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**MATRIX METHODS  
IN  
STRUCTURAL MECHANICS**

Proceedings of the Conference held at  
Wright-Patterson Air Force Base, Ohio  
26-28 October 1965

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**MATRIX METHODS  
IN  
STRUCTURAL MECHANICS**

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## PREFACE

The Conference on Matrix Methods in Structural Mechanics held at Wright-Patterson Air Force Base on 26-28 October 1965 was sponsored jointly by the Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command, and the Air Force Institute of Technology, Air University. The purpose of the conference was to discuss the recent developments in the field of matrix methods of structural analysis and design of aerospace vehicles.

The conference was attended by over two hundred invited scientists and engineers from Australia, Belgium, Germany, Netherlands, and United Kingdom, and the United States. The participation of representatives from overseas was a noteworthy highlight of this conference since it permitted a free interchange of ideas and information between scientists and engineers from the United States and other countries. Therefore this conference may be truly considered to be the First International Conference on the subject of matrix methods in structural mechanics.

The thirty-six papers presented were arranged into six sessions under five different themes; General Matrix Methods, Finite Element Properties, Nonlinear Effects, Dynamics, and Applications. The papers cover practically all major aspects of recent research and development work in the field of matrix methods of structural analysis and design.

The members of the Technical Committee for the conference, who also served as the Technical Editors, would like to express their appreciation to the authors and session chairmen for their contributions towards the success of the conference and also to all those who attended and participated in the discussions. They also wish to thank Brig. Gen. Ernest A. Pinson, Commander of the Office of Aerospace Research, for his keynote address to the Conference. Wholehearted thanks are also extended to Dr. Raymond L. Bisplinghoff, Special Assistant to the Administrator, National Aeronautics and Space Administration, for his outstanding address entitled "Aeronautics and Space - What Lies Ahead" which he presented at the conference banquet.

The editors would like to express their gratitude to Maj. F.M. Charette of the Air Force Institute of Technology, who served as the project officer and was responsible for the administrative details of the conference program.

Grateful acknowledgement is made to Mr. John W. Thomas, Publications Editor, Editorial Branch of the Technical Information Section, Systems Engineering Group, for sparing no effort to accomplish the publication of these proceedings.

December, 1965  
Wright-Patterson Air Force Base  
Dayton, Ohio

Technical Editors

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