procedure can be further modified through study of trade-offs between reliability, other performance requirements, weight and space, calendar time, and monetary limitations. The initial allocation is made on the basis of factors which can be quantified at this time. Interpretation and use of the allocated numbers can be effective only if they are related to the more subtle factors in R&D programs.

7.2 Development Program Factors Affecting Allocations

The procedures described in this document have been concerned only with those allocations based upon previous experience with system components and their anticipated use in future applications (essentiality). Ideally, these should be the only factors influencing reliability allocations. Practically, however, development program managers are faced with a continual series of compromises. Brief discussions of some types of compromises or trade-offs which frequently occur are given below. Since no two situations are identical, these trade-offs cannot practicably be reduced to numerical quantities in the allocation equations. To the extent that individual unit reliabilities are compromised by such trade-offs, it is essential that, when recombined, these reliabilities equal the system requirement.

7.2.1 System Requirements Versus State-of-the-Art

Perhaps the most common problem encountered by the program manager is that in which system requirements -- and, thus, unit requirements -- are too high relative to the current state-of-the-art and the calendar time and funding permitted for the R&D program. Several possible solutions must be investigated. One solution is to lower the system requirement; another is to extend the R&D schedule; a third is to simplify the operational requirements; and a fourth is to reallocate reliability, weight, space, time, and funds. These alternatives are discussed below.

(1) Lower the System Requirement. Operational commands tend to demand much higher reliabilities than are in reality required. Thus, it is appropriate to determine whether it is more vital to provide the operational command with a system having a lower reliability by a specified calendar time or to delay delivery until equipment with the required reliability is available. (National defense needs are often most dependent upon the time elements.)

(2) Extend the R&D Schedule. This solution is the complement of (1), above. In effect, it requires a decision that the system concept is not sufficiently advanced for fruitful developmental effort until preceded by considerable research which will increase the time and funding required. Although additional research before development may appear to cost more initially, the price of failure, rework, "late-in-the program" research, and unusual use problems may very well overcome the initial cost differential.

(3) Simplify the Operational Requirements. It is common practice to attempt over-sophistication in present-day systems. Requirements for auxiliary functions, for accuracy, and for versatility are often more rigorous than is actually required. It thus becomes necessary to review original operational needs in relation to planned system performance. If the system can be simplified, its reliability will show a definite increase -- assuming other factors remain constant. Figure 6-1 in Section 6.8 clearly shows the effects of complexity.

(4) Reallocate Reliability, Weight, Space, Time, and Funds. This solution builds imbalance into the program in the interest of concentrating improvements in selected units. Specific methods employed are:

(a) Use of redundancy, which will require reconsideration of weight allowances, volume limits, and available power sources. Redundancy entails consideration of switching devices, types of maintenance, and interaction effects that must be carefully treated if redundancy is indeed to effect an apparent increase in system reliability.

(b) Extensive R&D efforts on selected units, with an attendant reduction in effort on other units.

The risks associated with a concentration of effort are great. It is possible to concentrate on units providing no apparent gain and possibly suffer loss through reduction of effort on others. Conversely, if the right units are selected, the gain could be great.

7.2.2 Unit-Allocated Reliability Trade-Offs Versus Time and Funds

Section 7.2.1 (4), above, established the point that it is not usually desirable to concentrate effort on only a few areas in the interest of overcoming system deficiencies. Within any program, however, development time and monetary allocations often become out of balance relative to the initial reliability allocation. Circumstances which can contribute to such imbalance include the following:

(1) Unique or radically new approaches to functional design may dictate additional funding in a particular area.

(2) If procurement of relatively standard units from other programs is intended, these programs must be considered in conjunction with the initial allocation. Increased reliability allocations could perhaps be given to units produced under the more intensified reliability efforts.

(3) Development time has a marked effect upon reliability. The more design and test time provided, the more mature -- and thus reliable -- the design is likely to become. Here, too, an increase in the reliability apportionment may be permissible. A unit produced under conditions which provide relatively little time for testing and, consequently, for correction or improvement of deficiencies, should not receive the same reliability allocation as a unit which requires little design time and thus permits more than adequate time for testing and improvement.

7.2.3 Unit-Allocated Reliability Trade-Offs Versus State of Design

The state of the design of a particular unit relative to other units affects the allocation. A unit which requires only modification and restudy from the viewpoint of reliability can generally be expected to achieve a greater reliability improvement than a unit which must be newly designed, if the same amount of effort is applied to each. When accurate reliability data is available on a standard unit, it should, of course, be utilized in lieu of the initial reliability allocation.

7.2.4 Unit-Allocated Reliability Trade-Offs Versus Type of R&D Effort

Within a system program it is not unusual to find development efforts on different units varying in scope from straight fabrication to research on individual parts. A review of the R&D efforts will result in a redistribution of the allocation based on (1) part selection criteria, (2) the type of engineering effort (tolerance studies, stability studies, derating policies, packaging, etc.), (3) the quantity and type of development and reliability testing planned, and (4) the provisions made for correction of deficiencies. Units developed under programs specifically oriented toward reliability and employing good design practice should be assigned a higher reliability than the initial allocation would normally provide. Materials handling techniques, process controls, assembly techniques, and inspection methods all influence the quality of the deliverable product. Thus, the R&D effort and the fabrication process can be considered in the reallocation of unit reliability.

7.3 Updating the Allocation

Reliability as a designed performance parameter does not remain static throughout a development program. Therefore, the allocated requirements must be periodically reviewed for their current applicability. The review should take account of circumstances such as those described below.

(1) Changes in design philosophy may affect the initial allocation determined pursuant to this document.

(2) Changes in program plans -- e.g., in funding, in scheduling, or in design and test emphasis -- will necessitate a restudy of the trade-offs made in the establishment of unit requirements.

(3) Acquisition of applicable use data on units may permit substitution of more specific, current experience for estimates incorporated in the allocation.

(4) Reliability prediction and analysis studies may indicate that individual units can achieve more or less reliability than allocated. (A note of caution: Present-day predictions often show reliability potential rather than a realistic state of the design.)

(5) Availability of test data from the program will give early indications of how well units are complying with requirements. Valid test data can provide a current base for reallocation of reliability.

8. RELIABILITY TESTING

8.1 Introduction

A reliability allocation performed to establish contractual reliability requirements is of limited value if methods for demonstrating compliance with the requirementsare not also specified. In fact, it can be argued that the primary purpose of a reliability allocation is to determine the parameters of reliability tests.

This section is limited to reliability tests performed for the purpose of determining whether a submitted product should be accepted or rejected depending upon its conformance to specified reliability goals. The tests involved, therefore, are acceptance rather than evaluation tests, The latter are essentially a process for estimating the level of achieved reliability.

Reliability acceptance tests are usually quite expensive and time consuming. On the other hand, reliability test effort on a level that yields imprecise or inaccurate results will eventually lead to adverse consequences such as accepting an unreliable product or rejecting a satisfactory one. Therefore, an attempt to balance these factors to achieve an optimum level of test effort is natural. In doing so, one must consider that:

(a) only a few items may be available for testing -- especially in the research and development phases of the system-life cycle;

(b) life testing is deleterious and often destructive and therefore can be very expensive for all but the simplest of items;

(c) items with high reliability requirements require long testing periods; since the number of failures, rather than the number of items on test, determines the test "sample size," the waiting time involved in obtaining the required number of failures may be a limiting factor;

(d) complex items may require expensive test equipment, especially if simulation of environmental conditions is a requisite;

(e) a philosophy too frequently exists holding that reliability testing is of secondary importance; consequently budgets often fail to provide sufficient money and time for adequate levels of test effort.

It is not within the scope of this contract to develop optimization methods which are of immediate applicability to all types of reliability test situations. Rather than limit the discussion to a few special cases, a detailed discussion of the concepts of reliability testing is presented. Guidelines are given for choosing between such alternatives as variables and attributes tests, single, multiple or sequential sampling plans, non-truncated or truncated life tests, parametric or nonparametric assumptions, etc. Useful tables showing the effect of varying the test parameters on the amount of testing are presented for the more common types of reliability acceptance tests. The last section on testing describes the general decision theory approach for test effort optimization, where costs of testing and costs of wrong decision are considered explicitly.

8.2 Basic Concepts of Reliability Acceptance Testing

An acceptance test is a procedure for testing or inspecting samples of some product from a submitted lot and, on the basis of the results, deciding whether or not the whole lot may be accepted as being satisfactory. Before such a test can be performed, the following factors must be considered:

(a) The definition of a lot.

(b) Methods for selecting samples from this lot.

(c) The environmental test conditions.

(d) The definition of a satisfactory lot.

(e) The number of sample items to be selected.

(f) The limitations on expenditures of money, time, manpower and equipment.

Consideration of the foregoing matters usually yields enough information to confine the number of appropriate acceptance tests to a very few. The discussion which follows encompasses only such tests as are needed to support a determination that specified reliability requirements have been satisfied. Wrong decision cost considerations are deferred until Section 8.6 in order to develop the concepts of reliability acceptance testing without having to introduce the relatively new, but closely related, field of decision theory.

8.2.1 Lot Definition and Sample Selection

The definition of a lot is important in acceptance sampling since decisions based on a relatively small number of sample items apply to the whole lot. Lots should be formed in such a manner that homogeneity of items within it is achieved. This is best accomplished by forming "rational lots", or lots which contain units of sufficiently identical origin to minimize the likelihood of sharp variations in quality. This can be accomplished by including:

(a) products produced from the same batches of raw material, components, or subassemblies.

(b) products manufactured by the same production or assembly line.

(c) products manufactured within a specified unit of time such as a day or a week.

It is generally true that the larger the lot, the more economical is the sampling plan. Lots that are too large, however, will not likely be rational lots.

Rarely can all of the foregoing conditions be satisfied even for simple parts such as resistors. For equipments or subsystems, the level at which allocated reliability requirements usually apply, the formation of rational lots may become a most difficult problem because of the very limited production aspect. Moreover, in the early developmental stages, design, manufacturing, and procurement changes are frequently made.

For reliability acceptance tests in such cases, the items on test are generally regarded to represent not a specific lot but the producer's capability to meet the requirement, the "lot" then being his future production. Any major changes in future production must, of course, lead to a re-evaluation of earlier test decisions.

The selection of sample items which can be formed into rational lots should be made in such a manner as to ensure representativeness. This is accomplished by selecting items randomly, giving each item in the lot an equal chance of being included in the sample. In reliability testing at the developmental stages, sample selection in this manner is virtually impossible. For complex items of limited production -- only five or six -- a sample of perhaps not more than two may be drawn. Fortunately in many types of life tests the "sample size" is not the number of items tested but the number of failures observed or the number of test hours accumulated. Since complex items generally exhibit a constant failure rate (an assumption which should be verified), a few test items can generate the required sample size if failures are restored to new condition by repair. It is, therefore, even more important for this case that the items selected (at least two is the AGREE† requirement for the development stage) be representative of the current output.

8.2.2 Environmental Test Conditions

Although testing may be performed on items which are in the development or prototype stage, the reliability requirement is usually one which eventually applies to field operation. It is necessary that the environmental test conditions imposed are appropriate for the particular stage of development. For the early stages of the system life cycle, it is usually desirable to minimize the variety of imposed environments in deference to test economy and to expedite acceptance (if justified) of the reliability program. Environmental conditions usually considered are temperature, vibration, off-on cycling, and input voltage. As the system life cycle approaches the operational stage, the severity and scope of the test environment should be adjusted accordingly. The particular environmental test conditions employed must be considered in defining the reliability test specification.

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† Advisory Group on Reliability of Electronic Equipment, Reliability of Military Electronic Equipment, Task Group No. 2, 4 June 1957.

8.2.3 Statistical Concepts

The statistical basis of a reliability acceptance test is obtained from the theory of testing hypotheses. In reliability testing, the hypothesis under test -- called the null hypothesis -- is that the submitted lot conforms to the reliability requirement. An alternative hypothesis is also specified (or at least implied) that the submitted lot does not conform to the reliability requirement. Rejection of the null hypothesis is essentially equivalent to acceptance of the alternative hypothesis.

Since samples rather than whole lots are inspected, the possibility of incorrect inferences due to sampling fluctuations has to be considered. If H_0 represents the null hypothesis that the product conforms to the reliability requirement (e.g., the mean life is equal to a specified value) and H_1 represents the alternative hypothesis that the reliability of the lot is at some reliability level considered to be unacceptable, either of the following two incorrect inferences may be drawn.

(a) Type I Error: H_0 may be rejected when it is true

(b) Type II Error: H_0 may be accepted when H_1 is true.

The probabilities of making these errors are dependent on the sample size and decision criterion. If the probability of a Type I error is denoted by α and the probability of a Type II error by β, the following table can be used to summarize the relationships:

TABLE 8-1

RELATIONSHIPS BETWEEN TEST DECISION AND TRUE SITUATION

Test Decision	True Situation	
	H_0 True	H_1 True
Accept H_0	Correct Decision Prob. = $1 - \alpha$	Type II Error Prob. = β
Reject H_0	Type I Error Prob. = α	Correct Decision Prob. = $1 - \beta$

In acceptance sampling, the probability of a Type I error, α, is commonly called the producer's or Alpha risk as it represents the risk that a product conforming to the specification is rejected. The probability of a Type II error, β, is called the consumer's or Beta risk as it represents the risk of accepting product which should be rejected ($1 - \beta$ is known as the power of the test).

The expression "confidence" or phrase "confidence level" is sometimes seen in reliability specifications to indicate the degree of assurance required in the life test results. In the strict sense, confidence levels, such as 95%, apply only to estimation problems where one desires to construct from the sample data an interval estimate of the parameter for which there is 95% confidence that this interval contains the true value of the parameter. Such intervals can be used to test hypotheses by accepting or rejecting the null hypothesis according to whether the parameter value such as mean life associated with this null hypothesis is or is not contained in the computed interval. Although this approach has been generally rejected in favor of hypothesis tests with specified producer's and consumer's risks, it has the advantage that the closeness of the hypothetical or standard parameter value to the accept-reject limit gives a good idea of how firm the corresponding decision is.

In many cases, tests based on confidence intervals lead to exactly the same results as those based on the theory of hypothesis tests. A test specification in terms of confidence levels, however, can be subject to serious misinterpretation as the following example will illustrate. Assume the specification states: "...A sample shall be life tested to determine with 90% confidence that the submitted lot conforms to the requirement of a 100-hour mean life ...". Two reasonable test criteria are:

(a) Compute the 90% lower confidence limit, θ_L. Since one can be 90% confident that the true mean life is greater than θ_L, if $\theta_L \geq 100$, accept the lot, otherwise reject it.

or

(b) Compute the 90% upper confidence limit. Since one can be 90% confident that the true mean life is less than θ_U, if $\theta_U < 100$, reject the lot, otherwise accept it.

Test (a) is equivalent to one where the consumer's risk is 10% at a true mean life of 100 hours. Test (b) is equivalent to one where the producer's risk is 10% at a true mean life of 100 hours. The difference between the two tests is apparent; the former requires that a lot with a mean life of 100 hours be accepted only 10% of the time; the latter requires 90% acceptance of a lot with this same mean life value. It is therefore strongly recommended that test specifications written in terms of confidence statements not be used unless the statement uniquely determines the test criteria.

For all usual types of tests, the magnitude of α and β and the number of test observations n are interrelated in such a manner that specifying any two of the quantities determines the third. In the past, for non-sequential tests, α and n were usually specified and a test was chosen to minimize the β error. For acceptance testing, the trend now appears to specify β instead of α. If it is important that both α and β be specified, the sample size is no longer at the disposal of the experimenter. In sequential sampling, α and β must be specified in advance, and the sample size is a random variable since its value is not predetermined but will vary over successive tests.

8.2.4 The Operating Characteristic (O.C.) Curve

By specifying two of the three quantities, n, α, and β, the accept-reject criterion of the acceptance test is uniquely determined for a given family of tests (e.g., a life test under the exponential assumption truncated after 15 failures occur). It is then possible to generate the O.C. curve of the test plan. This is a curve which shows the probability of lot acceptance over all possible incoming reliability levels. Two points on the O.C. curve are already determined, namely the α and β points and their corresponding reliability levels, which are given by H_0 and H_1, respectively.

As an illustration, if the specification is in terms of a survival probability or reliability for a given period of time, the general shape of the O.C. curve would be as shown in Figure 8-1.

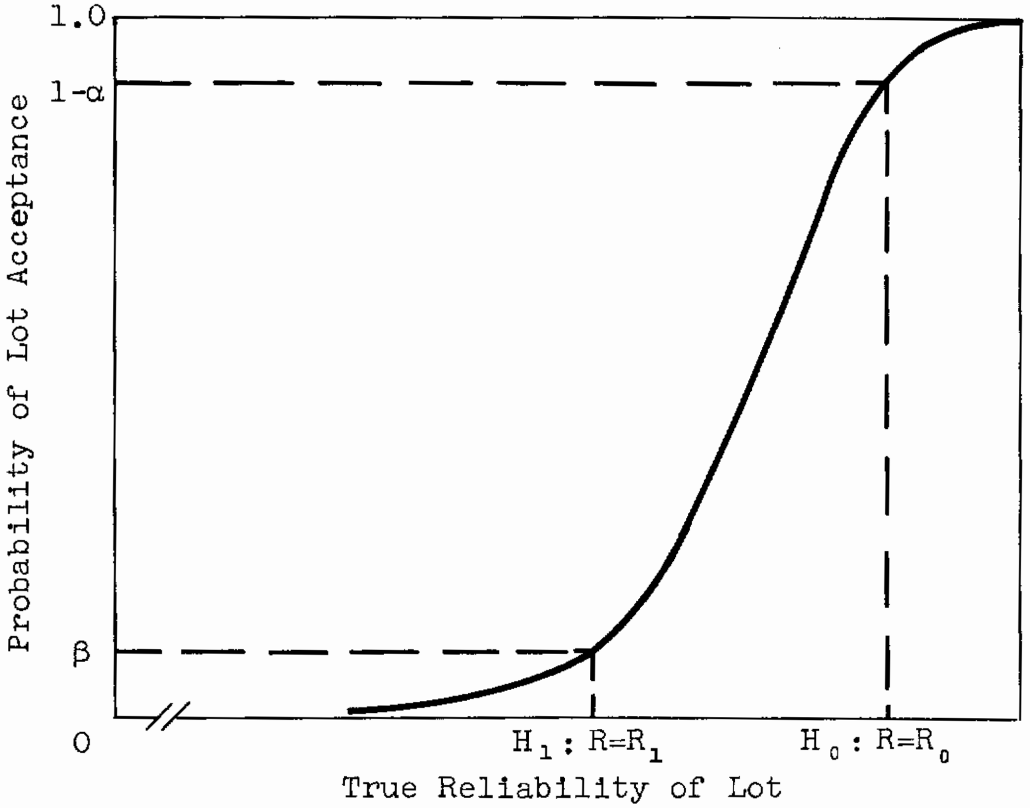

FIGURE 8-1

TYPICAL O.C. CURVE FOR RELIABILITY ACCEPTANCE TEST $H_0: R=R_0$, $H_1: R=R_1$, SPECIFIED α AND β

The probabilities of acceptance are interpreted as the long-run proportion of lots that will be accepted. If, for example, the O.C. curve shows that a lot with a reliability of .80 will be accepted with a probability of .65, then the interpretation is that, in the long run, 65% of all lots which are submitted with 20% defective items will be accepted.

