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THEORY OF STABILITY AND CONTROL FOR DISTRIBUTED PARAMETER SYSTEMS, AN ANNOTATED BIBLIOGRAPHY

by

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ABSTRACT: This bibliography provides an up-to-date collection of references pertaining to the theory of stability and control for distributed parameter systems.

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Up to the present time, most of the works in the theory of stability and control are devoted to systems whose dynamical behaviors are describable by a set of ordinary differential or difference equations. Recent attempts have been made in developing corresponding theories for distributed parameter systems whose dynamical behaviors are describable by partial differential equations, integral equations or functional differential equations. Most of the works in this area have been carried out in the Soviet Union. So far, it appears that their recent main efforts have been directed primarily toward the formulation of optimum control problems and the derivation of necessary and sufficient conditions for optimum controls in the form of maximum principles for particular classes of distributed parameter systems.

The objective of this bibliography is to provide an up-to-date collection of references pertaining to the theory of stability and control of distributed parameter systems. This bibliography only includes those works which deal directly with distributed systems in the form of partial differential equations or integral equations, and excludes all those works which deal only with the approximate forms of distributed systems without justification for the validity of approximation. Also excluded are works pertaining to distributed parameter systems which are describable by ordinary differential-difference equations. A comprehensive bibliography on this subject has been prepared by Choksy (see IRE Trans. on Autom. Control, Vol. AC-5, pp. 66-70, 1960).*

This bibliography is divided into two sections. The first section is devoted primarily to works on the theory of optimum control of distributed parameter systems and the second section is devoted to works on the theory of stability for such systems. This division is merely for the purpose of clarification and does not represent the intent of divorcing stability theory completely from optimum control theory.

In the section on stability theory, the selection of references has been heavily biased toward those works which pertain to the extension of Lyapunov's stability theory to partial differential equations, integral equations and denumerably infinite systems of ordinary differential equations. Also, among those works dealing with stability analysis for particular systems or special classes of systems, only those which are of potential interest to control are selected. The above selection tactics automatically exclude the enormous amount of literature on the application of classical analytical methods to the stability analysis of various specialized systems such as hydrodynamic

* An extended bibliography on this subject has been prepared by Choksy. It will appear in IEEE Trans. on Autom. Control.

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systems, elastic and aeroelastic systems.

It is hoped that this bibliography will be useful to those who are unfamiliar with the existing literature pertaining to the theory of stability and control of distributed parameter systems.

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(1) A. G. Butkovskii and A. Ya. Lerner, "The optimum control of systems with distributed parameters", Avtomatika i Telemekhanika, Vol. 21, No. 6, pp. 682-91, 1960; English translation: Automation and Remote Control, Vol. 21, No. 6, pp. 472-77, 1960.

This paper discusses the formulation of optimum control problems associated with certain classes of distributed parameter systems which are describable by a set of first-order partial differential equations and by diffusion equations. The main portion of the paper is devoted to the problem of controlling a particular system governed by a scalar linear partial differential equation of the form

$$c\,\frac{\partial Q(t,x)}{\partial t} + cv(t)\frac{\partial Q(t,x)}{\partial x} + Q(t,x) - u(t) = 0, \quad 0 \le x \le L$$

with boundary condition $Q(t,0) = 0$, where Q is the system state variable, u is the control variable, c is a constant and v is a specified function of t. The optimum control problem is to find u as a function of t satisfying constraint $u_1 \le u(t) \le u_2$, such that the functional

$$I = \int_0^T (Q_g - Q(t,L))^2 dt$$

attains its minimum value, where Q_g is a specified function of t or a constant. Explicit solution for u is obtained by transforming the system equations to a moving spatial coordinate system and applying Pontryagin's maximum principle.

(2) A. G. Butkovskii and A. Ya. Lerner, "Optimum control systems with distributed parameters", Doklady Akademii Nauk SSSR, Vol. 134, No. 4, pp. 778-81, 1960; English translation: Soviet Physics Doklady, Vol. 5, No. 5, pp. 936-39, 1961.

This paper is a condensed version of (1).

(3) A. G. Butkovskii and A. Ya. Lerner, "On the optimum control of systems with distributed parameters", Regelungstechnik, Vol. 5, pp. 185-88, 1961 (in German).

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The main content of this paper is identical to that of (1). The problem of time optimal control of heating a thick slab is taken as an example. Numerical results are obtained by first approximating the distributed system by a lumped parameter system using the method of spatial discretization and then by applying Pontryagin's maximum principle.

(4) A. G. Butkovskii, "Optimum processes in systems with distributed parameters", Avtomatika i Telemekhanika, Vol. 22, No. 1, pp. 17-26, 1961; English translation: Automation and Remote Control, Vol. 22, No. 1, pp. 13-21, 1961.

