CONTACT STRESSES IN CABLES DUE TO TENSION AND TORSION

Krishna Kumar†
IIT, Kanpur - 208016, India

John E. Cochran, Jr.* and Malcolm A. Cutchins*
Aerospace Engineering Department
Auburn University
Auburn University, AL 36849

ABSTRACT

The fundamental theory of wire ropes developed by Costello and Phillips is utilized to develop closed-form expressions for contact stresses in single strand cables with fibrous cores. These should be useful for gaining insight into the influence of various cable parameters on its strength and hence design. Furthermore, since the contact stresses can be often high and approach levels on the order of the ultimate stress of the material, these expressions may facilitate the analysis of energy dissipation in vibrating cables through hysteresis due to periodic variations of stresses.

†Professor, Department of Aeronautical Engineering.
*Professor, Department of Aerospace Engineering.

INTRODUCTION

The importance of stranded cables for engineering applications is universally recognized. Yet, until recently, empirical data generated through their extensive experimentation had been the sole basis for rope-selection and design.[1,2,3] Such a conspicuous absence of earlier interests in theoretical investigations of cable deformations and stresses may be attributed to perhaps what appears to be an "overwhelming" complexity of the problem. The lack of thorough understanding of stresses in cables and the mechanisms which cause failures leads to the use of rather large factors of safety, thus leaving much to be desired for an effective usage of material in cables. Recent considerations of use of cables in large space structures (e.g., stayed columns, damping devices which utilize cables for energy dissipation or in other space applications involving tethers) require greater knowledge of cable stresses and cable-dissipative mechanisms.

A paper by Hall[4] together with a series of three papers by Hruska[5,6,7] on estimation of the tensile, tangential and radial forces in wire rope seemed to mark the beginning of a new era directed at theoretical studies on cable stresses and deformations. However, little attention was given to the determination of actual stresses. Leissa[8] was probably the first one to undertake complete stress analysis of a 6x7 wire rope under tension. Starkey and Cress[9] improved his analysis by incorporating the effect of wire curvature. They also considered the crossed-wire case. Bert and Stein[10] presented a more general treatment for computation of contact stresses in the most severe contact regions and applied it to a 6x37 Warrington IWRC rope.

In the foregoing investigations, however, several simplifying assumptions of questionable validity were made. Costello, Phillips and other associates[11,20] gave a new direction to the subsequent theoretical investigations on cable stresses and deformations. They treated separately the equilibrium of each of the constituent helical wires, although the assumption of no frictional force was kept intact. Their approach, however, requires solving simultaneously the governing set of nonlinear equations using a Newton-Raphson algorithm. The use of a computer is a must to generate the results, numerically, using data specific to cables under investigation. The "Linear Theory" as used earlier by Phillips and Costello[21,22] and Velinsky, et al.,[23] although applicable to a cross-section of arbitrary geometry, has somewhat limited utility. They exploit it for relating changes in variables/parameters of helical wires to those of strands as well as the changes in variables/parameters of strands to those of cable in the loaded configuration. However, no attempt has been made to "integrate" these linearized relations in a concise form. In the process, their linearization merely achieves some economy of computational time without altering the basic numerical character of simulation involving the simultaneous solution of a set of usually a large number of algebraic equations (depending upon the geometry of the cable cross-section). In this manner, their approach does not provide any direct design insights as to the influence of the various cable parameters on elastic deformations unless one resorts to extensive numerical simulation covering the practical range of values for each of the system parameters. Kumar and Cochran[24,25] incorporated some suitable simplifying approximations to develop closed-form solutions for elastic multilayered strand cable deformations. This latter work is now extended to develop analytical expressions for critical contact stresses in cables of the single-strand fibrous-core type. Unlike the treatment of some of the earlier theoretical investigations, the effect of torsional moment is also incorporated along with that of simple tension.

It is felt that the explicit nature of the results developed here would be particularly useful in gaining fresh insights into the influence of the various cable parameters, thus facilitating design and rope-selection. The better understanding of the contact stresses may also be crucial to modeling of the energy dissipation phenomenon in the cables likely to be exploited in some new proposed applications, e.g., for augmentation of damping in large space structures. See Cutchins, et al.,[26] for other wire rope related models.

