PROGRESS IN THE DEVELOPMENT OF VIBRATION DAMPING MATERIALS

bу

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## I. INTRODUCTION

Considerable progress has been made in recent years in the development and technique of application of vibration damping viscoelastic materials which are used in sheet metal constructions as surface layers to dampen their bending vibrations. It is well known which requirements the dynamic-elastic properties of the materials must fulfill so that high damping can be obtained and how the damping depends on the thickness of the sheet metal panels and the damping layer.

First of all, the properties of coatings on one or both sides of sheet metal panels have been investigated (1,2,3,4). It was found that the damping effect is proportional to the imaginary part of the complex elasticity modulus, called loss modulus, which is equal to the product of the real part, the dynamic elasticity modulus, and the loss factor. This loss modulus is used primarily as an index of the internal damping in the material (see Section II). This means that high internal energy losses and simultaneous stiffness of the surface layer material are desirable. The damping increases rapidly with the ratio of the thickness of the coating and the sheet metal, irrespective of the density of the damping material. Only the dynamic-elastic characteristic values enter into the damping of the composite as material constants. This means, however, that in comparing two damping materials having the same dynamicelastic properties, and limited to a given mass, the lighter material is more favorable because a thicker coating can be used and in so doing achieve greater damping.

It was further determined how, by application of polymers, extra high damping capacity can be achieved systematically at a given temperature in a wide frequency range (5). In the investigation, no particular attention was given to the characteristic temperature range in which high damping of the materials used was exhibited. It was accepted as an inherent property.

In the latest development, after investigation of simple arrangements with a coating on one or both sides of the panels, configurations with several layers, and sandwich constructions



having a damping layer between two panels, were investigated (6,7,8,9) (particularly the "constrained layers" which were thoroughly investigated by Kerwin belong to these multi-layer coatings). These "constrained layers" consist of a stiff upper layer (with a small elongation) and a shear-loaded damping layer between the sheet metal and the upper layer. The sandwich arrangements, investigated by Kurtze and Watters, serve a special purpose. Here, with sufficient static bending stiffness, an adequate dynamic bending softness is desired, which means that the length of the bending waves in a large frequency range should be smaller than the length of the sound waves in air in order to assure sufficient attenuation of the composite in all frequencies. Adequate damping of the bending waves is the requirement here.

The requirements with respect to the dynamic-elastic properties of the materials used for these layers are modified in the latter cases in comparison with those mentioned first in the case of simple surface layers.

In the course of the development, the unsatisfactory temperature band width of high damping effect of the material was considered as a deficiency in connection with certain technical applications. Attempts to alleviate this deficiency are reported below. Today it is possible to manufacture materials of optimum effectiveness for most of the temperature ranges called for on account of technical reasons. The limits of the effectiveness, resulting from basic laws, were thoroughly investigated. There is hardly any doubt that the temperature band-width problem can also be solved systematically for multilayer configurations.

## II. THEORETICAL RESULTS

As already noted in connection with single layers, the dynamic-elastic characteristic value which must be optimized in respect to temperature level and temperature band-width is the so-called loss modulus.

The dynamic-elastic properties of a material under fluctuating tensile loading with an unrestrained transverse strain, a condition realized by a vibrating coated panel, are characterized by the complex modulus of elasticity E\* ( $\omega$ ) = E' ( $\omega$ ) + j E" ( $\omega$ ); where  $\omega$  =  $2\pi f$  is the angular velocity, f the frequency of the vibration, E' the dynamic-elastic modulus, and E" the loss modulus. The loss factor is d = E"/E"; it is equal to the tangent of the phase angle between strain and stress.

The dynamic-elastic behavior of high polymers depends a great deal on the molecular relaxation processes which, during deformation of the material, lead to a more or less fast stress relief. This phenomenon is the cause of the relatively strong frequency and temperature dependence of the dynamic-elastic characteristic values. E\* and its components E' and E" can always be expressed as integrals over the contributions of all molecular relaxation mechanisms in the following customary form (10)

$$E^{*}(\omega) = E^{!}(\omega) + jE^{"}(\omega) = \int_{0}^{\infty} H(\tau) \frac{j\omega T}{1+j\omega \tau} \frac{d\tau}{\tau}$$
(1)
$$E^{!}(\omega) = \int_{0}^{\infty} H(\tau) \frac{\omega^{2} \tau^{2}}{1+\omega^{2}\tau^{2}} \frac{d\tau}{\tau}$$
(1a)
$$E^{"}(\omega) = \int_{0}^{\infty} H(\tau) \frac{\omega \tau}{1+\omega^{2}\tau^{2}} \frac{d\tau}{\tau}$$
(1b)

In which  $\tau$  is the relaxation time, which indicates how a molecular mechanism with this time constant contributes temporally to the stress relief by first intermittent and subsequent constant elongation of a test specimen of the damping material, according to  $\exp(-t/\tau)$ , where t equals time. The distribution function  $H(\tau)$  defines the relaxation spectrum,  $H(\tau)$  d $\tau/\tau$  determines the contribution to  $E^*(\omega)$  of the molecular mechanisms with the relaxation time  $\tau$  in an interval  $d(\tau)$  around  $\tau$ .

