FOREWORD

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ABSTRACT

A theory for creep buckling of perfect flat plates under axial compression is presented which is based on the use of an equation of state to represent the incremental stresses that arise at buckling. The theoretical results which apply to any arbitrary creep properties are directly equivalent to inelastic buckling solutions where now the tangent and secant moduli are strain rate dependent quantities. Based on these results, a general theory for plates and shells under various types of loadings is presented as a logical extension of the compressed plate case.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

W. J. Trapp
Chief, Strength and Dynamics Branch
Metals and Ceramics Laboratory
Directorate of Materials and Processes

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LIST OF SYMBOLS

b	plate width
D	flexural rigidity, $D = E_s h^3/12(1-\nu^2)$
E	elastic modulus
E_s	secant modulus
$[E_s]_{\dot{\epsilon}}$	strain-rate dependent secant modulus
E_t	tangent modulus
$[E_t]_{\dot{\epsilon}}$	strain-rate dependent tangent modulus
$[E_u]_{\epsilon}$	$= \partial\sigma_i/\partial\dot{\epsilon}_i$
h	thickness
k	buckling coefficient
t	time
T	temperature
w	lateral deflection
x,y,z	coordinates
γ	shear strain
ϵ	normal strain
$\dot{\epsilon}$	strain rate
ν_e	elastic Poisson's ratio
ν	plastic Poisson's ratio
σ	normal stress
σ_a	applied creep stress
τ	shear stress

<u>Subscripts</u>

i	intensity
x,y,z	coordinate orientation

I INTRODUCTION

The results of a recent theoretical and experimental investigation of the creep buckling of columns (1) have indicated that the classical stability concepts utilized for elastic and inelastic stability can be successfully extended to the corresponding creep problem. The purpose of the present paper is to employ the theoretical concepts developed for the perfect column problem in formulating a general theory for the creep buckling of perfect flat plates and shells.

Available literature on the plate and shell problem is rather scanty. An hypothesis based on an extension of inelastic stability theory was presented in Ref. 2. Rabotnov and Shesterikov(3) introduced several valuable theoretical concepts which have made possible the present development of a creep buckling theory in sufficiently general form that results can be obtained directly from compressive creep data. In effect, the concepts advanced in Ref. 3 permitted the development of the creep buckling hypothesis of Ref. 2 in a formal rather than intuitive manner.

It should be mentioned that Lin (4) presented a theory for the creep deflection of a viscoelastic plate under compression in which the plate contained initial geometric imperfections. For reasons presented in Ref. 1, it appears that a theory for perfect plates has certain desirable features over initial imperfection theory in its ultimate usefulness in design and analysis.

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II CREEP BUCKLING OF PLATES UNDER AXIAL COMPRESSION

A perfect plate under axial compression undergoes axial deformation before buckling occurs and therefore follows the appropriate constant uniaxial stress creep relation. At buckling, an exchange of equilibrium configurations occurs from the straight to the laterally deflected form. Hence, a variable plane stress creep law is required to relate the incremental stresses and strains associated with bending in the presence of a relatively large axial compressive strain.

The essential difference between creep buckling of a plate and inelastic buckling results from the fact that the constant stress state existing prior to creep buckling produces time-dependent deformations whereas the increasing stress state associated with inelastic buckling produces time-independent deformations. For creep buckling, it is only the reduction of the extensional and flexural rigidities with time under a constant stress that can account for buckling of a perfect plate.

1. Properties of Materials Under Creep

In order to define the moduli associated with the extensional and flexural stiffnesses in a manner appropriate to the creep buckling problem, it is assumed that the following equation of state may be employed:

$$\varphi\ (\sigma_i,\ \epsilon_i,\ \dot{\epsilon}_i,\ T) = 0 \qquad (1)$$

In general form, the stress intensity σ_i, the strain intensity, ϵ_i and strain rate intensity $\dot{\epsilon}_i$ are defined according to the familiar plasticity relations for plane stress:

$$\sigma_i = (\sigma_x^{\ 2} + \sigma_y^{\ 2} - \sigma_x\sigma_y + 3\tau^2)^{1/2} \qquad (2)$$

$$\epsilon_i = [2/(3)^{1/2}]\ (\epsilon_x^{\ 2} + \epsilon_y^{\ 2} + \epsilon_x\epsilon_y + \gamma^2/4)^{1/2} \qquad (3)$$

$$\dot{\epsilon}_i = [2/(3)^{1/2}]\ (\dot{\epsilon}_x^{\ 2} + \dot{\epsilon}_y^{\ 2} + \dot{\epsilon}_x\dot{\epsilon}_y + \dot{\gamma}^2/4)^{1/2} \qquad (4)$$

Eq. (1) implies that creep data may be transformed into constant strain rate stress-strain data in terms of σ_i, ϵ_i and $\dot{\epsilon}_i$ in the form shown schematically in Fig. 1 for a constant temperature.

In buckling of plates, the secant modulus is associated with the extensional rigidity. In Fig. 1, the secant modulus for a prescribed value of strain rate intensity is defined as

$$[E_s]_{\dot{\epsilon}} = \sigma_i/\epsilon_i \tag{5}$$

Thus, for given values of σ_i and $\dot{\epsilon}_i$, $[E_s]_{\dot{\epsilon}}$ uniquely defines a point on the φ-surface of Fig. 1.

