SYSTEM LEVEL DESIGN AND ANALYSIS OF TRUSS STRUCTURES DAMPED BY VISCOUS STRUTS

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ABSTRACT

A procedure is presented to design passive damping into large truss structures using viscously damped struts to enhance vibration attenuation or stability of controls system. A method is derived from the equations of motion using Rayleigh-Ritz method to relate the approximate contributions of a viscously damped strut to the system level modal damping ratios and frequencies. Strut placement locations, the total number of struts required and the damping characteristics of struts can be easily identified and calculated. The procedure consists of three steps: 1) extract structural characteristics from the undamped baseline finite element model, 2) on a mode by mode basis, perform damping design using the derived equations to meet system level requirements and 3) update finite element model to include damping mechanism and perform verification analysis using complex eigensolution.

INTRODUCTION

Truss systems are often used for large space structures because of weight efficiency. These structures often have many flexible modes within the disturbance and control bandwidth. For stringent performance requirements, tight joints are required for precision truss structures. The intrinsic structural damping associated with this type of structure may be very low (less than 0.1% equivalent viscous damping[1]) and the dynamic responses under operational forces can be significantly amplified. Passive vibration control is a cost effective and reliable way to suppress dynamic responses and also provide additional stability margin to the controls system. Struts with good stiffness and damping characteristics can significantly enhance the performance of this class of structure.

Struts with imbedded viscoelastic materials have been successfully designed, tested, and integrated into truss structures[2,3]. Viscoelastic materials are often frequency and temperature dependent[4]. However, design procedure and approximate analytical methods[5] for this type of structure have been quite well established. Test results from demonstration structural articles compared favorably with analytical prediction[3].

Precision struts with build-in fluid viscous damping chamber have been built and tested. They were demonstrated to be quite effective in provided stiffness and damping[6]. This class of struts can be characterized by a small number of frequency independent physical parameters. The dynamics of this class of struts is well understood[7]. The analytical methods for structures with viscous damping, though more complicated and not commonly used, has a solid mathematical foundation. This paper presents a simple three-step procedure to design viscously damped struts into a large truss structure. Based on the baseline undamped structural model, the most effective strut placement locations, the key stiffness and damping strut parameters and the number of struts required are determined. Only simple design iterations are required to optimize the design. The engineering design is then verified by the rigorous analytical method.

ANALYSIS OF DAMPED STRUCTURES

It is essential to understand the analysis of a damped structure before designing such a structure to meet the design objectives. A complex structure is modeled by a finite element model with nxn matrices. The governing differential equations of a structure with viscous damping are given by:

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{C}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{p}\,g(t) \tag{1}$$

The damping matrix is due to viscous dashpots in the structure. The intrinsic damping is assumed to be negligible or added at the modal level. It is unlikely that the dashpot locations and characteristics result in a damping matrix which is mass or stiffness proportional, or satisfies Caughey's orthogonality condition[8]. Classical normal modes do not provide uncoupled scalar equations to Equation (1). The solution to Equation (1) is often obtained in the first order form by rewriting the equation as:

$$\begin{bmatrix} C & M \\ M & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \end{bmatrix} = \begin{bmatrix} p \\ 0 \end{bmatrix} g(t) \tag{2}$$

In order to uncouple the matrix equation, a complex eigenvalue problem for the large 2nx2n matrices[9] must be solved.

$$\lambda_i \begin{bmatrix} C & M \\ M & 0 \end{bmatrix} \begin{bmatrix} \phi_u \\ \phi_u \end{bmatrix}_i + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{bmatrix} \phi_u \\ \phi_u \end{bmatrix}_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3}$$

Both the eigenvalues and eigenvectors are complex. The corresponding undamped natural frequencies and modal damping can be computed from the complex eigenvalues:

$$\omega_i = \sqrt{\lambda^{R\,2}_i + \lambda^{I\,2}_i} \tag{4a}$$

$$\xi_i = \frac{-\lambda^R_i}{\omega_i} \tag{4b}$$

The modal damping is embedded in the real part of the eigenvalue. For a passive stable system, the real parts of the eigenvalues are always non-positive. The introduction of viscous dampers in the finite element model also results in the presence of overdamped modes with zero vibratory frequencies and large damping coefficients. The techniques in selecting an accurate and efficient algorithm for complex eigensolution computation is quite important but not elaborated upon here.