Various types of molecular mechanisms may contribute to the relaxation spectra of high polymer materials at a given temperature. In addition to larger segments of the main molecular chains which contribute a great deal to the modulus in the transition range above the freezing temperature of these chains (in the amorphous ranges), secondary mechanisms such as molecular side-chains, mechanisms in the crystalline ranges, etc., are to be considered.

In vibration damping materials considerable use is made of the marked relaxation processes in the main-chain segments of preponderantly amorphous materials in the above mentioned transition range. These processes lead to marked changes of the dynamic modulus of elasticity E' with frequency and temperature (elastic dispersion) and to extremely high maxima of the loss modulus, i.e., to great energy absorption in the material of which particular use is made here.

The sequence of a spectrum with  $\tau$  and the characteristic values E', E', and d with the angular frequency  $\omega$  for a high polymer material, predominantly having main-chain mechanisms, is shown in Fig. 1. In this case, we deal with an amorphous cross-linked material, a polyester (11,12). In Fig. 1 the characteristic curves for the polyester are plotted in loglog scale, and also a comparison is made with a hypothetical material, having a well defined relaxation time  $\tau$ .

It is obvious that the relaxation spectrum of the <u>real</u> high-polymer material is very broad and extends over many orders of magnitude of  $\tau/\tau_{\rm P}$ , where  $\tau_{\rm P}$  is a reference relaxation time. It may further be noted that  ${\rm H}(\tau)$  progresses like E" (  $\omega$  ) save for a constant factor (see Equation 8). This holds true for all high-polymers. Whereas in the case of the <u>hypothetical</u> material relatively steep dispersion gradients of E' and high but relatively narrow maxima of E" and d appear, the gradient and the maxima of the <u>real</u> material, however, are spread rather broadly.

The relaxation mechanisms become more mobile with increasing temperature and the relaxation spectrum is shifted within shorter times according to the laws of kinetics of chemical reactions (10). With the relaxation spectrum the frequency curves of E' and E" are shifted toward higher frequencies with increasing temperature. This shift has been thoroughly investigated by J. D. Ferry and his associates Landel and Williams for various amorphous non-cross-linked high-polymers in the transition range, and the following function of temperature was empirically determined (13)

$$\log \frac{\omega}{\omega_{S}} = 8.86 \frac{T - T_{S}}{101.6 + (T - T_{S})}$$
 (2)

in which T is the absolute temperature in degrees Kelvin,  $\omega_s$  is a reference frequency for the angular frequency which changes with T,  $T_s$  is a reference temperature, belonging to  $\omega_s$ , in the transition range (about 50°K above freezing temperature of the material) for which Equation (2) is valid;  $\log \omega/\omega_s$  defines the shift of the values E' and E", measured at temperature  $T_s$  and angular frequency  $\omega_s$ , in direction toward higher or lower frequencies with change of T in respect to  $T_s$ . Equation (2), which will be designated as equation WLF (according to Williams, Landel, and Ferry), is valid in the range of  $\pm$  50°K around  $T_s$ . Thereby also the range of validity is given for the estimation of the temperature bandwidth of the vibration damping materials (see below).

Ferry and his associates have based the coefficients in the WLF equation on molecular characteristic values and in so doing proved the universal character of the equation. This holds for all high-polymers with uniform main-chain mechanisms.

In the development of high-polymers with high vibration damping properties in a wide temperature range, the expansion of the temperature range is achieved by widening the relaxation spectrum. The relaxation mechanisms are no longer uniform in such materials. There are always molecular mechanisms present, having different freezing temperatures, which are distributed over a more or less broad temperature range. Mechanisms with lower freezing temperature generally have a lower activation energy than those with a higher one, and their relaxation spectra shift slower with increasing temperature in a shorter time than the mechanisms with a higher freezing temperature. The spectra of "broad-band-materials" therefore become somewhat "deformed" with increasing temperature, and as a further result the WLF equation is modified for these materials. These modifications may be determined qualitatively on the basis of the theoretical principles mentioned above. In this case of qualitative estimations, however, we forego this procedure and assume that the WLF equation can be considered a valid approximation for "broad-band-materials" as well. justification of this assumption is derived from experimental analysis on a large number of materials of the calculated relationship between the temperature band-width and the attainable maximum amount of the loss modulus E".

