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DEVELOPMENT OF A COMPUTER PROGRAM FOR THE ANALYSIS OF ONE-DIMENSIONAL MAGNETOHYDRODYNAMIC FLOW PROBLEMS

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NOTICES

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ABSTRACT

A general method has been developed for the analysis of one-dimensional magnetohydrodynamic channel flow problems and is presented in this report. The basic differential equations for one-dimensional flow (influence coefficient equations) are discussed, and it is shown how the magnetohydrodynamic body force and joule heating are included. Methods are also presented for including the effects of area change, external heat exchange and wall friction, and approximate methods for including real gas effects are mentioned.

The resulting set of equations has been programmed for solution on the IBM 7070 digital computer. A description of the computer program is given, and the complete Fortran listing of the main program and subroutines is presented in the appendices. The accuracy of the program was checked by duplicating existing analytic or numerical solutions and was found to be very good. Several typical calculations are presented as examples of the range of application of the method.

PUBLICATION REVIEW

This report has been reviewed and publication is approved.

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ILLUSTRATIONS

NOMENCLATURE

A	Area, m^2
A_w	Wall surface area, m^2
a	Speed of sound, m/sec
$\vec{B}$	Magnetic flux density vector of magnitude B, weber/m^2
c_p	Specific heat at constant pressure, joule/kg - °K
c_v	Specific heat at constant volume, joule/kg - °K
D	Mean hydraulic diameter, m
$\vec{E}$	Applied electric field intensity vector of magnitude E, volt/m
$\vec{E}'$	Net electric field intensity vector of magnitude E', volt/m
$\vec{F}$	Body force vector of magnitude F, nt
f	Friction coefficient
h	Specific enthalpy, joule/kg
$\vec{J}$	Electric current density vector of magnitude J, amp/m^2
K_1, K_2	Terms defined by Eqs. (28) and (29), volt/m and $volt^2/m^2$
K_3	Constant defined by Eq. (30), m/volt
L	Electromagnetic length interaction parameter
ℓ	Length, m
M	Mach number
$\dot{m}$	Mass flow rate, kg/sec
P	Power, watt
p	Pressure, nt/m^2
$\dot{Q}$	Rate of energy loss caused by heat transfer, watt
q	Net heat added per unit mass of gas, joule/kg
$\bar{R}$	Universal gas constant, joule/kg-mol °K
R	Gas constant = $\bar{R}/W$, joule/kg - °K
T	Absolute temperature, °K
$\vec{u}$	Velocity vector of magnitude u, m/sec
V	Volume, m^3
W	Molecular weight, 1/mol

X	Body force, nt
x	Distance (axial direction), m
$\bar{x}$	Friction-distance parameter, 4f x/D
Z	Compressibility factor
γ	Ratio of specific heats
η	Electromagnetic conversion efficiency
ρ	Density, kg/m^3
σ	Electrical conductivity, mho/m
τ_w	Wall shearing stress, nt/m^2

SUBSCRIPTS

eff	Effective
i	Initial
o	Total

SUPERSCRIPT

*	Conditions at M = 1.0

1.0 INTRODUCTION

A research and development program for a two-megawatt magnetohydrodynamic (MHD) accelerator is presently in progress at the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), to demonstrate the feasibility of the continuous MHD accelerator technique for low-density, hypervelocity wind tunnel application.

As a part of this project a computer program has been developed in order to be able to make parametric studies with respect to MHD channel geometry and operating conditions for the selection of the basic channel design parameters and power supply criteria. In this program, the quasi one-dimensional channel flow equations were written in the form of influence coefficient equations, which were developed by Shapiro (Ref. 1), and solved by iteration on the IBM 7070 digital computer.

One-dimensional flow methods have been studied previously by a number of authors. Shercliff (Ref. 2) has studied heating and friction effects on magnetohydrodynamic flows using a one-dimensional flow analysis. Liu (Ref. 3) has included the effects of friction for the case of an infinitely conducting plasma, and Gross (Ref. 4) has studied the effect of heat addition for various Mach number regimes. Mager and Baker (Ref. 5) and Baker and Rogers (Ref. 6) have applied the influence coefficient equations of Shapiro to the analysis of one-dimensional constant electromagnetic conversion efficiency MHD channel flows for ideal gases with constant specific heats. Baker and Rogers include the combined effects of friction and heat transfer by assuming that frictional losses are proportional to the MHD body force and that friction and heat-transfer effects are related through the Reynolds analogy. Additionally, quasi one-dimensional studies have been presented by Rosa (Ref. 7), Resler and Sears (Ref. 8), Kerrebrock and Marble (Ref. 9), and Oates (Ref. 10). In general, all of these analyses are restricted by the assumption of an ideal gas with constant specific heats and a gaseous discharge mechanism which follows the general form of Ohm's law. Also, simplifying constraints such as constant area, temperature, electric or magnetic fields, or electromagnetic conversion efficiency are usually assumed.

This report uses the basic influence coefficient equations of Shapiro which are applicable to the general analysis of one-dimensional internal

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flow problems, with the magnetohydrodynamic body force and joule heating included. Various calculations of the electric and magnetic field variations and the electrical conductivity are shown, and methods for including the effects of area change, friction, and heat transfer are given. In addition, approximate methods for including real gas effects are mentioned.

The influence coefficient equations are solved by iteration on the IBM 7070 digital computer. A discussion of the computer program is given, and several applications are noted. Complete Fortran listings of the main program and subroutines are given in the appendices.

2.0 THEORY

2.1 INFLUENCE COEFFICIENT EQUATIONS

A general method of analysis applicable to one-dimensional, internal compressible flow problems has been developed by Shapiro (Ref. 1) which permits consideration of the simultaneous effects of area change, wall friction, body forces, external heat exchange, chemical reactions, change of phase, mixing of injected gases, and changes in molecular weight and specific heat ratio. This method is based on the following assumptions:

1. The flow is one-dimensional and steady.
2. Changes in stream properties are continuous.
3. The gas is thermally perfect but not necessarily calorically perfect (semiperfect).

In addition to the above assumptions, the present analysis is also restricted by the assumption of constant mass flow rate. Thus, effects caused by mixing of injected gases will not be considered.

The results of Shapiro's analysis, based on the above restrictions, is the following set of simultaneous, non-linear differential equations:

$$\frac{dM^2}{M^2} = -\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2}\frac{dA}{A} + \frac{1+\gamma M^2}{1-M^2}\frac{dq}{c_p T} + \frac{\gamma M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2}\left(4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}\gamma p A M^2}\right) - \frac{1+\gamma M^2}{1-M^2}\frac{dW}{W} - \frac{d\gamma}{\gamma} \qquad (1)$$

$$\frac{du}{u} = -\frac{1}{1-M^2}\frac{dA}{A} + \frac{1}{1-M^2}\frac{dq}{c_p T} + \frac{\gamma M^2}{2(1-M^2)}\left(4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}\gamma p A M^2}\right) - \frac{1}{1-M^2}\frac{dW}{W} \tag{2}$$

$$\frac{da}{a} = \frac{(\gamma-1)M^2}{2(1-M^2)}\frac{dA}{A} + \frac{1-\gamma M^2}{2(1-M^2)}\frac{dq}{c_p T} - \frac{\gamma(\gamma-1)M^4}{4(1-M^2)}\left(4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}\gamma p A M^2}\right) + \frac{\gamma M^2-1}{2(1-M^2)}\frac{dW}{W} + \frac{1}{2}\frac{d\gamma}{\gamma} \tag{3}$$

$$\frac{dT}{T} = \frac{(\gamma-1)M^2}{1-M^2}\frac{dA}{A} + \frac{1-\gamma M^2}{1-M^2}\frac{dq}{T} - \frac{\gamma(\gamma-1)M^4}{2(1-M^2)}\left(4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}\gamma p A M^2}\right) + \frac{(\gamma-1)M^2}{1-M^2}\frac{dW}{W} \tag{4}$$

$$\frac{d\rho}{\rho} = \frac{M^2}{1-M^2}\frac{dA}{A} - \frac{1}{1-M^2}\frac{dq}{c_p T} - \frac{\gamma M^2}{2(1-M^2)}\left(4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}\gamma p A M^2}\right) + \frac{1}{1-M^2}\frac{dW}{W} \tag{5}$$

$$\frac{dp}{p} = \frac{\gamma M^2}{1-M^2}\frac{dA}{A} - \frac{\gamma M^2}{1-M^2}\frac{dq}{c_p T} - \frac{\gamma M^2[1+(\gamma-1)M^2]}{2(1-M^2)}\left(4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}\gamma p A M^2}\right) + \frac{\gamma M^2}{1-M^2}\frac{dW}{W} \tag{6}$$

These equations were obtained by expressing the equation of state, continuity equation, and definitions of Mach number and speed of sound in logarithmic differential form and combining with the one-dimensional steady flow energy and momentum equations. The term "influence coefficient" arises because the influence of each independent parameter $\left[dA/A,\ dq/c_pT,\ \left[(4f\,dx)/D + dX/(1/2\gamma pAM^2)\right]\ dW/W, \text{ and } d\gamma/\gamma\right]$ on the dependent variables (dM^2/M^2, du/u, da/a, dT/T, $d\rho/\rho$, and dp/p) is easily obtained by inspection of the coefficient of the independent parameter. Actually, since there are only five independent parameters in Eqs. (1) through (6), it is only necessary to solve any five of the equations.

One disadvantage in the use of the influence coefficient equations, however, is the existence of a singular point at $M = 1.0$. The existence of this singularity requires solutions to be carried out for either subsonic or supersonic flow, and it is not possible to solve continuously across $M = 1.0$ unless approximations are made for Eqs. (1) through (6). This procedure was not followed since, in general, magnetohydrodynamic accelerators are considered for operation at supersonic velocities only.

2.2 MAGNETOHYDRODYNAMIC EFFECTS

Previous analyses of magnetohydrodynamic flow problems by Petrie (Ref. 11), Cambel (Ref. 12), and others indicate that for small magnetic Reynolds numbers the induced magnetic field is negligible in comparison to the applied magnetic field. This allows the gasdynamic equations to be uncoupled from the electromagnetic (Maxwell's) equations. Thus the simultaneous effect of electric and magnetic fields on the flow of an electrically conducting gas can be accounted for by including the Lorentz body force and rate of energy addition in the basic momentum and energy equations. These effects are easily incorporated into the influence coefficient equations by including the body force term in the momentum parameter, $\dfrac{dX}{1/2\,\gamma pAM^2}$, and the MHD energy term in the energy parameter, $\dfrac{dq}{c_p\,T}$.

The Lorentz body force per unit volume and net rate of energy addition per unit volume added to an electrically conducting gas by crossed electric and magnetic fields are given by Resler and Sears (Ref. 8) as

$$\frac{\vec{F}}{V} = \vec{J} \times \vec{B} \tag{7}$$

$$\frac{P}{V} = \vec{J} \cdot \vec{E} = \vec{u} \cdot \vec{J} \times \vec{B} + \vec{J} \cdot \vec{E}' \tag{8}$$

where the power per unit volume $\vec{J} \cdot \vec{E}$ is composed of the rate at which work is done by the body force, $\vec{u} \cdot \vec{J} \times \vec{B}$, and joule heating, $\vec{J} \cdot \vec{E}'$. The net electric field, $\vec{E}'$, is given as the sum of the applied and induced fields

$$\vec{E}' = \vec{E} + \vec{u} \times \vec{B} \tag{9}$$

Equations (7) and (8) can be simplified considerably by the introduction of Ohm's law

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \tag{10}$$

where Hall and ion slip effects have been neglected. If in addition it is assumed that the electric and magnetic fields are oriented mutually perpendicular to the velocity vector (see Fig. 1), then Eq. (10) can be reduced to

$$J = \sigma(E - uB) \tag{11}$$

and Eqs. (7) and (8) become

$$\frac{\vec{F}}{V} = \sigma(E - uB)B \tag{12}$$

$$\frac{P}{V} = \sigma(E - uB)E \tag{13}$$

As written in the present form, Eqs. (10) and (11) appear to be valid only if Hall effects are completely neglected. Actually, though, the expressions are still correct if Hall effects are considered, provided that axial currents are suppressed from flowing by the use of segmented electrodes and insulated B-walls.

The Lorentz body force is included into the influence coefficient equations as

$$dX = -(\vec{J} \times \vec{B})Adx = -\sigma(E - uB)B\,Adx \tag{14}$$

The negative sign is necessary since dX was originally defined as an internal drag force in the derivation of the influence coefficient equations. The energy added to the gas per unit mass is given by

$$dq = \vec{J} \cdot \vec{E}\,\frac{dx}{\rho u} = \sigma(E - uB)E\,\frac{dx}{\rho u} \tag{15}$$

Equations (14) and (15) can be expressed in a more convenient form by introducing the electromagnetic conversion efficiency and length interaction parameter, which are defined by the following expressions:

$$\frac{\text{rate at which work is done by body force per unit volume}}{\text{total power added per unit volume}} = \frac{\vec{u} \cdot \vec{J} \times \vec{B}}{\vec{J} \cdot \vec{E}} = \frac{uB}{E} = \eta \tag{16}$$

$$\frac{\text{electromagnetic force per unit volume}}{\text{dynamic force per unit volume}} = \frac{|\vec{J} \times \vec{B}|}{\rho u^2}\, dx \approx \frac{\sigma u B^2}{\rho u^2}\, dx = \frac{\sigma B^2}{\rho u}\, dx = dL \quad (17)$$

where it is assumed that the current density J is on the order of magnitude σ uB.

After substituting Eqs. (16) and (17) into Eqs. (14) and (15) and nondimensionalizing, the magnetohydrodynamic momentum and energy parameters are obtained in the following form:

$$\frac{dX}{\frac{1}{2}\gamma p A M^2} = \frac{2(\eta - 1)}{\eta}\, dL \quad (18)$$

$$\frac{dq}{c_p T} = (\gamma - 1) M^2 \frac{(1 - \eta)}{\eta^2}\, dL \quad (19)$$

An alternate procedure to the one described above is proposed that permits the use of experimental data and removes the necessity of assuming Ohm's law. In this procedure, Eqs. (7) and (8) are modified directly to a form suitable for inclusion into the influence coefficient equations. The result is

$$\frac{dX}{\frac{1}{2}\gamma p A M^2} = - J(x)\, B(x) \frac{dx}{\frac{1}{2}\gamma p M^2} \quad (20)$$

$$\frac{dq}{c_p T} = J(x)\, E(x) \frac{dx}{\rho u c_p T} \quad (21)$$

where J (x), B (x), and E (x) are determined from experimental measurements.

2.3 CALCULATION OF ELECTRIC AND MAGNETIC FIELDS AND ELECTRICAL CONDUCTIVITY

Variation of the electric field strength with axial position or arbitrary variations of the electric field necessitated by restrictions such as constant η, constant current density, constant area and temperature, or maximum Mach number can be included in the calculations. General expressions for the required variation of E for the above cases are in the literature and are summarized below:

1. Constant η (Refs. 5 and 6)

$$E = \frac{uB}{\eta} \quad (22)$$

2. Constant current density

$$E = \frac{J}{\sigma} + uB \quad (23)$$

3. Constant area and temperature (Ref. 7)

$$E = uB \frac{\gamma M^2}{\gamma M^2 - 1} \tag{24}$$

4. Maximum Mach number (Ref. 13)

$$E = \frac{uB}{2}\left(1 + \frac{\gamma}{\gamma - 1}\right)\left[\frac{2 + (\gamma - 1)M^2}{1 + \gamma M^2}\right] \tag{25}$$

It should be noted that Eqs. (24) and (25) are based on the assumption of an ideal gas with no friction or heat-transfer effects included. Equation (23) is obtained simply by solving Eq. (11) for E.

