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ANALYTICAL STUDIES
OF
AIRCRAFT STRUCTURES EXPOSED TO TRANSIENT EXTERNAL HEATING

VOLUME IV

HEAT-TRANSFER COEFFICIENTS FOR SPEEDS UP TO $M=3$, WITH
EMPHASIS ON EFFECTS OF WALL TEMPERATURE VARIATIONS

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FOREWORD

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Robert Needham directed and was technically responsible for the research described in this report, whereas Neal Richardson acted as representative of Chairman of the Department L. M. K. Boelter.

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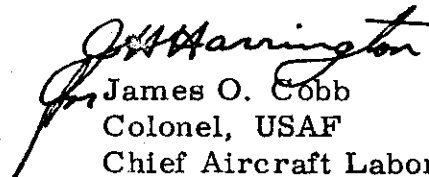
ABSTRACT

This report is the fourth of a series on analytical studies of aircraft structures exposed to transient external heating. Volumes I and II are concerned with the thermal analysis of the skin between supports, whereas volume III deals with the thermal analysis of the skin and support combination. The present volume complements its predecessors by reviewing and extending methods for the prediction of heat-transfer coefficients for external aircraft surfaces under various fluid-flow and wall-temperature conditions. For turbulent boundary layers, a new and simple integral expression is presented for predicting the effect on the heat-transfer rate of variations of the surface temperature in the fluid-flow direction. The rate at which the surface temperature may change subject to the restriction that the equations for steady-state heat transfer apply is determined analytically, and it is concluded that the equations for steady-state conditions may be used over a wide range of rates of change of surface temperature. Included also is a brief summary of the most pertinent information concerning stability of the laminar boundary layer.

PUBLICATION REVIEW

This report has been reviewed and is approved

FOR THE COMMANDER:


James O. Cobb
Colonel, USAF
Chief Aircraft Laboratory
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NOMENCLATURE

a	=	velocity of sound
c	=	specific heat (of metal)
c_p	=	specific heat, at constant pressure (of fluid)
C_f	=	friction coefficient, local
g	=	acceleration due to gravity
h	=	heat-transfer coefficient
i	=	specific enthalpy
J	=	mechanical equivalent of heat
k	=	thermal conductivity
ℓ	=	distance from leading edge to first stringer
Ma	=	Mach number = u/a
Nu	=	Nusselt number = hx/k
Pr	=	Prandtl number = $c_p \mu / k$
q	=	heat-transfer rate
r	=	recovery factor
Re	=	Reynolds number = $\rho u x / \mu$
St	=	Stanton number = $h / \rho u c_p$
t	=	time
T	=	temperature, absolute
u	=	velocity component parallel to surface
x	=	coordinate parallel to surface
y	=	coordinate normal to surface

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- y^+ = non-dimensional distance normal to surface = y/δ
- α = thermal diffusivity = $k/\rho c_p$
- γ = specific-heat ratio
- δ = boundary-layer thickness (in Couette flow, the fluid thickness)
- Δ = skin thickness
- ξ = distance from leading edge to location of temperature discontinuity
- Θ = non-dimensional temperature
- $$= \frac{T (1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}_s^2) - T_s (1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}_s^2)}{T_w(o) - T_s (1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}_s^2)}$$
- λ = spacing between stringers
- μ = absolute viscosity
- ν = kinematic viscosity = μ/ρ
- ρ = density (of fluid)
- $\bar{\rho}$ = density (of metal)
- τ = non-dimensional time = $\alpha t/\delta^2$

Superscript:

- * reference condition

Subscripts:

- r = recovery
- s = stream immediately outside boundary layer
- w = wall

INTRODUCTION

The design of an aircraft structure exposed to external transient heating requires information concerning expected values of stresses due to aerodynamic loads, stresses due to restraint of thermal expansions, and strengths of structural materials. These values depend, in part, on temperature distributions arising within the aircraft structure. In previous volumes (I, II, and III) of this series of reports are presented methods for determining the temperature distributions within various structural configurations assuming the heat-transfer coefficients at the external surface of the structure are known. The present volume complements its predecessors by reviewing and extending methods for the prediction of these heat-transfer coefficients.

In principle, the heat-transfer coefficient for a given situation can be calculated if the initial condition, boundary conditions, and fluid-property relations are known. In practice, however, this calculation is complicated frequently by the existence of some or all of the following conditions:

1. Space and time variations of surface temperature.
2. Space and time variations of free-stream velocity, pressure, and temperature.
3. Appreciable variations of the fluid properties.

Results of studies on the first complicating condition are presented in Sections II and III, where a new (and more simple) integral expression for the effect on heat-transfer rate of variations of surface temperatures in the flow direction is presented, and conditions under which the heat-transfer rate to surfaces with time variations of temperature may be treated as quasi-steady are determined. Literature relative to the second and third complicating

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conditions is reviewed briefly in Sections I and IV, whereas in Section V the most pertinent information concerning stability of the laminar boundary layer is summarized.

Although Section I, IV, and V are essentially reviews, the intention of the authors is not to provide an exhaustive survey of available literature on the subjects, but rather to call the attention of the reader to a typical useful combination of results applicable to the design of aircraft structures exposed to transient external heating. Obviously, many significant contributions to these subjects are neither reviewed nor mentioned herein.

It is hoped that the relations presented in this report are sufficiently simple and yet sufficiently useful to find general acceptance in the design room.

SECTION I

SURFACE HEATING WITH NEGLIGIBLE VARIATIONS OF
SURFACE TEMPERATURE AND PRESSURE (A Review)

Preliminary to discussing heat transfer for more complicated boundary layers, heat-transfer under steady conditions for boundary layers on isothermal flat plates is reviewed briefly.

Colburn (8) found in 1933 that, for turbulent flow of fluids with constant properties, heat transfer data could be correlated with skin-friction data by using the empirical relation

$$St = \frac{C_f}{2} \frac{1}{Pr^{2/3}}$$

where St is Stanton number, C_f is friction coefficient, and Pr is Prandtl number. This equation is an expression of the analogy which Reynolds (28) postulated to relate heat transfer and momentum transfer in shear flows, modified empirically to describe effects of moderate deviations of the Prandtl number from unity. For laminar flow of fluids with constant properties, Pohlhausen (27) had derived in 1921 already the following relation between the heat-transfer coefficient and the flow parameters

$$Nu = 0.332 Re^{1/2} Pr^{1/3}$$

where Nu is Nusselt number (based on local heat-transfer coefficient) and Re is Reynolds number. Since, for laminar flow, the local friction coefficient is related to the flow parameters by the relation

$$\frac{C_f}{2} = \frac{0.332}{\sqrt{Re}}$$

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Pohlhausen's expression may be written in the form

$$St = \frac{Nu}{RePr} = \frac{C_f}{2} \frac{1}{Pr}^{2/3}$$

This equation (derived for laminar flow) has exactly the same form as Colburn's equation (obtained using turbulent-flow data); it will be called the Pohlhausen-Colburn equation.

The equations discussed in the previous paragraph were obtained for fluids with negligible property variations. If the free-stream temperature differs greatly from the surface temperature, or if the free-stream Mach number is large, then variations of fluid properties are large and must be taken into account. Two methods are used for handling those variations - the first involving exact computations; the second being an engineering approximation.

In the first method, the fluid properties are expressed to the desired approximation as functions of temperature and pressure and the resulting expressions are introduced into the differential equations describing heat, mass and momentum transfer. These equations are solved then for the appropriate boundary conditions, automatic computing machines being used frequently to great advantage. Results are presented as tables or curves of friction and heat-transfer coefficients as functions of Mach number, free-stream temperature, and wall temperature. Typical works are those by Busemann (5), Karman and Tsien (18), Crocco (11), Hantsche and Wendt (14, 15), Brainerd and Emmons (4), Cope and Hartree (10), Van Driest (38), Young and Janssen (40), and Moore (24). Summaries of these works and more complete bibliographies are to be found in the texts by Schlichting (33), Howarth (16), Shapiro (34), and Pai (26).

