NON-LINEAR THERMAL STRESS ANALYSIS FOR NUCLEAR POWER PLANT BY FINITE ELEMENT METHOD

WATARU MIZUMACHI

Toshiba Electric Co.,
Atomic Power Division, Tokyo, Japan

This paper presents the non-linear thermal stress analysis for the nuclear power plant by the finite element method. The heat equilibrium equation becomes the non-linear differential equation when the thermal conductivity is dependent on the temperature of the body. And the heat flow due to the radiation is in proportion to the fourth power of the absolute temperature by Stafen-Boltzmann law. The finite element method has proven very successful in analyzing these non-linear problems.

Besides, there are two types of non-linearity as far as the stress analysis is concerned. One is the material non-linearity problem and the other is the geometrical non-linearity problem. The material non-linearity problem, namely the elastic-plastic analysis, is expected in the case of designing the nuclear power plant.

The non-linear heat problems and the material non-linearity problem can be analyzed by the same nodal points and the same elements with application of the finite element method.

AFLC—WPAFB—OCT 71 550

1. INTRODUCTION

The thermal loads cause severe stress conditions in the nuclear power plant. Therefore, a precise analysis is required. In the case of the thermal stress analysis, we have to first know the temperature distribution in the structure. Then, the thermal stress can be solved.

A remarkable advantage of the finite element method lies in the fact that both analyses can be done by the same meshes. In general, the non-linear problems are not very successful from the mathematical point of view. It has become convenient to use this method to solve the non-linear problems.

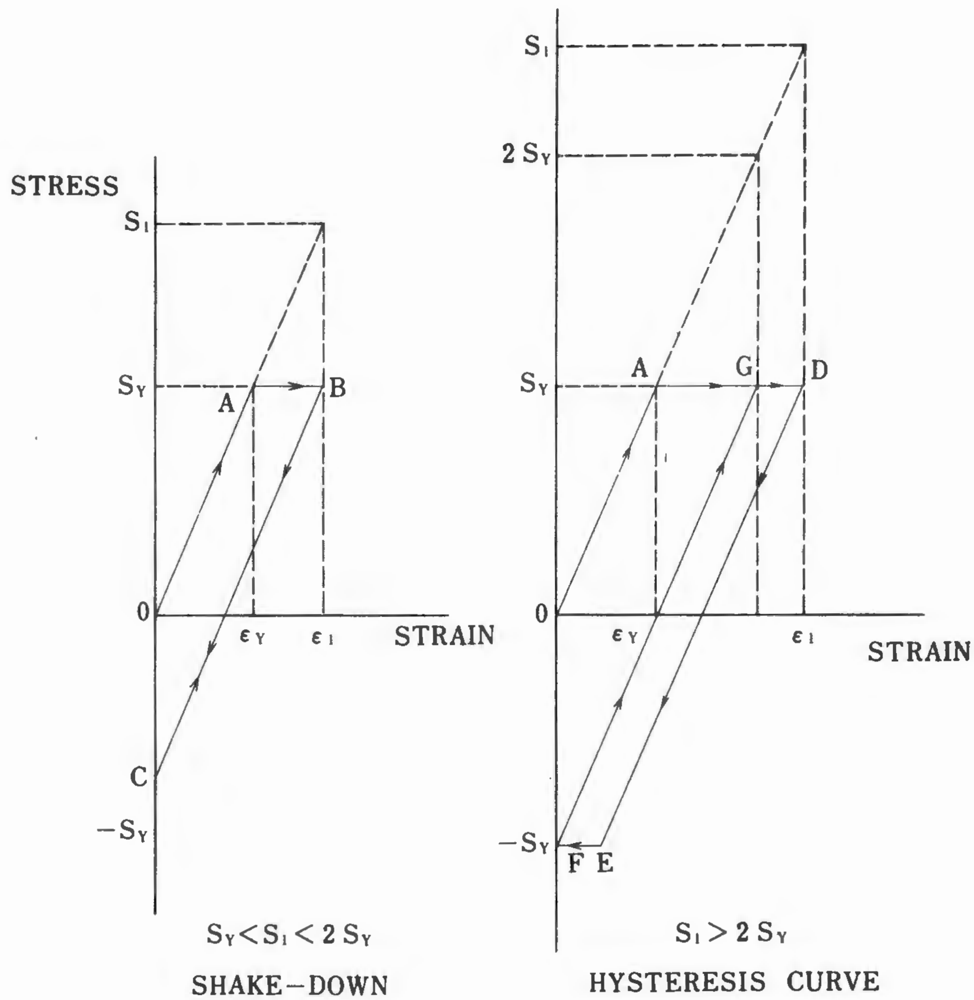
The thermal stress of the reactor pressure vessel, etc., is a self-balancing stress produced by a non-uniform distribution of temperature or by differing thermal coefficients of expansion. If the stress, neglecting stress concentrations, exceeds twice the yield strength of the material, the elastic analysis may be invalid and successive thermal cycles may produce incremental distortion. But if the stress does not exceed twice the yield point, the structure shakes down. The shakedown is the absence of a continuing cycle of plastic deformation. A structure shakes down if after a few cycles of load application, the deformation stabilizes and subsequent structural response is elastic, excluding creep effects. Therefore, not only the elastic behavior, but the plastic behavior should be analyzed.

