A REVIEW OF OPTIMIZING/COMPUTER CONTROL

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ABSTRACT

Optimizing control has as its principal objective the maintenance of the optimum performance of a multi-variable system subject to both disturbing and constraining influences. First, the system performance criteria must be defined. Then optimum performance can be achieved by either of two basically different methods: one, a direct approach in which the output performance is compared with the input manipulation to determine the system behavior and thus provide the direction for optimizing control of the system; this may be done with or without a perturbation or test signal. Two, a model method in which the model provides the basis for analytical definition of the optimizing control conditions for the system. The model is manipulated such that its behavior agrees with the observed behavior of the system.

Optimizing computer control is gaining attention as a further means for achieving better control of a complex physical system. The single negative feedback loop no longer suffices for control of systems with widely varying parameters or systems with parameters which are either undefined or not completely known. Large disturbances or load changes also emphasize the need for more advanced control.

The development of reliable computer techniques in both analog and digital form have made it possible to consider much more complex working methods in automatic control than the conventional proportional, integral,and derivative effects. These include nonlinear and higher order control functions, application of filtering, prediction and correlation techniques, repetitive computer methods, etc.

PERFORMANCE COMPUTATION

The system under control can generally be described, as in Figure 1, as having k outputs under the influence of i independent inputs (determined by factors external to the system under consideration), and under the influence of j dependent inputs which may be manipulated.

The system presumably has a utility of some sort; that is, the system is put into use with the outputs forming a valuable product or service. It is assumed that the utility can be judged by some appropriate method so that the performance of the system can be computed.

There are two general methods of specifying performance and these are first, economic and second, technical. Economic performance is often expressed as a linear combination of system variables

$$p = \sum_i K_i u_i + \sum_j K_j m_j + \sum_k K_k q_k$$

where p = performance criterion

u_i = independent input variables

m_j = dependent (manipulated) input variables

q_k = system output variables

K_i, K_j, K_k = Appropriate profit or cost coefficients

On the other hand, technical performance is often specified in such terms that an optimum or best value may exist. For example, in many industrial processes,

$$p = f(m_j, u_l, q_k)$$

The necessary conditions for the optimum are determined by setting

$$\frac{\partial p}{\partial m_j} = g(m_j, u_l, q_k) = 0$$
$$j = 1, 2, \ldots$$

Another example is the formulation of best performance in the traditional sense of minimum mean-square error criterion,

$$p = \lim_{T \to \infty} \frac{1}{T} \int_0^T \sum_k (w_k - q_k)^2 \, dt$$

where w_k represents the desired functions corresponding to system outputs, q_k. Whether performance is optimized as an extremum control in which a maximum or minimum value is maintained or is optimized as a feed-back control in which a near-zero difference is maintained is not important because one form can be readily converted into the other.

Constraining influences are very important and it is often necessary to subject the control system to bounds of the form,

$$Q_{1k} < q_k < Q_{2k}$$

or of the form

$$q_k \geq h(q_1, q_2, \ldots)$$

These constraints often make it necessary to find system performance at a limit which is in turn a function of other variables. The performance computer of Figure 1 is therefore employed to compute the necessary functions and obtain the variable (p) on which to base system control.

DIRECT OPTIMIZATION

Direct optimization proceeds with a minimum of knowledge about the system under control. As shown in Figure 2, the optimizer receives data on the variations in the manipulated variables (m_j) and the resulting changes in performance (p) and in turn manipulates each of the m_j inputs in the indicated direction for improving the performance. Sometimes a perturbation or test signal is employed in order to initiate changes upon which the control measurements are based.

Direct optimization is thus exploratory or experimental in nature in that the results of each manipulation is assessed and another manipulation is made. These may be done sequentially or simultaneously in a number of manipulated variables, depending upon the type of exploring scheme in use.

The direct method may be achieved through continuous measurements or by the use of sampled and/or quantized data. In either case, the general principle of the optimization system may be the same.

DIRECT OPTIMIZATION WITHOUT PERTURBATION

Direct optimization may proceed without employing a perturbation or test signal; one such method is shown in Figure 3. The divider generates the derivative of performance with respect to the manipulated variable (m_j) and thus determines when the performance is maximum. Analysis of the dynamics shows that the system is stable for a number of practical applications. When several manipulated variables are involved, the divider-integrator circuit is repeated for each variable and the manipulations are performed either sequentially or simultaneously.