Engineering approximations are obtained by examining results of exact

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computations and experimental tests. These examinations indicate (a) that the Pohlhausen-Colburn equation is a good approximation provided that all fluid properties are evaluated at the same temperature, and (b) that the equations for friction coefficients for fluids with negligible property variations are good approximations for fluids with large property variations provided that fluid properties are evaluated at the appropriate reference temperature. Methods for computing this reference temperature have been suggested by Rubesin and Johnson (30), Young and Janssen (40), Eckert (13), and Summer and Short (36). Since Eckert's suggestion has been compared favorably with a relatively large number of data points (13, 19), it is preferred over the other suggestions. Result is that the heat-transfer rate may be calculated to good approximation using the following relations:

$$q = St^* \rho^* u_s c_p^* (T_r - T_w)$$

$$St^* = \frac{C_f^*}{2} \frac{1}{(Pr^*)^{1/3}}$$

$$T_r = T_s + r \frac{\gamma - 1}{2} Ma_s^2$$

$$r = Pr^{*1/2} \quad \text{laminar flow}$$

$$r = Pr^{*1/3} \quad \text{turbulent flow}$$

$$\frac{C_f^*}{2} = \frac{0.332}{\sqrt{Re^*}} \quad \text{laminar flow}$$

$$= \frac{0.0296}{Re^{*0.2}} \quad \text{turbulent flow, } Re^* < 10^7$$

$$= \frac{0.185}{(\log_{10} Re^*)^{2.584}} \quad \text{turbulent flow, } Re^* < 10^9$$

$$T^* - T_s = 0.50 (T_w - T_s) + 0.22 (T_r - T_s)$$

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where

q	=	heat-transfer rate
ρ	=	density
u	=	velocity
c_p	=	specific heat
T	=	absolute temperature
r	=	recovery factor
γ	=	specific-heat ratio
Ma	=	Mach number

and the superscript * indicates fluid properties are to be evaluated at the reference temperature, whereas the subscripts r, s, and w refer respectively to recovery, stream immediately outside boundary layer, and wall. If the temperature profile in the boundary layer is such that the specific heat varies appreciably, then it is more accurate to calculate the heat-transfer rate using the relation

$$q = St^* \rho^* u_s^* (i_r - i_w)$$

the recovery enthalpy using the relation

$$i_r = i_s + \frac{r u_s^2}{2 g J}$$

and the reference temperature from the reference enthalpy using the relation

$$i^* - i_s = 0.50 (i_w - i_s) + 0.22 (i_r - i_s)$$

where

i	=	specific enthalpy
g	=	acceleration of gravity
J	=	mechanical equivalent of heat

These equations are suitable for the calculation of heat transfer to aerodynamic surfaces with small variations (along the surface) of temperature and pressure.

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Effects of temperature and pressure variations are discussed in the following sections of this report.

SECTION II

SURFACE HEATING WITH A VARIATION OF SURFACE TEMPERATURE
IN THE DIRECTION OF FLUID FLOW

For aircraft structures exposed to transient external heating, the isothermal flat plate discussed in Section I is frequently not a satisfactory model. Strengthening members, such as spars and ribs, act as heat sinks with the result that the rate of temperature change is smaller in the vicinity of these members than for the remainder of the surface. In many cases, the resulting non-uniform temperature distribution must be taken into account when calculating heat-transfer rates in order to obtain the desired accuracy.

Available information concerning effects of non-uniform surface temperatures on surface heating is reviewed by Tribus and Klein (37). Particularly interesting are the results obtained by Rubesin (29, 31), Scesa (32), and Eckert (12). For low-speed flow of a fluid having constant physical properties along a flat plate having a variation of surface temperature in the direction of fluid flow, Rubesin presents the following expression for the local heat-transfer rate

$$q(x) = h(x, 0) \left[T_w(0) - T_s \right] + \int_0^x h(x, \xi) \frac{dT_w(\xi)}{d\xi} d\xi$$

where $h(x, \xi)$ is the heat-transfer coefficient at point x (where $x > \xi$) for the case when $T_w = T_s$ for $x < \xi$ and $T_w = \text{const} \neq T_s$ for $x > \xi$. This equation is equally valid for both laminar and turbulent flows, Rubesin presenting the following approximate relations for $h(x, \xi)$:

$$\begin{aligned} \frac{h(x, \xi)x}{k} &= 0.304 \text{Re}^{1/2} \text{Pr}^{1/3} \left[1 - \left(\frac{\xi}{x} \right)^{3/4} \right]^{-1/3} && \text{laminar flow} \\ &= 0.0288 \text{Re}^{0.8} \text{Pr}^{1/3} \left[1 - \left(\frac{\xi}{x} \right)^{39/40} \right]^{-7/39} && \text{turbulent flow} \end{aligned}$$

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Although these relations are usable and sufficiently accurate as given by Rubesin, the authors suggest that these equations be modified slightly with the following general results:

1. Both equations modified reduce to the equations for an isothermal plate for $\xi = 0$.
2. The second equation modified is integrated much more easily than is this equation unmodified.
3. Both equations modified are still sufficiently accurate for engineering applications.

Consider first the equation for laminar flow. Since this expression should reduce, for $\xi = 0$, to the Pohlhausen relation for an isothermal plate, it is suggested that the constant coefficient be changed slightly to read 0.332, giving

$$\frac{h(x, \xi)x}{k} = 0.332 \text{Re}^{1/2} \text{Pr}^{1/3} \left[1 - \left(\frac{\xi}{x} \right)^{3/4} \right]^{-1/3}$$

comparing favorably with the relation given by Eckert (12, equation 138).

Two changes are suggested for the equation for turbulent flow. Since (a) $\frac{39}{40}$ is very nearly equal to unity and $\frac{7}{39}$ is very nearly equal to $\frac{1}{5}$, and (b) $\left[1 - \frac{\xi}{x} \right]^{-1/5}$ is much more easily integrated than $\left[1 - \left(\frac{\xi}{x} \right) \frac{39}{40} \right]^{-7/39}$, the authors considered replacing the exponents $\frac{39}{40}$ and $\frac{7}{39}$ by unity and $\frac{1}{5}$. In addition, since this expression should reduce, for $\xi = 0$, to the Blasius relation for an isothermal plate, the authors felt that the constant coefficient should be changed slightly to read 0.0296, giving

$$\frac{h(x, \xi)x}{k} = 0.0296 \text{Re}^{0.8} \text{Pr}^{1/3} \left[1 - \frac{\xi}{x} \right]^{-1/5}$$

An examination of the data obtained by Scesa (32) indicates that this expression correlates the data as well as does the expression given by Rubesin.

Hence, defining, the local heat-transfer coefficient as the local heat-transfer rate divided by the local temperature difference, one obtains

$$h(x) = \frac{h(x, 0)}{T_w(x) - T_s} \left\{ [T_w(0) - T_s] + \int_0^x F\left(\frac{\xi}{x}\right) \frac{dT_w(\xi)}{d\xi} d\xi \right\}$$

where

$$F\left(\frac{\xi}{x}\right) = \begin{cases} \left[1 - \left(\frac{\xi}{x}\right)^{3/4} \right]^{-1/3} & \text{for laminar flow} \\ \left[1 - \frac{\xi}{x} \right]^{-1/5} & \text{for turbulent flow} \end{cases}$$

An expression for the error in calculated heat-transfer rate made by using a uniform plate temperature equal to the leading edge temperature $T_w(0)$ and neglecting surface-temperature variations is obtained readily and is given by

$$\% \text{ error} = \frac{100}{1 + \frac{T_w(0) - T_s}{\int_0^x F\left(\frac{\xi}{x}\right) \frac{dT_w(\xi)}{d\xi} d\xi}}$$

Note that a small value of the integral and/or a large value of the temperature difference $T_w(0) - T_s$ results in a small error. Quantitative error information is obtainable only after the surface-temperature distribution is specified.

As an example of the computations involved in the calculation of heat-transfer coefficients using the aforementioned equations, consider turbulent flow over an aerodynamic surface (represented by a flat plate) exposed to thermal radiation and having strengthening members spaced as indicated in Figure 1. It is assumed that fluid properties are constant and that the temperature distribution is given by

$$T_w(\xi) = T_w(0) \quad \text{for} \quad 0 \leq \xi \leq l - \lambda/2$$

$$= T_{w_{ave}} - \left[T_w(0) - T_{w_{ave}} \right] \cos\left(2\pi \frac{\xi - l}{\lambda}\right) \quad \text{for} \quad l - \frac{\lambda}{2} \leq \xi$$

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Substituting into the expression for the turbulent heat-transfer coefficient, one obtains for $x > l - \frac{\lambda}{2}$

$$h(x) = h(x, 0) \left\{ \frac{T_w(0) - T_s}{T_w(x) - T_s} + \frac{T_w(0) - T_{wave}}{T_w(x) - T_s} \int_{l - \frac{\lambda}{2}}^x \frac{2\pi}{\lambda} \left[1 - \frac{\xi}{x} \right]^{-\frac{1}{5}} \sin\left(2\pi \frac{\xi - l}{\lambda}\right) d\xi \right\}$$

It is convenient to change to the dimensionless variable $\frac{\xi}{x}$, the integral reading then

$$I = \int_{\frac{l}{x} - \frac{\lambda}{2x}}^1 \frac{2\pi x}{\lambda} \left[1 - \frac{\xi}{x} \right]^{-1/5} \sin\left(\frac{2\pi x}{\lambda} \frac{\xi - l}{x}\right) d\left(\frac{\xi}{x}\right)$$

Integrating by parts, one obtains

$$I = -\frac{5}{4} \left(\frac{2\pi x}{\lambda}\right)^2 \int_{\frac{l}{x} - \frac{\lambda}{2x}}^1 \left[1 - \frac{\xi}{x} \right]^{4/5} \cos\left(\frac{2\pi x}{\lambda} \frac{\xi - l}{x}\right) d\left(\frac{\xi}{x}\right)$$

The function $\left(1 - \frac{\xi}{x}\right)^{4/5}$ may be expanded in a binomial series, convergent even for $\frac{\xi}{x} = 1$. Hence,

$$I = -\frac{5}{4} \left(\frac{2\pi x}{\lambda}\right)^2 \int_{\frac{l}{x} - \frac{\lambda}{2x}}^1 \left[1 - \frac{4}{5} \frac{\xi}{x} - \frac{4}{5} \cdot \frac{1}{5} \cdot \frac{1}{2} \left(\frac{\xi}{x}\right)^2 + \dots + \frac{\frac{4}{5} \left(\frac{4}{5} - 1\right) \dots (4/5 - n + 1)}{n!} \left(\frac{\xi}{x}\right)^n + \dots \right] \cos\left(\frac{2\pi x}{\lambda} \frac{\xi - l}{x}\right) d\left(\frac{\xi}{x}\right)$$

In this form, the integration is carried out readily, any desired accuracy being obtained by taking a sufficient number of terms of the series. (Note that, for many mathematical forms which $\frac{dT_w}{d\xi}$ might assume, the integration is not easy and more elaborate techniques, such as numerical integration, might be necessary.)

