Analysis of a Five-Layer, Viscoelastic, Constrained-Layer Beam

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ABSTRACT

This paper documents the analysis and application of the five-layer beam damping system which consists of: undamped beam (layer 1), adhesive layer (layer 2), stand-off spacing (layer 3), viscoelastic damping material (layer 4), and constraining layer (layer 5). It includes the derivation of equations and development of the general-purpose computer program to compute and provide graphical plots of modal loss factor, modal frequency, RMS response and peak resonance, each as a function of temperature. Parametric results are presented for variation in thickness of each layer (except the undamped beam layer). This technology can be used by designers as a means of estimating damping in beam-like structures with viscoelastic constrained-layer damping.

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1.0 INTRODUCTION

A traditional design approach to reduce resonant vibration in aircraft and spacecraft structures is to either stiffen the structure by increasing the thickness of members or add mass - both approaches typically result in an increase in vehicle weight. These approaches are not viable in today's design environment because demanding mission requirements dictate a need for lighter-weight structures. Two alternatives are to either properly integrate damping technology into the structure during design and manufacturing, or if necessary use add-on damping treatments to reduce resonant response and increase structural fatigue life without significant weight increases. In either case, simplified analytical equations are prerequisite to good structural designs.

A practical stand-off damping treatment for a beam structural element, which is one variation of constrained-layer damping [1 - 4], has been recently devised [5 - 8]. This treatment consists of a stand-off layer, the viscoelastic material layer, and a constraining layer attached to the base beam structure(Figure 1). The damping system considered in this report consist of the same treatment with another layer - an adhesive layer added between the base beam and the stand-off layer (Figure 2). The purpose of this report is to document the analysis and application of this five-layer-beam damping system.

The report includes the derivation of equations and a description of a general purpose computer program V5LBD (viscoelastic five-layer beam damping) to compute results and provide parametric analysis. V5LBD computes and provides graphical plots of modal loss factor, modal frequency, RMS response, and peak resonance, each as a function of temperature over the range of -50 to 250^0 F [9].

Sections 2.0 and 3.0 of this report present theory and derivation of the governing equations. Section 4.0 discusses the development of V5LBD which calculates results from the governing equations. Graphics capabilities and carpet plotting programs are presented in section 5.0. Parametric analysis is provided in section 6.0, and conclusions in section 7.0.

This technology* can be used by designers to predict damping characteristics of structures so that viscoelastic constrained-layer damping concepts can be effectively used. It can also be used by engineers to design add-on treatments to dampen structures that might experience vibration problems in service.

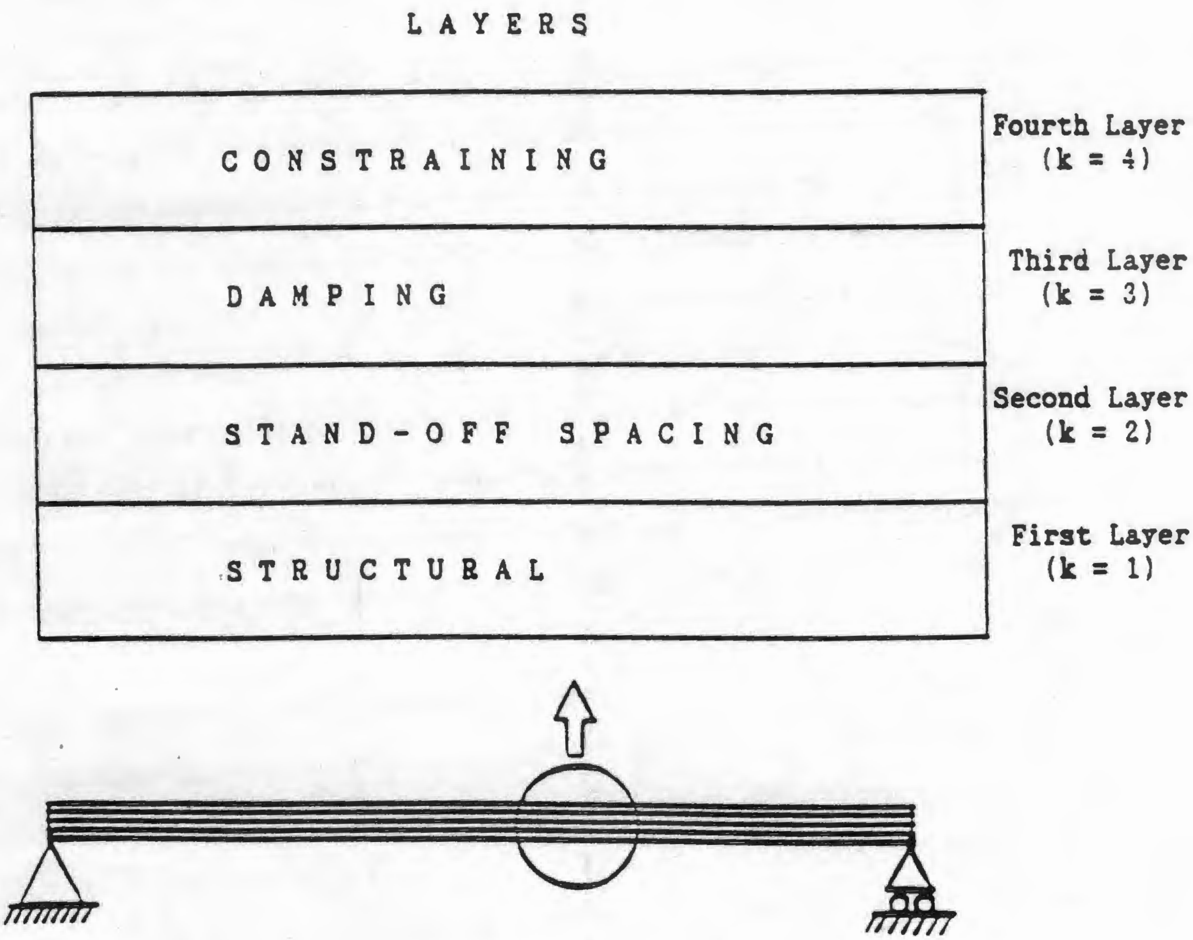

FIGURE 1 - SCHEMATIC OF FOUR-LAYER VISCOELASTIC CONSTRAINED LAYER DAMPING SYSTEM

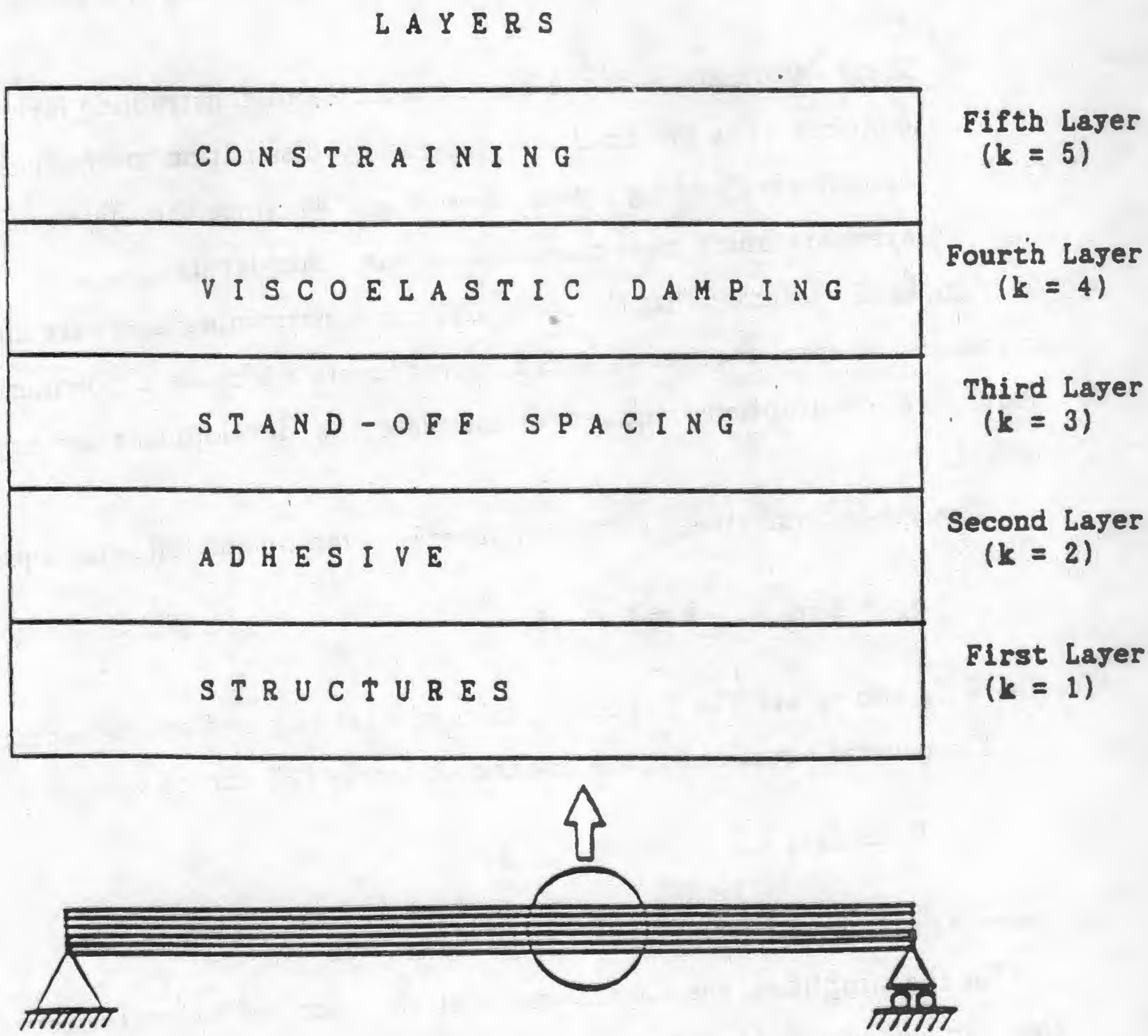

FIGURE 2 - SCHEMATIC OF FIVE-LAYER VISCOELASTIC CONSTRAINED LAYER DAMPING SYSTEM

2.0 DERIVATION OF GOVERNING EQUATIONS

The five-layer simply-supported beam model is depicted in Figure 2. The principal assumptions used to derive the governing equations are:

1. Only bending and shear deformations are considered. In-plane extensional strains are assumed to be small and negligible.
2. Bending deformations are governed by classical Euler-Bernoulli beam theory [10].
3. Shear deformations of the base structure and constraining layer (Figure 3) are identical. The principal shearing-energy dissipation mechanism occurs in the viscoelastic damping layers (layers 2 and 4) since the shear stiffnesses of these layers are much lower than those of the other layers.

It is implicitly assumed that the structural and constraining layers are made of metallic materials, whereas the spacing and damping layers are made of polymeric compounds. Additional assumptions required to complete the development are presented as required.

The extensional stress (σ) in each layer is given by the following equation:

$$\sigma_k = E_k \epsilon_k, \qquad k = 1, \ldots, 5 \tag{1}$$

where E_k and ϵ_k are The Youngs modulus of elasticity and strain respectively.

The general equation for the extensional forces (F) can be written in the form:

$$F_k = E_k \epsilon_k A_k, \qquad k = 1, \ldots, 5 \tag{2}$$

where A_k is the cross sectional area in kth layer.

For the simplified, one-dimensional analysis described below the sectional properties, namely, centroid location, Z_d, and flexural rigidity, $\overline{EI}$, are needed. As shown in Figure 3, Z_d is defined to be the distance from the mid-plane of the structural layer to the sectional centroid. The strain-displacement relations for the layers are:

$$\epsilon_1 = Z_d \phi' \tag{3}$$

$$\epsilon_2 = (H_{21} - Z_d)\phi' - \frac{H_2}{2}\psi_1' \tag{4}$$

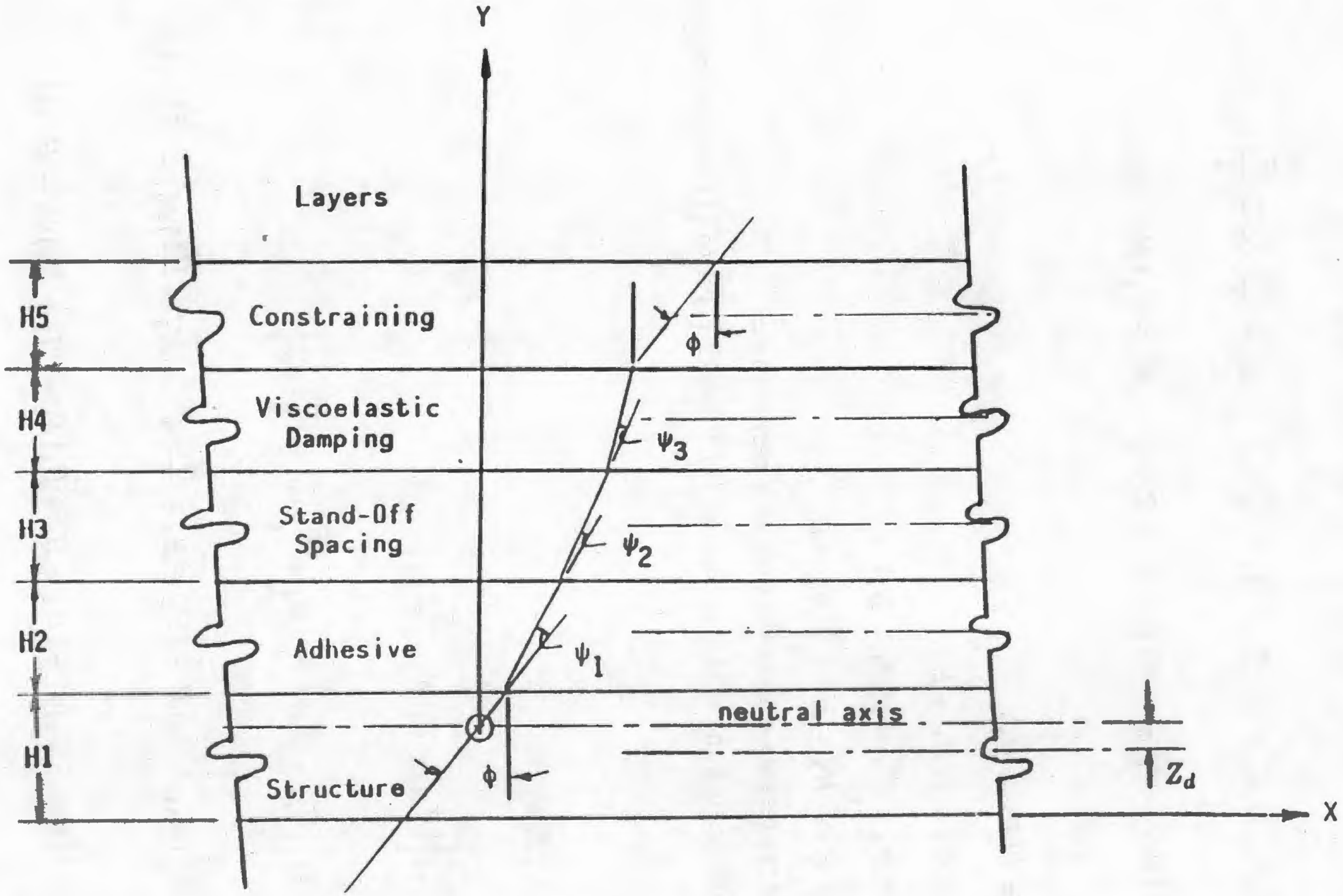

Figure 3 - Schematic of Five-Layer Viscoelastic Constrained Layer Damping System Due to Bending

$$\epsilon_3 = (H_{31} - Z_d)\phi' - (H_2 + \frac{H_3}{2})\psi_1' - \frac{H_3}{2}\psi_2' \tag{5}$$

$$\epsilon_4 = (H_{41} - Z_d)\phi' - (H_2 + H_3 + \frac{H_4}{2})\psi_1' - (H_3 + \frac{H_4}{2})\psi_2' - \frac{H_4}{2}\psi_3' \tag{6}$$

$$\epsilon_5 = (H_{51} - Z_d)\phi' - (H_2 + H_3 + H_4)\psi_1' - (H_3 + H_4)\psi_2' - H_4\psi_3' \tag{7}$$

where

$$H_{21} = \tfrac{1}{2}(H_1 + H_2)$$

$$H_{31} = H_2 + \tfrac{1}{2}(H_1 + H_3)$$

$$H_{41} = H_2 + H_3 + \tfrac{1}{2}(H_1 + H_4)$$

$$H_{51} = H_2 + H_3 + H_4 + \tfrac{1}{2}(H_1 + H_5)$$

The prime represents differentiation with respect to x.