It should be emphasized that the O.C. curve gives the probability of acceptance <u>if</u> a lot with a reliability level of R is submitted. It does <u>not</u> give the probability or relative frequency <u>distribution</u> of the reliability level of accepted lots. To illustrate this important concept, if every submitted lot had a reliability of .20, and the probability of acceptance was .05, then, on the average, 5 out of every 100 lots would be accepted. But, since all lots have a .20 reliability, every accepted lot would also have a reliability of .20 (ignoring the minor point that failures detected during inspection are discarded). This is one justification for the oft-made statement that "you cannot inspect reliability or quality into a product."

8.2.5 The Test Specification

Since reliability may be specified by various parameters such as probability of survival, mean life, failure rate, etc., the following terminology will be used in general discussion:

(a) Acceptable Reliability Level (ARL) is the level of reliability, measured by an appropriate parameter, considered to be acceptable and which represents the null hypothesis, i.e., H_0 = ARL. (The Acceptable Quality Level, AQL, is the analogous term for acceptance tests based on percent defective.)

(b) Unacceptable Reliability Level (URL) is the level of reliability which is considered to be unacceptable and which represents the alternate hypothesis, i.e., H_1 = URL. (The Lot-Tolerance-Percent-Defective is the analogous term for acceptance tests based on percent defective.)

(c) Discrimination Ratio (k) is a ratio measure of the difference between the ARL and URL. For mean life requirements, $k = \theta_0/\theta_1$, for failure rate requirements, $k = \lambda_1/\lambda_0$, and for survival probability requirements, $k = (1 - R_1)/(1 - R_0)$ where the "0" subscript refers to the ARL and the "1" subscript refers to the URL.

An immediate question is where does the allocated reliability requirement fit? For example, suppose the allocation procedure determined that a particular equipment should have a reliability of .96 for 10 hours or $R(10) = .96$. The acceptance test is to have an α and β both equal to .10. Associated with α is the ARL and associated with β is the URL.

If the ARL is selected to be .96, then values of reliability somewhat less than .96 would be accepted with a relatively high probability. If the URL was selected to be .96, the plan will give a high assurance of rejecting a product with R less than .96, but this may be too severe on the producer. Also, as will be shown later, the amount of testing required for a decision is proportional to either the ARL or URL and the discrimination ratio. Assuming a k of 2, i.e., $\frac{1-R_1}{1-R_0} = 2$, the O.C. curve for the two alternate plans presented above will be similar to those shown in Figure 8-2. If, for the sake of argument, a true reliability of .96 or more is totally acceptable and a true reliability of less than .96 is totally unacceptable, Plan A which permits lots with a reliability of about .94 to have a 50% chance of acceptance is not restrictive enough. On the other hand, if a true reliability of .92 is totally acceptable and less than .92 is totally unacceptable, Plan B is too restrictive, since lots with an average reliability of .94 would have very slight chance of passing the test.

It is therefore most important that the ARL and URL be made as consistent as possible with operational requirements within the limitations imposed by time and monetary constraints. Since the allocated equipment reliability requirements are derived from the contract specified system requirement, they are, in effect, also contract specified. The usual interpretation of the contract specified requirement is that it represents the reliability level for which a high probability of acceptance is desired. This level is therefore, the ARL with an associated probability of acceptance of $1 - \alpha$. However, as pointed out in the AGREE report, Task Group No. 3†, the contract specified reliability level should actually be higher than the minimum sufficient for tactical requirements. This true minimum is the URL which has an associated probability of acceptance of β. Since a lot with a reliability level lower than the URL will not be sufficient for tactical requirements,

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† Advisory Group on Reliability of Electronic Equipment, Reliability of Military Electronic Equipment, 4 June 1957.

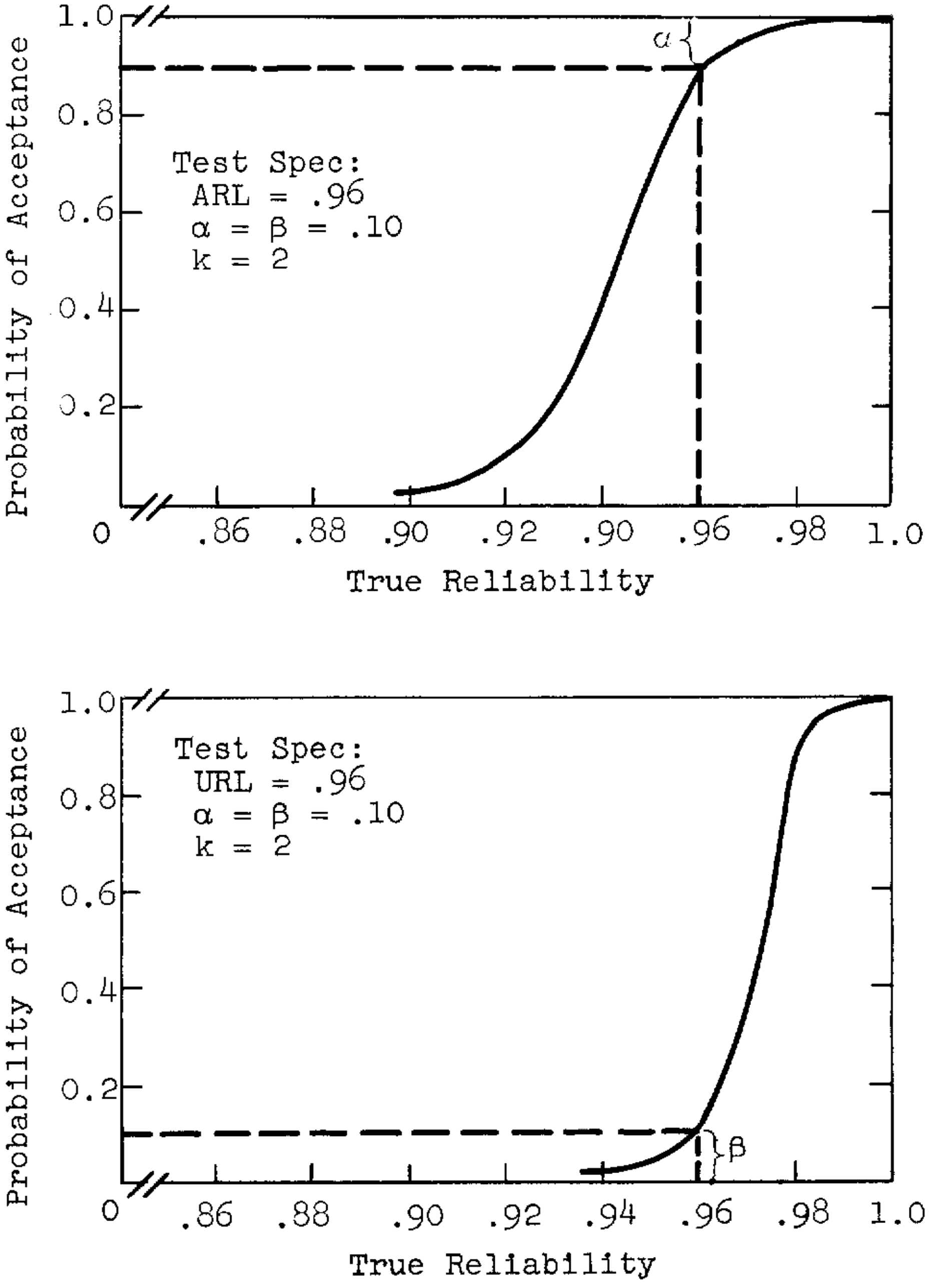

FIGURE 8-2

O.C. CURVES FOR TWO ALTERNATE
TEST SPECIFICATIONS

acceptance of such a lot is a risk the consumer or user attempts to minimize by choosing a plan with a low β. (Section 8.6 discusses the choice of the α and β risks in terms of minimizing total cost.)

If the above interpretation is adopted, the equipment requirements obtained from an allocation proceudre represent the ARL, provided the system contract reliability specification represents the ARL. Appropriate changes must be made if the system reliability requirement is based on some other criterion such as the minimum reliability sufficient for tactical requirements.

8.3 Types of Reliability Acceptance Tests

Numerous ways exists for classifying the types of reliability acceptance tests. The choice of which type of test to employ may be based on statistical considerations, physical aspects, economic factors, or combinations of these as well as others. The following lists of definitions will aid in describing the various types of acceptance tests. Note that the definitions with the same numerical prefix represent possible different alternatives for particular test characteristics.

1A Attributes Test - A test procedure where the items under test are classified into qualitative categories such as success or failure.

1B Variables Test - A test procedure where the items under test are classified according to quantitative characteristics such as power output.

1C Life Tests - A test conducted over time in which time to failure is measured.

2A Single Sampling Plan - An acceptance test where one sample of known size is taken to determine conformance to a specification.

2B Multiple Sampling Plan - An acceptance test in which, after each sample, a decision is made to accept, reject or take another sample. A maximum number of samples is specified which, if reached, must lead to an accept or reject decision.

2C <u>Sequential Sampling Plan</u> - A test of hypothesis or acceptance test in which the sample size is not determined in advance. Observations are made sequentially or in stages and the decision to terminate the test or obtain another observation depends, at each stage, on the results of observations previously made.

3A <u>Lot-by-Lot Sampling</u> - Acceptance tests which are based on a sample from an individual lot and for which the conclusions apply only to that lot.

3B <u>Continuous Sampling</u> - Acceptance tests for a continuous production process or items for which lot formation is highly artificial. Rejection involves 100% inspection of output for a prescribed number of items.

3C <u>Chain Sampling</u> - Lot-by-lot sampling in which results of previous acceptance tests on lots are included in the decision criterion.

4A <u>Non-Truncated Life Tests</u> - Tests in which all sample items are tested to failure.

4B <u>Truncated Life Tests</u> - Tests which are terminated before all sample items have failed.

5A <u>Parametric Tests</u> - Tests which assume an underlying probability distribution for the variable of interest.

5B <u>Nonparametric Tests</u> - Tests which require no assumption on the underlying probability distribution.

6A <u>Replacement Life Test</u> - Tests in which failed items are replaced or restored to new condition by repair.

6B <u>Non-Replacement Life Tests</u> - Tests in which failed items are not replaced or repaired.

7A <u>Standard Stress Life Tests</u> - Tests in which the test conditions simulate the existing use conditions.

7B <u>Accelerated Life Tests</u> - Tests in which the stress (external and internal) is controlled to induce early failures in order to reduce the amount of testing time.

Except for a few special cases, a selection of one characteristic from each of the seven groups represents a type of acceptance test. For example, selecting 1B, 2A, 3A, 4B, 5A, 6A, and 7B represents a single sampling-variables life test for a particular lot. The test is terminated by a truncation rule based on an assumed probability distribution. Accelerated stress conditions are to be applied. There are easily over 100 feasible types of acceptance tests which can be generated from the above list. Guidelines for choosing the most appropriate combination are given below.

8.3.1 Attributes, Variables, and Life Tests

An attribute reliability test is one where each of n items is tested and judged to be a success or failure. An attributes test usually applies to a one-shot item in which time or cycles are not involved. It is possible, however, to include time by testing each item for a specified period and counting the number of successes and failures. This type of test is not called a life test because the time when the failures occurred during the testing period is not considered and no underlying time-to-failure distribution has to be assumed.

A variables reliability acceptance test is one in which some characteristic of the test items is measured on a continuous scale such as amplitude or power output. If the items are each tested for a specified time period, the characteristic is measured at the end of this time period. Consideration is not given to the distribution of the characteristic over time.

A life test is one in which items are tested over time and are classified into success or failure categories. Consideration is given to when the failures occur and the accept/reject decision is based on an assumed underlying failure distribution or on a nonparametric method if appropriate for the stated reliability goal.

Which of the three alternatives should be used can often be logically decided by considerating the application of the item and the type of reliability goal. Items which are of the one-shot type such as an explosive switch, or a proximity fuse cannot be life tested. For the explosive switch the only characteristic is good or bad (detonate or dud) which must be tested by an attributes test. The proximity fuse, on the other hand, can be tested either by attributes or by variables, the latter measurement perhaps being the distance from target at detonation. A programmer is an example of an item whose operation is not independent of time but whose output at the end of a specified time period must meet some specified criteria. In this case, either an attributes or variables test at the end of the time period can be used.

If the reliability goal is stated in terms of probability of survival for a fixed time period, an attributes test at the end of the time period may be used as well as a life test. If the goal is in terms of mean time to failure, a life test is indicated since mean life is a function of failure times. An example would be a component in a long-life satellite.

For cases where two or all three types of tests could conceivably be employed, other factors such as type of information provided, degree of protection afforded, amount and cost of inspection and ease of administration should be considered. The following table summarizes the advantages and disadvantages of each type of plan with respect to these considerations.

TABLE 8-2

COMPARISONS BETWEEN ATTRIBUTES, VARIABLES AND LIFE TESTING

Factor	Attributes	Variables	Life Test
Use of Item	Single operation	Single operation	Repetitive or continuous operation over time
Type of Information Yielded	Number or percent of sample that failed to meet specified quality characteristics at a given point in time.	Distribution of some quantitative output at a given point in time. Provides most information for quality improvement.	Distribution of failures over time.
Reliability Goal	Percent of defective or probability of survival over a fixed time period.	Output tolerance limits which define success or failure possibly applying after a fixed period of operation.	Mean life, failure rate or probability of survival for a fixed time period.
Sample Size for Given Protection	Highest	Lower than attributes test for corresponding plan.	Lower than attributes test for corresponding plan.
Ease of Inspection	Requires relatively simple test equipment and less qualified personnel.	More complex test equipment and better trained people required than for attributes tests.	Continuous observation necessary for most types of tests. Highly trained people required. Difficult to maintain controlled test conditions.
Simplicity of Application	Data recording and analysis is fairly simple. Single set of attributes criteria applies to all quality characteristics.	More clerical costs than attribute plans. Variables criteria needed for each quality characteristic.	More clerical costs than attribute plans. Has one set of criteria for all quality characteristics.
Statistical Considerations	No assumptions on failure distribution required. Binomial distribution applies for most cases. Extensive tables are available.*	Requires a parametric assumption on the distribution of the characteristic considered. Tables for the normal distribution are available.**	Requires an assumption of a time-to-failure distribution. Tables available for exponential and Weibull distributions.†

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* MIL-STD-105B, Sampling Procedures and Tables for Inspection by Attributes, Superintendent of Documents, Government Printing Office, Washington, D.C., 1958.

Dodge, H.T. and Romig, H.G., Sampling Inspection Tables, John Wiley & Sons, New York 1944.

Sobel, M. and Tischendorf, J.A., "Acceptance Sampling with New Life Test Objectives", Proceedings of the Fifth National Symposium on Reliability and Quality Control in Electronics, 1959, pp. 108-118.

** MIL-STD-414, Sampling Procedures and Tables for Inspection by Variables for Percent Defective, Superintendent of Documents, Government Printing Office, Washington, D.C., 1957.

† For exponential distribution, Handbook H-108, Sampling Procedures and Tables for Reliability and Life Testing, Office of the Assistant Secretary of Defense (Supply and Logistics), Washington, D.C., 1960.

For Weibull distribution, Goode, H.P., and Kao, J.H.K., Sampling Plans Based on the Weibull Distribution, Tech Report No. 1, Department of the Navy, Office of Naval Research, [Nonr-41(403)] Astia No. AD 243881, 1960.

8.3.2 Single, Multiple and Sequential Sampling

Single, multiple and sequential sampling plans can be devised that have nearly identical O.C. curves. It shall therefore be assumed in the following discussion that equal protection is afforded by each of these types.

In single sampling, one sample of n items is tested. Accept or reject decisions are made on the basis of the results by comparing the number of observed failures or defects to a predetermined acceptance number, c. In multiple sampling, more than one sample may be necessary before a decision is reached, but the maximum number of samples and thus the maximum number of items to be tested is known. An example is a double sample plan with the following test criteria:

n_1(1st sample size) = 100, c_1(accept number for first sample) = 3

n_2(2nd sample size) = 200, c_2(accept number for both samples) = 7

A first sample of 100 items is taken. If 3 or less defectives are found, the lot is accepted. If 8 or more defectives are found, the lot is rejected. If 4 to 7 defectives are found on the first sample, a second sample of 200 items is taken and the lot is accepted if the total number of defectives is 7 or less.

Sequential sampling is an extension of multiple sampling in that decisions to accept, reject, or sample further can be made after each individual item (or possibly groups of items) is tested. No maximum number of sample items is specified although the probability of very large samples is usually quite small. The decision criteria of a sequential sampling plan can be presented graphically. Figure 8-3 illustrates a test for reliability or percent defective. As sampling progresses, the number of defectives is plotted against the number of items tested. Testing is continued until the plotted step function crosses one of the two decision lines. Since the step function may remain in the continuous testing region for a long period, especially for borderline lots, truncation or stopping rules can be specified so that the effect on the α and β errors are negligible.

