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at 650°C. The evolution equation was determined for a state variable that characterizes the resistance to plastic flow and some properties of a damage state variable were established. Analysis of the tensile and creep data strongly suggested that two different mechanisms were active during deformation. A limited number of experimental results were compared to the response calculated from the flow and evolution equations.

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FOREWORD

This interim Technical Report was prepared by the Department of Aerospace Engineering and Applied Mechanics of the University of Cincinnati. The report describes research results obtained on Contract No. F33615-78-C-5199 "Constitutive Modeling". This research is a further improvement of a previous model and satisfies the requirements of Tasks 4.2.7. The Air Force Project Engineer is Dr. T. Nicholas, AFWAL/MLLN. The research reported was conducted from (September 1978 to October 1980) by D. C. Stouffer and S. R. Bodner. The report was submitted for publication in October 1980.

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SECTION I
INTRODUCTION

The usefulness of any constitutive equation depends on three major factors: (1) how well the constitutive equation can predict the mechanical response of material to a broad range of load histories; (2) how easily the equation can be used in computer algorithms and, in particular, its adaptability to finite element analyses of structural components; and (3) how easily the parameters in the constitutive model can be determined from experimental data. Any equation that does not satisfy all the above conditions is severely limited for its scope of application.

This paper is concerned with procedures for the determination of evolution equations in a state variable constitutive theory, and a partial evaluation of the predictive capability of those equations with the determined constants. The equations are characterized by two internal state variables which represent the resistance of the material to plastic flow and internal damage. The particular set of equations used in this exercise are those developed by Bodner and Partom, Refs. [1,2], but other equations of the same general class could be treated by similar methods.

The method of developing the state variable evolution equations suggested in this paper permits direct evaluation of the material parameters from the experimental data and thereby minimizes subsequent trial and error computation. Since state variable constitutive equations predict the strain rate as a function (not functional) of the stress, temperature and state variable histories; the constitutive function may be algebraically

inverted to calculate the state variable history from stress, strain rate and temperature histories that are determined from an experiment. A systematic analysis of the experimental data can therefore contribute to the formulation of the evolution equations for the state variables as well as defining the material constants.

An interesting consequence of this technique is that it can be used to evaluate more than one state variable. This allowed for the consideration of a damage state variable, Ref. [3]. Although a general development for the damage evolution equation is not included; a few properties of the variable were deduced from the above analysis.

The procedure is demonstrated for wrought René 95 at 650°C using data currently available in the literature, Refs. [4, 5, 6]. The results are also compared to an earlier application of the Bodner Partom equation, Ref. [7], for the same material. Examples of the calculated response are compared to observed tensile and creep results.

SECTION II
THE FLOW EQUATION

The basic equations of Bodner-Partom will be summarized for reference. These equations were motivated by the concepts and equations of "dislocation dynamics", e.g., Refs. [8, 9], and were formulated in the context of continuum mechanics, Ref. [10]. For small strains, the strain rates are considered to be decomposable into elastic (reversible) and inelastic (non-reversible) components,

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p \quad (1)$$

where $\dot{\epsilon}_{ij}^e$ are given in terms of stress rates determined from the time derivative of Hooke's Law. Both components in Eq. (1) are always non-zero for non-zero stresses and stress rates which imply that a yield criteria and loading/unloading conditions are not required.

The plastic strain rate is written in a form similar to the Prandtl-Reuse flow law,

$$\dot{\epsilon}_{ij}^p = \dot{\epsilon}_{ij}^p = \lambda s_{ij} \quad (2)$$

where $\dot{\epsilon}_{ij}^p$ and s_{ij} are the deviatoric strain rate and stress tensors, respectively. Equation (2) satisfies the condition incompressibility. Squaring Eq. (2) leads to

$$D_2^p = \lambda^2 J_2 \quad (3)$$

where D_2^p and J_2 are the second invariants of the deviatoric plastic strain rate and stress tensors.