By substituting the strain equations (3) through (7) into (2) we get the force equations:

$$F_1 = X_1 Z_d \phi' \tag{8}$$

$$F_2 = X_2\Big[(H_{21} - Z_d)\phi' - \frac{H_2}{2}\psi_1'\Big] \tag{9}$$

$$F_3 = X_3\Big[(H_{31} - Z_d)\phi' - (H_2 + \frac{H_3}{2})\psi_1' - \frac{H_3}{2}\psi_2')\Big] \tag{10}$$

$$F_4 = X_4\Big[(H_{41} - Z_d)\phi' - (H_2 + H_3 + \frac{H_4}{2})\psi_1' - (H_3 + \frac{H_4}{2})\psi_2') - \frac{H_4}{2}\psi_3\Big] \tag{11}$$

$$F_5 = X_5\Big[(H_{51} - Z_d)\phi' - (H_2 + H_3 + H_4)\psi_1' - (H_3 + H_4)\psi_2' - H_4\psi_3'\Big] \tag{12}$$

where

$$X_k = E_k H_k \qquad k = 1, \ldots, 5 \tag{13}$$

Applying the requirement for equilibrium of in-plane forces, i.e.,

$$\sum_{k=1}^{5} F_k = 0 \tag{14}$$

and simplifying produces the equation

$$\begin{aligned}
&- \left[X_1 + X_2 + X_3 + X_4 + X_5\right] Z_d \phi' \\
&+ \left[X_2 H_{21} + X_3 H_{31} + X_4 H_{41} + X_5 H_{51}\right] \phi' \\
&- \left[X_2 \frac{H_2}{2} + X_3 (H_2 + \frac{H_3}{2}) + X_4 (H_2 + H_3 + \frac{H_4}{2}) + X_5 (H_2 + H_3 + H_4)\right] \psi_1' \\
&- \left[X_3 \frac{H_3}{2} + X_4 (H_3 + \frac{H_4}{2}) + X_5 (H_3 + H_4)\right] \psi_2' \\
&- H_4 \left[\frac{X_4}{2} + X_5\right] \psi_3' = 0
\end{aligned} \tag{15}$$

Equation (15) is rearranged to give:

$$\begin{aligned}
&\left[X_2 H_{21} + X_3 H_{31} + X_4 H_{41} + X_5 H_{51}\right] = \\
&\quad + \left[X_1 + X_2 + X_3 + X_4 + X_5\right] Z_d \\
&\quad + \left[X_2 \frac{H_2}{2} + X_3 (H_2 + \frac{H_3}{2}) + X_4 (H_2 + H_3 + \frac{H_4}{2}) + X_5 (H_2 + H_3 + H_4)\right] \frac{\psi_1'}{\phi'} \\
&\quad + \left[X_3 \frac{H_3}{2} + X_4 (H_3 + \frac{H_4}{2}) + X_5 (H_3 + H_4)\right] \frac{\psi_2'}{\phi'} \\
&\quad + H_4 \left[\frac{X_4}{2} + X_5\right] \frac{\psi_3'}{\phi'}
\end{aligned} \tag{16}$$

The equilibrium equations for the shear forces (τ) in the X-direction are:

$$\tau_3 = -G_2 \psi_1 = F_5' + F_4' + F_3' \tag{17}$$

$$\tau_4 = -G_3 \psi_2 = F_5' + F_4' \tag{18}$$

$$\tau_5 = -G_4 \psi_3 = F_5' \tag{19}$$

where G_k is the shear modulus and ψ_k the angle of deformation in the k^{th} layer (see Figure 3). The quantities F_3', F_4' and F_5' are easily obtained by differentiation of equations (10) through (12).

At this point in the derivation it is necessary to assume a sinusoidal mode shape (which satisfies the boundary conditions for a simply-supported beam) in order to determine ψ_1, ψ_2 and ψ_3 consistent with the definition of loss factor presented in [12]. Assuming for simplicity that

$$w = sinKx \tag{20}$$

then

$$\phi = w' = KcosKx \tag{21}$$

$$\phi' = w'' = -K^2 sinKx \tag{22}$$

and

$$\phi'' = w''' = -K^3 cosKx \tag{23}$$

Substituting equation (21) into (23) provides

$$\phi'' = -K^2 \phi \tag{24}$$

Assuming that ψ_1, ψ_2 and ψ_3 have the same distribution as w, ψ_1, ψ_2, and ψ_3 are related to ϕ by

$$\psi_1 = \alpha_1 \phi \tag{25}$$

$$\psi_2 = \alpha_2 \phi \tag{26}$$

and

$$\psi_3 = \alpha_3 \phi \tag{27}$$

where the $\alpha's$ are coefficients of proportionality. Subsequent differentiations of Equations (25) through (27) give

$$\alpha_1 = \frac{\psi_1'}{\phi'} = \frac{\psi_1''}{\phi''} \tag{28}$$

$$\alpha_2 = \frac{\psi_2'}{\phi'} = \frac{\psi_2''}{\phi''} \tag{29}$$

and

$$\alpha_3 = \frac{\psi_3'}{\phi'} = \frac{\psi_3''}{\phi''} \tag{30}$$

Returning to equation (24) and using the relations in equations (25) through (30), equation (17) can be written as

$$\begin{aligned}
-G_2\psi_1 = \frac{G_2\psi_1''}{K^2} &= \Big[X_5(H_{51} - Z_d) + X_4(H_{41} - Z_d) + X_3(H_{31} - Z_d)\Big]\phi'' \\
&- \Big[X_5(H_2 + H_3 + H_4) + X_4(H_2 + H_3 + \frac{H_4}{2}) + X_3(H_2 + \frac{H_3}{2})\Big]\psi_1'' \\
&- \Big[X_5(H_3 + H_4) + X_4(H_3 + \frac{H_4}{2}) + X_3\frac{H_3}{2}\Big]\psi_2'' \\
&- \Big[X_5H_4 + X_4\frac{H_4}{2} + X_3\frac{H_3}{2}\Big]\psi_3''
\end{aligned} \tag{31}$$

Finally, equation (31) can written in the form:

$$\begin{aligned}
&\Big[X_5(H_{51} - Z_d) + X_4(H_{41} - Z_d) + X_3(H_{31} - Z_d)\Big] = \\
&+ \Big[\frac{G_2}{K^2} + X_5(H_2 + H_3 + H_4) + X_4(H_2 + H_3 + \frac{H_4}{2}) + X_3(H_2 + \frac{H_3}{2})\Big]\frac{\psi_1''}{\phi''} \\
&+ \Big[X_5(H_3 + H_4) + X_4(H_3 + \frac{H_4}{2}) + X_3\frac{H_3}{2}\Big]\frac{\psi_2''}{\phi''} \\
&+ \Big[X_5H_4 + X_4\frac{H_4}{2} + X_3\frac{H_3}{2}\Big]\frac{\psi_3''}{\phi''}
\end{aligned} \tag{32}$$

Similarly, equation (18) is written as

$$\begin{aligned}
-G_3\psi_2 = \frac{G_3\psi_2''}{K^2} &= \Big[X_5(H_{51} - Z_d) + X_4(H_{41} - Z_d)\Big]\phi'' \\
&- \Big[X_5(H_2 + H_3 + H_4) + X_4(H_2 + H_3 + \frac{H_4}{2})\Big]\psi_1'' \\
&- \Big[X_5(H_3 + H_4) + X_4(H_3 + \frac{H_4}{2})\Big]\psi_2'' \\
&- \Big[X_5H_4 + X_4\frac{H_4}{2}\Big]\psi_3''
\end{aligned} \tag{33}$$

Rearranging equation (33) gives

$$\begin{aligned}
-G_4\psi_3 = \frac{G_4\psi_3''}{K^2} &= \Big[X_5(H_{51} - Z_d)\Big]\phi'' - \Big[X_5(H_2 + H_3 + H_4)\Big]\psi_1'' \\
&- \Big[X_5(H_3 + H_4)\Big]\psi_2'' - \Big[X_5H_4\Big]\psi_3''
\end{aligned} \tag{34}$$

Also equation (19) can be written as

$$-G_4\psi_3 = \frac{G_4\psi_3''}{K^2} = \Big[X_5(H_{51} - Z_d)\Big]\phi'' - \Big[X_5(H_2 + H_3 + H_4)\Big]\psi_1''$$
$$- \Big[X_5(H_3 + H_4)\Big]\psi_2'' - \Big[X_5 H_4\Big]\psi_3'' \tag{35}$$

Rearranging equation (35) gives

$$\Big[X_5(H_{51} - Z_d)\Big] = \Big[X_5(H_2 + H_3 + H_4)\Big]\frac{\psi_1''}{\phi''}$$
$$+ \Big[X_5(H_3 + H_4)\Big]\frac{\psi_2''}{\phi''} + \Big[\frac{G_4}{K^2} + X_5 H_4\Big]\frac{\psi_3''}{\phi''} \tag{36}$$

Thus equations (17), (18), and (19) are replaced by equations (32), (34), and (36) respectively.