This paper is concerned with the problem of optimum control of systems whose motion is describable by a set of nonlinear integral relations of the form:

$$Q_i(t) = \int_{t_o}^{t_1} K_i(t, \tau, u_1(\tau), \ldots, u_r(\tau))\, d\tau \quad, i = 1, \ldots, n$$

where Q_i are the system state variables and u_j are the control variables. The problem is to find the control function $u(t) = [u_1(t), \ldots, u_r(t)]$ belonging to a closed region Ω at any instant of time $t \epsilon [t_o, t_1]$ such that the performance index

$$Q_o = \int_{t_o}^{t_1} F(\tau, Q_1(\tau), \ldots, Q_n(\tau), u(\tau))\, d\tau$$

assumes the minimum possible value. A necessary condition for which the optimum control vector u must satisfy is derived. The result is applied to the problem of controlling the heating of a thick solid body and a heat exchange process.

(5) R. Bellman and R. Kalaba, "Reduction of dimensionality, dynamic programming, and control processes", Trans. of ASME, J. of Basic Eng. Vol. 8, Series D, No. 1, pp. 82-4, 1961.

A major difficulty in the way of a successful systematic approach to the study of control processes by way of the theory of

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dynamic programming is the occurrence of processes having state vectors of high dimension. However difficult the problem is for systems ruled by a finite set of differential equations, it is several orders of magnitude more complex for systems of infinite dimensionality and for systems with time lags. By combining a technique presented earlier for dealing with finite dimensional systems and various methods of successive approximations and quasi-linearization, certain classes of control processes associated with infinite dimensional systems can be treated. The ideas are illustrated by discussing control of a system involving a time lag and control of a thermal system. (Authors' summary)

(6) A. G. Butkovskii, "The maximum principle for optimum systems with distributed parameters", Avtomatika i Telemekhanika, Vol. 22, No. 10, pp. 1288-1301, 1961; English translation: Automation and Remote Control, Vol. 22, No. 10, pp. 1156-69, 1962.

This paper extends the results in reference (4) to systems whose motion is describable by a nonlinear integral equation of the form:

$$Q(P) = \int_D K(P, S, Q(S), u(S))\, dS$$

where Q is the system state vector (with values in E_n) and u is the control vector (with values in E_r). P and S are points in D, a subset of E_m. There are given $q + 1$ functionals $I^i(Q)$, $i = 0, 1, \ldots, \ell$, and $I^i(Q,u)$, $i = \ell + 1, \ldots, q$. The control problem is to find an admissible u such that I^p, $0 \le p \le q$, takes on its minimum value and the constraints $I^i = 0$, $i = 0, 1, \ldots, p-1, p+1, \ldots, q$, are satisfied. A maximum principle is derived. The result is applied to two examples. In one of the examples, Q is the temperature distribution of a thick slab, u is the temperature of one face of the slab satisfying constraint $A_1 \le u(t) \le A_2$ for all $t \in [0,T]$. Taking $Q = 0$ at $t = 0$, Q is related to u by

$$Q(x,t) = \int_0^T K(x,t,\tau)\, u(\tau)\, d\tau \quad , \quad x \in [0,1]$$

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The problem is to minimize

$$I^0 = \int_0^1 [Q^*(x) - Q(x,T)]^\gamma \, dx \qquad , \quad \gamma = \text{constant} > 0$$

where Q^* is specified.

(7) A. G. Butkovskii, "Some approximate methods for solving problems of optimal control of distributed parameter systems", Avtomatika i Telemekhanika, Vol. 22, No. 12, pp. 1565-75, 1961, English translation: Automation and Remote Control, Vol. 22, No. 12, pp. 1429-38, 1962.

Approximate methods for solving problems of optimal control of distributed parameter systems are discussed. For systems which are in the form of a set of partial differential equations, approximate solutions are obtained by first approximating the system equations by a set of ordinary differential equations through the use of spatial discretization, and then applying Pontryagin's maximum principle. For systems which are in the form of a linear integral equation, approximate solutions are obtained by first approximating the kernel by a degenerate kernel and reducing the control problem to the so called "L problem" in the theory of moments. No discussions are made on the errors resulting from the approximations.

(8) Ju. V. Egorov, "Certain problems in the theory of optimal control", Doklady Akad. Nauk SSSR, Vol. 145, No. 5, pp. 720-23, 1962; English translation: Soviet Math. Doklady, Vol. 3, No. 4, pp. 1080-84, 1962.

This paper discusses the existence and uniqueness of an optimum control for a series of problems associated with a linear diffusion equation of the form:

$$\frac{\partial u(t,x)}{\partial x} = \frac{\partial^2 u(t,x)}{\partial x^2}, \quad x \in (0,1)$$

One of the problems considered is the case where the initial and boundary conditions are:

$$u(0,x) = 0, \ \left.\frac{\partial u(t,x)}{\partial x}\right|_{x=0} = 0, \ \left.\frac{\partial u(t,x)}{\partial x}\right|_{x=1} = \alpha(p(t) - u(t,1)),$$

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where α is a positive constant and p is the control variable. The problem is to establish existence and uniqueness of an admissible control p(t) defined on [0, T], $|p(t)| \leq 1$, which minimizes the functional

$$I(p) = \int_0^1 (u(T,x) - u_o(x))^2 \, dx$$

where u_o is a prescribed function from $L_2(0,1)$. Other variations of this problem are also considered. The author also points out an incorrect statement made by Butkovskii (ref. (6)) pertaining to the uniqueness of optimum control in the above problem.