This procedure is mathematically rigorous and gives the correct solution to Equation (1). However, it is quite computationally intensive for large structures . Also, from the design point of view, it does not offer much insight into the behavior of the structures, and does not help synthesizing and optimizing passive damping design for structure. However, once the damping design is complete, the complex eigensolution should be performed to verify the passive damping design.

TRUSS STRUCTURES

If a undamped truss structure has n degrees of freedom, the equations of motion are given by:

$$M_{11}\, \ddot{u}_1 + K_{11}\, u_1 = p\, g(t) \tag{5}$$

The small amount of intrinsic damping in the structure is inserted at the modal level. A few elastic struts are replaced by viscously damped struts to enhance the damping in the structure. For design purposes, it is assumed that the truss behavior is governed by the axial properties of the struts. Then, a typical viscous strut can be characterized by a three-node model with an internal dashpot[7]. The modified structure requires additional degrees of freedom to model the dashpots in the finite element model. Let the additional nv degrees of freedom be represented by u_2. The governing differential equations are now given by:

$$\begin{bmatrix} M'_{11} & 0 \\ 0 & M'_{22} \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} C'_{11} & C'_{12} \\ C'_{21} & C'_{22} \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} K'_{11} & K'_{12} \\ K'_{21} & K'_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} p \\ 0 \end{bmatrix} g(t) \tag{6}$$

Equation (6) describes the behavior of a structure yet to be designed. The damping design will entail the locations, number, mass, damping and stiffness properties of the viscous struts. Unlike the viscoelastic struts, all the structural properties specified in Equation (6) are frequency independent. Using engineering assumptions, a design procedure can be derived to approximate the solution to these governing equations.

In order to utilize the information of the baseline structure to help the damping design, it is important to recast Equation (6) into the same number of degrees-of-freedom as the baseline structure. Assuming the mass at the internal degrees of freedom of the struts is small and the internal dynamics of the struts is not important to the solution, then for a harmonic force input this condition is summarized as:

$$\begin{bmatrix} \mathbf{M}'_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}'_{22} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_1 \\ \ddot{\mathbf{u}}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{C}'_{11} & \mathbf{C}'_{12} \\ \mathbf{C}'_{21} & \mathbf{C}'_{22} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}_1 \\ \dot{\mathbf{u}}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{K}'_{11} & \mathbf{K}'_{12} \\ \mathbf{K}'_{21} & \mathbf{K}'_{22} \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{p} \\ \mathbf{0} \end{bmatrix} e^{i\omega t} \tag{7a}$$

$$\mathbf{M}'_{22} = \mathbf{0} \tag{7b}$$

The $\mathbf{u}_2$ degrees of freedom can be condensed out by using the second matrix equation of Equation (7a):

$$\mathbf{u}_2 = (i\omega \mathbf{C}'_{22} + \mathbf{K}'_{22})^{-1} (i\omega \mathbf{C}'_{21} + \mathbf{K}'_{21})\, \mathbf{u}_1 \tag{8}$$

Backsubstitute $\mathbf{u}_2$ into the first matrix equation of Equation (7a) and collecting terms, Equation (7a) can now be represented by:

$$\tilde{\mathbf{M}}\, \ddot{\mathbf{u}}_1 + \tilde{\mathbf{K}}\, \mathbf{u}_1 = \mathbf{p}\, e^{i\omega t} \tag{9a}$$

In this form, the stiffness matrix is complex:

$$\tilde{\mathbf{K}} = \tilde{\mathbf{K}}^R + i\, \tilde{\mathbf{K}}^I \tag{9b}$$

It is also a function of both the stiffness and damping characteristics of the struts ($\mathbf{u}_2$ degrees of freedom). Despite the dissimilarity in appearance, Equations (7) and (9) are identical descriptions of the same system. Equation (7) is the preferred form for analytical computation while Equation (9) is very useful to guide the damping design. For damping design, there is no need to form $\tilde{\mathbf{K}}$ explicitly. Instead, the contribution of each strut element to $\tilde{\mathbf{K}}$ is evaluated individually. The contribution of each strut to system level damping can be assessed through its contribution to $\tilde{\mathbf{K}}^I$.