For an incremental departure from the point on this surface representing the bent state in the buckling process

$$\delta\sigma_i = (\partial\sigma_i/\partial\epsilon_i)\,\delta\epsilon_i + (\partial\sigma_i/\partial\dot{\epsilon}_i)\,\delta\dot{\epsilon}_i \tag{6}$$

The tangent modulus associated with the flexural rigidity is defined as

$$E_t = \delta\sigma_i/\delta\epsilon_i \tag{7}$$

Consequently, by use of Eq. (7), Eq. (6) becomes

$$E_t = \partial\sigma_i/\partial\epsilon_i + (\partial\sigma_i/\partial\dot{\epsilon}_i)\,(\delta\dot{\epsilon}_i/\delta\epsilon_i) \tag{8}$$

In a more convenient form

$$E_t = [E_t]_{\dot{\epsilon}} + [E_u]_{\epsilon}\,(\partial/\partial t) \tag{9}$$

2. Buckling Model

In the commonly accepted model for inelastic buckling of a column, incremental bending and axial loading proceed simultaneously at buckling so that the stress state on the convex side remains stationary while the concave side follows a loading path in the direction of the local tangent modulus. This implies an assumption concerning the strain rate of infinitesimal bending that was first pointed out in Ref. 5. In order for the loading path to be in the direction of the local tangent modulus, loading on the concave side must proceed at a strain rate associated with the compressive stress-strain curve. If this assumption were not implied, but instead instantaneous incremental bending were assumed, then the local bending stiffness would have a value of E and buckling of the column would not occur. Thus, incremental bending at the local strain rate leads to a lower limit to the bending stiffness which appears to be the appropriate value to use in determining instability.

In creep buckling of the column, a series of stress-strain curves at different strain rates reflect the time dependent nature of the problem (1). In the presence of an applied compressive stress, the local tangent modulus decreases with time as successive strain rate curves are crossed until buckling occurs. Thus, for creep buckling as for inelastic buckling the lower bound to the bending stiffness is taken as the tangent modulus appropriate to the strain rate conditions at buckling. In this sense, the creep buckling mechanism for plates as well as columns is a logical extension of that associated with inelastic buckling.

3. Creep Buckling Theory

Having discussed the moduli appropriate to the extensional and flexural rigidities, it is now pertinent to consider the equilibrium equation for stability of a flat plate. In the creep buckling process, the applied compressive stress σ_a remains fixed while the secant and tangent moduli defined by Eqs. (5) and (9) are strain rate dependent. As a consequence, the following equilibrium equation for compressive inelastic stability of a flat plate (6) can be used directly with the suitable definitions of the moduli.

$$\left(\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{\sigma_a h}{D}\frac{\partial^2 w}{\partial x^2} = 0 \qquad (10)$$

By substituting the value for E_t given by Eq. (9) into Eq. (10), and replacing E_s by $[E_s]_{\dot{\epsilon}}$ the following is obtained:

$$\left(\frac{1}{4} + \frac{3}{4}\frac{[E_t]_{\dot{\epsilon}}}{[E_s]_{\dot{\epsilon}}}\right)\frac{\partial^4 w}{\partial x^4} + \frac{3}{4}\frac{[E_u]_{\epsilon}}{[E_s]_{\dot{\epsilon}}}\frac{\partial^5 w}{\partial x^4 \partial t} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{\sigma_a h}{D}\frac{\partial^2 w}{\partial x^2} = 0 \qquad (11)$$

A solution of Eq. (11) can be given in the following form for a simply supported flat plate

$$w = w_o(t) \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b} \tag{12}$$

By substituting the appropriate derivatives of Eq. (12) into Eq. (11)

$$\left[\left(\frac{1}{4} + \frac{3}{4} \frac{[E_t]_\epsilon}{[E_s]_\epsilon}\right) \left(\frac{m\pi}{a}\right)^4 w_o + \frac{3}{4} \frac{[E_u]_\epsilon}{[E_s]_\epsilon} \left(\frac{m\pi}{a}\right)^4 \dot{w}_o + 2 \left(\frac{m\pi}{a}\right)^2 \left(\frac{\pi}{b}\right)^2 w_o + \left(\frac{\pi}{b}\right)^4 w_o - \frac{\sigma_a h}{D} \left(\frac{m\pi}{a}\right)^2 w_o \right] \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b} = 0 \tag{13}$$

For a nontrivial solution, the bracketed terms are set equal to zero with the following result:

$$\sigma_a = \frac{D}{h} \left[\left(\frac{1}{4} + \frac{3}{4} \frac{[E_t]_{\dot\epsilon}}{[E_s]_\epsilon}\right) \left(\frac{m\pi}{a}\right)^2 + 2\left(\frac{\pi}{b}\right)^2 + \left(\frac{a}{m\pi}\right)^2 \left(\frac{\pi}{b}\right)^4 + \frac{3}{4} \frac{[E_u]_\epsilon}{[E_s]_\epsilon} \left(\frac{m\pi}{a}\right)^2 \frac{\dot{w}_o}{w_o} \right] \tag{14}$$

For creep buckling, the Rabotnov-Shesterikov stability criterion (3), $\dot{w}_o/w_o = 0$ is applied to Eq. (14). As a consequence, the last term vanishes and Eq. (14) reduces to the creep buckling analog of the inelastic buckling solution.

$$\sigma_a = \frac{\pi k \eta E}{12(1-\nu_e^2)} \left(\frac{h}{b}\right)^2 \tag{15}$$

$$\text{where } \eta = \frac{(1-\nu_e^2)}{(1-\nu^2)} \frac{[E_s]_\epsilon}{E} \left[\frac{1}{2} + \frac{1}{4} \left(1 + 3 \frac{[E_t]_\epsilon}{[E_s]_\epsilon}\right)^{1/2} \right] \tag{16}$$

4. Application of Theory

It is the purpose now to demonstrate the use of the theory for the prediction of creep buckling times of aluminum alloy 2024-0 plates at 500°F. Compressive creep data obtained in the experimental program of Ref. 1 are used here for the development of the theoretical results.