Generally, multiple sampling requires less testing than single sampling, and sequential sampling requires less testing than multiple sampling. This is true because lots with very good or very poor quality will

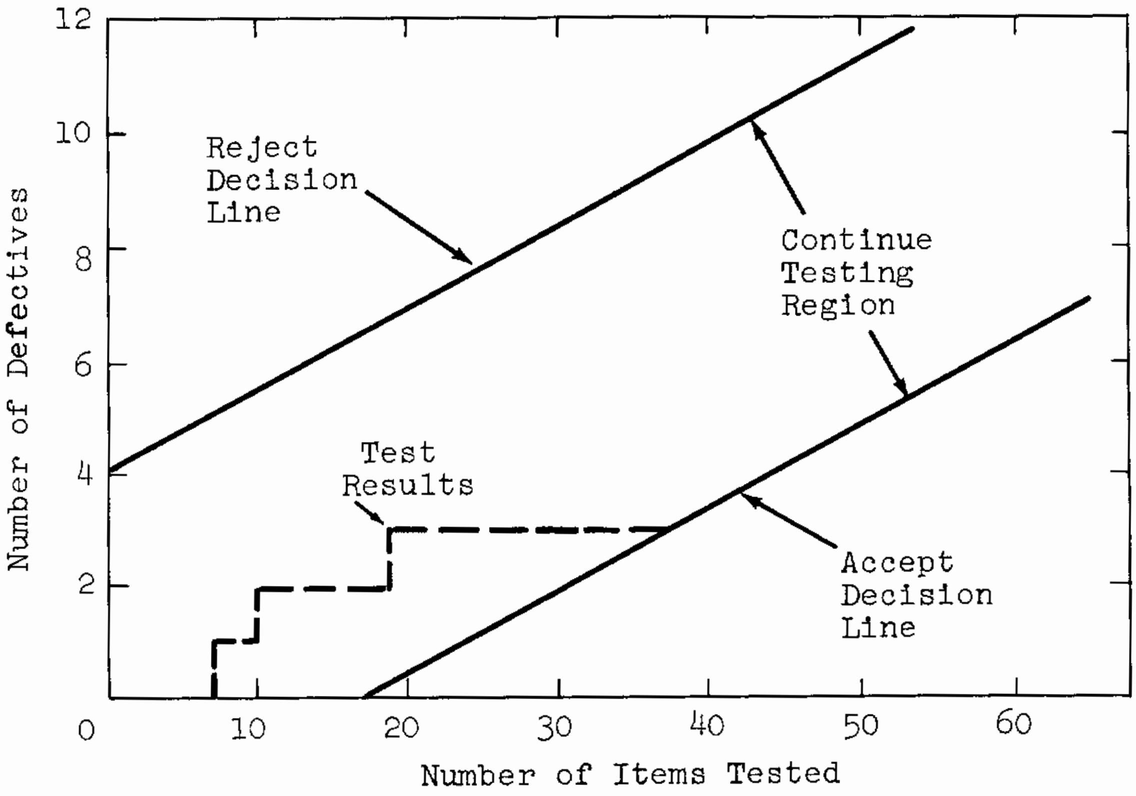

FIGURE 8-3

GRAPHICAL REPRESENTATION OF SEQUENTIAL ACCEPTANCE TEST FOR RELIABILITY OR PERCENT DEFECTIVE

exhibit such characteristics early in the testing, and decisions can be made before multiple samples or further samples in a sequential test are required. Since the first sample of a multiple sampling plan is always smaller than a single sample size and since decisions on sequential tests can be made after the results of each test item, such savings in sample size can be extensive. It should be noted that the exact sample size of multiple or sequential sampling plans is not predetermined but is a function of the true quality of the submitted product. The average sample for various levels of incoming quality can be computed, and the results can be plotted to yield an average sample number (ASN) curve. An example of these curves is shown in Figure 8-4 for plans approximately equivalent to the single sampling plan of n = 75, c = 1.

Table 8-3 compares some characteristics of single, multiple and sequential sampling plans.

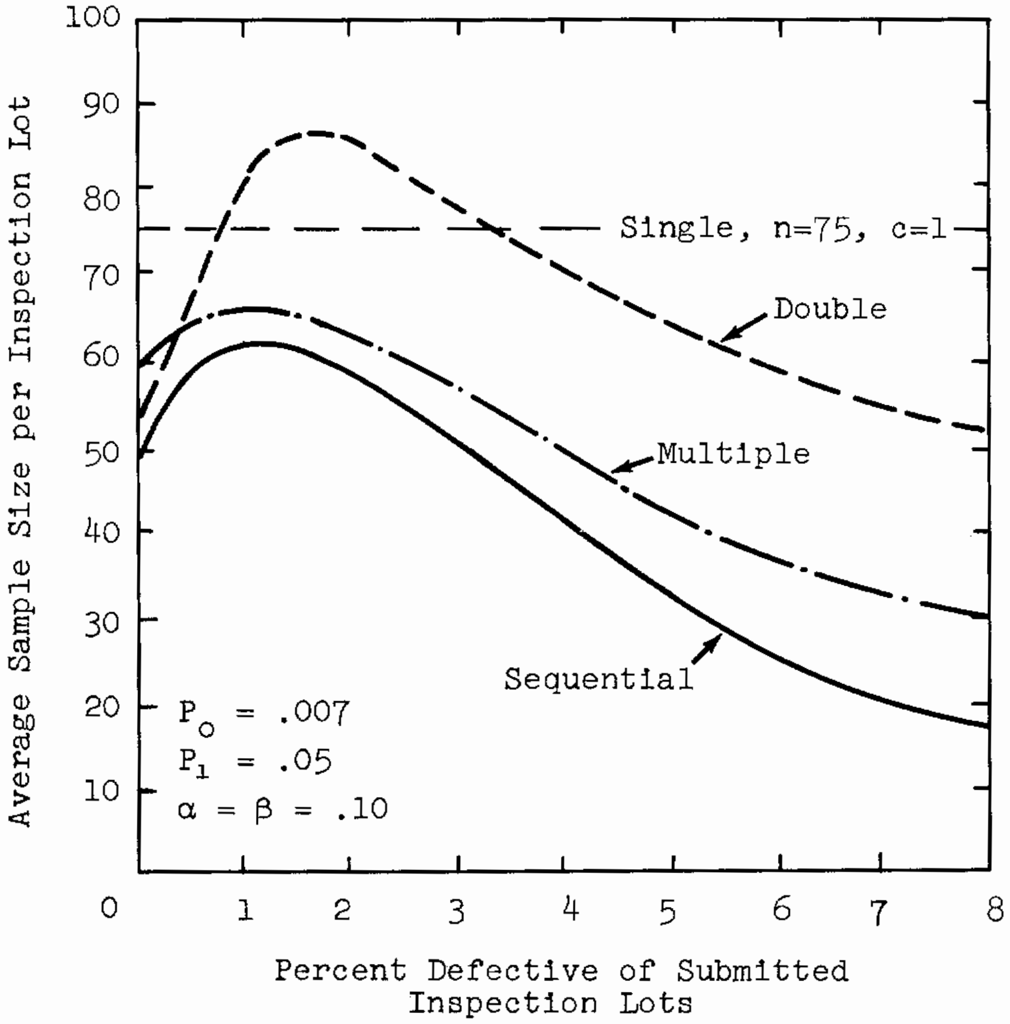

FIGURE 8-4

AVERAGE AMOUNT OF INSPECTION UNDER SINGLE, DOUBLE, MULTIPLE AND SEQUENTIAL SAMPLING (ASN CURVES)

TABLE 8-3

COMPARISON OF SINGLE, MULTIPLE AND SEQUENTIAL SAMPLING PLANS

Characteristic	Single	Multiple	Sequential
Sample Size	Known	Average can be computed for various incoming quality levels. Generally less than single.	Average can be computed for various incoming quality levels. Generally less than single and multiple
Decision Choices	Accept or reject	Accept, reject or take another sample until final sample is selected	Accept, reject or test another item
Predetermined Characteristics	Two of the three quantities n, α or β	Same as single	Fix α and β; n is a random variable
Statistical Considerations	Must know distribution of sample statistic	Same as single	Compare sample against two constants. No need to know sampling distribution
Personnel Training	Requires least training	More trained people required than for single	Requires most training
Ease of Administration	Easiest. Scheduling can be fairly precise and precise test-cost estimates can be made	More difficult than single since the exact number of tests is unknown. Only average test costs can be estimated	Most difficult in terms of testing, scheduling and overall administration. Most time consuming
Miscellaneous	Best used for testing situations where ease of administration is most important and cost of testing is relatively unimportant	Has psychological advantage in that supplier is given a "second chance" by taking further samples if first sample results indicate a marginal lot.	Most efficient test in terms of required sample size. Will require approximately 50% of sample size of single sampling plans. Best to use when test costs are most important.

8.3.3 Lot-by-Lot, Continuous and Chain Sampling Plans †

The choice between lot-by-lot and continuous sampling is usually dictated by the type of production process. If rational inspection lots can be formed, samples taken from the lot can be tested to determine lot conformance. Continuous sampling procedures apply when production is continuous and the formation of inspection lots is artificial. Chain sampling has features of both lot-by-lot and continuous sampling. It is used to make decisions on a submitted lot but may require the use of results of previous lot samplings for determining conformance. Therefore, the assumption of a continuous production of equal quality lots is necessary in order to use previous results.

Continuous sampling plans for reliability acceptance tests are rarely used because: (1) 100% inspection is required if a reject decision is made; and (2) the continuous production assumption is usually not realistic for complex equipments. Chain sampling, which is more economical than lot-by-lot sampling, is appropriate when performing costly and destructive testing but the required assumption of a continuous production of equal quality lots is again a serious limitation when complex equipments are involved.

8.3.4 Non-Truncated and Truncated Life Tests

Non-truncated life tests are defined as those in which a decision is made only after all items on test have failed. A truncated life test is defined here to be one in which testing is terminated after a preassigned number of failures occur or after a preassigned number of test hours have been accumulated. For practical reasons, most life tests are of the truncated type in order to control

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† Information and procedures for continuous sampling plans can be found in Handbook H106, Multi-Level Continuous Sampling Procedures and Tables by Attributes, Office of the Assistant Secretary of Defense (Supply and Logistics), Wash. D. C., 31 October 1958.

The basic chain sampling approach is described in an article by H.F. Dodge, "Chain Sampling Inspection Plans", Industrial Quality Control, Vol.XI, No.4, Jan. 1955, pp. 10-13.

economic and scheduling factors. Truncated life tests are especially suitable when the failure time distribution is exponential because of its constant failure rate. If a normal or log-normal failure time distribution exists, however, the mathematical difficulties of evaluating the results of truncated tests are quite formidable. This is also true for other failure distributions with non-constant failure rates which involve more than one parameter whose values are unknown.

The word truncation also applies to sequential tests in which a rule is specified for making a decision if the accept or reject line is not crossed before a predetermined number of failures or accumulated number of test hours are reached. The truncation can also take the form of converging reject and accept lines if truncation is to be based on both the number of failures and the accumulated test time. The rules for truncating the sequential plan must be such that the effects on α and β of the ordinary sequential plan are negligible.

8.3.5 Parametric and Nonparametric Tests

Parametric tests which involve an assumption or knowledge of an underlying failure law are used almost exclusively in life tests. For complex electronic items, the exponential failure law is usually assumed. Attributes tests which are conducted after a period of test operation are essentially nonparametric since no assumption of the failure distribution over the testing period is required. Generally, parametric tests are more efficient than nonparametric tests since, for a given amount of testing, more precise estimates or smaller probabilities of incorrect decisions will result than for nonparametric tests. The limitations on the types of statistics testable constitute a disadvantage of nonparametric tests in reliability conformance testing. For example, nonparametric tests of central tendency apply to median life, while the specification may be in terms of a mean life.

It should be noted that an incorrect assumption of the underlying failure distribution in a parametric test can lead to an O.C. curve which differs greatly from that planned, especially for small sample sizes. Also, nonparametric tests are generally easy to conduct and evaluate, often requiring only counting, adding, subtracting or ranking. Because of these two points nonparametric tests are now receiving much more consideration than in the past.

8.3.6 Replacement and Non-Replacement Life Tests

Replacement tests are those in which failures occurring during the test are replaced by new items. If the items are complex, replacement may be interpreted to mean restoration of the failed item to new condition by repair or replacement of failed components within the unit of product. In non-replacement tests, failed items are not replaced or repaired; hence the number of test items decreases as life testing progresses.

Generally, the effect of using a replacement test is to decrease the waiting time before a decision can be made over that of a non-replacement test with the same number of items on test originally.† This savings in time is at the cost of having to place more items on test. If a sequential test is used, it is usually preferable to plan for a replacement test since all items may fail in a non-replacement test before a decision is made, and more test items will have to be obtained.

8.3.7 Standard Stress and Accelerated Life Tests

Standard stress tests are those in which the internal and external stress conditions expected during operational use of the item are simulated as much as practical during the life test. An accelerated test is one in which the test conditions are adjusted so as to accelerate failure. While accelerated tests can be used to discover and evaluate critical weaknesses in the parts or design, their attractiveness in acceptance tests is that the amount of test time is reduced since the required number of failures for a decision will occur relatively early. This reduction in waiting time is most important for items which have very high reliability goals since the amount of required test timeto establish conformance can be prohibitive.

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† Table 8-6 in Section 8.4.3 presents some ratios of expected waiting time for replacement to non-replacement tests when the exponential distribution is assumed.

If the stress conditions are accelerated, the reliability goal under standard stress conditions has to be modified accordingly. Therefore the relationship (approximate at least) of reliability to the acceleration factor must be known in order to set up appropriate test criteria. Most accelerated life tests now performed are at the part level because of the high reliability requirements existing at this level (failure rates on the order of one per million hours) and stress/failure relationships are relatively easy to determine through experimentation.

8.4 Amount of Testing for Common Reliability Acceptance Tests

This section presents tables and formulas for determining the amount of testing required for various commonly used reliability acceptance tests. All tests are attribute tests in the sense that sampled items are determined to be either successes or failures. Those tests which involve measurement of time to failure are called life tests consistent with previous usage. The following tests are considered:

A. Attributes Tests - (For one-shot items or tests conducted over a fixed time period)

  A.1 Nonparametric (Binominal)

    A.1.1 Single Sampling
    A.1.2 Sequential Sampling

  A.2 Exponential Assumption

    A.2.1 Single Sampling

B. Life Tests - Exponential Assumption

  B.1 Truncated Non-Sequential Tests
  B.2 Sequential Tests

8.4.1 Attributes Tests - Nonparametric

For nonparametric attributes tests, the allocated reliability must be expressed by a probability of survival for a fixed time period (e.g., the mission length) or equivalently, the percent of items in the lot that are defective, i.e., the percent that will fail during operation over this fixed time period. If R_0 represents the A.R.L. and R_0 the U.R.L., the test specification would be of the following form:

$$H_0: \quad R(T) = R_0$$

$$H_1: \quad R(T) = R_1$$

Test duration - T hours (or cycles)
Producer's Risk - α
Consumer's Risk - β

For one-shot items where time is not involved, the test duration is equivalently 0 hours, i.e., the item is inspected for reliability conformance without any test time accumulation.

8.4.1.1 Single Sampling

If the lot size is assumed to be large relative to sample size, the binomial distribution can be used to generate the O.C. curve of a single sampling plan. Accept or reject decisions are made by testing n items for T hours. The lot is accepted if the number of failures is less than or equal to c, the acceptance number. The probability of acceptance if R(T) is the true reliability is

$$P_a[R(T)] = \sum_{x=0}^{c} \frac{n!}{x!(n-x)!} [1-R(T)]^x [R(T)]^{n-x} \qquad (8\text{-}1)$$

If R(T) is close to 1.0 and n is not too small, the Poisson approximation can be used to obtain P_a from the equation

$$P_a[R(T)] = \sum_{x=0}^{c} \frac{e^{-m} m^x}{x!} \qquad (8\text{-}2)$$

where $m = n[1-R(T)]$

In order to meet the test requirements, values of n and c must be chosen so that

$$P_a[R_0] = 1-\alpha \quad \text{and} \quad P_a[R_1] = \beta$$

Because c can only take on integral values, it usually is not possible to find a single sampling plan that satisfies the above requirements exactly. Since each of the R_0, R_1, α and β values is in some sense arbitrary (a requirement of exactly .95 probability of acceptance if $R = R_0$ is very rarely determined on purely logistical, economic, or other nonpersonal factors), a reasonable approach would be to select a plan with a minimum sample size that comes closest to meeting the requirement.

$$P_a[R_0] \geq 1-\alpha, \quad P_a[R_1] \leq \beta.$$

Single sampling plan tables are presented in Section 8.4.1.3.

8.4.1.2 Sequential Sampling

For sequential sampling, the number of items on test is not predetermined but is a random variable whose average is a function of the true reliability. The expected sample size or number of observations before a decision is reached for incoming reliability levels of $R = 1.0$, $R = R_0$ and $R = R_1$ is given by the formulas below.

$$E_{R=1}(n) = \frac{a_1}{b_2}$$

$$E_{R=R_0}(n) = \frac{(1-\alpha)a_1 + \alpha a_2}{(1-R_0)b_1 + R_0 b_2} \qquad (8\text{-}3)$$

$$E_{R=R_1}(n) = \frac{\beta a_1 + (1-\beta)a_2}{(1-R_1)b_1 + R_1 b_2}$$

where $a_1 = \log \frac{\beta}{1-\alpha}$, $a_2 = \log \frac{1-\beta}{\alpha}$

(natural logarithms)

$$b_1 = \log \frac{1-R_1}{1-R_0}, \quad b_2 = \log \frac{R_1}{R_0}$$

These formulas are based on plans where one item at a time is tested. For practical reasons it may be desired to test groups of items at one time; at worst, the α and β errors will be less than specified but at the expense of increased sample sizes.

8.4.1.3 Tables of Attribute Sampling Plans

Table 8-4 can be used to determine the single sampling plan (n and c) for various values of $k = (1-R_1)/(1-R_0)$ and various sets of α and β. This table is an extension of Table 2C-5 in Quality Control and Reliability Handbook H-108 which is referenced in Table 8-2.

Table 8-5 presents single sampling plans which approximate the O.C. curve requirements for various common sets of R_0, R_1, α and β. The expected sample sizes for equivalent sequential plans is also shown.

Comparison of the two tables will reveal some differences (usually minor) in n and c for an identical single sample test specification. Table 8-4 is based on the Poisson distribution and the plans are derived so that α is guaranteed and β is no more than specified. Table 8-5 is based on the binomial distribution and the criterion used was to meet both the α and β requirements as nearly as possible.

The tables illustrate the following points:

(1) Sample size varies inversely with α, β and k (R_0 fixed)

(2) Sample size varies directly with R_0 (k fixed).

(3) Sequential sampling generally will result in lower sample sizes than single sampling.