Other variations of the electric field might be included. For example, an expression for the required electric field variation to maintain constant temperature when heat-transfer losses are considered was obtained by setting dT/T equal to zero in Eq. (4) and solving for E. Thus, if the area, specific heat ratio, and molecular weight are assumed constant, friction is neglected, and it is assumed that heat-transfer rate losses can be represented by an arbitrary variation as a function of axial position, the following expression is obtained for E:

$$E = uB \frac{\gamma M^2}{\gamma M^2 - 1} + \frac{\dot{Q}/\ell}{A\sigma(E - uB)} \tag{26}$$

where $\dot{Q}/\ell$ is the rate of energy loss per unit length caused by heat transfer. This expression is similar to the one obtained by Rosa (Ref. 7) for the constant temperature, constant area case, except for the addition of the second term. Solving Eq. (26) explicitly for E results in the following expression:

$$E = -\frac{K_1}{2} + \frac{\sqrt{K_1^2 - 4K_2}}{2} \tag{27}$$

where K_1 and K_2 are defined by

$$K_1 = -uB\left(1 + \frac{\gamma M^2}{\gamma M^2 - 1}\right) \tag{28}$$

$$K_2 = (uB)^2 \frac{\gamma M^2}{\gamma M^2 - 1} - \frac{\dot{Q}/\ell}{A\sigma} \tag{29}$$

Similarly, the magnetic field can be varied as a function of axial position, or by suitable restrictions, variations similar to those obtained for the electric field strength could be obtained.

The electrical conductivity is usually assumed to be a scalar constant; however, it is possible to include more appropriate expressions if desired. For example, based on a limited amount of experimental data obtained at AEDC, it appears that the conductivity is strongly influenced by the current density of the discharge. Moreover, σ can be represented approximately as a linear function of J as follows:

$$\sigma = \sigma_i + K J \tag{30}$$

where σ_i is the conductivity of the gas with no applied electric field. In order to use an expression of this type, it would be necessary to determine both σ_i and K experimentally.

2.4 EFFECTS OF AREA CHANGE, FRICTION, AND HEAT TRANSFER

Area effects are included in the equations by calculating dA/A and multiplying by the appropriate influence coefficient. Therefore, arbitrary variations of area with axial position can be incorporated in the calculations; or the required area variations necessary to maintain constant temperature, pressure, density, velocity, or Mach number can be calculated. For example, by setting dT/T equal to zero in Eq. (4) and solving for dA/A, the following expression is obtained:

$$\frac{dA}{A} = -\left(\frac{1-\gamma M^2}{(\gamma-1)M^2}\right)\frac{dq}{c_p T} + \frac{\gamma}{2} M^2 \left(\frac{4f\,dx}{D} + \frac{dX}{\frac{\gamma}{2}\,pAM^2}\right) - \frac{dW}{W} \tag{31}$$

Similar expressions can be obtained by setting dM^2/M^2, du/u, $d\rho/\rho$, and dp/p equal to zero and solving for dA/A.

Friction effects are included by calculating the friction parameter, 4fdx/D, where the friction coefficient, f, and the hydraulic diameter, D, are defined by

$$f = \frac{\tau_w}{\frac{1}{2}\rho u^2} \tag{32}$$

$$D = \frac{4A}{dA_w/dx} \tag{33}$$

Heat-transfer effects are presently included by modifying the energy addition parameter dq/c_pT to include the net effect of joule heating minus the rate of heat loss from the gas caused by heat transfer. If it is assumed that arbitrary variations of the heat-transfer rate loss can be determined from heat-transfer calculations, then the net energy addition parameter is given as

$$\frac{dq}{c_p T} = (\gamma - 1)M^2\left(\frac{1-\eta}{\eta^2}\right)dL - \frac{\dot{Q}/\ell}{\dot{m}}\,\frac{dx}{c_p T} \tag{34}$$

where $\dot{Q}/\ell$ is the power loss per unit length caused by heat transfer.

2.5 APPROXIMATE METHODS FOR INCLUDING REAL GAS EFFECTS

The original derivation of the influence coefficient equations is based on the assumption of a thermally perfect (or semiperfect) gas. This permits variation in the specific heats to be considered provided that the thermodynamic relation

$$c_p - c_v = R \tag{35}$$

is satisfaied and the specific heats are functions of T only. For dissociated or ionized gases, however, according to Hilsenrath (Ref. 14), Eq. (35) should be replaced by

$$c_p - c_v = R\left\{1 + \frac{Z\left[1 + \frac{1}{Z}\left(\frac{\partial Z}{\partial \ln T}\right)_\rho\right]^2}{1 + \frac{1}{Z}\left(\frac{\partial Z}{\partial \ln \rho}\right)_T}\right\} \tag{36}$$

Thus to correctly account for equilibrium dissociation or ionization effects, it would be necessary to rederive the influence coefficient equations based on the relation between specific heats given by Eq. (36) and include the variations of specific heat ratio, molecular weight, and the dissociation or ionization reaction energy.

For gases which are only slightly dissociated, one possible procedure is to neglect the change in the dissociation reaction energy in comparison to the enthalpy change caused by area, heat-transfer, or MHD effects. This permits the simpler thermodynamic relation given by Eq. (35) to be used. Thus values of the molecular weight and specific heat ratio can be obtained from tables of Z and γ as functions of pressure and temperature (Ref. 15). The value of c_p can then be calculated from the relation

$$c_p = \frac{\gamma \bar{R}}{(\gamma - 1) W} \tag{37}$$

which is easily derived from Eq. (35). This procedure yields reasonably accurate results for nitrogen at pressures above 0.1 atm for temperatures up to about 5000°K. With air, however, the temperature range for which reasonable results are obtainable is limited to about 3500°K for pressures above 0.1 atm because of the early dissociation of oxygen.

Another approach which is useful, especially when the thermodynamics of the accelerator are specified, is obtained by defining an effective value of c_p and γ given by

$$c_{p_{eff}} \equiv \frac{\gamma_{eff} \bar{R}}{(\gamma_{eff} - 1) W} \equiv \frac{\frac{dh}{dx}}{\frac{dT}{dx}} \tag{38}$$

It should be emphasized that the effective values defined by Eq. (38) will not, in general, be equal to the ordinary values associated with c_p and γ. If these relations are used in the basic derivation and the speed of sound and Mach number are expressed in terms of γ_{eff} instead of γ, then the influence coefficient equations can be used for real gas solutions simply by replacing γ and c_p with γ_{eff} and $c_{p_{eff}}$. This procedure has been used, for example, to calculate accelerator channel flows with constant dissociation. For these solutions, the value of dh/dT was obtained from the Mollier diagram by following a path of constant Z.

3.0 DISCUSSION OF THE COMPUTER PROGRAM

3.1 METHOD OF SOLUTION

Equations (1) through (5) have been programmed for solution on the IBM 7070 digital computer. A discussion of the program is given below, and the complete Fortran listing of the program is presented in Appendix III. A list of symbols for the Fortran listing is given in Appendix I and a sample input card format in Appendix II.

In finite difference form, Eqs. (1) through (5) become

$$\frac{\Delta M^2}{\bar{M}^2} = \frac{1}{1-\bar{M}^2}\left[-2\left(1+\frac{\bar{\gamma}-1}{2}\bar{M}^2\right)\frac{\Delta A}{\bar{A}} + \left(1+\bar{\gamma}\bar{M}^2\right)\frac{\Delta q}{\bar{c}_p\bar{T}} + \bar{\gamma}\bar{M}^2\left(1+\frac{\bar{\gamma}-1}{2}\bar{M}^2\right)\left(\frac{4\bar{f}\Delta x}{\bar{D}} + \frac{\Delta X}{\frac{1}{2}\bar{\gamma}\bar{p}\bar{A}\bar{M}^2}\right) - \left(1+\bar{\gamma}\bar{M}^2\right)\frac{\Delta W}{\bar{W}} - \left(1-\bar{M}^2\right)\frac{\Delta\gamma}{\bar{\gamma}}\right] \tag{39}$$

$$\frac{\Delta u}{\bar{u}} = \frac{1}{1-\bar{M}^2}\left[-\frac{\Delta A}{\bar{A}} + \left(\frac{\Delta q}{\bar{c}_p\bar{T}}\right) + \left(\frac{\bar{\gamma}\bar{M}^2}{2}\right)\left(\frac{4\bar{f}\Delta x}{\bar{D}} + \frac{\Delta X}{\frac{1}{2}\bar{\gamma}\bar{p}\bar{A}\bar{M}^2}\right) - \frac{\Delta W}{\bar{W}}\right] \tag{40}$$

$$\frac{\Delta a}{\bar{a}} = \frac{1}{1-\bar{M}^2}\left[\left(\frac{\bar{\gamma}-1}{2}\right)\bar{M}^2\frac{\Delta A}{\bar{A}} + \left(\frac{1-\bar{\gamma}\bar{M}^2}{2}\right)\left(\frac{\Delta q}{\bar{c}_p\bar{T}}\right) - \left(\frac{\bar{\gamma}(\bar{\gamma}-1)\bar{M}^2\ \bar{M}^2}{4}\right)\left(\frac{4\bar{f}\Delta x}{\bar{D}} + \frac{\Delta X}{\frac{1}{2}\bar{\gamma}\bar{p}\bar{A}\bar{M}^2}\right) + \left(\frac{\bar{\gamma}\bar{M}^2-1}{2}\right)\frac{\Delta W}{\bar{W}} + \left(\frac{1-\bar{M}^2}{2}\right)\frac{\Delta\gamma}{\bar{\gamma}}\right] \tag{41}$$

$$\frac{\Delta T}{\bar{T}} = \frac{1}{1-\bar{M}^2}\left[(\bar{\gamma}-1)\,\bar{M}^2\,\frac{\Delta A}{\bar{A}} + (1-\bar{\gamma}\,\bar{M}^2)\left(\frac{\Delta q}{\bar{c}_p\,\bar{T}}\right) - \left(\frac{\bar{\gamma}(\bar{\gamma}-1)\,\bar{M}^2\;\bar{M}^2}{2}\right)\right.$$
$$\left.\left(\frac{4\,\bar{f}\,\Delta x}{\bar{D}} + \frac{\Delta X}{\frac{1}{2}\,\bar{\gamma}\,\bar{p}\,\bar{A}\,\bar{M}^2}\right) + (\bar{\gamma}-1)\,\bar{M}^2\,\frac{\Delta W}{\bar{W}}\right] \tag{42}$$

$$\frac{\Delta\rho}{\bar{\rho}} = \frac{1}{1-\bar{M}^2}\left[\bar{M}^2\,\frac{\Delta A}{\bar{A}} - \left(\frac{\Delta q}{\bar{c}_p\,\bar{T}}\right) - \left(\frac{\bar{\gamma}}{2}\,\bar{M}^2\right)\left(\frac{4\,\bar{f}\,\Delta x}{\bar{D}} + \frac{\Delta X}{\frac{1}{2}\,\bar{\gamma}\,\bar{p}\,\bar{A}\,\bar{M}^2}\right) + \frac{\Delta W}{\bar{W}}\right] \tag{43}$$

where Δ denotes the difference of any given variable across an increment and the bar denotes the average value of the variable on the increment (that is, dM^2/M^2 has been replaced by the approximation $\Delta M^2/\bar{M}^2$). An iterative solution is used to solve Eqs. (39) through (43), which is summarized in the following steps:

1. Initial values of each parameter (M_i, u_i, A_i, etc.) are calculated at the beginning of a specific increment (from inputs originally).
2. The difference or change of each of the dependent variables is estimated (initially at zero) and average values are calculated.
3. Based on the calculated average values of the dependent variables, the necessary influence coefficients are calculated and subroutines are called in which values of the independent parameters are calculated from the various expressions discussed in Sections 2. 2 through 2. 5.
4. Equations (39) through (43) are solved and final values of the dependent variables are calculated from linear approximations of the form

$$u = u_i + \Delta u \tag{44}$$

5. Convergence of the iteration is checked by comparing the estimated and calculated values of the change in the dependent variables. If these do not compare within the prescribed limit (0. 005 percent), the estimated values are reset equal to the calculated values and the iteration is repeated until the solution converges.
6. After convergence of the solution is obtained, the calculated final values of one increment are used as inputs for the next increment and the solution is continued.

Since an iterative finite difference solution of Eqs. (39) through (43) is used, both the accuracy and computation time are necessarily dependent on the increment size. Therefore, a procedure has been incorporated which varies the increment size in proportion with the change of the dependent variables over the increment. By making several check solutions at different increment sizes and comparing the results with analytic solutions, it was found that the analytic solution could be duplicated to within 0.1 percent by allowing a maximum variation of 10 percent in the dependent variables across any given increment. A lower bound of 0.1 percent change per increment step was added to increase computation speed in case the initial increment size is chosen too small. A calculation of the change of the dependent variables over the increment is made immediately after convergence of the solution is checked, and the increment size is increased or decreased if necessary. This procedure has been found to work very well for problems where a sudden change in the rate of change of a given variable with respect to position is likely to occur.

Although an iterative solution is used in the present program, it should be possible to use another technique such as the Runge-Kutta method to increase computation speed.

3.2 SUBROUTINES

The IBM 7070 Full Fortran is utilized so that individual subroutines may be used to calculate all of the independent variables in Eqs. (39) through (43). Eight basic subroutines are included which cover the variation of area, magnetic flux density, electric field strength, energy addition, specific heats, momentum addition, electrical conductivity, and molecular weight. For each of the eight basic subroutines, a set of general constants (inputs) is included so that as new individual subroutines are added they can be included in the program without making any changes in the main program. Each individual subroutine is assigned a code number, and a function has been included in the main program which searches the program tape for the desired subroutine. The entire program is on magnetic tape and includes the full Fortran package, main program, and all of the subroutines. This program could be modified if necessary so that the subroutines are read in on punched cards; however, this procedure becomes very unhandy when it is desired to run several consecutive solutions using different subroutines.

By using subroutines for the independent variables it is possible to use the basic main program for the solution of a wide number of one-dimensional flow problems since the necessary variations required for individual problems are accounted for in subroutines. This also permits the program to be extended to more complicated problems simply by adding a new subroutine. A Fortran listing of all of the subroutines which have been written to date is included in Appendix IV.

3.3 ACCURACY

An estimate of the computer program accuracy was obtained by duplicating the calculations for various internal flow problems for which existing analytic or numerical solutions are available. These solutions include:

1. Isentropic flow
2. Rayleigh flow
3. Fanno flow
4. Constant area flow with combined heating and friction
5. Constant η MHD channel flow
6. Isothermal, constant area MHD channel flow
7. Maximum Mach number MHD channel flow
8. MHD channel flow with constant E, B, A, and σ
9. Equilibrium real gas isentropic nozzle flow for nitrogen

Comparison plots of the existing analytic or numerical solutions and the computer solutions are shown in Figs. 2a through k.