2. FINITE ELEMENT METHOD

The finite element method was originally developed for the vibration problem of an airplane. And then it proved to be applicable to several other fields, as well as very useful in analyzing the non-linear problems.

However, the big problem is to solve the large matrix (the author calls it the Jumbo Matrix), the order of which is 10,000 to 100,000. It becomes so large because the geometrical shapes of structures in the nuclear power plant are complex and require a three-dimensional analysis. The author has used the conjugate gradient method to solve such complex systems.

Although this method has the remarkable advantages mentioned above, there exists no method without defects. The inherent disadvantage of this method lies in the fact that a lot of elements and nodal points are geometrically required to represent real structures so that reasonably accurate results are obtained. Consequently, the input data specifying the geometry and the topology are inevitably large in quantity and have to be accurate. In order to avoid the large quantiy of input data, the automatic mesh generation routine which provides for an automatic means of discretization for continuous domains is useful. Only the input data for geometrically irregular points are required. The advantages of automating the generation of input data are obvious. This facility can reduce the quantity of input data drastically, and so saves the hand-labor time and avoids the probability of human error involved in the preparation of data. Nevertheless, there might exist input errors. It is ridiculous to find the input errors after the data have been computed because the calculations to analyze the complex systems such as the nuclear power plant, take a couple of hours and need the large core memory in the computer. Therefore, any errors in geometry and topology should be previously detected by using a plotting system. This device also produces plots of the deformed structure, and stress contours.



A. S. M. E. BOILER AND PRESSURE VESSEL CODE

(SECTION – III)

"NUCLER VESSELS"

3. NON-LINEAR HEAT TRANSFER ANALYSIS

The finite element method is effecient in getting the solution for non-linear heat transfer problems because it is completely general with respect to geometry, material properties, and boundary conditions. In general, non-linear problems are idealized to linear ones in order to avoid difficult and clumsy calculations. However, non-linear problems can be solved as they are by the iteration of the same procedure by using this method.

The transient heat equilibrium equation is expressed as the first order non-linear equation.

$$C(\rho)\frac{\partial T(t)}{\partial t} + K(T)T(t) = Q(t) \qquad (3.1)$$

where

C : Heat Capacity Matrix
ρ : Density
K : Conductivity Matrix
Q : External Heat Flow
T : Temperature
t : Time

It is reasonable to assume that the flow-temperature curve consists of linear segments. Namely, the conductivity matrix is constant within each time increment. We consider equation (3.1) at time t and at time $(t + \Delta t)$. Then equation (3.1) can be written as follows:

$$C\frac{\Delta T_t - \Delta T_{t-\Delta t}}{\Delta t} + K\Delta T_t = Q_{t+\Delta t} - Q_t \qquad (3.2)$$

Then equation (3.2) becomes

$$K^* \Delta T_t = Q_t^* \qquad (3.3)$$

where

$$K^* = \frac{C}{\Delta t} + K \qquad (3.4)$$

$$Q^* = \frac{C}{\Delta t}\Delta T_{t-\Delta t} + (Q_{t+\Delta t} - Q_t) \qquad (3.5)$$

$$\Delta T_t = T_{t+\Delta t} - T_t \qquad (3.6)$$

$$\frac{\partial T_t}{\partial_t} = \frac{T_t - T_{t-\Delta t}}{\Delta t} \qquad (3.7)$$

The non-linear heat equilibrium equation yields equation (3.3), which can be solved directly for the temperature increments at the end of the time increment.

