APPLICATION OF THE RITZ PROCEDURE TO DAMPING PREDICTION USING A MODAL STRAIN ENERGY APPROACH

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ABSTRACT

An automated procedure is defined to derive modal damping values in constrained-layer damping problems. The procedure uses the NASTRAN finite element program with DMAP modifications to derive modal loss factors using a Modal Strain Energy (MSE) approach. The frequency-dependent properties of the constrained viscoelastic layer are taken into account in an iterative solution. The Ritz procedure, a specialized Lanczos method for eigenvalue extraction, is used in the procedure together with standard NASTRAN super-element techniques to increase eigenvalue solution efficiency. Sample problems are discussed to illustrate the accuracy and efficiency of the method.

INTRODUCTION

Vibration reduction in structures has been a subject of investigation for many years. One of the most weight-effective means of reducing vibration is to incorporate a viscoelastic material in the form of a constrained layer in a built-up structure. In this method, an elastomer is sandwiched between two metallic sheets and is bonded to both. Flexural vibration causes shearing strain in the core, which dissipates energy and thereby reduces vibration.

An additional advantage of constrained-layer damping is that analytical methods and modeling techniques exist to predict structural behavior of the damped system. Using these analytical techniques, studies can be performed to gauge the adequacy of different damping treatments in eliminating unwanted responses.

It is generally felt that the Modal Strain Energy (MSE) approach using commercially available finite element programs is the most computationally efficient for use in analyzing constrained-layer damping problems. One of the major problems confronting MSE, however, is the frequency-dependent material properties of the viscoelastic layer. An automated procedure to derive modal loss factors using the undamped mode shapes and the material loss factors of the frequency-dependent material is presented in this paper. The same concept can then be extended to solve the forced-response problem by evaluating modal stiffness and modal mass matrices from the resulting mode shapes and frequency-dependent system stiffness.

The Ritz procedure, first described by Wilson, et al. [1], has also been taken advantage of in this application to constrained layer damping problems. The Ritz procedure provides a means to reduce the number of eigenvectors used in a forced response analysis without reducing solution accuracy. It can also provide significant savings over other eigenvalue solution techniques.

OVERVIEW OF MODAL STRAIN ENERGY METHOD

In this approach, first suggested by Johnson, et al. [2], it is assumed that a standard mode superposition approach can be used to uncouple the equations of motion:

$$M\ddot{x} + C\dot{x} + Kx = p(t) \tag{1}$$

where

M, C, K = physical coordinate mass, damping and stiffness matrices (all real and constant)

$x, \dot{x}, \ddot{x}$ = vectors of nodal displacements velocities, and accelerations

p = vector of applied nodal loads

The damped structure can be represented in terms of the real normal modes of the associated undamped system if appropriate damping terms are inserted into the uncoupled modal equations of motion:

$$\ddot{\alpha}_r + \eta^{(r)} \omega_r \alpha_r + \omega_r^2 \alpha_r = p_r(t) \qquad (2)$$

$$x = \Sigma \phi^{(r)} \alpha_r(t) \qquad r = 1,2,3\ldots \qquad (3)$$

where

α_r = rth modal coordinate

ω_r = natural radian frequency of the rth mode

$\phi^{(r)}$ = rth mode shape vector of the associated undamped system

$\eta^{(r)}$ = loss factor of the rth mode

p_r = modal force vector for rth mode

It is implied that the damping matrix, C, of Eq. (1), need not be explicitly calculated, but that it can be diagonalized by the same real modal matrix that diagonalizes K and M.

Modal loss factors are calculated using the undamped mode shapes and the material loss factor for each material [3]. For a structure damped with a viscoelastic layer, the material loss factor of the metal sheet is very small compared with that of the viscoelastic layer. Hence, the modal loss factor is found from:

$$\eta^{(r)} = \eta_\upsilon \, [V_\upsilon^{(r)} \,/\, V^{(r)}] \qquad (4)$$

where η_υ is the material loss factor of the viscoelastic core evaluated at the rth calculated resonant frequency and $V_\upsilon^{(r)} / V^{(r)}$ is the fraction of elastic strain energy attributable to the sandwich core when the structure deforms in the rth mode shape.

Eq. (4) implies that damping of a structure can be described by associating a single number, the modal loss factor, with each undamped natural mode shape and frequency. The composite loss factor for each mode is taken to be proportional to the material loss factor for the viscoelastic portion of the structure. This approximation has been shown to be accurate for practical applications.

A basic difficulty with the modal strain energy method is that the modal properties are obtained from system matrices that are assumed to be constant. Viscoelastic materials, however, have storage moduli that vary significantly with frequency. To resolve this contradiction, a simple correction is made to the modal loss factor described in Eq. (4). The corrected value of modal loss factor is given as:

$$\eta^{(r)'} = \eta^{(r)} \sqrt{G_2(f_r)/G_{2,ref}} \qquad (5)$$

where

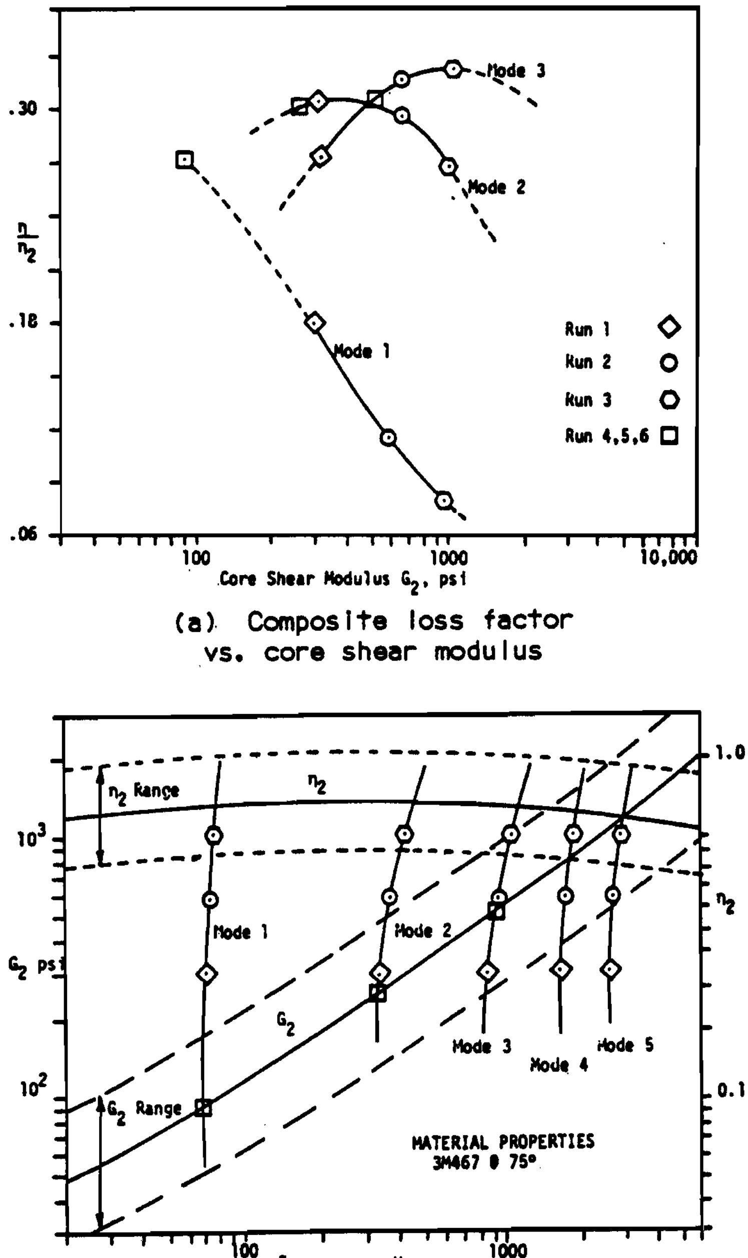
- $\eta^{(r)'}$ = adjusted modal damping ratio for the rth mode
- $\eta^{(r)}$ = modal damping ratio for the rth mode obtained by iteration
- $G_{2,ref}$ = core shear modulus used in final normal modes calculation to obtain modal frequencies, shapes, and masses
- $G_2(f_r)$ = core shear modulus at $f = f_r$, where f_r is the rth mode frequency calculated with $G_2 = G_{2,ref}$

To design a damping treatment, one begins by making several normal mode runs for a range of different core shear moduli. A set of natural frequencies and damping ratios is obtained for each value of the core shear modulus. Curves are drawn for each mode, and the intersections with the material property curve are found as shown in Figure 1, taken from [2]. Each intersection represents the shear modulus value which is appropriate for calculating the damping ratio of the associated mode. An intermediate value of the core shear modulus within the frequency range of interest is then selected as a source for the final values of modal stiffness, mass and mode shape which are used in subsequent forced response calculations. Additionally, modal damping ratios obtained by the iterative scheme are corrected according to Eq. (5).