As a first step in the development, the compressive creep data shown in Fig. 2 are transformed graphically into constant strain rate stress-strain data, as implied by Eq. (2), by a series of steps. From the latter $[E_s]_{\dot{\epsilon}}$ and $[E_t]_{\dot{\epsilon}}$ can then be evaluated. Further, by an inverse sequence, the critical times can be found graphically from the compressive creep curves. The entire development is illustrated in the following.

Tangents to the compressive creep curves of Fig. 2 represent the instantaneous values of strain rate. The strain rates for each creep curve may now be plotted as a function of strain as shown in Fig. 3. The intersections of the curves with any vertical in Fig. 3 represent the stress and strain data at a constant strain rate. It is now possible to construct constant strain rate stress-strain curves for any specified strain rate using these data as shown in Fig. 4.

In order to apply Eq. (15) conveniently, the secant and tangent moduli to the curves given in Fig. 4 are determined, combined according to Eq. (16) and plotted against stress as shown in Fig. 5. A straight line through the origin on this graph is associated with a particular b/h ratio for the plate since from Eq. (15) for long, simply supported plates with k = 4

$$\frac{4\pi^2}{12(1-\nu_e^2)} \left(\frac{h}{b}\right)^2 = \frac{\sigma_a}{\eta E} \tag{17}$$

Eq. (17) defines the slopes of the lines through the origin as shown in Fig. 5 for various b/h values. Each intersection of such a line with a constant strain rate effective modulus (ηE) curve represents an unique set of conditions for creep buckling.

It is most convenient to find strain rates corresponding to the correct effective modulus for specific b/h values from a cross plot of the data in Fig. 5 in terms of stress and strain rate as shown in Fig. 6. In order to predict creep buckling times, one first finds the appropriate strain rate for the applied stress and b/h value from Fig. 6. The creep strain corresponding to the strain rate is then found from Fig. 3 and the creep buckling time can be read off from the proper compressive creep curve in Fig. 2. Using this process, critical times were found for a number of applied stresses and b/h values and these results are given finally in Fig. 7.

It is important to note that the creep buckling results presented in Fig. 7 represent the critical times at which lateral deflections first develop in a perfect plate. Collapse of the plate or creep crippling is of course a later event in the same sense ordinary buckling and crippling are distinctly different phenomena.

III CREEP BUCKLING OF PLATES AND SHELLS

We return now to certain theoretical considerations that become apparent from the analysis of the flat plate under axial compression. The essential result of applying the Rabotnov-Shesterikov stability criterion, $\mathring{w}_o/w_o = 0$, is that the time-dependent term in the definition of the tangent modulus vanishes so that Eq. (9) reduces to

$$E_t = [E_t]_{\dot{\epsilon}} = \delta \sigma_i / \delta \epsilon_i \tag{18}$$

As a consequence, the creep buckling problem becomes quasi-static as pointed out in Ref. 3.

As defined by Eq. (5), the secant modulus is

$$[E_s]_{\dot{\epsilon}} = \sigma_i / \epsilon_i \tag{19}$$

Therefore, the strain-rate dependent tangent and secant moduli for a fixed stress in the creep buckling problem are directly analogous to the tangent and secant moduli associated with the increasing stress in the inelastic buckling problem.

As a consequence, the plasticity reduction factors obtained for inelastic buckling of flat plates and shells under various types of loadings may be used directly in the creep buckling problem with the understanding that the tangent and secant moduli are strain-rate dependent for the latter. Thus, the inelastic buckling results obtained in Ref. 6 for flat plates, in Ref. 7 for shells and in Ref. 8 for orthotropic plates and cylindrical shells may be employed for creep buckling of materials with arbitrary creep characteristics by following a graphical procedure similar to that presented herein for the flat plate under axial compression.

