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WADC TECHNICAL REPORT 54-237

**DEVELOPMENT OF EQUIPMENT FOR THE EXPERIMENTAL  
MEASUREMENT OF THE MOMENTS OF INERTIA, AND  
PRODUCT OF INERTIA OF AIRCRAFT**

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# Contracts

## FOREWORD

This report was prepared for the U. S. Air Force by the Cornell Aeronautical Laboratory, Inc., Buffalo, New York, as Cornell Aeronautical Laboratory Report TB-822-F-3 under Contract No. AF 33(616)-182, Supplemental Agreement No. 5, Project No. 1365, Task No. 13550, "Moment of Inertia Investigation," (formerly RDO No. 458-414).

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## ABSTRACT

The design and development of equipment for the experimental measurement of the moments of inertia, and the product of inertia of airplanes by the spring oscillation method are discussed.

Design information in the form of airplane jack point location, jacking loads, jack point clearances, estimated values of moment of inertia etc. for a variety of aircraft, are tabulated.

The design evolution is discussed, indicating the reasons for various design features, and features considered but later discarded. Structural design loads and margin of safety are given.

The calibration and shakedown of the equipment are discussed, and representative test data are included.

## PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

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## INTRODUCTION

The moment of inertia and product of inertia of aircraft are required for the prediction of airplane response characteristics and for the analysis of response data to determine parameters such as stability derivatives.

Experience has shown that inertial information calculated on the basis of airplane mass distribution data is of insufficient accuracy to permit proper prediction and analysis of airplane response characteristics. Experimentally measured values determined by techniques such as the spring-oscillation method have been found to have suitable accuracy.

The spring-oscillation method consists of supporting the aircraft at two jacking points, thereby defining an axis of oscillation, and placing a suitable spring at a third point of support. The airplane, if disturbed, will oscillate about the jacking points as a single degree of freedom, second order system. The moment of inertia about the oscillation axis is proportional to the oscillation period and the spring constant. This moment of inertia is then transferred to an axis passing through the airplane center of gravity.

Specific examples of the spring-oscillation method are presented in References 2, 3, 4 and 5.

The equipment required for test by the spring-oscillation method is relatively simple, consisting of aircraft jacks, springs, suitable retainers for the springs, jack point adapters, a steel measuring tape, and suitable timing device (stop watch).

The Air Force has awarded Cornell Aeronautical Laboratory, Inc. Contract AF 33(616)-182 for the purpose of designing and constructing a "standardized" test kit for determining the moments of inertia of aircraft by the spring-oscillation method. This equipment is to consist of a selection of springs, spring retaining cages, and a selection of jack-to-spring-cage adapters. Appendix III of this report contains the text of Exhibit B of the contract, and outlines in detail the objectives and aims of this program.

An earlier phase of this contract consisted of a study to determine the accuracies that would be required in aircraft inertial information. This plus an evaluation of several experimental methods for determining moments of inertia formed a background for the present phase. Reference 1 reports the results of these studies. Reference 2, prepared as part of the present phase is a handbook which describes the test equipment constructed and outlines specific procedures for its use.

This report discusses the design evolution, present design, calibration and shakedown of the test equipment.

### Design of Spring Cages

A basic aim of the program was to design equipment that would be usable with the greatest number of aircraft in the weight range considered. The experimental parameter requiring the greatest range of adjustment to accommodate a variety of aircraft and test configurations is the spring rate. The initial step in the design was therefore a determination of the required range of spring rates.

The spring rate range was determined by calculating the approximate rate that would be required for various airplanes in several different test configurations. This was accomplished for as many airplanes as possible and was based on technical information obtained from the respective aircraft manufacturers through the Aircraft Laboratory at Wright Air Development Center. These data include airplane gross weight, center of gravity position, jack point locations, allowable jacking loads, calculated moments of inertia and minimum jack point ground clearances and are tabulated in Tables I and II. Several airplanes for which data were requested are not included because sufficient information was not available.

Required spring rates and loads were calculated for pitching and rolling moment of inertia tests and are reported in Table I.

For the pitching case, the airplane was assumed to be supported at a pair of wing jack points symmetrically disposed with respect to the airplane center of gravity. The springs were assumed placed at a nose or tail jack point depending upon whether the wing jack points lay behind or ahead of the center of gravity.

To estimate the required spring rates for oscillating the airplane about an axis parallel to the roll axis, it was assumed that a jacking position existed on the fuselage aft of the center of gravity (for tricycle geared airplane) at the same vertical location with respect to the fuselage center line as the nose jack point. Such a jack point rarely exists; however with many airplanes this jack point may be furnished by the careful design of a structure which attaches to the airframe. Since the oscillation axis is assumed to be directly below the center of gravity, spring cages are assumed placed at equal spanwise station locations under each wing. The spring rates under each wing must be equal, and the sum of the two must equal the total required spring rate.

The originally proposed spring cage design provided for a maximum of seven springs within a single cage as pictured in figure 1. Combinations of from three to seven springs of various rates would be inserted into the cage to produce various total spring rates, which are equal to the sum of the rates of the individual springs.



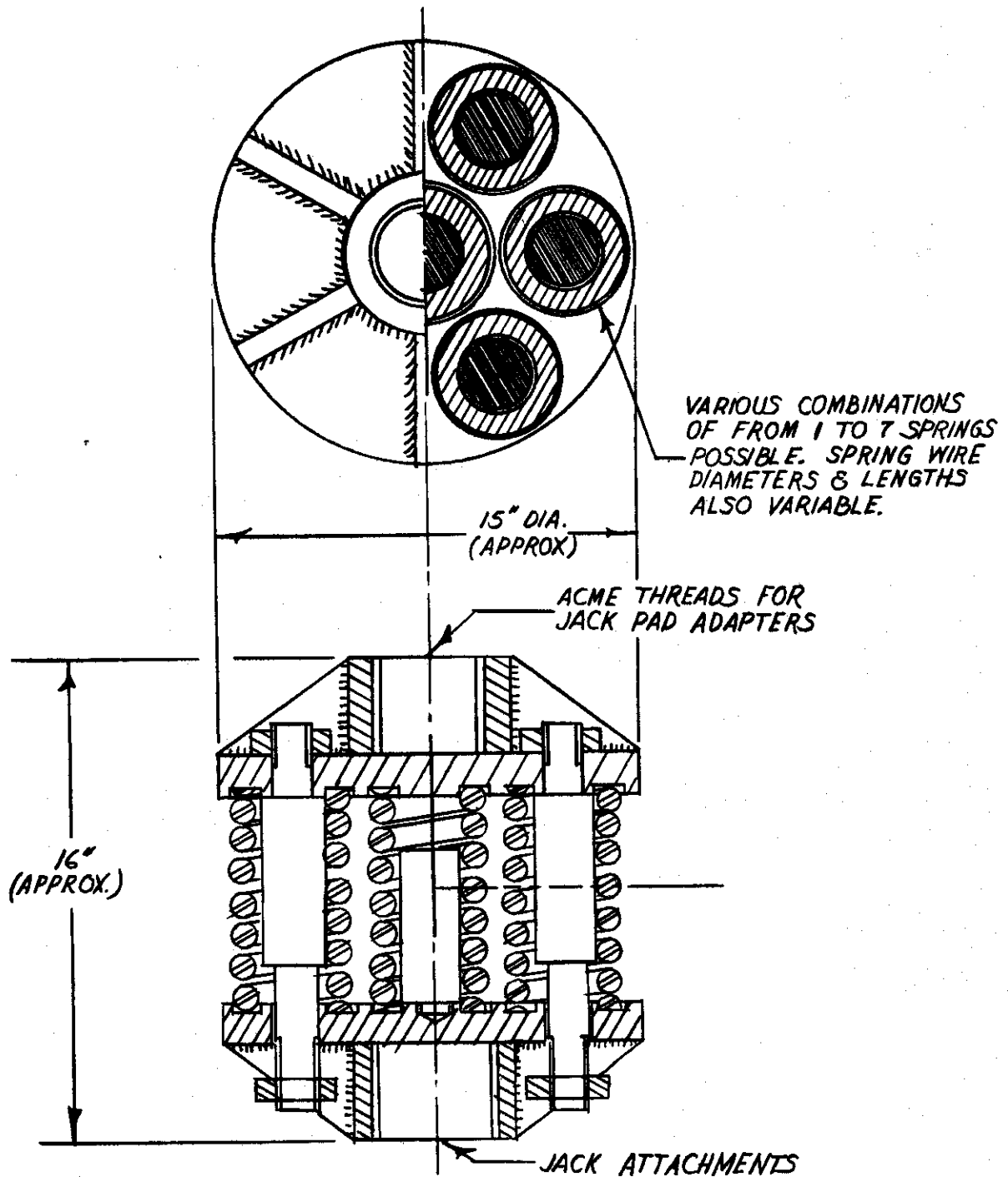


FIG. 1 PROPOSED SPRING CAGE UNIT

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Study indicated that it would be simpler to utilize a small number of individual spring rates with provision for a greater number of springs per cage than to provide a large number of individual spring rates and seven springs per cage. Since spring diameter and free length had to be identical for all springs regardless of rate, the only alterable characteristics would have been spring wire diameter and the number of active coils. The variety of spring rates required precluded any reasonable compromise on these terms.

The design was therefore modified, resulting in the final configuration shown in figure 2. This present design features two sizes of spring cage, each having seven spring positions. The larger of the two allows three different size springs to be placed concentrically at each position. The smaller cage allows two sizes of springs at each position. Thus, a total of twenty one springs may be installed in the larger cage and fourteen in the smaller. Three different spring rates are represented and a wide range of total spring rates may be selected by inserting various numbers and combinations of the basic springs in the cages, the total spring rate being the sum of the individual spring rates. Two cage sizes were chosen so that a smaller cage could be used with smaller airplanes when only a few springs might be needed. This is desirable since there are cases where there is limited clearance between the cages and adjacent airplane structure.

A cage consists of top and bottom plates which are aluminum castings. These plates are machined to provide locating bosses at each of the spring locations. Three of these are drilled to allow insertion of cage retaining bolts. The lower cage plate is fitted with Teflon bushings in the bolt holes to provide a low friction bearing surface for the bolts. The bolt diameter is smaller than the bushing so that they are in contact only if the cage should become misaligned under load. Provision is made for a standard cup type jack pad to be inserted in the upper casting to match the standard jack point installed on an airplane. A completely assembled cage is shown in figure 3. The cage assembly is installed on a jack by sliding the lower cage plate over the appropriate jack adapter screwed onto the jack strut. These adapters are designed to fit the standard jack types designated; i.e. B-3B, B-4, B-5, B-6.