OVERVIEW OF THE RITZ PROCEDURE

The Ritz procedure, first described by Wilson, et al. [1], provides an efficient way of solving large eigenvalue problems. The procedure has been implemented in both COSMIC and MSC/NASTRAN [4,5]. The algorithm is illustrated in Figure 2.

To start the procedure, a Krylov sequence is used to compute a set of mass-orthogonal starting vectors. A static load is used to derive the initial vector of the set. Note that cases involving singular stiffness matrices are also easily handled. Given this set of starting vectors, an eigenvalue problem of order L is solved (where L is the size of the starting vector set, or number of desired eigenvalues) to derive both the structure eigenvalues and the generalized eigenvectors, Z. These eigenvectors are then applied as a transformation matrix to the starting vector set to produce the final set of



(a) Composite loss factor vs. core shear modulus

(b) Material properties

Figure 1 Design method for sandwich beam with viscoelastic core (frequency-dependent material properties).

- Given mass matrix **M**, stiffness matrix **K** and load vector $\vec{p}$
- Triangularize **K** such that

  $\mathbf{K} = \mathbf{L}^T\mathbf{D}\mathbf{L}$
- Solve for starting vector $\vec{x}_1^*$

  $\mathbf{K}\vec{x}_1^* = \vec{p}$

  $\vec{x}_i^T\mathbf{M}\vec{x}_i = 1$
- Solve for additional vectors $i = 2, \ldots, L$, orthonormalizing with respect to **M**

  $\mathbf{K}\vec{x}_i^* = \mathbf{M}\vec{x}_{i-1}$

  $c_j = \vec{x}_j^T\mathbf{M}\vec{x}_{i-1}^*$, for $j = 1, \ldots, i-1$

  $$\vec{x}_i^{**} = \vec{x}_i^* - \sum_{j=1}^{i-1} c_j\vec{x}_j$$

  $\vec{x}_i^T\mathbf{M}\vec{x}_i = 1$
- Form $\mathbf{M}^*$ and $\mathbf{K}^*$

  $\mathbf{X} = [\vec{x}_1, \cdots, \vec{x}_L]$

  $\mathbf{M}^* = \mathbf{X}^T\mathbf{M}\mathbf{X}$

  $\mathbf{K}^* = \mathbf{X}^T\mathbf{K}\mathbf{X}$
- Solve the L by L eigenvalue problem

  $[\mathbf{K}^* - \omega_i^2\mathbf{M}^*]\vec{z}_i = 0$

  $\mathbf{Z} = [\vec{z}_1, \cdots, \vec{z}_L]$
- Compute final Ritz vectors by orthogonalizing **X** with respect to **K**

  ${}^0\mathbf{X} = \mathbf{X}\mathbf{Z}$

Figure 2 The Ritz Procedure

Ritz vectors which are both mass and stiffness orthogonal. The resulting Ritz vectors and eigenvalues contain no components which are orthogonal to the applied static displacement used as the initial starting vector. This is an important property of the Ritz procedure -- unwanted eigenvectors which would be recovered in a standard normal modes analysis, but which would show no participation in subsequent forced-response analysis, are eliminated in the Ritz procedure.

Studies using the Ritz procedure [4,6] have indicated that for normal modes analysis, it provides a reduction by a factor of three to ten in the eigenvalue extraction procedure when compared to the FEER method used in COSMIC/NASTRAN for the same number of modes. In addition, because the static load vector can eliminate recovery of unwanted modes in the eigenvalue solution, fewer Ritz modes can be used to obtain the same level of accuracy for subsequent forced-response analysis. Use of the static load vector also eliminates any need for a static correction factor in forced-response analysis.

Following Wilson's original publication of the Ritz procedure, it has been demonstrated that the procedure is identical to the Lanczos method with full reorthogonalization. In fact, the Lanczos method has recently been implemented in MSC/NASTRAN [7], and its efficiency is comparable to the Ritz procedure for general eigenvalue extraction problems. Still, the Ritz procedure offers some advantages for applications in which the dynamic loading imposed in subsequent forced-response analysis is spatially invariant and well defined. Such may be the case for evaluation of constrained-layer damping concepts. In these cases, the numerical efficiency of the Lanczos procedure is obtained in solving the eigenvalue problem, while at the same time, the number of modes recovered is limited only to those that participate in the forced-response problem by dictating the starting vector used in the sequence.

The NASTRAN implementation of the Lanczos method will be used in this paper as an efficient means to derive true natural mode shapes and frequencies. The Ritz procedure is used to derive Ritz modes and frequencies which may or may not correlate directly with the true natural modes. Both procedures simply provide a set of normal modes and frequencies which can be used for efficient forced-response analysis, as well as to predict values of modal damping for constrained-layer problems.

AUTOMATIC EXTRACTION OF MODAL LOSS FACTORS FOR CONSTRAINED-LAYER DAMPING PROBLEMS

An automated iterative procedure has been developed to derive modal loss factors for constrained-layer damping problems. The key issue in deriving the mode shapes and normal modes of the facesheet and constrained-layer assemblage is the ability to update the stiffness matrix of the viscoelastic layer at the beginning of each iteration.

The structure stiffness matrix of the assemblage is partitioned, as in a standard NASTRAN superelement approach, into a frequency-independent part (i.e., facesheets) and frequency-dependent part (i.e., viscoelastic constrained layer). Initially, with a starting value of G, the shear modulus of the viscoelastic layer, the stiffness matrix is formed and assembled with the rest of the structure. The natural frequencies and mode shapes of the assembly are then found using the Ritz procedure implemented in NASTRAN. A new estimate of shear modulus of the viscoelastic layer for the next iteration is found from the frequency vs. G table for the first natural frequency. With this new value of G, the stiffness matrix of the constrained layer is updated and assembled with that of the frequency-independent part, and the eigenvalue extraction proceeds again. The iterative operation for this mode is continued until the current estimate of the natural frequency is acceptably close to the previous estimate. This process is repeated for all modes requested by the user. The converged values for frequencies, eigenvectors, and corresponding shear moduli of the core are saved for each mode to determine the modal loss factors according to Eq. (4). Additionally, these values can be utilized for subsequent forced-response analysis of the candidate structure. Figure 3 shows the flow chart of the procedure described here.

A basic assumption in this method is that the mode shapes of the assemblage do not vary significantly with the change in core shear modulus. Each mode shape is derived using a different value for the shear modulus of the constrained layer. If there is no significant change in mode shape with shear modulus, then the diagonal terms of the generalized stiffness matrix, $\underline{K}_o$, will remain large in comparison to the off-diagonal terms:

$$\underline{K}_o = \Phi^T K_o \Phi \qquad (6)$$

where

$\Phi = [\phi_1, \phi_2 \ldots]$

ϕ_i = mode shape i, derived using stiffness matrix K_i

K_o = stiffness matrix assembled using intermediate value of core shear modulus

For truly normal modes, of course, $\underline{K}_o$ contains only diagonal terms. This assumption is implicit in any MSE approach, since the modal damping values which are derived are generally used in subsequent linear forced-response analysis.

FINITE ELEMENT MODELING METHOD

The method used for finite element modeling of a viscoelastic constrained layer is described in [2]. Briefly, the viscoelastic core is modeled with three-dimensional isoparametric solid elements called HEXA elements in NASTRAN. Each element has three translational degrees of freedom defined at each node. The face sheets are modeled with quadrilateral shell elements, QUAD4's, which have three translations and two rotational degrees of freedom at each corner node. Since the plate nodes are offset to one surface of the plate and coincident with the corner nodes of the adjoining solid elements, there exists a coupling between stretching and bending deformations of the plate elements. This membrane-bending coupling is defined via the property card of QUAD4's. After the model is assembled, a standard superelement normal mode extraction with the user-specified DMAP is performed. Specifically, the