Duplication of the simple isentropic, Rayleigh, Fanno, and combined heating and friction flows (Refs. 16 and 17) resulted in excellent agreement; in general, the maximum deviation was less than 0.1 percent (see Figs. 2a to e). Also, excellent agreement is obtained with the calculation of Baker and Rogers (Ref. 6) for constant η MHD channel flow (see Fig. 2f). The isothermal, constant area MHD channel flow solution of Rosa (Ref. 7) is compared in Fig. 2g. The curves were calculated from the following expressions given by Rosa:

$$\frac{1}{\eta} = \frac{E}{uB} = \frac{\gamma M^2 - 1}{\gamma M^2} \tag{45}$$

$$\frac{\sigma B^2}{\rho u} = \frac{\gamma}{2} M^2 - 2 \ln M - \frac{1}{2\gamma M^2} + \text{Constant} \tag{46}$$

Calculations using the expressions of Eckert and Weirick (Ref. 13) for maximum Mach number MHD flow and Oates (Ref. 10) for constant E, B, A, and σ MHD flow have been duplicated within 2 to 3 percent (see Figs. 2h and i).

The accuracy of the real gas procedure was checked by making a graphical real gas isentropic nozzle flow solution on a nitrogen Mollier diagram (Ref. 18) and then duplicating the solution with the computer. From the results presented in Fig. 2j, it is seen that very good agreement is obtained with the two methods; however, it should be noted that the solution was started from a stagnation temperature and pressure of 4000°K and 10 atm. For a similar solution with a stagnation temperature and pressure of 6500°K and 20 atm, the static temperature was underestimated by about 20 percent and the static pressure by about 60 percent at high velocities (see Fig. 2k). This is caused by errors involved in using Eq. (37) and neglecting the dissociation energy.

4.0 APPLICATIONS

As an example of the range of application of the present program, several typical calculations have been performed and the results are shown in Figs. 3 and 4. An example of the real gas effects on a simple MHD channel flow problem is shown in Fig. 3, where a constant E, B, A, and σ channel flow solution is compared both for real and perfect gases. It is seen that lower values of Mach number and temperature are predicted for the real gas solution.

An example of the heat-transfer effects on an isothermal constant area MHD channel flow problem is shown in Fig. 4. This solution was accomplished by using the electric field variation predicted by Eq. (27). An arbitrary heat-transfer rate loss of 5 kw/cm was assumed. It is noted from Fig. 4 that the solution with heat-transfer losses included results in higher velocity ratios and Mach numbers than the solution without heat transfer. This is explained by the fact that the solution with heat-transfer losses included requires more ohmic heating and therefore a larger electric field strength to maintain constant temperature. The resulting increase in current density produces a larger $\vec{J} \times \vec{B}$ accelerating force.

5.0 CONCLUDING REMARKS

A general method of analysis has been developed which is applicable to a wide range of one-dimensional magnetohydrodynamic and ordinary gasdynamic channel flow problems. Although attention has been placed primarily on developing a program applicable to MHD accelerators, with suitable modifications it is also possible to use the present program to obtain MHD generator solutions.

The computer program has been maintained flexible by dividing it into a basic main program in which the dependent variables are calculated and a set of subroutines in which all of the independent variables are calculated. With this technique, the program can be used for many different types of internal flow problems by choosing the proper set of subroutines. The accuracy of the numerical solution technique was checked by duplicating the calculations for a number of existing analytic or numerical solutions. The results of these comparison calculations indicate that the program is sufficiently accurate for the intended applications.

The program in its present form is suitable for conducting parametric studies of various accelerator configurations, examining the effects of friction, heat transfer, and variations in specific heat ratio and molecular weight on accelerator performance, or comparing theoretical computer solutions with experimental data.

REFERENCES

1. Shapiro, A. H. The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1. The Ronald Press Company, New York, 1953.
2. Shercliff, J. A. "Some Generalizations in One-Dimensional Magnetogasdynamics." Review of Modern Physics, Vol. 32, No. 4, April 1960, p. 980.
3. Liul, C. S. "On One-Dimensional Magnetogasdynamics." Ph. D. Thesis, Northwestern University, August 1961.
4. Gross, Robert A. "A Note on One-Dimensional Plasma Motion." Journal of the Aero/Space Sciences, Vol. 25, No. 12, December 1958, p. 788.
5. Mager, A. and Baker, J. "One-Dimensional Magneto-Aerodynamics." Marquardt Report No. MR 20,002, October 1958.
6. Baker, J. and Rogers, I. "One-Dimensional Magnetogasdynamic Channel Flow with Constant Conversion Efficiency." Marquardt Report No. MR 20,146, September 1961.
7. Rosa, Richard J. "Engineering Magneto-Hydrodynamics." Part 2 of Ph. D. Thesis, Cornell University, June 1956.
8. Resler, E. L., Jr., and Sears, W. R. "The Prospects for Magneto-Aerodynamics." Journal of the Aeronautical Sciences, Vol. 25, No. 4, April 1958, pp. 235-245.

9. Kerrebrock, Jack L. and Marble, Frank E. "Constant Temperature Magneto-Gasdynamic Channel Flow." Journal of the Aero/Space Sciences, Vol. 27, No. 1, January 1960, p. 78.

10. Oates, Gorden C. "Constant Electric Field and Constant Magnetic Field Magnetogasdynamic Channel Flow." Journal of the Aero/Space Sciences, Vol. 29, No. 2, February 1962, pp. 231-232.

11. Petrie, S. L. "The Basic Principles of Magnetogasdynamics." ARL-TN-60-121, March 1960.

12. Cambel, A. B. Plasma Physics and Magnetofluidmechanics. McGraw-Hill Book Company, Inc., New York, 1963.

13. Eckert, R. C. and Weirick, R. G. "Investigation of a Maximum Mach Number Plasma Acceleration Channel." ASD-TDR-61-224, May 1961.

14. Hilsenrath, Joseph, Klein, Max, and Woolley, Harold W. "Tables of Thermodynamic Properties of Air Including Dissociation and Ionization from 1,500°K to 15,000°K." AEDC-TR-59-20, December 1959.

15. Ahtye, Warren F. and Peng, T. C. "Approximations for the Thermodynamic and Transport Properties of High-Temperature Nitrogen with Shock-Tube Applications." NASA TN D-1303, August 1960.

16. Keenan, Joseph H. and Kaye, Joseph. Gas Tables. John Wiley and Sons, Inc., New York, 1960.

17. Noyes, R. N. "An Exact Solution of the Compressible Flow Characteristics in Constant-Area Passages with Simultaneous Friction and Constant Heat Flux." Journal of Heat Transfer, Trans. ASME, Vol. 83, No. 4, November 1961, pp. 454-456.

18. Humphrey, R. L., Little, W. J., and Seeley, L. A. "Mollier Diagram for Nitrogen." AEDC-TN-60-83, May 1960.

APPENDIX I: NOMENCLATURE FOR FORTRAN LISTINGS

A	Area or initial value of a quantity
AJ	Current density (Printout sheet only)
B	Magnetic flux density or average value of a quantity
C	Speed of sound
CP	Specific heat at constant pressure
CA(I)	Area subroutine constants
CB(I)	Magnetic field subroutine constants
CE(I)	Energy subroutine constants
CEF(I)	Electric field subroutine constants
CG(I)	Specific heat subroutine constants
CM(I)	Momentum subroutine constants
CW(I)	Molecular weight subroutine constants
CC(I)	General constants
E	Energy addition parameter
EF	Electric field strength
ETA	Electromagnetic conversion efficiency
GAMMA	Specific heat ratio
HUNTF	Function used to locate required subroutine
L	Electromagnetic length interaction parameter (Printout sheet only)
M	Mach number (Printout sheet only)
NACODE	Area subroutine code
NBCODE	Magnetic field subroutine code
NECODE	Energy subroutine code
NEFCODE	Electric field subroutine code
NGCODE	Specific heat subroutine code
NMCODE	Momentum subroutine code
NSCODE	Conductivity subroutine code
NWCODE	Molecular weight subroutine code

NREAD	Instruction to read real gas tables (NREAD $\neq$ 0)
NPRINT	Number of increments calculated between printouts
NTBLR	Number of rows in real gas tables
NTBLC	Number of columns in real gas tables
P	Pressure
PRESS(I)	Pressure values in real gas tables
RHO	Density
SIGMA	Electrical conductivity
T	Temperature
TEMP(I)	Temperature values in real gas tables
TBLX	Compressibility factor table
TBLY	Specific heat ratio table
U	Velocity
UM	Momentum addition parameter
W	Molecular weight
X	Axial distance
XL	Magnetic length interaction parameter
XM	Mach number
XT	Total length
Z	Compressibility factor

APPENDIX II: TYPICAL INPUT CARD FORMAT

JOB TITLE: Magnetohydrodynamic Accelerator Characteristics
PROJECT NO: PL2287

```
       1 3 5 7 9 11  15  19  23  27  31  35  39  43  47  51  55  59  63  67  71  75  79
        2 4 6 8 10  14  18  22  26  30  34  38  42  46  50  54  58  62  66  70  74  78 80
      1 CONST. E,B,A,SIGMA MHD FLOW - NITROGEN - REAL GAS
       4.00  -011.50   +00  .        4.00  +03 1.013 +05  .        1.00  -02
          XT        XM        U         T         P        RHO        A

ADELX  5.00  -03
CA     1.00  -02  .         .         .         .         .         .

CB     2.00  +00  .         .         .         .         .         .
CEF    1.00  +04  .         .         .         .         .         .

CE       .         .         .         .         .         .         .
CG       .         .         .         .         .         .         .
       1 3 5 7 9 11  15  19  23  27  31  35  39  43  47  51  55  59  63  67  71  75  79
        2 4 6 8 10  14  18  22  26  30  34  38  42  46  50  54  58  62  66  70  74  78 80
CM       .         .         .         .         .         .         .
CS     3.00  +02  .         .         .         .         .         .

CS       .         .         .         .         .         .         .
CW     2.80  +01  .         .         .         .         .         .

CC       .         .         .         .         .         .         .
            1   101   205   301   402   501   602   702     1    25    22    13
       NACODE NBCODE NEFCOD NECODE NGCODE NMCODE NSCODE NWCODE NREAD NPRINT NTBLR NTBLC

       1 3 5 7 9 11  15  19  23  27  31  35  39  43  47  51  55  59  63  67  71  75  79
        2 4 6 8 10  14  18  22  26  30  34  38  42  46  50  54  58  62  66  70  74  78 80
```