This method has the very great advantage of simplicity and easy realizability and has been applied to a number of industrial processes. The disadvantages of this method are that only a relatively few (four or five) manipulated variables can be accommodated and that its speed of response is limited by the dynamics of the system.

DIRECT OPTIMIZATION WITH PERTURBATION*

The perturbation or test signal method shown in Figure 4 employs the perturbation to disturb the system. The response of system performance (p) is then correlated to the perturbation to generate control signals for the system inputs (m_j). The particular method shown in Figure 4 employs the continuous time integral of the product of system performance and the perturbation signal. The perturbation signal may be a sinusoid or other periodic wave, white noise in continuous or discret form or an impulse sequence. Each of these forms of the signal has a particular use depending upon the system and control method employing it.

The perturbation method also has good stability in a number of practical applications. When several manipulated variables are involved, the multiplier-integrator circuit is repeated for each variable. In this case, it is sometimes convenient to use perturbation signals of differing time scale or frequency in order to discriminate the effects of simultaneous manipulations.

* See references 1 and 5

This method has the advantage of simplicity and may be used very adequately in multi-dimensional problems. It has been applied to engine control as well as industrial processes. The disadvantages of this method are that the perturbations may be undesirable (as in machine tool control) and often the system becomes unstable for too small a perturbation signal amplitude.

MODEL METHODS

Model methods provide an alternate approach to optimizing control. In general, the necessary conditions for optimum performance of the system under control are determined on the basis of an appropriate system model. The model may form an integral part of the control system or it may only be present in concept. It may range from some physical simulation or analog of the process to a mathematical abstraction manifested as a set of equations or a multi-dimensional surface describing the system behavior.

PREDETERMINED OPTIMIZATION

If the model is complete and exact, then the conditions for optimization can be determined completely and exactly. In particular, these conditions may be predetermined for any given set of constraints and boundary conditions.

A conceptual approach to Predetermined Optimization is given in Figure 5. An optimizing computer determines paths or functions for the manipulated variables, m_j, based on the predicted behavior of the system as described by the model variables, w_k. The actual behavior of the system is described by q_k; system performance is then gauged in terms of the q_k variables.

It is apparent that, once computed, the optimizing conditions can be stored on punched-tape, magnetic drum or even, in the simple two-dimensional case, on a mechanical cam. The system variables are then manipulated according to the playback of the appropriate stored program. Note that in systems manually operated or supervised, the operator is often guided by a predetermined optimizing program stored graphically or in tabulated form or stored mentally in the guise of experience.

The control scheme described above is essentially open-loop; i.e., there is no feedback of information to verify either that the resulting system performance is as specified, or that the model accurately describes the system behavior. Accordingly, if there are any factors tending to cause the system to deviate from the model as, for example, the influence of disturbances u_i, the system performance may be expected to deviate from the computed optimum.

REPETITIVE COMPUTED OPTIMIZATION

The predetermined optimization concept is modified by repetitive feedback of information describing the state of the system. Thus, as shown in Figure 6, the q_k variables are periodically sampled into the optimizing computer providing the basis for repetitive recomputation of the optimizing conditions. In this way, each computation is based on the most recent information describing the state of the process. As a result, deviations of the system from the postulated model do not cause cumulative errors. Indeed, the repetitive computer action tends to force the system to the desired performance despite significant inadequacies of the model.

SELF-CHECKING

In practical applications of optimizing computer control, it is expected that the postulated model will deviate significantly from the actual system behavior. There are several reasons for this:

1. The system may not be known in complete and accurate analytical form.

2. Some of the variables describing the state of the system may not be readily measured to provide feedback to the optimizing computer.

3. The system may be so very complex that a control program based on the complete model would be impractical from the standpoint of computer capacity.

The repetitive control concept described in the preceding section is effective in substantially reducing the effect of model deviations on system performance. Limitations are introduced, however, in terms of the repetitive period, measurement dead-time, limiting of the manipulated variables, and the general order of the approximations employed.

The self-checking concept is superimposed on the repetitive control action to extend its effectiveness and range of applicability. This is illustrated in Figure 7.