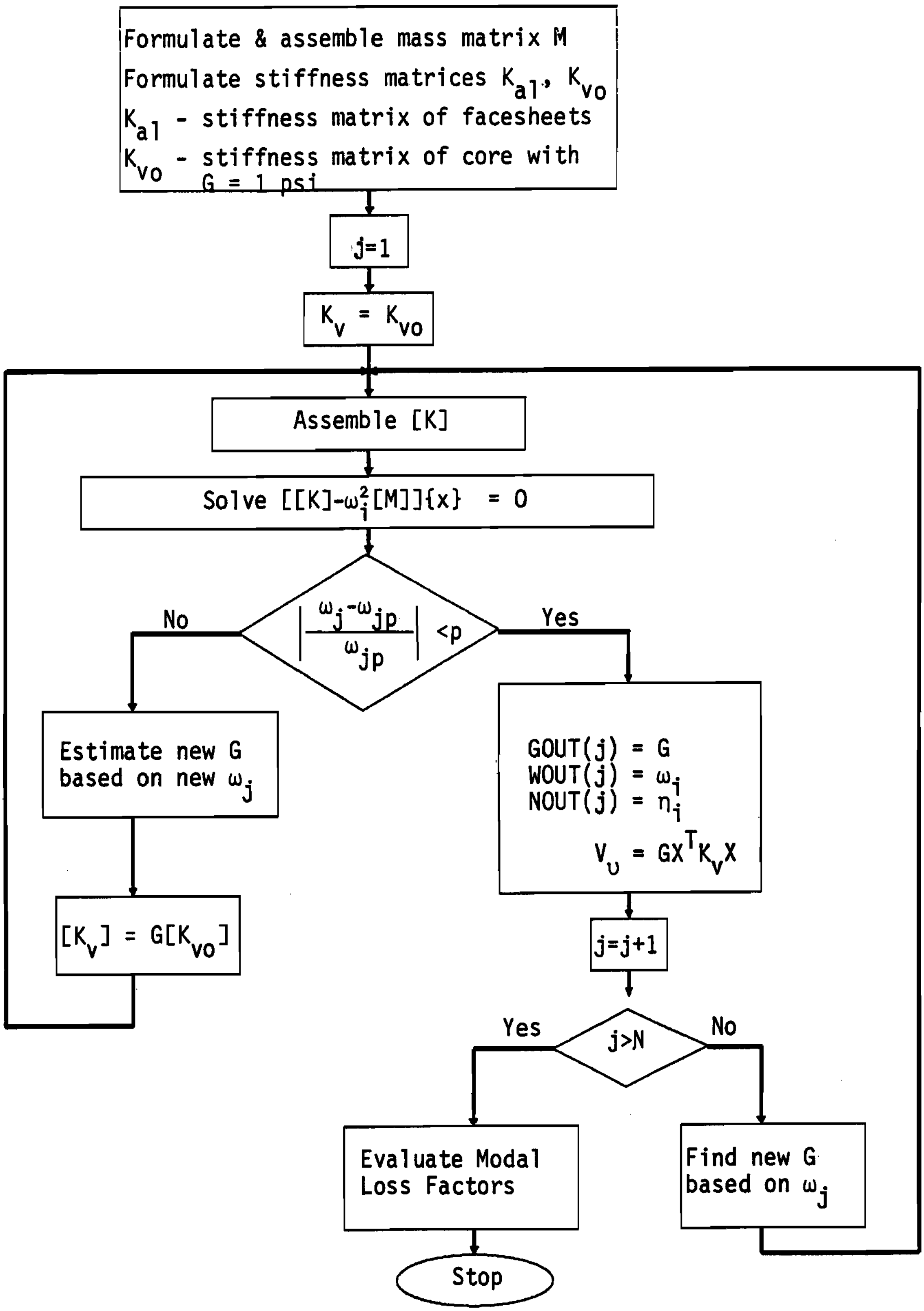


Figure 3 Flow chart for automatic extraction of modal loss factor.

facesheet elements are placed in an upstream superelement, whereas the solid elements making up the core are placed in the residual structure. This allows partitioning of the stiffness matrix into frequency-dependent and frequency-independent parts. The user is required to input a table defining the shear modulus, G, and the material loss factor, η, of the core as functions of frequency. Additionally, it is most convenient to set the initial value of G of the core to 1.0.

Calculation of elastic strain energy is performed using a standard option in MSC/NASTRAN. The fraction of total strain energy within a group of elements corresponding to the viscoelastic core for each normal mode will be output. Multiplying this value for each mode by the viscoelastic material loss factor yields the modal loss factor for that mode. The modal loss factors are output as a matrix print option in NASTRAN.

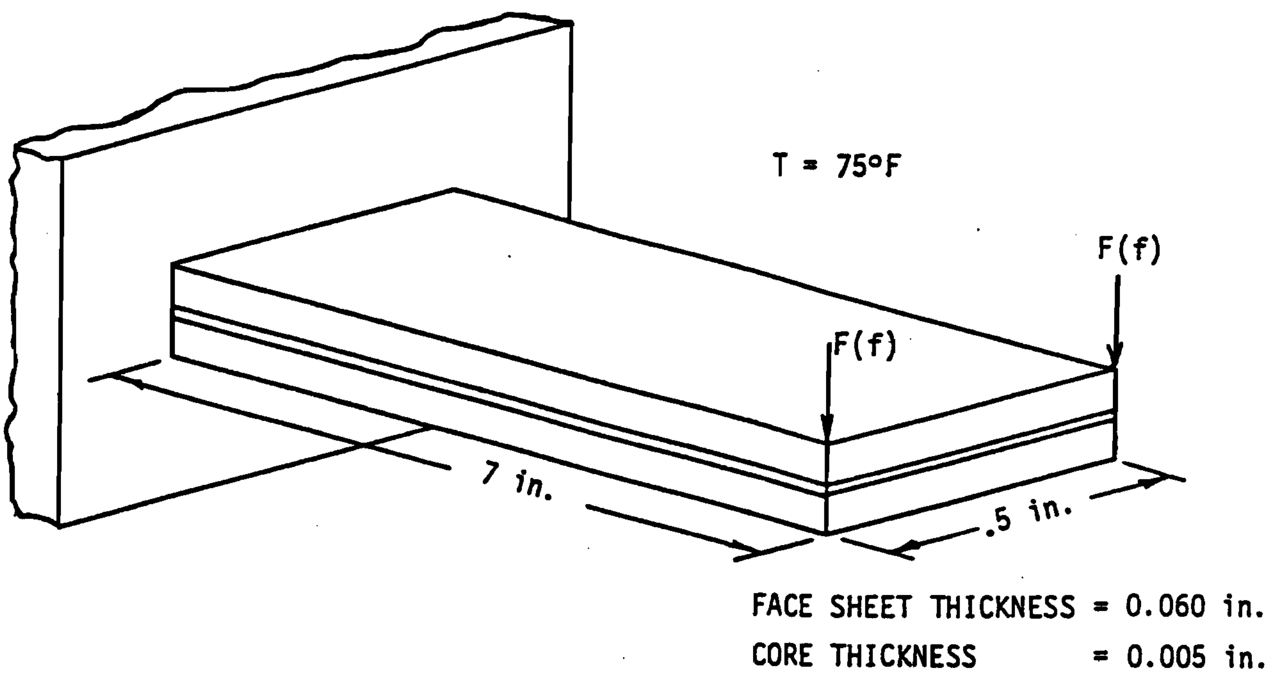
EXAMPLE PROBLEM

A cantilever beam similar to the one in [2] is analyzed using four different approaches to show the validation and advantages of the proposed solution method. The 7 inch long cantilever beam has identical aluminum face sheets 0.060 inch thick and a viscoelastic core 0.005 inch thick, as shown in Figure 4. The finite element model consists of 20 elements in the lengthwise direction and one element widthwise. All nodes are at element corners. Poisson's ratio of the core elements is taken to be 0.49. The viscoelastic material loss factor is assumed to be a constant (1.35) with respect to frequency. The tip of the cantilever beam is subjected to a random loading function as shown in Figure 5. The objective is to determine response functions which are accurate, in a cost-effective manner.

To establish a reference set of response functions, the sandwich beam is first analyzed using the direct frequency response (DFR) method. The viscoelastic core shear modulus is defined as a function of frequency as shown in Figure 6(a). The use of frequency-dependent material properties for direct frequency response analysis is described in [7]. Results from the other three approaches are compared with results from this method for validation.

The second approach is similar to the one used in [2]. Initially, a set of the lowest five modes and damping ratios is obtained for a range of different core shear moduli. For example, four shear moduli are examined in the present case, using the Lanczos method. The normalized structural damping factor and the first five natural frequencies are plotted versus G for each mode, as shown in Figure 6b. The intersection of the curve for each mode with the material property curve represents the G value which is appropriate for calculating modal damping of the associated mode. Subsequently, an intermediate value of G of 300 psi was used in the forced-response calculations to evaluate the responses. The damping ratios used for this analysis were adjusted with a correction factor obtained using Equation (5). The natural frequencies, corresponding shear moduli, and damping factors (after adjustment) used in the forced-response analysis are listed in Table 1.