Calculations are carried out for the following numerical values:

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Flight speed = 415 mph
Altitude = 40,000 ft.
 $T_w(o) - T_s = 300^\circ\text{F}$
Chord length = 122.5 in.
 $l = 0.12 \times \text{chord length}$
 $\lambda = 0.05 \times \text{chord length}$

Calculation results are plotted in Figure 2, where the heat-transfer coefficient for the non-uniform surface temperature divided by the heat-transfer coefficient for a uniform surface temperature is plotted as a function of distance from the leading edge for four values of the temperature rates $(T_w(o) - T_{w_{ave}}) / (T_{w_{ave}} - T_s)$. These results indicate that appreciable deviations from isothermal-plate heat-transfer coefficients are realized even though no temperature discontinuities exist. The maximum error made (for this example), if one assumes the heat-transfer coefficient equal to the coefficient for an isothermal plate, is obtained directly from these curves and is tabulated in Table I. Note also that the error made by neglecting the given temperature non-uniformity is conservative for some portions of the skin and non-conservative for other portions.

If the non-uniform temperature distribution can be replaced to satisfactory approximation by straight-line segments, then the integral appearing in the expression for the heat-transfer coefficient is evaluated easily. The gradient $dT_w/d\xi$ is constant and does not enter into the integration process. E. g., for the case of turbulent flow, the integral reduces then to a sum of terms having the following form:

$$-\frac{5}{4} \times \left. \frac{dT_w(x)}{dx} \right|_{ij} \left[\left(1 - \frac{x_j}{x}\right)^{4/5} - \left(1 - \frac{x_i}{x}\right)^{4/5} \right]$$

where $\left. \frac{dT_w(x)}{dx} \right|_{ij}$ is the slope of the segment between x_i and x_j .

The discussion in this section up to this point has been limited to low-speed flows of fluids with constant properties. This limitation is imposed

because the derivation of the first equation of this section includes a superposition of solutions of a linear equation. For flows involving either supersonic speed or large temperature differences, however, fluid property variations are not negligible. No simple (and yet accurate) method for taking these variations into account is known. It is believed, however, that for most practical applications involving smooth temperature variations (rather than abrupt temperature discontinuities) the error made by evaluating fluid properties at the reference temperature described in Section I is acceptable. Note also that, for high-speed flows, the free-stream temperature T_s would be replaced by the recovery temperature T_r as the driving potential.

In summary, expressions for heat-transfer coefficients for plates with non-uniform surface temperatures are presented in this section, the expression for turbulent flows being appreciably simpler than expressions suggested previously. Results obtained in a sample calculation indicate that if non-uniform surface-temperature distributions exist, then appreciable deviations from isothermal-plate heat-transfer coefficients may be realized even though no temperature discontinuities exist. Errors made by neglecting the given temperature non-uniformity may be conservative or non-conservative, depending upon the situation details.

SECTION III

SURFACE HEATING WITH APPRECIABLE TIME BUT WITH NO SPATIAL VARIATION OF SURFACE TEMPERATURE; PRESSURE VARIATIONS ARE NEGLIGIBLE

It is desired frequently to calculate the rate of heat transfer to a surface having a temperature changing with time. Typical examples of such surfaces are aerodynamic surfaces of aircraft subjected to heat radiation from nuclear explosions and external surfaces of missiles subjected to aerodynamic heating. If the surface temperature is changing slowly, then one may use equations for steady-state heat transfer (e. g., the equations mentioned in Sections I and II). The purpose of the work described in the present section is to indicate how rapidly the surface temperature may change subject to the restriction that these equations for steady-state heating apply to good approximation.

The simplest model embodying effects of time variations of surface temperature is perhaps laminar constant-shear (i. e., Couette) flow. Consider, therefore, high-speed Couette flow having one surface fixed and the other surface moving at constant velocity u_s . The most important of the time-dependent characteristics of the system are not altered if one assumes constant fluid properties. Hence, for simplicity, assume fluid properties are constant. Then the velocity profile does not change with time and

$$\frac{u}{u_s} = \frac{y}{\delta}$$

where u is local velocity, y is distance from the fixed surface, and δ is distance between the two surfaces. (See Figure 3.) If u_s is equal to the velocity immediately outside the boundary layer, and if δ is equal to the boundary-layer thickness, then the model has properties analogous to the boundary layer on an airfoil. In particular, the heat capacity (per unit area per degree temperature rise) and heat-transfer rate are of the same order

of magnitude for the two systems.

Consider first a wall temperature which does not vary with time. (Results for this limiting case will be used in the more general study of time-dependent wall temperature.) The temperature distribution in the fluid is described by the following differential equation:

$$k \frac{d^2 T}{dy^2} + \mu \left(\frac{du}{dy} \right)^2 = 0$$

where μ is viscosity. Then the first term represents the variation with respect to y of the heat transfer rate, whereas the second term represents the work done on the fluid. The boundary conditions are:

$$\begin{array}{lll} y = 0: & u = 0, & T = T_w \\ y = \delta: & u = u_s, & T = T_s \end{array}$$

An especially instructive form of the differential equation is obtained if one takes advantage of the following equalities holding for this model:

$$\left(\frac{du}{dy} \right)^2 = \left(\frac{u_s}{\delta} \right)^2 = \frac{d^2}{dy^2} \left(\frac{u^2}{2} \right)$$

Then, since fluid properties are assumed to be constant,

$$\frac{d^2}{dy^2} \left(T + \frac{\mu}{k} \frac{u^2}{2} \right) = 0$$

Introducing the Prandtl number

$$Pr = \frac{c_p \mu}{k}$$

and the Mach number

$$Ma = \frac{u}{a} = \frac{u}{\sqrt{\gamma gRT}}$$

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where $c_p = \frac{R \gamma}{\gamma - 1}$ = specific heat at constant pressure.

a = velocity of sound

R = gas constant

the differential equation becomes

$$\frac{d^2}{dy^2} \left[T \left(1 + Pr \frac{\gamma - 1}{2} Ma^2 \right) \right] = 0$$

It is apparent now that the parameter $T \left(1 + Pr \frac{\gamma - 1}{2} Ma^2 \right)$ plays an important role in high-speed Couette flow. Integrating twice

$$T \left(1 + Pr \frac{\gamma - 1}{2} Ma^2 \right) = Ay + B$$

From boundary condition at $y = 0$:

$$B = T_w$$

From boundary condition at $y = \delta$:

$$A = \frac{T_s \left(1 + Pr \frac{\gamma - 1}{2} Ma_s^2 \right) - T_w}{\delta}$$

Hence the temperature distribution is given by the relation

$$\frac{T \left(1 + Pr \frac{\gamma - 1}{2} Ma^2 \right) - T_w}{T_s \left(1 + Pr \frac{\gamma - 1}{2} Ma_s^2 \right) - T_w} = \frac{y}{\delta} = \frac{u}{u_s}$$

Heat-transfer rate at the stationary wall is

$$q_w = -k \left. \frac{dT}{dy} \right|_{y=0} = \frac{k}{\delta} \left[T_w - T_s \left(1 + Pr \frac{\gamma - 1}{2} Ma_s^2 \right) \right]$$

If no heat is transferred at the wall, then

$$T_w = T_s \left(1 + Pr \frac{\gamma - 1}{2} Ma_s^2 \right) \equiv T_r$$

where T_r is the adiabatic wall (recovery) temperature. Hence the heat-transfer

rate may be written in the form

$$q_w = \frac{k}{\delta} (T_w - T_r) \equiv h(T_w - T_r)$$

where h is the heat-transfer coefficient.