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APPENDIX III: FORTRAN LISTINGS OF MAIN PROGRAM

```
C        MAGNETOHYDRODYNAMIC ACCELERATOR CHARACTERISTICS PROGRAM
     1   DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
       1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
     2   COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
       1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
       2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
     3   TYPE 5
     4   WRITE OUTPUT TAPE 24,5
     5   FORMAT(17H PROGRAM IS 25011)
     7   READ 8,XT,XM,U,T,P,RHO,A,ADELX,(CA(I),I=1,7),(CB(I),I=1,7),(CEF
       1 (I),I=1,7),(CE(I),I=1,7),(CG(I),I=1,7),(CM(I),I=1,7),(CS(I),I=1,14
       2 ),(CW(I),I=1,7),(CC(I),I=1,7),NACODE,NBCODE,NEFCOD,NECODE,NGCODE,N
       3 MCODE,NSCODE,NWCODE,NREAD,NPRINT,NTBLR,NTBLC
     8   FORMAT(72H1
       1                 /(1P7E10.4/1P1E10.4/10(1P7E10.4/),12I6)
         DUMMY=HUNTF(NWCODE,NGCODE,NBCODE,NEFCOD,NSCODE,NECODE,NMCODE,NACOD
       1 E)
         NPAGE=1
     9   NPRNT=NPRINT-1
    10   IF(NREAD)11,13,11
    11   READ 12,(TEMP(I),I=1,NTBLR),(PRESS(I),I=1,NTBLC),((TBLX(I,J),I=1,
       1 NTBLR),J=1,NTBLC),((TBLY(I,J),I=1,NTBLR),J=1,NTBLC)
    12   FORMAT(1P7E10.4)
    13   WRITE OUTPUT TAPE 24,8
    14   WRITE OUTPUT TAPE 24,15,NACODE,(CA(I),I=1,7),NBCODE,(CB(I),I=1,7)
       1 ,NEFCOD,(CEF(I),I=1,7),NECODE,(CE(I),I=1,7),NGCODE,(CG(I),I=1,7)
    15   FORMAT(28H0 AREA SUBROUTINE           I6//80H     CA1         CA2
       1         CA3         CA4         CA5         CA6         CA7/1P7E12
       2 .4//28H0 MAGNETIC FIELD SUBROUTINE I6//80H     CB1         CB2
       3      CB3         CB4         CB5         CB6         CB7/1P7E12.4/
       4 /28H0 ELECTRIC FIELD SUBROUTINE I6//80H    CEF1        CEF2
       5  CEF3        CEF4        CEF5        CEF6        CEF7/1P7E12.4//28
       6 H0 ENERGY SUBROUTINE         I6//80H      CE1         CE2
       7 CE3         CE4         CE5         CE6         CE7/1P7E12.4//28H0
       8  GAMMA SUBROUTINE          I6//80H      CG1         CG2         CG
       9 3         CG4         CG5         CG6         CG7/1P7E12.4//)
    16   WRITE OUTPUT TAPE 24,17,NMCODE,(CM(I),I=1,7),NSCODE,(CS(I),I=1,14
       1 ),NWCODE,(CW(I),I=1,7)
    17   FORMAT(28H0 MOMENTUM SUBROUTINE       I6//80H     CM1         CM2
       1         CM3         CM4         CM5         CM6         CM7/1P7E12
```

```
   2 .4//28H0 SIGMA SUBROUTINE          I6//80H     CS1         CS2
   0      CS3         CS4         CS5         CS6         CS7/1P7E12.4/
   4 /80H     CS8         CS9        CS10        CS11        CS12
   6   CS13        CS14/1P7E12.4//30H0 MOLECULAR WEIGHT SUBROUTINE I4//
   7 80H     CW1         CW2         CW3         CW4         CW5
   8   CW6         CW7/1P7E12.4//)
     WRITE OUTPUT TAPE 24,18,(CC(I),I=1,7)
18   FORMAT(19H0 GENERAL CONSTANTS//80H     CC1         CC2         CC3
   1          CC4         CC5         CC6         CC7/1P7E12.4)
19   IF(NREAD)20,27,20
20   WRITE OUTPUT TAPE 24,21,(TEMP(I),I=1,NTBLR)
21   FORMAT(14H1 TEMPERATURES//(1P12E10.4)////)
22   WRITE OUTPUT TAPE 24,23,(PRESS(I),I=1,NTBLC)
23   FORMAT(11H0 PRESSURES//(1P12E10.4))
24   WRITE OUTPUT TAPE 24,25,((TBLX(I,J),I=1,NTBLR),J=1,NTBLC)
25   FORMAT(9H1 TABLE X//(1P12E10.4))
     WRITE OUTPUT TAPE 24,26,((TBLY(I,J),I=1,NTBLR),J=1,NTBLC)
26   FORMAT(9H1 TABLE Y//(1P12E10.4))
27   WRITE OUTPUT TAPE 24,28
28   FORMAT(1H1)
     BT=T
     BP=P
     CALL WEIGE
     CALL GAMME
29   C=SQRTF(GAMMA*8317.0*T/W)
30   IF(XM)32,31,32
31   XM=U/C
32   IF(U)34,33,34
33   U=C*XM
34   IF(RHO)36,35,36
35   RHO=P/(8317.0*T/W)
36   IF(P)38,37,38
37   P=RHO*T*8317.0/W
38   X=0.0
39   XL=0.0
41   DELX=ADELX
46   BA=A
47   BC=C
48   BXM=XM
     BMSQ=BXM*BXM
49   BRHO=RHO
```

```
50   BU=U
51   BX=X
52   BXL=XL
53   CALL BMAGE
54   CALL EFLDE
55   CALL SIGME
56   RATIO=1.0+((GAMMA-1.0)/2.0)*XM*XM
57   EE=GAMMA/(GAMMA-1.0)
58   PT=P*(RATIO)**EE
     SPT=PT
59   TT=T*RATIO
     STT=TT
60   PR=1.0
61   TR=1.0
62   ETA=B*BU/EF
63   AJ=SIGMA*(EF-BU*B)
64   WRITE OUTPUT TAPE 24,65,X,XL,XM,C,U,T,TT,P,PT,RHO,A,B,EF,SIGMA,W,G
   1 AMMA,ETA,PR,TR,AJ
65   FORMAT(116H0      X           L           M           C           U
   1            T         TT            P          PT          RHO/1P10
   2 E12.4/116H      A            B           E         SIGMA          W
   3        GAMMA       ETA           PR          TR           AJ/1P10E
   4 12.4)
66   MAXITR=20
67   AXM=XM
68   AU=U
69   AC=C
70   AT=T
71   ARHO=RHO
72   AX=X
73   X=AX+DELX
74   AXL=XL
75   AA=A
76   AP=P
77   AW=W
78   AGAMMA=GAMMA
80   SDELMQ=0.0
81   SDELU=0.0
82   SDELC=0.0
83   SDELT=0.0
```

```
 84     SDELRH=0.0
        NITR=0
 85     BP=(AP+P)/2.0
 86     BXM=(AXM+XM)/2.0
        BMSQ=BXM*BXM
 87     BU=(AU+U)/2.0
 88     BC=(AC+C)/2.0
 89     BT=(AT+T)/2.0
 90     BRHO=(ARHO+RHO)/2.0
 91     BX=(AX+X)/2.0
 92     BXL=(AXL+XL)/2.0
 93     CALL WEIGE
 94     CALL GAMME
 95     CALL BMAGE
 96     CALL EFLDE
 97     CALL SIGME
        DELXL=(SIGMA*DELX*B*B)/(BRHO*BU)
 98     XL=AXL+DELXL
 99     ETA=BU*B/EF
101     DM=1.0-BMSQ
102     GAM=GAMMA-1.0
103     GAMT=GAM/2.0
104     EMA=-2.0*(1.0+GAMT*BMSQ)
105     ECA=GAMT*BMSQ
106     ETTA=GAM*BMSQ
107     CALL ENRGE
108     EML=1.0+GAMMA*BMSQ
109     ECL=(1.0-GAMMA*BMSQ)/2.0
110     ETL=ECL*2.0
111     CALL MMNTE
112     EMLL=GAMMA*BMSQ*(-EMA/2.0)
113     EULL=GAMMA*BMSQ/2.0
114     ECLL=(-GAMMA*(GAMMA-1.0)/4.0)*BMSQ*BMSQ
115     ETLL=ECLL*2.0
116     ETW=GAM*BMSQ
117     CALL AREAE
118     BA=(A+AA)/2.0
119     DELW=(W-AW)*2.0
120     DELGAM=(GAMMA-AGAMMA)*2.0
121     DELA=A-AA
122     DELMSQ=(BMSQ/DM)*((EMA*DELA/BA)+EML*E+EMLL*UM-EML*DELW/W-DM*DELGAM
```

```
     1 /GAMMA)
      XMSQ=AXM*AXM+DELMSQ
 123  IF(ABSF(XMSQ-1.0)-.05)124,124,127
 124  WRITE OUTPUT TAPE 24,125
 125  FORMAT(18H0 PROGRAM DIVERGES)
 126  GO TO 6
 127  DELU=(BU/DM)*(-DELA/BA+E+EULL*UM-DELW/W)
 128  DELC=(BC/DM)*(ECA*DELA/BA+ECL*E+ECLL*UM-ECL*DELW/W+(DM/2.0)*DELGAM
     1 /GAMMA)
 129  DELT=(BT/DM)*(ETTA*DELA/BA+ETL*E+ETLL*UM+ETW*DELW/W)
 130  DELRHO=(BRHO/DM)*(BMSQ*DELA/BA-E-EULL*UM+DELW/W)
 131  XM=SQRTF(XMSQ)
 132  U=AU+DELU
 133  C=AC+DELC
 134  T=AT+DELT
 135  RHO=ARHO+DELRHO
 136  W=AW+DELW
 137  GAMMA=AGAMMA+DELGAM
 138  P=8317.0*RHO*T/W
      NITR=NITR+1
      IF(NITR-MAXITR)139,139,150
 139  IF(ABSF((DELMSQ-SDELMQ)/DELMSQ)-.00005)140,140,144
 140  IF(ABSF((DELU-SDELU)/DELU)-.00005)141,141,144
 141  IF(ABSF((DELC-SDELC)/DELC-.00005)142,142,144
 142  IF(ABSF((DELT-SDELT)/DELT)-.00005)143,143,144
 143  IF(ABSF((DELRHO-SDELRH)/DELRHO)-.00005)150,150,144
 144  SDELMQ=DELMSQ
 145  SDELU=DELU
 146  SDELC=DELC
 147  SDELT=DELT
 148  SDELRH=DELRHO
 149  GO TO 85
 150  IF(ABSF((XM-AXM)/AXM)-.10)151,155,155
 151  IF(ABSF((U-AU)/AU)-.10)152,155,155
 152  IF(ABSF((C-AC)/AC)-.10)153,155,155
 153  IF(ABSF((T-AT)/AT)-.10)154,155,155
 154  IF(ABSF((RHO-ARHO)/ARHO)-.10)158,155,155
 155  DELX=DELX/2.0
 156  X=AX+DELX
 157  GO TO 80
```

```
158    IF(ABSF((XM-AXM)/AXM)-.001)159,166,166
159    IF(ABSF((U-AU)/AU)-.001)160,166,166
160    IF(ABSF((C-AC)/AC)-.001)161,166,166
161    IF(ABSF((T-AT)/AT)-.001)162,166,166
162    IF(ABSF((RHO-ARHO)/ARHO)-.001)163,166,166
163    DELX=2.0*DELX
164    X=AX+DELX
165    GO TO 80
166    NPRNT=NPRNT+1
       IF(NPRNT-NPRINT)193,167,193
167    NPRNT=0
168    BX=X
169    BXL=XL
170    BXM=XM
       BMSQ=BXM*BXM
       BRHO=RHO
171    BC=C
172    BU=U
173    BT=T
174    BP=P
175    CALL WEIGE
176    CALL GAMME
177    CALL BMAGE
178    CALL EFLDE
179    CALL SIGME
180    ETA=B*BU/EF
181    AJ=SIGMA*(EF-BU*B)
182    RATIO=1.0+((GAMMA-1.0)/2.0)*XM*XM
183    EE=GAMMA/(GAMMA-1.)
184    PT=P*RATIO**EE
185    TT=T*RATIO
186    PR=PT/SPT
187    TR=TT/STT
188    NPAGE=NPAGE+1
189    IF(NPAGE-10)192,190,192
190    NPAGE=0
191    WRITE OUTPUT TAPE 24,28
192    WRITE OUTPUT TAPE 24,65,X,XL,XM,C,U,T,TT,P,PT,RHO,A,B,EF,SIGMA,W,G
     1 AMMA,ETA,PR,TR,AJ
193    IF(X-XT)66,7,7
194    END
```

APPENDIX IV: FORTRAN LISTING OF SUBROUTINES

```
        SUBROUTINE AREAE - 001
C       FIFTH ORDER POLYNOMIAL VARIATION OF A WITH X
     1  DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
       1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
     2  COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
       1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
       2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
        A=CA(1)+CA(2)*BX+CA(3)*BX*BX+CA(4)*BX**3+CA(5)*BX**4+CA(6)*BX**5
        RETURN
        END
```

```
        SUBROUTINE AREAE - 002
C       CONSTANT PRESSURE
     1  DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
       1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
     2  COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
       1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
       2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
        A=AA+BA*(E+((1.0+(GAMMA-1.0)*BMSQ)/2.0)*UM-DELW/W)
        RETURN
        END
```

```
      SUBROUTINE AREAE - 003
C     CONSTANT RHO
    1 DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2 COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
      A=AA+BA*(E/BMSQ+(GAMMA/2.0)*UM-DELW/(W*BMSQ))
      RETURN
      END


      SUBROUTINE AREAE - 004
C     CONSTANT TEMPERATURE
    1 DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2 COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
      A=AA+BA*(-((1.0-GAMMA*BMSQ)/((GAMMA-1.0)*BMSQ))*E+(GAMMA*BMSQ/2.0)
     1*UM-DELW/W)
      RETURN
      END


      SUBROUTINE AREAE - 005
C     CONSTANT VELOCITY
    1 DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2 COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
      A=AA+BA*(E+(GAMMA*BMSQ/2.0)*UM-DELW/W)
      RETURN
      END
```

```
          SUBROUTINE BMAGE  - 101
C         FIFTH ORDER POLYNOMIAL VARIATION OF B WITH X
      1   DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
         1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
      2   COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
         1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
         2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          B=CB(1)+CB(2)*BX+CB(3)*BX*BX+CB(4)*BX**3+CB(5)*BX**4+CB(6)*BX**5
          RETURN
          END
```

```
         SUBROUTINE EFLDE - 201
C        CONSTANT CURRENT DENSITY,CONSTANT SIGMA
     1   DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
       1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
     2   COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
       1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
       2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
         EF=CEF(1)/CS(1)+B*BU
         RETURN
         END
```

```
         SUBROUTINE EFLDE - 202
C        CONSTANT ETA
     1   DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
       1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
     2   COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
       1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
       2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
         EF=BU*B/CEF(1)
         RETURN
         END
```

```
         SUBROUTINE EFLDE - 203
C        ELECTRIC FIELD-MAXIMUM MACH NUMBER
     1   DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
       1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
         COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
       1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
       2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
         EF=BU*B/2.0*(1.0+(GAMMA/(GAMMA-1.0))*((2.0+(GAMMA-1.0)*BXM*BXM)/(1
       1 .0+GAMMA*BXM*BXM)))
         RETURN
         END
```

```
          SUBROUTINE EFLDE — 204
C         ISOTHERMAL, CONSTANT AREA CHANNEL
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
        1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
        1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
        2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          EF=BU*B*((GAMMA*BXM*BXM)/(GAMMA*BXM*BXM-1.0))
          RETURN
          END


          SUBROUTINE EFLDE — 205
C         FIFTH ORDER POLYNOMIAL VARIATION OF E WITH X
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
        1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
        1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
        2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          EF=CEF(1)+CEF(2)*BX+CEF(3)*BX*BX+CEF(4)*BX*BX*BX +CEF(5)*BX*BX*BX*
        1 BX +CEF(6)*BX*BX*BX*BX*BX
          RETURN
          END


          SUBROUTINE EFLDE — 206
C         ISOTHERMAL, CONSTANT AREA CHANNEL INCLUDING HEAT TRANSFER
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
        1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
        1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
        2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          CALL SIGME
          Z1=-(BU*B*(1.0+(GAMMA*BMSQ)/(GAMMA*BMSQ-1.0)))
          Z2=BU*BU*B*B*((GAMMA*BMSQ)/(GAMMA*BMSQ-1.0))-(CEF(1)+CEF(2)*BX+CEF
        1 (3)*BX*BX +CEF(4)*BX*BX*BX )*BRHO*BU/SIGMA
          EF=-(Z1-SQRTF(Z1*Z1-4.0*Z2))/2.0
          RETURN
          END
```

```
          SUBROUTINE ENRGE - 301
C         MAGNETOHYDRODYNAMIC HEAT ADDITION
      1   DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
        1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
      2   COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
        1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
        2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          DELXL=(XL-AXL)
          E=((1.0-ETA)/(ETA*ETA))*(GAMMA-1.0)*BMSQ*DELXL
          RETURN
          END
```

```
          SUBROUTINE ENRGE - 302
C         SIMPLE HEATING (DQ/DX)*DELX/(CP)(T)
      1   DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
        1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
      2   COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
        1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
        2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          E=(CE(1)+CE(2)*BX+CE(3)*BX*BX)*DELX/(CP*BT)
          RETURN
          END
```

```
          SUBROUTINE ENRGE - 303
C         MAGNETOHYDRODYNAMIC HEAT ADDITION WITH HEAT TRANSFER LOSSES
      1   DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
        1 TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
      2   COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
        1 P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
        2 M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          DELXL=(XL-AXL)
          E=((1.0-ETA)/(ETA*ETA))*(GAMMA-1.0)*BMSQ*DELXL
          E = E-(CE(1)+CE(2)*BX+CE(3)*BX*BX+CE(4)*BX*BX*BX)*DELX/(CP*BT)
          RETURN
          END
```

```
      SUBROUTINE GAMME - 401
C     CONSTANT SPECIFIC HEAT RATIO
    1 DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2 COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
      GAMMA=CG(1)
      CP=CG(2)
      RETURN
      END
```

```
      SUBROUTINE GAMME - 402
C     REAL GAS PROPERTIES
      DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
      COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
      J=0
    1 J=J+1
      IF(BP-PRESS(J))2,1,1
    2 I=0
    3 I=I+1
      IF(BT-TEMP(I))4,3,3
    4 BIT=((PRESS(J)-BP)/(PRESS(J)-PRESS(J-1)))*TBLY(I,J-1)+((BP-PRESS(J
     1-1))/(PRESS(J)-PRESS(J-1)))*TBLY(I,J)
      BJT=((PRESS(J)-BP)/(PRESS(J)-PRESS(J-1)))*TBLY(I-1,J-1)+((BP-PRESS
     1(J-1))/(PRESS(J)-PRESS(J-1)))*TBLY(I-1,J)
      GAMMA=((TEMP(I)-BT)/(TEMP(I)-TEMP(I-1)))*BJT+((BT-TEMP(I-1))/(TEMP
     1(I)-TEMP(I-1)))*BIT
      CP=GAMMA*8317.0/((GAMMA-1.0)*W)
      RETURN
      END
```

```
          SUBROUTINE MMNTE - 501
C         MAGNETOHYDRODYNAMIC MOMENTUM ADDITION
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1    TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1    P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2    M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          DELXL=(XL-AXL)
          UM=2.0*(ETA-1.0)/ETA*DELXL
          RETURN
          END

          SUBROUTINE MMNTE - 502
C         MAGNETOHYDRODYNAMIC MOMENTUM ADD. WITH FRICTION CIRCULAR CROSS SEC
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1    TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1    P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2    M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          DELXL=(XL-AXL)
          F=CM(1)+CM(2)*BX+CM(3)*BX*BX
          D=SQRTF(4.0*BA/3.1416)
          UM=(2.0*(ETA-1.0)/ETA*DELXL)+4.0*F*DELX/D
          RETURN
          END

          SUBROUTINE MMNTE - 503
C         MAGNETOHYDRODYNAMIC MOMENTUM ADD WITH FRICTION RECT CROSS SECTION
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1    TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1    P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2    M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          DELXL=(XL-AXL)
          F=CM(1)+CM(2)*BX+CM(3)*BX*BX
          D1 = CM(4) + CM(5) * BX
          D2 = CM(6) + CM(7) * BX
          D=2.0*BA/(D1+D2)
          UM=(2.0*(ETA-1.0)/ETA*DELXL)+4.0*F*DELX/D
          RETURN
          END
```

```
          SUBROUTINE SIGME - 601
C         CONSTANT CURRENT DENSITY
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
         1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
         1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
         2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          SIGMA=CS(1)/(EF-BU*B)
          RETURN
          END
```

```
          SUBROUTINE SIGME - 602
C         CONSTANT SIGMA
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
         1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
         1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
         2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          SIGMA=CS(1)
          RETURN
          END
```

```
          SUBROUTINE SIGME  - 603
C         SIGMA PROPORTIONAL TO J
    1     DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
         1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2     COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
         1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
         2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
          EUB=EF-BU*B
          XJ=(CS(1)*EUB)/(1.0-CS(2)*EUB)
          SIGMA=CS(1)+CS(2)*XJ
          RETURN
          END
```

```
      SUBROUTINE WEIGE - 701
C     CONSTANT MOLECULAR WEIGHT
    1 DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
    2 COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
      W=CW(1)
      RETURN
      END


      SUBROUTINE WEIGE - 702
C     REAL GAS PROPERTIES
      DIMENSION CA(7),CB(7),CEF(7),CE(7),CG(7),CM(7),CS(14),CW(7),CC(7),
     1TEMP(30),PRESS(30),TBLX(30,30),TBLY(30,30)
      COMMON CA,CB,CEF,CE,CG,CM,CS,CW,CC,TEMP,PRESS,TBLX,TBLY,XT,XM,U,T,
     1P,RHO,A,B,EF,E,GAMMA,UM,SIGMA,W,X,XL,AXL,AA,DELX,DELW,BA,BC,BXL,BX
     2M,BP,BRHO,BT,BU,BMSQ,BX,Z,ETA,CP,DELXL,AW,ARHO,AU,AT,AP,AXM
      J=0
    1 J=J+1
      IF(BP-PRESS(J))2,1,1
    2 I=0
    3 I=I+1
      IF(BT-TEMP(I))4,3,3
    4 BIT=((PRESS(J)-BP)/(PRESS(J)-PRESS(J-1)))*TBLX(I,J-1)+((BP-PRESS(J
     1-1))/(PRESS(J)-PRESS(J-1)))*TBLX(I,J)
      BJT=((PRESS(J)-BP)/(PRESS(J)-PRESS(J-1)))*TBLX(I-1,J-1)+((BP-PRESS
     1(J-1))/(PRESS(J)-PRESS(J-1)))*TBLX(I-1,J)
      Z=((TEMP(I)-BT)/(TEMP(I)-TEMP(I-1)))*BJT+((BT-TEMP(I-1))/(TEMP(I)-
     1TEMP(I-1)))*BIT
      W=CW(1)/Z
      RETURN
      END
```