Total accumulated test time (i.e., the total number of hours accumulated by all items on test) can also be determined for single sampling plans by simply multiplying the sample size by the test period, i.e.,

$$T^* = nT$$

where T* is total number of test hours accumulated. For these plans, T* is actually the maximum number of hours, which will occur only if the lot is accepted. If (c + 1) failures occur before nT hours are accumulated, the lot is rejected at that time. For sequential sampling the expected maximum total test time given $R = R_1$ and the lot is accepted is

$$E_{R_1}(T^*) = E_{R_1}[(n)T]$$

TABLE 8-4

ATTRIBUTE SAMPLING PLANS FOR SPECIFIED R_0, R_1, α AND β

$P_a(R_o) = 1-\alpha$, $P_a(R_1) \leq \beta$ (c = acceptance number)

$k=\frac{1-R_1}{1-R_o}$	$\alpha = .01$						$\alpha = .05$						$\alpha = .10$					
	$\beta = .01$		$\beta = .05$		$\beta = .10$		$\beta = .01$		$\beta = .05$		$\beta = .10$		$\beta = .01$		$\beta = .05$		$\beta = .10$	
	c	D	c	D	c	D	c	D	c	D	c	D	c	D	c	D	c	D
1.5	135	110.4	100	79.1	82	63.3	94	79.6	66	54.1	54	43.4	76	66.0	51	43.0	40	33.0
2	45	31.7	34	22.7	29	18.7	32	24.2	22	15.7	18	12.4	25	19.7	17	12.8	14	10.3
2.5	26	16.4	20	11.8	17	9.62	18	12.4	13	8.46	10	6.17	14	10.3	10	7.02	8	5.43
3	18	10.3	14	7.48	12	6.10	12	7.69	9	5.43	7	3.98	10	7.02	7	4.66	5	3.15
3.5	14	7.48	11	5.43	9	4.13	9	5.43	7	3.98	6	3.29	7	4.66	5	3.15	4	2.43
4	11	5.43	9	4.13	8	3.51	8	4.70	6	3.29	5	2.61	6	3.90	4	2.43	3	1.75
4.5	10	4.77	8	3.51	7	2.91	6	3.29	5	2.61	4	1.97	5	3.15	3	1.75	2	1.10
5	8	3.51	7	2.91	6	2.33	6	3.29	4	1.97	3	1.37	4	2.43	3	1.75	2	1.10
7.5	5	1.78	4	1.28	4	1.28	3	1.37	3	1.37	2	.818	3	1.75	1	.532	1	.532
10	4	1.28	3	.823	3	.823	3	1.37	2	.818	2	.818	2	1.10	1	.532	1	.532

To find the sample size for given R_o, R_1, α and β, divide the appropriate D value by $1-R_o$ and take the greatest integer less than the quotient. Example: $R_o = .95$, $R_1 = .80$, $\alpha = .10$, $\beta = .05$. k is equal to $\frac{1-.80}{1-.95} = 4$. Sample size n = [D/.05] = [2.43/.05] = 48. The acceptance number c is equal to 4.

TABLE 8-5

ATTRIBUTE SAMPLING PLANS FOR SOME COMMON TEST PARAMETERS

R_0	R_1	k	Single Sampling Plans		Sequential Test, Expected Sample Size			Single Sampling Plans		Sequential Test, Expected Sample Size		
			n	c	$E_{1.0}(n)$	$E_{R_0}(n)$	$E_{R_1}(n)$	n	c	$E_{1.0}(n)$	$E_{R_0}(n)$	$E_{R_1}(n)$
			$\alpha = .10,\ \beta = .10$					$\alpha = .10,\ \beta = .20$				
.99	.98	2	950	13	199	437	582	650	9	136	285	451
	.97	3	320	5	104	175	142	180	3	71	114	110
	.95	5	110	2	53	69	43	60	1	36	45	33
	.90	10	37	1	23	25	12	30	1	16	16	9.4
.95	.90	2	190	13	40	103	87	113	8	28	67	67
	.85	3	60	5	20	35	25	35	3	14	23	19
	.75	5	20	2	9.3	12	7.8	11	1	6.4	7.9	6.1
	.50	10	8	1	3.4	3.6	2.2	5	1	2.3	2.3	1.7
.90	.80	2	80	11	19	48	40	56	8	13	31.3	30
	.70	3	25	4	8.8	15	11	18	3	6.0	9.9	8.9
	.50	5	9	3	3.7	4.8	3.4	5	1	2.6	3.1	2.7
.85	.70	2	49	10	11	29	25	33	7	7.7	18.6	19
	.55	3	16	4	5.1	8.6	6.8	11	3	3.5	5.6	5.4
.80	.60	2	33	9	7.6	19	17	24	7	5.2	12	13
	.40	3	9	3	3.2	5.3	4.6	6	2	2.2	3.4	3.6
			$\alpha = .20,\ \beta = .10$					$\alpha = .20,\ \beta = .20$				
.99	.98	2	650	9	188	339	379	400	6	125	206.9	275
	.97	3	220	4	98	136	93	140	3	65	83	67
	.95	5	78	2	50	54	28	60	2	33	33	20
	.90	10	22	1	22	19	7.9	16	1	15	12	5.8
.95	.90	2	129	9	38	80	56	78	6	25	49	41
	.85	3	46	4	19	27	16	31	3	12	16	12
	.75	5	16	2	8.8	9.4	5.1	11	2	5.9	5.8	3.7
	.50	10	4	1	3.2	2.8	1.4	4	1	2.2	1.7	1.0
.90	.80	2	59	8	18	37	26	39	6	12	23	19
	.70	3	23	4	8.3	11	7.5	9	2	5.5	7.2	5.4
	.50	5	8	2	3.5	3.7	2.2	5	2	2.4	2.3	1.6
.85	.70	2	35	7	11	22.1	16	21	5	7.1	14	11
	.55	3	10	3	4.8	6.6	4.5	6	2	3.2	4.1	3.3
.80	.60	2	24	7	7.2	15	11	16	5	4.8	9.1	8.0
	.40	3	7	3	3.0	4.1	3.0	4	2	2.0	2.5	2.2

$E_{R_i}(T^*)$ will be smaller than this value if lots are rejected since individual failures will occur before they accumulate T hours of test time. Table 8-5 can be used to determine $E_{R_i}(T^*)$ for $R_i = 1.0, R_0, R_1$.

8.4.2 Attributes Tests - Exponential Assumption

This type of test is similar to the nonparametric case in terms of test operation and criteria. The major difference is that $R(t)$ is replaced by the exponential formula $e^{-\lambda t}$ where t is time and λ, the failure rate, is equal to the reciprocal of the mean life. Also, the "amount of testing" can be measured in various ways as listed below:

n - the number of items on test
r - the number of failures
WT - the waiting time before a decision (time elapsed from start of test to the time a decision is reached)
T* - the total number of accumulated test hours before a decision is reached.

If the ARL and URL are specified in terms of mean life or failure rate, this type of test is appropriate if a mission length or significant time period T can be determined for the item. This, in turn, will yield $R_0(T) = e^{-\lambda_0 T} = e^{-T/\theta_0}$ and $R_1(T) = e^{-\lambda_1 T} = e^{-T/\theta_1}$, the ARL and URL, respectively for a reliability specification for T hours corresponding to the specified values $\lambda_0 = 1/\theta_0$ and $\lambda_1 = 1/\theta_1$.

The conversion of specified failure rates or mean lives to probability of survival specifications will lead to exactly the same types of tests discussed in the previous section, that is, n items are put on test each for T hours. If c or less failures occur, conformance to the reliability requirement is accepted. However, because of the exponential assumption, the expected waiting time before a decision is made can be calculated.

Table 8-4 can be used to determine n and c. Epstein† has shown that the expected waiting time before a decision is reached (as a function of true reliability R) is equal to

$$E_R(WT) = \sum_{k=1}^{c-1} \frac{n!}{r!(n-k)!} R^{n-k} (1-R)^k E_R[X_{k,n}] \qquad (8\text{-}4)$$

where

$$E_R[X_{k,n}] = \frac{-T}{\log R} \sum_{j=1}^{k} \frac{1}{n-j+1}$$

The term $\sum_{j=1}^{k} \frac{1}{n-j+1}$ is extensively tabulated in the above cited reference for many sets of k and n.

8.4.3 Life Tests - Exponential Assumption

The tests discussed in this section are based on the assumption that the underlying distribution of failures with time follows the exponential law,

$$\begin{aligned} f(t) &= \frac{1}{\theta} e^{-t/\theta} \\ &= \lambda e^{-\lambda t} \end{aligned} \qquad (8\text{-}5)$$

where t = failure time

f(t) is the failure time probability density function

θ is the mean failure time or mean time between failures (MTBF)

λ is the constant failure rate.

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† Epstein, B. Statistical Techniques in Life Testing, Chapt.III, Testing of Hypotheses, Wayne State University Technical Report, No. 3, ASTIA No.AD 21145, October, 1958.

For convenience, the following discussions are for tests on θ. The translation to failure rate λ is easily made from Equation 8-5. Appropriate test specifications are those for which the ARL and URL are or can be converted to exponential mean life or constant failure rate values. Two general types of tests are considered: truncated and sequential life tests.

8.4.3.1 Truncated Tests

Truncated tests as used here are those tests which are terminated before all test items fail. The truncation rule can be based on a preassigned number of failures or preassigned number of test hours. Truncated tests are usually more economical than non-truncated in the sense that the maximum amount of testing is known beforehand and the expected waiting time is lower. Table 8-6† illustrates this second point for a non-replacement test where r failures are required before a decision can be made. If only r items are put on test, one has a non-truncated test since all items must fail before a decision is made. If n>r items are on test, a decision can be made before all test items fail. The values in the table are the expected relative savings in time when using a truncated test with n > r items versus a non-truncated test with r items.

TABLE 8-6

EXPECTED RELATIVE SAVING IN TIME BY USING A TRUNCATED TEST

r \ n	1	3	10	20
1	1	.33	.10	.050
3		1	.18	.087
10			1	.230

Example: If 3 failures are required, by placing 10 items on test (non-replacement), the waiting time before a decision is, on the average, reduced to .18 of the waiting time of a non-truncated test.

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† Condensed from Handbook H-108 (See Reference in Table 8-2.)

Only replacement tests which are terminated after a specified number of test hours will now be considered. For a given test specification (ARL, URL, α , β), a sample size of n and a critical number of failures can be computed. If T_0 is the test termination time (the maximum number of test hours each item will accumulate) and r_0 is the critical number of failures, the decision rule is as follows:

If r_0 failures occur before T_0 test hours, reject the lot.

If T_0 test hours are accumulated before r_0 failures occur, accept the lot.

Replacement tests terminated by a preassigned number of failures will lead to approximately the same amount of testing for plans with identical O.C. curves. Nonreplacement tests for either type will, on the average, require more test hours but fewer test items.

The O.C. curve for the plan described above can be obtained from the Poisson formula

$$P_a(\theta) = \sum_{x=0}^{r_0-1} \frac{e^{-nT_0/\theta}\left(\frac{nT_0}{\theta}\right)^x}{x!} \qquad (8\text{-}6)$$

where $P_a(\theta)$ is the probability of accepting items with a mean life of θ,

n is the number of items put on test,

r_0 is the critical (reject) number of failures,

T_0 is the test termination time.

If the ARL is denoted by θ_0 and the URL by θ_1 , r_0 is determined so that

$$P_a(\theta_0) \geq 1-\alpha, \quad P_a(\theta_1) \leq \beta$$

It should be noted that for a replacement test, $T_0^* = nT_0$ represents the total amount of test hours and, therefore, can be used in lieu of T_0 to index a set of sampling plans.

In order to compare the amount of testing for various plans, the following statistics can be used:

$E_\theta(r)$ = expected number of failures given mean life θ

$E_\theta(T^*)$ = expected total test time given mean life θ

$E_\theta(WT)$ = expected waiting time given mean life θ

For replacement tests with n items, the following important relationships hold:

$$\begin{aligned} E_\theta(T^*) &= \theta\ E_\theta(r) \\ E_\theta(WT) &= \frac{\theta}{n} E_\theta(r) = \frac{E_\theta(T^*)}{n} \end{aligned} \tag{8-7}$$

Therefore, computing $E_\theta(r)$ enables one to determine the other two statistics for a given plan. The formula for the expected number of failures if the mean life is θ has been shown by Epstein† to be

$$E_\theta(r) = m \sum_{x=0}^{r_0-2} \frac{e^{-m} m^x}{x!} + r_0 \left[1 - \sum_{x=0}^{r_0-1} \frac{e^{-m} m^x}{x!} \right] \tag{8-8}$$

where m is the Poisson mean $\frac{n\ T_0}{\theta}$

Values of $E_{\theta_0}(r)$ and $E_{\theta_1}(r)$ are given for various common test plans in Table 8-7 of Section 8.4.3.3.

8.4.3.2 Sequential Life Tests - Testing with Replacement

Sequential life tests under the exponential distribution are commonly used for complex equipment in order to minimize the amount of testing. For a given θ_0, θ_1, α and β, the following values are computed:

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† Epstein, B. Statistical Techniques in Life Testing, Chapt. III, Testing of Hypothesis, Wayne State University, Technical Report No. 3, ASTIA No. AD 21145, October, 1958.

$$h_0 = \frac{-\log \frac{\beta}{1-\alpha}}{\frac{1}{\theta_1} - \frac{1}{\theta_0}}, \quad h_1 = \frac{\log \frac{1-\beta}{\alpha}}{\frac{1}{\theta_1} - \frac{1}{\theta_0}}, \quad s = \frac{\log (\theta_0/\theta_1)}{\frac{1}{\theta_1} - \frac{1}{\theta_0}} \qquad (8\text{-}9)$$

If n items are put on test, continuous decisions can be made as follows:

If after t test hours are accumulated with r observed failures

$$\begin{aligned} \text{reject if:} \quad & nt \le -h_1 + rs \\ \text{accept if:} \quad & nt \ge h_0 + rs \qquad (8\text{-}10) \\ \text{continue testing if:} \quad & -h_1 + rs < nt < h_0 + rs \end{aligned}$$

This decision criterion is shown graphically in Figure 8-5 for a specific test plan. The expected number of failures before a decision is reached can be shown to be as follows:

$$E_{\theta_0}(r) = \frac{(1-\alpha) \log \frac{\beta}{1-\alpha} + \alpha \log \frac{1-\beta}{\alpha}}{\log k - (k-1)}$$

$$(8\text{-}11)$$

$$E_{\theta_1}(r) = \frac{\beta \log \frac{\beta}{1-\alpha} + (1-\beta) \log \frac{1-\beta}{\alpha}}{\log k - \frac{k-1}{k}}$$

where $k = \frac{\theta_0}{\theta_1}$.

The relationships between $E_\theta(r)$, $E_\theta(T^*)$ and $E_\theta(WT)$ are the same as those for the truncated non-sequential replacement test given by equation (8-7).

If θ is between θ_0 and θ_1, the expected number of failures is greater than both $E_{\theta_0}(r)$ and $E_{\theta_1}(r)$. In order to avoid having to test a substantial number of items, a truncation rule can be set on the number of failures, the total time accumulated, or a combination of the two. This leads to a pair of decision lines which at some point begin to converge.

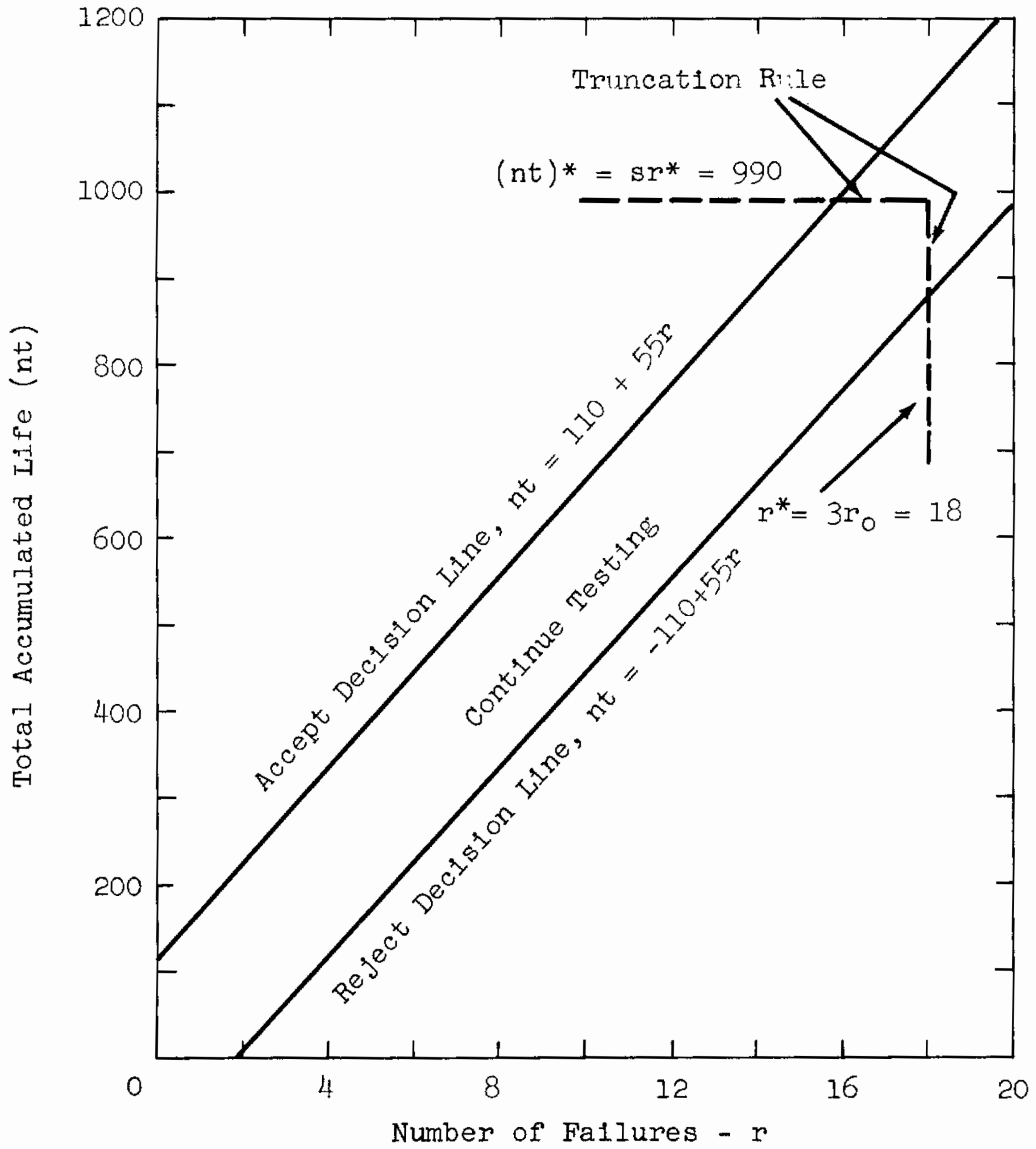

FIGURE 8-5

SEQUENTIAL REPLACEMENT LIFE TEST
$\theta_o = 100, \theta_1 = 33, \alpha = \beta = .10$

It has been determined that the following truncation rule (based on the number of failures) will have a negligible effect on the α and β errors.

(a) The maximum number of failures is set at three times the number of failures required for an equivalent non-sequential test. Hence r^*, the truncated number, equals $3r_0$.

(b) The maximum total accumulated test-hours, nt^*, is set at sr^* where s is the slope of the decision lines.

Figure 8-5 also illustrates the decision criterion based on this truncation rule.

Although the truncation number of failures may be large, it does prevent the possibility of a seemingly never-ending test. Also, the probability that r^* failures will occur is usually very small. As an example, if $\alpha = .05$, $\beta = .05$ and $k = 3$, the rejection number for a non-sequential test is 10. The truncated sequential test would therefore be terminated at a maximum of $3(10) = 30$ failures. It can be shown, however, that if $\theta = \theta_0$, the probability of requiring more than 7 failures is approximately .10; if $\theta = \theta_1$ only 10% of the time more than 12 failures will be required; and for the worst possible case (a value of θ between θ_0 and θ_1) 26 failures would be exceeded with a probability of .10.

Table 8-7 of the next section presents values of $E_{\theta_0}(r)$ and $E_{\theta_1}(r)$ for the same plans as the truncated replacement tests. The truncation number for failures described above is also given.