In the third approach, the natural frequencies and unadjusted damping ratios for the first five modes are automatically obtained with an iterative procedure discribed earlier using the Lanczos method for the eigenvalue



• Upper and lower face sheets are aluminum.

Figure 4 Cantilever Sandwich Beam Subjected to Random Loading.

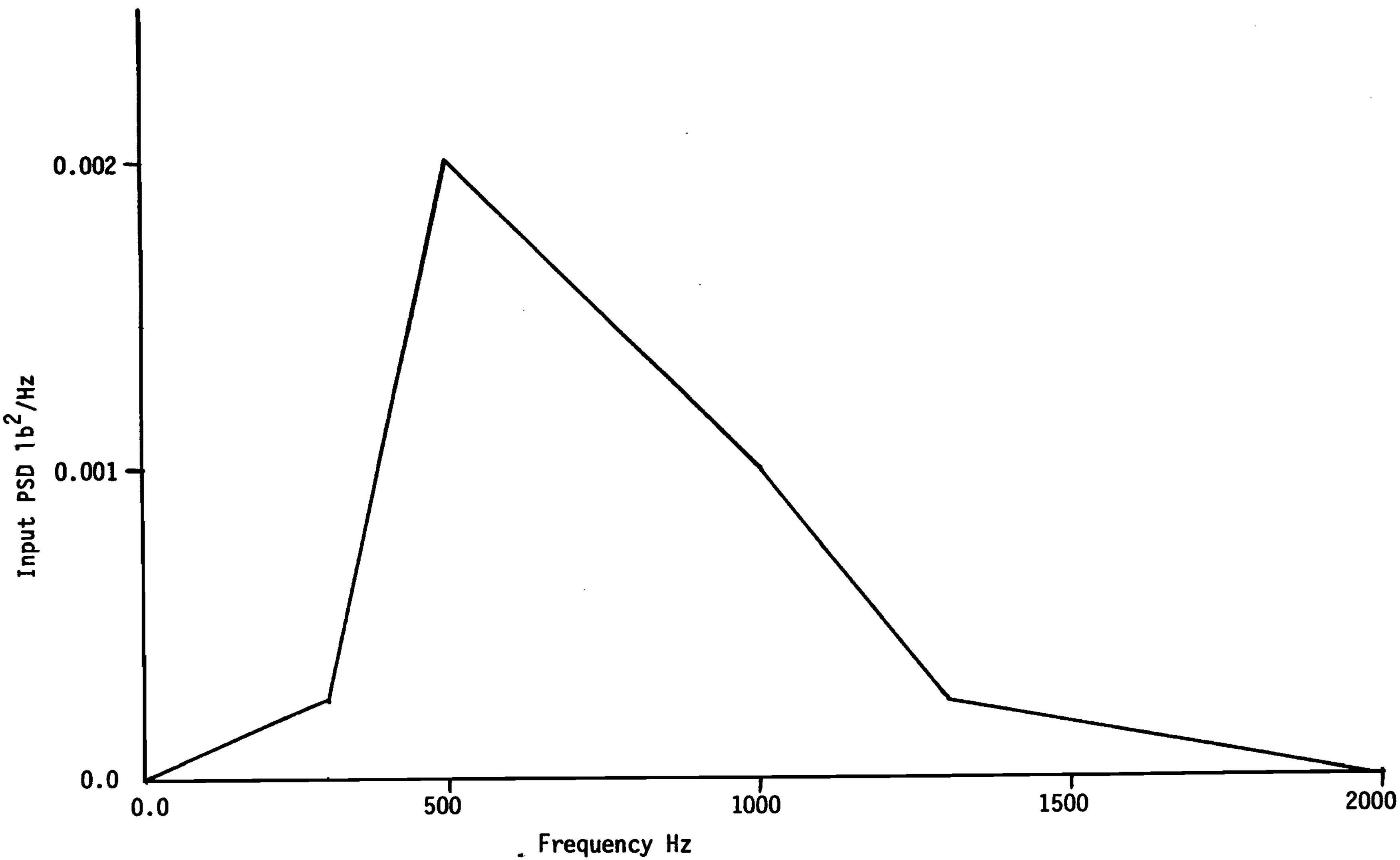


Figure 5 Random Loading Function.

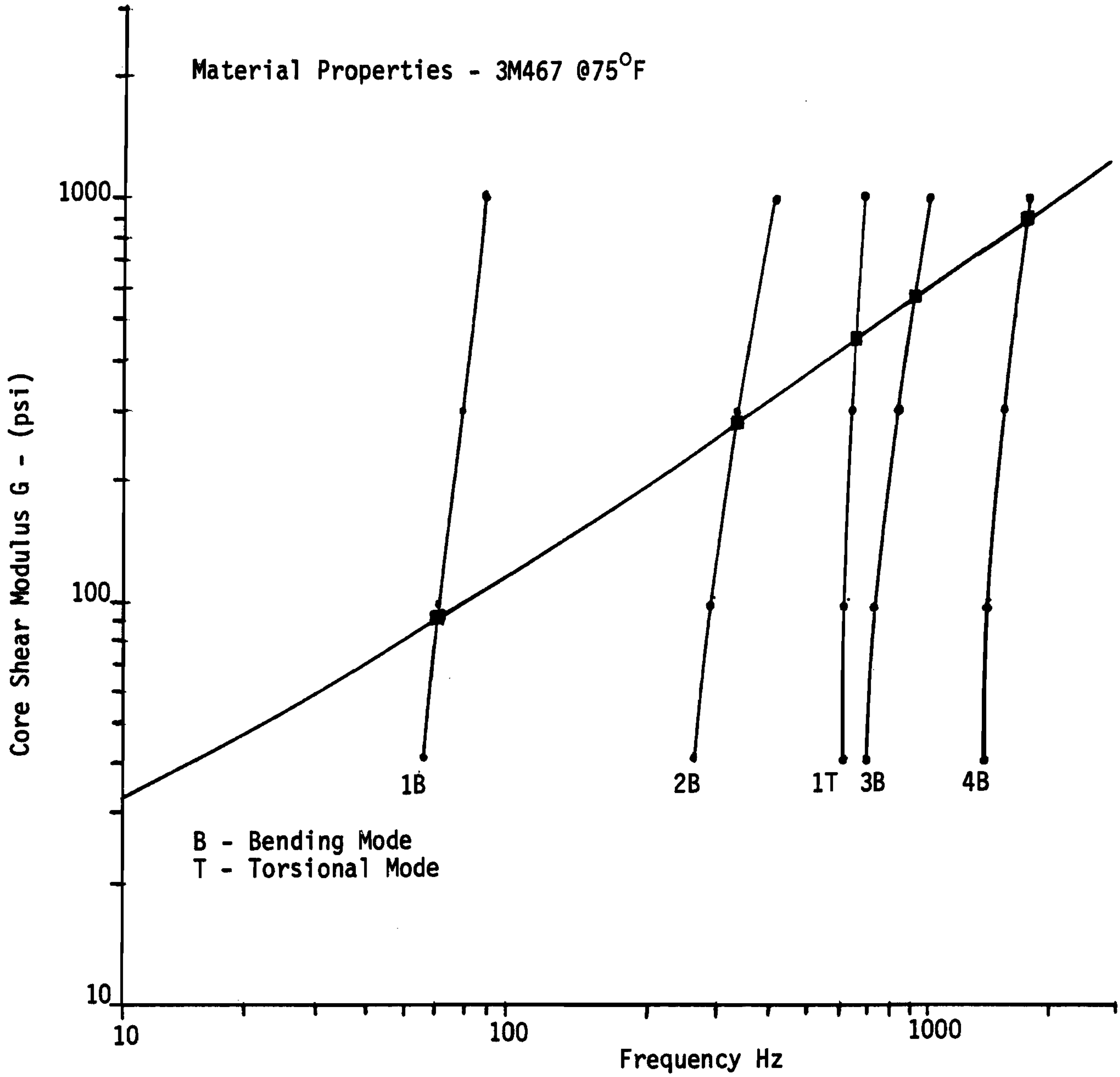


Figure 6(a) Material Properties vs. Frequency.