Self-checking refers to the periodic adjustment of the parameters of the model (y_k) such as to force the model to agree (at least within the neighborhood of the operating point) with the observed behavior of the system. The behavior of the system is represented by variables q_k (see Figure 7); the predicted behavior based on the model is represented by w_k.

An important conceptual distinction is noted here: whereas most of the discussions on adaptive control considered a fixed model and adjusted the parameters of the control system to fit this model, the basic idea presented here is that of a model adjusted to fit the system.

EXAMPLE: OPTIMIZING CONTROL OF A BATCH CHEMICAL PROCESS

The model method concept has been applied to the optimizing control of a batch chemical process. A simple prototype of the process studied is described very briefly* as follows:

The reaction mixture is made up of three chemical components identified as X, Y, and Z. Hydrogen under pressure and in the presence of catalyst reacts with X and Y according to the following reaction scheme:

$$X + H_2 \xrightarrow{k_1} Y$$

$$Y + H_2 \xrightarrow{k_2} Z$$

where k_1 and k_2 represent kinetic reaction coefficients.

A reasonable approximation to the kinetic behavior of this process is given by the equations,

$$\frac{dx}{dt} = - k_1 \; x \tag{1a}$$

$$\frac{dy}{dt} = k_1 x - k_2 y \tag{1b}$$

$$x + y + z = 1 \tag{1c}$$

where x, y, z represent molar concentrations of components X, Y, Z, respectively.

The kinetic coefficients are functions of the operating conditions: pressure, temperature, catalyst, agitation, etc. Assuming only pressure is to be manipulated and that all other influencing factors are relatively constant, the coefficients may be expressed,

* See references 4, 7, and 8 for a more detailed description of the process and the computer control application.

$$k_1 = A_1 p^{N_1} \quad (2a)$$

$$k_2 = A_2 p^{N_2} \quad (2b)$$

where A_1, A_2, N_1, N_2 are assumed constant

p = process pressure.

Based on a mathematical model consisting of Equations (1, 2), the necessary conditions for optimum process performance may be derived. In the particular case under study, control to a specified product composition consistent with minimum processing time is established as the performance criterion. By means of the calculus of variations, the following optimizing control equations are derived:*

$$\frac{du}{dv} = (1 - k)u + 1 \quad (3a)$$

$$\frac{dk}{dv} = \left(\frac{N_1}{N_2} - 1\right)\frac{k}{u} \quad (3b)$$

where $u = y/x$

$$v = \log_e x_o/x$$

$$k = \frac{k_2}{k_1} = \frac{A_2}{A_1} p^{N_2 - N_1}$$

Equations (3a, b) are solved by the optimizing computer such as to satisfy the boundary conditions (x_o, y_o, z_o) representing the initial composition and (x_f, y_f, z_f) representing the desired final composition.**

If equations (1, 2) described the process behavior exactly, then one computation based on Equations (3a, b) and the specified boundary conditions would suffice to define the optimum control path, p(t). Thus, the p(t) schedule could be recorded on tape or other storage medium and played back through appropriate transducers and pressure controller to manipulate the process pressure according to the schedule. In the example under consideration, however, the model only approximates the process kinetics because of the neglect of such factors as variations in catalyst activity, other components in the reaction mixture, higher order terms in the kinetic equations, etc.

* See references 4 and 8 for derivations.
**In terms of the u, v coordinates, these boundary conditions are expressed as (u_o, O) and (u_f, v_f), respectively.

Open-loop control of the process would lead, therefore, to very significant deviations from the desired end-point.

The repetitive control concept was applied here. Equations (3a, b) are solved for the optimum control path leading from the current state of the process (based on the most recent composition measurement of the reaction mixture) to the specified final composition. Thus, each time a new composition measurement is made available to the computer, a new control path is computed. This technique was demonstrated to be very effective in forcing the process to the prescribed performance.

Application of the self-checking concept to this system is currently being implemented. It is assumed, in this approach, that the inadequacies of the model may be absorbed in the parameters of Equations (2a, b). Thus, the actual progress of the reaction is compared with that described by Equations (1, 2). The parameters A_1, A_2, N_1, and N_2 are adjusted periodically such as to minimize the discrepancy between system behavior and model prediction. Coupled with this self-checking is a statistical smoothing technique by which random errors in measurement are filtered out.