8.4.3.3 Summary of Exponential Life Test Plans

Table 8-7 summarizes the characteristics of the truncated and sequential life test plans discussed in the previous two sections. All combinations of $\alpha = .05, .10, .25$; $\beta = .05, .10, .25$, and $k = 1.5, 2, 3$ and 5, are included.

For truncated replacement life tests, one can use this table to completely describe all the test characteristics since the rejection number r_0 and $E_{\theta_0}(r)$ is given in the table; the sample size n, the test termination time T_0, and the termination on total test hours T^* are functionally related to each other by $E_{\theta_0}(r)$, as follows:

TABLE 8-7

TEST PARAMETERS AND EXPECTED NUMBER OF FAILURES FOR VARIOUS TRUNCATED AND SEQUENTIAL REPLACEMENT LIFE TESTS

$k=\frac{\theta_0}{\theta_1}$	α	β	Truncated Tests-replacement			Sequential Tests-replacement		
			Rejection Number, r_0	Expected Number of Failures †		Truncation Number, r^*	Expected Number of Failures †	
				$E_{\theta_0}(r$	$E_{\theta_1}(r)$		$E_{\theta_0}(r)$	$E_{\theta_1}(r)$
1.5	.05	.05	67	54	67	201	28	37
	.05	.10	55	43	55	165	19.7	31
	.05	.25	35	26	34	105	12.0	24
	.10	.05	52	43	52	156	25	28
	.10	.10	41	33	41	123	18.5	24
	.10	.25	25	18.8	24	75	10.1	16.5
	.25	.05	32	28	32	96	18.0	15.7
	.25	.10	23	19.6	23	69	14.0	14.5
	.25	.25	12	9.4	11.4	36	5.8	7.6
2	.05	.05	23	15.7	23	69	8.6	13.7
	.05	.10	19	12.4	18.7	57	7.0	13.1
	.05	.25	13	7.6	12.3	39	3.7	8.8
	.10	.05	18	12.8	12.0	36	7.7	10.3
	.10	.10	15	10.2	14.8	45	6.2	9.7
	.10	.25	9	5.4	8.4	27	3.1	6.2
	.25	.05	11	8.6	10.9	33	5.5	5.9
	.25	.10	8	5.8	7.8	24	4.0	5.1
	.25	.25	5	3.2	4.6	15	1.8	2.8
3	.05	.05	10	5.4	9.9	30	2.9	6.1
	.05	.10	8	3.8	7.8	24	2.3	5.6
	.05	.25	6	2.6	5.6	18	1.3	3.9
	.10	.05	8	4.6	7.9	24	2.6	4.6
	.10	.10	6	3.0	5.8	18	2.0	4.2
	.10	.25	4	1.6	3.5	12	1.1	2.8
	.25	.05	5	3.2	5.0	15	1.9	2.6
	.25	.10	4	2.4	3.9	12	1.2	2.1
	.25	.25	2	0.8	1.6	6	0.61	1.3
5	.05	.05	5	2.0	5.0	15	0.78	3.3
	.05	.10	4	1.4	3.9	12	0.59	2.9
	.05	.25	3	0.8	2.7	9	0.33	2.1
	.10	.05	4	1.6	3.9	12	0.70	2.5
	.10	.10	3	1.2	2.9	9	0.52	2.2
	.10	.25	3	1.0	2.8	9	0.28	1.5
	.25	.05	2	1.0	2.0	6	0.50	1.4
	.25	.10	2	0.8	1.9	6	0.35	1.2
	.25	.25	1	0.4	0.86	3	0.16	0.68

† If either n, T_0, or T^* is specified, the other two test parameters can be determined from $E_{\theta_0}(r)$ by the relationships:

$$E_{\theta_0}(r) = \frac{nT_0}{\theta_0} = \frac{T^*}{\theta_0}.$$

For expected total accumulated test hours: $E_{\theta_0}(T^*)=\theta_0 E_{\theta_0}(r)$; $E_{\theta_1}(T^*)=\theta_1 E_{\theta_1}(r)$

For expected waiting time for n items on test: $E_{\theta_0}(WT)=\frac{1}{n}E_{\theta_0}(T^*)$; $E_{\theta_1}(WT)=\frac{1}{n}E_{\theta_1}(T^*)$

$$E_{\theta_0}(r) = \frac{nT_0}{\theta_0} = \frac{T^*}{\theta_0} .$$

Hence if any one of n, T_0, or T^* are specified (as well as θ_0), the other two can be determined. As an example, assume that a maximum of 75 hours of test time is available for each item on test. If $\theta_0 = 200$ and $\theta_1 = 100$, what is the appropriate plan if $\alpha = .10$, $\beta = .05$? From the table, for $k = 2$, $\alpha = .10$, $\beta = .05$, we have

$E_{\theta_0}(r) = \frac{nT_0}{\theta_0} = 12.8$. Since $T_0 = 75$ and $\theta_0 = 200$,

$n = (12.8)\ (200)/75 = 34$. Hence, 34 items are put on test, each for a maximum of 75 hours. Replacements or repairs are made upon failure. If less than 18 failures occur before $(75)\cdot(34) = 2550$ total test hours are accumulated, the lot is accepted. If 18 failures occur before 2550 test hours are accumulated, the lot is rejected.

For sequential life test plans, the truncation number and the expected number of failures for $\theta = \theta_0$ and $\theta = \theta_1$ are given in the table. The test criterion can be determined from equations (8-9) and (8-10) given in the previous section.

Inspection of Table 8-7 reveals that for both truncated and sequential tests, the amount of testing increases as α and β decreases and the amount of testing decreases as k (for fixed θ_0) increases. Also, the table shows that the amount of testing on sequential tests will generally be less than that of nonsequential tests.

The choice of which life-test plan to use depends on individual circumstances, and generalization is difficult. A thorough understanding of the ARL and URL concepts and the associated α and β risks will often reduce the number of satisfactory choices. Further reduction is usually possible when cost of testing in terms of dollars, materials, and time is considered. The cost of testing can be reduced by increasing the α and β risks as shown in Table 8-7. This, of course, must be balanced against wrong decision costs. The next section presents an approach for choosing β risks on unit, equipment, or subsystem tests when a system URL and system β risk is specified. In Section 8.6 the balancing problem -- test costs versus test risks -- is treated from a decision theory point of view.

8.5 An Approach For Choosing Sampling Risks For Unit, Equipment Or Subsystem Tests

The allocation procedure determines unit, equipment or subsystem reliability requirements from a specified system reliability requirement. For complex systems, there are usually several subcontractors responsible for supplying the units, equipments or subsystems. The prime contractor or the military activity is charged with the task of submitting to the operational user a system that meets a stated reliability goal. With respect to system tests, this goal is logically the URL (unacceptable reliability level) since it represents the minimum reliability level sufficient for tactical or operational purposes. The consumer or Beta risk for the system represents the probability that if the integrated system is truly at the URL, only $\beta\%$ of the time will such a system be accepted by the test. The prime contractor has to translate this system test requirement into equivalent requirements for unit, equipment or subsystem tests. The system ARL (acceptable reliability level) which is associated with the producer's risk, has, in this case, only a vague meaning since there are several unit producers which define the "system producer". Alpha risks assigned for unit tests are usually fixed by contract or negotiation and therefore are independent of a system Alpha risk associated with a system ARL.

From the definition of the URL, if each unit in the system had a reliability level equal to its URL, the system would have a reliability level equal to its URL. If there exists a consumer's or Beta risk, β_S, associated with the system URL, the problem then is to determine the Beta risks on the unit tests such that there is $(1-\beta_S)\%$ confidence that systems composed of units accepted by these tests will have a reliability level better than the URL. The producer or Alpha risks on these unit tests shall be assumed to be fixed either through contract or negotiation. An alternative for the system consumer is to allow each producer to set his own Alpha risk as long as a specified unit Beta risk is maintained.

The method herein described for determining the unit Beta risks is limited to systems and units which have exponential failure densities. For convenience, the reliability requirements will be stated in terms of failure rates. The conversion of probability of survival or mean life requirements to failure rates is easily made by using the relationship

$$\lambda = \frac{-\log R(T)}{T} = \frac{1}{\theta}$$

The approach is based primarily on intuitive reasoning, and no attempt is made to rigorously justify the argument. It is important to note, however, that consideration was given to the fact that, in practice, unit quality from a production line has a distribution, i.e., the failure rate of the units will vary over a range of values according to some frequency distribution. Of importance is the outgoing distribution of system failure rate when acceptance tests are performed on the units, the system being generated by a random mating of accepted units. It is conjectured that, for the more common types of unit quality distributions, the outgoing system quality distribution based on the unit tests described below will be at least as satisfactory as the system quality-distribution based on system tests. Further work must be done before this conjecture can be rigorously supported. For several assumed unit failure-rate distributions, the analysis of the system failure-rate distribution for the example given below, as well as others, has shown the conjecture to be reasonable.

Assume that the system is composed of two units A and B. The specified system failure rate is $\lambda^* = .02$. Through the allocation procedure, failure rates of $\hat{\lambda}_a = .012$ and $\hat{\lambda}_b = .008$ have been allocated to units A and B, respectively. Also assume that λ^* represents the URL and that the ARL is equal to $(1/2)\lambda^*$ or .01. If a system test were to be performed, one would have as the test hypothesis:

$$\left.\begin{aligned} H_0 &: \lambda_{os} = (1/2)\lambda^* = .01 \\ H_1 &: \lambda_{1s} = \lambda^* = .02 \end{aligned}\right. \quad (k = 2)$$

with specified α_s and β_s risks, both of which shall be assumed to be equal to 0.10. The unit tests are based on the following hypotheses:

	Unit A	Unit B
H_0:	$\lambda_{oa} = .006$	$\lambda_{ob} = .004$
H_1:	$\lambda_{1a} = .012$	$\lambda_{1b} = .008$

For a truncated replacement test, one can determine an appropriate system test for a fixed n, T_0 or T^*. From Table 8-7, $T^*/\theta_{os} = T^*\lambda_{os} = 10.2$ for $\alpha = \beta = .10$ and $k = 2$. Therefore a total of $\frac{10.2}{0.01} \approx 1000$ system test hours is required. The rejection number given in the table is $r_{os} = 15$. Therefore, if a total of 1000 system test-hours is accumulated and less than fifteen failures occur, the system is accepted.

Neglecting the interactions that may occur when units are integrated into a system†, β risks on independent unit tests of A and B must be determined to assure with 90% confidence that systems composed of accepted units will have a failure rate lower than .02. A logical approach appears to be to test each unit for a total of 1000 hours and to assign unit rejection numbers so that they sum to fifteen or less. Lowering the rejection number for a fixed test time will decrease the Beta risk, but since the unit's allocated failure rates (URL) are lower than the system failure rate, the net effect is that unit Beta risks will be greater than that of the system Beta risk. The increased unit Beta risks have the important consequence of greatly reducing the overall amount of unit testing as compared to tests in which unit Beta risks are equal to the system Beta risk.

Unit rejection numbers can be assigned in many ways. Such factors as unit essentiality‡, cost of testing, or cost of wrong decision, may be the overriding considerations. The simplest approach is to assume that all such factors are approximately equal and to assign unit rejection numbers based on relative unit contribution to the total system failure rate. If this is done, the rejection numbers for A and B are

$$r_{oa} = \frac{\lambda_{1a}}{\lambda_{1s}} = \frac{.012}{.020}\,(15) = 9$$

$$r_{ob} = \frac{\lambda_{1b}}{\lambda_{1s}} = \frac{.008}{.020}\,(15) = 6$$

The assignment of unit rejection numbers in this fashion is a conservative approach since the maximum acceptable number of equivalent system failures is 8 + 5 = 13, or one less than the maximum acceptable for a system test. For n subsystems, this assignment leads to a decrease of n-1 allowable failures from that of a system test. A less conservative approach would be to assign unit acceptance numbers based on the system acceptance number of $r_{os} - 1$.

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† See Section 8.5.2.

‡ See section 8.5.1

The β values for the unit tests can be obtained as follows:

For each unit, the following is known:

$$\lambda_0, \lambda_1, T^*, r_0.$$

Since β is the probability of less than r_0 failures if λ is equal to λ_1,

$$\beta = \sum_{x=0}^{r_0-1} \frac{e^{-\lambda_1 T^*} (\lambda_1 T^*)^x}{x!}$$

Hence, using tables of the cumulative Poisson distribution†,

$$\text{Unit A:} \quad \beta_a = \sum_{x=0}^{8} \frac{e^{-12}(12)^x}{x!}$$

$$= .155$$

$$\text{Unit B:} \quad \beta_b = \sum_{x=0}^{5} \frac{e^{-8}(8)^x}{x!}$$

$$= .191$$

Unit tests based on the above parameters may severely penalize a satisfactory producer because of a high α risk. This can be resolved by devising new unit tests where the β_j's are found by performing the above computations, but α_j is kept at a desired level. It should be noted that the above procedure is relatively insensitive to values of α_s and k. (Because r_{os} can only take

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† Molina, E.C. Poissons Exponential Binomial Limit, D. Van Nostrand Co., Inc., 1942

Ryswick,,R. and Weiss, G., Tables of the Incomplete Gamma Functions of Integral Order, Navweps Report 7292, U.S. Naval Ordnance Laboratory, White Oak, Maryland, November 1960.

on discrete values, plans given in Table 8-7 do not exactly satisfy the α_s and β_s requirements and, therefore, β_j computed for various α_s will differ by relatively small amounts.) This enables one to use a general procedure for finding β_j as outlined below:

1) Using the specified values of λ_{1s} and β_s, find an appropriate sampling plan in terms of system test hours and rejection number of failures (T^*, r_{os}). (Reasonable values of α_s and k can be assumed for determining T^* and r_{os} from Table 8-7 or from other sources.)

2) Unit rejection numbers to be used for obtaining β_j only, are computed from the equation†

$$r_{oj} = \frac{\lambda_{1j}}{\lambda_{1s}} (r_{os}) \qquad \left(\sum_j \lambda_{1j} = \lambda_{1s}\right) \qquad (8\text{-}12)$$

where λ_{1j} is the (allocated) URL of the jth unit

3) Using tables of the cumulative Poisson distribution, β_j can be obtained from the equation;

$$\beta_j = \sum_{x=0}^{r_{oj}-1} \frac{e^{-\lambda_{1j}T^*}\left(\lambda_{1j}T^*\right)^x}{x!} \qquad (8\text{-}13)$$

4) An appropriate unit test plan is then based on the allocated parameters λ_{oj} and λ_{1j}, the fixed α_j and the computed β_j.

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† A less conservative approach for obtaining unit rejection numbers is to use the formula

$$r_{oj} = 1 + \frac{\lambda_{1j}}{\lambda_{1s}} (r_{os} - 1)$$

Figure 8-6 presents the approximate β_j values for $\beta_s = .05$ and $\beta_s = .10$ for ratios of $Z_j = \lambda_{1j}/\lambda_{1s}$ from 0.05 to 1.0. (For any system, $\sum_j Z_j = 1.0$) The example described above can be used to illustrate the use of the chart. For unit A, $Z_a = \frac{.012}{.020} = .6$. The $\beta_s = 0.10$ curve shows that β_a should be approximately equal to .15. For unit B, $Z_b = \frac{.008}{.020} = .4$ and the $\beta_s = 0.10$ curves shows that β_b should be approximately equal to .185. Assuming that the producers risk on unit tests is fixed at .10 for an ARL of $2\lambda_1$, one has the following test parameters:

Unit A:	Unit B:
$\text{ARL} = \lambda_0 = .006$	$\text{ARL} = \lambda_0 = .004$
$\text{URL} = \lambda_1 = .012$	$\text{URL} = \lambda_1 = .008$
$\alpha = .10$	$\alpha = .10$
$\beta = .15$	$\beta = .185$

8.5.1 Inclusion of Unit Essentiality

Unit essentiality can be incorporated into this procedure by an appropriate adjustment of the unit rejection numbers. The above example shall be used to illustrate the approach by assuming that Unit A has an essentiality of $E_a = 1.0$ and Unit B an essentiality of $E_b = 0.5$. From the system Beta risk requirement, r_{os} was determined to be equal to 15. On an assumption of essentiality of 1.0 for both units, $r_{oa} = 9$, $r_{ob} = 6$. However, since $E_b = 0.5$, only 50% of Unit B failures would result in system failures by the definition of essentiality.[†] Therefore, the rejection number 6 for Unit B can be increased by 150% to 12 since system trials where 9 Unit A failures and 9 Unit B failures occurred, would, on the average, lead to only 15 system failures.

[†] Essentiality of a unit is defined in Section 4 to be the probability that the system fails if the unit fails. See Section 5.3.3 for a discussion of the situation where two or more units with essentialities less than one fail in the system.

The general method for incorporating unit essentiality is, therefore, almost identical to that where unit essentialities are equal to one. Step (1) on Page 138 is used to determine the rejection number for system failures, r_{os}. Unit rejection numbers are then obtained from the equation

$$r_{oj} = \frac{1}{E_j} \frac{\lambda'_{1j}}{\lambda_{1s}} r_{os}$$

where λ'_{1j} = equivalent allocated URL (minimum failure rate) of the jth unit if essentiality were equal to one.

Since λ'_{1j} is approximately equal to $(E_j)(\lambda_{1j})$, i.e., the essentiality of the unit multiplied by its allocated URL, the above equation reduces to

$$r_{oj} = \frac{\lambda_{1j}}{\lambda_s} r_{os}. \tag{8-14}$$

To illustrate the method, from the allocation procedure where $E_a = 1.0$, $E_b = 0.5$, one would have for $\lambda_{1s} = .020$

$$\lambda_{1a} = .012$$

$$\lambda_{1b} = .016$$

From Step (1), $r_{os} = 15$. Using equation (8-14) one obtains

$$r_{oa} = \frac{.012}{.020}(15) = 9$$

$$r_{ob} = \frac{.016}{.020}(15) = 12$$

These two rejection numbers and the respective URL's of .012 and .016 can be used to find the Beta risks on the unit tests for fixed unit Alpha risks. Figure 8-6 is still appropriate except that a Z may be greater than one; in these cases equation (8-13) has to be used to compute the B_j. The unit tests are then based on the parameters given in Step (4).