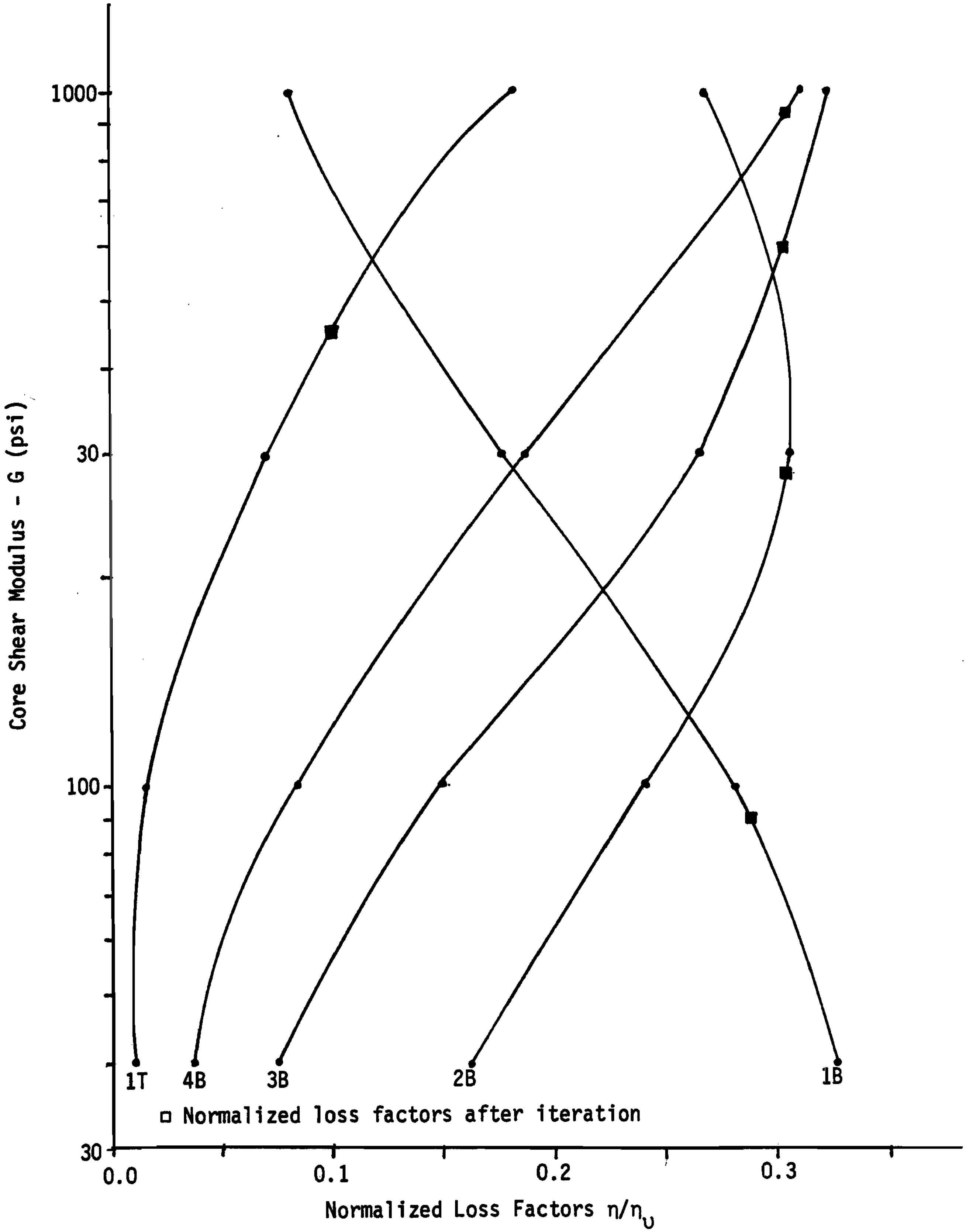


Figure 6(b) Composite Loss Factors vs. Core Shear Modulus Method to Obtain Damping Factors for Frequency-Dependent Material Properties

TABLE 1

MODAL DAMPING RATIOS USING
NORMALIZED LOSS FACTORS
OBTAINED FROM A SET OF NORMAL MODE RUNS
WITH LANCZOS METHOD

Mode No.	Type	Frequency (Hz)	Core Shear Modulus psi	Normalized Loss Factor η/η_u	Modal* Damping Ratio
1	1B	64	90	0.290	0.214
2	2B	350	280	0.305	0.398
3	1T	650	450	0.095	0.157
4	3B	925	575	0.30	0.561
5	4B	1,720	920	0.30	0.709

* Used $G_{2,ref}$ = 300 psi; refer to Eq. (5).

extraction. The output eigenvalues, shear moduli, and damping ratios are as shown in Table 2. Since the output values of Table 1 and Table 2 are virtually identical, output from forced-response calculations will also be similar to the one obtained from the first approach.

The fourth approach is similar to the third approach, except that the Ritz procedure is used for eigenvalue extraction. The output from the eigenvalue extraction run is listed in Table 3. Note that the first torsion mode at 650 Hz. is not extracted. Evidently, this is because the starting load vector used for eigenvalue extraction does not contain any components of the torsional mode. The forced-response analysis is then performed using modal loss factors derived from the Ritz procedure eignenvalue extraction, which are markedly different for higher modes compared to those obtained using the Lanczos method.

The response at the tip of the beam obtained by forced-response analyses using the first, second, and fourth approaches is presented in Figures 7 through 9. Note that the response from all three approaches is similar up to 500 Hz., at which the peak of the forcing function occurs. Moreover, the responses obtained from the Ritz procedure compare quite well with those from the Lanczos method throughout the frequency spectrum.

The differences in response at higher frequencies for the Ritz and Lanczos method compared to the DFR method can be be attributed to two factors. First, at higher frequencies, the error due to the correction factor (Eq. (5)) increases as the difference between the true shear modulus and the reference modulus increases. Secondly, at higher frequencies, modes six and higher contribute more significantly to the response. However, the contribution from higher modes is ignored in the Lanczos mode superposition analysis with just five modes, and only approximated in the Ritz modes.

For a further comparision between used Ritz modes and Lanczos natural modes, a set of mode superposition analyses (third and fourth approaches) were performed using only the first three modes. The output from an eigenvalue extraction run using the Ritz method is shown in Table 4. The eigenvalues and modal loss factors from the Lanczos method, which remain the same since it provides true natural modes, are in Table 2. The response from the forced-response analyses is plotted with that from the original DFR analysis in Figures 10 through 12. Note that the response from both methods, Ritz and Lanczos, compare well with that from DFR up to 500 Hz. At higher frequencies, however, the responses from the Lanczos method diverge considerably from DFR, whereas the responses from the Ritz method continue to correlate well with responses from the DFR method up to 1300 Hz.

This close correlation between the Ritz procedure results and the DFR is due to the use of a static load vector to derive the initial Ritz vector in the algorithm. The third Ritz mode (which is not the same as the third natural mode) contains components of all of the higher modes that are not orthogonal to the initial Ritz vector. Thus, the contribution to response from higher modes is approximated when the Ritz procedure is used, in the same manner as a static correction factor.