Since the assumption that plane sections remain plane is being used, the bending moment M can be related to deflection by

$$\overline{EI}\phi' = M \tag{37}$$

where $\overline{EI}$ is the flexural rigidity. The total bending moment is expressed by

$$M = \sum_{k=1}^{5} M_{kk} + \sum_{k=1}^{5} F_k(H_{k1} - Z_d) \tag{38}$$

where M_{kk} is the bending moment of the k^{th} layer given by

$$M_{kk} = \phi' E_k I_k \tag{39}$$

Equations (37) and (38) define the flexural rigidity as follows:

$$\overline{EI} = \Big[E_1 I_1 + E_2 I_2 + E_3 I_3 + E_4 I_4 + E_5 I_5$$
$$+ X_1 Z_d{}^2 + X_2(H_{21} - Z_d)^2 + X_3(H_{31} - Z_d)^2 + X_4(H_{41} - Z_d)^2 + X_5(H_{51} - Z_d)^2\Big]$$
$$- \Big[E_2 I_2 + E_3 I_3 + E_4 I_4 + \frac{X_2 H_2}{2}(H_{21} - Z_d) + X_3(H_{31} - Z_d)(H_2 + \frac{H_3}{2})$$
$$+ X_4(H_{41} - Z_d)(H_2 + H_3 + \frac{H_4}{2}) + X_5(H_{51} - Z_d)(H_2 + H_3 + H_4)\Big]\frac{\psi_1'}{\phi'}$$
$$- \Big[E_3 I_3 + E_4 I_4 + X_3(H_{31} - Z_d)\frac{H_3}{2} + X_4(H_{41} - Z_d)(H_3 + \frac{H_4}{2}) + X_5(H_{51} - Z_d)(H_3 + H_4)\Big]\frac{\psi_2'}{\phi'}$$
$$- \Big[X_4(H_{41} - Z_d)\frac{H_4}{2} + X_5(H_{51} - Z_d)H_4 + E_4 I_4\Big]\frac{\psi_3'}{\phi'} \tag{40}$$

To compute the loss factor η ($\eta = 2\xi$, the fraction of critical damping) for the five-layer system, it is necessary to replace the elastic (E_2, E_4) and shear moduli (G_2, G_4) of the viscoelastic layers by the following complex moduli:

$$E_2^* = E_2(1 + i\eta_2) \tag{41}$$

$$G_2^* = G_2(1 + i\eta_2) \tag{42}$$

and

$$E_4^* = E_4(1 + i\eta_4) \tag{43}$$

$$G_4^* = G_4(1 + i\eta_4) \tag{44}$$

where $i = \sqrt{-1}$.

The complex flexural rigidity $\overline{EI}^*$ can be found by substituting equations (28) through (30) into equations (16),(32), (34) and (36) and solving for Z_d, α_1, α_2, and α_3. By substituting these values into equation (40) and replacing the moduli with equations (41) through (44),

$$\overline{EI}^* = EI_{real} + EI_{imag} \tag{45}$$

The intermediate manipulations required to determine EI_{real} and EI_{imag} are straightforward but very lengthy; consequently, they are not included. The system modal loss factor, η_m, is found from

$$\eta_m = \frac{EI_{imag}}{EI_{real}} \tag{46}$$

Using the boundary conditions for a simply-supported beam leads to the equation for the eigenvalues $sinKL = 0$ where $KL = \pi, 2\pi, 3\pi, ...$ and the wave number is $K_N^2 = \frac{(N\pi)^2}{L^2}$.

The modal frequency (f_N) for Nth mode of vibration, is calculated from

$$f_N = \frac{1}{2\pi} \times K_N^2 \sqrt{\frac{EI_{real}g}{\sum_{k=1}^{5} H_k \rho_k}} \tag{47}$$

where:

$H_k = k^{th}$ layer thickness:

$\rho_k = k^{th}$ layer density, mass/unit volume:

N = mode number; and

g = acceleration of gravity.

In practice the complex modulus is evaluated for a given temperature and an estimated modal frequency, f_e. The modal frequency is calculated from f_N and compared to the convergence criteria

$$\left|1 - \frac{f_e}{f_N}\right| \leq \epsilon_{FREQ} = 0.01 \tag{48}$$

If this condition is not met, the new estimated frequency is taken as the old calculated frequency and the process repeated.

First-order verification of the equations derived for the five-layer beam damping system was made by comparing predictions with those for a degenerate case. This comparison was made by setting the four-layer beam system thicknesses for layers three and four equal to zero (stand-off and viscoelastic layers) in the five-layer program and thickness for layer two equal to zero (stand-off layer) in the four-layer program. Predicted outputs for modal loss factor, modal frequency ratio, RMS response and peak resonance were compared and found to be identical (See Figures 4 and 5). This comparison partially validates the five-layer equations, but other extensive comparisons beyond the scope of this study could be made to totally validate them.

3.0 COMPARISON OF RESPONSE

The undamped/damped amplitude ratio for a single-degree-of-freedom system undergoing sinusoidal excitation is

$$\frac{X_{peakundamp}}{X_{peakdamp}} = \frac{m_d}{m_u} \times \left(\frac{f_d}{f_u}\right)^2 \times \frac{\eta_d}{\eta_u} \tag{49}$$

where m is the weight density of the structure. The subscript "u" refers to the undamped base structure and the subscript "d" the response with damping treatment.

The root-mean-square amplitude response W_{rms}, is obtained by using the equation

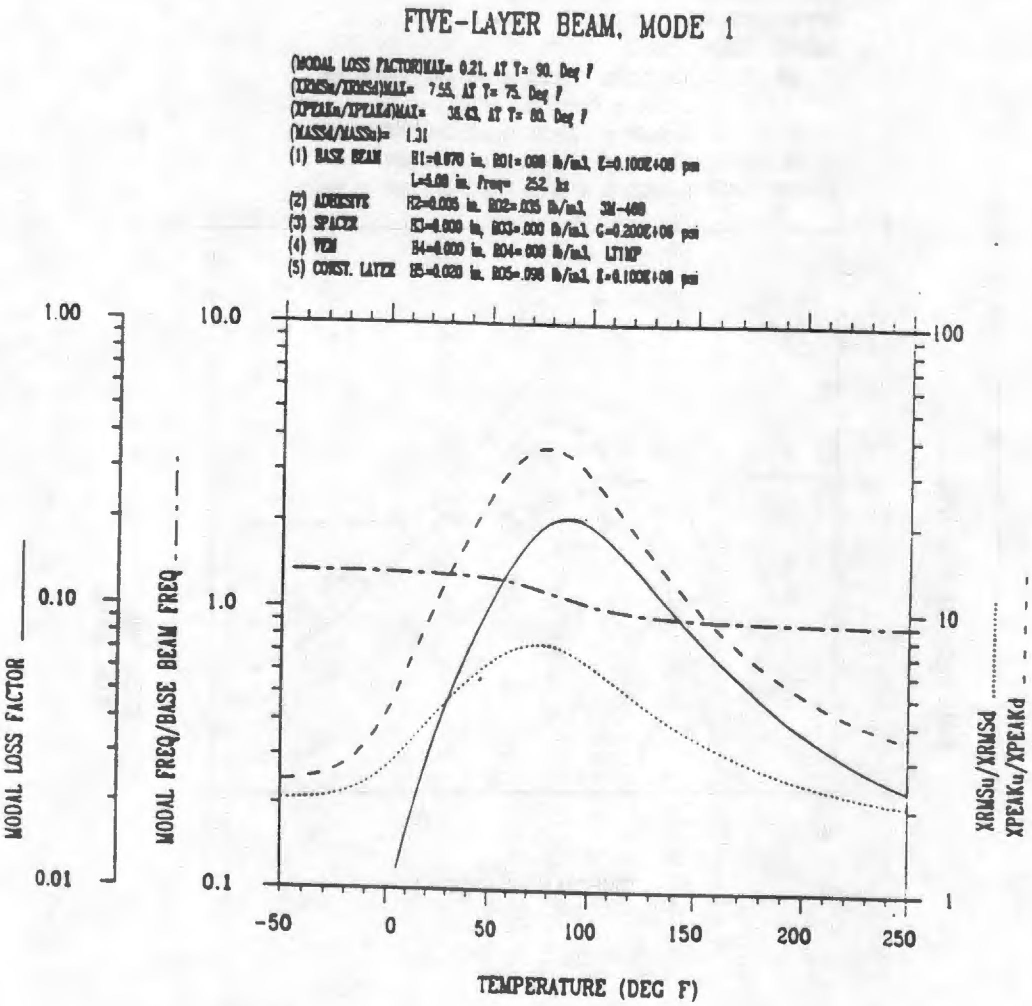

Figure 4 - Output Plot for the Five-Layer Beam Computer Program Where Stand-Off (H3) and the Viscoelastic Layer (H4) are Set at Zero