A basic assumption of the formulation of Bodner-Partom is that all inelastic deformations are controlled by a relationship between D_2^P and J_2 which involves other state variables, i.e.

$$D_2^P = f(J_2, T, Z_i) \quad (4)$$

where T is the temperature and Z_i are history dependent internal state variables. Combining Eqs. (2), (3) and (4) leads to a representation for the inelastic strain rate tensor that can be written as a function of stress, temperature and internal state variables; i.e.

$$\dot{\epsilon}_{ij}^P = \dot{\epsilon}_{ij}^k = f^{1/2} [J_2, T, Z_i] \frac{S_{ij}}{\sqrt{J_2}} \quad (5)$$

A particular form for Eq. (4) was established from the relationship between dislocation velocity and stress, Refs. [8, 9], by Bodner and Partom. In a constant temperature environment this representation is

$$f^{1/2} = D_0 \exp[-(Z^2/3J_2)^n (n+1)/(n)] \quad (6)$$

In Eq. (6), D_0 is the limiting strain rate in shear, n is a material constant that controls strain rate sensitivity and also influences the overall level of the stress-strain curves, and Z is interpreted as an internal state variable that governs resistance to plastic flow. That is, an increase of Z would correspond to work hardening and would require an increase of stress to maintain a given plastic strain rate. Physically, Z can be interpreted as a measure of the stored energy of cold work, Refs. [11, 12].

In the earlier references it was proposed that Z should be taken to be a scalar function of plastic work for isotropic hardening conditions. This was later extended to include both isotropic and kinematic hardening by using different values of Z for positive and negative stress in uniaxial reversed loading conditions, Ref. [12]. The model was subsequently generalized by Stouffer and Bodner, Ref. [13], to deformation induced anisotropic flow. In this case Z was shown to be a second order tensor in a six dimensional vector space. The discussion in this paper is limited to isotropic hardening for which Z is a single valued scalar for reversed plastic flow.

Let us also briefly note some recent work, Ref. [3], where isotropic damage was included as an additional state variable in the flow equations. Using the Kachanov definition, Ref. [14], let ω be a history dependent variable that represents the deterioration in the ability of a material to support a stress. In this formulation stress, σ_{ij} , is replaced by a pseudo stress $\sigma_{ij}/(1-\omega)$. Introducing this result into the flow equations ultimately reduces to replacing Z by $Z (1-\omega)$.

Applying the preceding model to the case of an uniaxial stress σ , the one dimensional flow equation can be written as

$$\dot{\epsilon}^p = \frac{2}{\sqrt{3}} \left(\frac{\sigma}{|\sigma|} \right) D_0 \exp \left\{ -\frac{1}{2} \left[\frac{Z (1-\omega)}{\sigma} \right]^{2n} \left(\frac{n+1}{n} \right) \right\} \quad (7)$$

for the uniaxial inelastic strain rate, $\dot{\epsilon}^p$. A direct method of developing the evolution equations for the state variables Z and ω is introduced in the next section. The method is general and can be applied to any state variable equation. Equation (7) will be used in this particular example.

SECTION III
STRUCTURE OF THE EVOLUTION EQUATIONS

Let us consider the evolution equation for the state variable Z in the absence of damage. Therefore tertiary creep and the large strain range of a tensile test are excluded from this analysis. The evolution equation for the state variable Z is generally sought in the form of a differential equation;

$$\dot{Z} = f(\sigma_{ij}, Z) \quad . \quad (8)$$

A more specific representation is based on the concept that plastic work, W^P , controls the hardening process, and that W^P and \dot{W}^P are functions of σ_{ij} and $\dot{\epsilon}_{ij}^P$ which can be determined from σ_{ij} and Z . That is, assume

$$\dot{Z} = f_1' (W^P) \dot{W}^P - f_2(Z) \quad (9)$$

where f_1 and f_2 are material functions and $()'$ denotes differentiation with respect to plastic work. Equation (9) is a generalization of the specific representation developed by Bodner. The underlying philosophy in assuming the structure of Eq. (9) is to allow for the assignment of physically useful properties to the functions f_1 and f_2 . Specifically, f_2 is chosen so that the second term is negligible during short time histories. Thus, f_1 is selected so that Eqs. (1) and (7) will reproduce the family of stress-

strain curves run at high strain rates; the function f_2 is neglected in this situation. Conversely, both f_1 and f_2 are required to predict secondary creep. Mathematically, f_2 is a perturbation term that must be added for long time response.

The main thrust of this work is directed toward establishing representations for f_1 and f_2 ; and developing the relationships between these representations and the experimental response of the material. The first step in achieving this goal is to recognize that for any experiment, the stress and inelastic strain rate histories can be determined from the experimental data. Then, by inverting Eq. (7),

$$Z(1-\omega) = \sigma(t) \left[\frac{2n}{n+1} \ln\left(\frac{2D_0}{\sqrt{3} \dot{\epsilon}^p(t)}\right) \right]^{1/2n} \quad (10)$$

the history of the state variable product $Z(1-\omega)$ can be calculated for each particular experiment. This provides a powerful technique to directly determine the functions f_1 , f_2 and ω .