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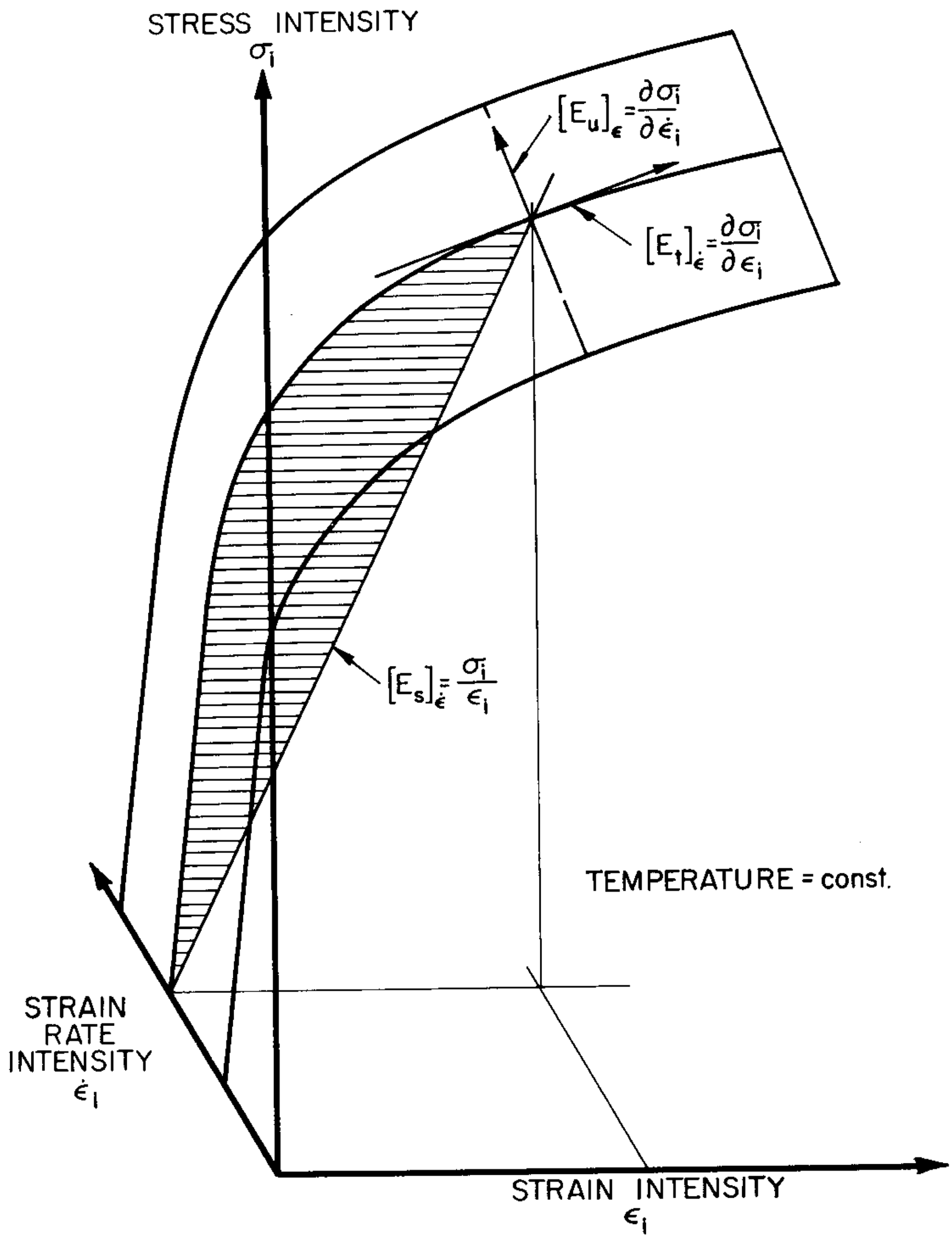