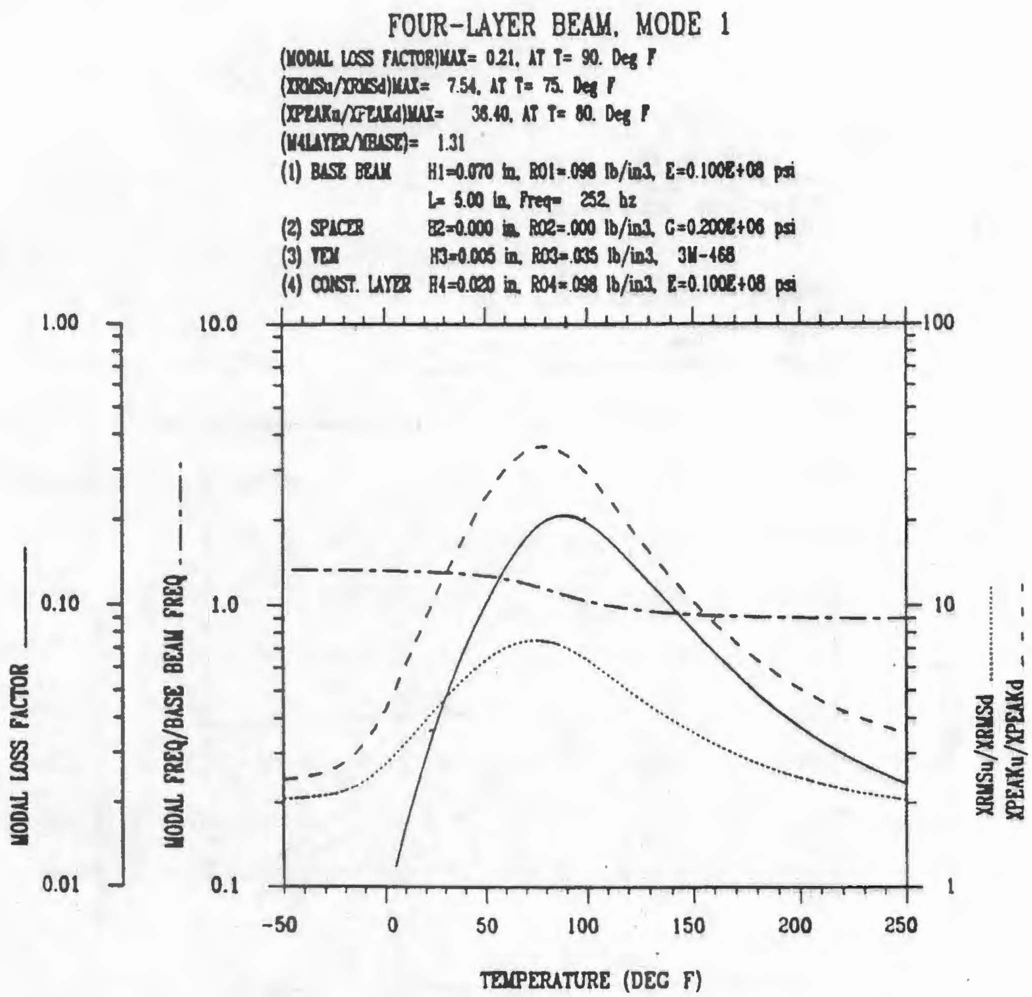

Figure 5 - Output Plot for the Four-Layer Beam Computer Program Where the Stand-Off Layer (H2) is Set at Zero

derived in [13]; which is

$$W_{rms} = \frac{\pi \overline{F}(w)}{2\sqrt{2} m^{1/4} K^{3/4} \eta^{1/2}} \times \left[\frac{1 + \sqrt{1 + \eta^2}}{\sqrt{1 + \eta^2}} \right] \tag{50}$$

Noting that

$$K = mw^2 = m(2\pi f)^2 \tag{51}$$

The undamped/damped response ratio is given by

$$R_{rms} = \frac{(W_{rms}/\overline{F}(w))_u}{(W_{rms}/\overline{F}(w))_d} = \frac{m_d}{m_u} \times \frac{\sqrt{1+\eta_d^2}}{\sqrt{1+\eta_u^2}} \times \frac{\left[1+\sqrt{1+\eta_u^2}\right]}{\left[1+\sqrt{1+\eta_d^2}\right]} \times \sqrt{\frac{\eta_d}{\eta_u}} \times \left(\frac{f_d}{f_u}\right)^{\frac{3}{2}} \tag{52}$$

the response with damping treatment.

In summary the four main governing equations derived are: modal loss factor equation which results by solving equation (45) and calculating the response ratio (46), modal frequency equation (47), peak amplitude equation (49), and RMS response equation (52).

4.0 PROGRAM DEVELOPMENT AND COMPUTATIONS

An overview of the computer program V5LBD computations scheme is presented in the flow diagram shown in Figure 6. The geometry of the base beam, thickness and material properties for each layer, and the viscoelastic damping parameters are input variables. The following quantities are calculated for a specified temperature: frequency estimate (approximate frequency), temperature shift function, reduced frequency, shear modulus, and the material damping of the viscoelastic layer. Next, modal damping and frequency are calculated. The calculated modal frequency is then compared to the estimated frequency using the convergence criterion. If the convergence criterion is not satisfied, the calculation is iterated for an improved value of the frequency estimate. Once convergence is achieved, the RMS and peak response values are calculated.

The five-layer damping system analyzed in this report includes two different viscoelastic materials for one damping application. When using only one viscoelastic material the effective temperature range of that material may be narrower than the re-