ANALYSIS

The cable under consideration is a single strand composed of a fibrous core wrapped around helically by an arbitrary number of wires, hereafter denoted by m. The radius of the wire, its helix radius and helix angle are denoted by R, r and α, respectively. The case investigated in this paper is that of the cable subjected to an external axial pull or tensile load F, and an axial or torsional moment M. It is assumed that under this loading, the helical character of the wires is retained. The corresponding deformed variables are represented by using primes as superscripts.

From simple geometric considerations of the cable cross-section it can be shown that:

$$(R/r) = [\sin^2\alpha/(\sin^2\alpha + \cot^2\pi/m)]^{1/2}$$

which, for practical situations, can be approximated, to within 1% error, as

$$R/r \approx \sin\alpha \sin \pi/m \,[1 + (1/2) \cos^2\alpha \sin^2\pi/m] \qquad (1)$$

The corresponding relation for the deformed configuration can be stated as

$$R'/r' \approx \sin\alpha' \sin \pi/m \,[1 + (1/2) \cos^2\alpha' \sin^2\pi/m] \qquad (2)$$

where

$$R' \approx R(1-\nu\xi) \qquad (3)$$

ν = Poisson's ratio

ξ = tensile strain in the helical wires

Using these three relations, we can show

$$(r/r') = 1 + \delta \tag{4}$$

where

$$\delta = \nu\xi + a \cot\alpha \, \Delta\alpha$$

$$a = 1 - \sin^2\alpha \sin^2 \pi/m.$$

Strain Relations: The consideration of compatibility of the cable deformations leads to the following strain relations:

Axial strain of the cable, $\varepsilon = (1+\xi) \sin\alpha'/\sin\alpha - 1$ (5)

Rotational strain of the cable, $\beta = (r/r')(1+\varepsilon)\cot\alpha' - \cot\alpha$ (6)

where

ξ = longitudinal strain of helical wires

Denoting the change in the wire helix angle (i.e., $\alpha'-\alpha$) by $\Delta\alpha$, Eqs. (5) and (6) can be written to first order in $\Delta\alpha$ as:

$$\xi \approx \varepsilon - \Delta\alpha \cot\alpha$$

$$\Delta\alpha \approx [\varepsilon(1+\nu)\sin\alpha \cos\alpha - \beta \sin^2\alpha]b \tag{7}$$

where

$$b \approx 1 + (\cos^2\pi/m - \nu)\cos^2\alpha + (1-2\nu) \cos^4\alpha \tag{8}$$

Force and Moment Relations: We denote by T and G the resultant internal forces and moments in the helical wires, respectively. The subscripts n, b and a are added to represent the corresponding components of T and G along the normal, binormal and axial directions, then,

$$T_a = \pi R^2 \xi E$$

$$G_n = (\pi R^4 E/4)(\lambda_n' - \lambda_n)$$

$$G_b = (\pi R^4 E/4)(\lambda_b' - \lambda_b)$$

$$G_a = (\pi R^4 E/4)(\lambda_a' - \lambda_a)$$

where

E = the modulus of rigidity of the material of the helical wires.

$\lambda_n, \lambda_n' = 0$; the initial and final components of curvature of the helical wires along the normal.

$\lambda_b, \lambda_b' = (\cos^2\alpha/r), (\cos^2\alpha'/r')$, the initial and final components of curvature of the helical wires along the binormal.

$\lambda_a, \lambda_a' = (\sin\alpha \cos\alpha/r), (\sin\alpha' \cos\alpha'/r')$; the initial and final components of curvatures of the helical wire along the axis.

Through some suitable simplifying approximations, it can be shown that

$$T_a = \pi R^2 E[\varepsilon\{1-(1+\nu)b \cos^2\alpha\} + \beta\ b \sin\alpha \cos\alpha]$$

$$G_a = [\pi R^4 E/\{4(1+\nu)r\}][\nu\varepsilon + \{a - \nu - \tan^2\alpha\} \cot\alpha\ \Delta\alpha]\sin\alpha \cos\alpha$$

$$G_b = [\pi R^4 E/\{4r\}][\nu\varepsilon + (a-\nu-2 \tan^2\alpha)\cot\alpha\ \Delta\alpha]\cos^2\alpha$$

$$G_n = 0$$

Now considering the equilibrium of helical wires, we obtain:[13]