Under the already mentioned and still further simplifying assumptions, which do not represent a substantial curtailment of the general validity of the principles found, the dependence of the maximum values of  $E^{\rm ll}$  from the temperature bandwidth was calculated. Here the limit value  $E_{\rm co}$  at a given temperature plays quite an important role as "determinant" which is approximated by  $E^{\rm ll}$  ( $\omega$ ) at high frequencies (see Fig. 1). According to equation (1)

$$E_{\infty} = E* (\omega)_{\omega \to \infty} = \int_{H(\tau)}^{\infty} \frac{d\tau}{\tau}$$
 (3)

This limit value  $E_{\infty}$  is obtained when the relaxation mechanisms are unable to follow the fast deformation cycles, i.e., when the internal stresses, caused by fast changing deformation, cannot be relieved to an appreciable extent. Under these conditions, all "not-filled" high-polymers have values  $E_{\infty}$  which are

very close together and above  $10^{10}$  dyn/cm<sup>2</sup>. Through the addition of suitable filler  $E_{\infty}$  can be increased. Nevertheless, values of  $E_{\infty} > 10^{11}$  dyn/cm<sup>2</sup> hardly occur, so that this value may be considered as the upper limit in conjunction with the following estimations. According to equation (3),  $E_{\infty}$  corresponds to the area (integral value) limited by the  $H(\tau)$ -curve ( $H(\tau)$  in linear scale) and the abscissa (in log scale). As a further simplification, the relaxation spectrum, in a semilog presentation, is approximated by a "box function" as follows:

$$H(\tau) = 0$$
 for  $\tau < \tau_1$  and for  $\tau > \tau_2$   
 $H(\tau) = H_m = \text{const for } \tau_1 \le \tau \le \tau_2$  (3a)

Then, using equation (3) one obtains

$$\frac{H_{\rm m}}{E_{\rm co}} = \frac{1}{\ln(\tau_2/\tau_1)} \tag{4}$$

With this expression one derives from equation (1)

$$\frac{\mathrm{E''}(\omega)}{\mathrm{E}\,\infty} = \frac{1}{\ln(\tau_2/\tau_1)} \quad \text{arc } \tan\frac{\omega}{1-\omega^2} \frac{(\tau_2-\tau_1)}{\tau_1} \tag{5}$$

If, in the case of the hypothetical material in Fig. 1, the spectrum deterioriates to a spectral line, i.e., the relaxation times contract to a sharply defined relaxation time  $\tau_r$ ,  $H_m$  increases beyond all limits and  $E''(\omega)/E_{\infty}$  approaches the limit function

$$\frac{E''(\omega)}{E_{\infty}} = \frac{\omega \tau_{r}}{1 - \omega^{2} \tau_{r}^{2}}$$
 (6)

In Fig. 2a, idealized in the indicated manner, relaxation spectra of various width, based on  $E_{\infty}$ , are plotted over log  $(\tau/\tau_r)$ ; where  $\tau_r$  is the mean relaxation time (geometric mean). In Fig. 2b, the relative loss moduli  $E''(\omega)/E_{\infty}$ , belonging to these spectra, are plotted as functions of log  $(\omega/\omega_r)$ , which are calculated according to equations (5) and (6), with the angular frequency  $\omega_r = 1/\tau_r$ .

Figs. 2a and 2b demonstrate that, with given value  $E_{\infty}$ , the maximum values of  $E''(\omega)$  become smaller with increasing width of the relaxation spectrum and consequently with the frequency band of  $E''(\omega)$ .

Also the area which, in semi-log presentation is limited by the E"-curve and the frequency coordinate, is proportional to E<sub>∞</sub>. When E"( $\omega$ ) is plotted over  $\ln(\omega/\omega_r)$ , this area, according to equation (1b) and (3), is

$$\int_{-\infty}^{+\infty} \mathbb{E}^{n}(\omega) \, d(\ln \omega / \omega_{r}) = \frac{\pi}{2} \int_{-\infty}^{+\infty} \mathbb{H}(\tau) \, d(\ln \tau / \tau_{r}) = \frac{\pi}{2} \, \mathbb{E}_{\infty} \quad (7)$$

This relationship generally holds true for any high-polymer. It establishes the limits of the maximum damping capacities which can be obtained with a given width of the spectrum or the  $E''(\omega)$ -curves. Therefore, one has to strive for the highest possible  $E_{\infty}$  values. In this connection, however, the influence of the density, changing with  $E_{\infty}$ , on the damping capacity of the damping material on the sheet metal has to be considered (see Section III). With real high-polymers, the distribution function  $H(\tau)$  always changes relatively little with  $\tau$ . It is expressed in a usable approximation (10) as follows

$$E^{II}(\omega) = \frac{\pi}{2} H(\tau) |_{\tau = 1/\omega}$$
 (8)

This demonstrates that  $E''(\omega)$  progresses similar to the distribution function  $H(\tau)$  if it is assumed that  $\omega = 1/\tau$ . The E''-values are greater than the corresponding H-values by a factor  $\pi/2$  (see Fig. 1).

If one defines the frequency band-width as the "half-amplitude-width" of the E" ( $\log \omega / \omega_r$ )-curves, i.e., the interval of the  $\log (\omega / \omega_r)$ -values at which the E"-values decrease to half of the maximum values, then the correspondingly defined "half-amplitude-width" of the spectrum is equal to the "half-amplitude-width" of the E"-curve (see Fig. 2).