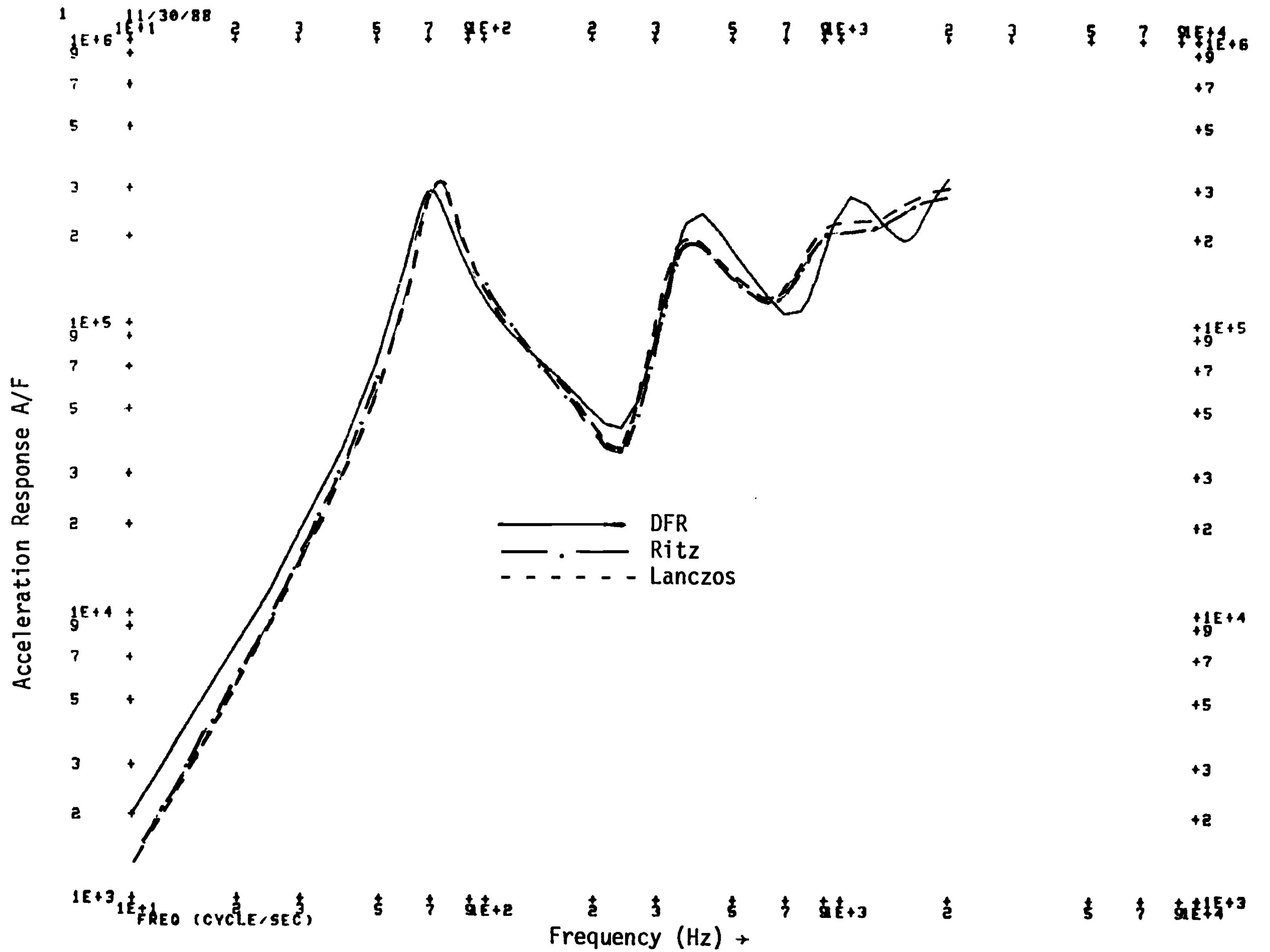


Figure 7 Comparison of Acceleration Frequency Responses at the Tip of the Beam Obtained from Direct Frequency Response Method & Modal Strain Energy Method using Lanczos & Ritz Procedure (with 5 Modes).

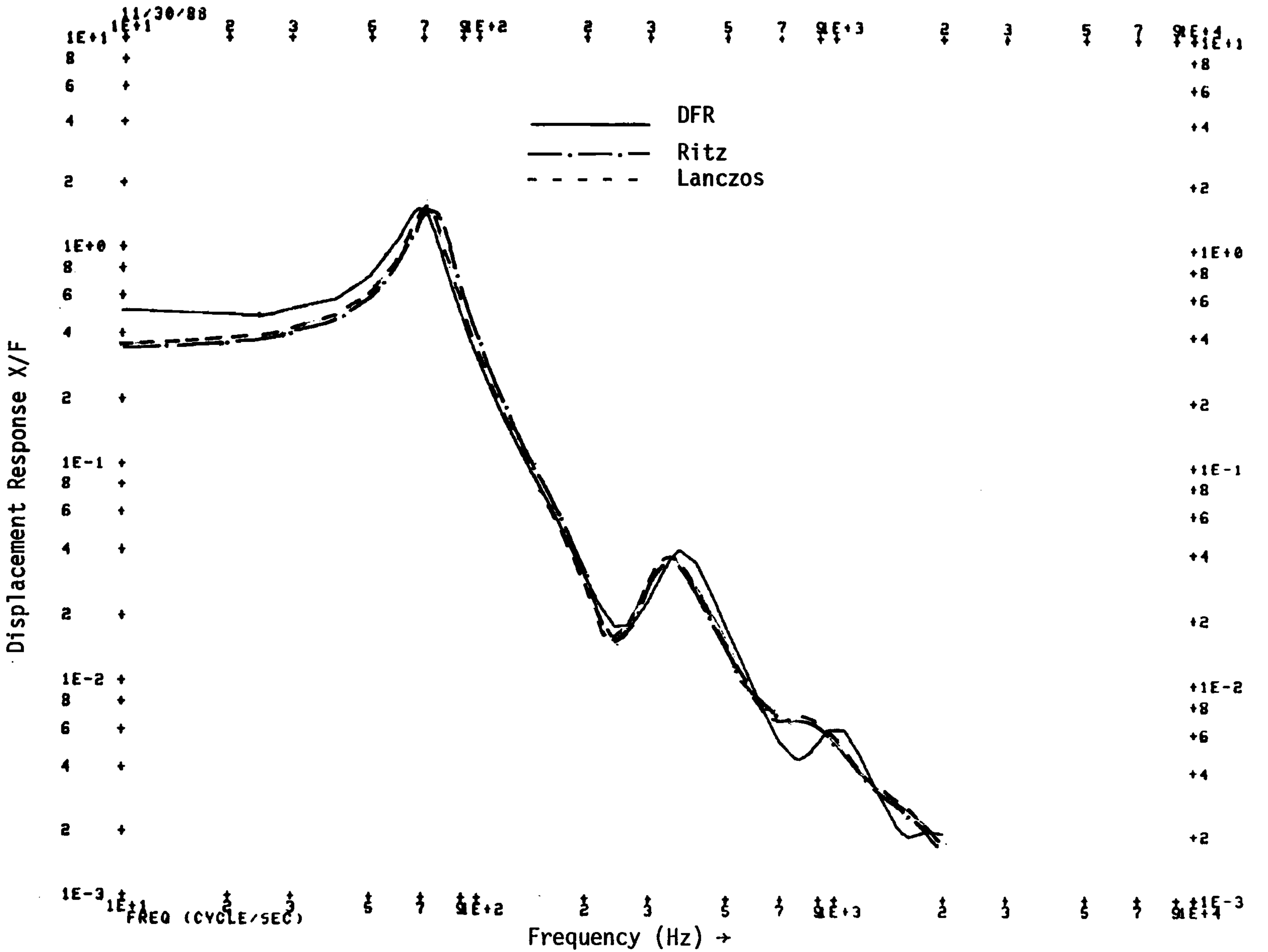


Figure 8 Comparison of Displacement Frequency Responses at the Tip of the Beam Obtained from Direct Frequency Response Method & Modal Strain Energy Method using Lanczos and Ritz Procedure (with 5 Modes).