The combination of repetitive computer control and self-checking provides the basis for wide practical applicability of the model approach to optimizing control. In particular, the control effectiveness becomes very much less dependent on the accuracy or completeness of the model employed, providing there is an adequate information feedback to the control computer. Thus, the optimizing control for typically very complex systems may be realized with computer control facilities within the bounds of economic justification.

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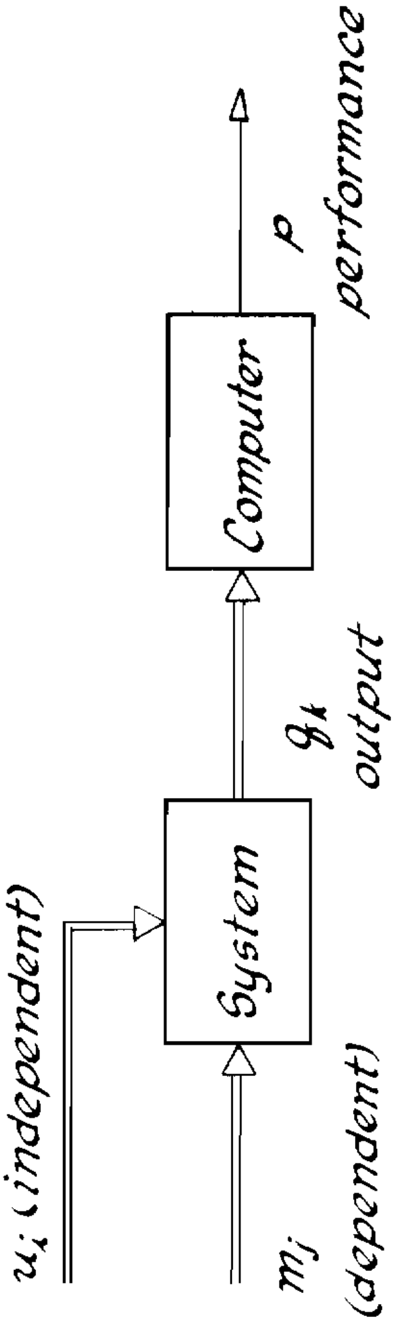