$$T_b \approx G_a \lambda_b' - G_b \lambda_a'$$

$$q_n \approx - T_a \lambda_b' + T_b \lambda_a'$$

where q_n is the normal component of the resultant contact force per unit length. These two relations lead to:

$$T_b \approx [b\ \pi R^4 E/(4r^2)]\ [\varepsilon\{(1+2\nu)\sin^2\alpha - \nu^2-\nu \cos^2\alpha\} \cos\alpha - \beta\{\sin^2\alpha - \nu_f \cos 2\alpha\} \sin\alpha] \sin\alpha \cos^2\alpha \quad (9)$$

$$q_n \approx - (\pi R^2 E/r)\cos^2\alpha[\varepsilon\{1-(1+\nu)\ b \cos^2\alpha\} + \beta\ b(1 + c \sin^4\alpha\} \sin\alpha \cos\alpha] \quad (10)$$

where

$$\nu_f = \nu/(1+\nu)$$

$$c = (1/4)(1+\nu_f)(R^2/r^2)$$

The angle γ that locates the lines of action of the line contact loads of a helical wire with adjacent helical wires is given by

$$\cos\gamma = \sin\alpha \sin \pi/m[1 - \sin^2 \pi/m \cos^2\alpha]^{1/2} \quad (11)$$

giving the following expression for the contact force per unit length denoted by Q:

$$q_n = -2Q \cos\gamma \tag{12}$$

Using the earlier expressions for q_n and $\cos\gamma$, we get

$$Q = [\pi R^2 E/(2r)][\cos^2\alpha/(\sin\alpha \sin \pi/m)][1 + 1/2 \sin^2 \pi/m \cos^2\alpha]$$

$$[\varepsilon\{1-(1+\nu)b \cos^2\alpha\} + \beta\, b\{1 + c \sin^4\alpha\}\sin\alpha \cos\alpha] \tag{13}$$

Here, the strains ε and β are related to the tensile force (F) and torsional moment (M) as follows:[24,25]

$$\bar{F} = F_\varepsilon \varepsilon + F_\beta \beta \tag{14}$$

$$\bar{M} = M_\varepsilon \varepsilon + M_\beta \beta \tag{15}$$

where the nondimensionalized force and moment represented by $\bar{F}$ and $\bar{M}$ are defined as follows:

$$\bar{F} = F/(m\pi R^2 E)$$

$$\bar{M} = M/(m\pi R^3 E) \tag{16}$$

and the expressions for force and moment derivatives, (F_ε, F_β. M_ε and M_β) developed by Kumar and Cochran[24] and reproduced below, are now utilized to solve Eqs. (14) and (15) for ε and β:

$$F_\varepsilon = e \sin\alpha$$

$$F_\beta = [b - (1+\nu)/4 \sin^2 \pi/m \sin^4\alpha \cos^2\alpha] \sin^2\alpha \cos\alpha$$

$$M_\varepsilon = (r/R)[e-(1/4)(1-\nu_f \cos^2 2\alpha)\sin^2 2\pi/m \sin^2\alpha]\cos\alpha \tag{17}$$

$$M_\beta = (r/R)[b \cos^2\alpha + (1/4)(1+f)] \sin\alpha$$

where

$$e = 1-(1+\nu) \cos^2\alpha -(1+\nu)(\cos^2 \pi/m - \nu) \cos^4\alpha$$

$$f = \nu_f \cos 2\alpha +(1+\nu) \sin^2\alpha \cos^2\alpha \tag{18}$$

Substituting the resulting expressions for ε and β into Eq. (13) followed by nondimensionalization leads to

$$\bar{Q} = Q_F \bar{F} + Q_M \bar{M} \tag{19}$$

with

$$\bar{Q} = Q/(E\pi R)$$

$$Q_F = (1/2)\, g_1 \sin\alpha \cos^2\alpha$$

$$Q_M = (1/\nu)(1+\nu)^2\, g_2 \sin^3\alpha \cos^3\alpha \sin \pi/m(1+1/2 \sin^2 \pi/m \cos^2\alpha)$$

$$g_1 = 1-2\nu \cos^2\alpha + (1+4\nu+9\nu^2)\cos^4\alpha + (1+3\nu-20\nu^2-33\nu^3)\cos^6\alpha$$