Consider now high-speed Couette flow with a steady velocity profile but with a non-steady wall temperature. The temperature distribution in the fluid is described by the following partial differential equation:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2$$

where t = time and ρ = density. Rearranging the right-hand side of this equation using procedures established in the previous paragraph,

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2}{\partial y^2} \left[T \left(1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}^2 \right) \right]$$

where $\alpha = k/\rho c_p$ = thermal diffusivity. Since the term $T \text{Pr} \frac{\gamma-1}{2} \text{Ma}^2$ does not vary with time, the equation may also be written in the form

$$\frac{\partial}{\partial t} \left[T \left(1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}^2 \right) \right] = \alpha \frac{\partial^2}{\partial y^2} \left[T \left(1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}^2 \right) \right]$$

The partial differential equation has now the form of the thermal-diffusion equation, and solutions for the thermal-diffusion equation may be taken as solutions for the equation describing temperature distributions in high-speed Couette flow with non-steady wall temperatures. The boundary and initial conditions are:

$$\begin{array}{ll} y = \delta, t > 0: & T \left(1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}^2 \right) = T_r \\ y = 0, t > 0: & T \left(1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}^2 \right) = T_w \\ 0 \leq y \leq \delta, t = 0: & T \left(1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}^2 \right) - T_w(0) = \frac{y}{\delta} \frac{T_r - T_w(0)}{\delta} \end{array}$$

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For convenience, introduce the dimensionless variables

$$\Theta = \frac{T(1 + \text{Pr} \frac{\gamma-1}{2} \text{Ma}^2) - T_r}{T_w(0) - T_r}$$

$$\tau = \alpha t / \delta^2 \qquad y^+ = y / \delta$$

The following partial differential equation, boundary conditions, and initial condition result:

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial y^{+2}}$$

$$\begin{aligned} y^+ &= 1, \quad \tau > 0: & \Theta &= 0 \\ y^+ &= 0, \quad \tau > 0: & \Theta &= f(\tau) \\ 0 \leq y^+ \leq 1, \quad \tau = 0: & \Theta &= 1 - y^+ \end{aligned}$$

Carslaw and Jaeger (6, p 86) give the solution

$$\Theta = 2 \sum_1^{\infty} e^{-n^2 \pi^2 \tau} \sin n \pi y^+ \left[\int_0^1 (1 - y^+) \sin n \pi y^+ dy^+ + n \pi \int_0^{\tau} e^{n^2 \pi^2 \tau'} x f(\tau') d\tau' \right]$$

This expression may be put into a more convenient form by integrating the first integral directly and the second integral by parts,

$$\Theta = 2 \sum_1^{\infty} e^{-n^2 \pi^2 \tau} \frac{\sin n \pi y^+}{n \pi} \left[e^{n^2 \pi^2 \tau} f(\tau) - \int_0^{\tau} e^{n^2 \pi^2 \tau'} \frac{df(\tau')}{d\tau'} d\tau' \right]$$

The series

$$2 \sum_1^{\infty} \frac{\sin n \pi y^+}{n \pi}$$

is the Fourier Sine Series for the function $1 - y^+$. Therefore one obtains the

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convenient form

$$\Theta = (1 - y^+) f(\tau) - 2 \sum_1^{\infty} \frac{\sin n \pi y^+}{n \pi} \int_0^{\tau} e^{-n^2 \pi^2 (\tau - \tau')} \frac{df(\tau')}{d\tau'} d\tau'$$

The first term is a linear (quasi-steady state) temperature distribution whereas the second term is the deviation from the linear distribution caused by the finite heat capacity of the fluid. The heat-transfer rate is given by

$$q_w(\tau) = - \frac{k}{\delta} \left[T_w(o) - T_r \right] \frac{\partial \Theta}{\partial y^+} \Big|_{y^+ = 0}$$

$$= \frac{k}{\delta} \left[T_w(\tau) - T_r \right] + \frac{k}{\delta} \left[T_w(o) - T_r \right] 2 \sum_1^{\infty} \int_0^{\tau} e^{-n^2 \pi^2 (\tau - \tau')} x \frac{df(\tau')}{d\tau'} d\tau'$$

or

$$q_w(t) = \frac{k}{\delta} \left[T_w(t) - T_r \right] + \rho c_p \delta 2 \sum_1^{\infty} \int_0^{\tau} e^{-n^2 \pi^2 \frac{\alpha(t-t')}{\delta^2}} x \frac{dT_w}{dt} \frac{\alpha dt'}{\delta^2}$$

The first term is a quasi-steady state heat-transfer rate whereas the second term is the deviation from the quasi-steady caused by the finite heat capacity of the fluid. The relative magnitudes of these two terms are of interest; if the magnitude of the second term is small compared with the magnitude of the first term, then one may calculate the heat-transfer rate to good approximation using equations derived for steady-state systems.

Suppose it is known that, at some time $t = 0$, the temperature distribution in the boundary layer may be approximated closely by steady-state temperatures. Suppose further that, at time $t > 0$, the variation of wall temperature with time

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may be approximated closely by a linear temperature-time relation. Then dT_w/dt' is constant for $0 \leq t' \leq t$ and the heat-transfer rate is given by

$$q_w(t) = \frac{k}{\delta} [T_w(t) - T_r] + \rho c_p \delta \frac{dT_w}{dt} \sum_{n=1}^{\infty} \frac{2}{(n\pi)^2} (1 - e^{-n^2 \pi^2 \frac{\alpha t}{\delta^2}})$$

$$= q_w \text{ quasi-steady} + \rho c_p \delta \frac{dT_w}{dt} g\left(\frac{\alpha t}{\delta^2}\right)$$

Where $g\left(\frac{\alpha t}{\delta^2}\right)$ is a smooth function having as a lower limit zero, corresponding to $\frac{\alpha t}{\delta^2} = 0$, and as an upper limit $1/3$, approached closely already for $\frac{\alpha t}{\delta^2} = 1/3$. See Figure 4. The relative magnitudes of the two terms on the right-hand side of this equation could be determined now by introducing the value of the boundary-layer thickness δ . This procedure would be used, however, only if the "thin skin" assumption does not hold. If the thin skin assumption holds, then the rate of change of wall temperature is related to the heat-transfer rate by the relation

$$q_w(t) = \bar{\rho} c \Delta \frac{dT_w}{dt}$$

where $\bar{\rho}$ = density of skin material
 c = specific heat of skin material
 Δ = thickness of skin

The heat-transfer rate is given then by the relation $q_w(t) = \frac{q_{w \text{ quasi-steady}}}{1 - \frac{\rho c_p \delta}{\bar{\rho} c \Delta} g\left(\frac{\alpha t}{\delta^2}\right)}$

The ratio of the heat capacities

$$\frac{\rho c_p \delta}{\bar{\rho} c \Delta}$$

appears as an important parameter in the result. In fact, one can make now the following general statement:

If the thin skin assumption is valid, then the error made by calculating the rate of heat transfer from the skin to the boundary layer using the

equations for steady-state heat transfer is bounded by the relation

$$\frac{q_{\text{exact}} - q_{\text{quasi-steady}}}{q_{\text{exact}}} \leq \frac{1}{3} \frac{\rho c_p \delta}{\bar{\rho} c \Delta}$$

Since the analysis of this section is made for laminar flow, a few comments concerning turbulent flow seem appropriate. The main effect of turbulence on heat-transfer is to increase (in fluid not immediately adjacent to a wall) the effective thermal conductivity and hence the effective thermal diffusivity. Hence the general equation for the heat-transfer rate $q_w(t)$, holding for thick skins as well as for thin skins, could be used also as a guide for turbulent flows if the molecular thermal conductivity were replaced by an effective (turbulent) thermal conductivity. The relation obtained for the case of a linear variation of wall temperature with time is especially interesting. Here the thermal diffusivity appears in the function $q(\frac{\alpha t}{\delta^2})$, whose upper limit is independent of the value of α . Hence it is concluded that the general statement made for a laminar boundary layer on a thin skin is, in its present form, a useful guide also for the case of a turbulent boundary layer on a thin skin.

The error made by using equations for steady-state conditions is small usually, as is indicated by the following example for turbulent flow on a thin skin:

$$\begin{aligned} \rho c_p &= 0.0069 \text{ B/ft}^3 \text{ } ^\circ\text{R (air at 30,000 ft altitude)} \\ \bar{\rho} c &= 37 \text{ B/ft}^3 \text{ } ^\circ\text{R (aluminum)} \\ \delta &= 0.068 \text{ ft (velocity = 1000 ft/sec, running length = 5 ft)} \\ \Delta &= 0.0050 \text{ ft} \end{aligned}$$

For these numbers,

$$\frac{q_{\text{exact}} - q_{\text{quasi-steady}}}{q_{\text{exact}}} \leq \frac{1}{3} \frac{0.0069 \times 0.068}{37 \times 0.0050} = 8.5 \times 10^{-4}$$

i. e. , the error made by using equations for steady-state conditions is less than a tenth of one percent. This error is acceptable certainly.