(9) R. Bellman and R. Kalaba, "Dynamic programming applied to control processes governed by general functional equations", Proc. of U. S. Acad. of Science, Vol. 48, pp. 1735-37, 1962.

This paper discusses briefly the application of dynamic programming to derive the functional equation associated with the control problem: Minimize with respect to the control variable v the functional

$$J(u,v) = \int_0^T \int_R g(u,v) \, dA dt$$

where the scalar functions u and v are connected by the equation $u_t = Lu + h(u,v)$, $u(t=0) = c(p)$, where $p \in R$; $u, v \in R$ and L is a linear operator with certain prescribed properties.

(10) I. McCausland, "On optimum control of temperature distribution in a solid", J. of Electronics and Control, Vol. 14, No. 6, pp. 655-68, 1963.

The paper describes a method of determining the switching instants for the input to a particular distributed-parameter system, namely a metal slab which is heated on one face and has the opposite face insulated. The switching instants are determined on the basis of achieving a desired uniform temperature in the slab in the shortest possible time, given certain limitations on the temperature which can be applied to the heated face. A previously known method of solving this problem was by sub-dividing the slab onto sections and using lumped-parameter methods; the

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present method makes use of a Fourier-series representation of the temperature distribution, the input being such as to bring all the space harmonics of the error distribution to zero at the same time. (Author's Summary)

(11) P.K.C. Wang and F. Tung, "Optimum control of distributed parameter systems", Proc. of 1963 Joint Automatic Control Conference, Minneapolis, Minn., pp. 16-32, 1963; also appeared in Trans. of ASME, J. of Basic Eng. Vol. 86, Series D, No. 1, pp. 67-79, 1964.

This paper presents a general discussion of the optimum control of distributed-parameter dynamical systems. The main areas of discussion are: (a) The mathematical description of distributed parameter systems, (b) the controllability and observability of these systems, (c) the formulation of optimum control problems and the derivation of a maximum principle for a particular class of systems, and (d) the problems associated with approximating distributed systems by discretization. In order to illustrate the applicability of certain general results and manifest some of the properties which are intrinsic to distributed systems, specific results are obtained for a simple, one-dimensional, linear-diffusion process.

(12) A. G. Butkovskii, "The broadened principle of maximum for optimum control problems", Avtomatika i Telemekhanika, Vol. 24, No. 3, pp. 314-27, 1963; English translation: Automation and Remote Control, Vol. 24, No. 3, pp. 292-304, 1963.

In this paper, the author generalizes his results in reference (6) to integral equations defined on a Banach space. A maximum principle is derived for the following problem: to determine a function $w \in W(P)$, $P \in D$, such that for the condition

$$\Phi(W(P), \int_D K(P, S, W(S))\, dS) = \Theta,$$

the functional $\Phi^o(\int_D K(P_1, S, W(S))\, dS)$ attains its minimum possible value, where $D \subset E_s$, P_1 is a fixed point in D, and it is assumed that the set of functions $w \in W(P)$, $w \in m \subset b$ forms a convex set M in a certain Banach space B, Θ is the null element

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in B, and the functional Φ^o has a continuous strong derivative with respect to $W \in M$. In the case where Φ is a linear operator and Φ^o is a linear functional, it is shown that the maximum principle is also the sufficient optimality condition.

(13) Ju. V. Egorov, "Optimal control in Banach space", Doklady Akad. Nauk SSR, Vol. 150, No. 2, pp. 241-4, 1963; English translation: Soviet Math. Doklady, Vol. 4, No. 3, pp. 630-3, 1963.

This paper discusses the problem of optimum control of a system described by

$$\frac{dx(t)}{dt} = f(x(t), u(t)), \quad x(a) = x_o, \quad a \le t \le b$$

defined on a Banach space, where x is the state vector, u is the control. It is required to select an admissible control $u \in U$ -- a given set of a certain topological space, such that a functional of the form

$$\int_a^b f^0(x(t), u(t))dt$$

is minimized. A maximum principle analogous to that of Pontryagin is stated without proof for each of the following cases (i) $x(b) = x$, (ii) $x(b) \in S$ -- a smooth manifold in a Banach space, (iii) $x(b)$ belongs to a given convex set in a Banach space. In all cases, the time b is not fixed. Examples of systems associated with each of the above cases are given.

(14) I. McCausland, "On-off control of linear systems with distributed parameters", Ph.D. dissertation, Cambridge University, England, Oct. 1963.

This dissertation considers the problem of time optimal control of the temperature distribution in a solid. The main results are published in reference (10). The question of controllability of a simple linear heat equation for a slab with control at one of the faces is also discussed.

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(15) K. A. Lur'e, "On the Hamilton-Jacobi method in variational problems of partial differential equations", Prikladnaya Matematika i Mekhanika, Vol. 27, No. 2, pp. 255-64, 1963; English translation: J. of Applied Math. and Mech., Vol. 27, No. 2, pp. 378-91, 1964.