$$-4\{\cos^8\alpha/(\sin^4\alpha \sin^2 \pi/m)\}\{1-3\nu \cos^2\nu + (1+4\nu+12\nu^2) \cos^4\alpha\}$$

$$g_2 = 1-\nu_f^2 + 1/2 \sin^2 \pi/m \cos^2\alpha - \nu(5-3\nu^2)\cos^2\alpha - (1-3\nu-17\nu^2)$$

$$\cos^4\alpha - 3(1-2\nu+9\nu^2+23\nu^3)\cos^6\alpha + 4(1-2\nu)\{\cos^6\alpha/(\sin^4\alpha$$

$$\sin^2 \pi/m)\}\{1-3\nu \cos^2\alpha + (1+4\nu+12\nu^2) \cos^4\alpha\}$$

Contact Stresses:

To compute the contact stresses, direct use can be made of the results given by Seely and Smith[27] in their book.

The expressions for the three principal compressive stresses due to the contact force are given below:

$$\sigma_x = 2\nu[\sqrt{1 + (z/p)^2} - (z/p)](p/\Delta)$$

$$\sigma_y = [(\sqrt{1 + (z/p)^2} - z/p)^2/\{1 + (z/p)^2\}^{1/2}](p/\Delta)$$

$$\sigma_z = [1 + (z/p)^2]^{-1/2}[p/\Delta]$$

where

$\sigma_x, \sigma_y, \sigma_z$ = principal compressive stresses in the axial, tangential and radial directions, respectively.

z = distance below the surface to the point under consideration.

$$p = [2\ Q\ \Delta/\pi]^{1/2}$$

$$\Delta = [2(1-\nu^2)R/E]$$

Since the above compressive stresses are maximum when z=0, i.e., on the surface of the wires at the contact points, we get

$$(\sigma_x)_{max} = 2\nu\ p/\Delta = 2\nu[1/(1-\nu^2)]\}^{1/2}\ [EQ/(\pi R)]^{1/2}$$

$$(\sigma_y)_{max} \approx (\sigma_z)_{max} \approx p/\Delta = [1/(1-\nu^2)]^{1/2}[EQ/(\pi R)]^{1/2}$$

The latter of these is the maximum normal contact stress which we denote by σ; hence

$$\sigma \approx [1/(1-\nu^2)]^{1/2}[QE/(\pi R)]^{1/2} \qquad (20)$$

Substitution of the earlier expression for Q followed by nondimensionalization leads to the maximum contact stress:

$$\bar{\sigma} \approx [(1/2)\ \sin\alpha]^{1/2}\ \cos\alpha[h_1\ \bar{F} + h_2(1+\nu)^2\ \sin^2\alpha\ \cos\alpha\ \sin(\pi/m)\bar{M}] \qquad (21)$$

where

$$\bar{\sigma} = \sigma/E$$

$$h_1 = 1+\nu^2/2-2\nu\ \cos^2\alpha + (1+4\nu + 9\nu^2)\ \cos^4\alpha\ + (1+3\nu-20\nu^2-33\nu^3)$$
$$\cos^6\alpha\ -4\{\cos^8\alpha/(\sin^4\alpha\ \sin^2\ \pi/m)\}\{1-3\nu\ \cos^2\alpha +$$
$$(1 + 4\nu + 12\nu^2)\ \cos^4\alpha\}$$

$$h_2 = [1+\nu^2/2-\nu_f^2 + (\sin^2\ \pi/m\ \cos^2\alpha)/2\ -\nu(5-3\nu^2)\ \cos^2\alpha$$
$$-\ (1-3\nu-17\nu^2)\ \cos^4\alpha\ -3(1-2\nu+9\nu^2+23\nu^3)\{\cos^6\alpha/$$
$$(\sin^4\alpha\ \sin^2\ \pi/m)\}\ \{1-3\nu\ \cos^2\alpha\ +(1+4\nu+12\nu^2)\ \cos^4\alpha\}]$$

and $\bar{F}$, $\bar{M}$ are given by Eq. (16).