SECTION IV
Contrails
EVALUATION OF TENSILE RESPONSE

The coefficient n in Eq. (7) controls the strain rate sensitivity of the model with increases in n corresponding to a decrease in strain rate sensitivity. The properties of Eq. (7) are such that for each choice of n , a family of rate dependent stress strain curves can be predicted from a single representation of Z . Thus, the family of Z histories calculated from Eq. (10) using experimental data can be reduced to a single response curve Z with the correct value of n . The relationship $Z(W^P) = Z_0 + f_1(W^P)$ is valid for this analysis provided the time duration of the tensile tests are short compared to the creep response and material damage is not significant.

The response of René 95 at 650°C published in Ref. [5] was used for this analysis. This work contains a family of four tensile response curves run at strain rates of 8.3×10^{-3} , 8.3×10^{-4} , 8.3×10^{-6} and $1.6 \times 10^{-7} \text{ sec}^{-1}$. The observed tensile response at the highest strain rate had a 0.2% yield stress of about 1200 MPa and almost no strain hardening with the ultimate stress less than 1300 MPa. The other tensile response curves were similar with a lower value of yield stress. From a family of creep curves, it was observed that for the highest value of creep stress, 1200 MPa, the corresponding minimum creep rate was $2.7 \times 10^{-5} \text{ sec}^{-1}$. Thus it was concluded that the two tensile curves run at 8.3×10^{-3} and $8.4 \times 10^{-4} \text{ sec}^{-1}$ had a negligible amount of creep and could be used for evaluating the rate sensitivity constant, n . Unfortunately, the highest rate is only a factor of ten greater than the next rate, and the resulting tensile curves crossed.

Thus, we used the value of $n = 3.2$ reported in the literature, Ref. [7], for René 95 at 650°C.

The history of Z as a function of plastic work was established with $n = 3.2$ for the tensile test run at a strain rate of $8.3 \times 10^{-4} \text{ sec}^{-1}$. The state variable history is shown in Fig. 1. The curve appears to be of an exponential form with a saturation value $Z_1 = 318 \text{ MPa}$. The condition of a saturation value of Z agrees with the observed behavior that the stored energy of cold work increases with deformation at a decreasing rate. A further investigation showed that $\ln(Z_1 - Z)$ was very close to a linear function of the plastic work, w^P . Thus the representation

$$Z = f_1 + C_0 = Z_1 - (Z_1 - Z_0) \exp(-mW^P) \quad (11)$$

was established from the data with the constants $Z_0 = 1682 \text{ MPa}$ and $m = 0.37 \text{ MPa}^{-1}$ determined using a least squares analysis. The initial hardness Z_0 was determined by extrapolating $\ln(Z_1 - Z)$ to zero plastic work. Data could not be accurately determined for small values of plastic work; however, it is expected that values of Z exist below Z_0 as indicated by the dotted line in Fig. 1.

The first term of the evolution equation, Eq. (9), for tensile response can be obtained by differentiating Eq. (11) to obtain

$$\dot{Z} = m(Z_1 - Z_0) \exp(-mW^P) \dot{W}^P = f_1'(W^P) \dot{W}^P. \quad (12)$$

An alternative formulation for the tensile data was established by Bodner, Ref. [7], by combining Eqs. (11) and (12) to get

$$\dot{Z} = m(Z_1 - Z) \dot{W}^P = f_1'(Z) \dot{W}^P. \quad (13)$$

Even though $f_1'(W^P)$ and $f_1'(Z)$ are equivalent for tensile response alone, the two representations for f_1' give two different forms for the evolution equation with different properties. In this presentation we elect to investigate the consequences of using the second representation, Eq. (13).

In closing, it should be noted that Eq. (11) is different from the representation reported in Ref. [5] for the same material. Both models essentially the same value through the range of interest; however the asymptotic behavior of Eq. (11) appears to be more consistent with the observed response of the material.