In determining the temperature curves of E" for a given reference angular frequency  $\omega_n$ , one proceeds from the WLF equation (2). In Fig. 3a, the dependence of log ( $\omega$  /  $\omega_s$ ) on the temperature difference T - T<sub>s</sub>, according to equation

(2), is shown. In the materials, which satisfy the WLF equation, experience shows that at a certain reference angular frequency  $\omega_r$  of the range of application the maximum of the E"( $\omega$ )-temperature-curve lies at a temperature  $T_r$  about 20°K below  $T_s$  (11).  $T_r$  therefore is selected as a reference temperature for the temperature curves in Fig. 2 with the aid of

below  $T_s$  (11).  $T_r$  therefore is selected as a reference temperature for the temperature curves in Fig. 2 with the aid of the WLF equation. The value  $E''/E_{\infty}$ , belonging to a temperature T of the validity range of the WLF equation and to the fixed angular frequency  $\omega_r$ , is equal to the ordinate value (Fig. 2b) for a log ( $\omega$  /  $\omega_r$ )-value which is derived from the WLF-curve in Fig. 3b, for the temperature T. The family of temperature curves, corresponding to the family of frequency curves, are presented in Fig. 4. In this case also the areas limited by  $E''/E_{\infty}$ -curves and the temperature coordinate are still approximately constant.

The "half-amplitude-width" of the E"(T)-curves for fixed frequency is defined as temperature band-width  $\Delta T$  for high damping. The desired relationship between  $\Delta T$  and E"\_max/E\_ $\infty$  can finally be extracted from the family of curves in Fig. 4. In Table 1 the values for various band-widths of the relaxation time or frequency (band-widths according to Fig. 2 based on log  $\tau/\tau_{P}$  and log  $\omega$  /  $\omega_{P}$ ) can be found together with values of the products  $\Delta T$  E"\_max/E\_ $\infty$ .

Materials with sharply defined relaxation time  $\tau_r(\Delta \log \tau/\tau_r=0)$  do not exist. High-polymers with uniform main-chain mechanisms have a band-width  $\Delta T$  of about 20°K (see Figure 5 below). Toward the upper end the range of the band-width, for which the relation between  $\Delta T$  and  $E''_{max}/E_{\infty}$  is valid, is limited by the range of validity of the WLF equation to approximately 100°K (compare Figs. 3 and 4). With increasing bandwidth  $\Delta T$  this value of  $\Delta T$   $E''_{max}/E_{\infty}$  changes only slightly between 20° and 100°K. It increases linearly from 5.5° to 6.5°K. This analysis shows that no substantial error is made if we suppose that

$$\Delta T E''_{\text{max}}/E_{\infty} \approx 6.0^{\circ} \text{K}$$
 ( $\Delta T \text{ in } \circ \text{K}$ ) (9)

This relation, derived under simplified conditions, can only represent a more or less good approximation. In the following section, the validity of this relation is tested by experiment on a number of materials.

Fig. 5 shows temperature curves of the loss modulus E" with a reference frequency of 200 cps for a number of high-polymer materials, of which the temperature ranges with maximum damping have been adjusted to different values. Each of the materials examined by the author in the following figures is designated by a number. The curves have been determined by the bending-wave method (2,11) on small strips whose damping layers were produced either by spraying of an aqueous dispersion with or without Vermiculite filling (material with temperature band-widths up to 60°C) or by bonding a sheet of high-polymer material to the metal strips (material with large band-width).

The upper part of Fig. 5 shows curves of materials with relatively small band-width  $\Delta T$ , the freezing temperatures of which are distributed over a wide temperature range. It is obvious that E" may be increased considerably by Vermiculite filling (Material 1, 5, and 6). This important finding will be further treated below. Also noteworthy is the increase of E" with increasing freezing temperature (see temperature curves of loss factor for a sheet metal panel with damping layer in Fig. 9).

The curves in Fig. 5b are for materials which were developed for industrial use and for ground vehicles (temperature ranges about  $-10^{\circ}$  to  $+60^{\circ}$ C).

The materials for which the curves are shown in Fig. 5c were specifically adjusted for large temperature band-width.

In Fig. 6 the maximum values of the loss modulus  $E''_{max}$  of all materials referenced in Fig. 5 and several others are plotted over the temperature band-width  $\Delta T$ . The plotted family of curves with the parameter  $E_{\infty}$  corresponds to the relation in equation (9). Because of the simplifying assumptions made in the derivation of this relation, it cannot be expected that the  $E_{\infty}$  values, which can be extracted from Fig. 6 for the various materials, correspond exactly to those determined experimentally. Apart from these insignificant variations, Fig. 6 confirms excellently the applicability of equation (9).

As expected, all limit values  $E_{\infty}$  of the materials are above  $10^{10}$  dyn/cm<sup>2</sup> (compare Section II). The comparatively low  $E_{\infty}$  -values of the unfilled materials with band-widths

 $\Delta T$  below 70°C (E"  $\approx$  2 x 10<sup>10</sup> dyn/cm<sup>2</sup>) are caused by the fact that the methods of spraying a filler-free aqueous dispersion seldom creates bubble-free damping layers. These bubbles result in a certain loss of rigidity of the material. In specimens cut out of unfilled high-polymer sheet material with a band-width  $\Delta T$  above 100°C, the  $E_{\infty}$  -values are normally close to 5 x 10<sup>10</sup> dyn/cm<sup>2</sup>. The stiffening effect of the Vermiculite filling is especially noticeable in Fig. 6. In certain individual cases,  $E_{\infty}$  can reach the limit of 10<sup>11</sup> dyn/cm<sup>2</sup>.