In summary, the rate at which the surface temperature may change subject to the restriction that the equations for steady-state heating apply to good approximation is examined in this section. The boundary layer is approximated by Couette flow the heat-transfer rate is expressed as the sum of (1) the quasi-steady rate and (2) the deviation from this quasi-steady rate. The deviation is found to be a linear function of the heat capacity per unit area per unit temperature rise of the boundary layer. For many cases of practical interest, this heat capacity is small compared with the heat capacity of the skin with the result that the deviation from the quasi-steady heat-transfer rate is also small.

SECTION IV

SURFACE HEATING WITH PRESSURE GRADIENTS
IN FLOW DIRECTION (A Review)

The surfaces of most aerodynamic components are curved rather than flat with the result that the pressure, velocity, and temperature vary along a streamline in the fluid outside the boundary layer. Such gradients influence the growth of the boundary layer, affecting thereby the magnitudes of the friction force and heat-transfer rate. The designer of these aerodynamic components is interested consequently in the effect of pressure gradients (resulting from surface curvatures) on boundary-layer characteristics.

Pertinent experimental information has been obtained by Chapman and Kester (7). They tested a cone-cylinder body at a Mach number of 2 in a wind tunnel with flexible-plate nozzle walls in order to obtain turbulent friction data with various pressure distributions along the cylinder. The pressure distributions for two of the tests differed from each other by approximately the same amount that the pressure distributions on a 3 percent thick biconvex airfoil differs from a constant pressure. Average friction coefficients were determined, using conditions at the outer edge of the boundary layer averaged over the length of the model as reference conditions. No effect of the variation of pressure distribution on this average friction coefficient was observed. It is concluded, therefore, that the pressure gradients realized along the surfaces of slender air foils (excluding the region immediately adjacent to the stagnation point) for turbulent flow at low supersonic speeds do not alter appreciably the friction forces from flat-plate values. Since a pressure gradient affects the velocity profile directly but the temperature profile only indirectly, the effect of a pressure gradient on the heat-transfer rate is even less than the

effect on the shear force.

Pertinent analytical results have been reported by Morduchow and Grape (25). They present first a method for determining the effect of a pressure gradient on the heat-transfer rate from a compressible laminar boundary layer to an isothermal wall, representing the velocity profile by a sixth-degree polynomial and the stagnation-enthalpy profile by a seventh-degree polynomial. Then, as an example of the use of their method, they present calculated heat-transfer rates to a biconvex circular-arc airfoil of thickness ratio 0.04. Their calculation results indicate that, if conditions immediately behind the shock wave are taken as reference conditions, then the local Nusselt number drops appreciably below the flat-plate value as distance from the leading edge increases. E. g., for a Mach number of 3, the value of Nu/\sqrt{Re} for the trailing edge is 40% lower than the value of this parameter for a point the same distance from the leading edge of a flat plate. Inspection of their calculation results indicates, however, that local conditions (pressure, temperature, and velocity) outside the boundary layer vary appreciably with distance from the leading edge and that, if one uses local conditions immediately outside the boundary layer as reference conditions, then the local Nusselt number does not differ significantly from flat-plate values, even at the trailing edge.

In summary, the following design procedure is suggested for handling the effects of a pressure gradient on the heat-transfer rate for a slender airfoil at low supersonic speeds:

1. Determine local conditions outside the boundary layer, taking into account effects of shock waves (if supersonic flow occurs) and surface curvatures.
2. Using local conditions as reference conditions and the wetted length from the leading edge as the characteristic distance, calculate the heat-transfer rate using equations derived for flow over a flat plate.

SECTION V

STABILITY OF THE LAMINAR BOUNDARY LAYER (A Review).

Since the rate of heat transfer for a turbulent boundary layer may be several times the rate for a laminar boundary layer (see Table II), the design of a high-speed aircraft depends upon whether the boundary layer over a given aerodynamic surface is laminar or turbulent. Therefore, a brief review of available information concerning the location of the point at which a boundary layer becomes turbulent seems appropriate here.

The many experimental analytical studies on boundary layer stability have indicated that transition to turbulent flow has its origin in either (1) finite disturbances propagated into the boundary layer from the free stream outside the boundary layer or from the surface over which the fluid is flowing, or (2) infinitesimal disturbances present within the boundary layer which are amplified for certain distributions of fluid velocity and density. Effects of the finite disturbances are dismissed with the statements that (1) free-stream disturbances encountered in flight through the atmosphere have usually a negligible effect on transition, and (2) surface-roughness disturbances can be made negligible by making the surface sufficiently smooth. Effects of fluid velocity and density profiles on the amplification (or damping) of infinitesimal disturbances are the subject of the remainder of this section.

Lees and Lin (21, 22) extended earlier calculations of criteria for flow stability to include effects of compressibility and heat transfer in viscous boundary layer flows. They reached the following conclusions:

1. For any value of the free-stream Mach number, if the product of density and vorticity has an extremum (i. e., if $\frac{d}{dy} (\rho du/dy)$ vanishes) at any location within the boundary layer where the magnitude of the fluid velocity relative to the free-stream velocity is

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less than the sound velocity outside the boundary layer (i. e. , where $(u_s - u) < a_s$), then the flow is always unstable at sufficiently high Reynolds numbers. (In such a boundary layer, inertia forces are destabilizing for all Reynolds numbers and viscous forces are destabilizing at high Reynolds numbers.)

2. If the free stream velocity is subsonic, then the flow is always unstable at sufficiently high Reynolds numbers. (In subsonic boundary layers, inertia forces may be stabilizing or destabilizing. Viscous forces are destabilizing at high Reynolds numbers and dominate over inertia forces at sufficiently high Reynolds numbers.)
3. If the free-stream velocity is supersonic and if $\frac{d}{dy} \left(\rho \frac{du}{dy} \right)$ does not vanish for some $(u_s - u) < a_s$, then the flow is stable at all Reynolds numbers for certain conditions (inertia forces are stabilizing and large enough to dominate over destabilizing viscous forces at high Reynolds numbers) and unstable at sufficiently high Reynolds numbers for other conditions (inertia forces are stabilizing but not large enough to dominate over destabilizing viscous forces at high Reynolds numbers). Velocity and density distributions necessary for complete stabilization are achieved by (1) transferring heat at a sufficiently high rate from the boundary layer to the aerodynamic surface, and/or (2) accelerating the main-stream gas at a sufficiently high rate.

In applying these analytical results of Lees and Lin, it is to be kept in mind that the calculated critical Reynolds number gives the location of the point at which infinitesimal disturbances are first amplified; complete transition to turbulence takes time and occurs downstream of this point.

Van Driest (39), using these results of Lees and Lin as a basis, calculated the stability of the laminar boundary layer in a compressible fluid as a function of Mach number and wall-to-free-stream temperature ratio, but limited to the case of no acceleration of the free stream (i. e. , limited to flow along a flat plate). His results are summarized in Figure 5, where the minimum critical Reynolds number is given as a function of free-stream Mach number and wall-to-free-stream temperature ratio. Note that, for Mach numbers greater than 1 and less than 9, wall temperatures exist for which the boundary layer is stable at all Reynolds numbers. Morduchow and Grape (25), Low (23), and Shapiro (35),

also using the results of Lees and Lin as a basis, extended Van Driest's calculations to include effects of pressure gradients on the stability of laminar boundary layers in compressible fluids. Morduchow and Grape, e. g. , calculated for flow over a thin biconvex circular-arc airfoil of thickness ratio 0.04 in supersonic flow at zero angle of attack. Results indicate quantitatively the favorable effect of a negative pressure gradient on boundary-layer stability, as shown in Figure 6, where wall-to-free-stream temperature ratio required for infinite critical Reynolds number is given as a function of Mach number immediately behind shock wave. Results emphasize the importance of the geometry of the aerodynamic surface in affecting local Mach numbers and free-stream temperatures (and affecting, consequently, the boundary-layer velocity distribution and stability).

The obtaining of data which may be compared with the aforementioned analytical results is today one of the most important fluid-mechanics problems; several experimental studies are being conducted at present in the United States to obtain such data. Investigators having made already significant contributions include D. Coles (9), R. Korkegi (20), and J. Jack and N. Diaconis (17). In order to indicate the magnitude of the transition Reynolds number encountered, the data of Coles and Korkegi are presented as Figure 7, where transition Reynolds number is plotted as a function of Mach number for insulated flat plates. Until the results of further studies become available, magnitudes given in Figure 7 and trends indicated by Figures 5 and 6 provide guides indicating the effects of free-stream Mach numbers, wall-to-free-stream temperature ratio, and pressure gradient on the location of the point of transition from laminar to turbulent flow.

SUMMARY

Methods for the prediction of forced-convection heat-transfer coefficients are reviewed and extended, keeping foremost in mind the needs of aircraft designers dealing with speeds up to Mach number 3.