STRUT CHARACTERISTICS

In order to design damping at the system level, the damping and stiffness characteristics of the damped struts must be totally understood. A class of viscously damped struts can be represented by three frequency independent parameters[7] as shown in Figure 1.

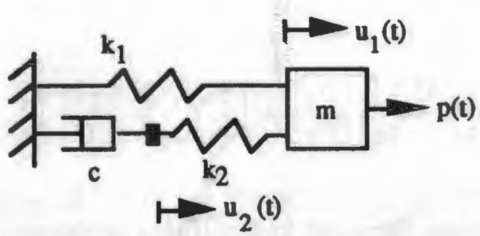

Figure 1. 3-Parameter 2 DOFs Viscous Strut Model

The dynamic characteristics of this class of strut were derived based on a similar method such that the results can be used for system level damping design. The stiffness and damping characteristics of the struts are summarized here. The strut is represented by a complex stiffness:

$$k = k^R (1 + i\,\eta) \tag{10a}$$

where,

$$k^R = k_1 \left[\frac{\kappa^2 k_1{}^2 + (1+\kappa)(c\omega)^2}{(c\omega)^2 + \kappa^2 k_1{}^2} \right] \tag{10b}$$

$$\eta = \frac{\alpha^2 (c\omega) k_1}{\kappa^2 k_1{}^2 + (1+\kappa)(c\omega)^2} \tag{10c}$$

$$\kappa = \frac{k_2}{k_1} \tag{10d}$$

Normalized design curves, optimum strut damping and damping bandwidth can be found in Reference 7. The maximum loss factor, η_{op}, is governed by κ only:

$$\eta_{op} = \frac{\kappa}{2\sqrt{1+\kappa}} \tag{11a}$$

and the frequency at which this maximum loss occurs, ω_{op}, is governed by the damping coefficient:

$$\omega_{op} = \frac{\kappa}{\sqrt{1+\kappa}} \frac{c}{k_1} \tag{11b}$$

A simple 3-parameter viscous strut model allows a simple strut representation for system level design and a simple strut performance specification for component level design.

APPROXIMATE ANALYSIS OF DAMPED STRUCTURES

From a practical standpoint, if a few struts are replaced by damped struts to increase system damping, say to around 10%, the basic undamped structural characteristics should not be changed significantly. Based on this assumption, the Rayleigh-Ritz method can be used to compute the approximate solution to Equation (1)[10]. This not only expedites the computation significantly but it also provides a direct physical insight into the "modal" damping synthesis of the structure. The undamped normal modes are used as the basis vectors (generalized coordinates):

$$u \approx \sum_{i=1}^{m} \phi_i\, q_i = \Phi\, q \tag{12}$$

where the eigenvalue problem is performed at a selected frequency of interest:

$$\tilde{K}^R \phi_i \approx \tilde{\omega}_i^2 \tilde{M} \phi_i \tag{13}$$

In practice, the mode shapes of the baseline structure are used to start the design process. Then Equation (9) is approximated by:

$$\Phi^T \tilde{M} \Phi \ddot{q} + \Phi^T \tilde{K} \Phi q \approx \Phi^T p\, g(t) \tag{14}$$