In order to determine the conductivity matrix, the temperature distribution within each element is assumed to be linear to the coordinates of the nodal points. Dr. E.L. Wilson (Univ. of California, Berkeley) gives this method.

For radiation problems, the net flow transferred from one surface to another is given by Stafen-Boltzmann law.

$$Q = \sigma(T_1^4 - T_2^4) \qquad (3.8)$$

where

$\sigma = 4.88 \times 10^{-8}$ kcal/m^2 h °K^4
(Stafen-Boltzmann Coefficient)
T : Absolute Temperature

Assuming that the difference between T_1 and T_2 is small, the pseudo-conductivity matrix is given. Then the temperature distribution due to the radiation can be solved by the finite element method.

4. STRESS ANALYSIS

The element stiffness matrix of the finite element method can be written as follows by applying the principle of virtual work and the Castigliano theory.

$$[K]_e = [A^{-1}]^T [\iiint [B]^T [D] [B] \, dV] [A^{-1}] \qquad (4.1)$$

where

$[K]_e$: Element Stiffness Matrix

$[A]$: Matrix of Displacement Functions (Coordinate Matrix)

$[B]$: Strain Coefficient Matrix

$[D]$: Stress Strain Matrix (Material Matrix)

The stiffness matrix of the structure is given by assemblying the element stiffness matrices.

$$[K] = \sum_{e=1}^{N} [K]_e \qquad (4.2)$$

As far as the thermal stress is concerned, the thermal load is given as follows:

$$\{F\} = -\iiint [B]^T [D] \{\epsilon_0\} \, dV \qquad (4.3)$$

where

$$\{\epsilon_0\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \alpha_x T \\ \alpha_y T \\ \alpha_z T \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (4.4)$$

$\{F\}$: Thermal Load

$\{\epsilon_0\}$: Thermal Strain

α : Coefficient of Thermal Expansion

At first displacements of each nodal point due to the thermal load can be solved.

$$\{u\} = [K]^{-1}\{F\} \qquad (4.5)$$

The stiffness matrix $[K]$ of equation (4.5) has the order of 10,000 to 100,000 to analyze structures with geometrically complicated shapes. This order is too large to apply normal methods, for example, the Gauss method. Therefore, the special method to solve equation (4.5) is expected for the finite element method..

The conjugate gradient method is successful in solving the large sparse matrix. It is inherently required to store the addresses of non-zero elements in the computer core. On the other hand, the addresses of non-zero elements should be stored when the finite element method is applied, because this method requires knowing each number of nodal points. This fact shows that the conjugate gradient method is especially nice for solving the finite element method. Besides, there is one more big advantage. The order of the stiffness matrix becomes so large because the element stiffness matrices are assembled by equation (4.2). In the case of using the conjugate gradient method, the values and addresses of non-zero elements are needed. In other words, the element matrices need not be assembled.

The following is the procedure by which the stress analysis can be done directly by the element matrices without having the assembled stiffness matrix of a structure. Assume the initial vector to solve equation (4.5).

$$\{u\}_1 = \{\text{Random Number}\} \qquad (4.6)$$

Or unless specified, it is assumed to be equal to the thermal load vector defined by equation (4.3).

$$\{u\}_1 = \{F\} \qquad (4.7)$$

Iteration

$$\{u\}_{i+1} = \{u\}_i + \alpha_i \{S\}_i \qquad (4.8)$$

where

i : Iteration Number

The remainder $\{G\}$ is

$$\{G\}_{i+1} = \{\{F\} - [K]\{u\}_i\} \qquad (4.9)$$

The direction of movement $\{S\}$ is

$$\{S\}_{i+1} = \{G\}_{i+1} + \beta_i \{S\}_i \qquad (4.10)$$

where

$$\beta_i = \frac{\{G\}^T_{i+1} \{G\}_{i+1}}{\{G\}^T_i \{G\}_i} \qquad (4.11)$$

The step length α along the direction is

$$\alpha_i = \frac{\{S\}^T_i \{G\}_i}{\{S\}^T_i [K] \{S\}_i} \qquad (4.12)$$

The convergence condition is

$$\frac{\sqrt{\sum_{j=1}^{n} (\{u_j\}^2_{i+1} - \{u_j\}^2_i)}}{\sqrt{\sum_{j=1}^{n} \{u_j\}^2_i}} < \epsilon \qquad (4.13)$$