This paper considers the problem of finding a function z for which the functional

$$\int\int_G L(x, Y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})\, dx\, dy$$

is stationary. Using the notion of a functional (Fréchet) derivative, an analog of the Hamilton-Jacobi equation is derived. The notion of a complete integral of the resulting equation and the problem of obtaining z therefrom are also discussed.

(16) A. I. Egorov, "On optimal control of processes in distributed objects", Prikladnaya Matematika i Mekhanika, Vol. 27, No. 4, pp. 688-96, 1963; English translation: J. of Applied Math. and Mech., Vol. 27, No. 4, pp. 1045-58, 1964.

This paper considers the problem of optimal control of a distributed parameter process governed by a system of quasi-linear partial differential equations of the form:

$$\frac{\partial^2 u_i}{\partial x\, \partial t} + b_i(x, t)\frac{\partial u_i}{\partial x} + c_i(x, t)\frac{\partial u_i}{\partial t} = f_i(x, t, u_1, \ldots, u_n, v),$$

$$i = 1, \ldots, n$$

where b_i and c_i have continuous second derivatives with respect to x and t in a region G $(0 \le x \le \ell,\ 0 \le t \le T)$, v is a control variable with its values in V -- a convex set in E_r. The functions f_i are continuous in x and t and twice continuously differentiable with respect to $u_1, \ldots, u_n$ and v. The initial and boundary conditons are:

$$u_i(x, 0) = \phi_i(x), \quad u_i(0, t) = \psi_i(t), \quad i = 1, \ldots, n.$$

The optimum control problem is to determine an admissible

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control $v \epsilon V$ so that the functional

$$S = \sum_{i=1}^{n} [\; A_i u_i(\ell, T) + \int_0^{\ell} \alpha_i(x) u_i(x, T) dx + \int_0^{T} \beta_i(t) u_i(\ell, t)\, dt$$

$$+ \int_0^{\ell} \int_0^{T} \gamma_i(x, t) u_i(x, t) dt dx \;]$$

takes on its minimum value, where A_i are specified real numbers, α_i, β_i and γ_i are given functions continuous in G. A maximum principle for this problem is derived. For linear systems, it is shown that the maximum principle is a necessary and sufficient condition for optimality. Three examples are worked out to illustrate the application of the derived maximum principle.

(17) K. A. Lur'e, "The Mayer-Bolza problem for multiple integrals and the optimization of the performance of systems with distributed parameters", Prikladnaya Matematika i Mekhanika, Vol. 27, No. 5, pp. 842-53, 1963; English translation: J. of Applied Math. and Mech., Vol. 27, No. 5, pp. 1284-99, 1964.

In this paper, the optimal control problem for distributed parameter systems governed by partial differential equations is formulated as a Mayer-Bolza problem for multiple integrals. Necessary conditions for stationarity and Weirstrass condition are derived via classical calculus of variations. From the latter condition, an analog of the Pontryagin's maximum principle is derived. The analysis is carried out for a system of first order partial differential equations defined on a one-dimensional spatial domain.

(18) P.K.C. Wang, "Optimum control of distributed parameter systems with time delays", Research Note No. NJ-40, IBM Research Laboratory, San Jose, Calif., April, 1963; also in IEEE Trans. on Automatic Control, Vol. AC-9, No. 1, pp. 13-22, 1964.

This paper discusses the optimum control of distributed parameter systems with time delays which are governed by a set of partial differential-difference equations of the form:

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$$\frac{\partial U(t, X)}{\partial t} = G(U(t, X), U(t-\tau, X), F(t, X))$$

defined for $t > 0$ on a spatial domain $\Omega \subset E_M$, where τ is a fixed delay time, X is the spatial coordinate vector, U is a system variable with values in E_N, F is a control variable with values E_K. The optimum control problem is to find a distributed control law which minimizes a specified performance index. The technique of dynamic programming is used to derive the functional equations associated with optimization. Specific results are derived for linear systems with quaratic performance index. Also, an explicit condition for the complete null controllability of linear systems is given. The paper concludes with a discussion on the approximate solutions for the minimum energy control of a simple linear parabolic system with a time delay.

(19) A. G. Butkovskii, "Optimal control of systems with distributed parameters", Proc. of the Second International Congress on Automatic Control, to be published by Butterworths Pub. Co., London; preprint no. 513

In essence, this paper is a summary of the results contained in the author's previous papers (references (1) - (4), (6), (7)).

(20) A. G. Butkovskii, "The method of moments in the theory of optimal control of systems with distributed parameters", Avtomatika i Telemekhanika, Vol. 24, No. 9, pp. 1217-25, 1963; English translation: Automation and Remote Control, Vol. 24, No. 9, pp. 1106-13, 1964.

In this paper, methods for solving the problem of optimal control of linear distributed parameter systems by using results pertaining to the L-problem in the theory of moments are discussed. The relation between the results obtained by this method and those obtained by using the maximum principle are considered. On the basis of the results described, a numerical algorithm for determining the optimum control is proposed.