For design purposes, assume the coupling between generalized coordinate does not significantly affect the dynamic response. The approximate uncoupled equations of motion are therefore:

$$\phi_i^T \tilde{M} \phi_i \ddot{q}_i + \phi_i^T \tilde{K} \phi_i q_i \approx \phi_i^T p\, g(t) \tag{15a}$$

$$\left[\left(-\omega^2 \phi_i^T \tilde{M} \phi_i + \phi_i^T \tilde{K}^R \phi_i\right) + i\left(\phi_i^T \tilde{K}^I \phi_i\right)\right] q_i e^{i\omega t} \approx \phi_i^T p\, e^{i\omega t} \tag{15b}$$

The equivalent "modal" characteristic of the generalized coordinate can be found by equating the complex stiffness to a single degree of freedom system at resonance frequency:

$$\left[\left(-\omega^2 + \omega_o^2\right) + i\left(2\xi_o \omega \omega_o\right)\right] q_o e^{i\omega t} = \frac{p}{m} e^{i\omega t} \tag{16}$$

The "modal" frequency of the modified structure can therefore be approximated by:

$$\tilde{\omega}_i = \sqrt{\frac{\phi_i^T \tilde{K}^R \phi_i}{\phi_i^T \tilde{M} \phi_i}} \tag{17}$$

but the change in structural weight of the struts with respect to the overall structural and non-structural weight is often very small such that $\phi_i^T \tilde{M} \phi_i \approx 1$. The equivalent damping near resonance can therefore be approximated by:

$$\xi_i = \frac{1}{2} \frac{\phi_i^T \tilde{K}^I \phi_i}{\tilde{\omega}_i^2} \tag{18}$$

ELEMENT MODAL CONTRIBUTION

Based on this approximate analysis method, it is possible to assess the contribution of a viscous strut at a given location to the system level damping and stiffness change. Decompose the global stiffness into element stiffness contributions (ne = number of elastic elements and nv = number of damped struts):

$$\tilde{K} = \left(\sum_{j=1}^{ne} k_j + \sum_{j=1}^{nv} \tilde{k}^R_j(\omega)\right) + i \sum_{j=1}^{nv} \eta_j(\omega)\, \tilde{k}^R_j(\omega) \tag{19}$$

The "modal" stiffness is given by:

$$\tilde{\omega}_i^2 = \phi_i^T \left(\sum_{j=1}^{ne} k_j + \sum_{j=1}^{nv} \tilde{k}^R{}_j(\omega) \right) \phi_i \tag{20}$$

The normalized "modal" stiffness contribution of the j-th viscous strut to the system is given by:

$$\varepsilon_{ij} = \frac{\phi_i^T \tilde{k}^R{}_j \phi_i}{\tilde{\omega}_i^2} \tag{21a}$$

The normalized "modal" stiffness contribution is also identical to the "modal" stain energy (MSE) ratio:

$$\varepsilon_{ij} = \frac{w_{ij}}{\sum_j w_{ij}} = \frac{\frac{1}{2}\phi_i^T \tilde{k}^R{}_j \phi_i}{\frac{1}{2}\tilde{\omega}_i^2} \tag{21b}$$

The "modal" damping ratio contribution of the j-th viscous strut to the system is given by:

$$\xi_{ij} = \frac{\eta_j}{2} \frac{\phi_i^T \tilde{k}^R{}_j \phi_i}{\tilde{\omega}_i^2} = \frac{\varepsilon_{ij}\,\eta_j}{2} \tag{22}$$

The system level damping from all the viscous struts is therefore simply given by:

$$\xi_i = \frac{1}{2} \sum_{j=1}^{nv} \varepsilon_{ij}\,\eta_j \tag{23}$$

The assumptions used in deriving these approximations provides a very simple concept for damping design. It is clear from Equation (23) that there are three key parameters in system level modal damping design: the strut locations, ε_{ij}, the strut loss factors, η_j, and the total number of viscous struts, nv. For a given mode, the strut location with the highest strain energy ratio is the most effective location in providing damping. This location has the maximum relative displacement, hence relative velocity, to activate the viscous damper. The strut with higher loss factor also provides higher system level damping. The contribution of each damped strut to the system level damping is proportional to the strut loss factor and the modal strain energy ratio. System level damping can also be increased by incorporating more struts. Of course, as the most effective locations are occupied, the effectiveness of an additional strut is diminishing as the modal strain energy ratio is declining.