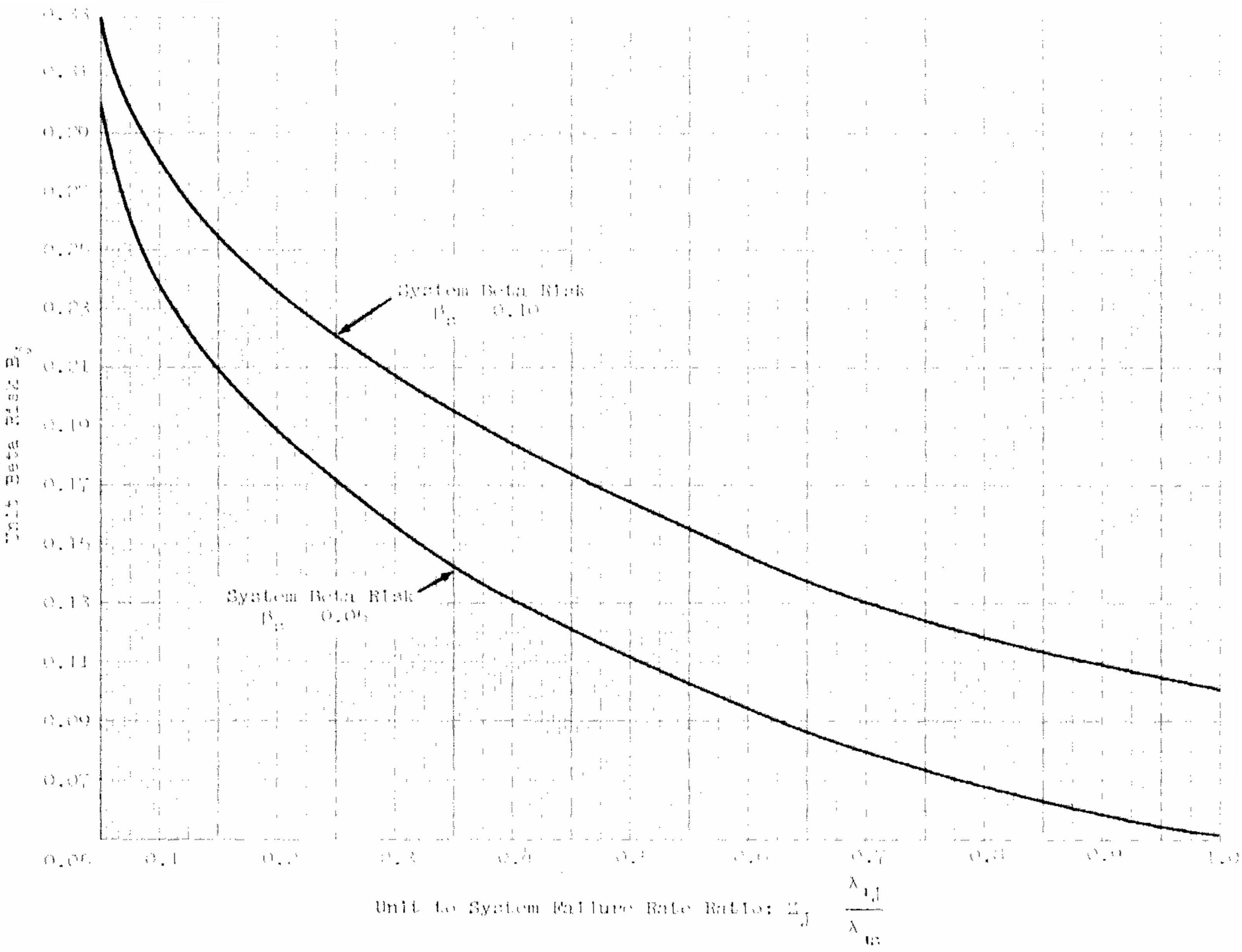


FIGURE [illegible] 6

CHART FOR DETERMINING UNIT BETA RISKS

It should be noted that since essentiality is already incorporated into the allocation procedure to determine allocated unit URL's, each allocated URL is "equally important" regardless of the essentiality. The above procedure incorporates the interpretation of an allocated failure rate based on an essentiality of less than one to define an appropriate test. For example, Unit B with an essentiality of one should have a Beta risk of approximately 0.19 for a URL of .008 and a rejection number of 6. If the essentiality of Unit B was 0.50, the allocated URL is equal to approximately .016 and for a rejection number of 12, the Beta risk for a URL of .016 is approximately 0.13. For this same plan (r_{ob} = 12) the acceptance probability for a Unit B failure rate of .008, is, however, greatly increased from 0.19 to approximately .88. To summarize, for a unit with an essentiality of less than one, the rejection number is increased and, therefore, so is the Beta risk. The URL (in terms of failure rate) is also increased, however, for decreasing essentiality; this has the effect of lowering the Beta risk. The net effect of incorporating essentiality will generally be to reduce the amount of testing on the unit but maintain a Beta risk that is approximately equivalent to tests where unit essentialities are equal to one.

8.5.2 Engineering Considerations

The above approaches which lead to a decreased amount of unit testing through increased unit Beta risks are based solely on a statistical viewpoint. An important engineering consideration is the interface that may result when accepted units are integrated into a system. For example, on independent tests, two units may satisfy the failure rate criteria imposed. When these two units are integrated into a system, the failure rate of the combination may be higher than the sum of the individual failure rates because each unit may be at its extreme tolerance limit with respect to an engineering parameter common to both.

An obvious approach to this problem is to determine unit tolerance intervals, which, for all combinations, will lead to a satisfactory system output. This is often impractical and, in many cases, the interface is not solely a matter of tolerances, e.g., transients in a generator, undetected by test equipment, may cause transients or shorts in the primary equipment.

In defining the Beta risks on unit tests, it is therefore necessary that possible interface be considered. The Beta risks determined by the procedures presented in this section can be interpreted to be the maximum allowable. The amount of necessary decreases in these computed risks will depend on the expected amount of interface problems resulting from system integration.

8.6 The Decision Theory Approach to Optimum Testing

The decision theory approach towards optimizing reliability test effort involves recognition of the fact that the purpose of such testing is to provide a basis for acceptance or rejection with respect to a specified reliability requirement. In the usual case, two types of wrong decisions may result: (a) the decision to reject a product that in reality has met its requirement, and (b) the decision to accept a product that in reality has failed to meet its requirement. In conventional sampling, these wrong decisions are represented by the α and β risks, respectively. These two risks, however, are associated with only two specific levels of reliability, namely the ARL and URL. The general decision theory approach toward optimum testing embraces the concept of the complete O.C. curve of a sampling plan and specifically includes the costs of the wrong decisions that can be made.

Figure 8-7 is a very simplified representation of the optimization problem if testing is to be performed on a product with a fixed but unknown reliability level. It is seen that test costs have a direct relationship to the amount of test effort, while wrong decision costs have an inverse relationship. At some value of test effort (measured on an appropriate scale), the total cost, which is the sum of test and wrong decision costs, is a minimum. For the reliability level to which the particular set of curves apply, this value of test effort is optimal.

The test-cost and wrong-decision-cost curves, however, are related to the true but unknown reliability level of the submitted product. It is theoretically possible to have an infinite number of optimum levels of test effort corresponding to the infinite number of possible reliability levels. It is therefore necessary to select an appropriate method for obtaining a single optimum level of test effort through some kind of "averaging" process. In decision theory, this averaging process is performed through use of the minimax expected loss criterion or through use of Bayes strategies. These concepts are discussed in Section 8.6.3

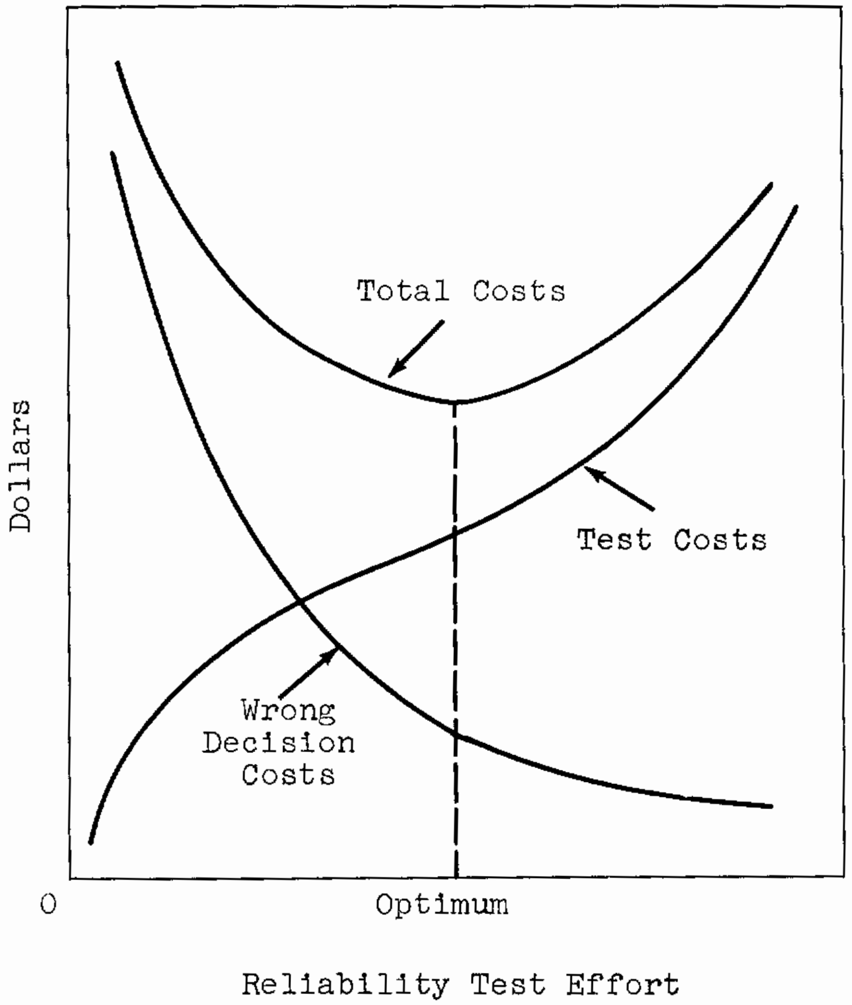


FIGURE 8-7

REPRESENTATION OF RELIABILITY TEST EFFORT OPTIMIZATION FOR ONE INCOMING RELIABILITY LEVEL

8.6.1 Cost Factors

Figure 8-7 indicates that the basic problem of optimizing test effort is essentially economic in nature. It is therefore necessary to establish figures -of-merit for the various economic factors to be considered, specifically the cost relationships between reliability, testing, and decisions. Most of these factors, including time, can be expressed in monetary terms. However, intangibles such as safety and tactical efficacy cannot be wholly disregarded.

The cost factors involved must be treated in a judicious manner in order to avoid complex relationships which might nullify their usefulness. The construction of cost models, imperfect though they may be, has a great advantage over intuitive approaches because model construction compels examination of all the prospective differences between alternative levels of reliability test effort.

Reliability test costs include the cost of items destroyed by testing, the amortized cost of test equipment and instrumentation, the cost of delays due to testing, and all other costs involved in performing the acceptance test. Wrong decision costs are the costs of rejecting good products or accepting poor products. The former includes unnecessary delay, rework, failure analysis, and all such costs resulting from unnecessary further development effort. The cost of accepting poor products may be expressed in terms of mission failure, logistics, maintenance, morale, etc., and it is definitely related to unit essentiality in terms of a specific system mission.

8.6.2 The Loss or Risk Function

The following table shall be used to represent the cost components associated with the accept/reject decision.

True Reliability	Decision	
	Accept Item	Reject Item
$R < R^*$	$C_1(R)$	0
$R \geq R^*$	0	$C_2(R)$

R^* is the specified reliability level (probability of survival, mean life, etc.)

$C_1(R)$ is the cost of accepting unsatisfactory product with reliability $R < R^*$.

$C_2(R)$ is the cost of rejecting satisfactory product with reliability $R \geq R^*$.

(In terms of decision theory, $C_1(R)$ and $C_2(R)$ are called risks, since the minimum (correct decision) costs are subtracted out from each row.)

The total loss or risk function for acceptance test τ is defined to be

$L_\tau(R)$ = (Probability of wrong decision given R) X (Losses due to wrong decision) + (Test costs given R)

or

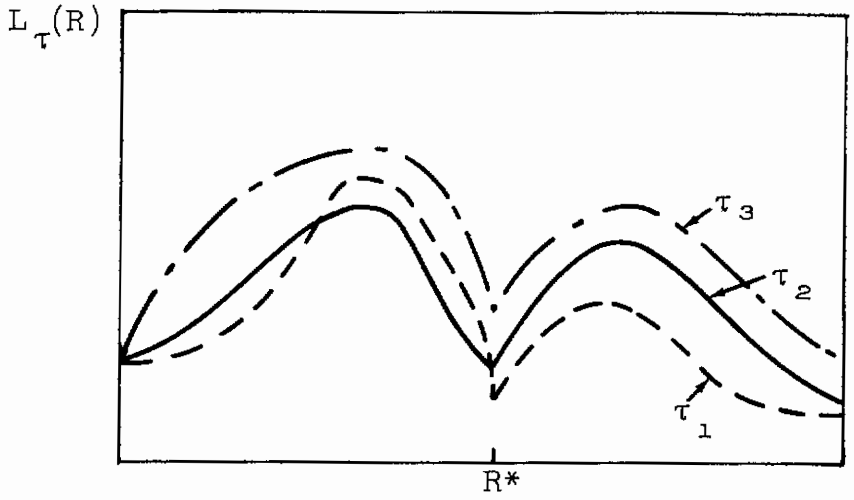
$$L_\tau(R) = \begin{cases} C_1(R)\,\epsilon_1(R) + C_\tau(R) & \text{if } R < R^* \\ C_2(R)\,\epsilon_2(R) + C_\tau(R) & \text{if } R \geq R^* \end{cases} \qquad (8\text{-}15)$$

where $\epsilon_1(R)$ is the probability of accepting a lot with test τ if the reliability is R

$\epsilon_2(R)$ is the probability of rejecting the lot with test τ if the reliability is R

$C_\tau(R)$ is the cost of test τ if the reliability is R.

A graph of this function for three possible tests τ_1, τ_2 and τ_3 might take the form shown below.



True Reliability Level

FIGURE 8-8

HYPOTHETICAL LOSS FUNCTIONS
FOR THREE TESTS; τ_1, τ_2, τ_3

8.6.3 Criterion for Choosing an Optimum Test

For a given acceptance testing problem, a family of loss function curves can be constructed for different tests (e.g., by varying the truncation on total accumulated test hours). If the minimax criterion is used, the appropriate test is one for which the largest total loss is a minimum, i.e., the maximum total loss is minimized. (In Figure 8-8 this is seen to be test τ_2)

Another decision criterion is one based on expected rather than maximum total loss. Referring to Figure 8-8, if it was known that the reliability level of a submitted product would most likely be greater than R*, then τ_1 appears to be the optimum test. This criterion can be expressed as follows:

Choose that test which minimizes the weighted average of the total loss function,

$$E_\tau[L(R)] = \int_R f(R)\, L_\tau(R)\, d(R) \qquad (8\text{-}16)$$

$E_\tau[L(R)]$ is the weighted average of the loss function for test τ where $f(R)$ is the a priori probability distribution of incoming reliability levels which correspond to the weights.

In decision theory, this criterion is known as a Bayes strategy.

The assignment of a priori probabilities to values of R by the function f(R) is equivalent to making an a priori hypothesis on the distribution of incoming quality. This can be done through analysis of test data on similar items or by analysis of previous developmental and reliability test data on the product itself.

The minimax losscriterion and the Bayes strategy may result in different optimum tests. The minimax criterion is generally quite pessimistic, while the Bayes strategies, on the other hand, may not give sufficient guarantee against avoiding extreme losses. Generally, Bayes strategies are used when possible, i.e., when fairly accurate estimates of incoming quality are available.

8.6.4 Acceptance Sampling Applications - Determination of α and β

The general decision theory approach described above for choosing an optimum reliability test is, in practice, quite difficult to implement. The need for cost models and a priori distributions often cannot be satisfied. The fact that test costs are usually a function of the incoming reliability level (e.g., the expected waiting time to reach a decision in a life-test is a function of the true mean-life) leads to difficulties in establishing correct and workable mathematical relationships.

A simplified approach is to use the concepts of decision theory to determine the appropriate α and β errors of the usual reliability acceptance tests for a fixed amount of available test effort. This, is equivalent to assigning an a priori probability of zero to all reliability levels except those for the ARL and URL. Cost functions apply only to these two levels, and the set (α, β) is chosen either by the minimax or Bayes strategy criterion.The outline of the procedure follows.

(1) Select a type of test (fixed, sequential, etc.) and specify the appropriate amount-of-testing parameters (e.g., maximum sample size, truncation on total accumulated test hours, sequential truncation rule, etc.)

(2) Select the ARL (say R_0) and the URL (say R_1).

(3) Determine the following wrong decision costs:

C_0 = cost of rejecting if $R = R_0$

C_1 = cost of accepting if $R = R_1$.

(4) Determine the test costs as a function of the degree of testing. In this case, relative costs of C_0 and C_1 may be used.

For the minimax criterion, compute for all combinations of α and β, the loss function

$$L_{ij}(R) = \begin{cases} C_0\alpha_i + C_{ij}(R_0) & \text{if } R = R_0 \\ C_1\beta_j + C_{ij}(R_1) & \text{if } R = R_1 \end{cases} \qquad (8\text{-}17)$$

where $C_{ij}(R)$ is the expected test cost for set (α_i, β_j) given R. This cost will depend on the expected amount of testing for life or sequential tests.

The appropriate set of (α_i, β_j) is that which leads to a minimum set of values for $L_{ij}(R)$.

For the Bayes strategy criterion, use past data to obtain estimates of the probability that $R = R_0$ and $R = R_1$. Let these probabilities be P_0 and $P_1 = 1-P_0$, respectively. The set (α_i, β_j) to choose is one that minimizes the function

$$E_{ij}[L(R)] = P_0[C_0\alpha_i + C_{ij}(R_0)] + P_1[C_1\beta_j + C_{ij}(R_1)] \qquad (8\text{-}18)$$

Illustrative Example:

Assume that optimum sampling risks are desired for a unit having a mean life requirement of 300 hours which can be translated into the following hypothesis:

$$H_0: \quad \theta = \theta_0 = 300 \text{ hours (ARL)}$$

$$H_1: \quad \theta = \theta_1 = 100 \text{ hours (URL)}$$

A truncated replacement test is to be conducted and, because of time and money factors, no more than 900 total test hours are to be accumulated. The following costs have been determined through consideration of the consequences of wrong decisions and the costs of testing:

C_0 = cost of rejection if $(\theta = 300) = \$1000$

C_1 = cost of acceptance if $(\theta = 100) = \$2000$

C_{ij} = expected cost of testing for plan $(\alpha = \alpha_i, \beta = \beta_j)$
$= (\$.20)\ [T_{ij}^*(R)]$

where $T_{ij}^*(R)$ = expected total test hours for error combination (α_i, β_j).