1. Expressions for steady-state heat transfer in boundary layers on isothermal flat plates, including simple engineering approximations for the handling of the temperature-dependence of fluid properties, are reviewed briefly.
2. For turbulent boundary layers, a new integral expression (more simple than expressions suggested previously) is presented for the effect on the heat-transfer rate of variations in the fluid-flow direction of the surface temperature.
3. The rate at which the surface temperature may change subject to the restriction that the equations for steady-state heating apply is determined analytically for laminar boundary layers, and it is concluded that the equations for steady-state conditions may be used over a wide range of rates of change of surface temperature.
4. Available information on the effect of a pressure gradient on the heat-transfer rate is reviewed, and it is concluded that, for slender bodies, equations for flow over flat plate may be used to good approximation if local conditions are used as reference conditions.
5. Effects of compressibility, heat transfer, and pressure gradient on the stability of laminar boundary layers are reviewed briefly.

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TABLE I

MAXIMUM ERROR MADE (For Example of Section II) IF ONE ASSUMES
ISOTHERMAL-PLATE HEAT-TRANSFER COEFFICIENT

Curve No.	$\frac{T_w(o) - T_{w_{ave}}}{T_{w_{ave}} - T_s}$	Max. Error (%)
1	0.129	16
2	0.148	19
3	0.169	23
4	0.196	28

TABLE II

COMPARISON OF TURBULENT AND LAMINAR FRICTION
COEFFICIENTS FOR SEVERAL REYNOLDS NUMBERS

Reynolds Number	C_f laminar = $\frac{0.664}{Re}$	C_f turbulent = $\frac{0.370}{(\log_{10} Re)^{2.584}}$	$\frac{C_f \text{ turbulent}}{C_f \text{ laminar}}$
5×10^5	0.00094	0.0041	4.4
5×10^6	0.00030	0.0027	9.0
5×10^7	0.000094	0.0019	20.2

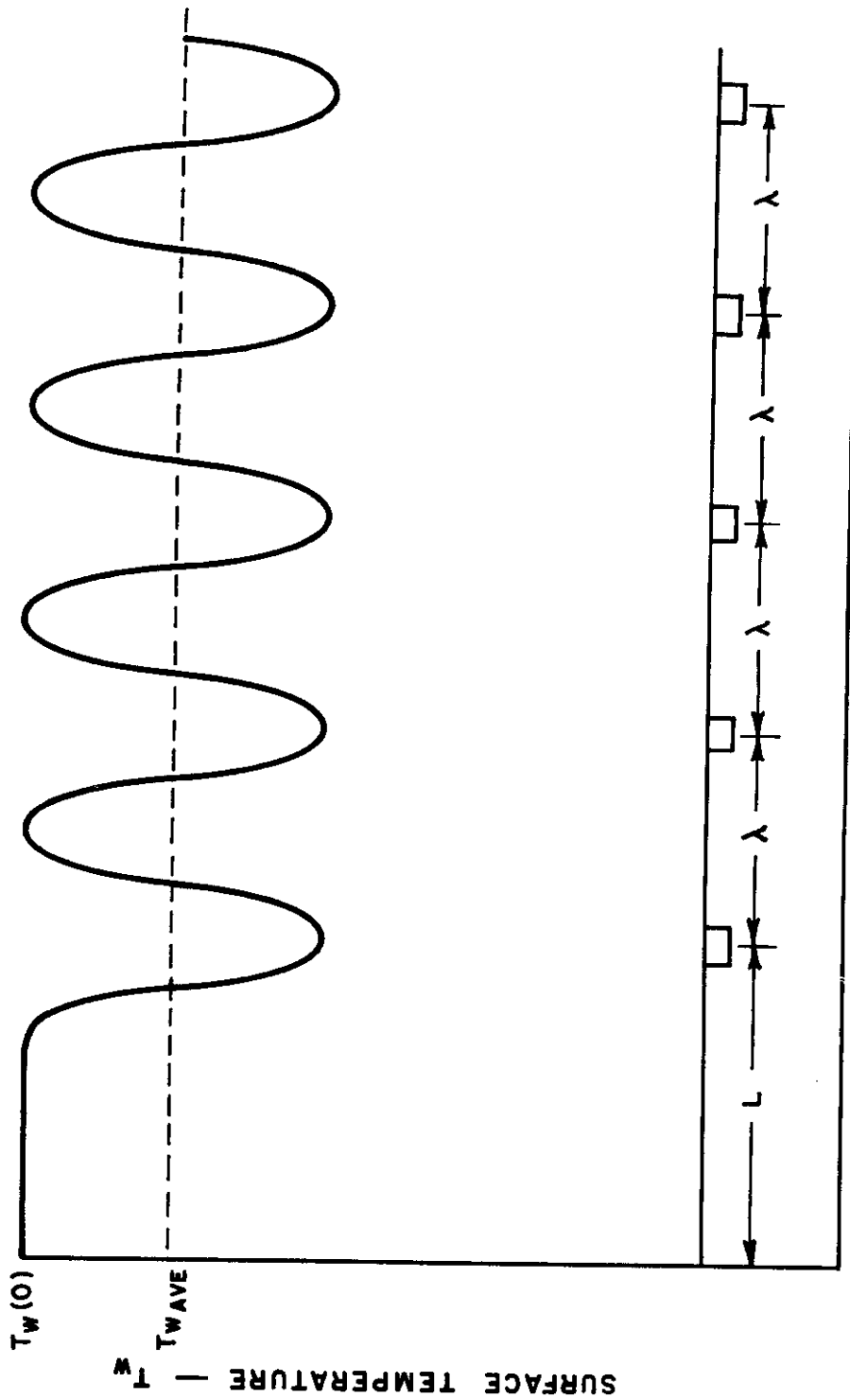


FIGURE I SURFACE TEMPERATURE VS DISTANCE FROM LEADING EDGE OF WING (AS ASSUMED FOR EXAMPLE IN SECTION II)

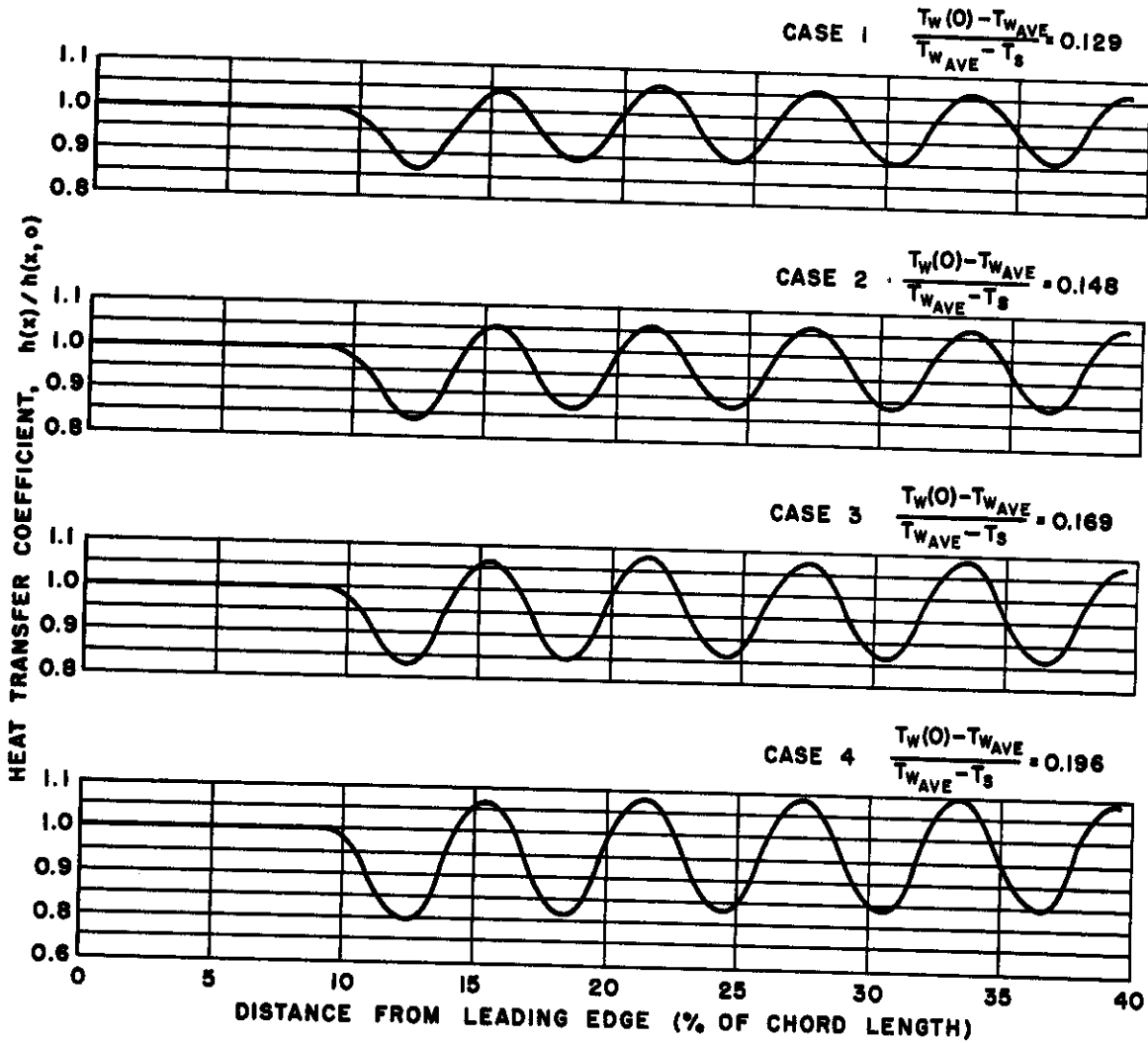


FIGURE 2 HEAT-TRANSFER COEFFICIENTS VS DISTANCE FROM LEADING EDGE FOR WING WITH SINUSOIDAL DISTRIBUTION OF SURFACE TEMPERATURE