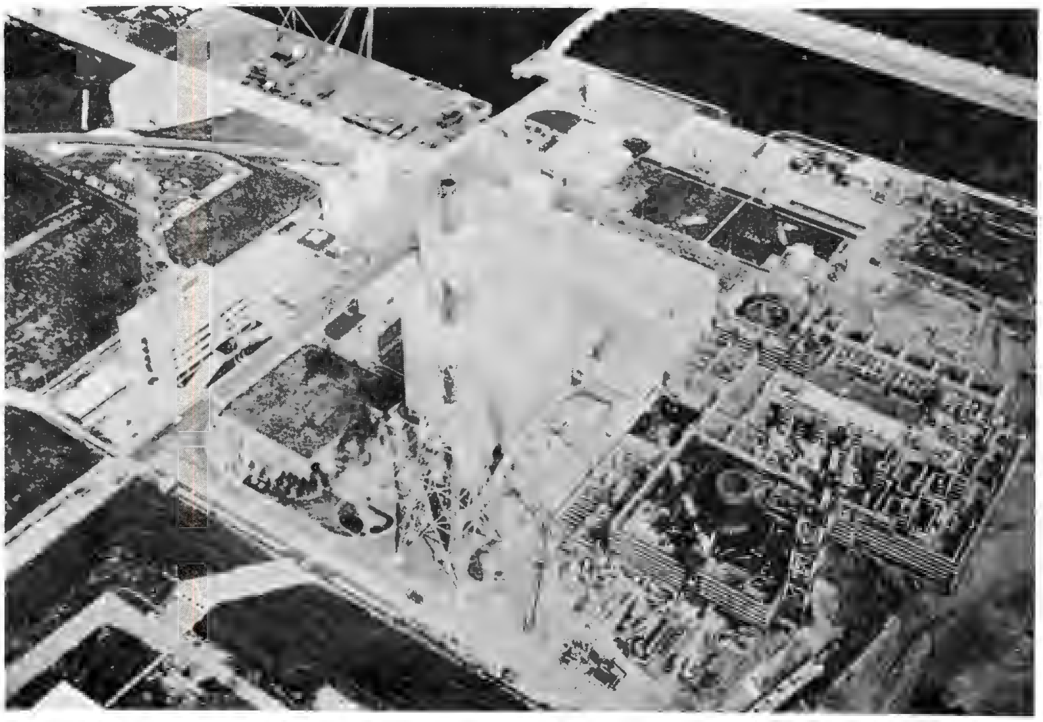
5. PLOTTER OUTPUTS OF THERMAL ANALYSIS

(1) Atomic Reactor Building

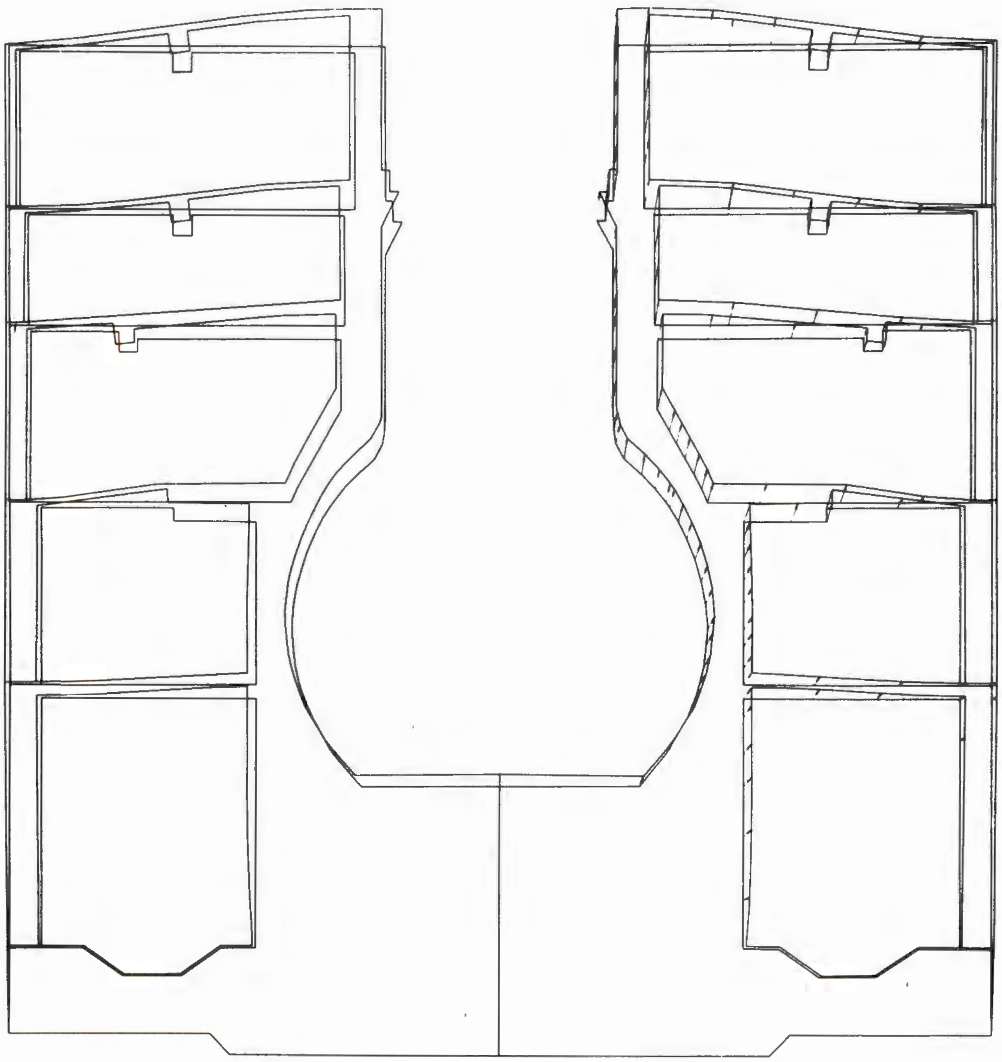
① TSURGA–I

This photograph is the nuclear power station at Tsurga, site of the Japan Atomic Power Co., which provided electricity for Expo '70 held near Osaka. The type is the Boiling Water Reactor (BWR).