RESULTS AND DISCUSSION

The expressions obtained above are of considerable significance. The results are dimensionless and explicit. These closed-form solutions provide useful information concerning the influence of the cable parameters and its properties. For a better assessment of the influence of the cable parameters and material properties, the variation of the rather involved functions, h_1 and h_2, was studied over the practical range of α (i.e., $\pi/3 \leq \alpha \leq \pi/2$), covering a number of cases with different values of m and ν. Some typical plots are presented here (Fig. 1). It is interesting to note that for "relatively large" α, the variables h_1 and h_2 do not show any significant variations and that both, to first order can be approximated as unity. This approximation simplifies Eq. (21) to

$$\bar{\sigma} = \{(1/2) \sin\alpha\}^{1/2} \cos\alpha[\bar{F} +(1+\nu)^2 \sin^2\alpha \cos\alpha \sin(\pi/m)\bar{M}].$$

This result is of considerable importance, particularly for preliminary cable design. It clearly brings out the role of α and m in affecting the contact stresses when the cable is subjected to tension and/or torsional moment. However, in view of the specified constraint on α for the above expression to be valid, it would be advisable to stick to the more accurate earlier results in the final analysis. Some typical results based on this more accurate equation, Eq. (21), are presented in Fig. 2 and Fig. 3. These plots show the effect of varying the wire helix angle α on the maximum contact stress $\bar{\sigma}$. Three different loading conditions with pure tension, pure torsional moment and combined tension and torsion are considered. In general, in the practical range of α, the dimensionless stress, $\bar{\sigma}$, continuously decreases with increasing α, vanishing altogether at $\alpha = \pi/2$ as expected. However, an exception to this general trend may be observed in the lower range of α, particularly when m (i.e., the number of helical wires in the cable) is large. From Fig. 3, it is evident that the changes in ν within practical limits for steel cables do not influence the contact stress characteristics to any perceptible extent.

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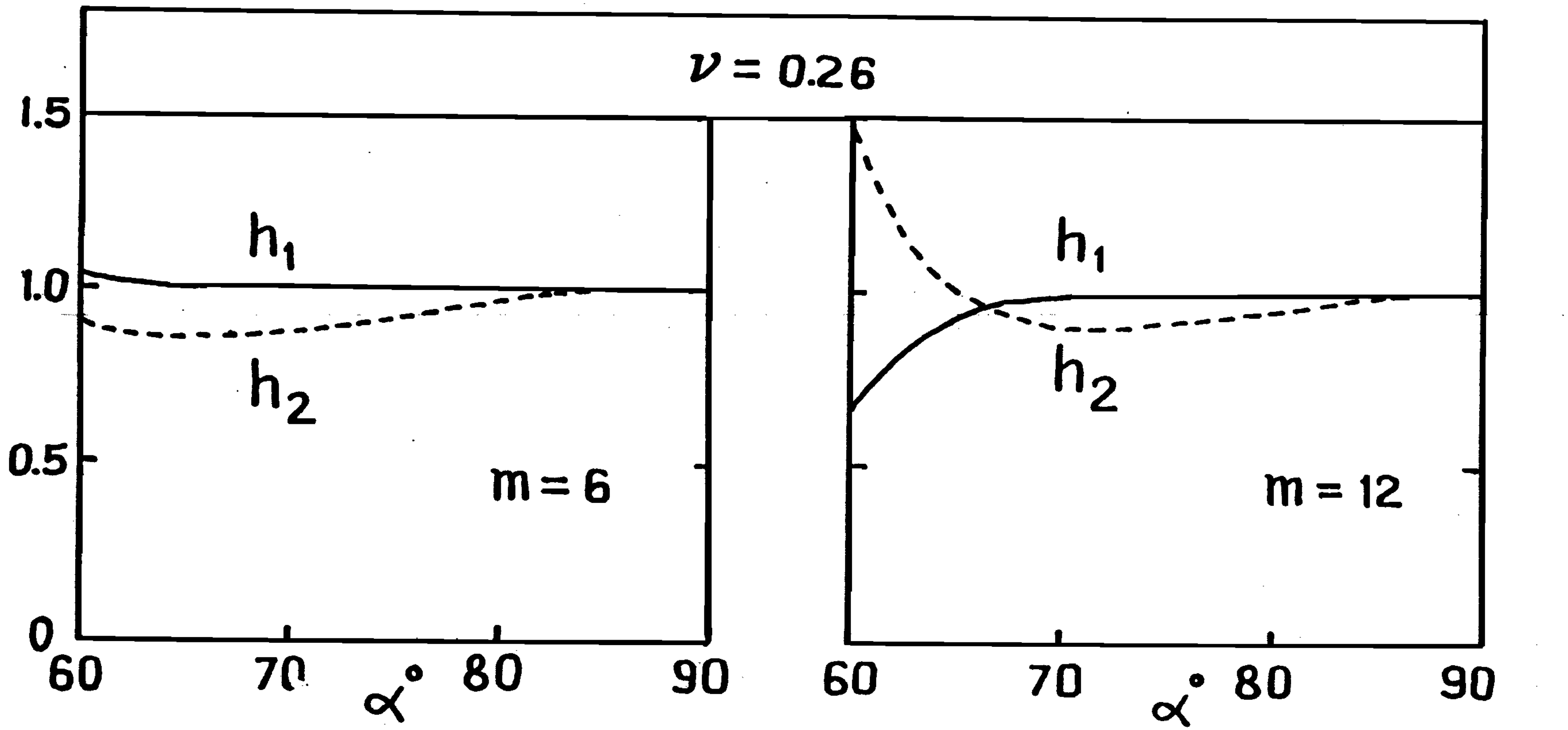