## Ball Joint Assembly

For some airplane configurations, it was considered desirable to be able to tilt the spring cage with respect to the jack. This may be necessary to insure that the forces produced by the springs act normal to a line drawn perpendicularly from the spring cage location to the airplane oscillation axis. This orientation is necessary if the spring rate about the oscillation axis is to be constant. The theory of the spring oscillation method as presented in the Handbook, Reference 2, assumes that the spring rate about the oscillation axis is independent of the angular deflection, i.e. a plot of restoring torque vs. angular deflection is linear. If the spring force vector is not normal to

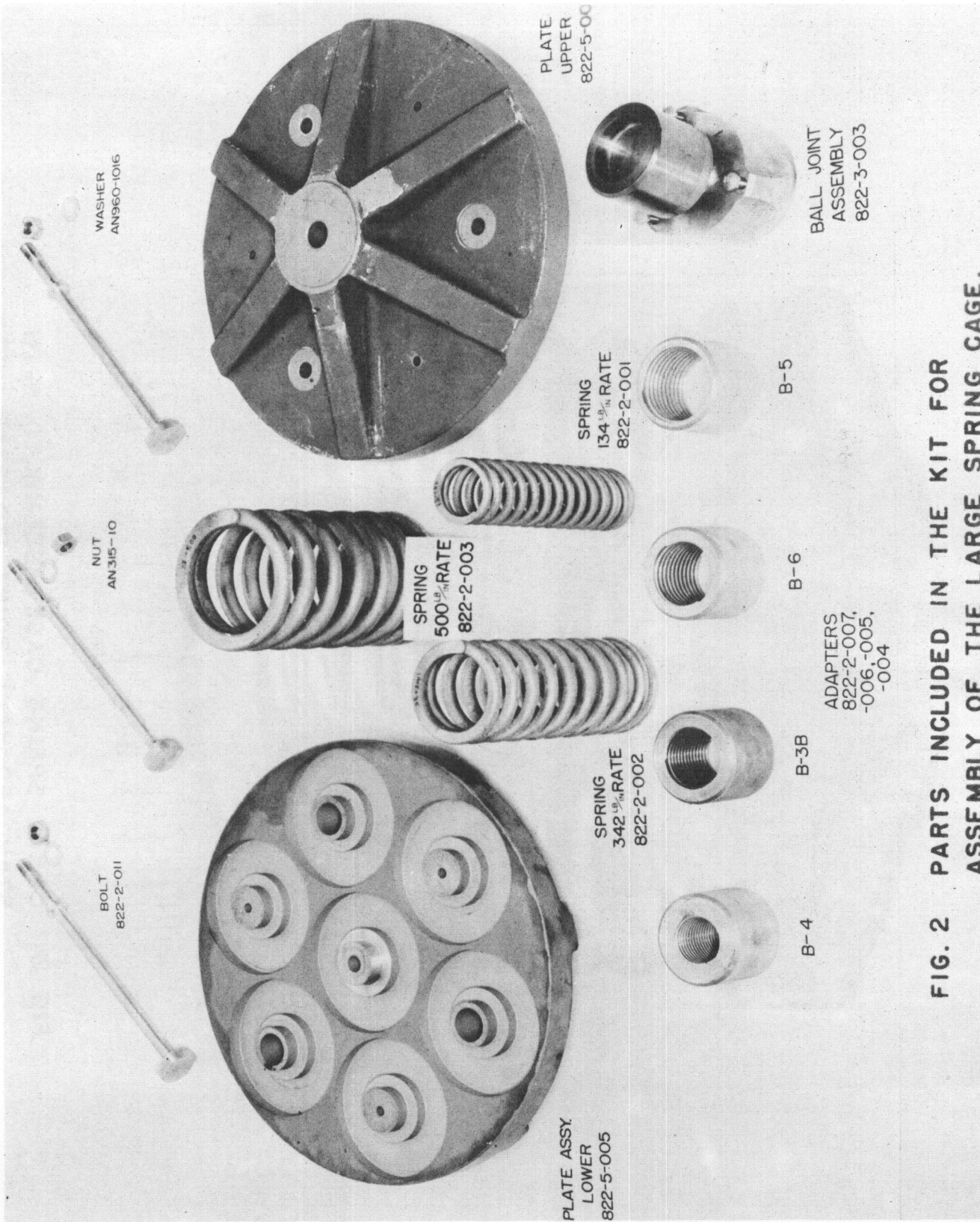
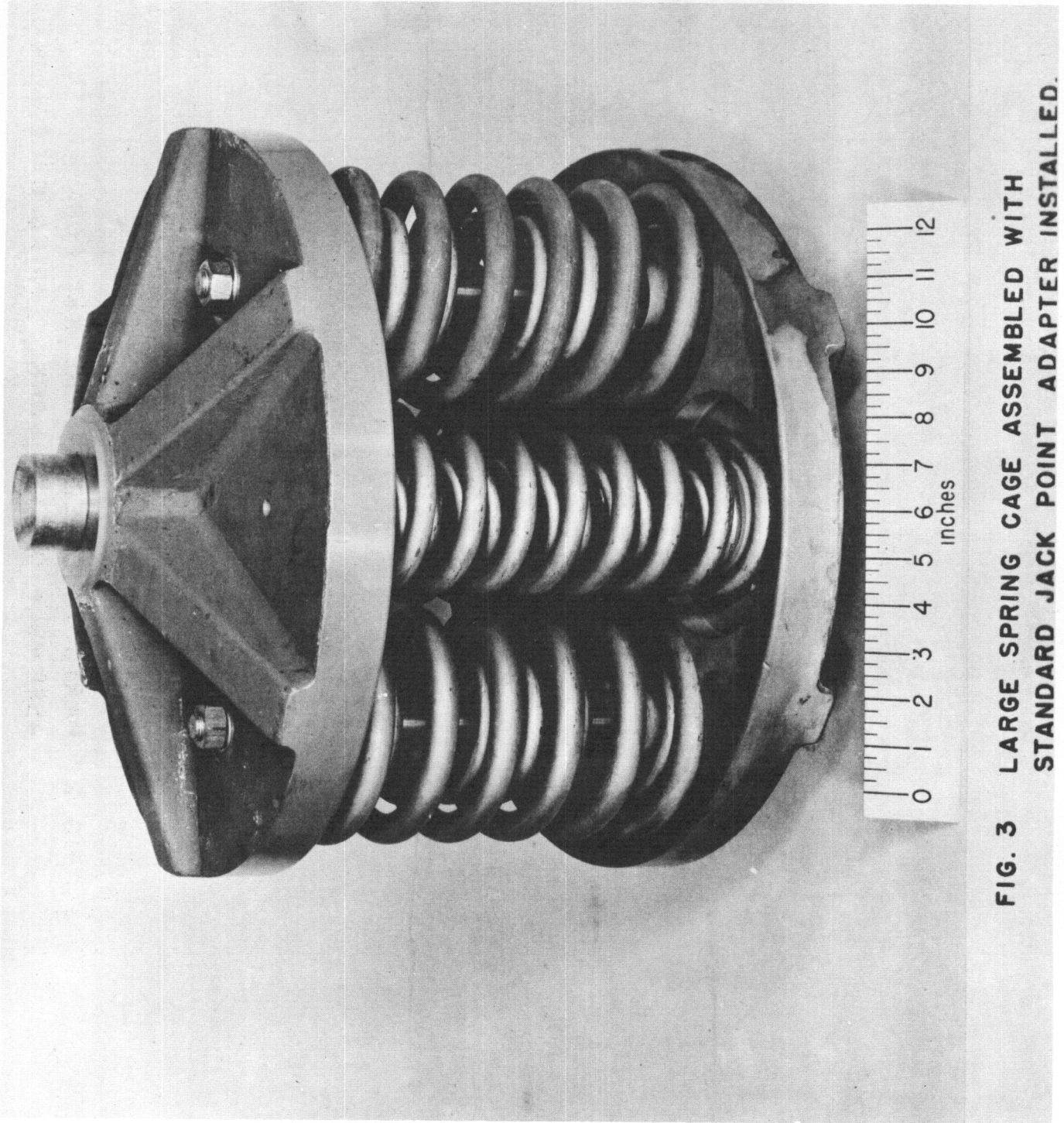


FIG. 2 PARTS INCLUDED IN THE KIT FOR ASSEMBLY OF THE LARGE SPRING CAGE.



**FIG. 3 LARGE SPRING CAGE ASSEMBLED WITH  
STANDARD JACK POINT ADAPTER INSTALLED.**

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the radius, a correction must be made for the initial angularity. The radius length will, however, change as the airplane oscillates, resulting in a variable spring rate about the oscillation axis. Corrections for this latter effect are difficult since the equations for moment of inertia determination would become non-linear.

A ball joint assembly, shown in figure 2, has been designed and is furnished as part of the test equipment. This is installed between the lower spring cage plate and the jack adapter and permits tilting the spring cage axis up to  $10^{\circ}$  in any direction with respect to the jack. The tilt is limited to  $10^{\circ}$  to minimize the likelihood of upsetting a jack. Available information on jack design criteria indicates that for a fully extended jack the normal load should not be misaligned with respect to the jack strut axis by more than  $8^{\circ}$ - $10^{\circ}$ . For less than full extension, the tolerance would of course be greater. The ball joint is intended for use only when necessary and particular applications should be investigated for possible jack instability. For situations requiring more than  $10^{\circ}$  of tilt, corrections should be applied to the spring rate or some means of tilting and anchoring the jack provided.

The ball joint contains a bronze bushing and may require some form of lubrication occasionally. A light graphite grease is recommended.

## Stress Analysis Criteria

### Spring Cages

The upper and lower spring cage plates were designed so that the yield stress of the material would not be exceeded for an applied load equal to twice that which would be applied by compressing a fully equipped (i.e. all springs installed) cage to the springs solid height.

These design loads are 26,700 lbs. for the small cage and 54,800 lbs. for the large cage. The casting material is 40-E aluminum alloy, and has a yield stress of 20,000 psi and an ultimate stress of 32,000 psi.

The stress analysis involved several assumptions, all of which lead to a conservative design. In terms of the yield strength there is a small positive margin of safety, and in terms of the ultimate stress the margin is 1.65, this being the ratio of ultimate to yield stress.

The design load should never be encountered in practice, since it would require that a spring cage be jacked with all springs bottomed. Both large and small spring cages were satisfactorily proof loaded to full deflection, with all springs installed.

## Springs

The springs were designed so that the stresses in the spring wire are no greater than approximately 67% of the yield strength of the wire when the spring is compressed to its solid height.

## Spring Design

The data in Table 1 indicate the required maximum and minimum spring rates and loads. The maximum required spring rate shown is 6175 lb/in (RB-66, roll inertia test). A maximum rate of 6200 lb/in was therefore selected for initial design analysis. On the basis of seven spring locations and a maximum desired rate of 6200 lb/in, a rate of approximately 900 lb/in would be required at each of the seven positions in the large cage to make up the maximum desired total. It was felt that three different spring types were an optimum and that total spring rate increments of approximately 100 lb/in were desirable. It was found by cut and try that individual springs with rates of approximately 100 lb/in, 300 lb/in and 500 lb/in would give a desirable compromise. This choice of approximate rates permitted the following design requirements to be met: standard spring wire sizes could be used, the free length of the two larger springs would be identical (the smaller spring is shorter but this is compensated for by the height of the locating bosses in the upper and lower cage plates), the proper number of active spring coils is provided, the maximum fiber stress of the material will not be exceeded if the spring is compressed to its solid height, and lastly, spring internal and external diameters would be such that all three could be mounted concentrically without interference between coils. A deflection of approximately four inches is required to deflect the springs to solid height. The maximum working deflection during use is not expected to exceed approximately 3.4 inches, however.