SUBROUTINE HUNTF

```
101  O RIGIN     CNTRL0325
102  *    SUBROUTINE  HUNTF
103  *    PURPOSE.  TO CALL IN THE SEVEN SUBROUTINES -AREAE,
104  *     BMAGE,EFLDE,ENRGE,GAMME,MMNTE,SIGME,WEIGE- AS THEY
105  *     ARE NEEDED SINCE THE 7074 MEMORY COULD NOT HOLD ALL
106  *     THE DIFFERENT VARIATIONS OF EACH SUBROUTINE AT ANY
107  *     GIVEN TIME.
109  H UNTF      PC   -1111110011
110              XU   52,X52
111              ZA1  0150
112              ZST1 ONEFIFTY
113              ZA1  BRTERROR
114              ZST1 0150
115              ZA1  *(2)
116              STD1 0125
117              XU   50,X50
118              XU   51,X51
119              ZA1  101
120              ZST1 SW101
121              ESF  1
122              XL   50,+10009     READ PAST TAPE MARK BETWEEN
123              PTR  25,CARD       MAINPROG.AND SUBR.
124              B    *
125  L OOP       ZA1  0+X94         PICK UP SEARCH ARG.
126              BM1  *+2           LAST ONE
127              B    *+2         NO.
128              ESN  1
129              STD1 *(2,9)+1
130              ZA3  *
201              ZST3 SUBR          STORE SUBR.NO.
202  L OOK       PTR  25,CARD       START SEARCH FOR NEEDED
203              B    *             SUBR.
204              ZA1  CARD(0,1)+7
205              C1   +99           LABEL CARD
206              BE   *+2           YES.
207              B    LOOK          NO.
208              C3   CARD+1        NEEDED SUBR
209              BE   LOAD          YES.
210              B    LOOK          NO
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211  L OAD      ZST3 SUBR            LOAD FOUND SUBR.
212            ZA3  1314(6,9)+X50    1314=MAINPROGRAM ORIGIN
213            PTR  25,CARD
214            B    *
215            ZA2  CARD(0,1)+7
216            C2   +91             SUBR.TITLE CARD
217            BE   LOAD+2          YES. READ ANOTHER CARD.
218            C2   +92
219            BE   *+2
220            B    *+3
221            ZA1  1317(6,9)       LOC.OF SORTF
222            STD1 CARD(6,9)+1
223            S3   CARD(6,9)+7
224            ZST3 RELOCBASE
225  L OADLOOP1 ZA3 CARD(4)+7
226            BZ3  EXECUTE
227            XL   52,9993
228            XA   52,1
229            ZA3  CARD(6,9)+7
230            A3   RELOCBASE
301            XLIN 51,9993
     L OADLOOP2 ZA1 CARD+X52
302            ZA2  52(2,5)
303            S2   +1
304            STD2 *(4)+2
305            STD2 *(5)+1
306            ZA3  CARD+6
307            BZ3  NEXTFIELD
3071           CD   9993(9),5
3072           BE   NEXTFIELD
3073           ZA3  9991(2,5)
3074           MSP  9993
3075           A3   RELOCBASE
3076           STD3 9991(2,5)
308  N EXTFIELD A2  +5
309            STD2 *(4)+2
310            STD2 *(5)+1
311            ZA3  CARD+6
312            BZ3  STORE
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313                  CD    9993(9),5
314                  BE    STORE
315                  ZA3   9991(6,9)
3151                 MSP   9993
3152                 A3    RELOCBASE
316                  STD3  9991(6,9)
317    S TORE        ZST1  0+X51
318                  BIX   52,ADVANCE
319                  PTR   25,CARD
320                  B     *
321                  B     LOADLOOP1
322    A DVANCE      XA    51,1
323                  B     LOADLOOP2
324    T HATSALL     XL    50,X50
325                  XL    51,X51
326                  XL    52,X52
327                  ZA1   SW101
328                  ZST1  101
329                  PTSB  25,BACKUP
330                  B     *
401                  ZA1   ONEFIFTY
402                  ZST1  0150
403                  B     1+X94
404    E XECUTE      ZA1   SUBR(7)
405                  SL1   2
406                  A1    SUBR(8)
407                  SL1   2
408                  A1    SUBR(9)
409                  A1    +909090
410                  MSA   9991
411                  ZST1  SUBR
412                  TYP   SUBRMSG
413                  NOP
414                  XA    50,1
415                  CD    50(5),3
416                  BE    *+2
417                  B     *+2
418                  XA    50,1
419                  XA    94,1
```

```
420              BES  1,THATSALL
421              B    LOOP
422  B RTERROR   B    TERROR
423  T ERROR     CD   0(1)+X99,2
424              BE   RETURN
425              BL   ERROR
426              CD   0(1)+X99,5
427              BE   EOF
428              BH   RETURN
429  E RROR      TRB  25
430              XS   97,1
501              PR   97
502  R ETURN     XA   97,1
503              PR   97
504  E OF        TYP  EOFMSG
505              NOP
506              PR   *+1
507              HB   *
508  M SG        DC
509                   -CANNOT FIND     -
510  E OFMSG     DRDW +MSG,MSG+2
511  S UBRMSG    DRDW -SUBR,SUBR
512              DA   1
513  O NEFIFTY        00,09
514  X 50             10,19
515  X 51             20,29
516  X 52             30,39
517  S UBR            40,49
518  S W101           50,59
519  R ELOCBASE       60,69
520  C ARD       DA   1,-RDW
521                   00,79
522  B ACKUP     DC
523                   +0
525  E ND        CNTRL0325
```

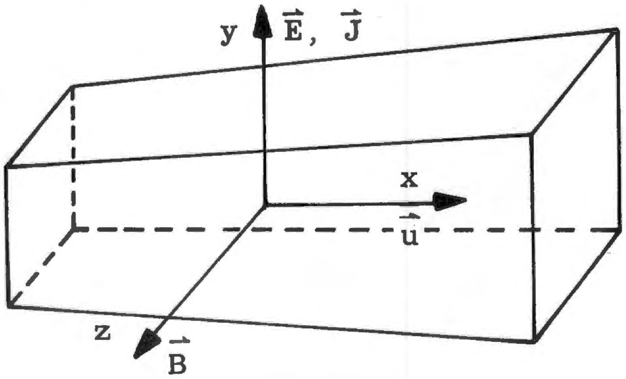
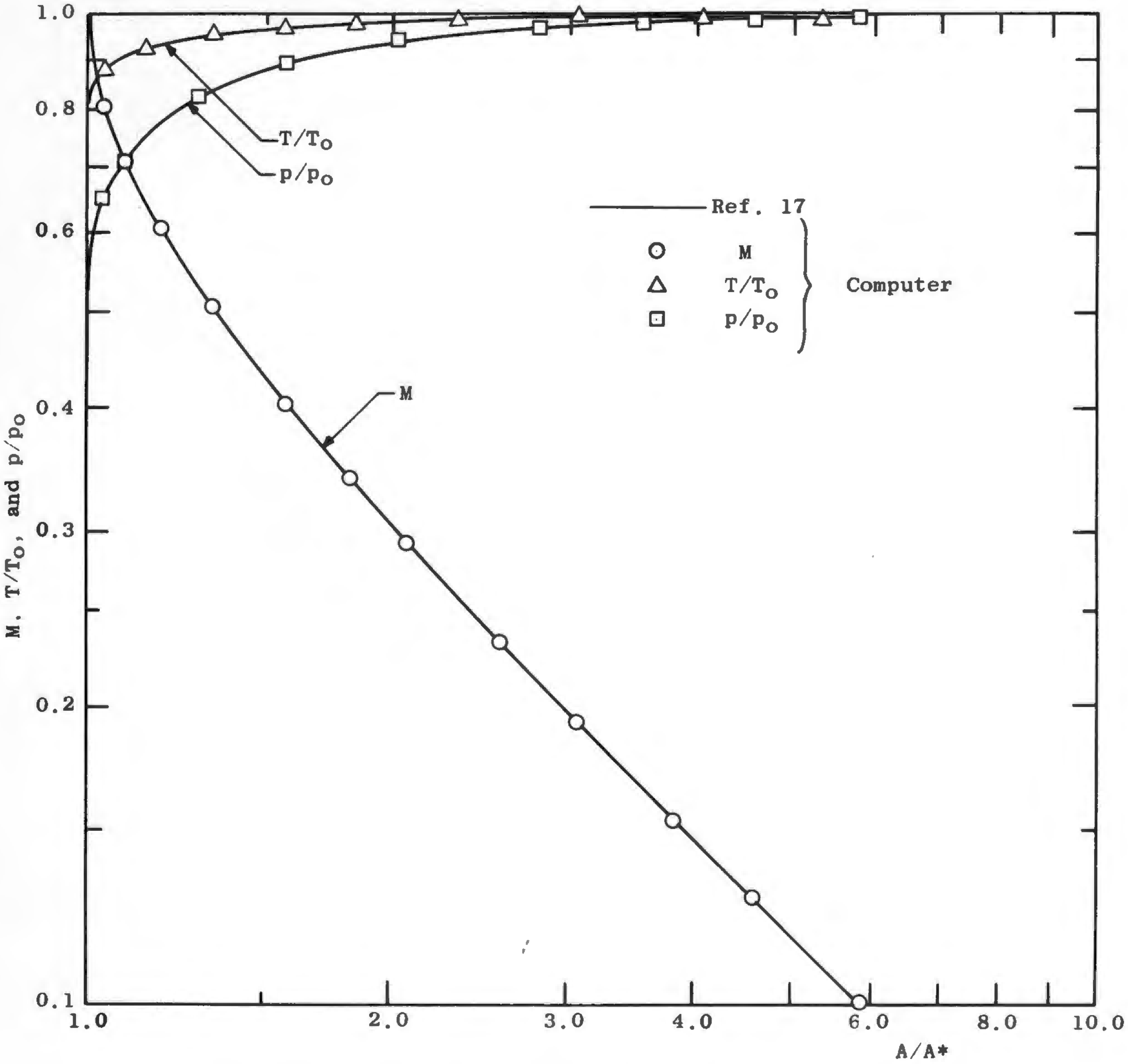
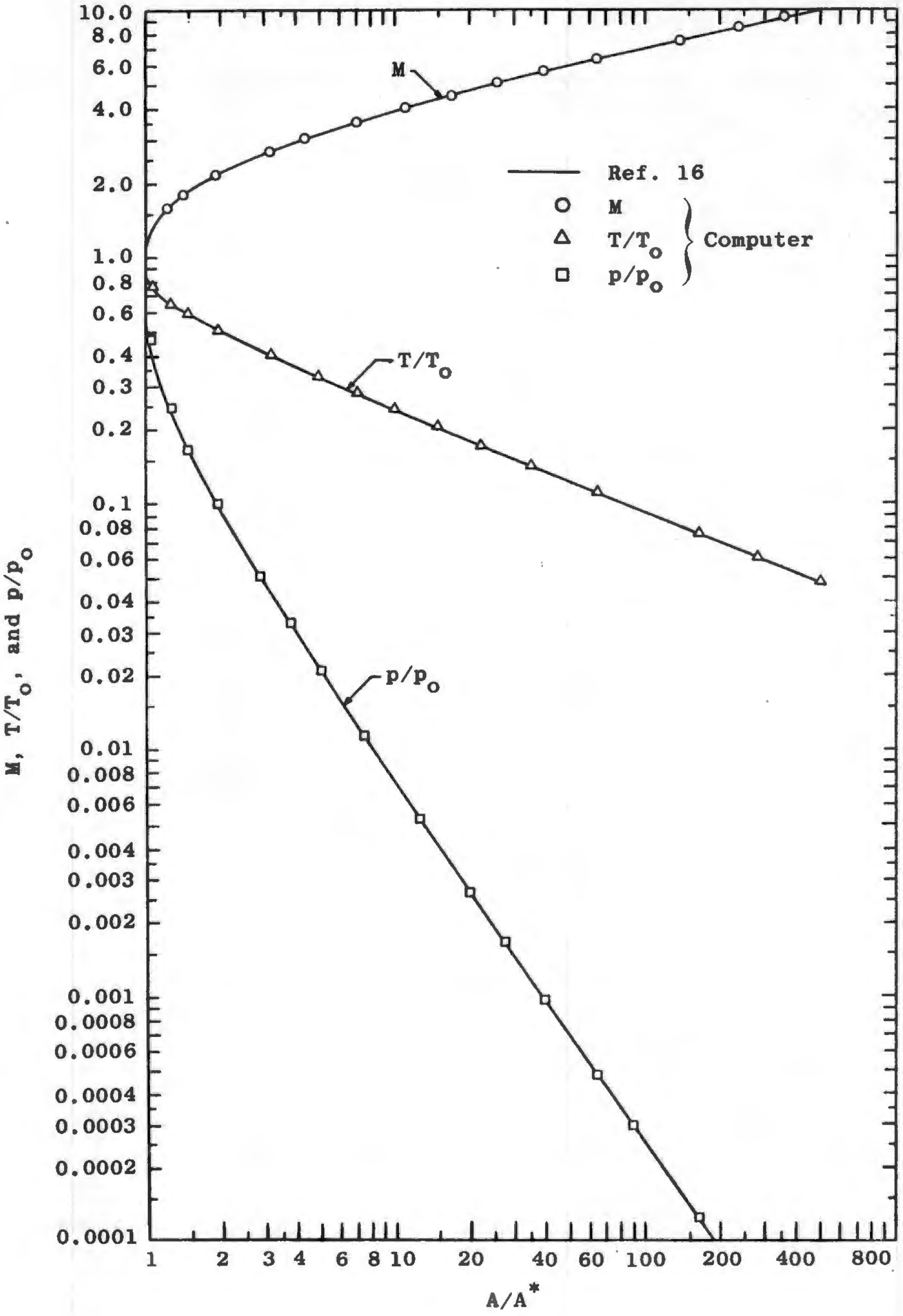