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Fig. 1 Typical plots showing the effect of α on h_1 and h_2

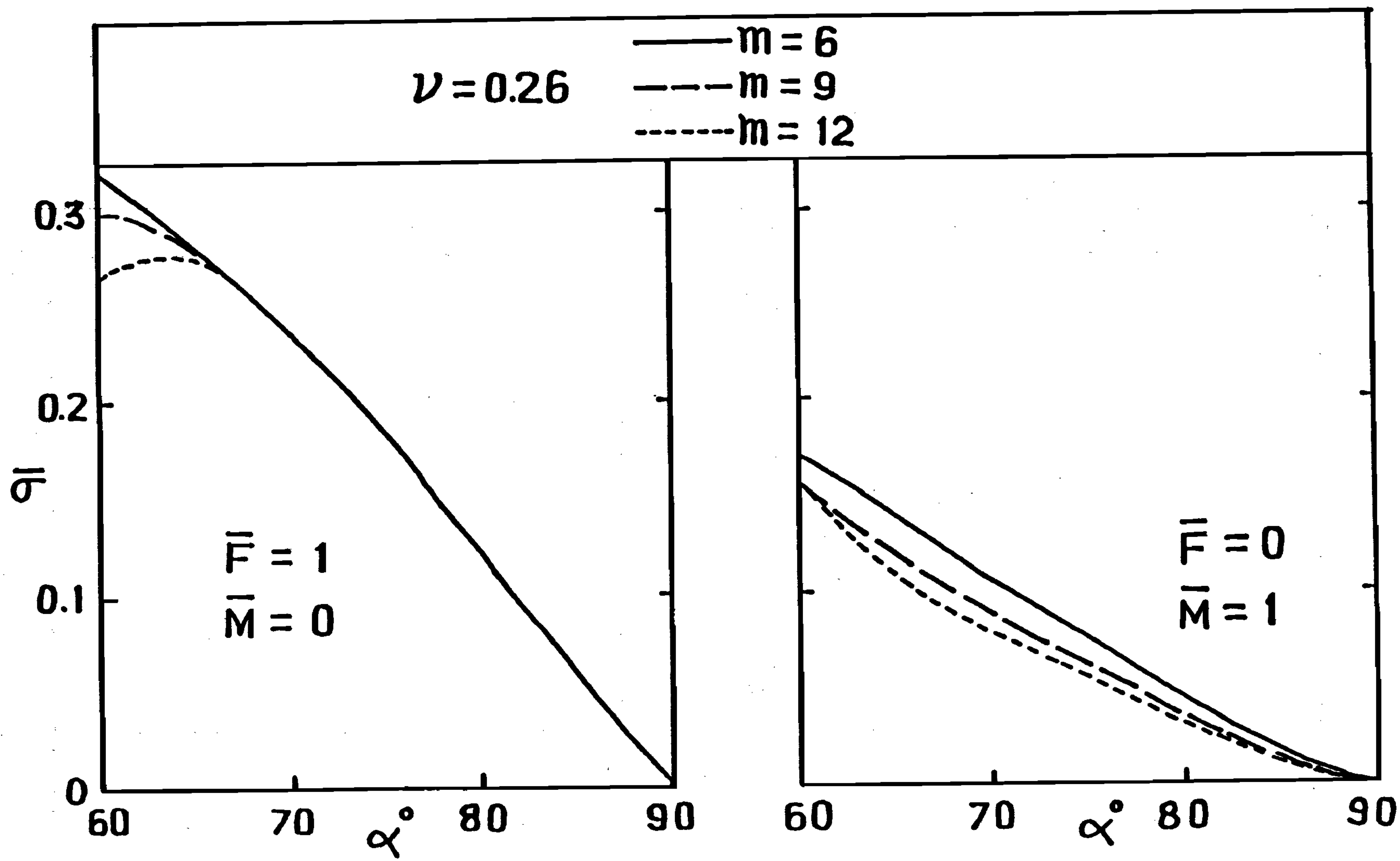

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Fig. 2 System plots showing the effect