② TEPCO–I, II, and III (FUKUSHIMA)



③ Thermal Stress Analysis



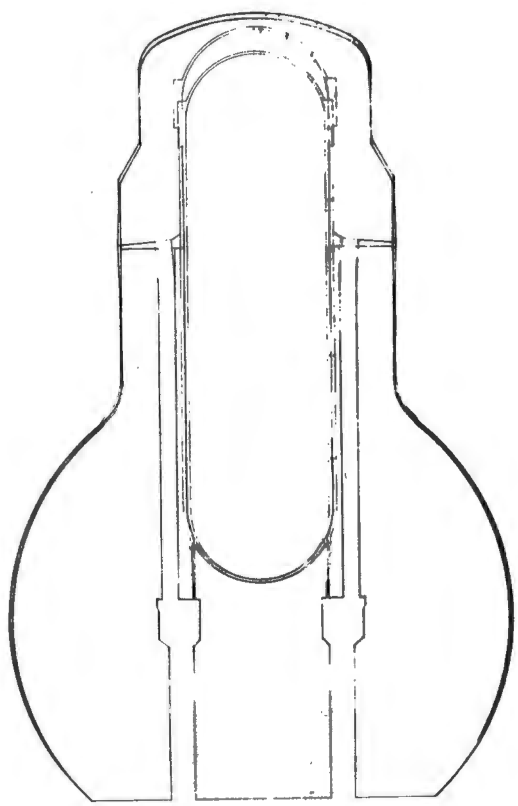
THERMAL STRESS ANALYSIS
ATOMIC REACTOR BUILDING

(2) Reactor Pressure Vessel, Primary Containment Vessel and Pedestal

① Reactor Pressure Vessel (TEPCO–I)

② Primary Containment Vessel (TSURGA–I)