SYSTEM LEVEL DAMPING DESIGN PROCEDURE

In the beginning of a design cycle, the baseline structure is modeled and analyzed. The performance of the structure is not satisfactory and higher damping is required in a few modes to reduce the dynamic responses or stabilize the control system. Consequently modal damping ratios are specified as design requirements. The modal properties of the baseline structural model can be used to start the design process. The modal strain energy ratio of each strut member is computed:

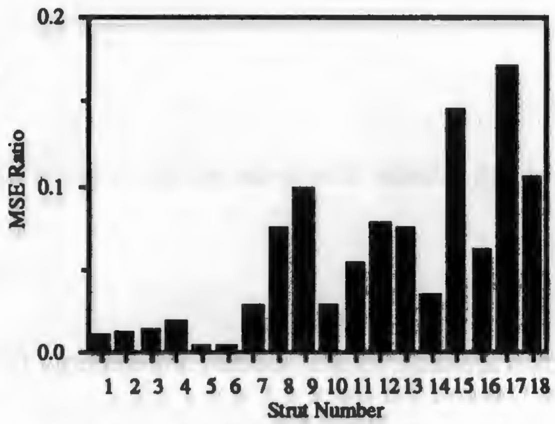

Figure 2 Modal Strain Energy Distribution

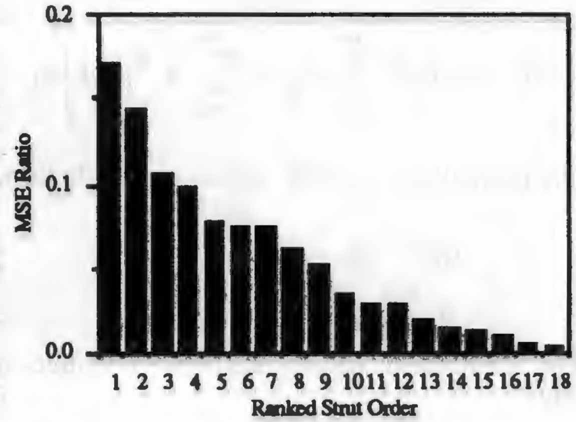

Figure 3 Rank Modal Strain Energy

$$\varepsilon_{ij} = \frac{\phi_i^T k_j \phi_i}{\tilde{\omega}_i^2} \tag{24}$$

A typical modal strain energy distribution is shown in Figure 2. For the mode of interest, rank strut members in descending order of modal strain energy ratios as shown in Figure 3. The order of the struts should be noted as shown in Table 1.

Table 1 Strut Modal Strain Energy Data

Strut Order	1	2	3	4	5	6	7	8	9	10	11	12
Strut No.	17	15	18	9	12	8	13	16	11	14	7	10
MSE	0.170	0.143	0.105	0.098	0.077	0.073	0.073	0.061	0.052	0.034	0.027	0.027
Cum MSE	0.170	0.314	0.418	0.516	0.593	0.666	0.739	0.800	0.852	0.886	0.914	0.941

If only one type of strut, with component loss factor η, is used, compute the cumulative sum of the the ranked modal strain energy ratios:

$$\varepsilon_{il} = \sum_{j=1}^{l} \varepsilon_{ij} \tag{25}$$

A typical plot of the cumulative strain energy of the ranked struts is shown in Figure 4.

Assuming a realistic strut loss factor, and working with realistic static and dynamic strut stiffnesses, find the suitable k_1 and k_2 by using Equations (10) and (11). Determine the frequency characteristics of the strut by defining the c parameter in Equation (11b). Iterate if necessary to optimize the strut design. Compute the dynamic stiffness and loss factor at the frequencies of interest. A typical strut loss factor curve is shown in Figure 5.