Inspection of Table 8-7 for k = 3 reveals that (α, β) combinations (.05, .05), (.05, .10) (.10, .05) and (.25, .05) have test truncation times $T^* = \theta_0 E_\theta(r)$ greater than 900 hours and hence can be eliminated from consideration. For the remaining five plans tabled, the loss functions are as follows:

$$L_{ij}(\theta) = \begin{cases} C_0\alpha_i + C_{ij}(\theta_0) & \text{if } \theta = \theta_0 \\ C_1\beta_j + C_{ij}(\theta_1) & \text{if } \theta = \theta_1 \end{cases}$$

The results of the computations are shown in the following table:

TABLE 8-8

LOSS FUNCTION COMPUTATIONS FOR ILLUSTRATIVE EXAMPLE

Plan		For $\theta = \theta_0$				For $\theta = \theta_1$			
α_i	β_i	$C_0\alpha_i$	$T_{ij}^*(\theta)$	$C_{ij}(\theta_0)$	$L_{ij}(\theta_0)$	$C_1\beta_j$	$T_{ij}^*(\theta)$	$C_{ij}(\theta_1)$	$L_{ij}(\theta_1)$
.05	.25	50	780	156	206	250	560	112	$\underline{372}$
.10	.10	100	900	180	280	200	580	116	$\underline{316}$
.10	.25	100	480	96	196	250	350	70	$\underline{320}$
.25	.10	250	720	144	$\underline{394}$	200	390	78	278
.25	.25	250	240	48	$\underline{298}$	250	160	32	282

The maximum values of each plan are underlined. It is seen that, for this limited number of plans, the combination ($\alpha = .25$, $\beta = .25$) has the minimum of these maximum values and, therefore, by the minimax criterion, would be selected as the optimum plan.

To illustrate the Bayes strategy criterion, assume that past experience with the producer of this unit indicates that 80% of the time, submitted quality is such that $\theta \approx 300$ hours while only 20% of the time $\theta \approx 100$ hours. Hence $P_0 = .8$, $P_1 = .2$. The expected loss for each of the five plans is shown below.

$$E_{(.05,\ .25)} = .8(206) + .2(\$372) = \$239$$

$$E_{(.10,\ .10)} = .8(280) + .2(\$316) = \$287$$

$$E_{(.10,\ .25)} = .8(196) + .2(\$320) = \$221$$

$$E_{(.25,\ .10)} = .8(394) + .2(\$278) = \$371$$

$$E_{(.25,\ .25)} = .8(298) + .2(\$282) = \$295$$

Hence, by the minimum expected loss criterion the plan $\alpha = .10, \beta = .25$ would be selected. The reason for the difference between the minimax and Bayes strategy criteria is fairly obvious. Plan ($\alpha = .10, \beta = .25$) has a loss of \$196 for $\theta = \theta_0$ which is a minimum for all plans. Since a relatively high probability exists that θ will actually equal θ_0, the high loss of \$320 for this plan when $\theta = \theta_1$, is a small contribution to the total expected loss.

With a priori information on incoming quality, the expected cost without testing can also be evaluated. If no testing were performed and submitted lots were always accepted, the loss would be equal to $P_1C_1 = (.20)\ (2000) =$ \$400. For products where testing is extremely costly, and past history indicates a high probability of satisfactory products, this type of evaluation might indicate that it is economically wiser to eliminate tests or perhaps to perform them on only a limited quantity of products to detect a maverick lot.

Obviously, the approach described above is subject to criticism since only two possible incoming quality levels are considered. Conventional sampling plans, however, do not usually involve any more than these same two levels. The O.C. curve which describes the plan over all possible levels is predetermined by these two levels and their associated risks. The inclusion of cost factors and a priori information will yield better tests from an overall economic viewpoint provided these inputs can be reasonably approximated.

9. SUMMARY OF RESULTS AND RECOMMENDATIONS

As a result of the study, a general reliability-allocation model has been developed, which is applicable to four basic types of system configurations: serial, modified serial, redundant and bimodal. The model incorporates those factors which have a direct and important bearing on achievable reliability of units, equipments and subsystems. The factors include:

System and failure definitions.
System reliability requirements.
Unit state-of-the-art.
Unit/system failure relationships.
Unit essentiality.
Unit duty cycles.

Data inputs required for implementing the allocation model have been developed from available failure data. As part of the allocation model, a method has been established for determining the feasibility of the overall system reliability requirement to provide assurance that the allocated unit requirements are attainable.

The complete allocation model has been applied to existing and hypothetical systems and found to be a practical and rigorous approach for determining realistic and consistent reliability requirements for units, equipments, and subsystems.

An investigation of existing reliability-testing procedures has led to the development of guidelines for choosing appropriate tests for demonstrating compliance to the allocated requirements. An approach has been developed for choosing confidence levels (Beta risks) on unit tests to assure, with prescribed confidence, that systems composed of units accepted by the tests will meet the overall system-reliability requirement. The procedure also allows for the incorporation of unit essentiality. The decision-theory approach to optimum reliability testing was also investigated and a relatively simple approach is described for choosing producer and consumer risks on unit tests, based on test costs and wrong-decision costs.

With respect to the reliability allocation problem, the following recommendations are offered:

(1) The allocation model and data inputs developed in this study provide a reasonable approach for allocating weapon system reliability. They should be incorporated into general Air Force reliability documents as a means for determining reliability requirements for units, equipments, and subsystems, given an overall weapon-system requirement.

(2) The input data provided by this study should be continually refined and revised. The "Active Element Group" concept of functional grouping requires further development in non-electronic categories in order that the level of analysis for such portions of the system might be better established. Additional data analysis is also required for refinement and extension of the coverage of suitable adjustment factors to the basic data inputs, and for determination of feasibility.

(3) Procedures need to be developed for relating system operational requirements -- presently stated in a variety of ways -- to reliability goals stated in a manner suitable for reliability studies and consistent with desired ojectives.

(4) Mathematical models should be developed for optimizing the assignment of reliability-improvement efforts, in conjunction with the utilization of Air Force reliability-allocation procedures.

With respect to testing for reliability compliance, the following recommendations are offered.

(1) A detailed review and consolidation of existing Air Force directives for specifying reliability-test parameters should be conducted, and recommendations for improvement made. The recommendations would be based on known relationships between types of tests, amount of testing, cost of testing, and cost of wrong decisions.

(2) The decision-theory approach should be investigated further with a view toward choosing optimum tests, and models for obtaining the required cost functions and <u>a priori</u> distributions should be developed.

APPENDIX A

AN EXAMPLE OF RELIABILITY ALLOCATION

APPENDIX A

AN EXAMPLE OF RELIABILITY ALLOCATION

This appendix presents an example of reliability allocation methods based on the model and data inputs developed in the preceding pages. The data inputs and a simplified step-by-step procedure for reliability allocation are presented in Volume II.

The system chosen for discussion is an existing one, slightly modified in this example to protect its identity and to better illustrate the allocation methods. It is a satellite system containing telemetry and communications equipment. Temperature and attitude control are necessary for successful completion of the mission, and self-contained power generation is required. The overall system effectiveness requirement is assumed to be a 0.50 probability of survival for 72 hours. The system will be described in more detail as the analysis progresses.

The units to which system reliability will be allocated are chosen on the basis of physical and functional independence. The design information indicates that the following division of the system would be logical, and would provide units whose failure probabilities can be considered independently of the other units:

(1) Electric Power
(2) Tracking, Telemetry and Command (TTC)
(3) Structure and Temperature Control
(4) Attitude Control
(5) Useful Payload (UP)

The system's useful payload has two identical active parallel functions, either of which must operate for 72 hours. Therefore the system is redundant, and its reliability block diagram is as shown in Figure A-1.

The numbers assigned to the units are used to identify them throughout this example. Each unit is analyzed in detail to determine its essentiality, its operating time, and the number of AEG's it contains in each functional category. The number of AEG's for each unit is entered in the data worksheet (see Table A-7).

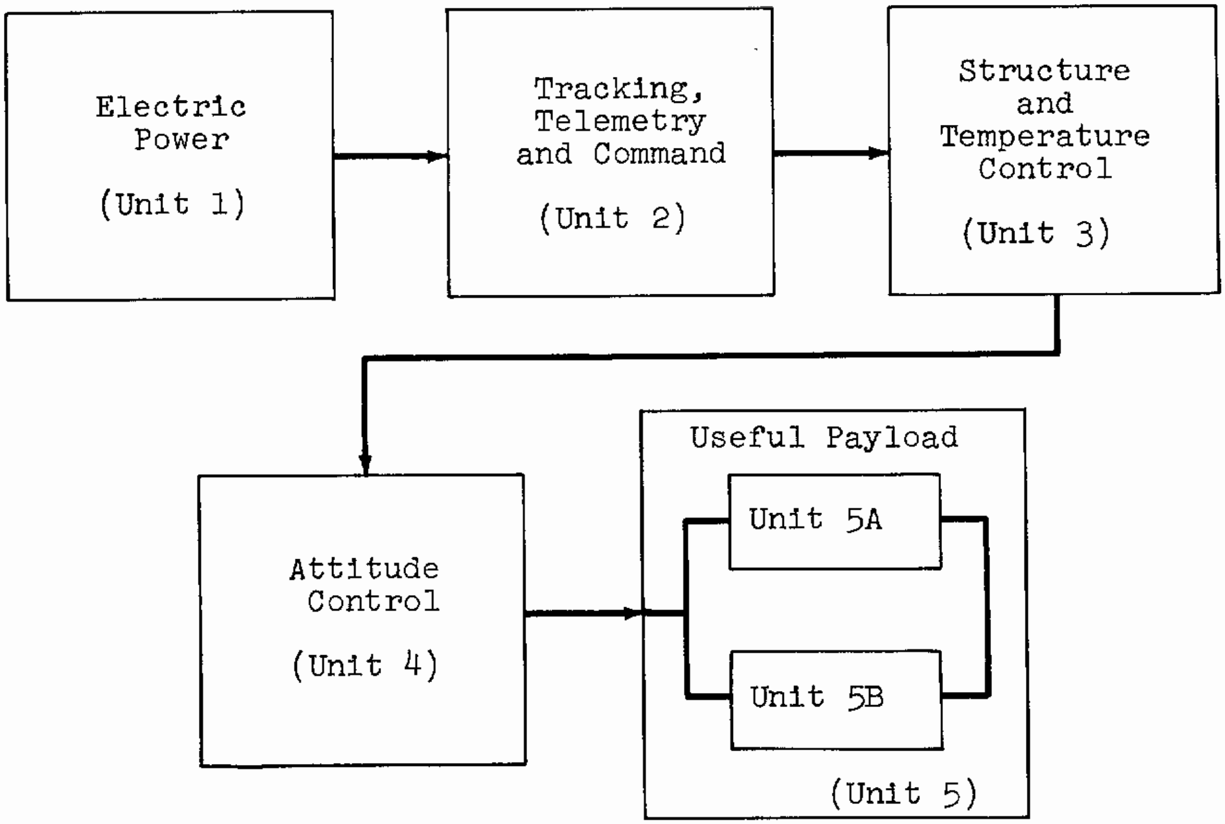


FIGURE A-1

RELIABILITY BLOCK DIAGRAM OF ILLUSTRATIVE SYSTEM

1. Unit 1, Electric Power

Failure of the power supply causes mission failure, so the essentiality of this unit is equal to one. Unit 1 must operate for the required system life of 72 hours. Power is generated by a solar array, which charges batteries feeding a power control unit through a regulator. No design information is available on the solar array and batteries, so they will not be included in this analysis. The regulator and power control unit are completely transistorized, and contain a total of 30 AEG's in the primary power (transistor) category.

2. Unit 2, Tracking, Telemetry and Command

2.1 Description

The tracking, telemetry and command function is required for mission success, so the essentiality of this unit is equal to one. Unit 2 is required to operate only during the first 24 hours of orbit. The unit has internal power regulation and control capabilities which contribute 105 AEG's in the primary power (transistor) category.

2.2 Relative Failure Rates of Digital Functions

The remainder of Unit 2 consists of digital circuitry for which no relative failure rates are available. These relative failure rates, k_i', must therefore be determined by the method described in Section 6.4.1. Briefly, a relative failure rate, k_d', for the digital functions is determined by first obtaining an average failure rate, $\overline{\lambda}_e$, for a standard electronic AEG in the system, and then estimating an average failure rate, $\overline{\lambda}_d$, for the digital functions; the relative failure rate is equal to the ratio $\overline{\lambda}_d/\overline{\lambda}_{ec}$.

2.2.1 Average Failure Rate of a Standard AEG

An average part class distribution for a standard AEG in the system can be obtained from a part count for the non-digital portion of the system. (The function for which a relative rate is being determined is digital; therefore, all digital AEG's are considered non-standard for the purpose of obtaining an average failure rate of a <u>standard</u> electronic AEG in the system.) Care must be taken in using part counts, because digital and analog parts may be listed together. Use of information containing counts in which digital parts could be identified permits calculation of the part class distribution of the passive parts as well as the proportion of tubes and transistors. This data is presented in Table A-1.

TABLE A-1

PART CLASS DISTRIBUTION FROM PART COUNT

Part	Number per AEG
Resistor	3.71
Capacitor	2.00
Transformer	0.21
Diode	2.10

Active Element	Proportion per AEG
Tube	0.01
Transistor	0.99

Combining the part class distribution with available part failure rates,[†] the average failure rate of this standared AEG is determined as indicated in Table A-2.

TABLE A-2

AVERAGE FAILURE RATE OF STANDARD AEG BASED ON SYSTEM PART COUNT

v	Part	n_v	λ_v (Multiply by 10^{-6})	$n_v\lambda_v$ (Multiply by 10^{-6})
1	Resistor	3.71	0.18	0.668
2	Capacitor	2.00	0.10	0.200
3	Transformer	0.21	0.31	0.065
4	Diode	2.10	0.45	0.945

$$\Sigma n_v\lambda_v = 1.878$$

i	Active Elements	w_i	λw_i
1	Tubes	0.01	20.7[‡]
2	Transistors	0.99	0.61

$$\begin{aligned}\bar{\lambda}_{e1} &= 0.01(20.7 + 1.878) \\ &\quad + (0.99)(3.3)(0.61 + 1.878)\end{aligned}$$

$$\bar{\lambda}_{e1} = 8.354 \times 10^{-6} \text{ per hour}$$

[‡] Tubes are of recent design. Failure rate of dual triode is assumed.

[†] "Reliability Stress Analysis for Electronic Equipment," Radio Corporation of America Publication TR 59-416-1, dated January, 1959.

As discussed in Section 6.4, the total failure rate of the passive elements ($\Sigma n_v \lambda_v = 1.878$) is first added to the failure rate of a tube and multiplied by the proportion of tubes in the system; then added to the failure rate of a transistor and multiplied by 3.3 times the proportion of transistors in the system. The sum of these two terms gives the estimated average failure rate of a standard AEG in the system, corrected to a tube active element.

Another method of obtaining an average part class distribution might be to consider a similar system. A part class distribtion for transistorized equipment is also available.† This distribution is based on a transistor active element, so that the proportion of tubes in the system is zero. As indicated in Table A-3, the average failure rate for this standard AEG is determined by adding the total failure rate of the passive elements ($\Sigma n_v \lambda_v = 1.972 \times 10^{-6}$ per hour) to the failure rate of the transistor, and multiplying the sum by 3.3 to adjust for non-tubed AEG's.

TABLE A-3

AVERAGE FAILURE RATE OF STANDARD AEG BASED ON A SIMILAR SYSTEM

v	Part	n_v	λ_v (Multiply by 10^{-6})	$n_v \lambda_v$ (Multiply by 10^{-6})
1	Resistor	5.7	0.18	1.026
2	Capacitor	2.0	0.10	0.210
3	Transformer	0.3	0.31	0.093
4	Diode	1.3	0.45	0.585
5	Switch	0.1	0.58	0.058

$\Sigma n_v \lambda_v = 1.972 \times 10^{-6}$ per hour

1 Transistor, $\lambda w = 0.61 \times 10^{-6}$ per hour

$\bar{\lambda}_{e2} = 3.3(1.0)(0.61 + 1.972)10^{-6} = 8.521 \times 10^{-6}$ per hour

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† WADD Technical Report 60-330, "Compilation of Component Field Reliability Data Useful in Systems Preliminary Design," Wright Air Development Center, Wright-Patterson Air Force Base, dated March 1961.

The average failure rate used for a standard AEG in the system of this example is the average of the two independently determined rates $\overline{\lambda}_{e_1}$ and $\overline{\lambda}_{e_2}$, yielding $\overline{\lambda}_e = 8.44 \times 10^{-6}$ per hour.

2.2.2 Average Failure Rate for Digital Functions

A parts distribution for an average digital AEG must be determined in order to obtain an average failure rate for digital functions. Because of a lack of schematics for the system being analyzed here, a parts distribution was obtained from four general sources: (1) design handbooks for transistorized digital circuitry, (2) textbooks on solid-state digital circuitry, (3) current literature in the field of digital circuitry, and (4) several design drawings for proposed or existing systems other than the one being analyzed. For eighteen typical circuits found in these sources, the parts distribution, as a function of the number of diode inputs (x) associated with a transistor inverter, is shown in Table A-4. These circuits include several types of diode-transistor gating and inversion, flip-flops (parts distribution being given on a per-transistor basis), level shifters and resistor gating. These are considered to be typical circuits since care was taken, especially in the textbook and current-literature sources, to exclude experimental, theoretical, and developmental circuits.

Field tests have shown that for digital applications, the failure rate of transistors and diodes is much less than for analog applications.† The ratio of digital-transistor failure rates to analog-transistor failure rates is 0.06, and the ratio of digital-diode failure rates to analog-diode failure rates is 0.008. The RCA rates used for this study †† are therefore modified by these ratios to obtain failure rates for digital application. The failure rate for transistors in digital applications becomes 0.036×10^{-6} per hour, and the failure rate for diodes in digital applications becomes 0.004×10^{-6} per hour. Using these rates, the average failure rate for a digital function $\overline{\lambda}_d(x)$, is determined as a function of the number of diode inputs, x, as shown in Table A-5.

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† ARINC Research Corporation Publication No. 114-5-256, "Reliability of Semiconductor Devices," dated August 31, 1961.

†† See footnote, Page 4.