(21) A. G. Butkovskii, A. Ya Lerner and S. A. Malyi, "Problems of optimal controlling of drawing from molten metal", Doklady Akademii Nauk SSSR, Vol. 153, No. 4, pp. 772-5, 1963; English translation: Soviet Physics Doklady, Vol. 8, No. 12, pp. 1149-51, 1964.

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The authors derive a mathematical model for the process of drawing from molten metal and give two formulations for the optimum control problem associated with such a process. No solutions are presented.

(22) S. Katz, "A general maximum principle for end-point control problems", J. of Electronics and Control, Vol. 16, No. 2, pp. 189-222, 1964.

A very broad class of end-point control problems is formulated in general terms, and correspondingly general procedures in the calculus of variations are developed for their solution:

The generality of the treatment lies in a systematic application of the point of view of functional analysis. The state of the system being controlled is described by an abstract vector in a suitable linear space; the evolution of the system state with time is described in terms of an operator on the state vector; the measure of system performance to be maximized or minimized is taken as a functional of the final state. The linearized system operator and its adjoint with respect to an inner product in the state space play control roles in the variational arguments.

The variational methods are in the spirit of those leading to the familiar Pontrjagin maximum principle, and indeed this principle appears as the finite-dimensional case of the present results. These results however apply equally to infinite-dimensional cases such as systems with distributed parameters whose mathematical description is given in terms of partial rather than ordinary differential equations. The methods are illustrated here for a problem in heat exchanger control. General numerical procedures are discussed, but for distributed parameter systems the calculations required to solve practical problems will be extremely formidable. (Author's Summary)

(23) P.K.C. Wang, "Control of distributed parameter systems", in Advances in Control Systems -- Theory and Applications" (book), Academic Press, Inc., N. Y., 1964.

In this paper, the author attempts to present a general unified discussion of various problems associated with the control of distributed parameter systems. First, the description of distributed parameter systems is discussed from both physical and mathematical standpoints. Simple examples of various types of distributed

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systems are given. The mathematical description is focused on the form of system equations and the establishment of consistency conditions for which a given mathematical model has the properties of a dynamical system. In particular, linear systems are discussed in detail. Next, various intrinsic properties of a distributed system which are of importance to control are discussed. The particular properties considered are stability, controllability and observability. Here, only stability in the sense of Lyapunov is discussed. Stability theorems due to Zubov and Massera are stated without proofs. Their applications are illustrated by examples. The discussions on controllability and observability are devoted primarily to the extensions of Kalman's definitions for finite-dimensional systems to distributed systems. Necessary and sufficient conditions for complete null-controllability and observability of linear distributed dynamical systems are given. The following section is devoted to the optimum control of distributed systems. Various problem formulations are given. Functional equations associated with optimization of systems in the form of partial differential equations are derived via dynamic programming. Also, Butkovskii's maximum principle for systems in the form of a set of nonlinear integral equations are discussed. The problem of optimum control of linear distributed systems with generalized quadratic performance index is discussed in detail. Finally, various problems associated with approximation and computation, and various practical aspects of control are discussed briefly.

(24) T. K. Sirazetdinov, "Concerning the theory of optimum processes with distributed parameters", Avtomatika i Telemekhanika, Vol. 25, No. 4, pp. 463-72, 1964.

This paper considers the problem of optimum control of a distributed parameter system governed by a scalar quasi-linear partial differential equation of the form:

$$\frac{\partial v(t,x_i)}{\partial t} = f_o(t, x_i, v, u_j) + \sum_{k=1}^{n} f_k(t, x_i, v, u_j)\frac{\partial v(t, x_i)}{\partial x_k} ,$$

$$i = 1, \dots, n,$$

$$j = 1, \dots, m$$

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with initial condition $v(t_o, x_i) = v_o(x_i)$, defined on a region $\tau \subset E_n$, where v is the system state variable, x_i are spatial coordinate variables, u_i are the control variables. The problem is to select a control satisfying constraints:

$$\phi_k(u_1, \ldots, u_m) = 0, \quad k = 1, \ldots, r < m$$

such that the functional

$$I = \int_{t_o}^{T} \int_{\tau} [G_o(t, x_i, v, u_j) + \sum_{k=1}^{n} G_k(t, x_i, v, u_j) \frac{\partial v}{\partial x_k}] \, d\tau \, dt$$

attains its minimum value. A maximum principle is derived. In the case where the system is linear, it is shown that the maximum principle is also a sufficient condition for optimum.

(25) P. L. Falb, "Infinite dimensional control problems I: On the closure of the set of attainable states for linear systems", J. of Math. Analysis and Applications, Vol. 9, pp. 12-22, 1964.

This paper considers optimum control problems in which the state space is infinite dimensional. Attempts are made to develop results analogous to those for finite dimensional systems. In particular, the problem of transferring a linear system from one state to another in minimum time with a limitation on the available power is considered. It is shown that the time-optimal control is (generally) bang-bang. A crucial step in the proof of this result is the demonstration of the closure of the set of states which are attainable from a given state within a specified time.