Fig.1 The System Performance

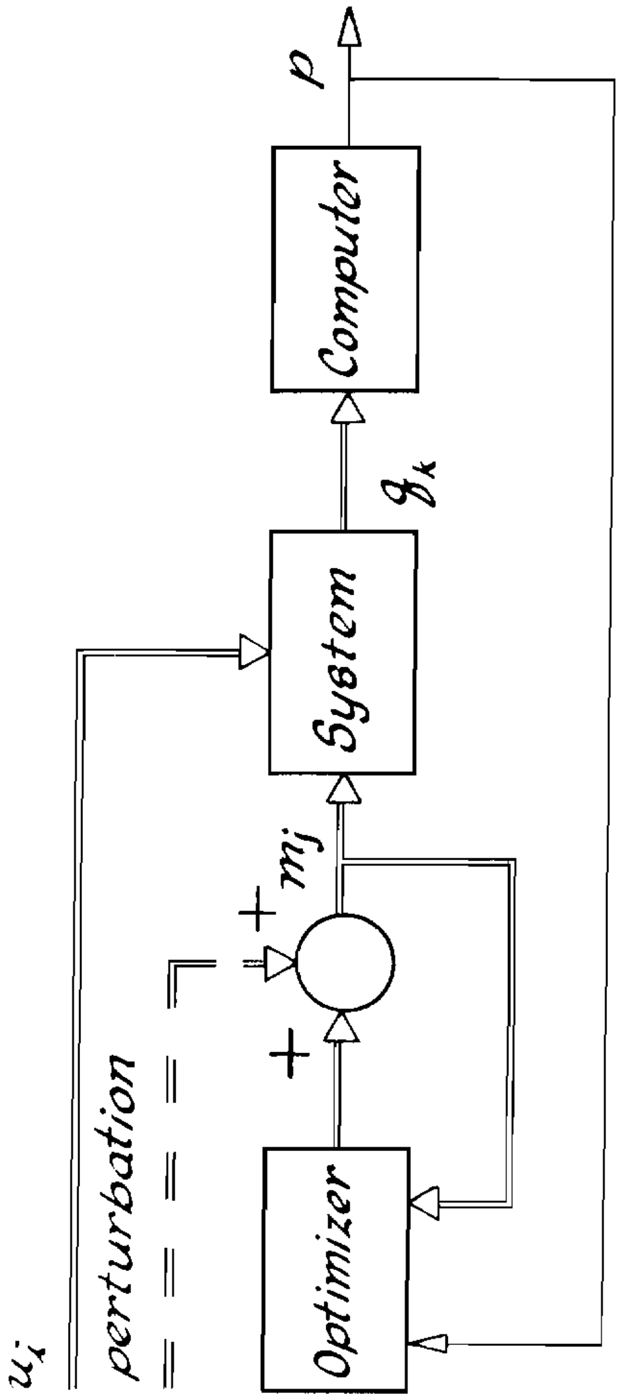

Fig. 2 Direct Optimization

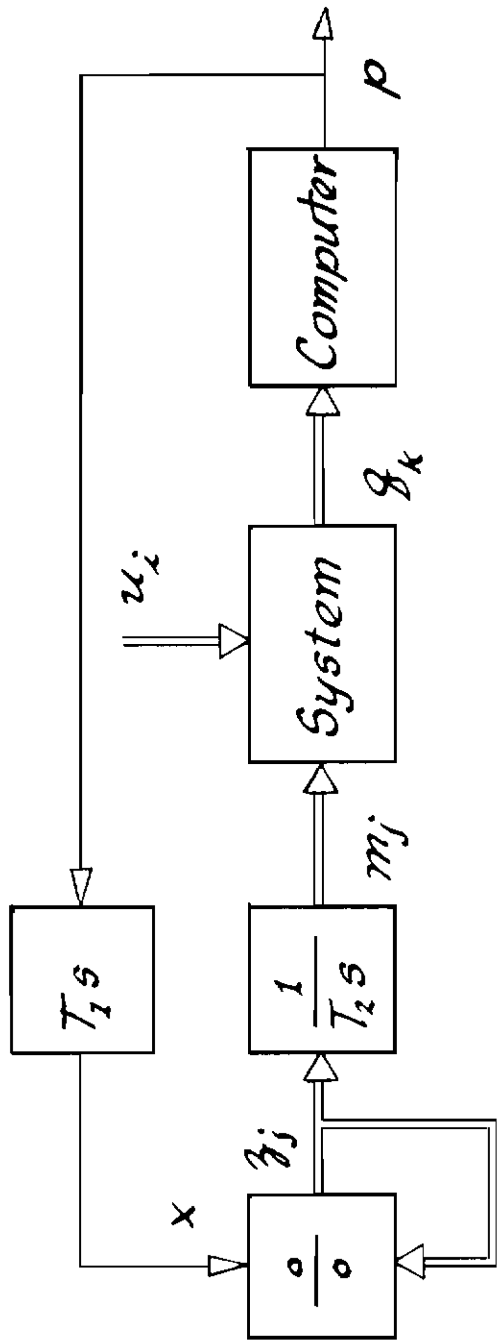

Fig. 3 Direct Optimization (without perturbation)