As additional insurance against possible interference between the concentrically mounted springs, the specification required that the medium size spring be coiled in a direction opposite to that of the large and small size springs. Thus in the event of interference, no coils of one spring could become meshed with compressing coils of an adjacent spring. It may be noted that the medium and small size springs in the kit are coiled in directions opposite to those specified in the detail drawings, due to an error by the manufacturer. (See drawings 16 and 17 as listed in Appendix IV). Tests were made to determine whether the coiling direction was sufficiently important to warrant returning the springs to correct the discrepancy. The tests showed that the diametral clearances were sufficient and that the coiling feature was not absolutely necessary to prevent interference. The delay which would have been involved in obtaining another set of springs was thus avoided.

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Final specification for the springs were worked out with the manufacturer, the Muehlhausen Spring Division of the Rockwell Spring and Axle Co., Logansport, Indiana. The actual design spring rates were 128  $\pm$ 8 lb/in, 322  $\pm$ 20 lb/in and 455  $\pm$ 23 lb/in. The springs upon calibration were found to have mean rates of 134 lb/in, 343 lb/in and 502 lb/in respectively. These are not within the design specifications; however, it was found that none of the springs of any given type deviated from these mean rates by more than 1.5%, a far better tolerance than originally specified. This is very desirable because this small deviation from the mean rate assumes that more uniform loading will be obtained with any given combination of springs and that tilt of upper and lower cage plates with respect to each other will be minimized. This also permits rapid estimation of total spring rates for various combinations of springs. The fact that the actual spring rates are not as specified is of no concern since the total rate will always have to be computed on the basis of the actual springs used in the cage in any instance.

The springs are cadmium plated and were baked following plating to minimize the effect of hydrogen embrittlement.

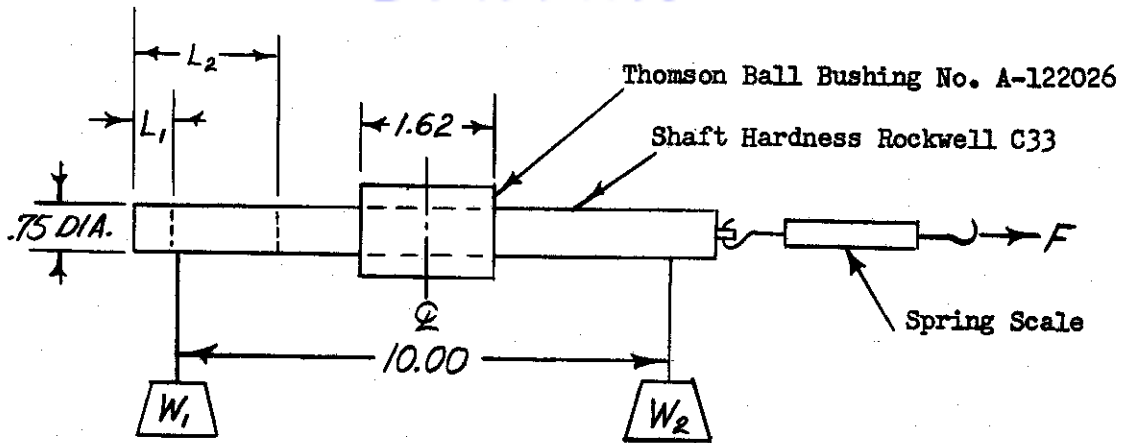
## Linear Ball Bushings

The preliminary design of the spring cage castings provided for a spindle bolt at each of the six outer spring locations. Each bolt was to slide through a bushing in the lower cage plate. It was felt that these bushings would be necessary to preserve proper alignment of upper and lower spring cage plates and springs. After some study it was decided that linear ball bushings would be used, since they gave the best aligning properties with the lowest friction. The number was reduced to three to correspond with the cage redesign which featured three instead of six retaining bolts.

During discussion of the preliminary design, Wright Air Development Center personnel expressed concern regarding the desirability of ball bushings, since little was known concerning their performance under the side loadings and moments that would be imposed. It was suggested that if possible some other form of bushing be substituted, or that tests be made to establish the suitability of a ball bushing.

Since no other type of bushing seemed to be superior, a test was conducted on a Thompson linear ball type bushing to determine axial friction or drag forces and wear characteristics when subjected to side loads and moments. The bearing was inserted in a test fixture as shown schematically in the diagram of figure 4. Appropriate loads and moments of various magnitudes were imposed and the force required to move the rod through the bushing was recorded. The table accompanying figure 4 shows the results obtained.

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- $F_1$  = Force registered by spring scale over a travel of  $L_1$ .
- $F_2$  = Force registered by spring scale at a travel of  $L_2$ .
- $L_1$  = Shaft travel when a substantial increase in  $F$  is noticed.
- $L_2$  = Maximum shaft travel = 3 inches.

Each test was begun with the weights equidistant from the centerline of the bushing.

Ball Bushing Test Data

$W_1$ Lbs.	$W_2$ Lbs.	$L_1$ Inches	$F_1$ Lbs.	$L_2$ Inches	$F_2$ Lbs.	Remarks
0	0					Action was a bit rough over full range of travel.
9.5	9.5	1.00	.75	3.00	2.00	Increasing roughness.
14.5	14.5	1.50	2.00	3.00	6.00	Increasing roughness.
19.5	19.5	1.25	2.00	3.00	6.00	Increasing roughness.
24.5	24.5	1.25	3.00	3.00	8.00	Rough and jerky.
29.5	29.5	1.00	3.50	3.00	14.00	Increasingly rough and jerky.
34.5	34.5	1.00	4.00	3.00	18.00	Extremely rough and jerky.
39.5	39.5	.75	4.00	3.00	18.00	Extremely rough and jerky.
44.5	44.5	.50	4.00	3.00	28.00	Extremely rough and jerky.
						Shaft began to show grooves from ball pressure so tests were discontinued.

Figure 4. BALL BUSHING TEST SETUP



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These tests indicated that the bushing was unsatisfactory in this form, and that possibly further tests with a harder shaft might be in order. Commercially available ball bushings were further investigated to ascertain whether a more suitable type could be found. The lack of manufacturer design information on the effect of side loadings made it apparent that no conclusions could be drawn without conducting lengthy tests on a variety of bushings. Since there was some doubt as to the actual necessity for these bushings to maintain cage alignment, (previous experimental determinations of aircraft moments of inertia, References 3, 4, 5, had used the equivalent of the spring cages without aligning bushings), it was decided to discontinue bushing tests and to design the spring cage so that one central ball bushing could be installed if necessary, but to construct and test the cage without any bushings. The need for bushings was to be determined during the shakedown tests.

As will be discussed in the section, "Calibration and Shakedown Tests", the spring cages were found to perform satisfactorily, and the cages are therefore furnished without ball bushings. The "Teflon" bushings installed in the lower cage plate, as discussed above under "Design of Spring Cages" provide a low friction surface for the retaining bolts in the event that the cage becomes misaligned but do not provide any alignment.

## CALIBRATION AND SHAKEDOWN TEST

The equipment was calibrated by determining the spring rates of all springs and also the spring rates of fully equipped cages. The method consisted of loading springs in compression up to their maximum working deflection in approximately eight or nine increments. A plot of force versus deflection was then made. A Baldwin-Southwark tester model 60-TE-474 was used to apply the load, and force dial readings were recorded. Spring deflection was measured with a dial gauge capable of being read to within .0005 inch up to 1 inch. Since the maximum spring deflection was approximately 3.4 inches, the gauge was reset each successive inch. The stated accuracy of force measurement was  $\pm 1.7\%$  of full scale. The three scale ranges used were 0 to 1200 lbs, 0 to 12,000 lbs, and 0 to 60,000 lbs.

Typical calibration curves are shown in figures 5, 6 and 7 for three individual springs. It will be noted from the curves that a slight zero error is present and that it is greatest for the stiffest spring. This is believed to be due primarily to slight misalignment of the spring ends. The spring ends are not perfectly square with respect to the spring center line and thus some initial deflection will occur for a very slight load. These effects are removed during assembly of the cages by tightening the nuts on the cage retaining bolts which provide one quarter of an inch preload deflection. The rates determined and marked on each spring were the result of taking the total applied force and dividing by the difference in deflection between the total and that indicated by the intersection of the faired straight line through the points and the abscissa.

The spring cages were tested in the same manner. Two dial gauges were used diametrically opposed in order to obtain a good average reading of deflection. All cages were fully equipped with springs with the exception of the smallest spring at the cage centers. These were not installed because material had been left unmachined for the boss to allow later installation of a ball bushing if this was found necessary during the shakedown tests. The cage calibration data are shown in figures 8 and 9.

A slight degree of non-linearity will be noted in the calibration curves of figures 8 and 9. This is considered to be a normal characteristic of coil springs having squared ends. The magnitude of this effect is such that the spring rate at maximum deflection is approximately 2-5% higher than that at very small deflections. This same effect is detectable to a certain extent in the calibrations of the individual springs (figures 5, 6 and 7). In practice a spring rate corresponding to the actual static deflection should be used. It was assumed that in the majority of cases this deflection would be between 1.75 and 3.0 inches. Several methods of defining individual spring rates in terms of the calibration data were investigated to find one which would give a cage rate closest to that corresponding to the assumed average deflection, the cage rate being the sum of the individual spring rates. This is the definition that is given above, and should give the spring rate at the average deflection within approximately  $\pm 1\%$ .