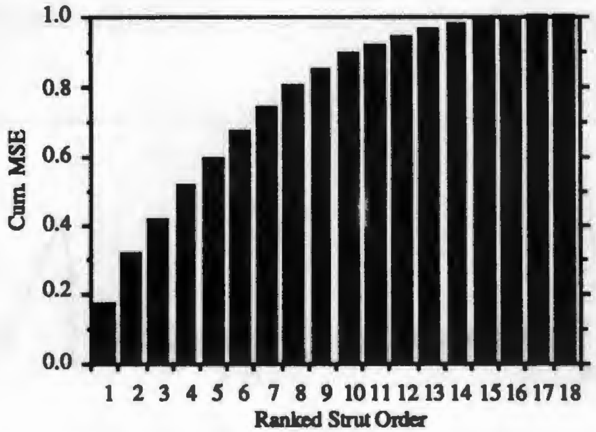

Figure 4 Cumulative Strut Modal Strain Energy

The system level modal damping for l number of struts is given by:

$$\xi_{ij} = \frac{\eta}{2} \varepsilon_{il} \tag{26}$$

The system level damping is a product of strut loss factor and contributions from participating struts, i.e. higher strut damping requires less members and vise versa. Iterate to determine the necessary component loss factor and number of struts to meet the design requirement on a mode by mode basis. Candidate struts for each modes are identified.

Candidate struts from all the modes are included in the final design to meet the design requirements. The modal strain energy ratios, ε_i, of these struts are computed as shown in Figure 6. The approximate system level damping is given by:

$$\xi_i = \frac{1}{2} \eta \, \varepsilon_i \tag{27}$$

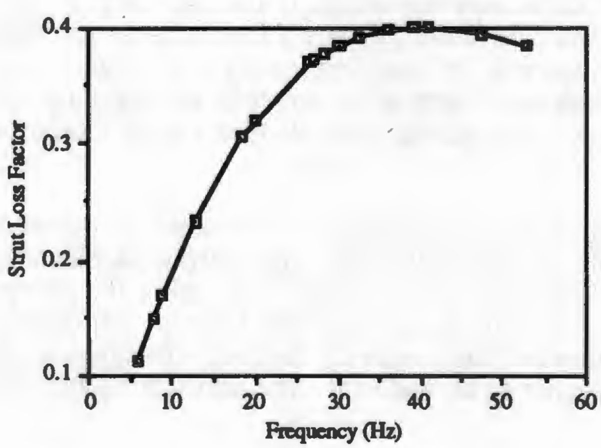

Figure 5 Strut Loss Factor vs Frequency

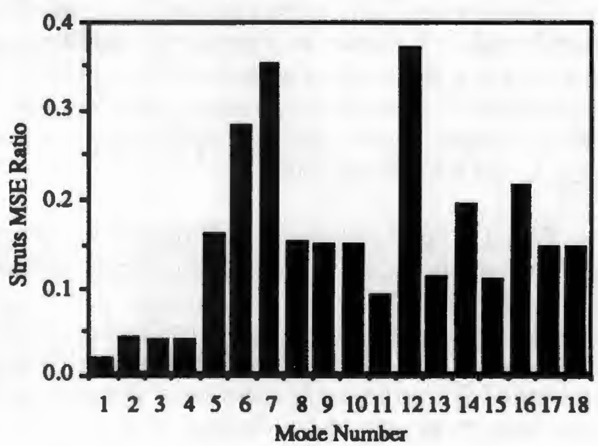

Figure 6 Struts MSE Ratio vs Mode Number

The system level damping is a product of the strut loss factor at the modal frequency and the participation of the selected struts at the system level. The modal strain energy ratios with respect to frequency are shown in Figure 7. The system level damping curve, Figure 8, is simply the product of Figures 6 and 7.