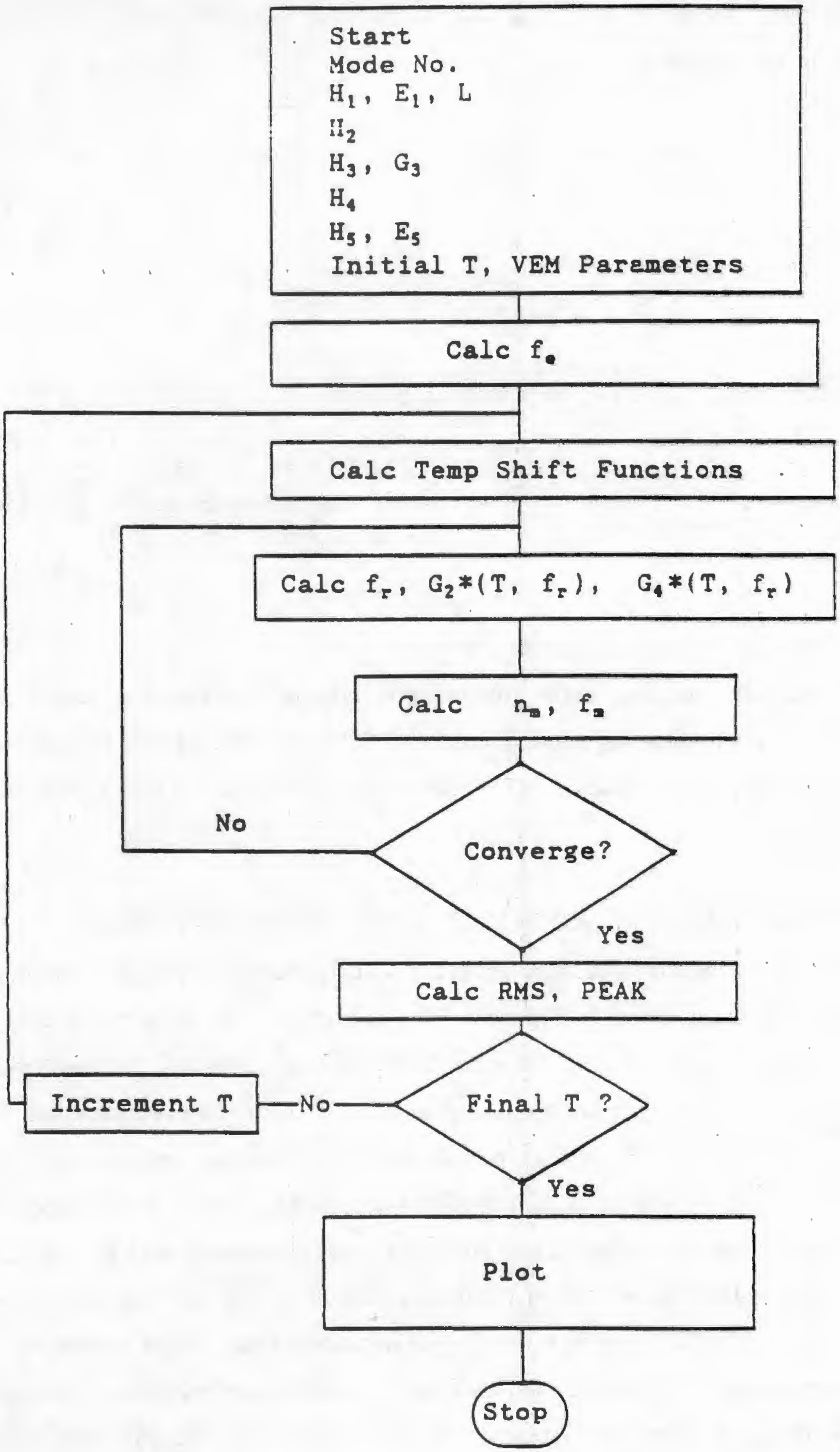

Figure 6 - Flow Diagram for the Five-Layer Model Computer Program

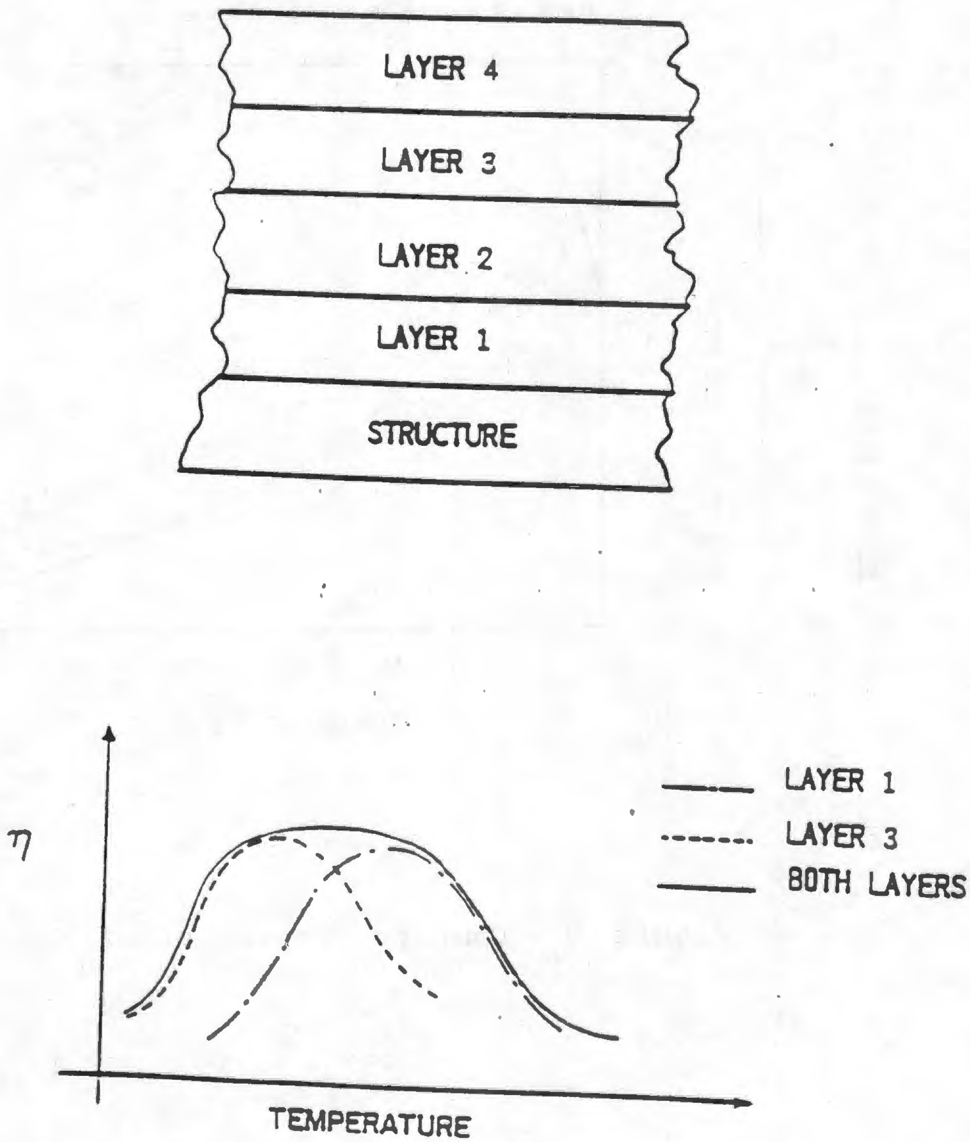

Figure 7 - Multiple Layer Temperature Broadening Effect

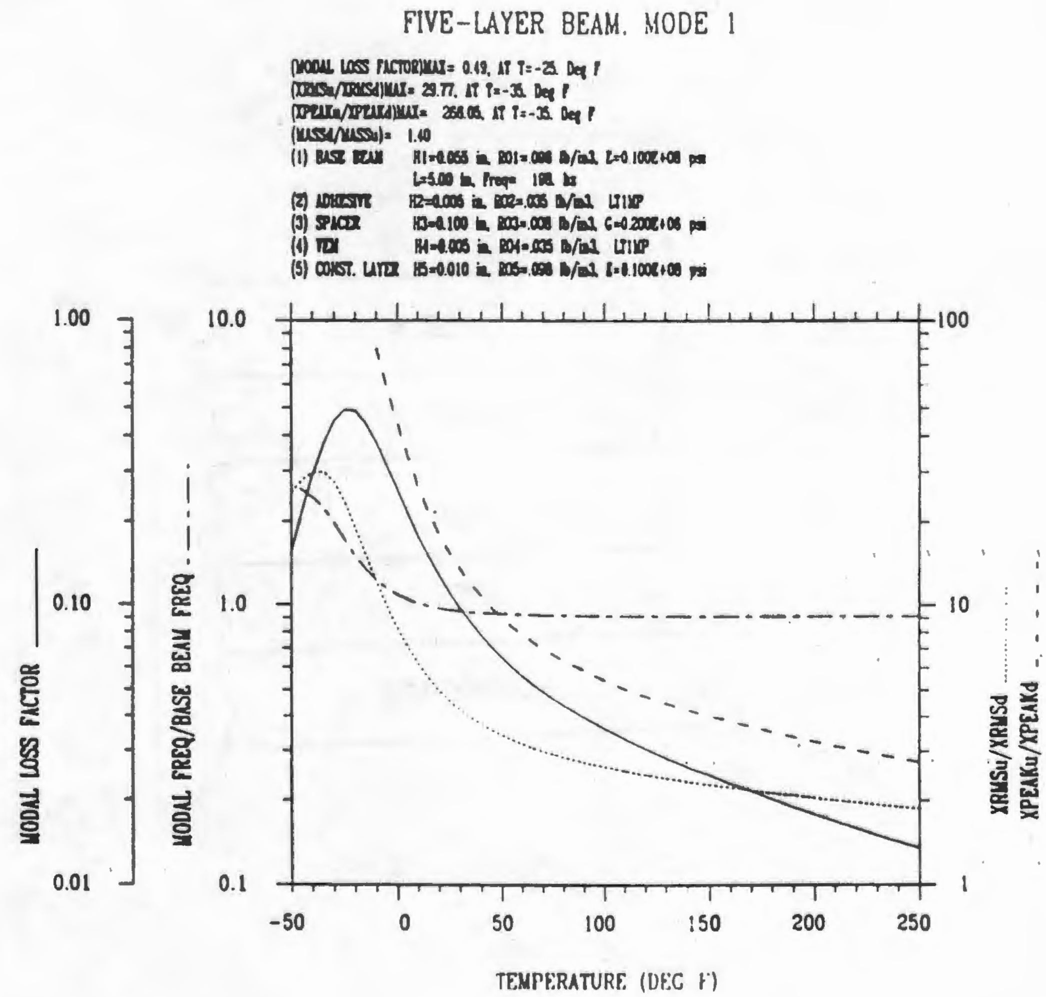

Figure 8 - Damping Properties of Material LT1MP for Layers 2 and 4

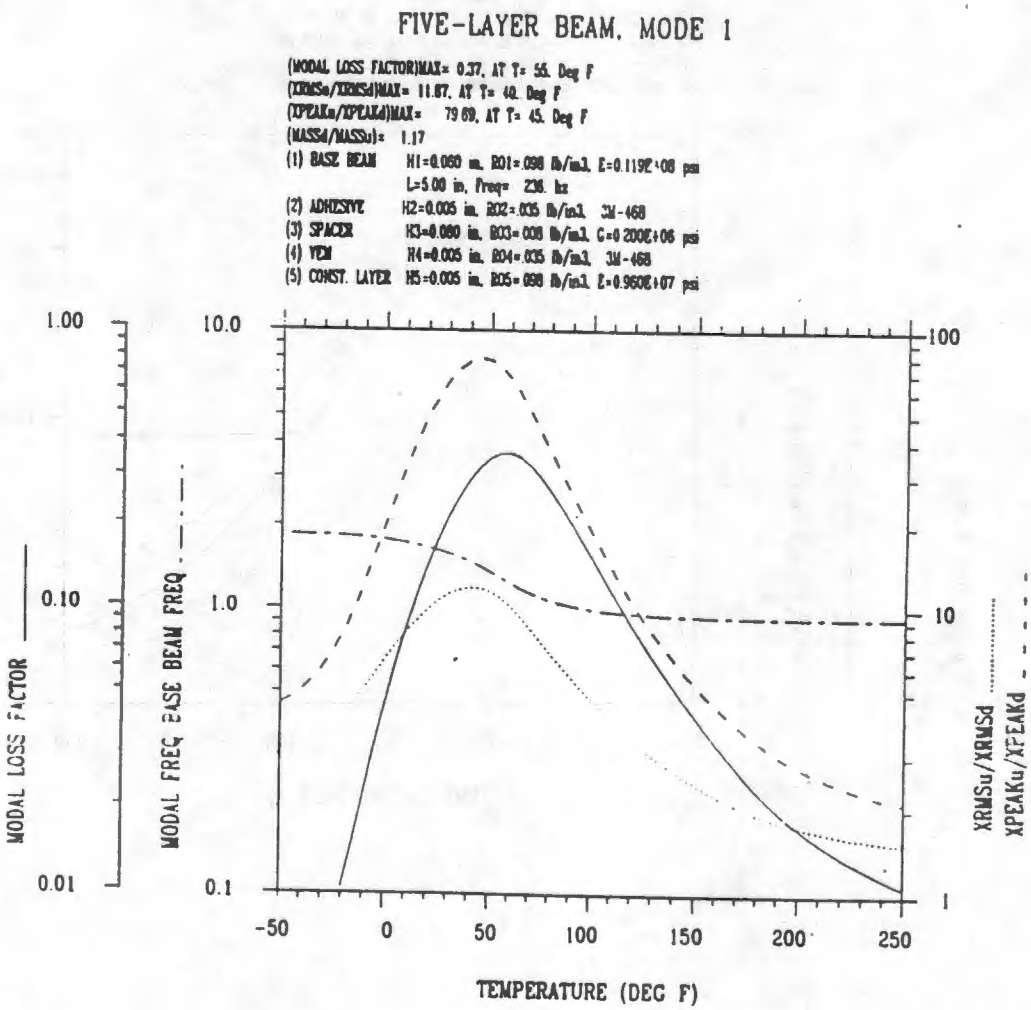

Figure 9 - Damping Properties of Material 3M-468 for Layers 2 and 4

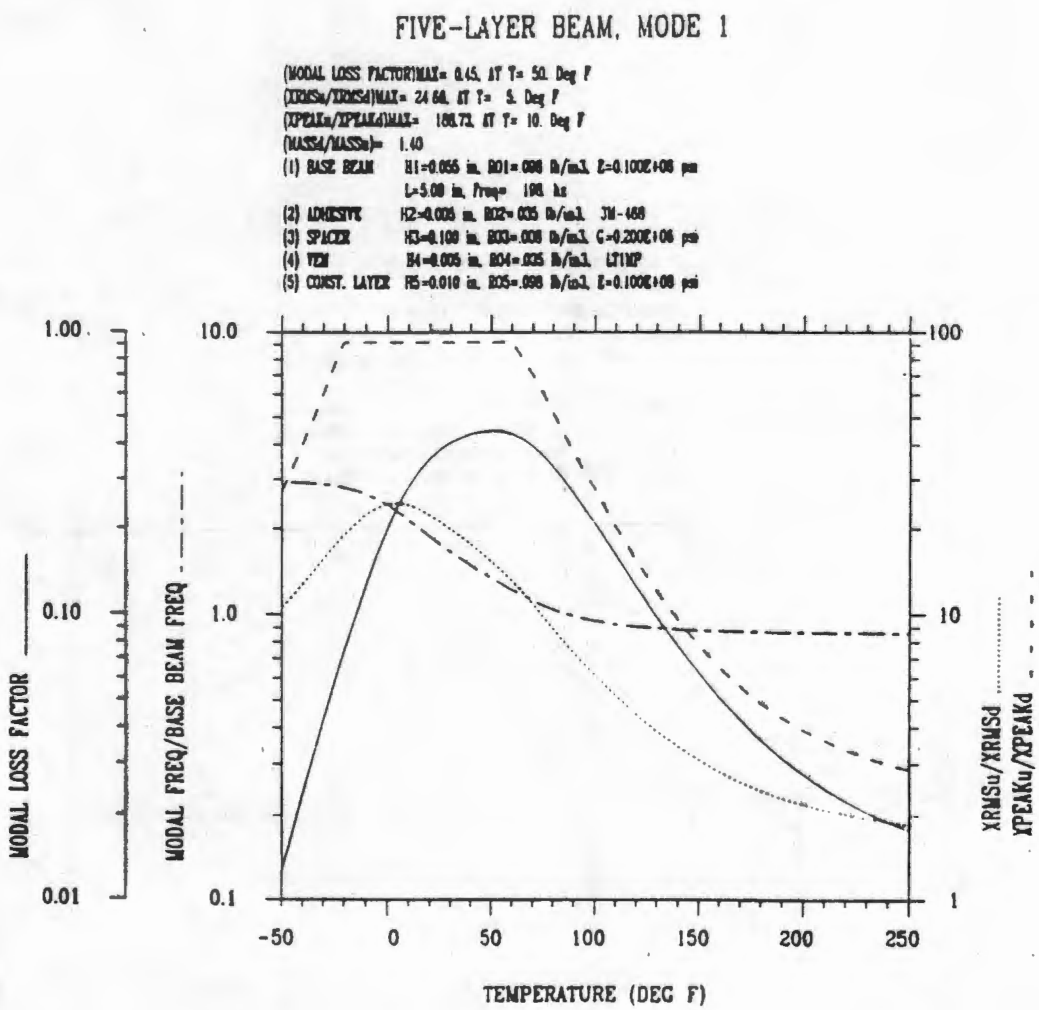

Figure 10 - Combined Effect of the Two Materials, 3M-468 (for Layer 2) and LT1MP (for Layer 4)

quired temperature range for an effective damping design. One way of broadening the temperature range over which maximum damping can be achieved is to use multiple viscoelastic materials with peaks in loss factor occurring at different temperatures (see Figure 7). As an example, suppose the temperature range for which the damping design has to operate is from 0 to 150^0 F. Using the analysis procedure in section 2.0, two materials are selected: LT1MP and 3M-468. The first material has its peak damping at -25^0 F (Figure 8) the second at 55^0 F (Figure 9). By placing the two materials in the order shown in Figure 10 the combined effect gives the desired results over a broader temperature range than provided by either material used separately.

The order in which the viscoelastic layers are applied is also important. Analysis has shown that the layer nearest the structure has to have the higher temperature damping properties to get a wider range of damping performance. To illustrate this concept