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(1) R. Bellman, "The boundedness of solutions of infinite systems of linear differential equations", Duke Math. J., Vol. 14, pp. 695-706, 1947.

This paper discusses the existence and uniqueness of solutions of an infinite system of homogeneous linear differential equations of the form $dx_i/dt = \sum_{j=1}^{\infty} a_{ij}x_j$, $i = 1, 2, \ldots$, where a_{ij} are constants. A sufficient condition for boundedness of solutions is derived for the case where the coefficient matrix is triangular.

(2) K. Persidskii, "On the stability of solutions of denumerable systems of differential equations", Izvestiya Akad. Nauk Kazah. SSR., Vol. 56, Ser. Mat. Meh. No. 2, pp. 3-35, 1948.

In this paper, the author extends Lyapunov's stability theory to a denumerably infinite system of ordinary differential equations of the form: $dx/dt = w(t, x)$, where x and w are infinite dimensional vectors. Also, w has the property that (i) $w(t, 0) = 0$, (ii) its components w_i satisfy $|w_i| < B(t)$ for all x whose components are bounded, (iii) w satisfies a Lipschitz-like condition. Examples are provided for illustrating the main results.

(3) M. G. Krein, "On some questions related to the ideas of Lyapunov in the theory of stability", Uspehi Matem. Nauk, Vol. 3, No. 3, pp. 166-69, 1948.

The author extends certain results of Persidskii and Malkin to differential equations of the form $dx/dt = F(t, x)$, where x is an element of a Banach space and F is an operator with its domain and range in the same space.

(4) K. Persidskii, "On the characteristic numbers of the solution of an infinite system of linear differential equations", Doklady Akad. Nauk SSSR, Vol. 63, pp. 229-32, 1948.

The author extends Lyapunov's notion of a characteristic number of a solution of a linear differential equation to infinite systems of linear differential equations of the form

$$dx_i/dt = \sum_{j=1}^{\infty} p_{ij}(t)x_j, \quad i = 1, 2, \ldots \; .$$

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(5) K. P. Persidskii, "On the stability of the solution of an infinite system of equations", Akad. Nauk SSSR. Prikl. Mat. Meh., Vol. 12, pp. 597-612, 1948.

The author considers infinite systems of differential equations of the form: $dx_i/dt = w_i(x_1, x_2, \ldots, t)$, $i = 1, 2, \ldots$, where w_i satisfies

$$|w_i(x_1, x_2, \ldots, t) - w_i(x_1', x_2', \ldots, t)| \leq A(t) \left| \sum a_{ij} |x_j - x_j'| \right|,$$

$$i = 1, 2, \ldots$$

in the region $t \geq 0$, $|x_i| \leq R > 0$; $\sum_{j=1}^{\infty} a_{ij} \leq L$; and $w_i(0, 0, \ldots, t) = 0$ for all $i = 1, 2, \ldots$ First the existence of solutions is established. Then, the stability of the trivial solution is discussed in the framework of Lyapunov stability theory.

(6) S. Goršin, "On the stability of the solutions of a denumerable system of differential equations with constantly acting disturbances", Izvestya Akad. Nauk Kazah. SSR, Vol. 60, Ser. Mat. Meh., No. 3, pp. 32-38, 1949.

The notion of stability in the presence of constantly acting disturbances for finite dimensional systems is extended to countable systems similar to those considered by Persidskii (ref. (2), (5)).

(7) K. P. Persidskii, "Uniform stability in the first approximation", Akad. Nauk SSSR, Prikl. Mat. Meh. Vol. 13, pp. 229-240, 1949.

The infinite system $dx_i/dt = \sum_{j=1}^{\infty} p_{ij} x_j + g_i(x_1, x_2, \ldots)$ is considered. It is assumed that $|g_i(x)| \leq ||x|| \; g(||x||)$ and $g(||x||) \to 0$ as $||x|| \to 0$, where $||x|| = \sup_i \{ |x_1|, |x_2|, \ldots \}$. Theorems pertaining to the relation between the uniform and asymptotic stability of the solution of the linear approximation and that of the original equation are given.

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(8) M. R. Resetov, "On the stability of the solutions of a denumerable system of differential equations, the linear parts of which have triangular form", Izvestiya Akad. Nauk Kazah. SSR, Vol. 60, Ser. Mat. Meh. No. 3, pp. 39-76, 1949.

First, the author considers the linear system $dx_i/dt = \sum_{j=1}^{i} p_{ij}(t)x_n$, $i = 1, 2, \ldots$, with p_{ij} real or complex and continuous in t for $t > 0$ and satisfy

$$\sum_{k=1}^{i-1} |p_{ij}| < a, \quad |p_{ii}| < L(t), \quad i = 1, 2, \ldots$$

where a is a constant > 0 and L continuous in t for $t > 0$. A theorem on the asymptotic stability of the trivial solution is given. Then, stability and instability theorems for the system $dx_i/dt = \Sigma\, p_{ij}(t)\, x_j + L_i(t, x_1, x_2, \ldots)$ with $L_i(t, 0, 0, \ldots) = 0$ are given.