FIGURE 1 STRESS-STRAIN-STRAIN RATE SURFACE FOR A MATERIAL AT A GIVEN TEMPERATURE

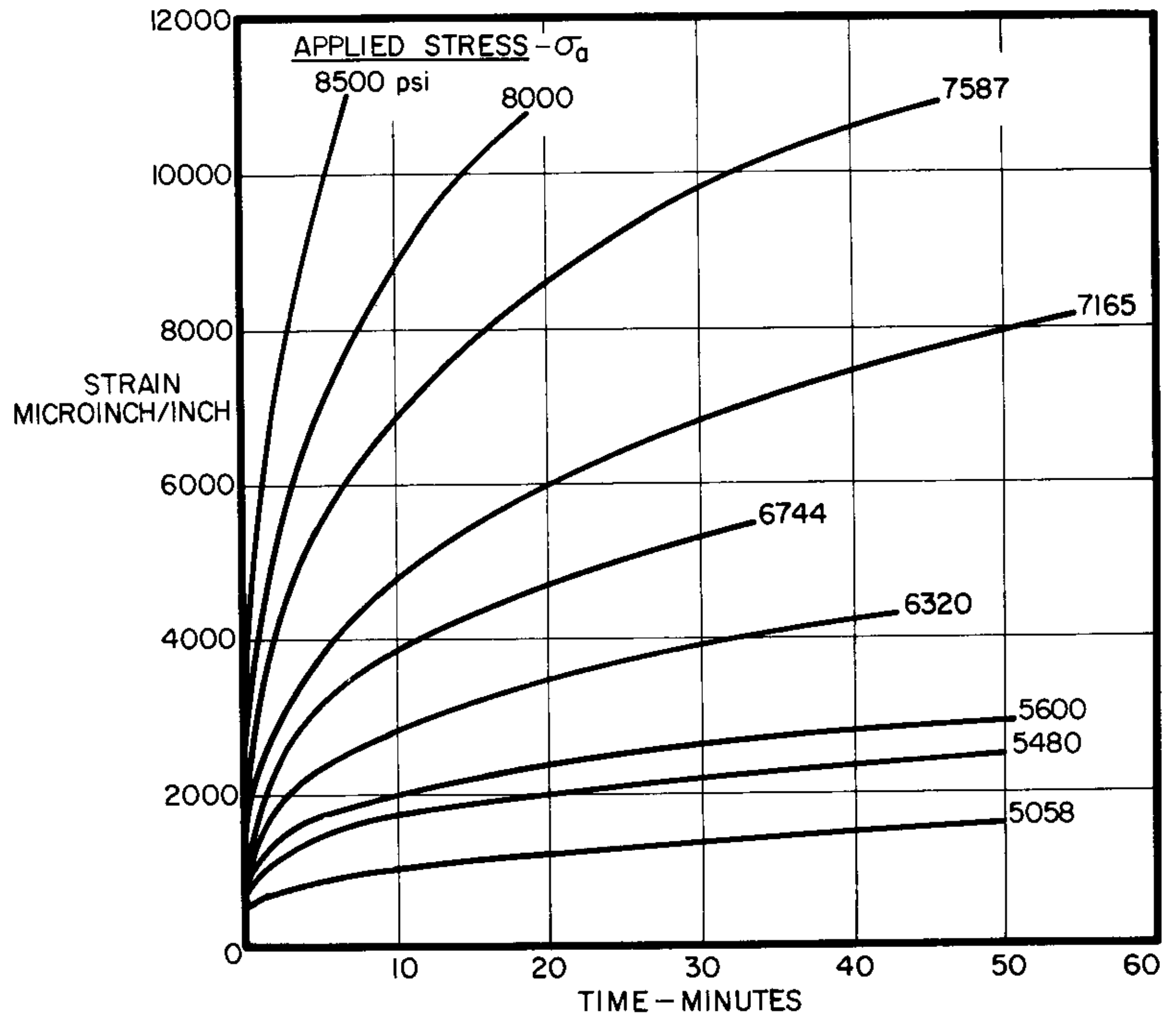

FIGURE 2 COMPRESSION CREEP DATA FOR 2024-0 ALUMINUM ALLOY AT 500°F (REF. 1).

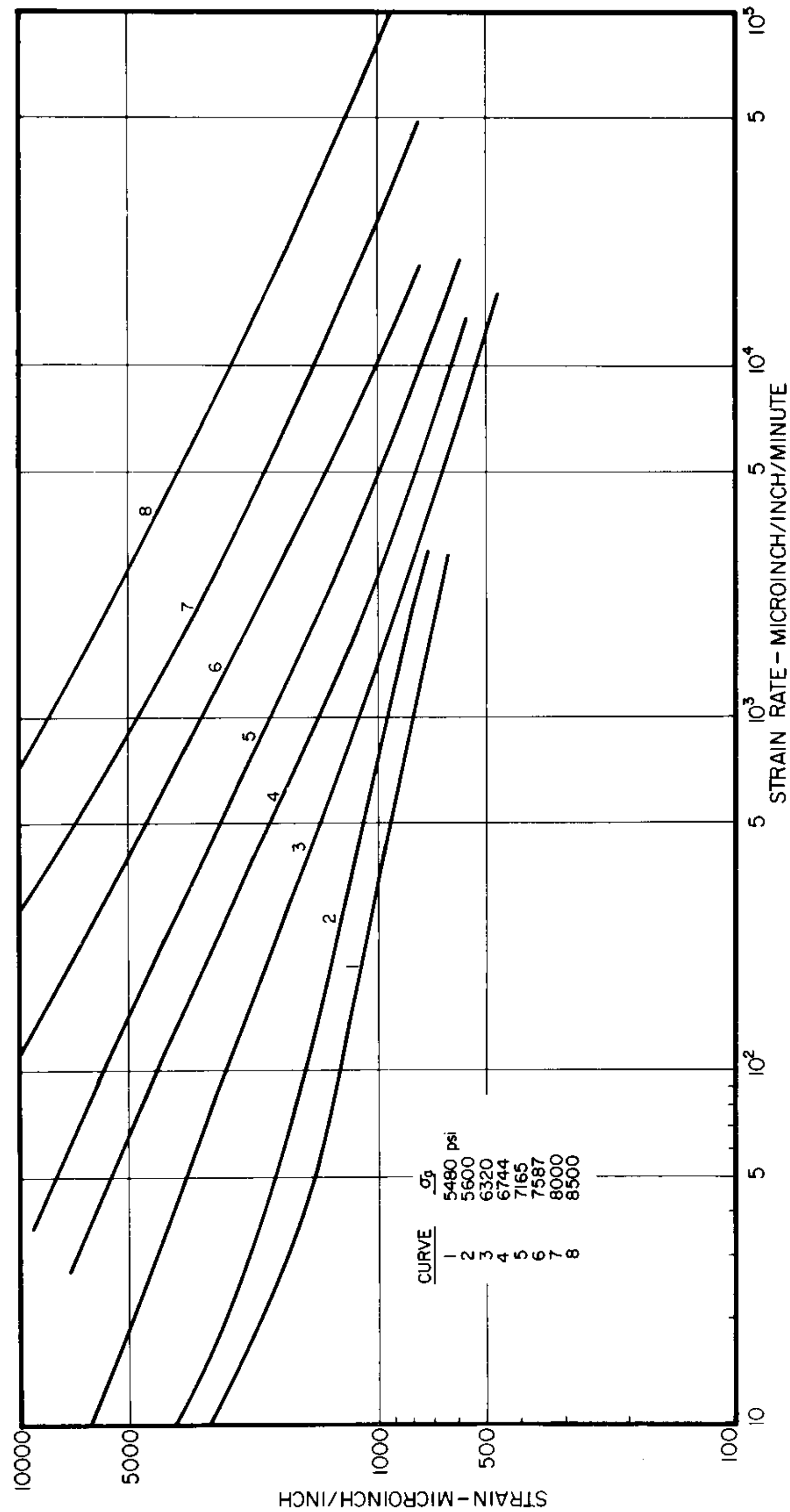

FIGURE 3 STRAIN-STRAIN RATE DATA DERIVED FROM FIG. 2.