Fig. 1 Orientation of Electric and Magnetic Fields for One-Dimensional MHD Approximation



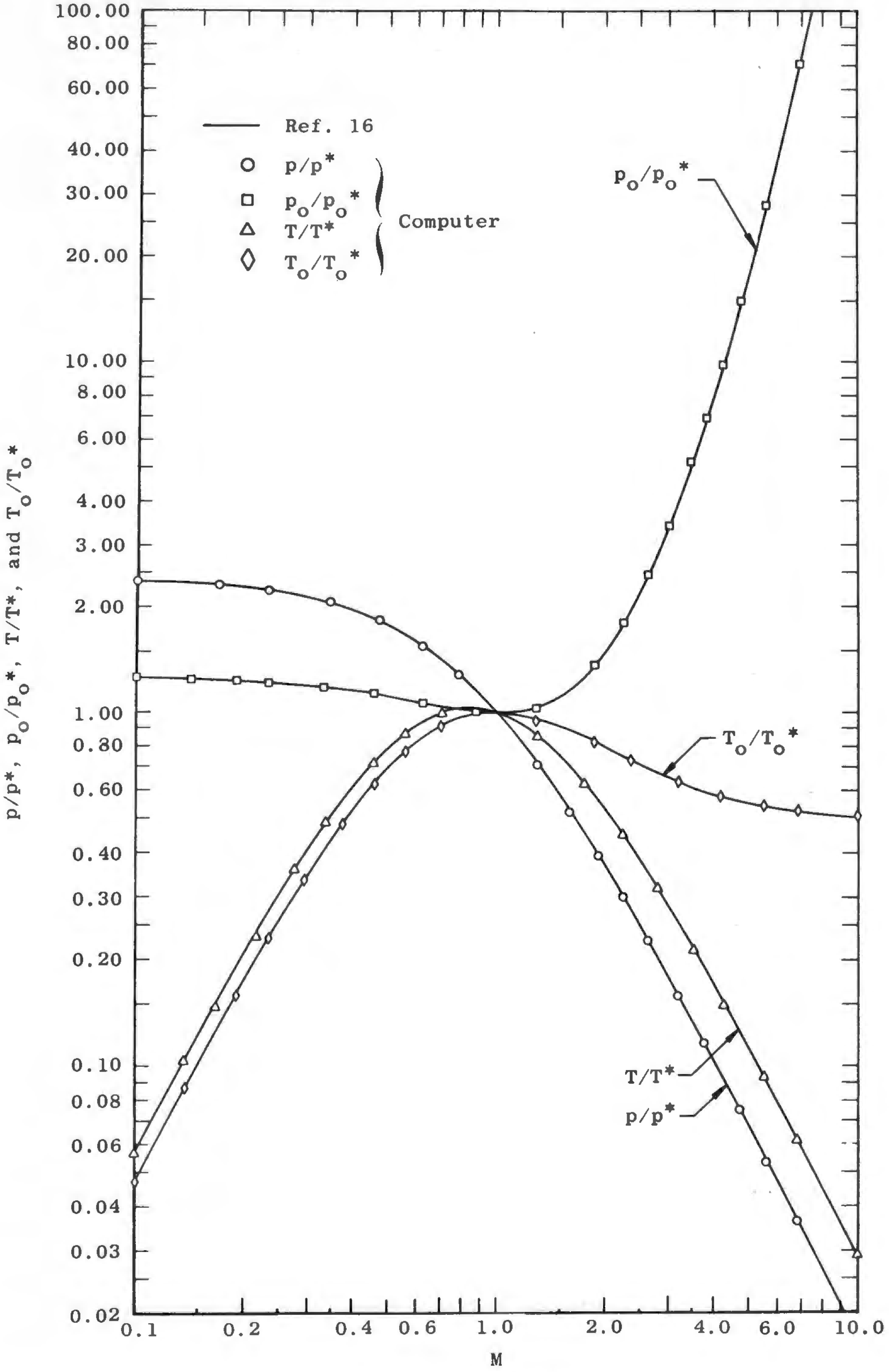
a. Subsonic Isentropic Flow, $\gamma = 1.4$

Fig. 2 Comparison of Computer Solutions with Existing Analytic or Numerical Solutions



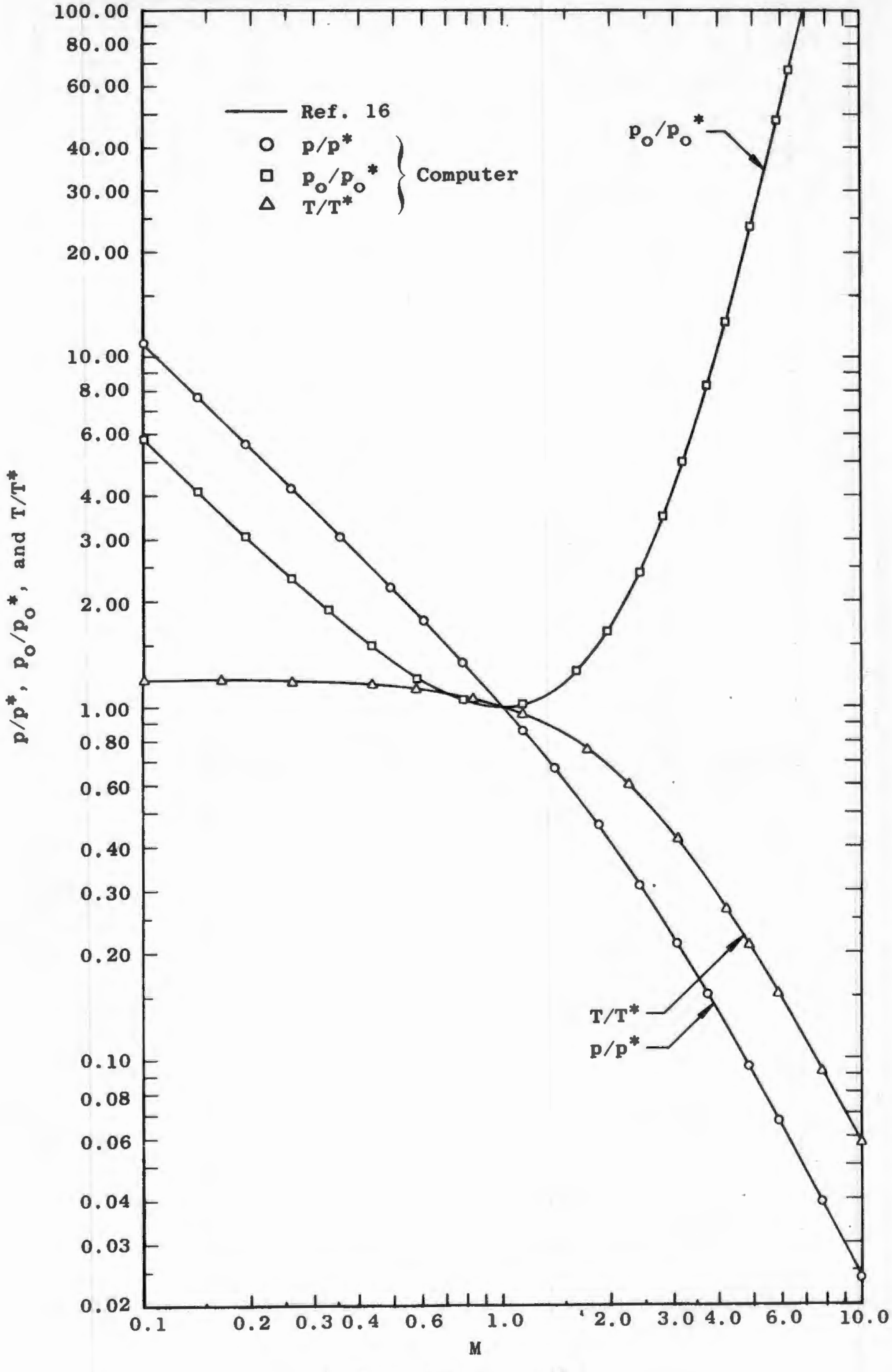
b. Supersonic Isentropic Flow, $\gamma = 1.4$