The stiffening of the damping material by the Vermiculite filling is illustrated by the temperature curves for 200 cps of the dynamic-elastic characteristic values of one of the materials samples in Fig. 7. The E'-curves approach a limit value with decreasing temperature, which corresponds to the limit value  $E_{\infty}$  of the  $E^{\dagger}(\omega)$ -curves for fixed temperature with increasing frequency. It is obvious that  $E^{\dagger}$  is already greatly increased in the frozen range by the Vermiculite filling, whereas the loss factor d drops somewhat. This behavior can be demonstrated by means of a spring model in which the filler granules are considered as embedded hard springs in a range of not too great differences of the E-moduli of the carrier and the filler materials (14). The stiffening effect of the filler material becomes considerably greater in the transition range above the freezing point where E' drops fast with increasing temperature. The increase of the loss factor d, in comparison, remains more limited. Consequently, the maximum of the loss modulus  ${\tt E}^{\tt H}$  becomes greatly increased and is shifted toward high temperatures.

The loss factor decomposed as sheet with damping layer, which is considered as a homogeneous unit, also increases with E". In addition to stiffening the carrier material, the light, air containing Vermiculite grains reduce the density of the filler in comparison to the unfilled material. This means, however, that with a limitation of the permissible mass of the layer, the filled material may be applied thicker. This again would increase the damping effect. These are the reasons why Vermiculite, with its granules acting like armoured air bubbles with high inherent stiffness, is especially well suited as a filler for vibration damping materials.

Fig. 8 shows the dynamic-elastic properties of a damping material with extremely large temperature band-width. The width of the relaxation spectrum obtained in this case is reflected in the large width of the range of relatively high

loss-modulus values of the E"-curve (compare with relatively sharp E"-maximum in Fig. 7). E', accordingly, drops only slightly in the transition range with increasing temperature, and the loss-factor increases gradually to the maximum.

The damping effects which are obtained with the various Vermiculite filled dispersions of prescribed ratio of the masses  $m_2$  of the coating and  $m_1$  of the sheet metal (here  $m_2/m_1 = 20\%$ ), are illustrated in Fig. 9. In this figure the temperature curves of the loss factor dombof the sheets with damping coatings are compiled for a number of materials with different temperature band-width, which already was considered in Fig. 5. If corresponding E" and dombcurves in Figs. 5 and 9 are compared, it becomes obvious that the great differences in the maxima E"max of the loss modulus E" are equalized in the maxima of the doombcurves (particularly significant for the materials 4 and 6). The reason for this is that the materials with especially high E"max (for instance, material 6) at the same time also show a comparatively high density: the air content in the Vermiculite granules is relatively low and their stiffness correspondingly high. In lighter materials (for instance, material 4), at a given ratio of  $m_1/m_2 = 20\%$ , the layer thickness is greater so that the loss in damping, caused by the comparatively low loss-modulus values, is compensated by the greater layer thickness.

In Fig. 9 is shown the maximum panel damping which can be attained with "broad-temperature-band-materials" which cover the range of technical service temperatures and also what reductions in damping have to be tolerated in comparison with "small-temperature-band-materials." The damping levels obtained with broad-band materials can still be called considerable.

In conclusion, it can be said that it is possible today to manufacture vibration damping materials with high damping and broad width of the ranges of high damping as required inservice. Even if the level of damping obtainable with the required temperature band-width is limited at the upper end of the band, for the physical reasons discussed in this paper, this restriction, nevertheless, should not curtail the usefulness of the damping material in the majority of technical applications.