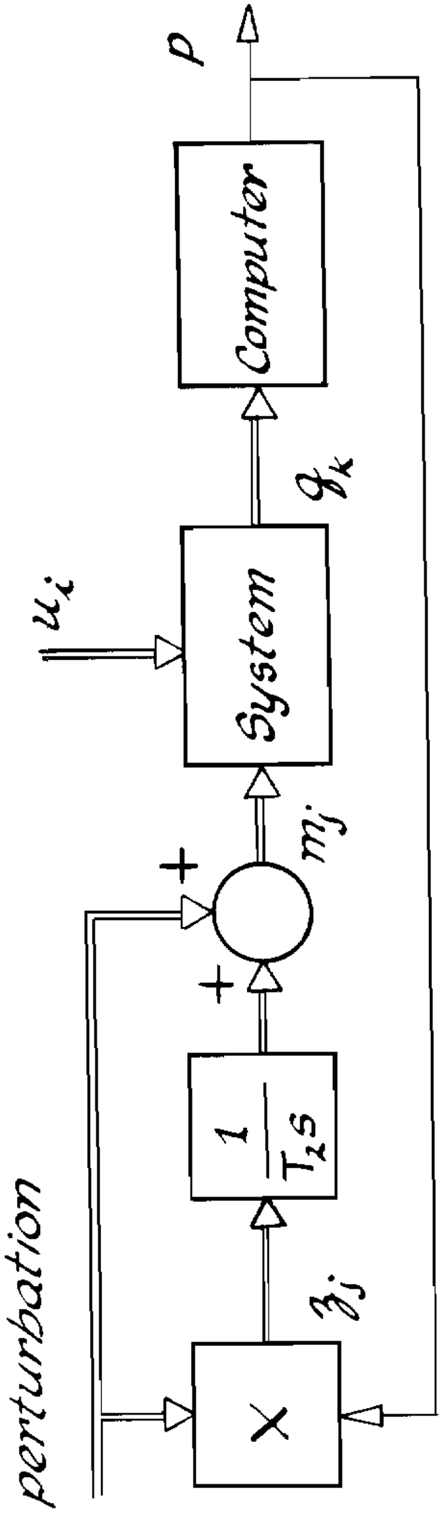

Fig. 4 Direct Optimization by Perturbation

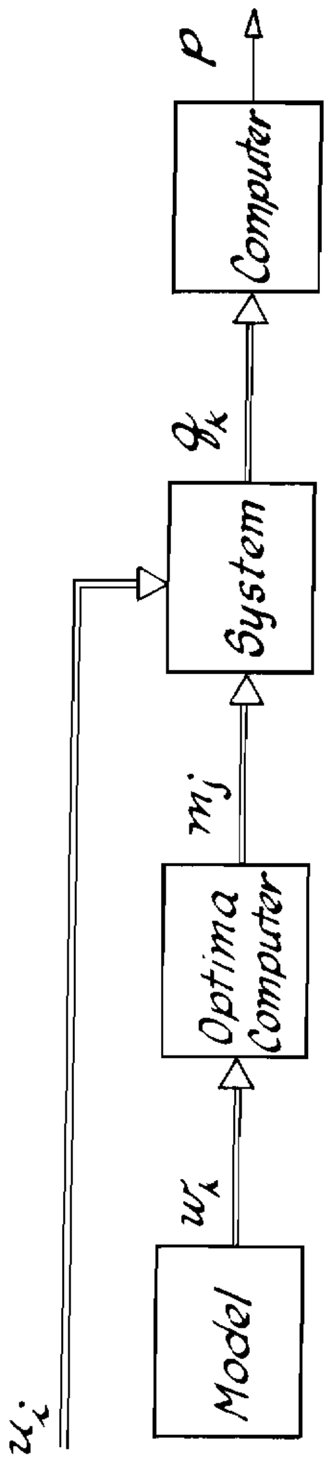

Fig. 5 Predetermined Optimization