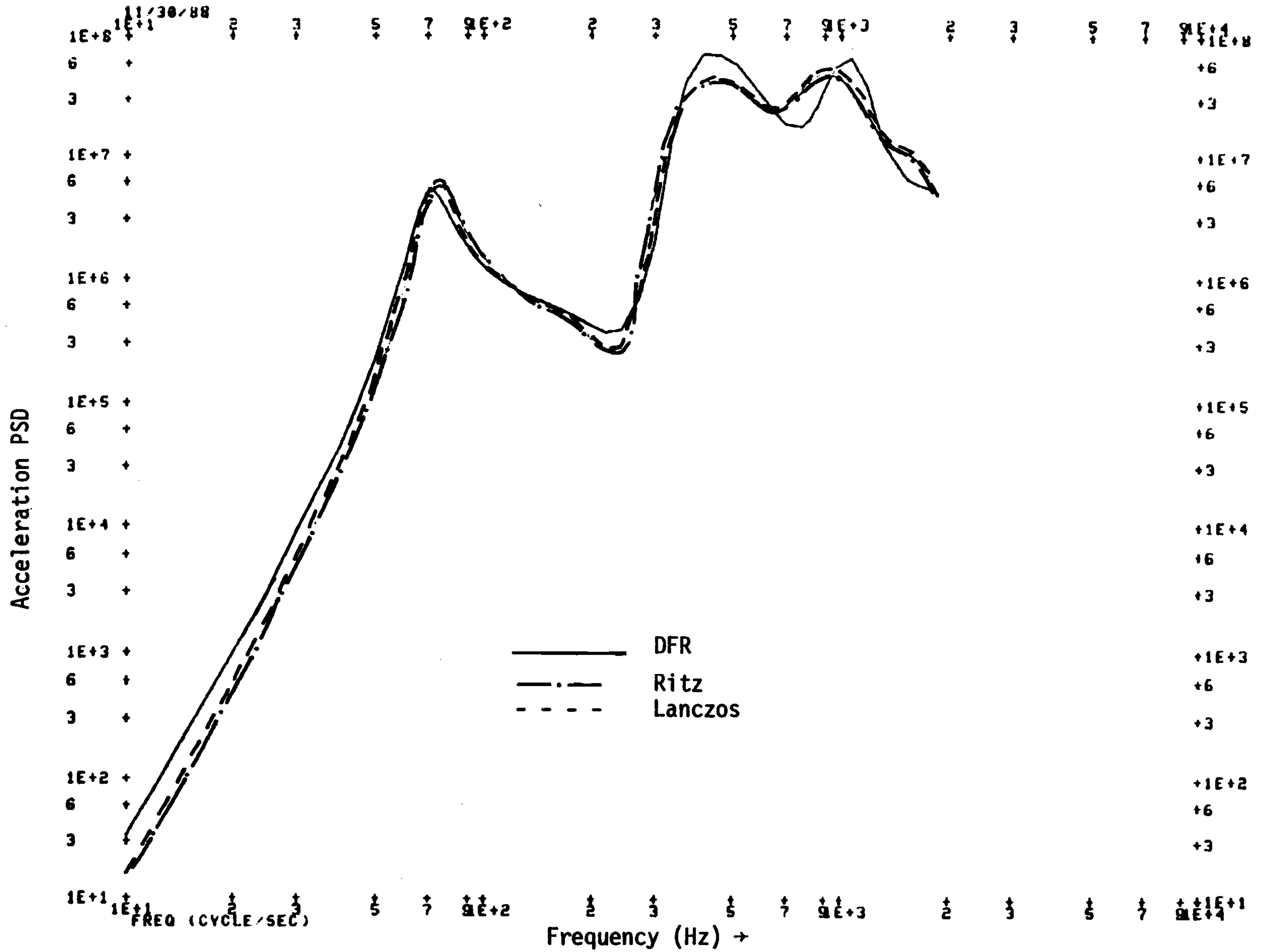


Figure 9 Comparison of Acceleration PSD Frequency Responses at the Tip of the Beam Obtained from Direct Frequency Response Method & Modal Strain Energy Method using Lanczos and Ritz Procedure (with 5 Modes).

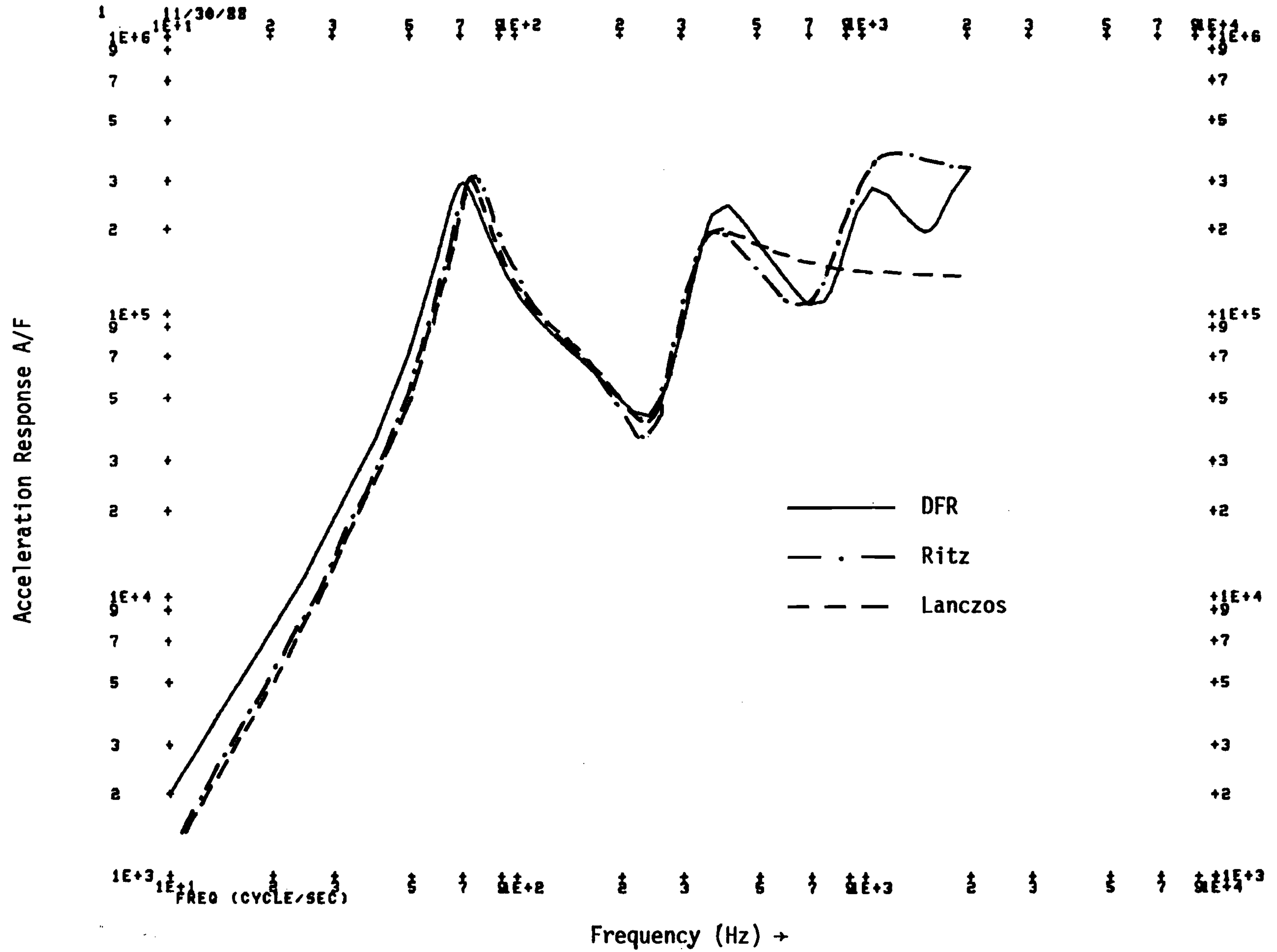


Figure 10 Comparison of Acceleration Frequency Response at the Tip of the Beam Obtained from DFR Method and MSE Method Using Lanczos and Ritz Procedure with 3 Modes

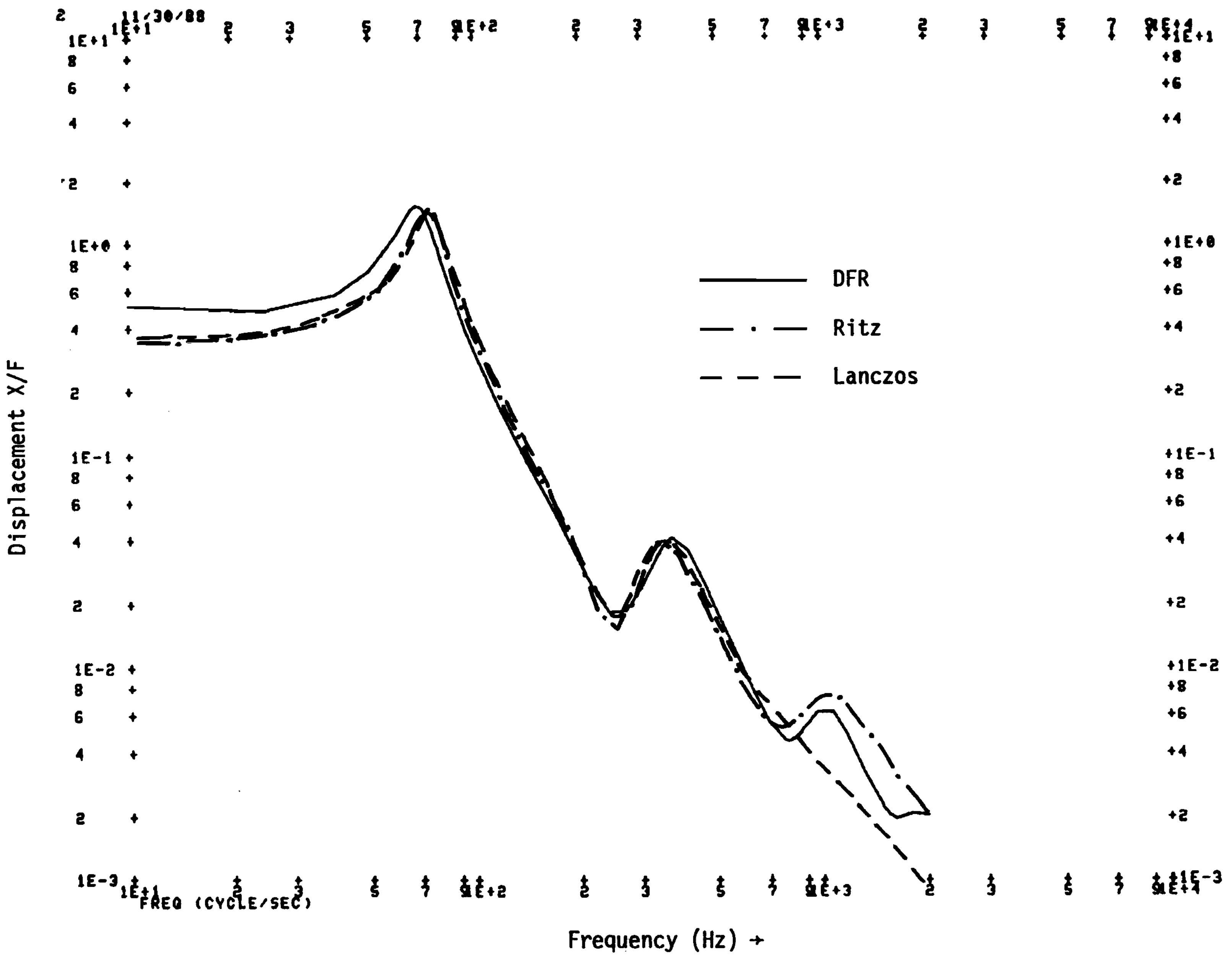


Figure 11 Comparison of Displacement Frequency Response at the Tip of the Beam Obtained from DFR Method and MSE Method Using Lanczos and Ritz Procedure with 3 Modes