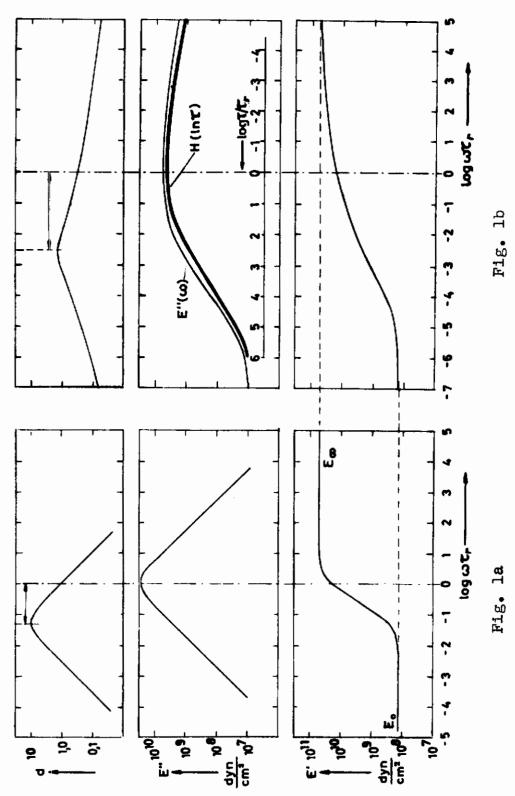


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TABLE I

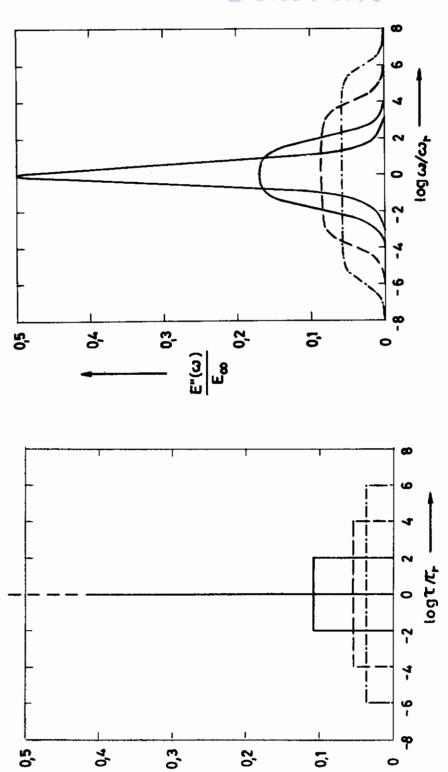
$\Delta \log \tau/\tau_{ m r} = \Delta \log \omega/\omega_{ m r}$	E" max /Eo	ΔT in <sup>O</sup> K	$\Delta T = E'' \max / E_{\infty}$ $1 n^{O} K$
0	0.500	<b>7.</b> 6	4.85
ħ	0.170	32.5	5.50
8	0,085	71.0	6.05
12	L50°0	0.611	6.80



- Dynamic Modulus E', Loss Modulus E" and Loss Factor d as Function of Angular Frequency & of:

and - Hypothetical Material With Well Defined Relaxation Time  $\tau_{\rm r}$  ;  $\rm E_{\rm O}$  . Limit Values of E' at Low and High Frequencies

Polyester With Distribution Function H(ln r) (Relaxation spectrum); Tr, Reference Relaxation Time According to G. W. Becker ı **Q** 



H H H

Value  $E_{\infty}$  of the Dynamic-Elastic-Modulus at High Frequencies. ( $\tau$  = Relaxation Time,  $\tau_{\rm r}$  = Reference Value in Center of Spectrum, | W r = 1/Tr = Reference Angular Frequency Idealized Relaxation Spectra  $H(\tau)$  and Corresponding Frequency Curves of the Relative Loss-Modulus  $E''(\omega)$  Based on the Limit Based on the Limit w = Angular Frequency
at Maximum of E"(ω) t ~ Figure

Fig. 2b - Frequency Curves for Loss Modulus

Fig. 2a - Relaxation Spectra

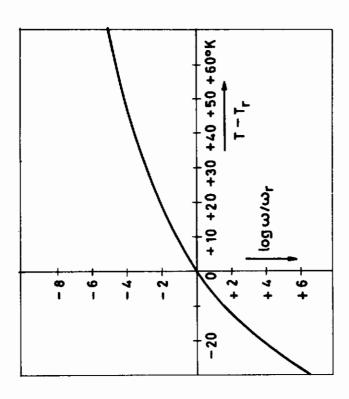


Fig. 3b - Corresponding Abscissa Values of the Frequency and Temperature Curves of the Relative Loss-Modulus E"/EQ(T<sub>r</sub> = Temperature of Loss-Modulus Maximum at Reference-Angular-Frequency  $\omega_{\mathbf{r}}$ )

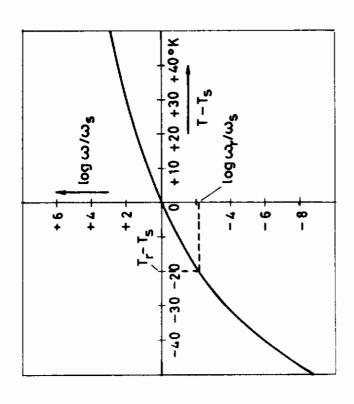


Fig. 3a - Shift of the Frequency Curves of the Dynamic-Modulus E'(\omega) and the Loss-Modulus E''(\omega) with Change of Absolute Temperature T According to WLF Equation