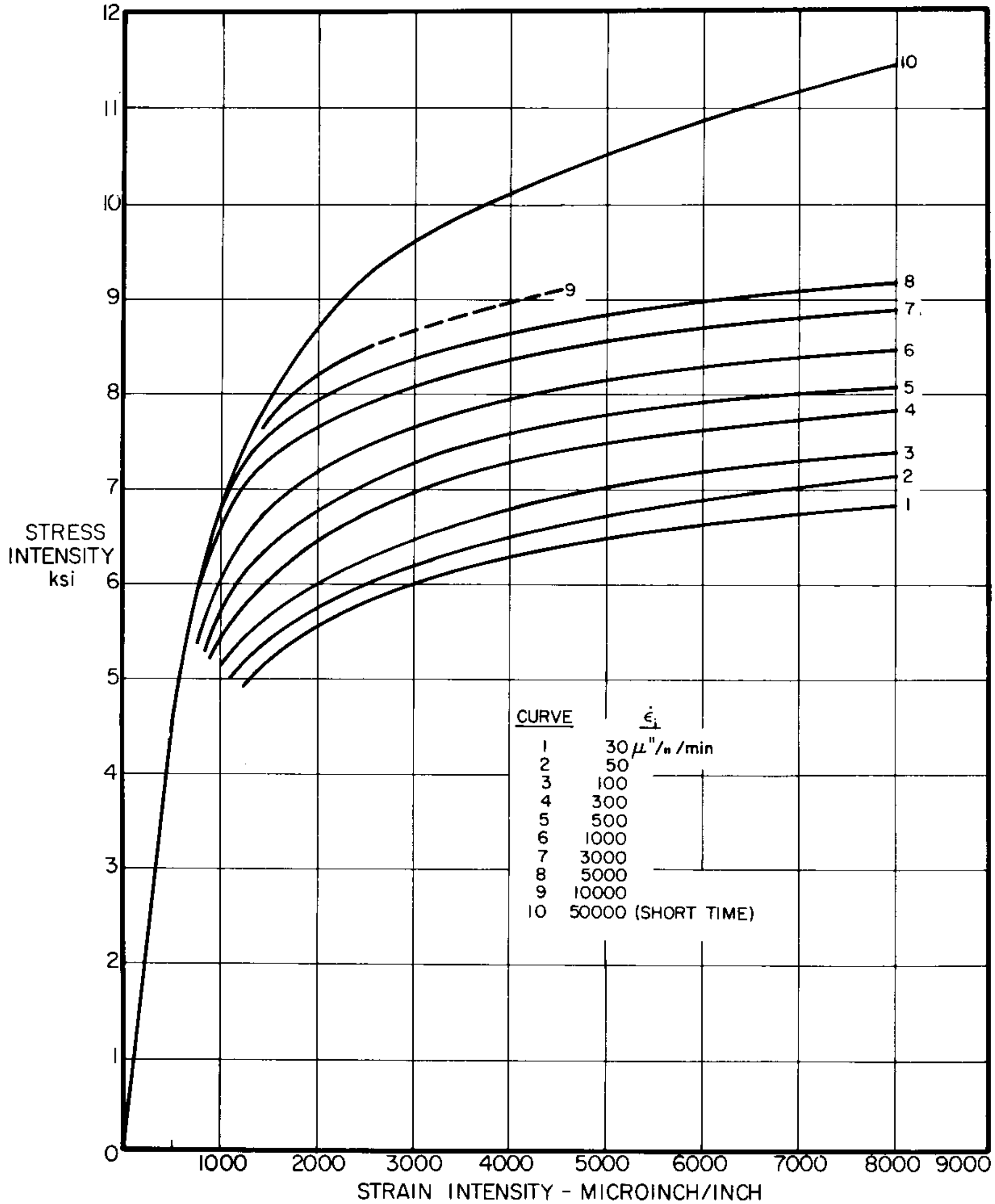

FIGURE 4 CONSTANT STRAIN RATE STRESS-STRAIN DATA DERIVED FROM FIG. 3.