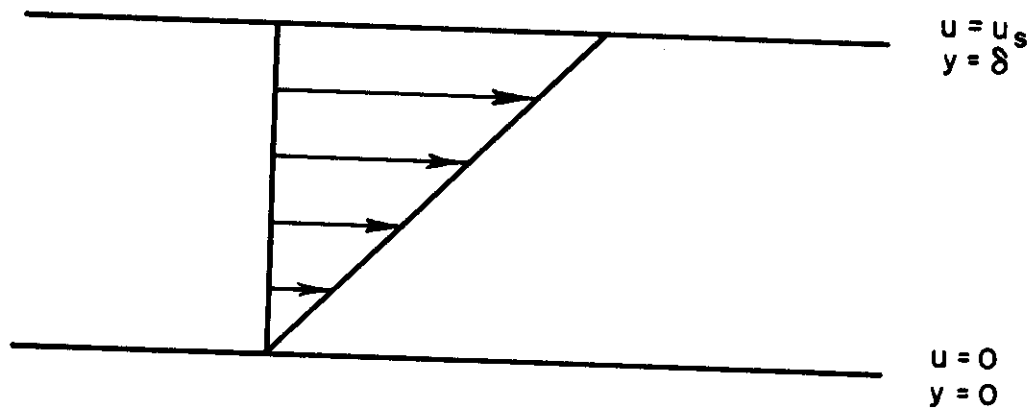


FIGURE 3 COUETTE FLOW WITH CONSTANT FLUID PROPERTIES

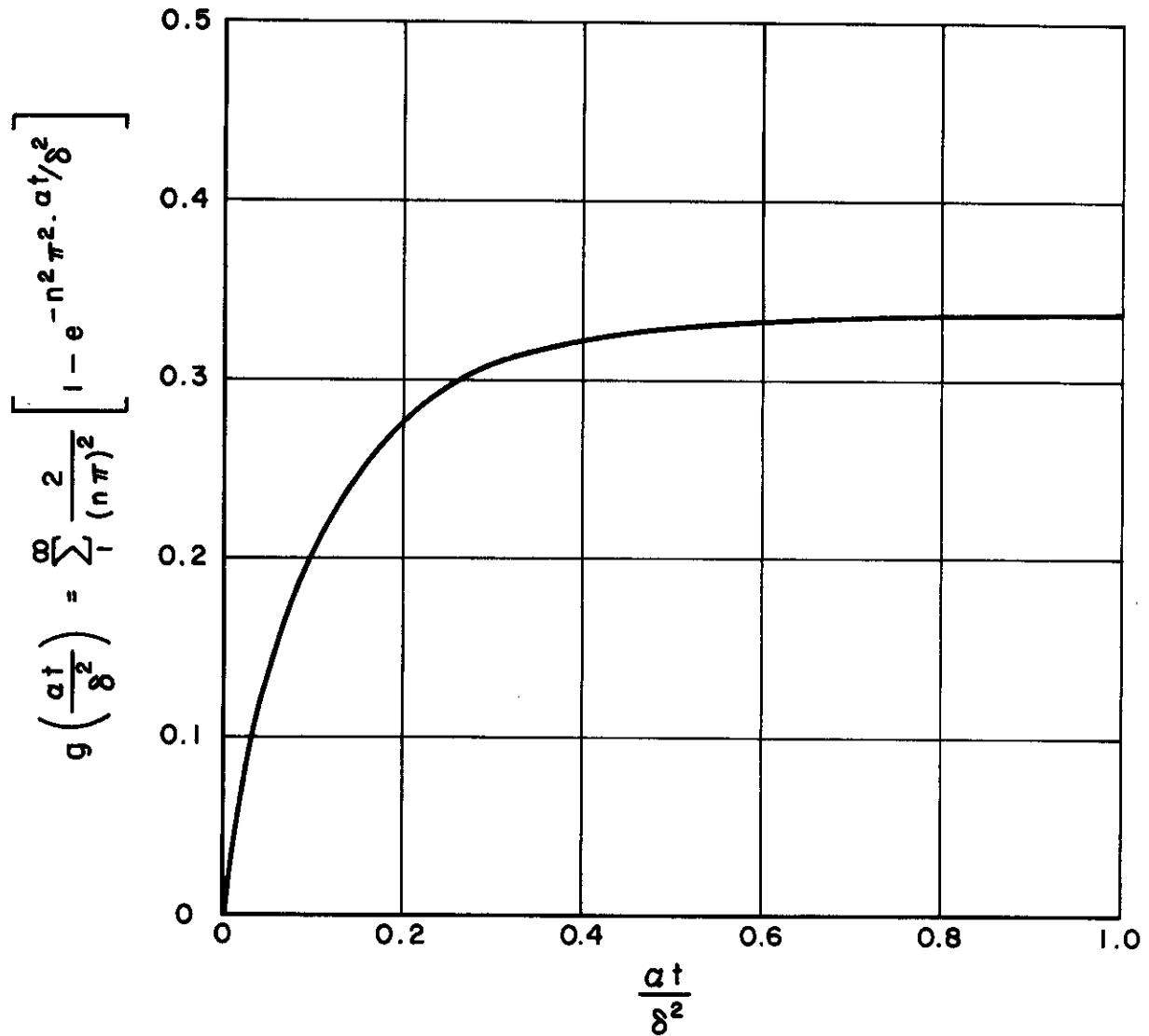


FIGURE 4 THE FUNCTION $g(\alpha t/\delta^2)$ APPEARING IN SOLUTION OF EQUATION FOR NON-STEADY HEAT TRANSFER

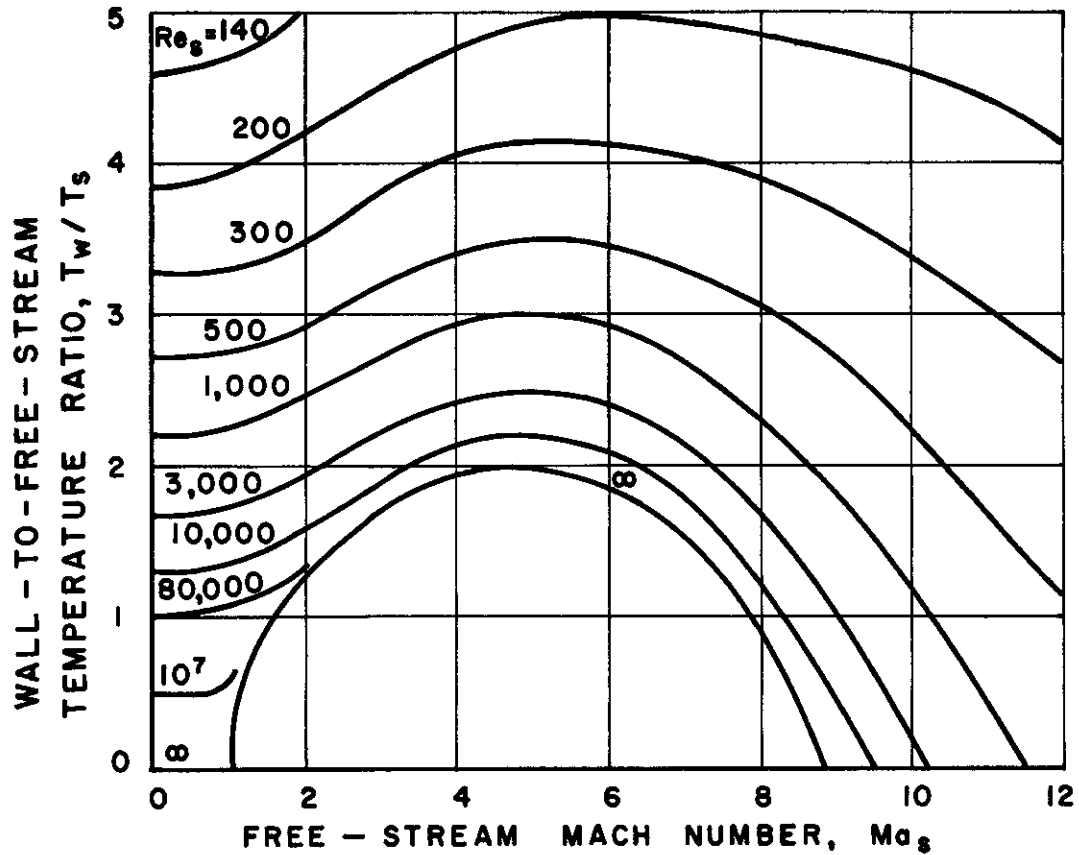


FIGURE 5 MINIMUM CRITICAL REYNOLDS NUMBER AS A FUNCTION OF FREE - STREAM MACH NUMBER AND WALL - TO - FREE - STREAM TEMPERATURE RATIO. PRANDTL NUMBER = 0.75, SUTHERLAND VISCOSITY LAW. (VAN DRIEST, REF. 39)