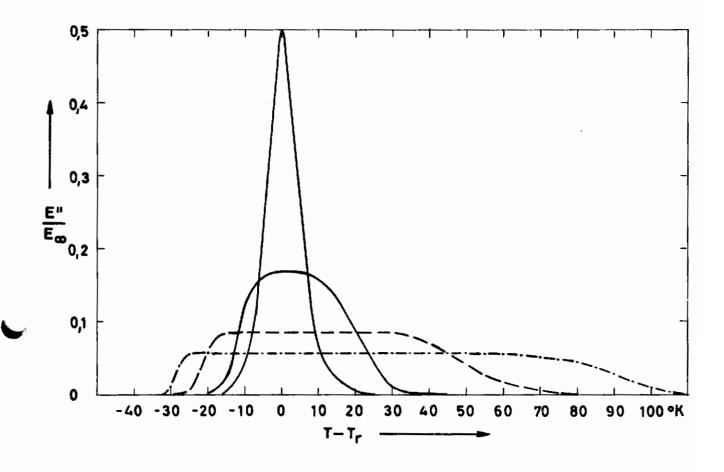


Fig. 4 - Temperature Curves of the Relative Loss-Modulus  $E''/E_{\infty}$  for Angular Frequency  $\omega_r$   $T_r = \text{Temperature of the Maximum of } E'' \text{ for } \omega_r$ 

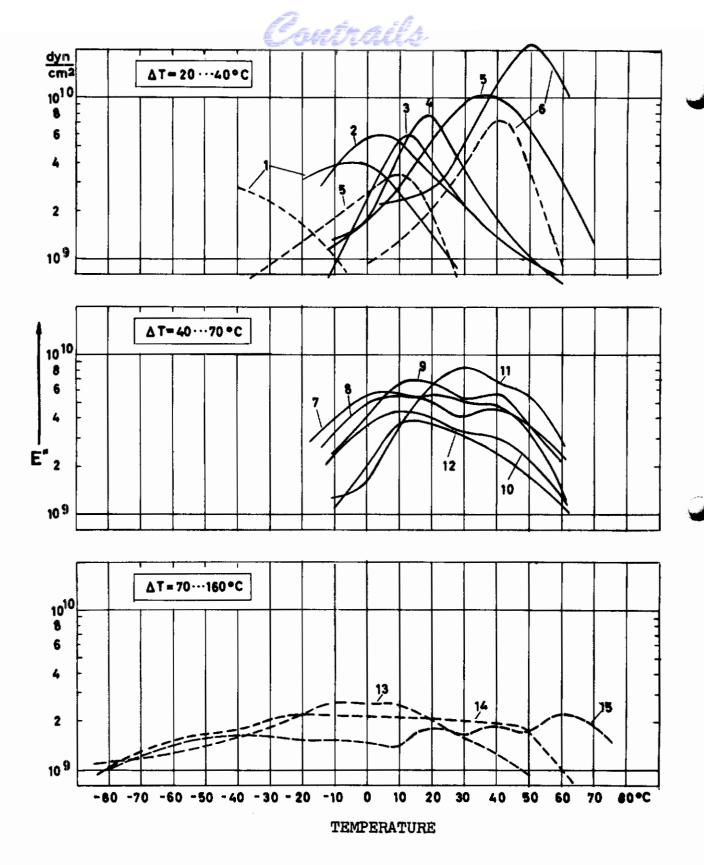


Fig. 5 - Temperature Curves of Loss-Modulus E" of Materials With Different Temperature-Band-Width  $\Delta T$  for 200 cps

--- Unfilled Material

----- Vermiculite-Filled Material

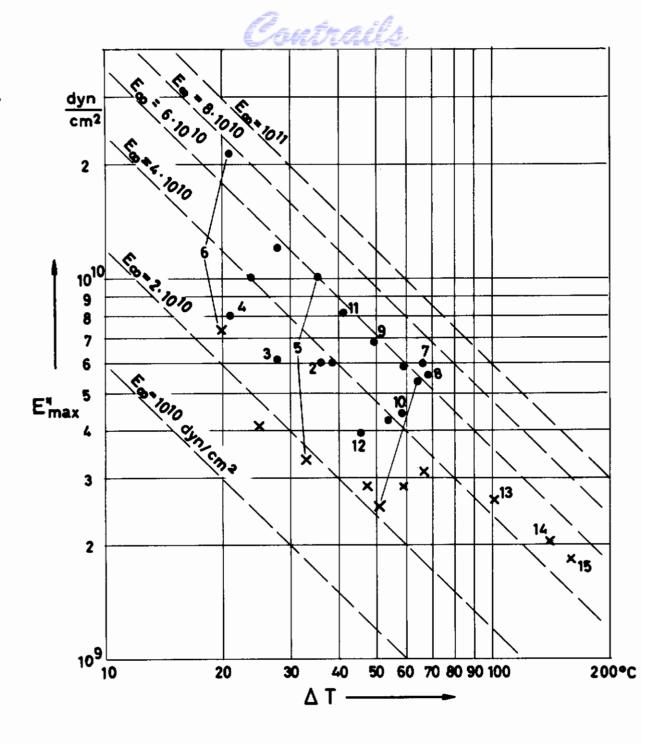


Fig. 6 - Maximum Values of Loss-Modulus for Various Damping Materials as Function of Temperature-Band-Width  $\Delta T$  for 200 cps