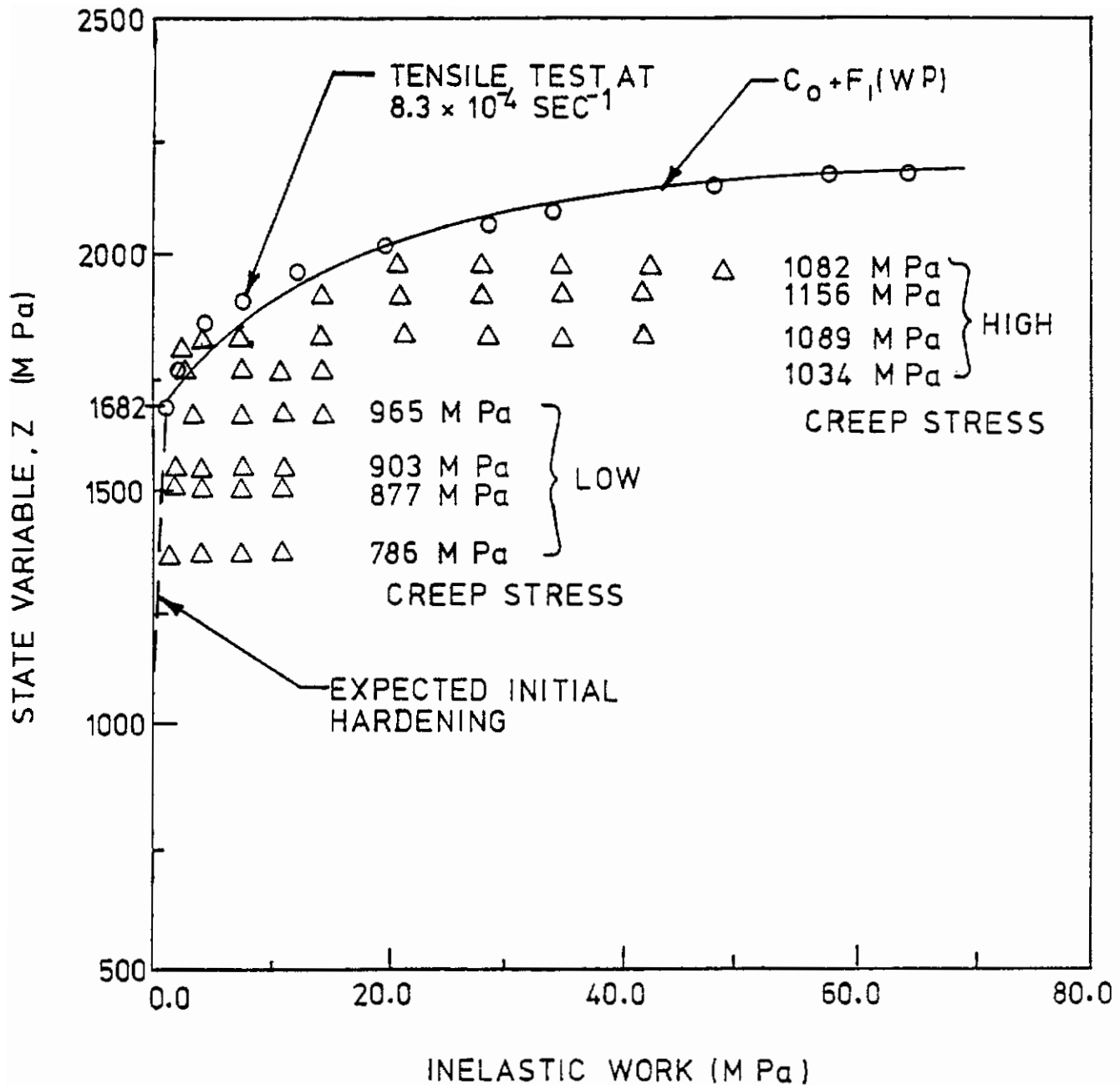


Figure 1. Internal state variable history for one tensile test and eight creep tests. The representation for tensile function $f_1(W^P) + C_0$ is compared to the experimental data.

EVALUATION OF THE CREEP RESPONSE

During secondary creep (when the strain rate is approximately constant) the value of Z must be constant if Eq. (7) is to be satisfied. Thus for secondary creep, Eq. (9) becomes

$$\dot{Z} = 0 = f_1'(Z) \dot{W}^P - f_2(Z) \quad (14)$$

The stationary value of Z corresponds to a stable material microstructure during the secondary creep regime.

In general $f_1'(Z)$ is known from the tensile data and the stationary value of Z can be calculated from Eq. (9) using the creep response data. The Z histories from eight creep tests are also shown in Fig. 1. It is interesting to note that for the duration of each test, Z is nearly constant. This occurs because the amount of primary creep during a typical test is very small compared to amount of secondary creep.