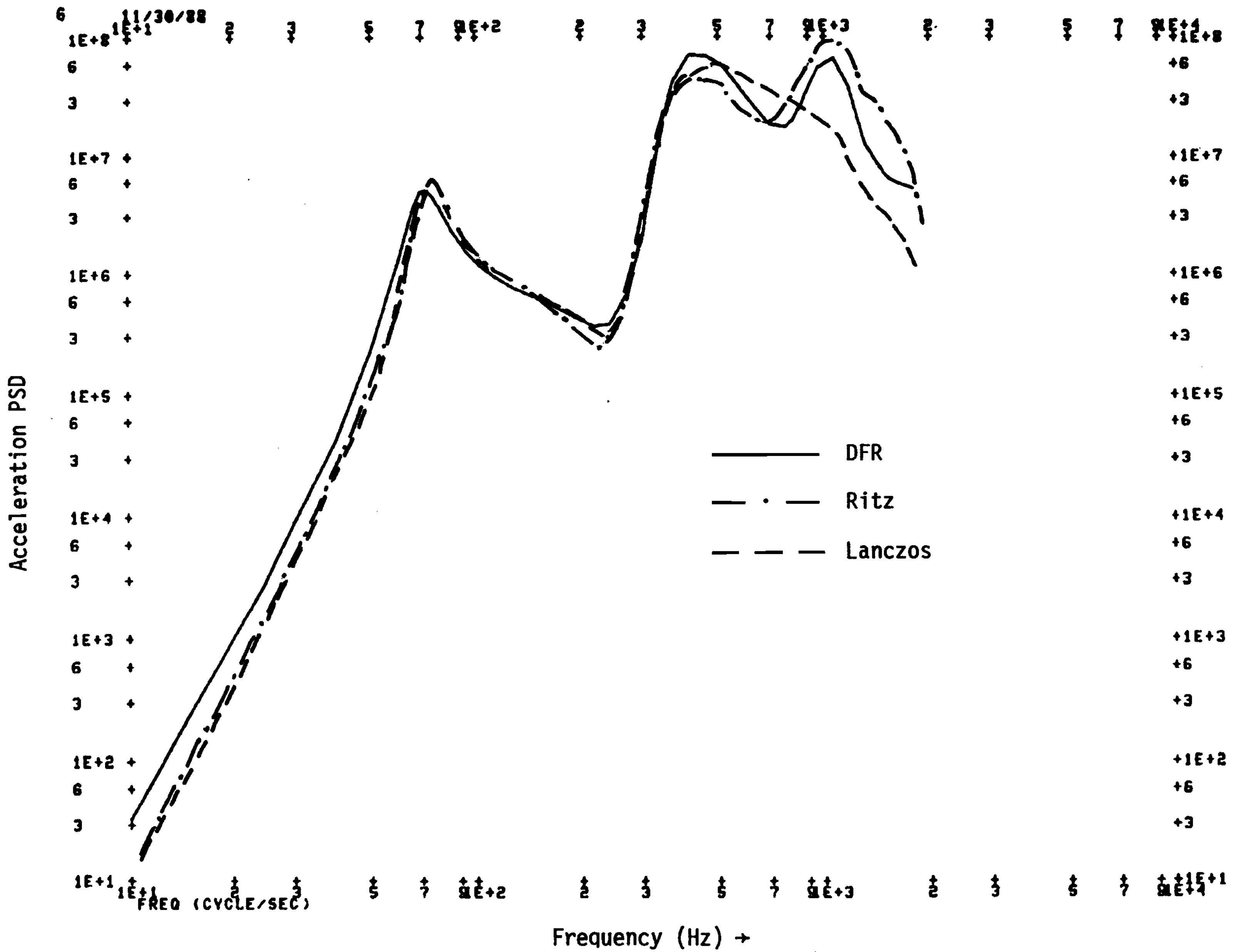


Figure 12 Comparison of Acceleration PSD Frequency Response at the Tip of the Beam Obtained from DFR Method and MSE Method Using Lanczos and Ritz Procedure with 3 Modes

TABLE 2

MODAL DAMPING RATIOS USING
OUTPUT FROM THE AUTOMATIC PROCEDURE
WITH LANCZOS METHOD

Mode No.	Type	Frequency (Hz)	Core Shear Modulus psi	Normalized Loss Factor η/η_v	Modal* Damping Ratio
1	1B	63.3	88.5	0.291	0.213
2	2B	343.8	282.3	0.302	0.395
3	1T	654.6	441.7	0.0995	0.163
4	3B	920	560.2	0.311	0.574
5	4B	1,748	930.9	0.305	0.726

* Used $G_{2,ref}$ = 300 psi; refer to Eq. (5).

TABLE 3

MODAL DAMPING RATIOS (FIVE MODES ONLY)
USING OUTPUT FROM THE AUTOMATIC PROCEDURE
WITH RITZ METHOD

Mode No.	Type	Frequency (Hz)	Core Shear Modulus psi	Normalized Loss Factor η/η_v	Modal* Damping Ratio
1	1B	63.1	86.9	0.291	0.211
2	2B	343.1	278.6	0.302	0.393
3	3B	918.8	556.8	0.311	0.572
4	4B	1,733.8	927.4	0.303	0.719
5	-	3,763.5	1,584.5	0.287	0.890

* Used $G_{2,ref}$ = 300 psi; refer to Eq. (5).

TABLE 4

MODAL DAMPING RATIOS (THREE MODES ONLY)
USING OUTPUT FROM THE AUTOMATIC PROCEDURE
WITH RITZ METHOD

Mode No.	Type	Frequency (Hz)	Core Shear Modulus psi	Normalized Loss Factor η/η_v	Modal* Damping Ratio
1	1B	63.1	86.9	0.29	0.211
2	2B	343.4	278.8	0.303	0.391
3	3B	1,182	682.2	0.33	0.672

* Used $G_{2,ref}$ = 300 psi; refer to Eq. (5).

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TABLE 5

COMPARISON OF CPU SECONDS FOR VARIOUS METHODS

	Direct Frequency Response (DFR) Method	Modal Strain Energy Method					
			Automatic				
		Manual	Lanczos		Ritz		
		Lanczos	5 Modes	3 Modes	5 Modes	3 Modes	
Eigenvalue Extraction	-	2,200	5,062	3,306	3,533	2,242	
Modal Superposition	-	1,199	1,199	1,054	1,070	928	
TOTAL	3,838	3,399	6,261	4,360	4,603	3,170	

Table 5 shows the CPU seconds used for the analysis of the 360 DOF model which was used with the various approaches described above. The Ritz procedure requires 27% less CPU time compared to the Lanczos method. The manual method with five modes requires less CPU than either the Ritz or the Lanczos method, but the time required to submit and examine five computer runs, and then to plot contours to obtain the modal loss factors can be substantial. In addition, the manual method is more vulnerable to the possibility of human error.

CONCLUSIONS

The modal strain energy method has numerous attractive features as a means to estimate damping in structures with constrained viscoelastic layers. The procedure presented in this report automates the extraction of modal loss factors. Instead of making several normal modes extraction runs, the user needs to execute the finite element model only once, thus saving costly engineering time. Additionally, the automatic extraction of the loss factors minimizes the introduction of human error inherent in the manual method. Finally, a substantial savings in CPU time accrues due to the Ritz method, making the automatic extraction of loss factors and subsequent forced-response analysis using the Ritz procedure an attractive alternative for designs using constrained-layer damping.

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