Fig. 2 Continued



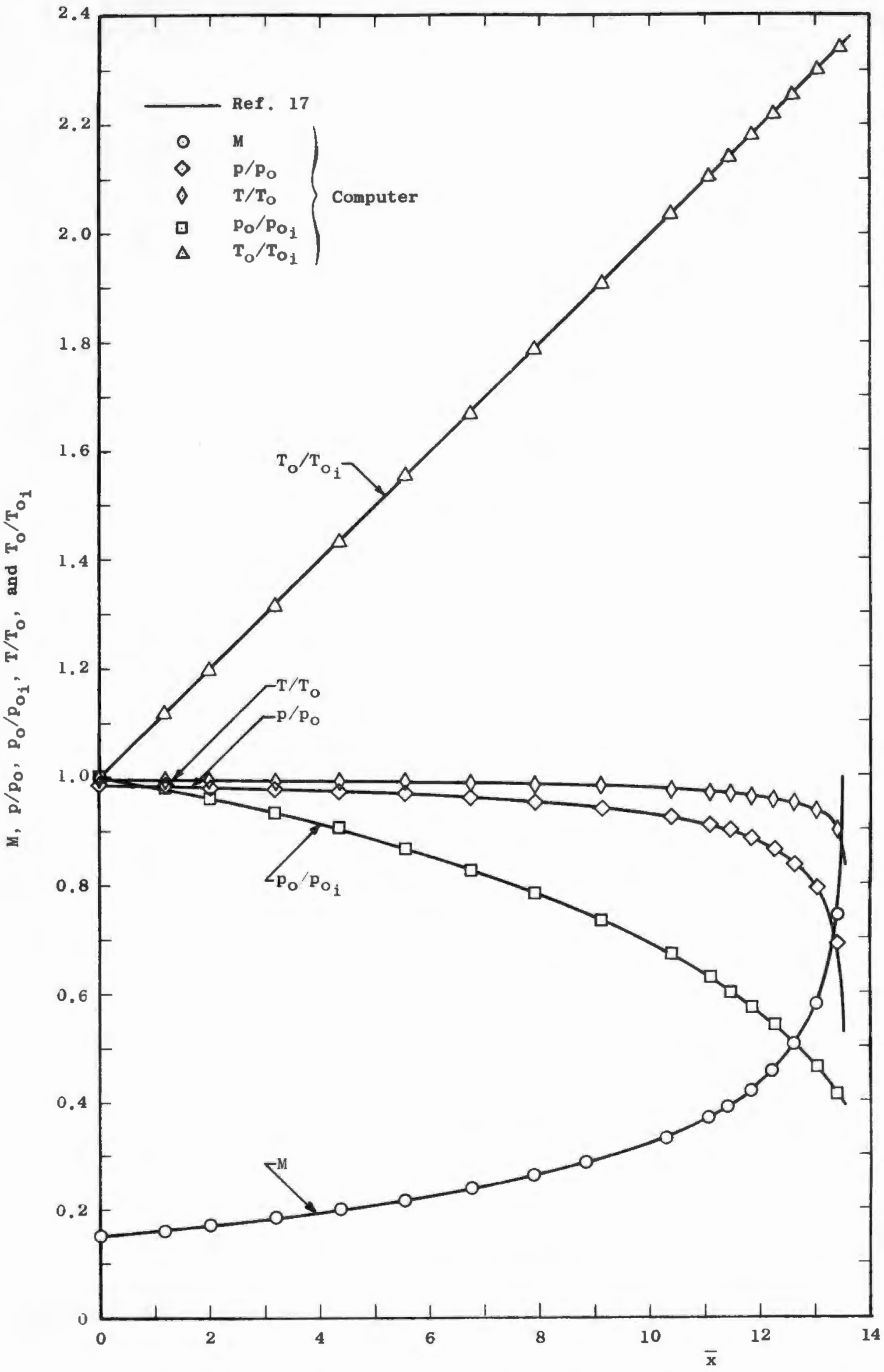
c. Rayleigh Flow, $\gamma = 1.4$

Fig. 2 Continued



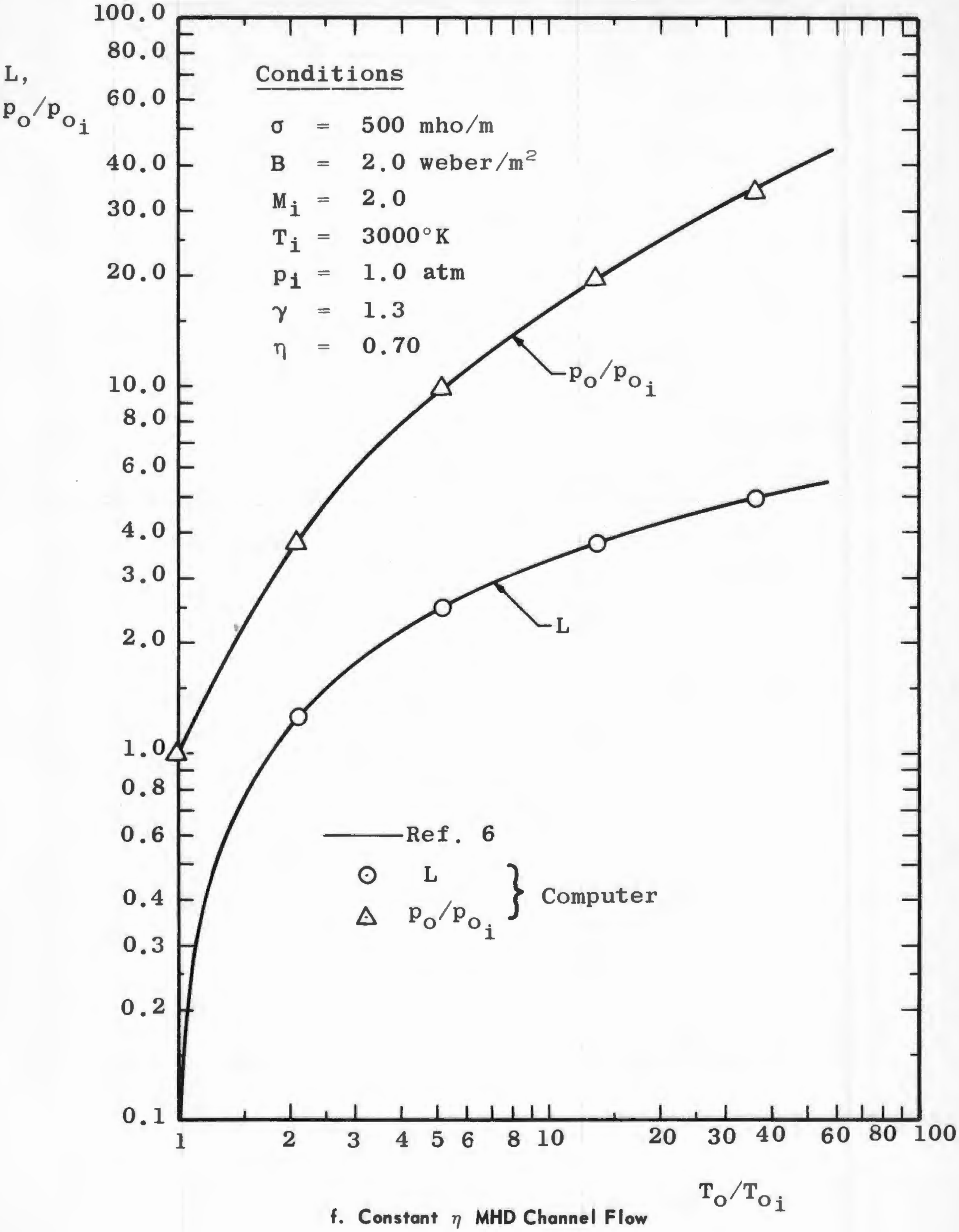
d. Fanno Flow, $\gamma = 1.4$

Fig. 2 Continued



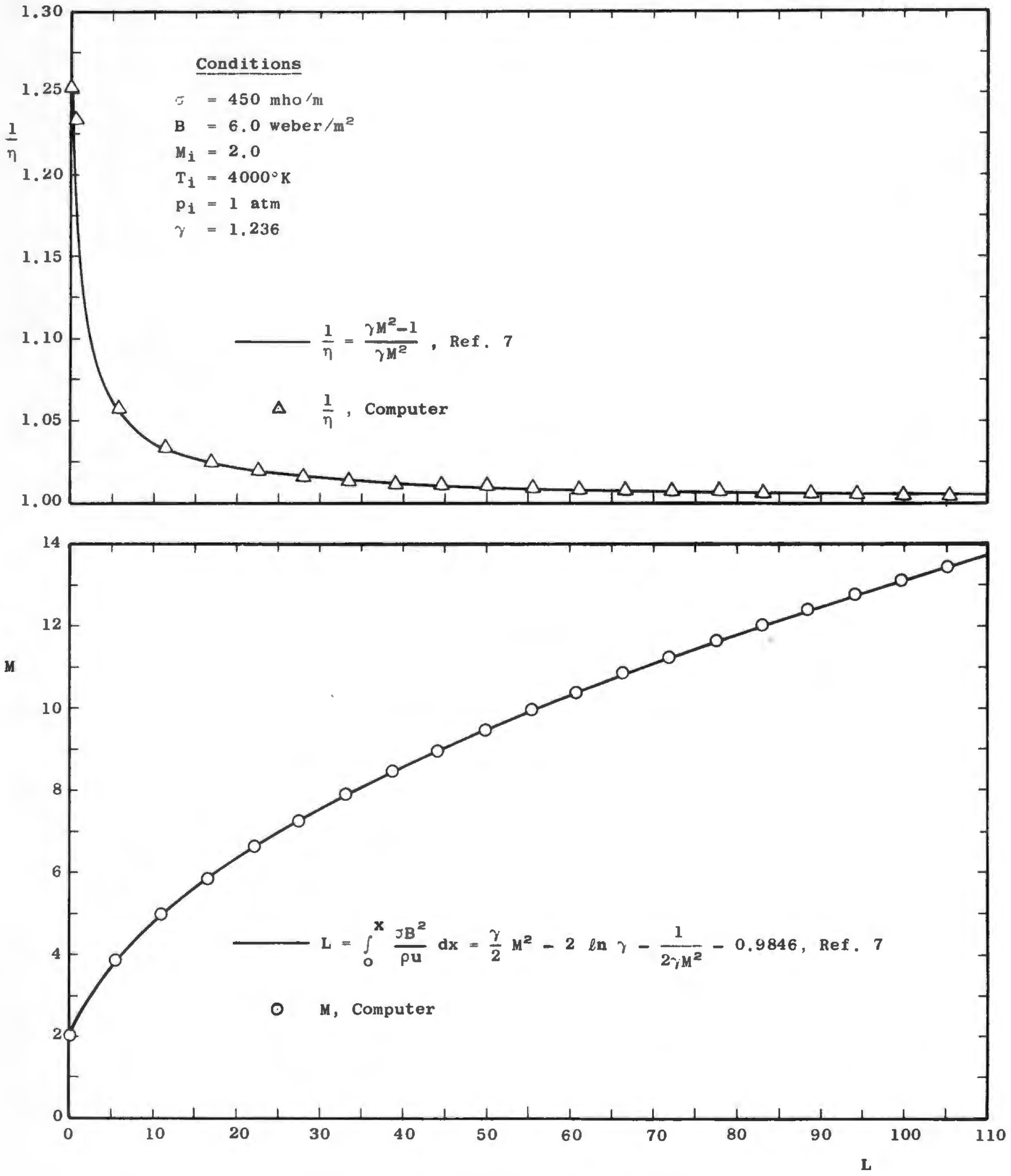
e. Constant Area Flow with Combined Heating and Friction

Fig. 2 Continued



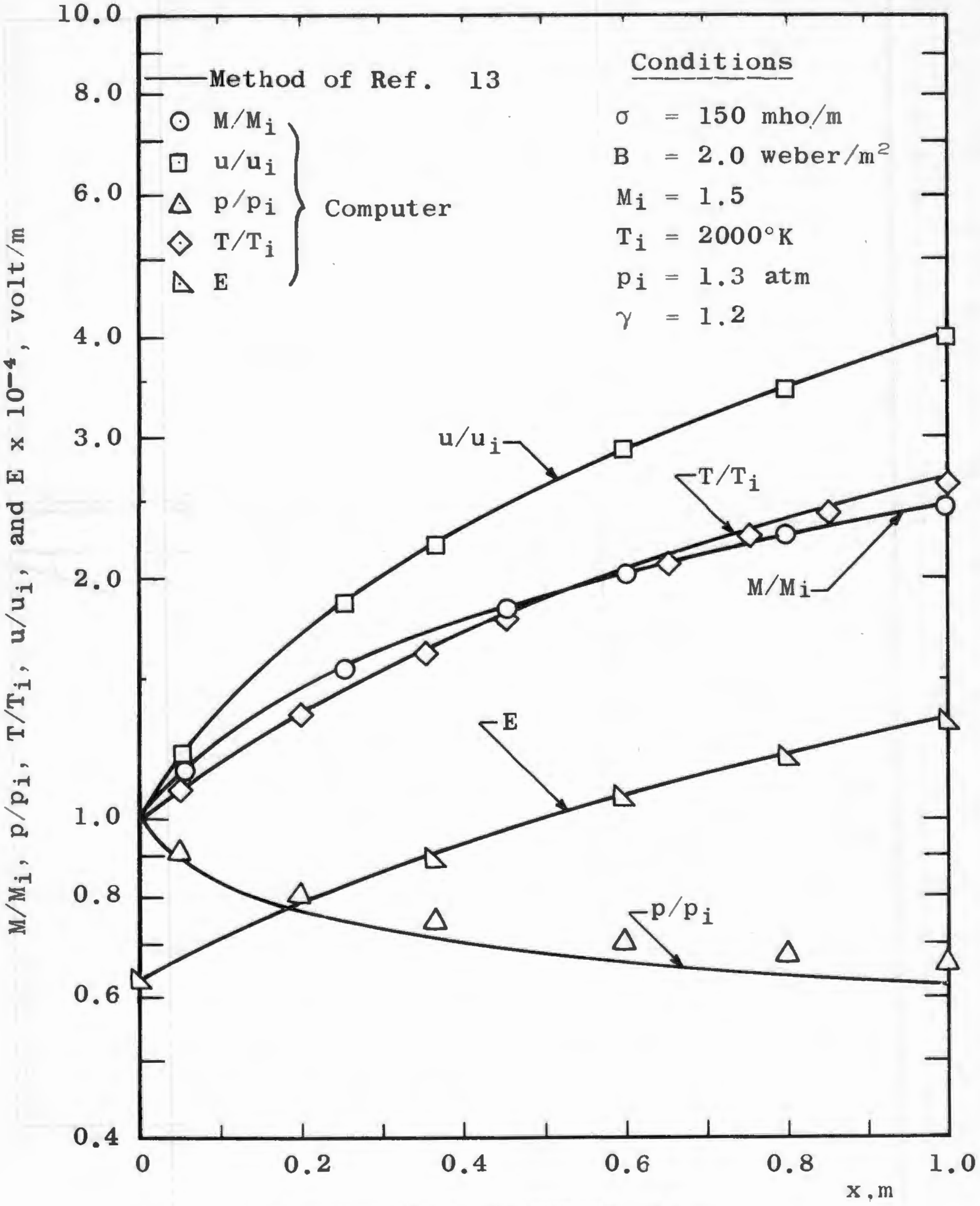
f. Constant η MHD Channel Flow

Fig. 2 Continued



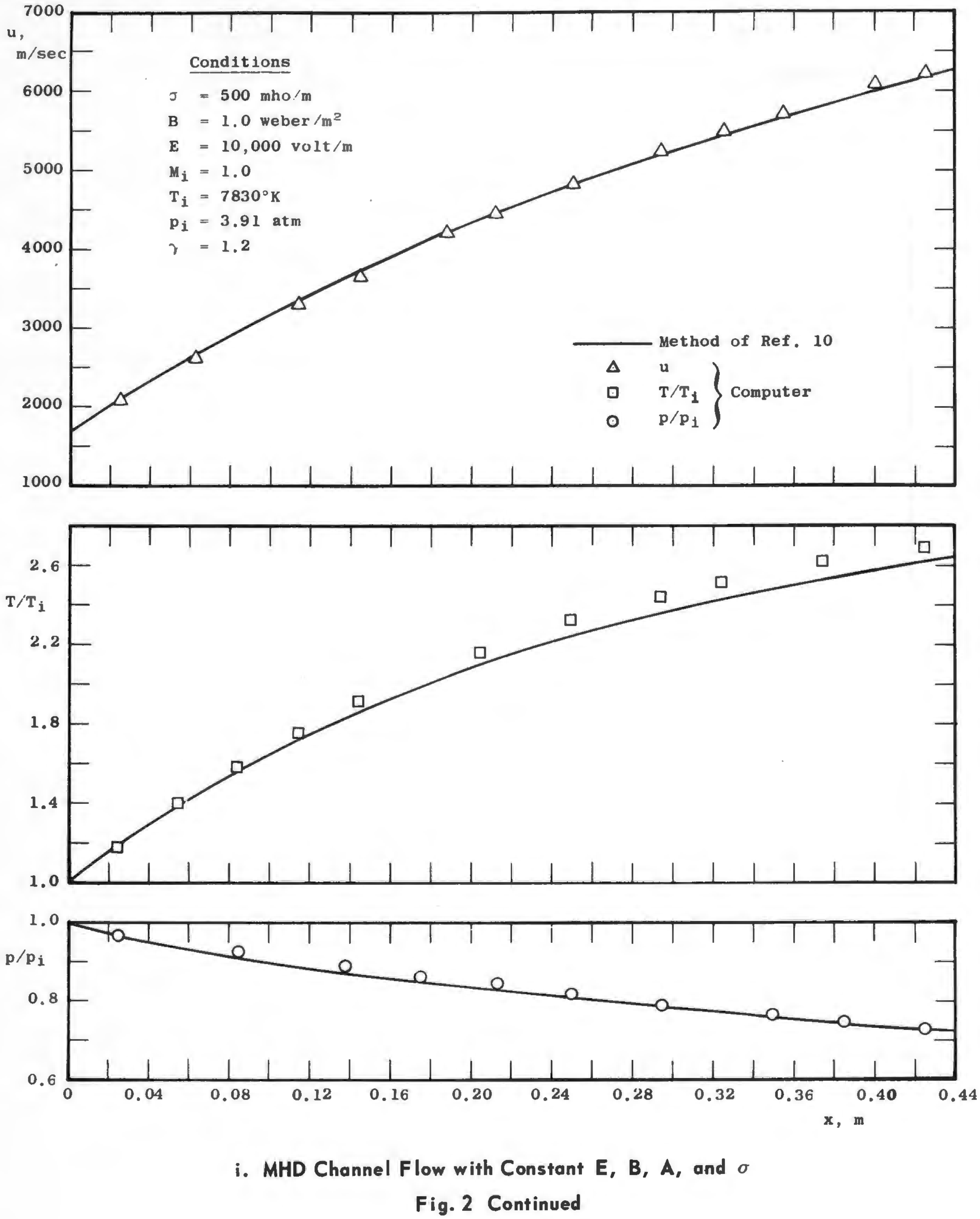
g. Isothermal, Constant Area, MHD Channel Flow