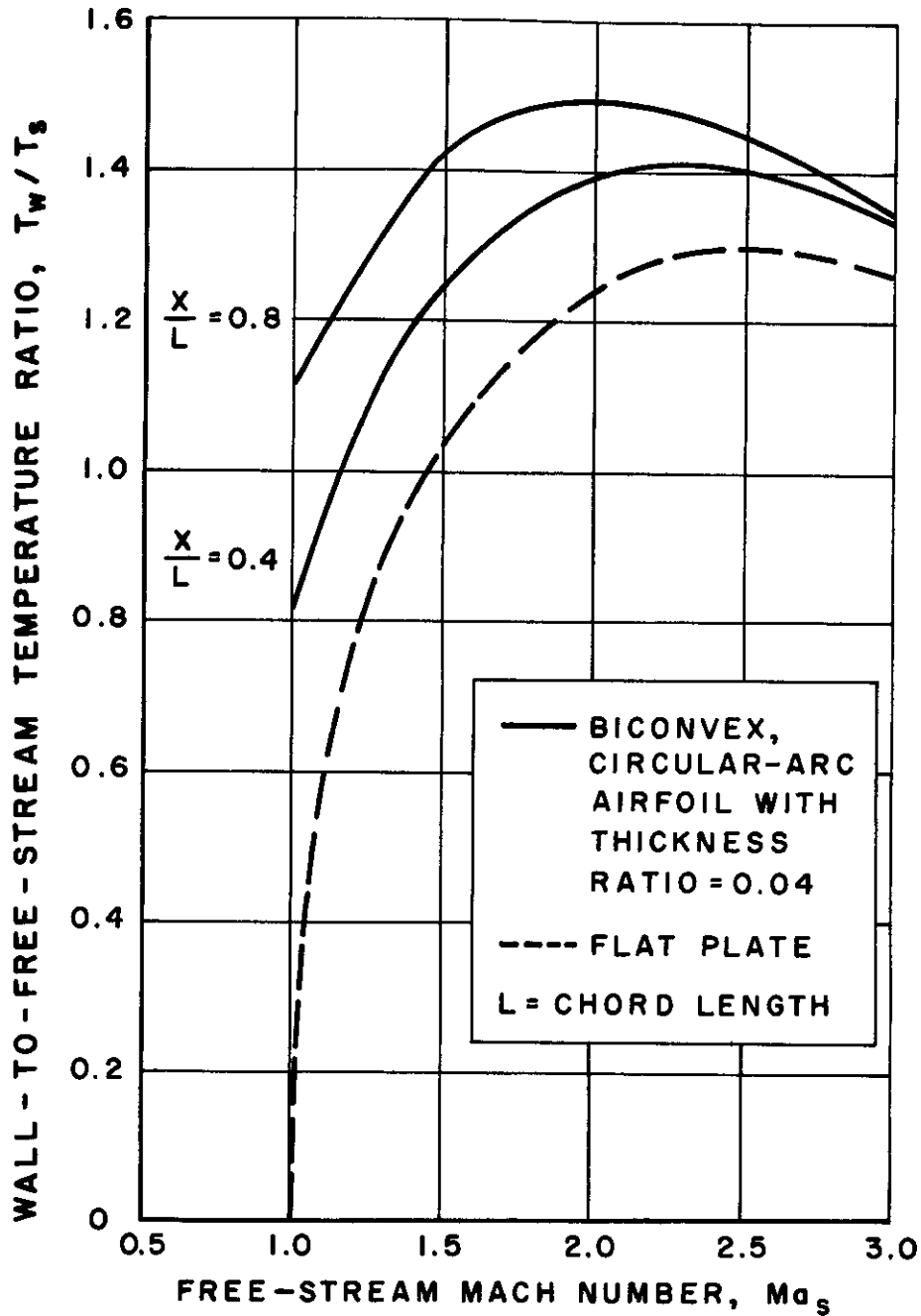


FIGURE 6 WALL-TO-FREE-STREAM TEMPERATURE RATIOS REQUIRED FOR INFINITE MINIMUM CRITICAL REYNOLDS NUMBER. PRANDTL NUMBER = 1, SUTHERLAND VISCOSITY LAW. (MORDUCHOW AND GRAPE, REF. 25)

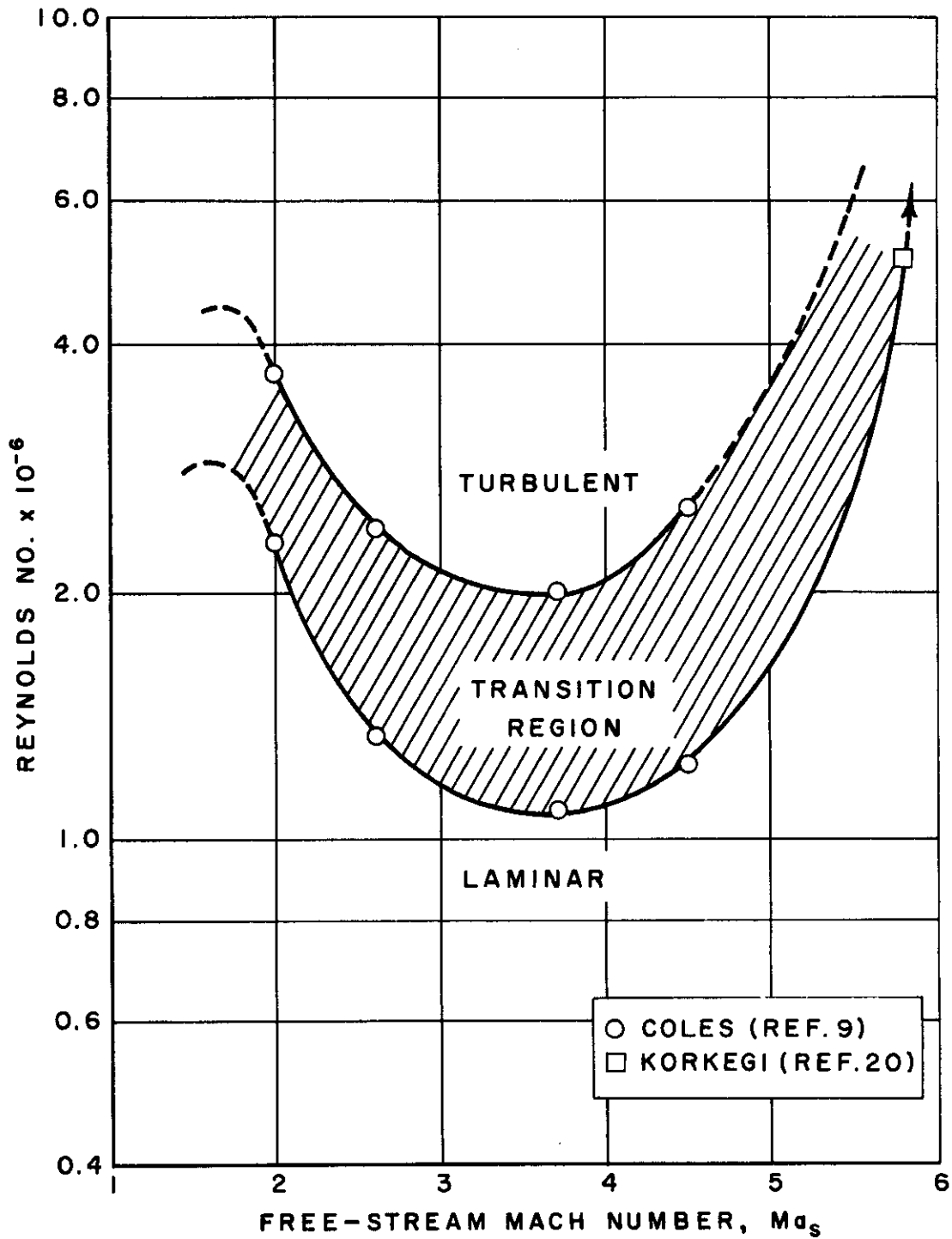


FIGURE 7 TRANSITION REYNOLDS NUMBER VS. MACH NUMBER FOR INSULATED FLAT PLATE IN WIND TUNNEL

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