TABLE A-4

PART CLASS DISTRIBUTION OF AVERAGE DIGITAL AEG

Circuit Type	Part Type			
	Resistors	Capacitors	Diodes	Coils
1	3		x	
2	2		x + 1	
3	5	2		1
4	x + 2		1	
5	x + 3		1	1
6	2		x + 1	
7	3		x + 5	
8	3		x + 4	
9	3	1	x + 2	
10	4	1	x + 1	
11	5	1	x + 5	
12	4	1	x	
13	5		x + 4	
14	3	1	1	
15	2		4	
16	3		3	
17	3	1		
18	5	1	x + 1	
Total	2x + 60	9	11x + 34	2
Average	0.11x + 3.3	0.50	0.61x + 1.89	0.11

TABLE A-5

AVERAGE FAILURE RATE OF AN AVERAGE DIGITAL AEG AS A FUNCTION OF DIODE INPUTS

Part	λ_v (Multiply by 10^{-6})	$\bar{n}_{iv}$, Average Number per AEG	$\lambda_v \bar{n}_{iv}$ (Multiply by 10^{-6})
Transistor	0.036	1.00	0.036
Resistor	0.18	0.11x + 3.30	0.020x + 0.594
Capacitor	0.10	0.50	0.050
Diode	0.004	0.61x + 1.89	0.002x + 0.008
Coil	0.08	0.11	0.009

$$\bar{\lambda}_d(x) = \sum_{v=1}^{5} \lambda_v \bar{n}_{iv} = (0.022x + 0.697)\, 10^{-6} \text{ per hour.}$$

2.2.3 Determining the Relative Rate

From the previous calculation, the average failure rate ($\overline{\lambda}_e$) for an electronic AEG in the system is 8.44 x 10^{-6} per hour. Using this and the values of $\overline{\lambda}_d(x)$ for values of x from 2 through 9, the relative failure rate, k_d', can be determined as a function of x from the formula $k_d'(x) = \dfrac{\overline{\lambda}_d(x)}{\overline{\lambda}_e}$. Table A-6 gives the values of k_d' for values of x from 2 through 9.

TABLE A-6

RELATIVE FAILURE RATES FOR DIGITAL AEG'S IN THE SYSTEM

x	$\overline{\lambda}_d = \Sigma \lambda_v \overline{n}_{iv}$ (Multiply by 10^{-6})	$k_d'(x) = \overline{\lambda}_d / \overline{\lambda}_e$
2	0.741	0.087
3	0.763	0.090
4	0.785	0.093
5	0.807	0.096
6	0.829	0.098
7	0.851	0.101
8	0.873	0.103
9	0.895	0.106

2.2.4 Using the New Relative Rate

The new functional category for which a relative failure rate was estimated will be called "Digital (x)" where x can be an integer from 2 through 9. The method of estimating the number of AEG's is as follows:

(1) For any digital system with diode gating and transistor inversion, estimate the average number (x) of diode inputs associated with an inverter. Consider flip-flops as two inverters,

each with one input. In most cases there will be sections of the computer containing only one order of complexity of logic; the x for this portion can be assigned, and a different x can be assigned to another portion. The range of x permits eight different functional categories, but this method of estimating is believed to give adequate results without the requirement for counting the exact number of inputs associated with each inverter.

(2) Count the total number of inverters in each of the categories assigned in Step (1). This is the number of AEG's in that category. (Remember to count flip-flops as two inverters, hence as two AEG's.)

Using these categories,the number of digital AEG's are counted and entered in the worksheet of Table A-7. In addition to the AEG's that are specifically digital, there are 70 magnetic core devices in this unit. Magnetic core devices are generally considered more reliable than other parts used in digital circuits. In this application, however, it is necessary to have groups of matched cores. Because of this, and the effects of vibration and shock, the magnetic core devices have been assigned to the "Digital (2)" functional category and are included in the count entered in the worksheet.

Parts leading up to the antenna (rigid coax and tuned cavity) are considered, but the antenna itself is stationary and will be considered failure-free.

3. UNIT 3 - Structure and Temperature Control

The airframe itself is considered to have a reliability of unity and will not be considered here.

Preliminary design information indicates that the temperature control unit has sensors that actuate bellows to control a rack and pinion, which moves hinged shutters. The preliminary design required ten such assemblies. These are assigned to the functional category, "Pneumatic-Bellows with Potentiometer Pick-off." This may at first seem to be a severe assignment. It should be noted, however, that the failure rate for this category (as for all categories) is considered sensitive to environment if all units are not in the same gross environment. Since all units of this system are in the same gross environment, adjustment of the relative failure rates is not necessary. This assumes that all failure rates would receive the same adjustment for satellite environment. As described in Section 6.7.3, a factor of 0.5 is used to adjust for satellite environment. Engineering judgment indicates, however, that because of cold-vacuum welding, the rack and pinion may suffer a higher failure rate in a satellite environment than in a ground environment. This assignment, then, does not seem so severe, and may, indeed, be conservative.

4. UNIT 4 - Attitude Control

Always necessary for mission success, Unit 4 has an essentiality equal to one. Its operating time is equal to that of the system (72 hours).

This unit has both mechanical and electronic AEG's. The major parts of its mechanical portion are estimated to be 15 valves, 6 motors, and 1 gyro. The valves are used for 4 minutes during each 24-hour interval.

The electronic portion consists of 6 sensors, 2 digital data processors, 5 amplifiers, and a power supply, and is completely transistorized. The only information available to help determine functional categories is a part count from the contractor. Fortunately, the part count is given for each component of the system, so it is possible to estimate the number of AEG's and assign them to functional categories. The appropriate "Digital (x)" category was estimated by considering the transistor-diode ratio in the digital processors, keeping in mind that "x" relates to the average gate inputs per transistor, and not simply to the transistor-diode ratio.

5. UNIT 5 - Useful Payload

The useful payload consists of two identical units, either of which must operate properly for 72 hours. Each unit includes a power converter, translator, power amplifier, receiver multiplexer, and a transmitter multiplexer. The contractor's circuit schematics were available, so an accurate determination of the number of AEG's was possible. As indicated on the worksheet, most of the unit is transistorized. The entry in the "special" category arises from the use of four tubes of fairly recent design.

6. Conversion of Relative Rates to Standard Form

Table A-7 now contains (1) the number, f_{ij}, of AEG's in the i^{th} functional category within the j^{th} unit, and (2) either the relative failure rate, k_i, for the electronic AEG's, or the relative failure rate, k_i', for the non-electronic or non-standard categories. The k_i are relative failure rates with respect to an audio AEG failure rate. The k_i' are relative failure rates with respect to an average electronic AEG, and must be converted to rates relative to the audio AEG failure rate as described in Section 6.5.

The table provides a column to list the total AEG's, f_i, in the i^{th} standard electronic functional category. When multiplied by the relative failure rate for the i^{th} category, the system electronic failure index, K_e, results. Summing the f_i column gives the total number of electronic AEG's, F_e, in the system. The ratio K_e/F_e is assumed equal to the ratio of the average AEG failure rate to the audio AEG failure rate, denoted $\overline{K}_e$ in the worksheet. Since k_i' is the ratio of the i^{th} functional failure rate to the average AEG failure rate, the product $\overline{K}_e k_i'$ gives the equivalent k_i, the ratio of the i^{th} functional failure rate to the audio AEG failure rate. The k_i' listed in Table A-7 are adjusted in the above manner to obtain a k_i for functions 5 through 13.

7. Modification of Relative Failure Rates

For each f_{ij} (number of AEGss in the i^{th} functional category within the j^{th} unit) a relative failure rate $\overline{k}_{ij}$ is entered in the worksheet. The value of $\overline{k}_{ij}$ is determined by modifying $k_1 = 4.3$ by the factor 0.3 (the ratio of transistorized AEG failure rate to tube AEG failure rate as discussed in Section 6.6.1). This same modifier is applied to k_2 and k_3 to form $\overline{k}_{24}$ and $\overline{k}_{35}$. Any factors affecting failure rates are introduced in this manner, usually on a unit basis; e.g., a unit may have a different gross environment so that the $\overline{k}_{ij}$ for that unit would be modified by the appropriate factor discussed in Section 6.6. For simplicity, these modifications should be made on the basis of a total unit. This is not always possible. In this example, there is one additional modification. The value of k_7 is reduced to half its value because of the extremely low operating period of the values. In such cases, engineering judgment is required.

TABLE A-7

DATA WORKSHEET FOR DETERMINING UNIT FAILURE INDICES, K_j

	i	Function		k_i	Unit 1		Unit 2		Unit 3		Unit 4		Unit 5A		Unit 5B		Category Totals	
					$\overline{K}_{i1}$	f_{i1}	$\overline{K}_{i2}$	f_{i2}	$\overline{K}_{i3}$	f_{i3}	$\overline{K}_{i4}$	f_{i4}	$\overline{K}_{i5A}$	f_{i5A}	$\overline{K}_{i5B}$	f_{i5B}	f_i	$f_i k_i$
Group a	1	Primary Power (Trans)		4.3	1.29	30	1.29	105			1.29	22	1.29	20	1.29	20	197	847.1
	2	Pulse, Low Power(Trans)		3.0							0.90	178					178	534.0
	3	RF, Low Power(Trans)		3.4									1.02	58	1.02	58	116	394.4
	4	Special		58.0									58.0	4	58.0	4	8	464.0
																	$\Sigma f_i = 499$	$\Sigma f_i k_i = 2239.5$
																	$\overline{K}_e = \frac{2239.5}{499}$ $\overline{K}_e = 4.488$	
			k_i'	$k_i = \overline{K}_e k_i'$														
Group b	5	Rigid Coax	0.30	1.35			1.35	1										
	6	Tuned Cavity	0.25	1.12			1.12	1										
	7	Values	1.80	8.08							4.04	15						
	8	Motor	1.50	6.73							6.73	6						
	9	Gyro	12.10	54.30							54.30	1						
	10	Bellows w/Pot Pickoff	23.50	105.47					105.47	10								
Group c	11	Digital (2)	0.087	0.390			0.390	298										
	12	Digital (3)	0.090	0.404							0.404	174						
	13	Digital (6)	0.098	0.440			0.440	90										
Unit Failure Index, $K_j = \sum_{i=1}^{13} f_{ij}\overline{K}_{ij}$					$K_1=38.7$		$K_2=293.7$		$K_3=1054.7$		$K_4=414.2$		$K_{5A}=317.0$		$K_{5B}=317.0$			

8. Unit Failure Indices

A failure index, K_j, used in the allocation model can be formed for the j^{th} unit by summing the product of the number of AEG's and the corresponding failure rate for each category in the unit. The results shown in Table A-7 are repeated here for convenience:

$$K_1 = 38.7$$

$$K_2 = 293.7$$

$$K_3 = 1054.7$$

$$K_4 = 414.2$$

$$K_{5A} = K_{5B} = 317.0$$

9. Feasibility of the System Reliability Requirement

The overall system effectiveness requirement is given as a 0.50 probability of successful operation for 72 hours. The system design adequacy is estimated to be equal to one. Therefore, the system reliability requirement is equal to the overall system effectiveness requirement, since

$$\text{System Reliability Requirement} = \frac{\text{System Effectiveness Requirement}}{\text{System Design Adequacy}}$$

The feasibility of this reliability requirement, $R^*(72) = 0.50$, can be estimated by determining a feasible requirement for an equivalent series system, and adjusting this requirement to account for redundancy.

The feasible requirement for an equivalent series system is determined by considering first the electronic and then the non-electronic portions of the system, and combining the results.

9.1 Electronic Portion

The electronic portion of the basic series system contains the following numbers of active element types:

413 Transistors, analog.
562 Transistors, digital.
4 Tubes.
186 Diodes, digital AEG.

The 70 magnetic core devices have been included with "Transistors, Digital" and ten digital diodes are counted as one Digital Diode AEG. The total number of active elements is 1165.

Reference to Figure 6-1 and use of the upper boundary of the ground band for the satellite system gives an estimation of the feasible mean life of the electronic portion of a tubed system with 1165 active elements of approximately 32 hours. Applying the correction factors explained in Section 6.8.2, the system has an equivalent number of active elements, $\sum_i N_i B_i$, as shown in Table A-8.

TABLE A-8

EQUIVALENT ACTIVE ELEMENTS

i	Type	N_i	B_i	$N_i B_i$
1	Transistor, Analog	413	0.3	123.9
2	Transistor, Digital	562	0.02	11.2
3	Diodes, Digital AEG	186	0.06	11.2
4	Tubes	4	1.0	4.0
			$\sum_i N_i B_i$ =	150.3

The estimated feasible mean life, $\tilde{\theta}_e$, of the electronic portion of this system is found by modifying the estimated feasible mean life of the electronic portion of an equivalent tubed system in the following manner:

$$\tilde{\theta}_e = 32 \text{ hours} \times \frac{1165}{150.3} = 248 \text{ hours}.$$

9.2 Non-Electronic Portion

The feasible mean life, $\tilde{\theta}_{ne}$, of the non-electronic portion of the basic series system can be determined as explained in Section 6.8.3. The total failure index, K_{ne}, of the non-electronic portion of the system is, from Table A-7,

$$K_{ne} = \sum_{i,j} f_{ij}k_i' = 1(0.30) + 1(0.25) + 15(1.80) + 6(1.50) + 1(12.10) + 10(23.50) = 283.6.$$

The feasible mean life of the non-electronic portion of the system, then, is

$$\tilde{\theta}_{ne} = \frac{32 \times 1169}{283.6} = 132 \text{ hours.}$$

9.3 Feasible Requirement for the Redundant System

Having determined the feasible mean life of the electronic portion of an equivalent series system, $\tilde{\theta}_e$, and the feasible mean life of the non-electronic portion of an equivalent series system, $\tilde{\theta}_{ne}$, the total feasible mean life of an equivalent series system, $\tilde{\theta}_s$, is

$$\tilde{\theta}_s = \frac{\tilde{\theta}_{ne}\ \tilde{\theta}_e}{\tilde{\theta}_{ne} + \tilde{\theta}_e} = 86 \text{ hours.}$$

The degree of redundancy, γ, is estimated by computing the failure index, K_a, for all series units, and the equivalent failure index, K_5', for each redundant unit to form the ratio $\gamma = \dfrac{K_5'}{K_a + K_5'}$. The failure index for the series units is, from Table A-7, $K_a = K_1 + K_2 + K_3 + K_4 = 1801.3$. Since the redundant units are identical, $K_5 = K_{5A} = K_{5B} = 317.0$. Then the degree of redundancy is $\gamma = \frac{317.0}{2118.3} = 0.15$.

The feasible system reliability requirement is, as discussed in Section 6.8.1,

$$\tilde{R}(T) = 2\,e^{-T/\tilde{\theta}_s} - e^{-(1+\gamma)T/\tilde{\theta}_s}$$

For this system, $T = 72$ hours and $\tilde{\theta}_s$ was determined to be 86 hours, so $T/\tilde{\theta}_s = 0.837$ and

$$\tilde{R}(72) = 2\,e^{-0.837} - e^{-(1.15)(0.837)}$$

$$\tilde{R}(72) = 0.48.$$

The feasible reliability requirement compares favorably with $R^*(72) = 0.50$, the system reliability requirement.

10. Allocation to the Units

Before the allocation to the units is made, an equivalent series failure index, K_5, of the redundant configuration is determined. This is explained in detail in Section 5.4, but for the purpose of this example, the value of $Z(\alpha, R^*)$ will be determined from the graph of Figure 9 of Volume II. The ratio $\alpha = \frac{K_a}{K_5'} = 5.68$ and the reliability requirement, $R^*(72) = 0.50$ are used, and Figure 9 of Volume II gives $Z(5.68, 0.50) = 0.0185$. The equivalent series failure index, K_5, of the redundant configuration is

$$K_5 = (0.0185)\ K_a = 33.3$$

and the total system failure index, is

$$K = K_a + K_5 = 1834.6$$

The failure index ratio for each of the series units is

$$w_1 = K_1/K = 0.0211$$

$$w_2 = K_2/K = 0.1601$$

$$w_3 = K_3/K = 0.5749$$

$$w_4 = K_4/K = 0.2258$$

For the redundant configuration, the failure index ratio is $w_5 = K_5/K = 0.0181$, and for the redundant units the failure index ratios are

$$w_{5A} = K_{5A}/K = 0.1728$$

$$w_{5B} = K_{5B}/K = 0.1728.$$

Since each series unit and the redundant configuration has an essentiality equal to 1, the allocated reliabilities are $\hat{R}(t_j) = R^*(T)^{w_j}$ ($j = 1, 2, 3, 4$ and 5). So for the series units the allocated reliabilities are

$\hat{R}_1(72) = 0.50^{0.0211} = \text{antilog}\ (-0.0146) = 0.986$

$\hat{R}_2(24) = 0.50^{0.1601} = \text{antilog}\ (-0.1110) = 0.895$

$\hat{R}_3(72) = 0.50^{0.5749} = \text{antilog}\ (-0.3985) = 0.671$

$\hat{R}_4(72) = 0.50^{0.2258} = \text{antilog}\ (-0.1565) = 0.855,$

and for the redundant configuration,

$\hat{R}_5(72) = 0.50^{0.0181} = \text{antilog}\ (-0.0125) = 0.988.$

For the redundant units, the allocated reliabilities are $\hat{R}(t_i) = R^*(T)^{W_i}$ (i = 5A and 5B). Therefore,

$\hat{R}_{5A}(72) = \hat{R}_{5B}(72) = 0.50^{0.1728} = \text{antilog}\ (-0.1198) = 0.887$

The allocated mean life for each series unit and for the redundant units is $\hat{\theta}_i = -t_i/\log \hat{R}\ (t_i)$ (i = 1, 2, 3, 4, 5A and 5B). Then,

$$\hat{\theta}_1 = \frac{72}{0.0146} = 4932 \text{ hours}$$

$$\hat{\theta}_2 = \frac{24}{0.1110} = 216 \text{ hours}$$

$$\hat{\theta}_3 = \frac{72}{0.3985} = 181 \text{ hours}$$

$$\hat{\theta}_4 = \frac{72}{0.1565} = 460 \text{ hours}$$

$$\hat{\theta}_{5A} = \hat{\theta}_{5B} = \frac{72}{0.1198} = 601 \text{ hours}$$

The allocated mean life of the redundant configuration is

$$\hat{\theta}_5 = \hat{\theta}_{5A} + \hat{\theta}_{5B} - \frac{\hat{\theta}_{5A}\ \hat{\theta}_{5B}}{\hat{\theta}_{5A} + \hat{\theta}_{5B}}$$

or, since

$$\hat{\theta}_{5A} = \hat{\theta}_{5B},\ \hat{\theta}_5 = \tfrac{3}{2}\hat{\theta}_{5A} = 902 \text{ hours}.$$

Assuming constant failure rates for the series and redundant units the allocated failure rate for each unit is

$$\hat{\lambda}_j = \frac{\log \hat{R}(t_j)}{t_j} \quad .$$

Then

$$\hat{\lambda}_1 = 203 \times 10^{-6} \text{ per hour}$$

$$\hat{\lambda}_2 = 4625 \times 10^{-6} \text{ per hour}$$

$$\hat{\lambda}_3 = 5535 \times 10^{-6} \text{ per hour}$$

$$\hat{\lambda}_4 = 2174 \times 10^{-6} \text{ per hour}$$

$$\hat{\lambda}_{5A} = \hat{\lambda}_{5B} = 1664 \times 10^{-6} \text{ per hour}$$

The average failure rate for the redundant configuration is

$$\hat{\lambda}_5 = \frac{1 - \hat{R}_5(72)}{72}$$

$$= 167 \times 10^{-6} \text{ per hour}$$

APPENDIX B

LIST OF FIGURES AND TABLES

APPENDIX B

LIST OF FIGURES AND TABLES