Fig. 2 Continued



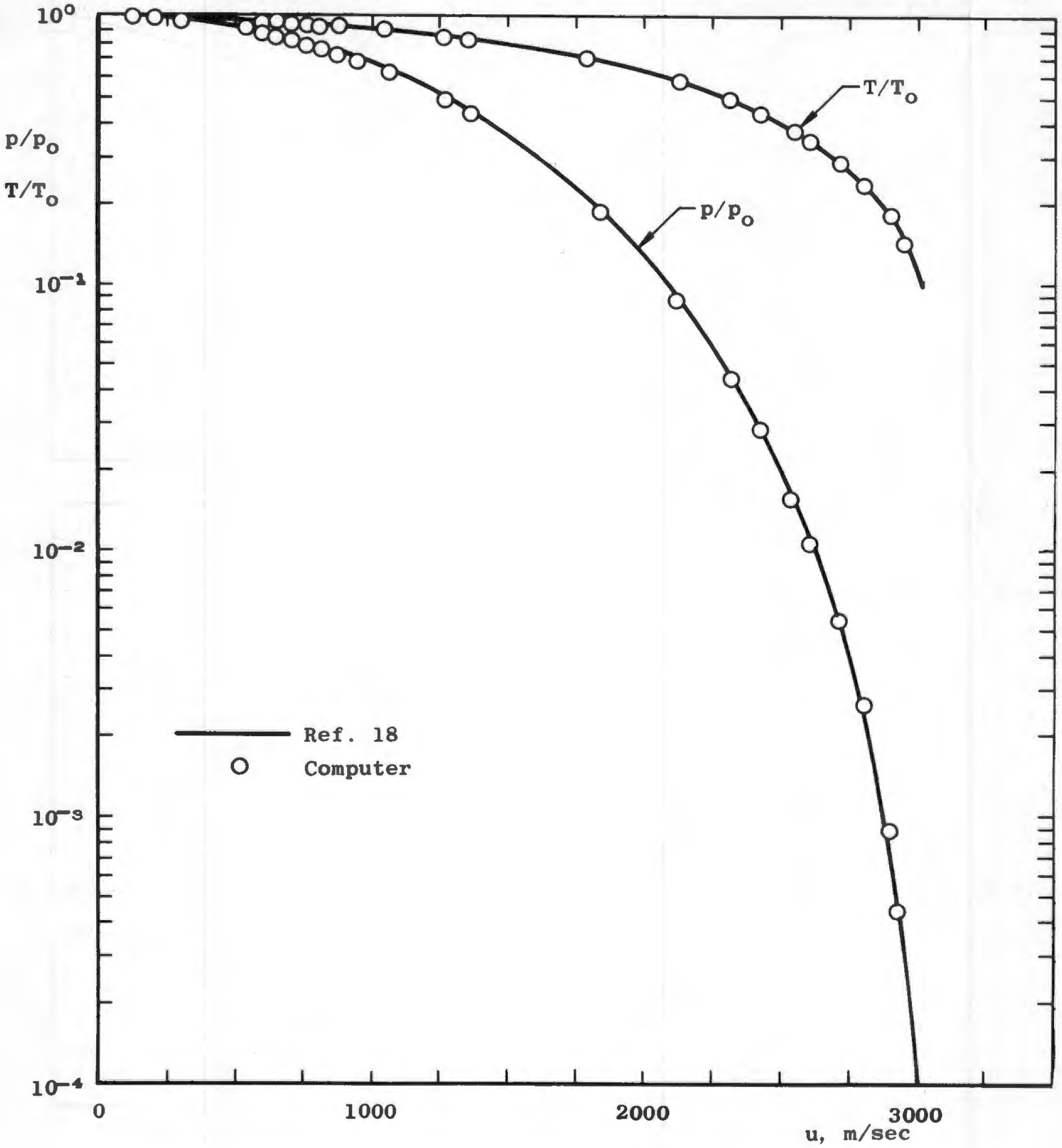
h. Maximum Mach Number MHD Channel Flow

Fig. 2 Continued



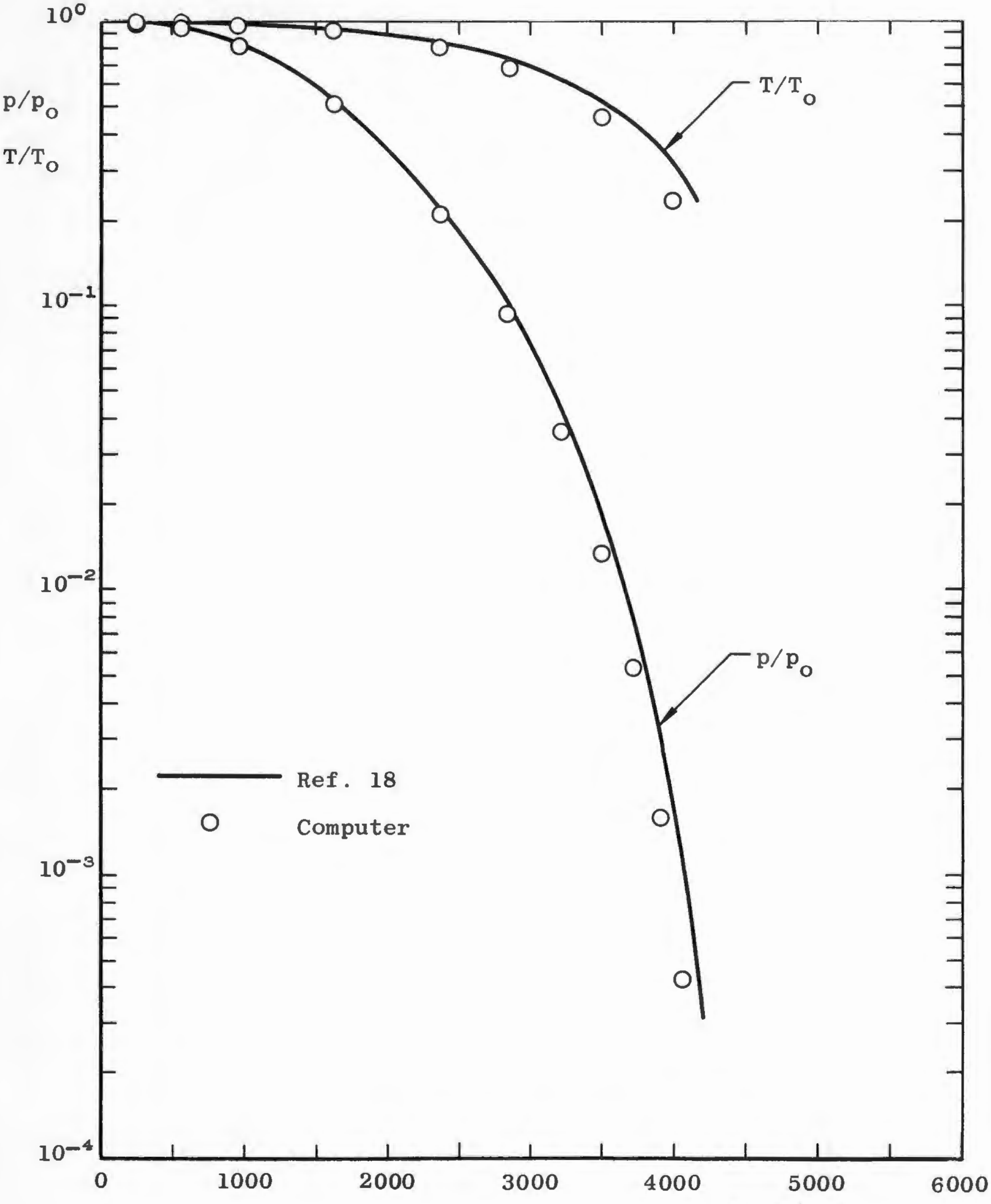
i. MHD Channel Flow with Constant E, B, A, and σ

Fig. 2 Continued



j. Equilibrium Real Gas Isentropic Nozzle Flow (Nitrogen), T_o = 4000°K, p_o = 10 atm

Fig. 2 Continued



k. Equilibrium Real Gas Isentropic Nozzle Flow (Nitrogen), T_o = 6500°K, p_o = 20 atm

Fig. 2 Concluded

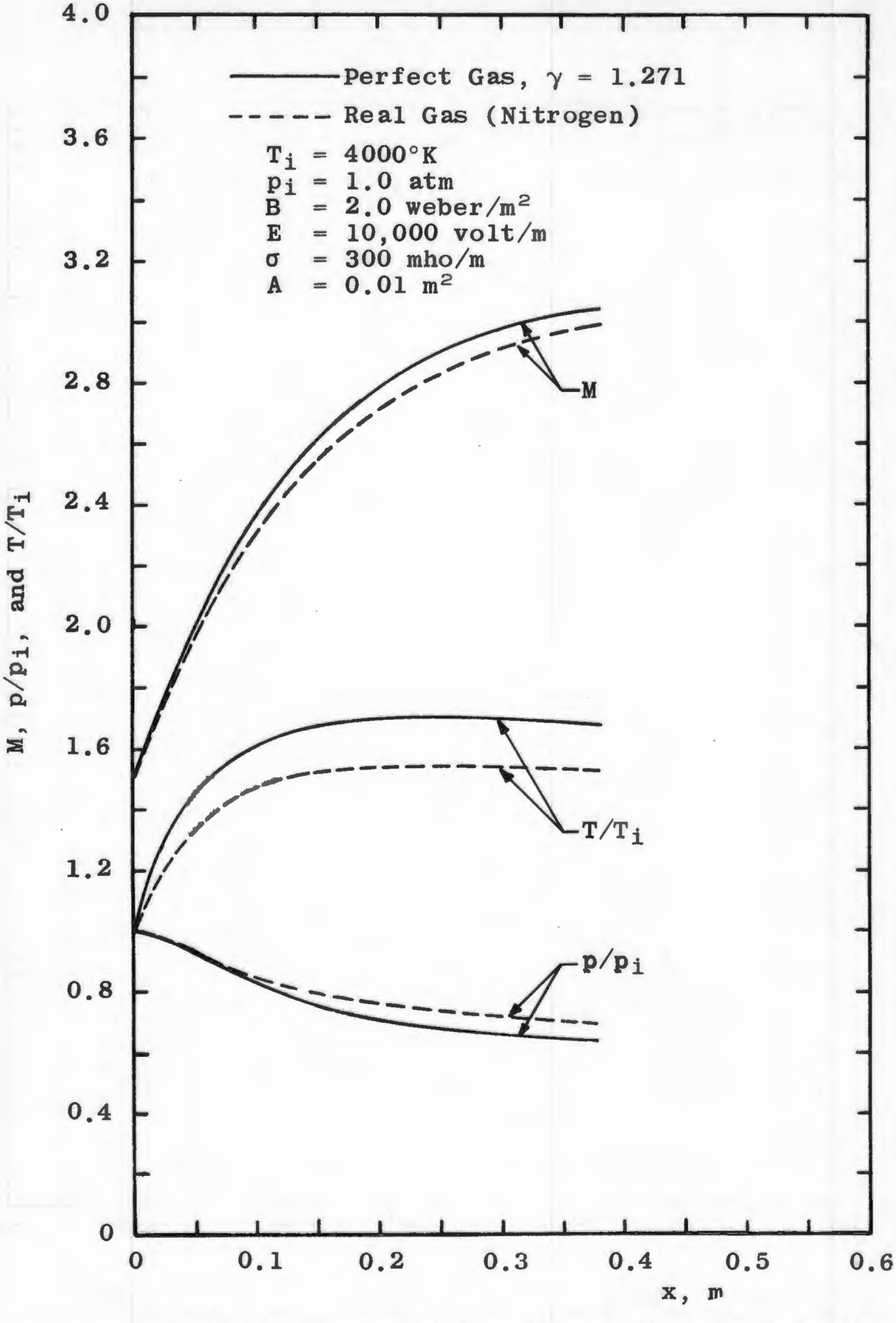


Fig. 3 Example of Real Gas Effects on MHD Channel Flow with Constant E, B, A, and σ

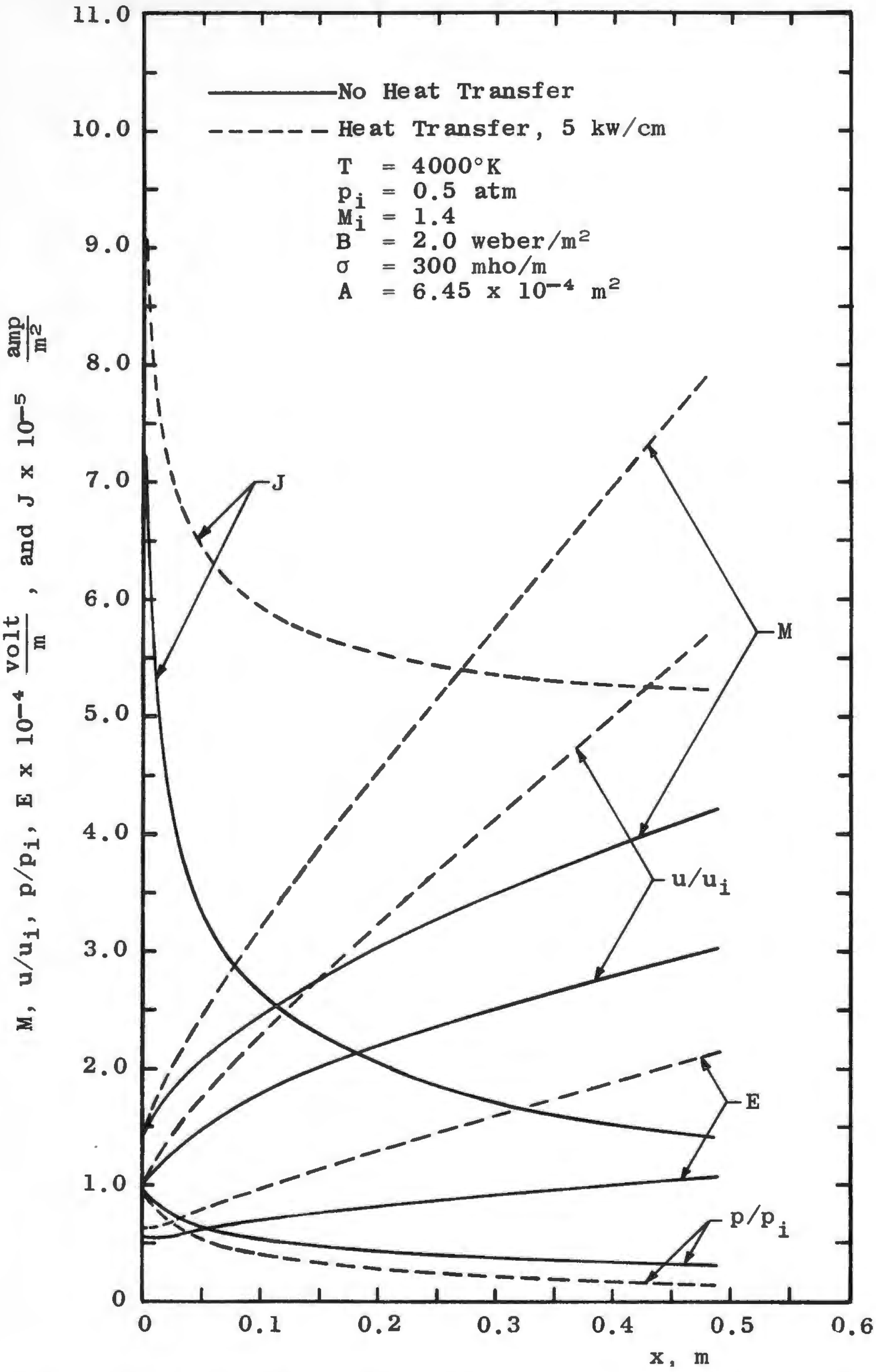


Fig. 4 Example of Heat-Transfer Effects on Isothermal, Constant Area, MHD Channel Flow