- ★ Unfilled Material
- Vermiculite-Filled -Material

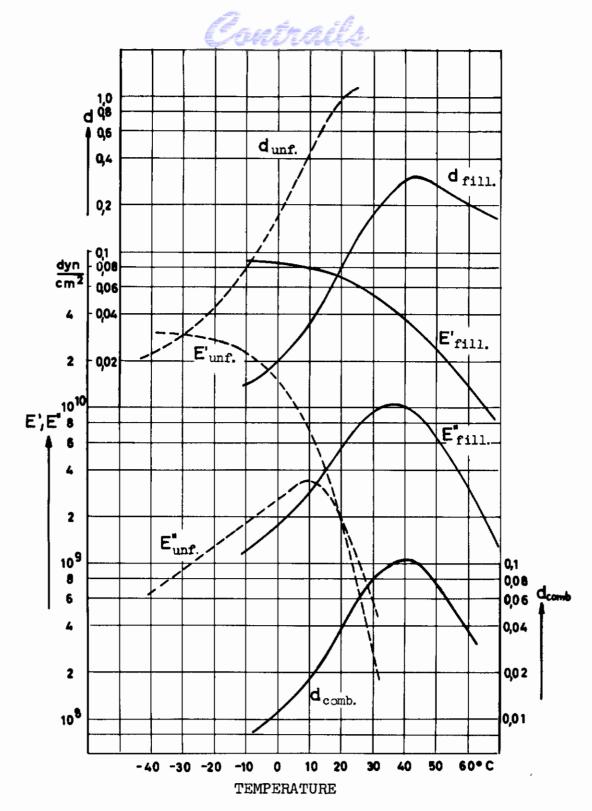


Fig. 7 - Dynamic Modulus of Elasticity E', Loss-Modulus E" and Loss-Factor d of a Damping Material and Loss-Factor d<sub>Comb</sub> of a Steel Sheet with Filled Layer as Function of Temperature for 200 cps and a Mass Ratio of Layer to Steel = 20 %

— — Unfilled Material
— Vermiculite-Filled Material

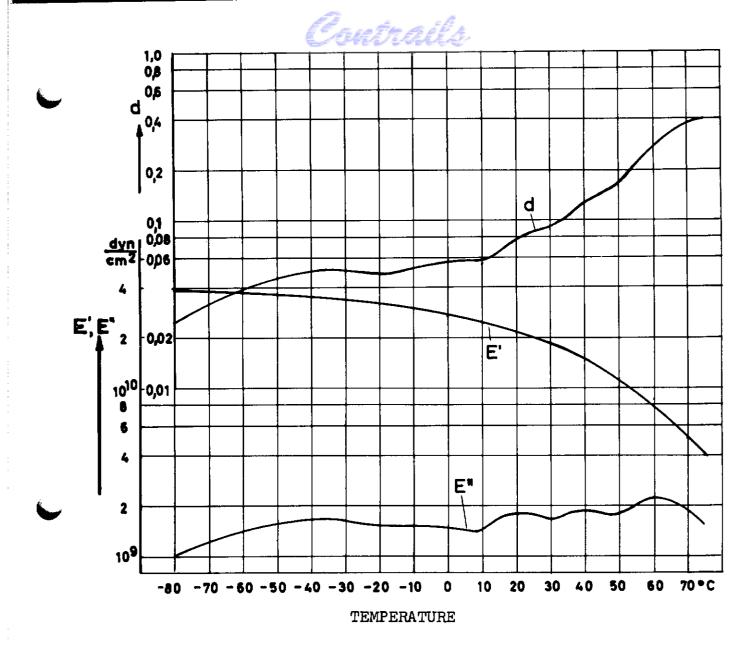


Fig. 8 - Dynamic Modulus of Elasticity E', Loss Modulus E" and Loss Factor d of a Non-Filled Temperature Broad-Band Damping Material as Function of Temperature for 200 cps

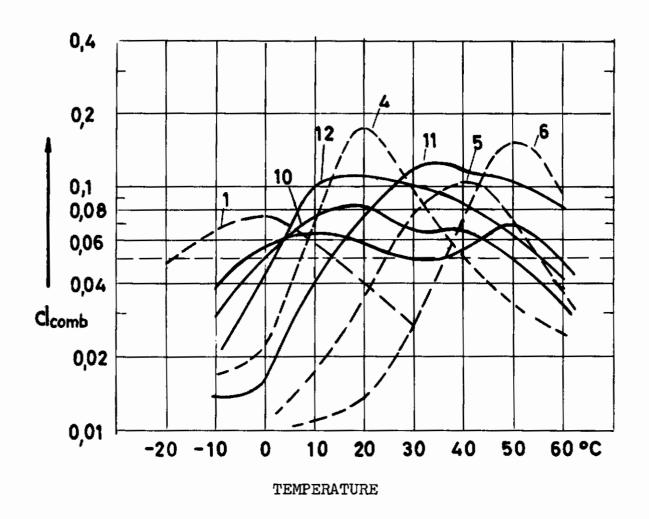


Fig. 9 - Loss Factor  $d_{\text{comb}}$  of Steel Sheet With Vermiculite-Filled Damping Layer for 200 cps and a Mass Ratio of Layer to Steel = 20 %