To start to establish a representation, let us assume the Bodner representation for f_2 ; i.e.

$$f_2(Z) = A Z_1 \left[\frac{Z - Z_2}{Z_1} \right]^r \quad (15)$$

where Z_2 is constant defining the minimum value of Z . If Eq. (15) is valid, then $\ln(f_1'(Z) \dot{W}^P)$ must be a linear function of $\ln((Z - Z_2)/Z_1)$ for Eq. (14) to be satisfied. The experimental data was reduced and plotted as shown in Fig. 2. The value $Z_2 = 1300$ MPa was chosen (arbitrarily) so that $Z - Z_2$ is always positive.

Many of the creep properties of René 95 are summarized in Fig. 2. First notice that the data is bilinear with a sharp change in slope. The constants for each region were calculated using a least squares analysis and it was found that the curves intersected at $Z = 1682$ MPa; a value that corresponds to a creep stress of approximately 980 MPa. Since $r = 12.0$ and $r = 1.34$ for stress above and below 980 MPa, it appears that different physical properties control deformation in the "high stress" and "low stress" domains. Further observe that the intersection point, $Z = 1682$ MPa, is the initial value of hardness, Z_0 , determined in the tensile tests. Thus the two response domains are segregated by the apparent initial value of hardness Z_0 .

Next recall that Z_0 was estimated by extrapolation to zero plastic work; but it was expected that values of Z below Z_0 existed at very small values of plastic work. The creep data gives further evidence that Z_0 is not the initial value of Z . Equation(7) requires the initial value of Z in a creep test must be below the stationary value during secondary creep in order to predict a primary creep rate greater than the secondary creep rate. This indicates that the initial rate of hardening, \dot{Z} , is very high for values of Z below $Z_0 = 1682$ MPa. This further confirms the existence of different deformation properties being active above and below a stress of 980 MPa.

It is difficult to accurately model the total creep and tensile response of René 95 when the response data is not available for very small values of plastic work. However, a reasonable approximation can

be constructed by observing that the low stress data in Fig. 2 is nearly constant when compared to the high stress data. Let us set $Z_2 = Z_0$ and neglect changes in Z below 1682 MPa. In this case $r = 3.0$ and $A = 5.34 \text{ sec}^{-1}$.

If damage is assumed to be the major mechanism that produces tertiary creep, an estimate of the value of the damage, ω , at failure can be obtained from the creep data. A total evaluation of the damage variable is not included since the creep curves were established using constant load rather than constant stress conditions. However, after adjustment for area reduction, it was found that the value of $Z(1-\omega)$ decreased about one to four percent between secondary creep and failure. It follows the value of ω at failure can be expected to be very small in most cases. Thus, the popular approach of assuming $\omega = 1$ at failure, Refs. [15, 16], is not applicable to René 95. This indicates that failure occurs due to a mechanism other than the Kachanov damage parameter. In René 95 this probably unstable crack growth.