consider two cases. In the first case, 3M-468 was used as the second layer and LT1MP as a fourth layer. In the second case these materials were reversed. It was found for a loss factor of $\eta \geq 0.1$, that the first case gave a wider temperature coverage -20 through 130^0 F (see Figure 10), while the second case resulted in a more narrow temperature coverage, -60 through 34^0 F, (see Figure 11).

5.0 GRAPHICS

Graphical plotting capability was built into the program using DI3000 software. The software allows users to plot the following four curves on the same graph: modal loss factor, modal frequency, RMS response ratio, and peak resonance ratio, each as a function of temperature. Users also have the option of inputting the required range of temperatures and choosing whether to plot results on the printer or only display them on the screen.

A general purpose computer program CP5LB (carpet plot for five-layer beam) to generate carpet plots for the five-layer damping system was also developed [9]. The first part of the program consists of developing carpet plots of maximum modal loss factor values versus temperature (Figure 12) . The second part of the program generates carpet plots of maximum RMS response ratios versus temperature for different combinations of layer thicknesses of viscoelastic (H4) and constraining (H5) layers (Figure 13). The input data for the program consists of the material properties; geometry

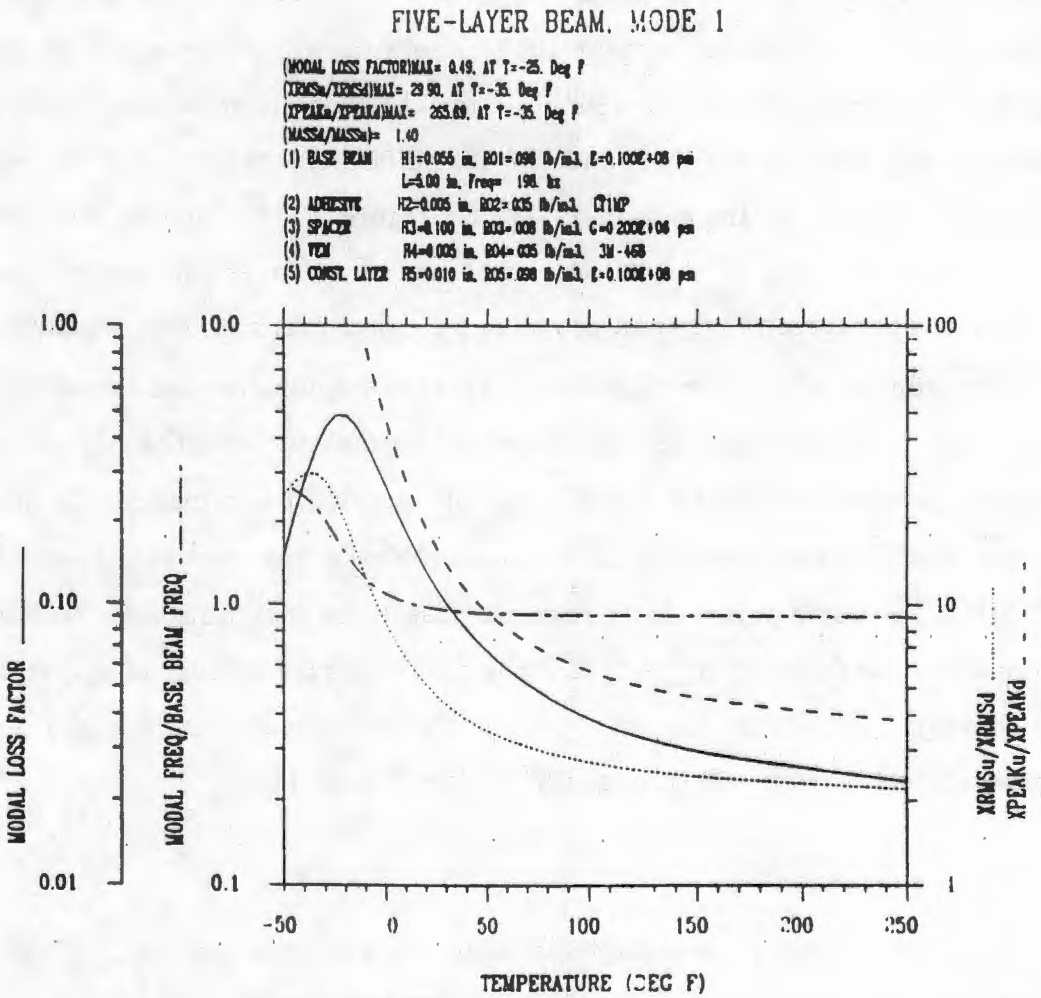

Figure 11 - Combined Effect of the Two Materials, LT1MP (for Layer 2) and 3M-468 (for Layer 4)