(9) V. Harasahal, "On the stability in the first approximation of the solutions of denumerable systems of differential equations", Izvestiya Akad. Nauk Kazah. SSR. Vol. 60, Ser. Mat. Meh. No. 3, pp. 77-84, 1949.

The author considers the same nonlinear system described by Resetov (Ref. (8)), except here it is assumed that L_i are of higher order than unity near the origin. Stability theorems analogous to those of Lyapunov are given.

(10) I. P. Makarov, "Conditions for the approach to zero of the solutions of an inhomogeneous infinite system of differential equations", Doklady Akad. Nauk SSSR, Vol. 68, pp. 225-28, 1949.

The system $dx_i/dt = \sum_{j=1}^{\infty} p_{ij}(t)\, x_j + \phi_i(t)$, $i = 1, 2, \ldots$, is considered, where p_{ij} satisfy certain boundedness conditions. Conditions for which any solution whose initial values are sufficiently close to the origin tends to zero as $t \to \infty$ are given.

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(11) K. Persidskii, "On the stability of solutions of differential equations", Izvestiya Akad. Nauk Kazah. SSR, Ser. Mat. Meh. No. 4, pp. 3-18, 1950.

The stability in the sense of Lyapunov for differential equations defined on a Banach space is discussed. Existence of solutions is also discussed.

(12) K. P. Persidskii, "Some critical cases of denumerable systems", Izvestiya Akad. Nauk Kazah. SSR, Ser. Mat. Meh. No. 5, pp. 3-24, 1951.

This paper considers the system:

$$dx/dt = H(t, x, y) \quad , \qquad dy/dt = F(t, x, y)$$

with $F(t, 0, 0) = H(t, 0, 0) = 0$. Also, F and H satisfy

$$\left.\begin{array}{l} \| F(t, x, y) - F(t, x', y') \| \\ \| H(t, x, y) - H(t, x', y') \| \end{array}\right. < B(t) \, \| x - x', y - y' \|$$

where B is a continuous function of t. Both stability and instability theorems are given.

(13) S. F. Feščenko, "Asymptotic solution of an infinite system of differential equations with slowly varying parameters", Dopovidi Akad. Nauk Ukrain. RSR Vol. 1954, pp. 82-86, 1954.

$$\frac{d^2 z_n}{dt^2} + \omega_n^2 z_n = \epsilon \sum_{j=1}^{\infty} A_{nj}(t)\, z_j + \epsilon\, B_n(t) \exp(i\,\Theta) \quad , \quad n = 1, 2, \ldots$$

where ϵ is a small parameter and ω_n are real numbers such that $\omega_n \to \infty$ as $n \to \infty$. Certain boundedness conditions are imposed on A_{nj}, B_n and their derivatives. Solutions are found for the resonance and non-resonance cases.

(14) V. I. Zubov, Methods of A. M. Lyapunov and their applications, Pub. House of Leningrad Univ. 1957, English Transl.: U.S. Atomic Energy Comm., Transl. No. AEC-tr-4439.

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In Chapter IV of this book, the author extends the stability theory of Lyapunov to a general dynamical system. The application of the extended results to stability problems in distributed parameter dynamical systems is discussed in Chapter V. Many specific examples are given.

(15) J. L. Massera, "Contributions to stability theory", Ann. Math., Vol. 64, pp. 182-206, 1956.

The author extends Lyapunov's stability theory to differential equations defined on a Banach space. Both stability and instability theorems are given. He also points out that certain well-known results for finite dimensional systems fail to hold for infinite dimensional systems. For example, for infinite dimensional time-invariant linear systems, asymptotic stability does not imply uniform asymptotic stability.

(16) J. L. Massera and J. J. Schäffer, "Linear differential equations and functional analysis", Parts I-III, Ann. of Math., Vol. 67, pp. 517-73, 1958; Vol. 69, pp. 88-104, 1959; Vol. 69, pp. 535-74, 1959, respectively.

In Part I, the authors apply Banach space techniques to the problem of determining properties of solutions of the nonlinear system $dx/dt = A(t)x + h(t, x)$ from those of the linear system $dx/dt = A(t)x + h(t)$. In Part II, using a generalized Floquet representation, the authors establishes the existence of periodic solutions of the above linear system where A is periodic in t. In Part III, the authors extend Lyapunov's second method to the above linear system defined on a Banach space. Direct and converse theorems are given, thus establishing the equivalence between the existence of certain generalized Lyapunov functions with the asymptotic behavior of the bounded and unbounded solutions of the system under consideration.

(17) J. L. Massera and J. J. Schäffer, "Linear differential equations and functional analysis", Part IV, Math. Ann., Vol. 139, pp. 287-342, 1960.

This is a continuation of the previous papers (Ref. 16). Discussions are primarily devoted to establishing exponential bounds for the solutions of the linear system in Ref. (16).

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(18) J.J. Schäffer, "Linear differential equations and functional analysis", Part V, Math. Ann., Vol. 140, pp. 308-21, 1960.

This is a continuation of the previous papers (Ref. (16) and (17)).