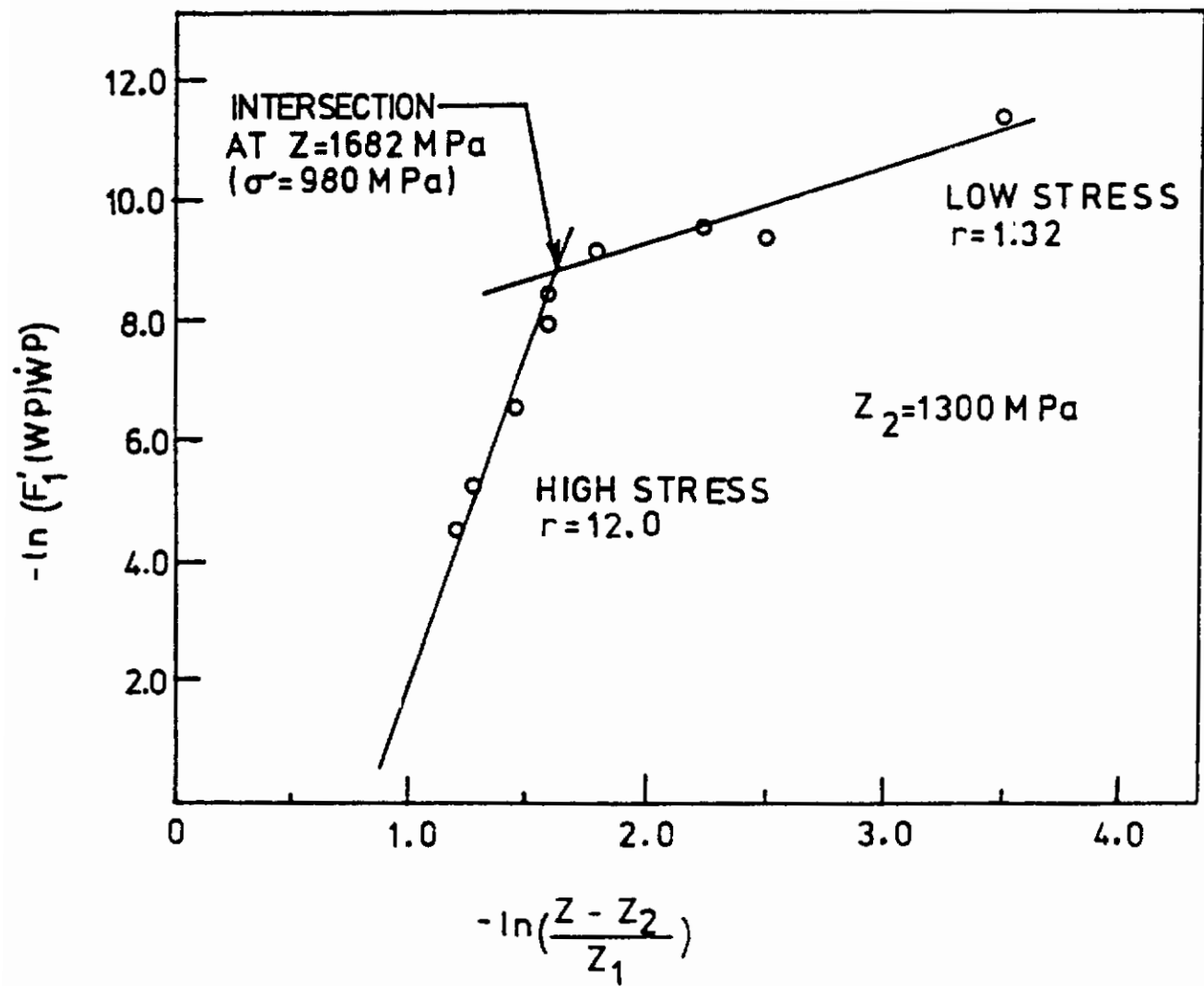


Figure 2. Bilinear representation for the stationary values of the internal state variable during secondary creep.

SECTION VI
COMPARISON OF THE MODEL AND DATA

The hardening law for René 95 at 650°C can be written by combining Eqs. (9), (12) and (14) to get

$$\frac{\dot{Z}}{Z_1} = m \left[1 - \frac{Z}{Z_1} \right] \dot{W}^p - A \left(\frac{Z - Z_2}{Z_1} \right)^r \quad (16)$$

with Z_0 the initial value Z . This can be used in combination with Eqs. (1) and (7) to calculate the response of the material assuming $\omega = 0$. The constants developed for René 95 are:

$D_0 = 10^4 \text{ sec}^{-1}$ (assumed)	$m = 0.37 \text{ MPa}^{-1}$
$n = 3.2$	$Z_2 = 1682 \text{ MPa}$
$Z_0 = 1682 \text{ MPa}$	$A = 5.34 \text{ sec}^{-1}$
$Z_1 = 2193 \text{ MPa}$	$r = 3.0$
$E = 1.77 \times 10^5 \text{ MPa}$	

where E is the elastic modulus. The final formulation for the hardening rule, Eq. (16) is identical to that proposed by Bodner, Ref. [7]. The constants Z_0 , Z_1 and m that were determined from the tensile test are very close to those reported by Bodner. The constants Z_2 and r developed from the creep data are different. The constant $Z_2 = Z_0$ was used as an approximation.

The above equations were used to predict the tensile response of René 95 at constant stress rate. Recall the constants were developed from a tensile test run at constant strain rate. The predictions are shown with two test results in Fig. 3. The agreement between theory and experiment is acceptable. Observe

that the stress, $\sigma = 980$ MPa, corresponding to intersection point in Fig. 2, is very close to the yield stress.

The creep response of René 95 was also calculated for three stress levels, 903 MPa, 1034 MPa and 1206 MPa. These three stresses are at the high and low ends of the two curves in Fig. 2 and also the intersection. The calculated curves and experimental data are shown in Fig. 4. The minimum creep rate for the 1206 MPa and 1034 MPa tests are very close; but the secondary creep rate for the 903 MPa was under predicted. This resulted from the approximation that was used for Z at the low stress levels.