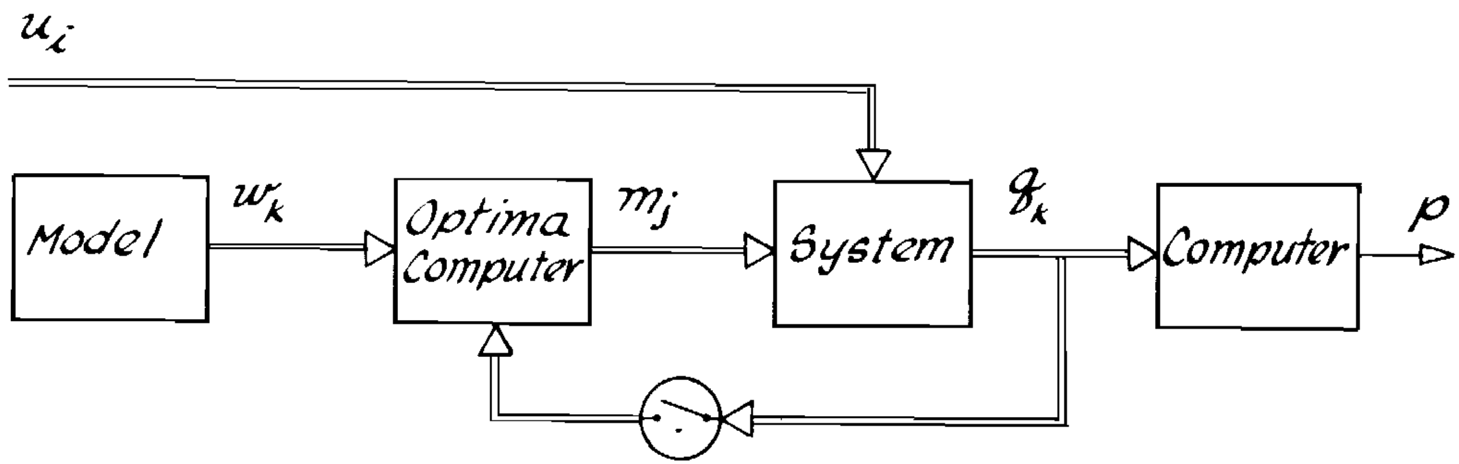

Fig. 6 Repetitive Computed Optimization

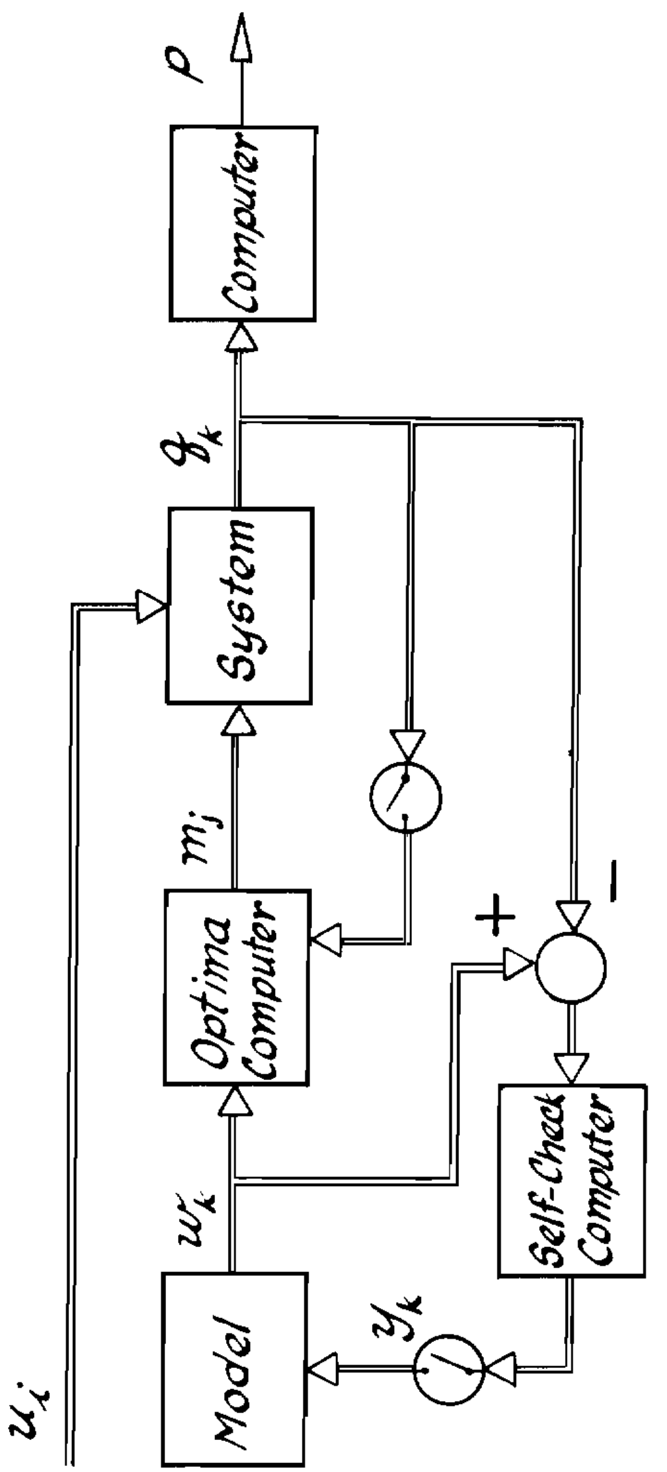

Fig. 7 Repetitive Computed Optimization with Self Checking