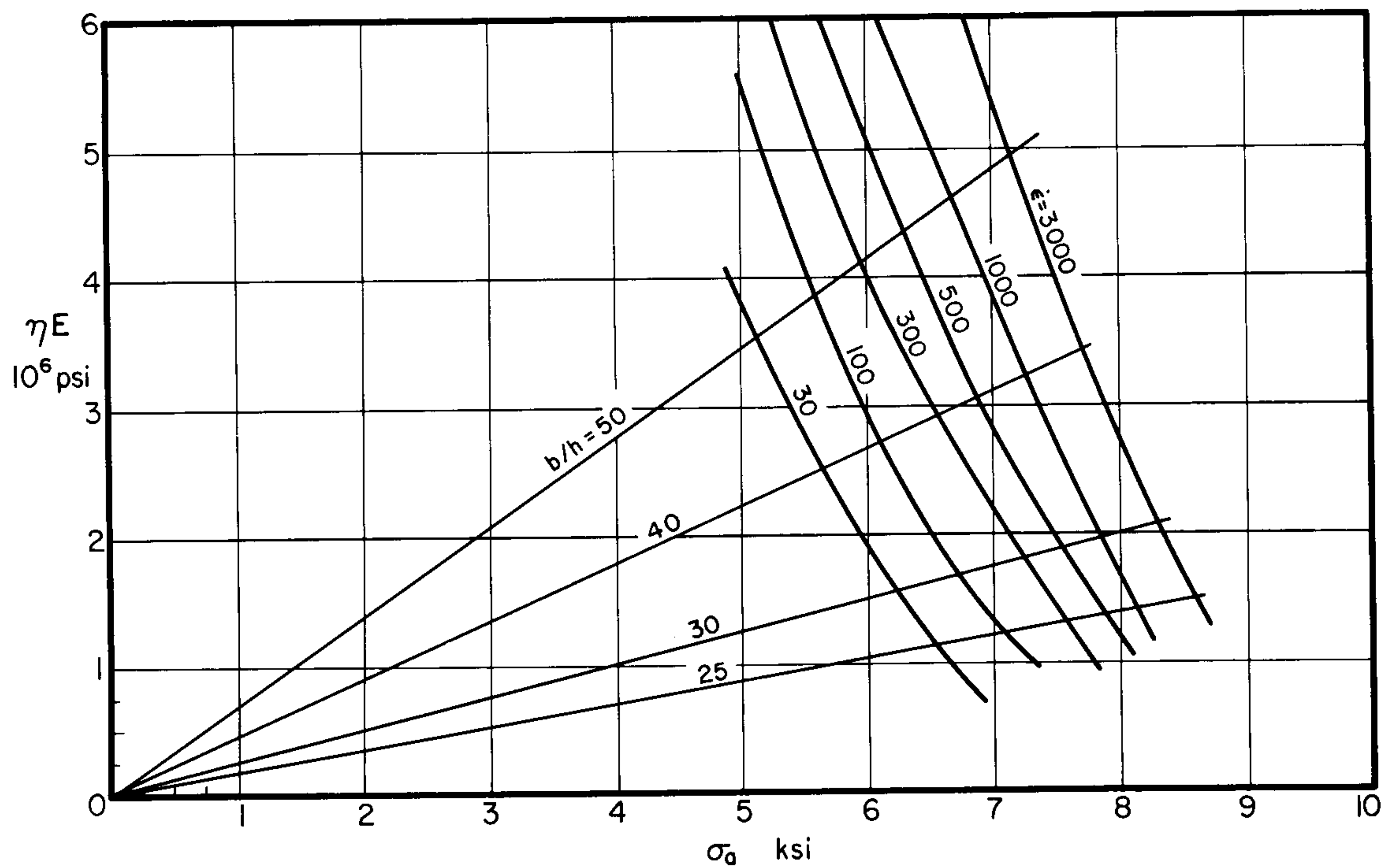

FIGURE 5 EFFECTIVE MODULUS-STRESS DATA FOR LONG FLAT PLATES UNDER AXIAL COMPRESSION DERIVED FROM FIG. 4.