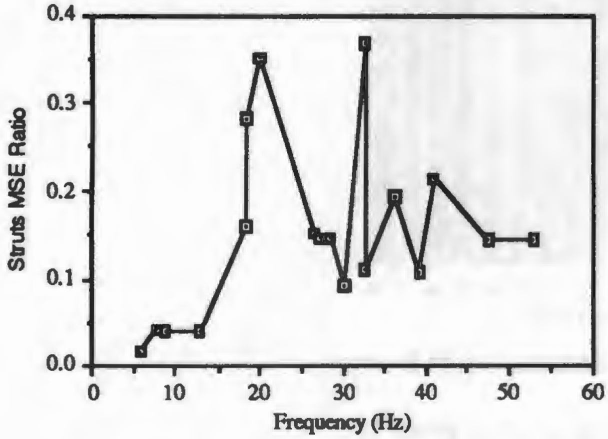

Figure 7 Struts MSE vs Frequency

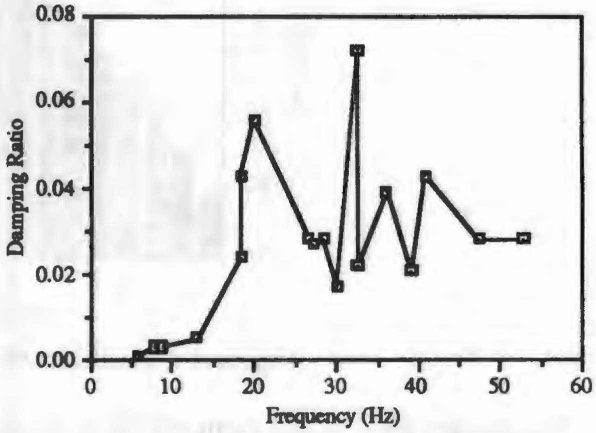

Figure 8 Predicted System Level Damping

The approximate system frequencies can be predicted by considering the "modal" stiffness contribution of the viscous struts and the relative change in dynamic axial stiffness:

$$\tilde{\omega}_i = \sqrt{1 + \left(\frac{k^R}{k_o} - 1\right)\varepsilon_i}\;\omega_i \tag{28}$$

As can be seen from the derivation of this procedure, quite a few assumptions were used in order to establish this simple procedure. As in any design process, iterations are required to refine the initial design. The number of struts and the strut parameters may be optimized. Also, for better damping prediction at different frequency ranges, the baseline finite element model can be updated to reflect the dynamic strut stiffness in accordance with Equation (10b) so that the strain energy distribution is better represented at the frequency ranges of interest. The effect of modal damping coupling can also be evaluated if necessary. However, it may be more expedient to let the verification analysis provide the final verdict. Generally speaking, if the damping is well distributed, it is closer to a proportional damping case. However, if only a few dampers are used to provide a substantial amount of damping to the system, the damping matrix can be quite non-proportional. If the strut placement also changes the mode shapes of the structure substantially, the original mode shapes are not a good approximation. An updated finite element model should be used as the baseline model.

When a good, practical and balanced damping design is in hand, the finite element model is updated to include all the viscous struts which are modeled by elastic and viscous elements as shown in Equation (6). This model removes all the assumptions imposed during the design process and provides the best engineering predictions of the behavior of the structure damped by viscous struts. The system damping and frequencies of the passively damped structure can be computed from the complex eigenvalues using Equations (4a) and (4b). The verification analysis is an important step to the design process.

There can be many variations to the method presented. Different struts with different stiffness and damping characteristics can be added to the structure due to the geometric difference of struts to be replaced. If damping is to be optimized over a wide frequency range, struts with different frequency characteristics can be used. However, the basic principles are still the same. The procedure can be modified to accommodate such special circumstances. A summary of the design and analysis procedure is provided in Table 2.