The results in the previous two sections suggest that two different micromechanical mechanisms are active at high and low levels of stress. This effect can also be seen in the creep response curves displayed in Fig. 4. The stress increments of 131 MPa and 172 MPa between the three creep tests are approximately equal. However, the minimum creep rate changes by factors of four and 140, respectively. This difference in secondary creep rate reflects the consequences of a possible change in micromechanical mechanism.

In closing, it is important to point out that the above results show how all phenomenological theories must be used with caution. That is, the value of the parameters that are determined for a material under a given range of conditions may not apply in another range of conditions. For example, if the value of 12.0 assigned to r for the high stress region, the predicted response in the low stress region would be poor. However, the above system of equations could be modified to account for the physical change in the response and extrapolation in both the high and low stress regions may be possible. Of course, the new formulation should be experimentally verified before application.

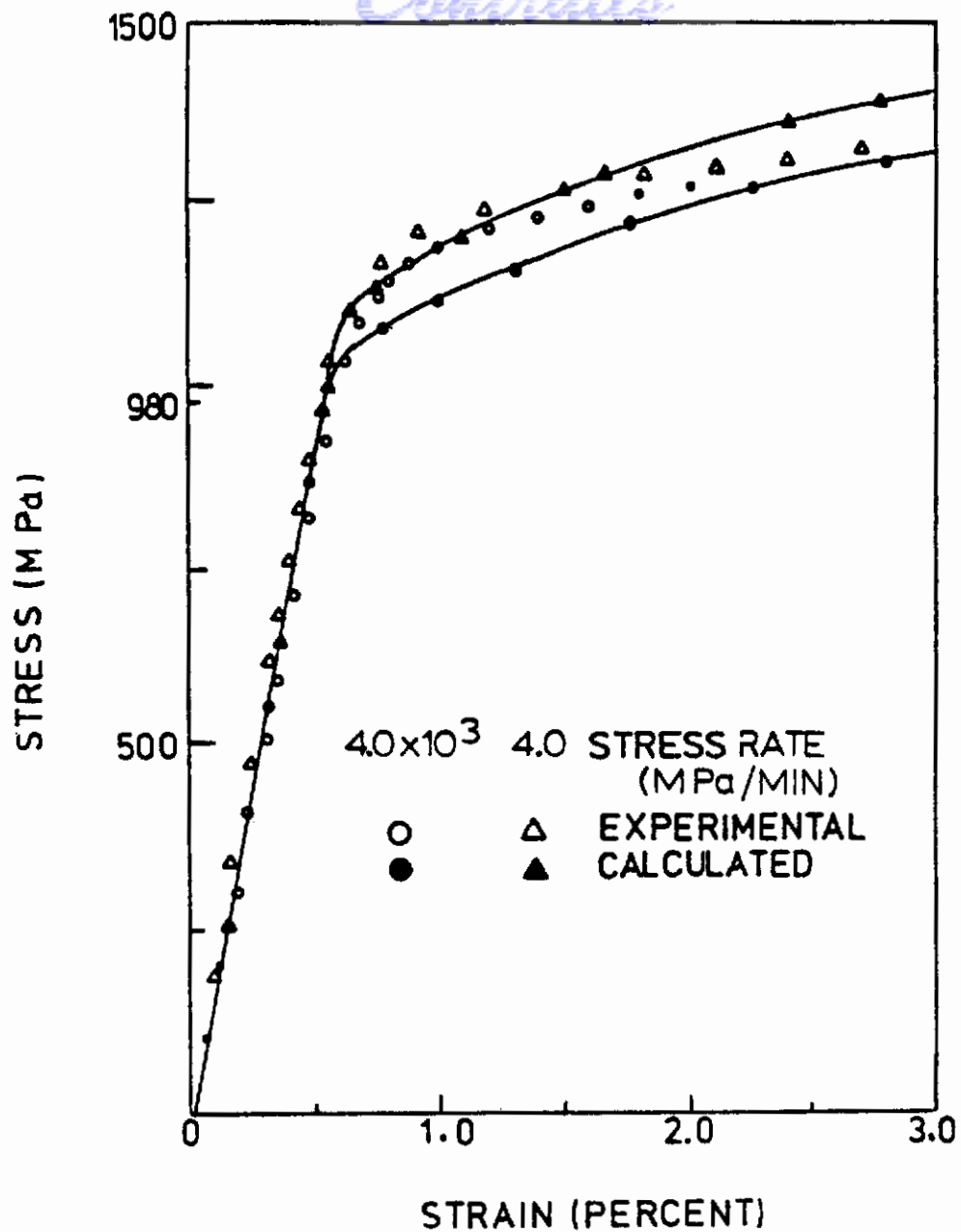


Figure 3. Calculated and experimental tensile response of René 95 at 650°C under constant stress rate control.