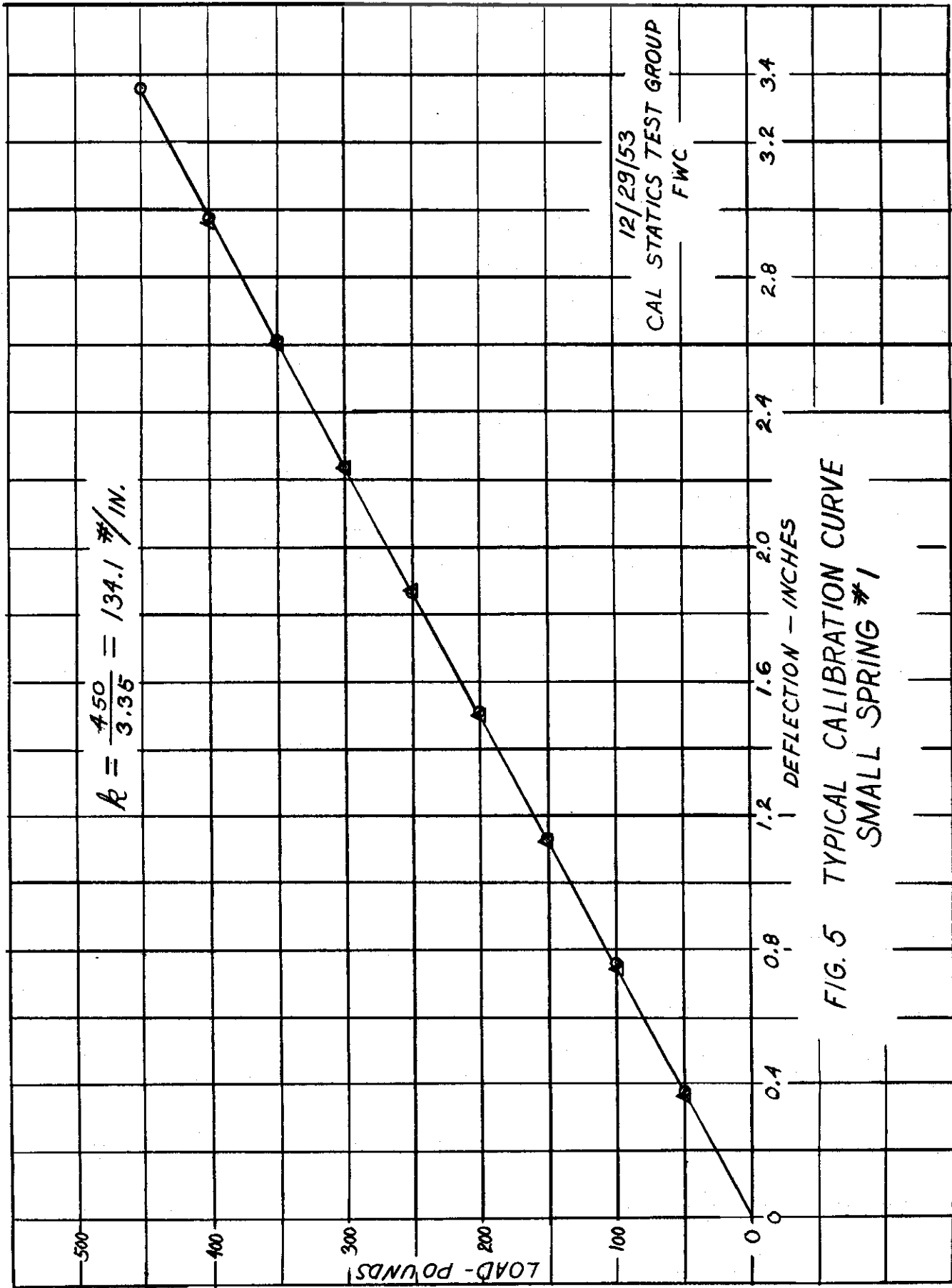
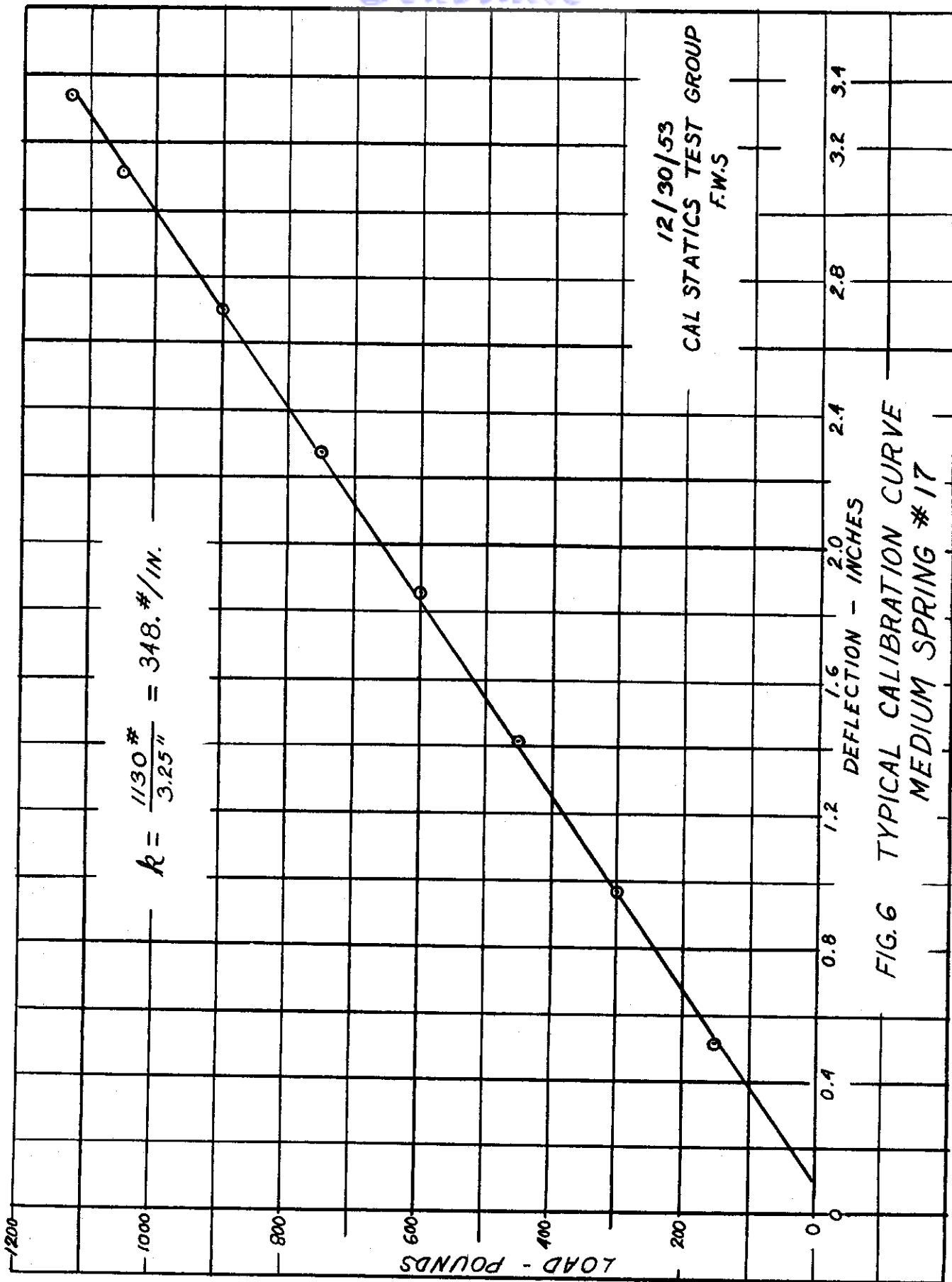