Table 2. Summary of Design and Analysis Procedure

1. Understand damping design requirements.
2. Perform eigenvalue analysis of baseline structural model to get ω_i and ϕ_i. Compute modal strain energy ratios of strut members, ε_{ij}.
3. Rank strut members in descending order of modal strain energy ratios for modes of interest.
4. Compute cumulative sum of the ranked modal strain energy ratios, ε_{i1}, for modes of interest.
5. Assume a physically achievable strut loss factor, η_j. Estimate required dynamic stiffness. Iterate to find the k_1, k_2 and c parameters to obtain static stiffness, maximum loss factor and frequency characteristics of struts. Compute the loss factors at the frequencies of interest, $\eta(\omega_i)$.
6. Find the number of struts required to meet the damping requirements for the given modes. For each mode, locate the struts. The set of viscous struts is all the members required for all the modes.
7. Compute the modal strain energy ratios of the set of viscous struts, ε_i.
8. Compute the predicted system level damping, $\eta(\omega_i)\varepsilon_i$.
9. Iterate upon the number of struts, strut locations, and strut parameters as necessary.
10. Iterate upon the accuracy of mode shapes and modal strain energy distributions at selected frequencies if necessary.
11. Update finite element model to include dashpots and perform complex eigenvalue problem to verify damping design.
12. Iterate as necessary to correct for any deficiency from complex eigensolution.

TRUSS DESIGN EXAMPLE

A small example is included here to illustrate the method. A 3 bay truss with 13 nodes and 60 active degrees of freedom is used. The first two bending modes of the structure are 21.3 Hz. The goal is to design 5% viscous damping into the system. For the mode of interest, a bottom longeron member has significant amount of modal strain energy. One damped strut is used to replace the original strut. The damped strut is designed to have k_1 = 110,491 lb/in, k_2 = 114,955 lb/in, c = 839.88 lb-sec/in. This corresponds to a loss factor of 0.36 in the member at the frequency of interest. The updated finite element model now has 122 equations in the first order form. The solution from the complex eigenvalue problem is summarized in the Table 3. The results are very satisfactory considering that only very little amount of computation required to arrive at this design.

Experience in working with large truss structures showed that the design procedure is quite effective and the design prediction and analytical solution are often quite close.

Table 3 Comparison of Design Prediction and Analytical Solution

Parameters	Design Goal	Eigensolution	Error
Frequency	21.3 Hz	21.95 Hz	3%
Damping	5%	5.45%	9%

CONCLUSION

A comprehensive design and analysis method for integration of viscously damped struts into large precision truss structure is presented. The method is based on an approximate solution to the governing differential equations using the Rayleigh-Ritz method. Simplification to a practical design procedure is facilitated by making relevant engineering assumptions for the struts and the truss behavior. The method effectively uses the modal data from the baseline structural model. A simple design procedure is use to determine the strut placement locations, the strut stiffness and damping parameters, and number of struts required to meet the design objectives. Upon completion of a damping design, a rigorous verification analysis is performed to check the passive design. Therefore all the assumptions used in the design process will not affect the accuracy of the analytical prediction. The method is simple, efficient and accurate, and has been used for large structures with good success.

NOMENCLATURE

Symbols

C,c = viscous damping matrix, coefficient
g = forcing function
i = imaginary unit, $\sqrt{-1}$
I = identity matrix
K, k = stiffness matrix, stiffness coefficient, strut axial stiffness
M, m = mass matrix, mass
p = spatial force vector
t = time, second
u = displacement vector
w = element strain energy
β = non-dimensional frequency parameter for viscous strut
ε = strain energy ratio
ϕ, Φ = eigenvectors, eigenvector matrix
η = loss factor
κ = non-dimensional stiffness ratio viscous strut
ξ = viscous damping ratio
λ, Λ = eigenvalue, eigenvalue matrix
ω = forcing frequency or natural frequency when used with index, radian/second
$\sim$ = denoting modified elements

Subscripts

i = for the i-th mode
j = the j-th strut element
l = number of viscous struts in descending order of modal strain energy
m = number of modes in solution
o = single degree of freedom system, pertaining to original component
op = condition at maximum loss factor
v = viscoelastic
u = displacement
$\dot{u}$ = velocity
ξ = damping ratio
1 = baseline degrees of freedom, or outer spring of viscous strut
2 = additional degrees of freedom for dashpots, or inner spring of viscous strut

Superscripts

I = Imaginary
R = Real
T = matrix transpose

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