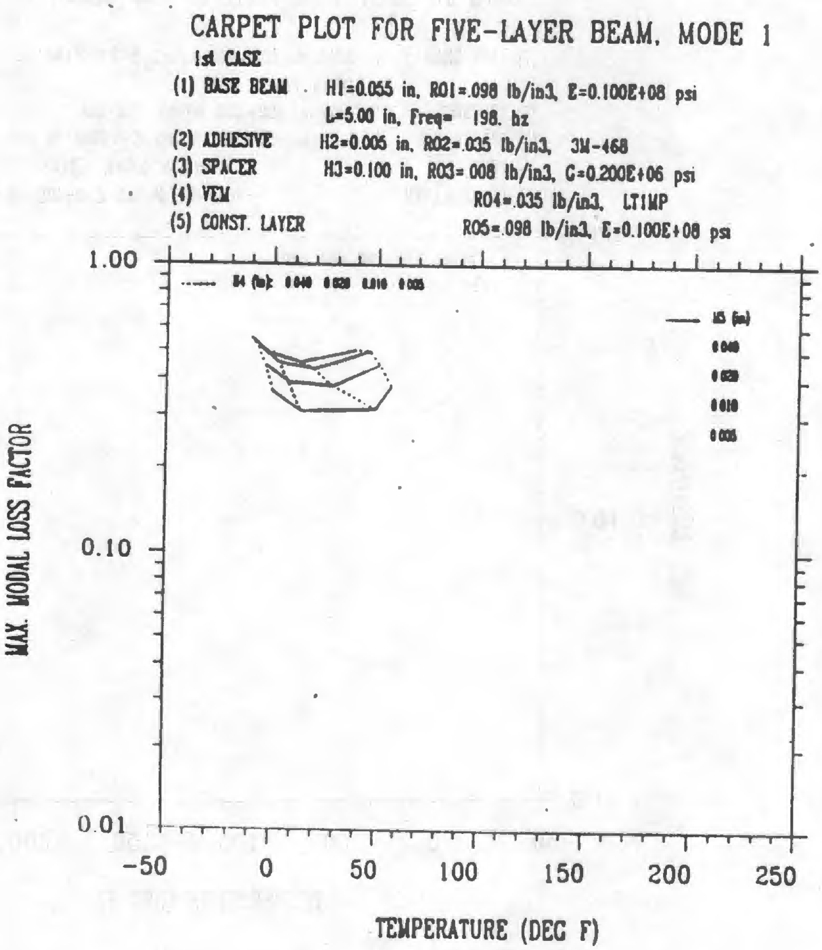

Figure 12 - Carpet Plot of Maximum Modal Loss Factor vs. Temperature (Degrees Fahrenheit)

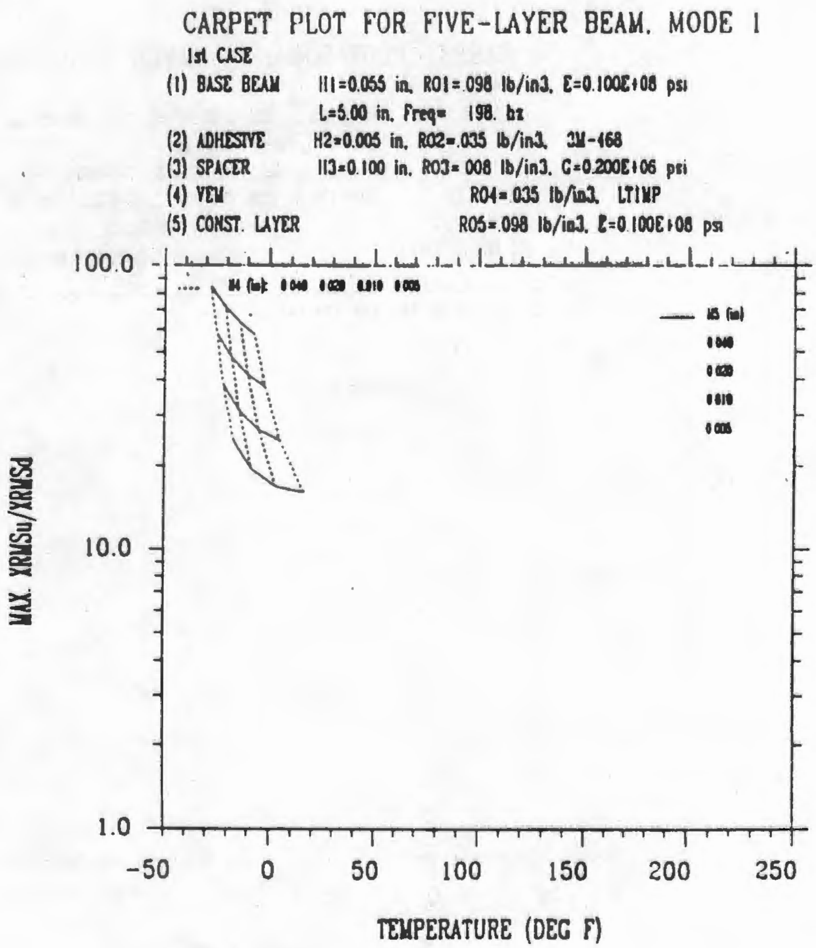

Figure 13 - Carpet Plot for Maximum RMS Ratio vs. Temperature (Degrees Fahrenheit)

of the undamped beam structure, the adhesive layer, and stand-off layer; and desired thickness values for H4 and H5 (four different thickness values for each H4 and H5 are used, see Figure 12). The program calculates the maximum modal loss factor at different combinations of H4 and H5 and uses an IMSL subroutine to fit a quadratic function to the data which forms the carpet plot curves. These carpet plots can then be used to choose optimum damping configurations for the five-layer systems. This shows the effect on damping properties created by different combinations of system layer thicknesses.

The computer program, CP5LB which generates carpet plots, can provide a convenient way to show trends in the maximum values of modal damping and RMS response ratio as functions of temperature.

6.0 AN EXAMPLE PARAMETRIC STUDY

To obtain an understanding of the behavior of the five-layer beam damping system, an example parametric study was conducted by varying the thickness of each layer (except the base beam) by 20 %. The baseline model consists of: a base beam 0.005 inches thick and 5.0 inches long; 0.005 inches of 3M-468 adhesive; a 0.10 inches stand-off layer; 0.005 inches of LT1MP viscoelastic material; and an aluminum constraining layer 0.01 inches thick. The results of the example parametric study are shown in Table 1. These results indicate that the stand-off layer is the most significant parameter to effect damping. It decreased the damped response by 19.18 % compared to 0.89 % for the adhesive-layer.

7.0 CONCLUSIONS

This damping technology is very effective in helping designers predict damping characteristics in beam-like structures. The temperature range can be broadened over which maximum damping is achieved by applying two different viscoelastic materials with peak in loss factors occuring at different temperatures. By using the computer program V5LBD, which utilizes the developed governing equations, it was shown that the order in which the viscoelastic layers are applied is very important. The viscoelastic layer nearest the structure must be the layer that has peak damping at the higher temperature, in order to achieve maximum damping over the broadest temperature range.

The carpet plotting program. CP5LB is an effective tool to show trends in damping characteristics as a function of temperature for parametric changes in the geometry of applied layers. It was demonstrated by conducting parametric studies for the five-layer system by varying the thickness of each layer (except the base beam) by 20 % that the stand-off layer had the most significant damping effect on response: decreasing RMS response by 19.18 % compared to 0.89 % for the adhesive-layer. Additional verifications of V5LBD need to be done as well as a comparison of the program output with actual test data. The five-layer system presented can be a very effective technique in helping the designer to select proper damping treatments for reducing resonant vibrations.

TABLE 1 - Parametric

H2 (in.)	H3 (in.)	H4 (in.)	H5 (in.)	E5 (psi)	XRMSu/XRMSd (MAX)	TEMP (°F.)	% CHANGE
0.005	0.10	0.005	0.010	10E6	24.66	5	BASE
0.006	0.10	0.005	0.010	10E6	24.88	5	0.89
0.005	0.12	0.005	0.010	10E6	29.39	5	19.18
0.005	0.10	0.006	0.010	10E6	24.98	0	1.30
0.005	0.10	0.005	0.012	10E6	27.52	0	11.60
0.005	0.10	0.005	0.010	12E6	27.06	0	9.73

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