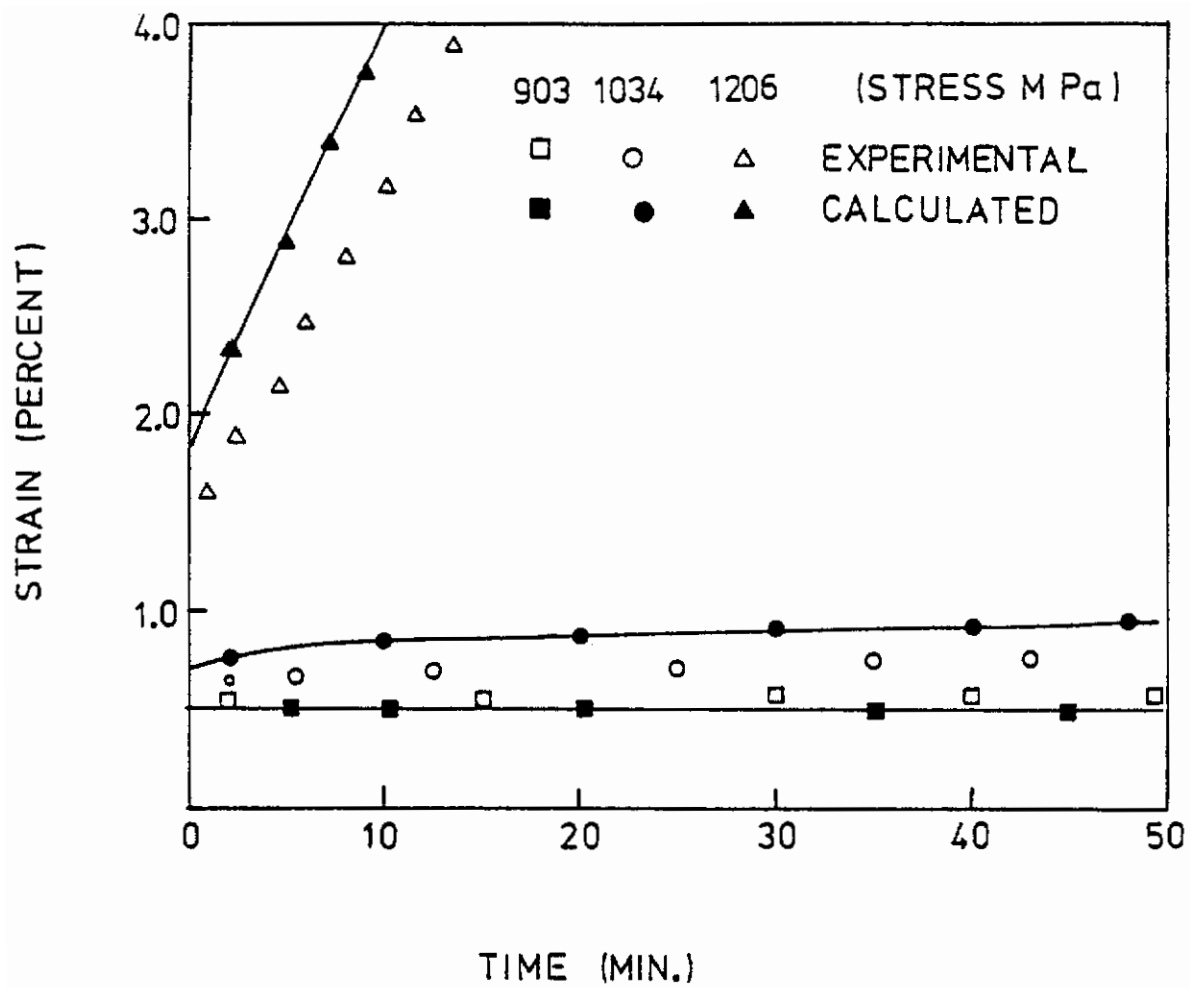


Figure 4. Calculated and experimental creep response of René 95 at 650°C at high, low and intermediate values of stress.

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