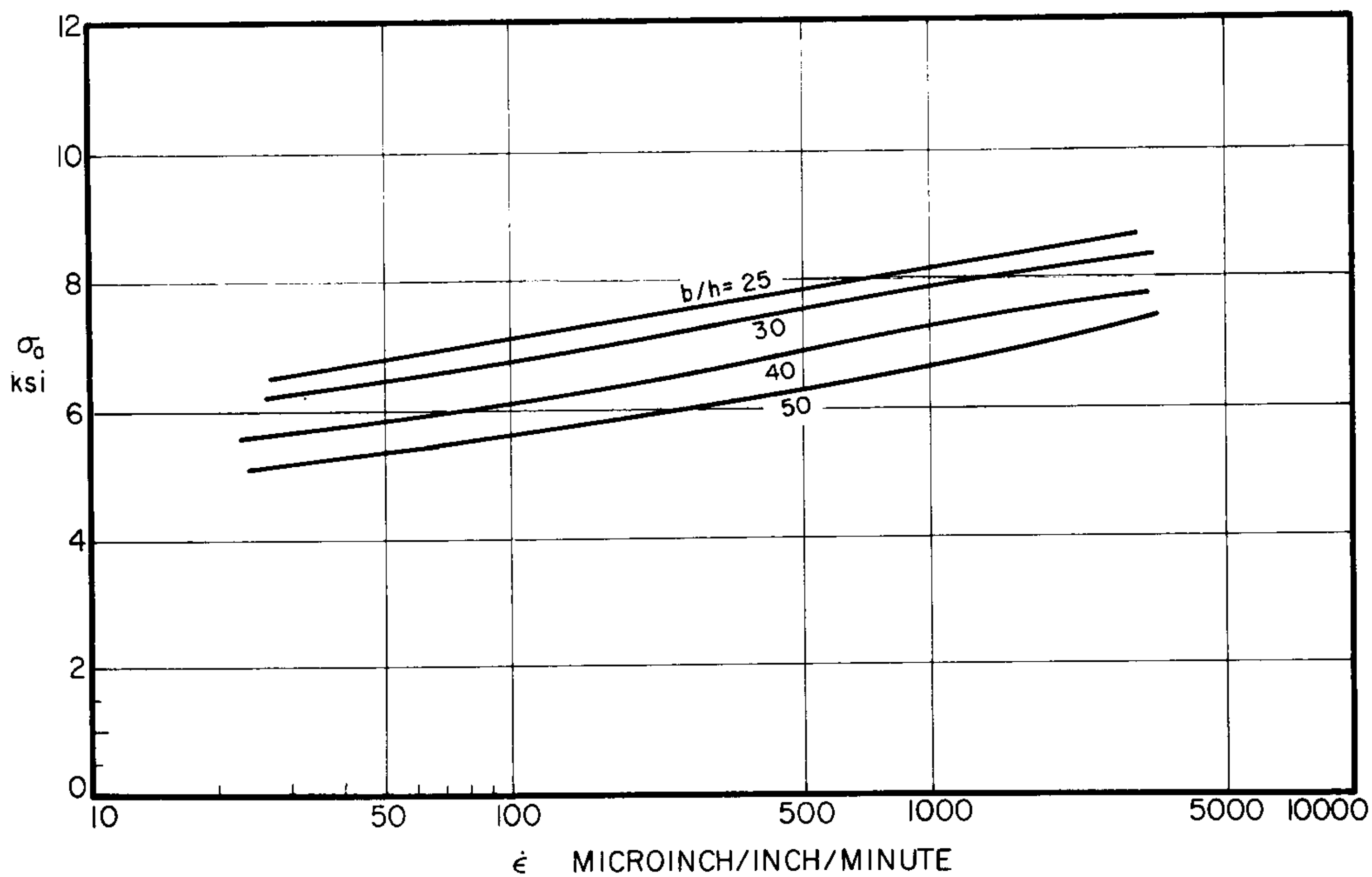

FIGURE 6 CONDITIONS FOR CREEP BUCKLING OF LONG FLAT PLATES UNDER AXIAL COMPRESSION FROM FIG. 5.

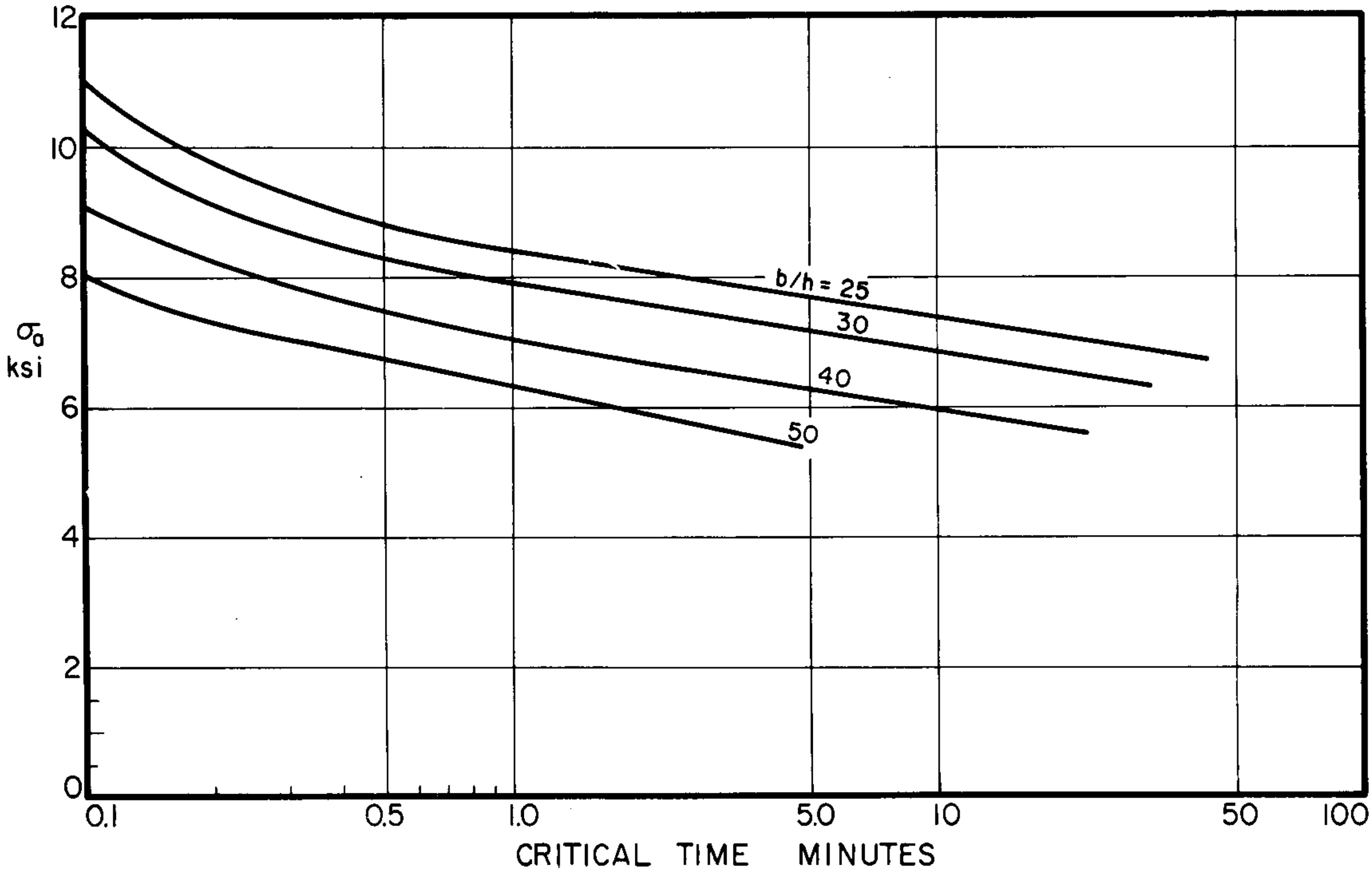

FIGURE 7 CRITICAL TIME FOR CREEP BUCKLING OF LONG FLAT PLATES UNDER AXIAL COMPRESSION. ALUMINUM ALLOY 2024-0 AT 500^{0}F.