(19) J.L. Massera and J.J. Schäffer, "Linear differential equations and functional analysis", Bol. Soc. Mat. Mexicana, Vol. 5, pp. 42-48, 1960.

This paper summarizes the main results in Ref. (16) - (18). The theorems are stated without proofs.

(20) Z.I. Halilov, "Stability of solutions of an equation in a Banach space", Dokl. Akad. Nauk SSSR, Vol. 137, pp. 797-99, 1961; English Transl.: Soviet Math. Dokl., Vol. 2, pp. 362-64, 1961.

This paper outlines a generalization of one of the theorems of Massera and Schäffer (Ref. (16) - (17)).

(21) W.K. Ergen and J.A. Nohel, "Stability of a continuous-medium reactor", J. of Nuclear Energy, Part A, Reactor Science, Vol. 10, pp. 14-18, 1959.

Sufficient conditions for the stability of a continuous-medium reactor are given under the assumption that there is no mechanical vibration coupled with the oscillation of reactor power; that delayed neutron effects may be neglected; and that hydrodynamic flow does not affect dynamic stability. The conditions given involve only known reactor parameters, and numerical examples are considered to complement the theory. (Author's summary)

(22) S.I. Gorshin, "On the stability in the large of the solutions of a denumerable system of differential equations under continuously acting disturbances", Prikl. Mat. i Mekh., Vol. 26, No. 2, pp. 212-17, 1962.

The author considers a denumerable system of differential equations of the form $dx_s/dt = \omega_s(t, x_1, x_2, \dots) + f_s(t, x_1, x_2, \dots)$, $s = 1, 2, \dots$, where f_s are considered disturbances. Stability theorems for this system based on Lyapunov's ideas are given.

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(23) P.K.C. Wang and M.L. Bandy, "Stability of distributed parameter processes with time-delays", J. Electronics and Control, Vol. 15, pp. 343-62, 1963.

In this paper, it is shown that time-delayed variables can enter into distributed parameter processes due to the presence of both internal and external delayed action energy sources. The dynamic behavior of such processes is describable by a system of partial differential-difference equations. Particular attention is focused on the class of equations which admit product solutions so that their time-dependent equations are reducible to denumerably infinite systems of ordinary differential difference equations. An extended version of Lyapunov's stability theory for such equations is given and its application is illustrated by the study of a one-dimensional diffusion process with non-linear delayed action sources. (Authors' summary)

(24) P.K.C. Wang, "Asymptotic stability of a time-delayed diffusion system", J. of Applied Mechanics. Trans. of ASME, Series E, Vol. 30, No. 4, pp. 500-504, 1963.

This paper discusses the asymptotic stability of the equilibrium states of a nonlinear diffusion system with time delays. It is assumed that the system is describable by a partial differential-difference equation of the form:

$$\frac{\partial u(t, X)}{\partial t} = \mathcal{L}u(t, X) + f(t, X, u(t, X), u(t - T, X), \ldots,$$

$$\frac{\partial u(t, X)}{\partial x_i}, \ldots, \frac{\partial u(t - T, X)}{\partial x_i}, \ldots) \quad i = 1, \ldots, M$$

where $\mathcal{L}$ is a linear operator uniformly elliptic in X defined for all $X \in \Omega \ldots$ a bounded, open M-dimensional special domain; f is a specified function of its arguments. In the development of this paper, the physical origin of the foregoing equation is discussed briefly. Then, conditions for asymptotic stability of the trivial solution are derived via an extended Lyapunov's direct method. Specific results are given for a simple one-dimensional linear heat equation with time-delayed arguments.

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(25) P.K.C. Wang and M.L. Bandy, "An application of Lyapunov's direct method to distributed parameter control system design", Research Note No. NJ-29, IBM Research Lab., San Jose, Calif., Jan. 1963.

The authors extends an idea of Bass to the design of a class of distributed parameter systems describable by a system of non-linear, uniformly strongly parabolic partial differential equations. Two examples are given.

(26) R.K. Brayton and W.L. Miranker, "A stability theory for nonlinear mixed initial boundary value problems", Research Report No. RC 1021, IBM Research Center, Yorktown Heights, N. Y. 1963.

In this paper, a method for constructing Lyapunov functionals is devised. The method is based on a canonical form in which many nonlinear mixed initial boundary value problems may be written. Problems of this type are interpreted as generating dynamic systems in function space. This interpretation is combined with the Lyapunov functional to give a stabiloty theorm. Several examples of the method are included. (Authors' summary)

(27) P.K.C.Wang and M.L. Bandy, "On the stability of equilibrium of a diffusion system with feedback control", Research Note No. NJ-56, IBM Research Lab., San Jose, Calif., June, 1964.

The authors apply Nirenburg's maximum principle for parabolic partial differential equations to derive a sufficient condition for stability of a linear diffusion system with feedback control.

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This work was undertaken in order to gain a more fundamental understanding of the control of distributed parameter systems, with emphasis on the control of flexible aerodynamic vehicles. The main effort was on the development of a control theory for distributed parameter systems rather than the development of techniques for approximating distributed parameter systems by lumped parameter systems for which existing control theory is applicable.

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