FIG. 5 TYPICAL CALIBRATION CURVE  
SMALL SPRING #1



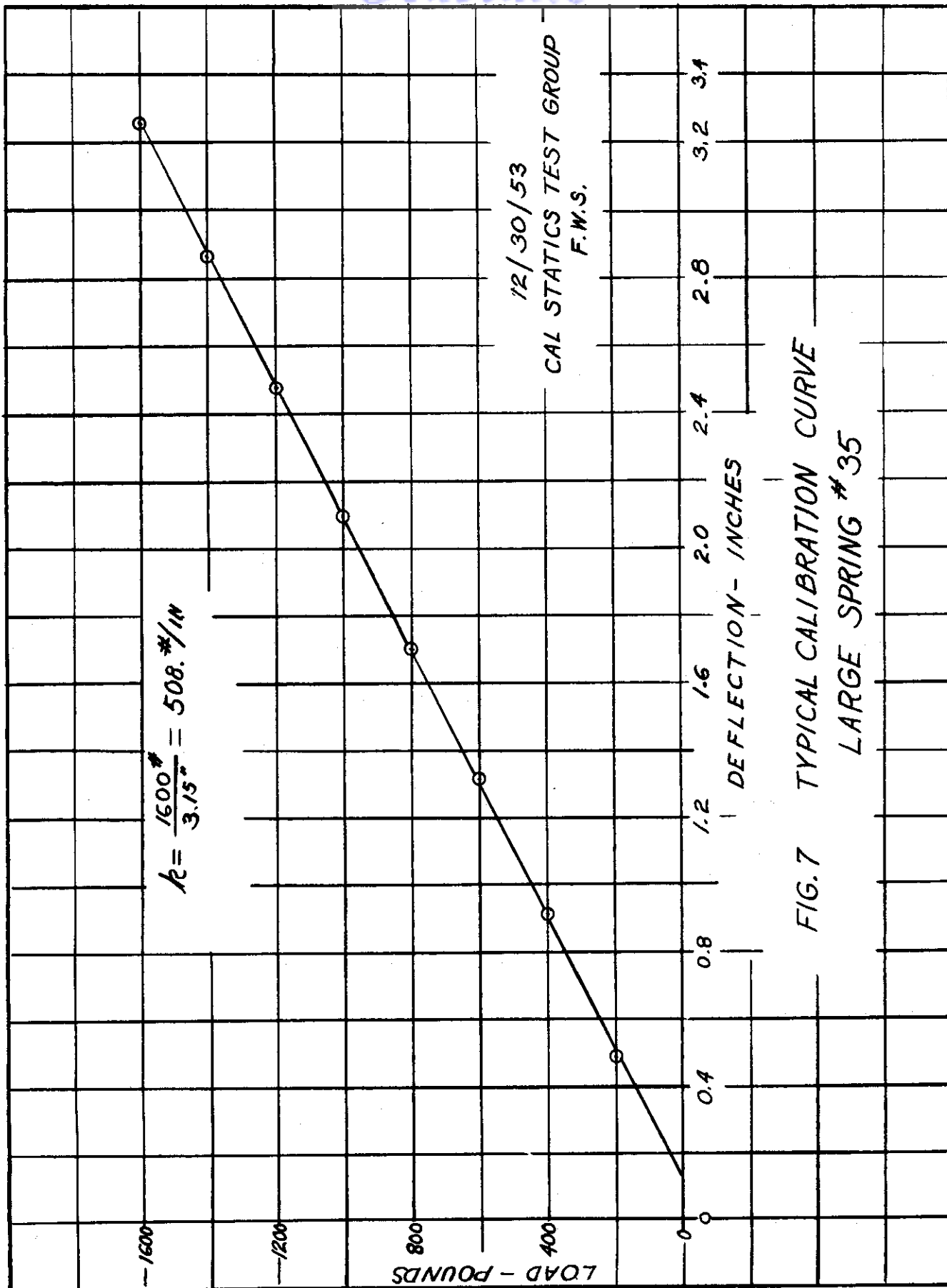


FIG. 7 TYPICAL CALIBRATION CURVE  
LARGE SPRING # 35

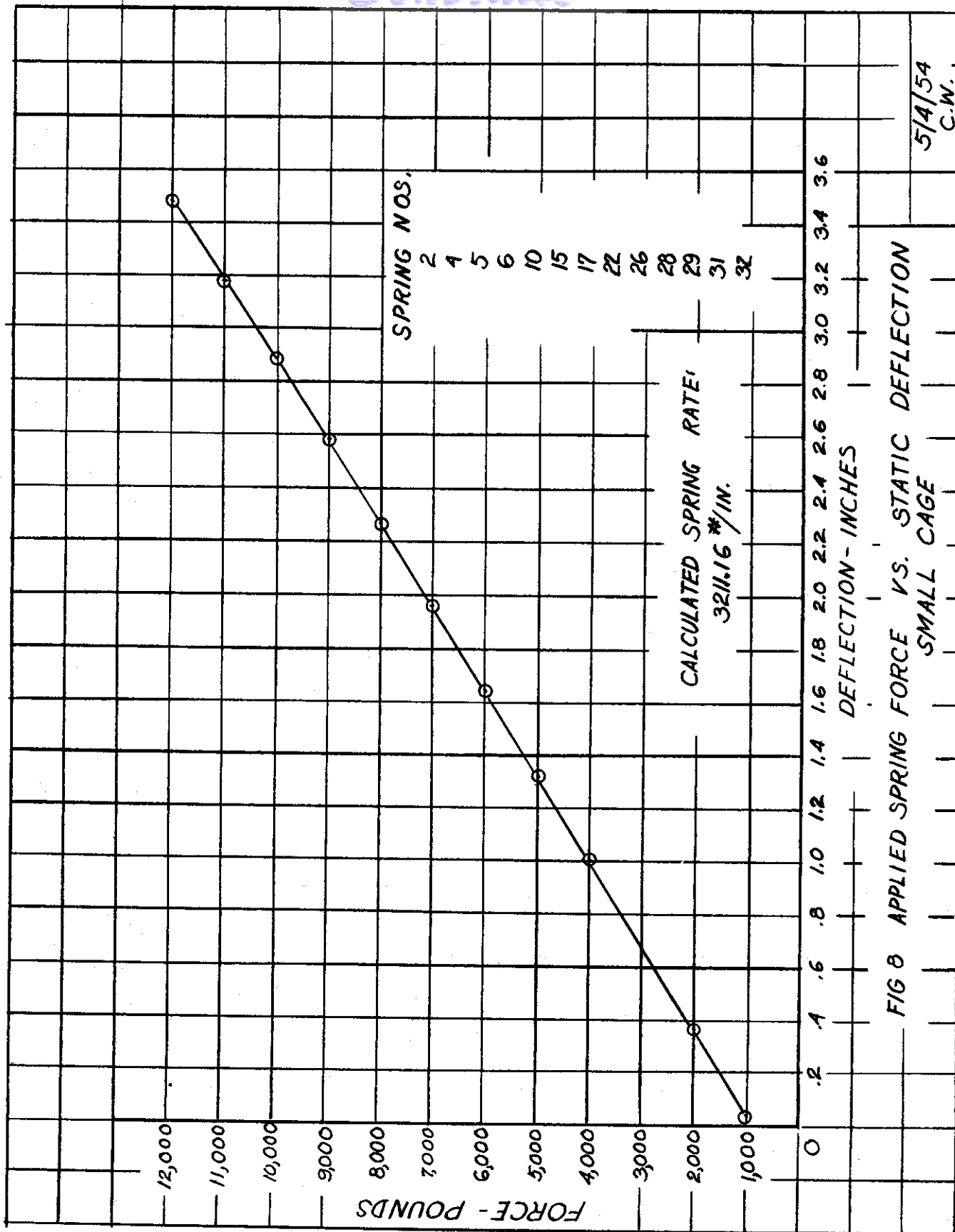


FIG 8 APPLIED SPRING FORCE VS. STATIC DEFLECTION  
SMALL CAGE

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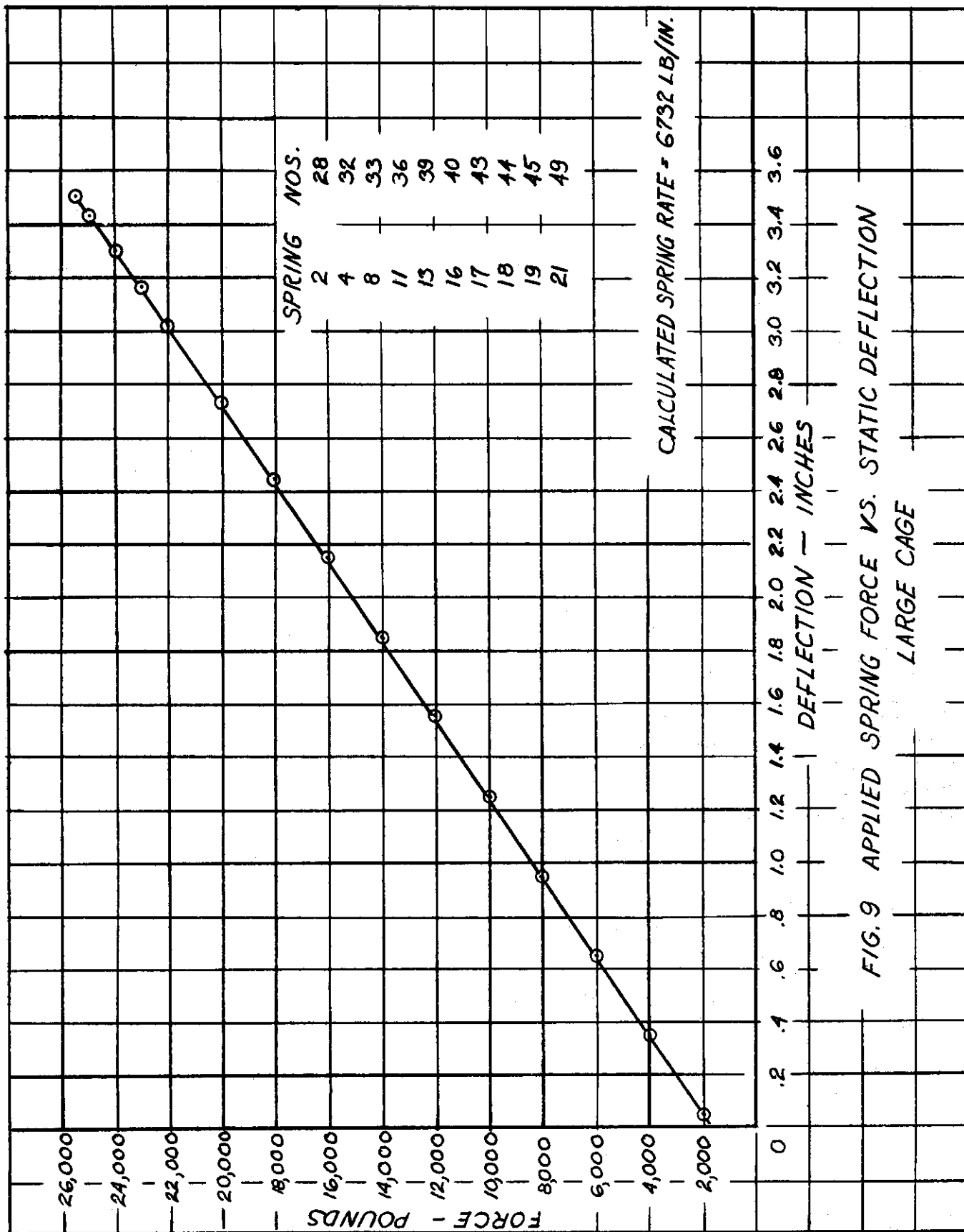


FIG. 9 APPLIED SPRING FORCE VS. STATIC DEFLECTION

LARGE CAGE

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The shakedown test consisted of determining the moment of inertia of an airplane in pitch, roll, yaw and the product of inertia. An F-6F airplane with full fuel load was used. This test, incidentally, serves to illustrate how additional jack points can be installed on an airplane to provide the proper oscillation axes for moment of inertia determination. The particular airplane had no suitable jack points, but did have two bomb rack installation holes at equal and opposite wing stations on both fore and aft spars which were capable of supporting the airplane weight in its test configuration, with a suitable margin of safety. Thus four jack points were designed and installed at these positions. This allowed the airplane rolling moment of inertia to be determined by placing two jack supports under one side of the wing and two spring supports under the opposite side. Jacking was accomplished by first raising the airplane to a level attitude with a lift truck at the tail, using a special steel bar installed on the airplane for that purpose. The airplane was then lifted until clear of the ground by simultaneously raising the four hydraulic wing jacks. The lift truck was then removed and the test begun.

The two forward jack points provided support and defined the oscillation axis for pitching inertia tests. An additional jack point was machined and screwed into the lifting bar at the tail for the jack mounted spring cage. Before jacking, the tail was raised with the lift truck until the airplane was level. The jack with spring cage was then placed under the tail jack point and all jacks were raised simultaneously until the landing gear cleared the floor. The lift truck was removed and the test begun. Data was taken with the landing gear up and down.

In order to provide two skewed axes for determining the yawing moment of inertia and product of inertia, special jack points were fitted to the axles, after removing the wheels and brakes. Jack points on one landing gear and at the tail determined one axis, with the spring cage located at a jack point on the opposite gear. This test was made with the airplane in a three point attitude so that the oscillation axis and spring jack point lay in a horizontal plane. The advantage of this was that the springs could be used in the vertical position and no tilting of the cage would be required nor would any side loads be experienced. The other skewed axis was defined by one forward wing jack point and a landing gear jack point on the opposite side of the airplane. Spring support was improvised by using a steel beam, one end attached to the lift truck for an anchor point and the other end supporting the tail of the airplane through a three foot cable. A jack mounted spring cage was placed at a jack point screwed to the center of the beam. This allowed the airplane to oscillate about the skewed axis with the support point at the tail moving through a path lying in a skewed plane, while at the same time the spring cage was able to be used in a vertical position, hence no side force. This naturally changes the value of the spring constant, but the angular spring constant of the system is calculated using the known spring rate of the cage and the length of the beam. A small deflection of the tail was used (1/8 in.) so that the assumptions of linearity would not be violated. Neither skewed axis test was



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performed with the airplane in a level attitude. The attitude was measured in each case and corrections to jack point location made accordingly.

These tests portray what can be done in the way of supporting an airplane whose jack point configuration (or lack of) is such as to normally discourage the determination of moments of inertia.

Proper alignment of the spring cages with respect to the jack points required care and usually several adjustments of the jack position. Lateral motion of the jack points with respect to the jacks due to unequal jack extension, deflection of the airframe, and possible unevenness of the floor caused the cage to become misaligned as the load was applied. This was corrected by moving the jack slightly after some load had been applied. This was accomplished by striking the jack near its base with a 4in. x 4in. timber. In one or two cases where the misalignment due to these causes was large, the full load was applied and the direction and extent of misalignment noted. The load was then removed and the jack purposely misaligned in a direction such that when the full load was next applied, proper alignment was obtained.

With proper alignment the spring cages performed satisfactorily and cage retaining bolts seldom if ever came in contact with the "Teflon" bushings. On the basis of this performance it was concluded that linear ball bushings were not necessary to preserve cage alignment. In fact it was felt that their use might actually be detrimental, since this would tend to conceal misalignment between jack and jack point. The cage and bushing would then be operating under a steady side load of unknown magnitude which would produce friction forces in the bearing, accelerate bearing wear and increase the likelihood of the airplane slipping off the jack.

In some instances the upper spring cage plate rotated with respect to the lower plate as the springs were compressed. This is due to a twisting or coiling of the spring as it is compressed. This effect is minimized by proper cage alignment and choosing individual springs such that two different coiling directions are represented in the combination. In most cases friction between the airplane jack point and adapter in the upper cage plate was sufficient to prevent this rotation.

Many precautions were taken during the shakedown program to prevent possible damage to the aircraft. The jacking procedure is more hazardous than usual since there are only two points of support capable of resisting side loads instead of the usual three or more. Ample use was made of safety jacks and shoring timbers to "catch" the airplane in the event of mishap. The actual operations were performed by personnel thoroughly familiar with aircraft jacking procedures. Such procedures are generally recommended when using this equipment.

A more detailed discussion of shakedown program particulars together with appropriate photographs is included in the Handbook, Reference 2.

# Contrails

## HANDBOOK

A handbook was prepared and is furnished with the equipment describing its operation and use. Sections included cover the following: definitions of moments of inertia and product of inertia, reasons for experimental determination, detailed description of the equipment, methods of estimating required spring rates, procedure for determining oscillation axes and jack loads, and the analytical procedure for determining the moments of inertia from measurements of distance, time and spring constants. Sample data sheets are included and a sample problem worked out for a particular airplane. Methods for estimating the accuracy of the test results are also given.