of α on $\bar{\sigma}$ when the cable is subjected to pure tension or torsional moment.

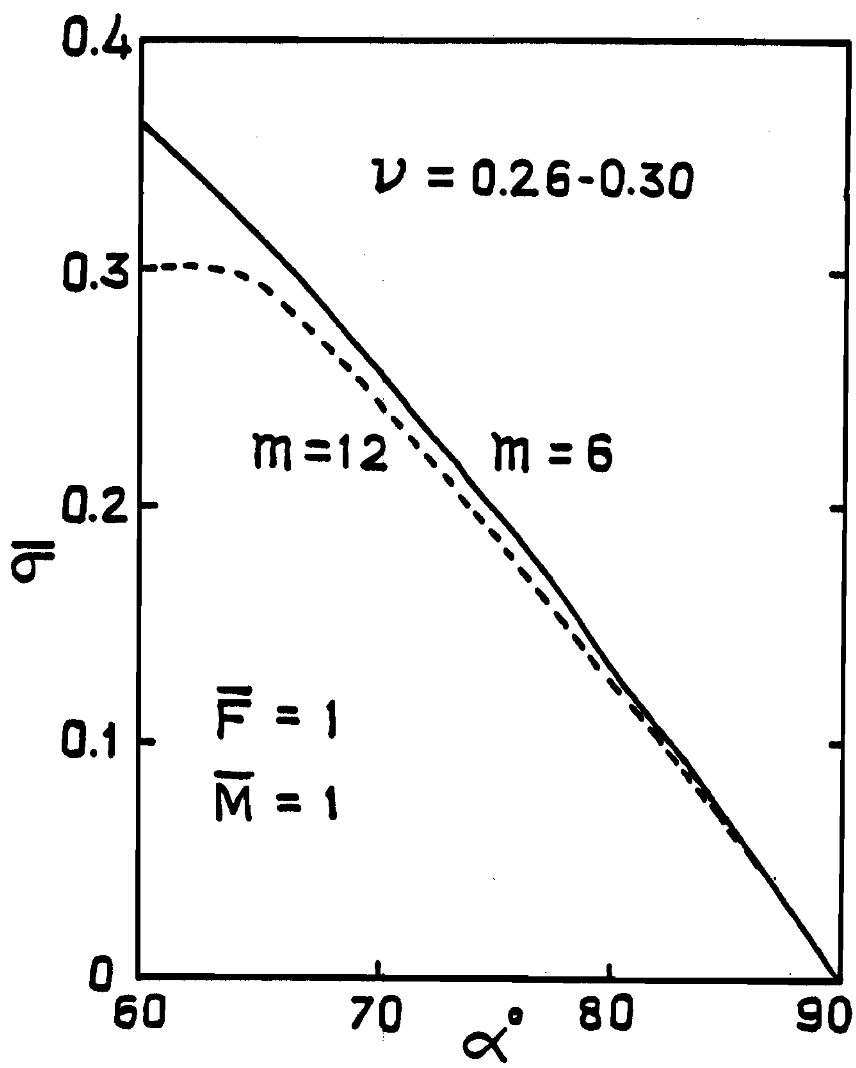

Fig. 3 System plots showing the effect of α on $\bar{\sigma}$ when the cable is subjected to tension as well as torsion.