The handbook is intended to be a complete and detailed description of the equipment and its use and is written for personnel with sufficient engineering background to perform airplane weight and balance measurements and calculations.

The equipment is intended for use with the jacking points which are normally furnished with an aircraft. The usefulness of the equipment may be extended considerably, however, if it is possible to provide additional points of support on the aircraft. This will establish additional axes of oscillation and spring locations and increase the number of test configurations that may be used. For the more experienced engineer having a knowledge of airplane structures and stress analysis, a section has been included in the handbook which suggests methods for providing these additional points of support. The primary considerations involved are selecting a part of the airframe which can support the load, and designing a fitting which will properly distribute the jacking load to the airplane structure as well as support the load itself. Diagrams and photographs of several schemes are included.

The results of the shakedown tests, in which the equipment was used to measure the moments and product of inertia of a Navy F6F aircraft, are included in the handbook as an illustrative example of the equipment's use. The F6F aircraft was chosen primarily because of its availability to this program.

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PACKAGING MEANS

Two boxes were built for packaging the equipment. One container holds all forty-eight springs within sixteen compartments, three different spring sizes located concentrically in each. The other container is also divided into special compartments and contains the spring cage plates, ball joints, jack adapters, and cage retaining bolts, nuts and washers. An inventory list is included in each box, and in addition a position diagram for each part is included in the cage parts box. These boxes are also used as shipping containers, and have skids under them which provide sufficient ground clearance for easy handling by a fork lift truck.

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HYDRA SPRING INVESTIGATION

A basic problem in the application of the spring oscillation method for moment of inertia measurement is the flexibility of the aircraft being tested. If airframe vibration modes are excited during the tests, the measurements will be in error since the assumption of a single degree of freedom system is no longer valid. This problem has been discussed at some length in Reference 1.

The best method of minimizing airframe vibrations is to set the test frequency low enough so that these modes are not excited. The frequency is reduced by decreasing the spring rate. However, the spring must support some static load with a practical deflection so that there will be some limit below which the spring rate may not be reduced without bottoming the spring. Thus there will be a lower limit to the oscillation frequency for any particular airplane test setup. The spring cage equipment is usable therefore in all cases where this lowest practical frequency is significantly different from a structural vibration frequency. A frequency separation of from 5 to 10 to 1 is desirable.

If the frequencies are insufficiently separated it may be possible to make corrections to the data, as was done in Reference 5, having given information on airframe stiffness, mass distribution etc. Since these data are not necessarily available, however, and the determination of such is not a part of the spring oscillation technique as described in Reference 2, the equipment is simply not recommended for use in cases where this frequency separation is judged insufficient.

At the start of this program it was agreed, however, that any promising means for extending the usefulness of the equipment in this instance would be studied and evaluated.

One practical solution to the above problem would be a spring of low rate, but capable of supporting a high static load with a small deflection as compared to a conventional wire coil spring. A pneumatic spring consisting of a piston in a cylinder connected to a tank filled with compressed air would meet these requirements. The air pressure and the area of the piston control the static load that can be supported while the ratio of the piston displacement per inch of stroke to tank volume determines the spring rate. While such a device sounds feasible, reduction to practice would probably be difficult and the size would certainly be many times greater than an equivalent coil spring. Serious consideration has not been given to such a scheme.

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During the course of this program, a newly developed hydraulic spring came to our attention. This was immediately investigated for possible use as part of the test equipment. Initially it appeared that this hydraulic spring could provide the equivalent of a spring cage in a smaller volume and might also be capable of providing characteristics similar to the pneumatic spring described above.

The hydraulic spring is manufactured by the Wales-Strippit Corporation of Tonawanda, New York as the "Hydra-Spring". The primary application of this device is in die stamping machines for stripping the material from the dies after the stamping operation. The spring consists of a piston-cylinder arrangement which is filled with a compounded silicone oil, referred to as "Comproil". This oil exhibits a compressibility factor of approximately 12%, (i.e. its volume may be reduced by 12% under load) and provides excellent sealing and lubricating properties as well. Several comproils are used depending upon the spring characteristics desired.

"Hydra Springs" have been found to be highly reliable in die stamping applications and consistently outlast coil springs used in similar applications. They are also considerably smaller than coil springs of equivalent rating.

At Cornell Aeronautical Laboratory's request, the Wales-Strippit Corp. prepared a design proposed for a "Hydra Spring" which would be the equivalent of one of the spring cages which had been designed. This "Hydra Spring" was to incorporate means for independently varying chamber pressure and volume, so that static load and spring rate could be adjusted independently.

The design proposal is pictured in figure 10. The device would consist of three sections, the main cylinder, a cylinder and piston arrangement for varying volume and a reservoir to store excess fluid.

From the curve of figure 11 it can be seen that the force is a function of the piston displacement but that the relationship is nonlinear throughout the entire range.

Adjustment of spring rate would be accomplished by first opening the valve from the reservoir to the main cylinder. The adjusting screws located on top of the reservoir and variable volume cylinder would then be turned in and out or vice-versa respectively, the same number of turns. This operation controls the amount of fluid in the working volume. The spring rate is controlled by the number of screw turns from a reference, and the pressure gauge reading. The spring rate would be determined by an initial calibration and plotted on a curve or chart in terms of these variables, as in figure 11.

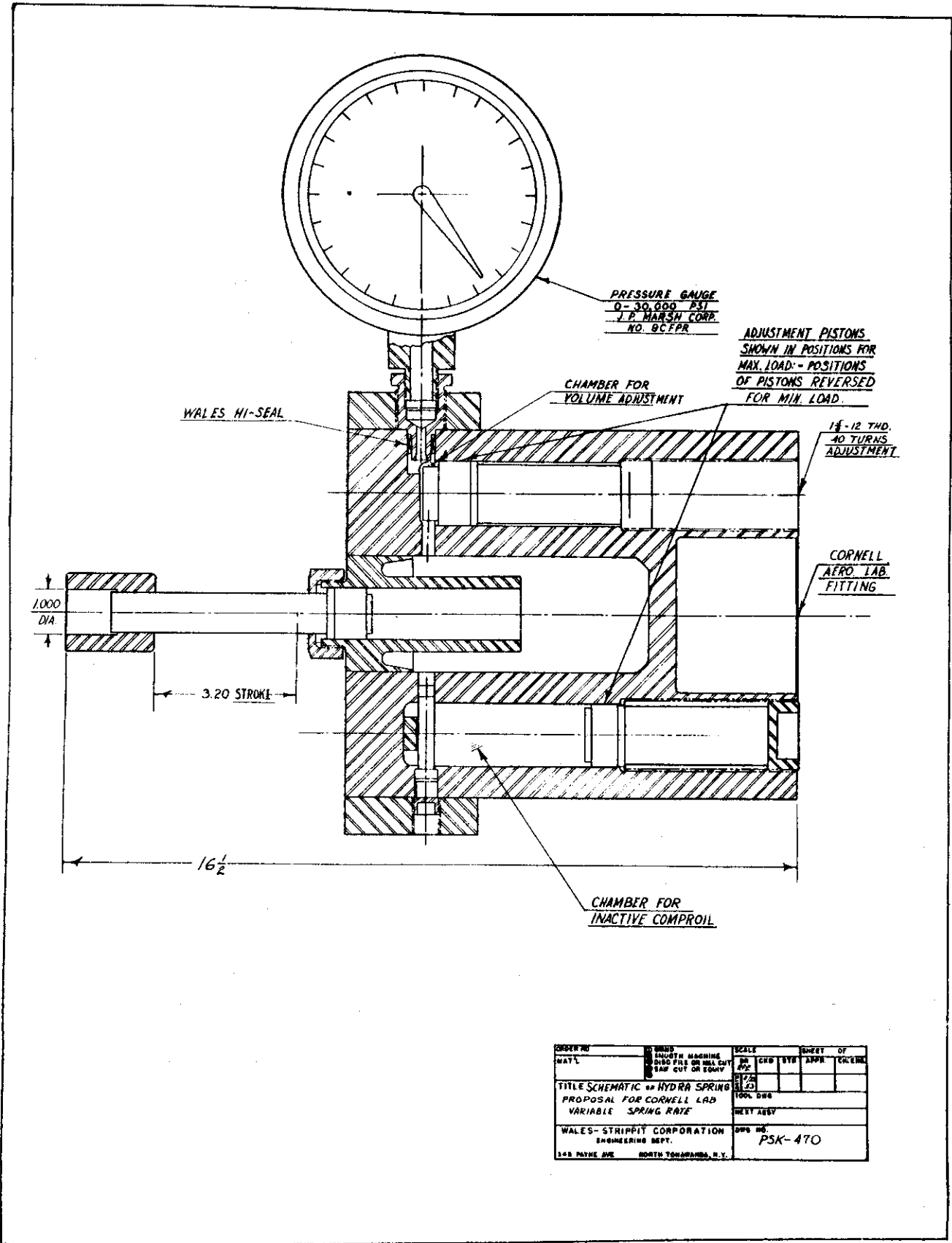


Figure 10. Schematic of "Hydra Spring" Proposal for Variable Spring Rate

# *Contrails* LOADS

PRELOAD 500 LB. CONST.

STROKE 3.20 IN. CONST.

NO. TURNS		LOAD IN LB.	SPRING RATE LB/IN
NO.	ADJUSTER		
1	0	16,500	5000
2	4	16,980	5150
3	8	17,460	5300
4	12	17,940	5450
5	16	18,420	5600
6	20	18,900	5750
7	24	19,380	5900
8	28	19,860	6050
9	32	20,340	6200
10	36	20,820	6350
11	40	21,300	6500

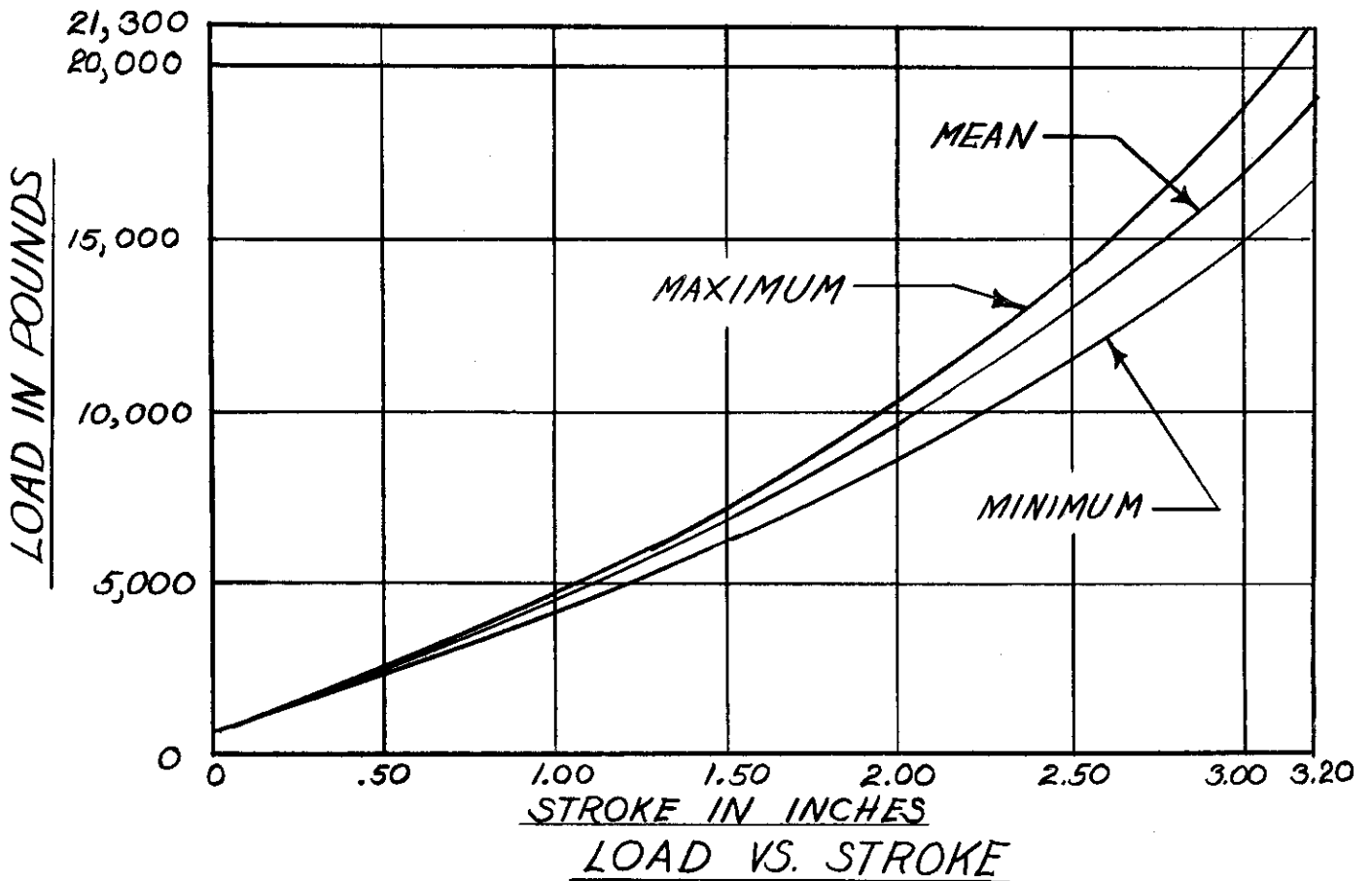


FIG. 11 HYDRASPRING - CALCULATED CALIBRATION.

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The static load rating may be adjusted by varying the chamber pressure independent of the volume. This would be accomplished by turning the screw on the working volume chamber until the required pressure was indicated on the gauge. This adjustment will make a small change in spring rate which would in turn require some minor adjustments of volume.

This basic idea of a hydraulic spring has several attractive features. Due to the nonlinearity of the calibration curve however, it would not be satisfactory in its present form. It was thought that perhaps some method might be found to linearize the performance with mechanical linkages or by using some mechanism within the cylinder itself. None of these modifications appeared to be simple and it was concluded that further research on the "Hydra Spring" would be required to develop it sufficiently for use with the spring-oscillation method. It should be pointed out that a linear force-deflection characteristic is not essential to the success of the "Hydra Spring" in its present industrial application. Thus such a development should not be expected as a result of research by the manufacturer. It is recommended that a special research program be initiated to further investigate the potentialities of the "Hydra Spring".



# Conrails

## CONCLUSIONS

The equipment designed and constructed during this program provides a practical and simple means of measuring the moments of inertia and product of inertia of aircraft.

The design and the equipment items furnished reflect many compromises in terms of performance requirements and the stipulated program scope. The equipment has been "standardized" as much as possible and should be usable with a wide variety of airplanes with gross weights between 10,000 and 120,000 lbs. The ability may be increased greatly by the design and use of suitable supports and fixtures to provide extra jacking points on some airplanes. Fixtures of this nature have not been furnished on test equipment since they are generally usable with one particular airplane only and cannot therefore be regarded as standard test equipment.

Test results of satisfactory accuracy should be obtained when the equipment is used and with reasonable care, diligence and proper attention to the precision of experimental measurements.

Safety precautions comparable to or better than those usually taken when jacking aircraft should be taken. The airplane has two instead of the usual three points of support capable of providing lateral restraint, since the springs provide vertical support only. Especially critical are situations wherein the airplane is jacked in unusual attitudes, as with skewed axes.

The overall success of the spring oscillation method using this equipment depends primarily upon the engineer conducting the tests. The equipment has been standardized for general applicability, and the test procedures outlined in the handbook have necessarily been generalized. Each specific airplane to be tested must be studied to ascertain the extent of applicability and to choose test configurations and procedures which provide maximum accuracy of results and safety of operation.

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2. Woodard, C., Handbook of Operating Instruction for Moment of Inertia Testing Equipment. Cornell Aeronautical Laboratory Report TB-822-F-2
3. Pauly, U. J.; Meyer, R. J. and Infanti, N. L., The Determination of the Moment of Inertia About the Lateral Axis of a B-25J Airplane Cornell Aeronautical Laboratory Report TB-405-F-9. 17 February 1948.
4. Turner, Howard L., Measurement of the Moments of Inertia of an Airplane by a Simplified Method. NACA TN 2201 1950.
5. Boucher, R. W.; Rich, D. A.; Crane, H. L. and Matheny, C. E., A Method for Measuring the Products of Inertia and the Inclination of the Principal Longitudinal Axes of Inertia of an Airplane. NACA TN 3084 April 1954.

EQUIPMENT DESIGN CRITERIA

AIRPLANE	PITCHING MOMENT OF INERTIA DETERMINATION			ROLLING MOMENT OF INERTIA DETERMINATION			CALCULATED MOMENTS OF INERTIA		
	Spring Rate	Force (Pounds)	Spring Defl. (In.)	Spring Rate (Lb/In)	Force (Pounds)	Spring Defl. (In.)	Roll, I <sub>XX</sub>	Pitch, I <sub>YY</sub>	Yaw, I <sub>ZZ</sub>
	(Lb/In)	(Pounds)	(In.)	(Lb/In)	(Pounds)	(In.)	Slug-Ft <sup>2</sup>	Slug-Ft <sup>2</sup>	Slug-Ft <sup>2</sup>
F-80C	620	1,255	2.01	600	900	1.5	11,070	13,880	24,420
F-84	400	775	1.95	700	1870	2.75	9,610	5,940	11,200
F-86	786	1,790	2.28	275	412	1.5	8,200	19,400	26,000
F-89	3400	8,550	2.52	615	615	1	334,700	70,022	399,400
F-94B	555	1,032	1.86	465	930	1	51,425	27,155	77,620
F-100A	2185	4,890	2.24				22,085	64,226	81,456
B-25	2475	3,675	1.48	955	1432	1.5	55,000	68,600	105,000
B-45	686	296	1.32	660	660	1	586,325	315,918	886,875
B-47B	3400	15,750	4.7	700	700	1	1,130,000	1,109,000	2,240,000
B-50	4440	7,320	1.65	1770	3500	2	1,750,000	1,030,000	2,780,000
RB-66	1425	3,780	2.65	6175	6175	1	289,000	407,000	682,600
C-45	238	252	1.06	1122	1122	1	8,284	8,773	16,466
C-54	2000	5,400	2.7	4500	4500	1	526,400	366,900	882,000
C-82	3840	8,750	2.28	3225	3225	1	253,388	187,654	389,984
KC-97E	1165	3,150	2.7	4575	4575	1	1,103,900	1,398,000	2,502,000
C-119B	3440	12,500	1.62	2925	2925	1	490,000	304,000	232,000
C-123	900	2,230	1.27	760	760	1	239,200	192,900	404,400
T-34	918	918	1	755	755	1	1,155	1,934	2,800
T-36	1860	1,890	1.012	385	385	1	89,000	151,000	70,500
T-37	1335	1,990	1.49	397	397	1	7,206	2,372	9,356

TABLE 1. Required Spring Rates for Rolling and Pitching Moment of Inertia, Spring Force, Spring Deflection and Manufacturer Calculated Moments of Inertia.

	WING		FUSELAGE		NOSE		LANDING GEAR		C.G. Sta. Location (In.)	Weight (Lb)
	Clearance (Ft)	Load (Lb)	Clearance (Ft)	Load (Lb)	Clearance (Ft)	Load (Lb)	Main Load (Lb)	Auxiliary Load (Lb)		
F-80C	2.5	4,700	2.7	9,240	3	1,350			205.8	15,243
F-84		5,680	3.51	7,100	2.7	5,250			190.13	17,970
F-86	2.67	10,300			2.4	8,200		1,460	185.19	14,500
F-89		69,300				38,400		29,700	270.4	44,574
F-94B		13,200				3,900			234.3	17,020
F-100A		23,983		19,693		17,736		3,949	310.8	28,561
B-25	7.5	25,000			5	2,500				26,660
B-45		82,000				5,580		12,400	336.3	82,600
B-47B	11.25	66,900	11.15	18,900	5.16	14,700		70,500*	615.7	100,000
B-50	5.6	67,000	5.7	3,000	3.9	20,180		28,400	430.9	120,000
RB-66									435.4	83,000
C-45	3.5	21,000	1.08						113	7,850
C-54	7.75	45,987			6.25	13,000		34,500	387.9	61,842
C-82	11.66	67,700			3.1	13,500		45,151	335.6	48,000
KC-97E	6.35	77,253	6.16	20,000	4	19,700		28,400	327.9	130,000
C-119B	11.8	81,000	1.44	60,900	3.01	31,531		59,650	335.2	64,000
C-123	11.4		1.83		1.9	27,600		34,000	323	54,000
T-34	2								87.6	2,900
T-36		28,500	1.25			21,189			241.6	23,458
T-37	2.56								133.2	5,650

\* Fore  
\*\* Aft

TABLE 2. Minimum Jack Point Ground Clearances, Allowable Jack Loads, Airplane Center of Gravity Positions and Gross Weights Used for Estimating Spring Rates of Table 1.

	JACKING LOADS		JACK POINT SUPPORT LOCATIONS			
	Pitching Moment of Inertia	Rolling Moment of Inertia	Pitching Moment of Inertia		Rolling Moment of Inertia	
			Jacks	Springs	Jacks	Springs
	Wing Nose Jack Jacks	Nose Fuselage (Aft)				
F-80C	4,700	1,255	9,415	Nose	Nose & Fuselage*	Wings
F-84	8,210	1,550	16,420	Nose**	Fuselage Side*	Wings
F-86	6,350	1,790	14,500	Nose	Nose & Fuselage*	Wings (Outer)
F-89	18,000	8,550	36,000	Nose	Nose & Fuselage*	Wings (Outer)
F-94B	6,784	1,032	13,568	Nose	Nose & Fuselage*	Wings
F-100A	11,885	4,890	19,511	Nose	Fuselage	Wings
B-25	11,490	3,675	23,000	Nose	Nose & Fuselage*	Wings (Inner)
B-45	41,300	1,000	81,600	Nose	Nose & Fuselage*	Wings
B-47B	42,200	15,750	84,250	Wings (Inner)	Landing Gears	Wings (Inner)
B-50	57,500	15,000	100,000	Wings (Inner)	Nose & Fuselage*	Wings (Inner)
RB-66	39,610	3,780	79,000	Wings	Fuselage*	Wings
C-45	3,799	252 (Tail)		Tail	Fuselage*	Wings
C-54	28,970	5,000		Nose	Nose & Fuselage*	Wings (Inner)
C-82	19,675	8,750		Nose	Fuselage*	Wings
KC-97E	55,360	19,280	110,720	Nose	Nose & Fuselage*	Wings (Inner)
C-119B	25,750	12,500	51,500	Nose	Nose & Fuselage*	Wings
C-123	25,900	2,230		Nose**	Fuselage*	Wings
T-34				Tail	Nose & Tail	Wings
T-36	10,800	1,890	21,600	Nose	Fuselage*	Wings
T-37	1,830	1,990		Nose	Fuselage	Wings

\* Jack points assumed for calculations would have to be furnished

\*\* Two spring cages required

TABLE 3. Calculated Jack Loads and Jack Point Locations Used for Estimating the Required Spring Rates for Table 1.

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## APPENDIX II

### SPRING CALIBRATION DATA

Number of Springs	Number of Springs	Number of Springs	Rate of Combination	Working Load	Airplane		Cage Size
					Pitch	Roll	
134 Lb/In Rate	343 Lb/In Rate	500 Lb/In Rate	Lb/In	Pounds			
3			402	1,368	F-84*	T-36, F-86	Small Spring Cage
4			536	1,820	F-94B	F-94	"
5			670	2,280	F-80	F-80, F-84, F-89	"
6			804	2,730	F-86, B-45	C-123, T-34	"
7			938	3,185		B-25	"
1	2		820	2,785	B-45	B-47, F-89	"
2	2		945	3,240	C-123*, T-34		"
	3		1029	3,500		B-25	"
1	3		1163	3,960	KC-97E	C-45	"
2	3		1297	4,410			"
	4		1362	4,640	T-37		"
1	4		1496	5,080	RB-66,		"
2	4		1630	5,550			"
	5		1715	5,840		B-50	"
1	5		1849	6,280	T-36		"
2	5		1983	6,750			"
	6		2058	6,990	C-54		"
1	6		2192	7,450	F-100		"
2	6		2326	7,910			"
	7		2401	8,160	B-25		"
1	7		2535	8,610			"
2	7		2669	9,060			"
3	7		2803	9,540			"
4	7		2937	9,960		C-119	"
5	7		3071	10,420			"
6	7		3205	10,900			"
7	7		3339	11,510			Large Spring Cage
		5	2500	8,040			"
1		5	2778	8,890			"
2		5	2912	9,310			"
	1	5	2853	9,120			"
1	1	5	2987	9,550	F-89		"
2	1	5	3121	10,000			"
		6	3012	9,640			"

\* Two spring cages required.

TABLE 4. Possible Spring Rates Using Various Combinations of Springs Furnished in the Moments of Inertia Measuring Equipment Kit, Showing Some Airplanes For Which They Could Be Used.  
(The table is continued on the following page)

# Contrails

Number of Springs	Number of Springs	Number of Springs	Rate of Combination	Working Load	Airplane		Cage Size
					Pitch	Roll	
134 Lb/In Rate	343 Lb/In Rate	500 Lb/In Rate	Lb/In	Pounds			
1		6	3146	10,080			Large Spring Cage
2		6	3280	10,500		C-82	
	1	6	3355	10,720	F-89 B-47, C-119		"
1	1	6	3489	11,180			"
2	1	6	3623	11,600			"
		7	3514	11,240			"
1		7	3648	11,680			"
2		7	3782	12,100			"
	1	7	3857	12,320	C-82		"
1	1	7	3991	12,780			"
2	1	7	4125	13,200			"
	2	7	4200	13,440			"
1	2	7	4334	13,880	B-50		"
2	2	7	4468	14,300			"
	3	7	4543	14,520			C-54, C-97
1	3	7	4677	14,980			"
2	3	7	4811	15,400			"
	4	7	4886	15,620			"
1	4	7	5020	16,080			"
2	4	7	5154	16,500			"
	5	7	5229	16,720			"
1	5	7	5363	17,190			"
2	5	7	5497	17,600		"	
	6	7	5572	17,820		"	
1	6	7	5706	18,260		"	
2	6	7	5840	18,700		"	
	7	7	5915	18,900		"	
1	7	7	6049	19,310			"
2	7	7	6183	19,800		RB-66	"
3	7	7	6317	20,200			"
4	7	7	6451	20,650			"
5	7	7	6585	21,050			"
6	7	7	6719	21,500			"
7	7	7	6853	21,950			"

TABLE 4. (cont'd)

# Contrails

SMALL SPRINGS		MEDIUM SPRINGS		LARGE SPRINGS	
Spring No.	Spring Rate Lb/In	Spring No.	Spring Rate Lb/In	Spring No.	Spring Rate Lb/In
1	134.1	17	348	35	508
2	135	18	344	36	507
3	133.5	19	337	37	497
4	133.5	20	342	38	500
5	134	21	347	39	508
6	133.5	22	340	40	508
7	134	23	346	42	507
8	135.9	25	345	43	496
9	133.1	26	346	44	491
10	133.5	28	348	45	502
11	134.2	29	344	46	507
12	136	30	343	47	491
13	135	31	340	48	483
14	134.7	32	341	49	507
15	134.7	33	340	50	494
16	134.2	34	346	51	500
mean value 134 Lb/In		mean value 343 Lb/In		mean value 502 Lb/In	

TABLE 5. Spring Calibration



EXHIBIT B, CONTRACT AF 33(616)-182

Design and Construction of Equipment for Measuring  
the Moments of Inertia of Aircraft

1. Introduction

1.1 The Cornell Aeronautical Laboratory is, at the present time, conducting a research study for the U. S. Air Force to evaluate various methods for experimentally measuring the moments and products of inertia of aircraft and to assess the effect of errors in these measurements on the aircraft's computed response characteristics.

1.2 The study has thus far shown the following:

1.2.1 Rolling moment of inertia may be determined when it is convenient to place springs at either wing or landing gear jack points and support the aircraft at two other points. Possible combinations would include: Springs under the wing jack points, with the aircraft restrained at nose and tail jack points such that the axis through the nose and tail support points is parallel to the OX axis of the aircraft; Springs under one main landing gear jack point with restraint at the other main gear and the nose gear, provided that these three points lie in a plane parallel to the OX axis. This is a skewed axis arrangement and requires that the pitching inertia be known independently. If the plane of the support points is not parallel to the OX axis the yawing inertia and the product of inertia become involved. If the inclination of test axis with respect to the principal axis is not large, the error in the moments of inertia will be small. In the few instances where the aircraft is fitted with a pair of jack points on each side, oriented in a fore and aft direction, one pair may be used for support and the springs placed on the opposite side to permit a rolling oscillation.

1.2.2 The spring oscillation method is well adapted to the measurement of pitching moment of inertia, wherein the airplane is oscillated about a lateral axis passing through a pair of wing jack points, with the spring located at a nose or tail jack point.

1.2.3 The yawing moment of inertia is not directly measurable by these methods, but may in some cases be determined by oscillating the aircraft about skewed axes, i.e. axes which are not parallel to a principal axis and which in general do not lie in a principal plane. This, however, requires the solution of simultaneous equations involving the three moments of inertia and the product of inertia and is, therefore, inherently less accurate than a direct measurement and applicable to few airplanes.

1.2.4 The product of inertia is not directly measurable by the spring oscillation method, except where skewed axes are used. This, however, requires the solution of simultaneous equations as mentioned above for the yawing moment of inertia.

# Contrails

1.2.5 The aircraft must be oscillated at frequencies below that of any airframe elastic mode, otherwise the airframe will exhibit secondary oscillations and the rigid body inertia will be difficult to obtain. These airframe frequencies generally decrease as the size of the aircraft increases so that the very largest aircraft must be oscillated at very low frequencies (approx. 0.2 cps) which would require relatively flexible springs, i.e. spring constants of the order 100-200 lbs/inch. Since each support point would carry upwards of 50,000 lbs., static deflections would be of the order 40 ft., an obviously impractical situation in terms of the proposed test method. Therefore, an upper limit on frequency and, hence, an upper limit on aircraft weight and/or flexibility seems necessary in utilizing this type of equipment, unless relatively complex and expensive components are employed, i.e. some form of pneumatic spring etc., or unless the airplane can be balanced with light static loads on the spring support points.

## 2. Program

### 2.1 Design and Construction of Spring Oscillation Test Equipment:

2.1.1 Design and construct testing equipment for measuring the inertia characteristics of aircraft by the spring oscillation method. This would be usable with aircraft in the weight range 10,000 lbs. to 120,000 lbs., provided that all airframe elastic modes have natural frequencies greater than 2 - 3 cycles per second. A suitable means may be found for measuring moments of inertia of such airplanes.

2.1.2 The equipment consists of springs, spring cage units and special jack point adapters, to be used in conjunction with various standard aircraft jacks. The aircraft would pivot about the jack point supports. The springs will provide the restoring forces for the oscillation. The spring cage units will be designed to hold combinations of up to seven springs, in order to provide a variety of spring constants. Some means of packaging the equipment will be constructed.

2.1.3 The equipment will be designed to be usable with the greatest possible number of aircraft in this category. A study will be made of jack point locations, loads, etc, for a variety of different aircraft in this category to insure that a sufficient number and variety of springs, jack fittings, adapters, etc. are provided.

2.1.4 The pitching inertia will be measurable in most cases. Rolling inertia may be measured when suitable jack point locations are available as noted above. The yawing inertia and the product of inertia cannot be determined unless skewed axes are used. The use of skewed axes is limited to those cases where the jack point locations and the allowable loads thereon will provide a suitable number of distinct axis locations about which the aircraft may be oscillated. Generally two to three axes will be required and the solution of simultaneous transcendental equations would be required to ascertain the inertia characteristics, when using skewed axes.

# Contrails

2.2 After completing construction of the equipment, a short test program will be performed to measure the inertias of one typical aircraft in order to evaluate the equipment, the experimental methods developed, and the accuracy of the results.

2.3 A handbook will be prepared describing the equipment, explaining its use in obtaining the data, and giving ample instruction on the method of reducing the data to obtain the moments of inertia and for estimating the accuracy of the results. Sample data sheets and computation forms will be included along with drawings and photographs which will supplement the text in its explanations. Data from the test program mentioned above will be used as a numerical example in the handbook.

2.4 The contractor shall provide for a one man month of consulting time to assist in familiarizing AF personnel with the equipment and procedures.

## List of Design Drawings for Moment of Inertia Test Equipment

	Quantity	Dr. No.	Description
1.	2	822-2-004	B-5 Adapter
2.	2	822-2-005	B-6 Adapter
3.	2	822-2-006	B-3B Adapter
4.	2	822-2-007	B-4 Adapter
5.	6	822-2-010-2	Bolt
6.	6	822-2-011-2	Bolt
7.	2	822-3-003-1	Ball Joint Assembly
8.	2	822-5-002-2	Plate, Upper
9.	2	822-5-003-1	Plate Assembly, Lower
10.	2	822-5-004-2	Plate, Upper
11.	2	822-5-005	Plate Assembly, Lower
12.	6	AN 960-916	Washer
13.	6	AN 960-1016	Washer
14.	6	AN 315-9	Nut
15.	6	AN 315-10	Nut
16.	16	822-2-001	Spring
17.	16	822-2-002	Spring
18.	16	822-2-003	Spring