③ Thermal Stress Analysis



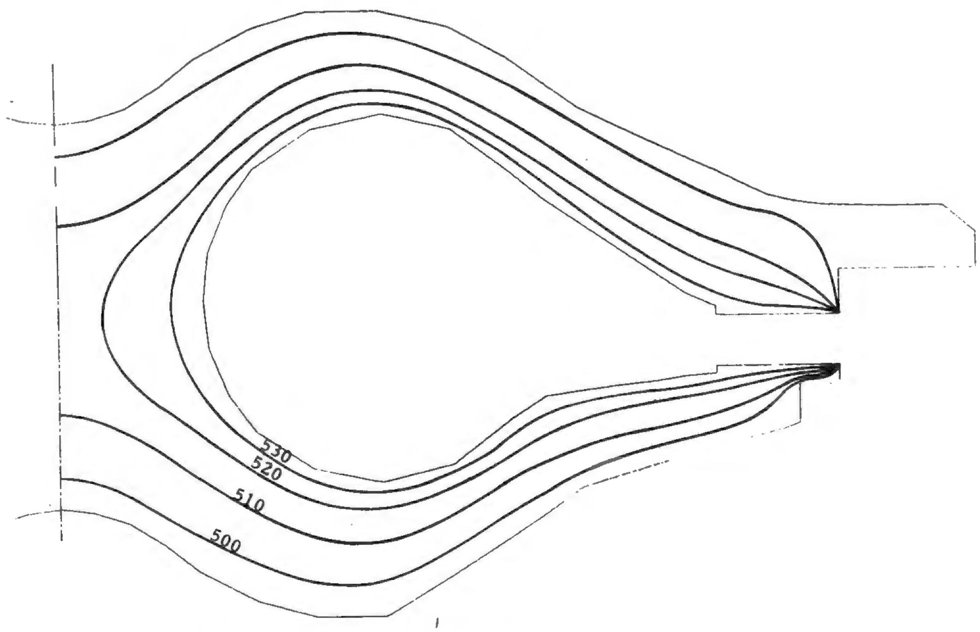
STRESS, ANALYSIS
THERMAL, PRESSURE, GRAVITY
RPV PCV SHIELD WALL

(3) Plastic Analysis for Turbine Nozzle Box

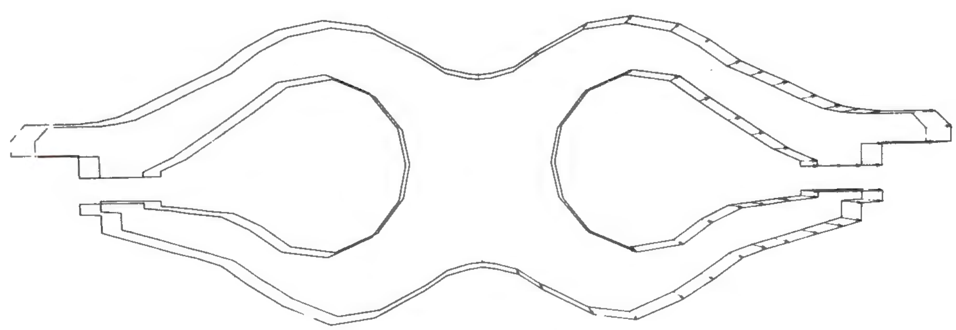
① Turbine Nozzle Box

The stress of the turbine nozzle box due to the thermal load and pressure is very severe. The stress should not exceed the yield stress. However, it is important to know the plastic behavior. Following are the results of the material non-linear analysis and those of the temperature distribution.

② Temperature Distribution



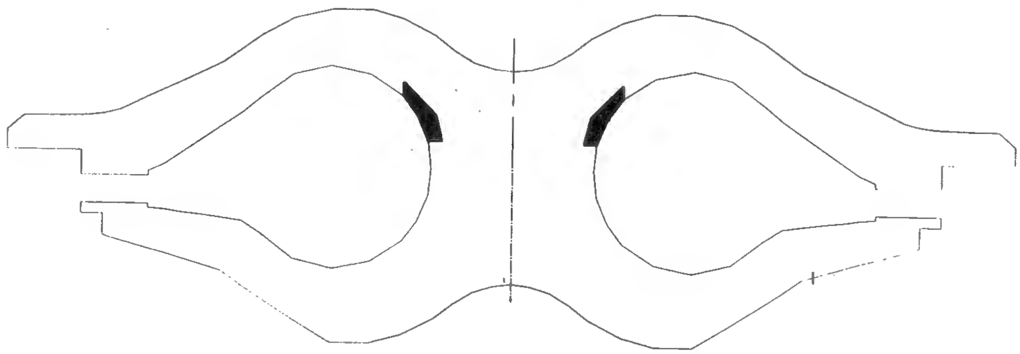
③ Elastic Thermal Stress Analysis

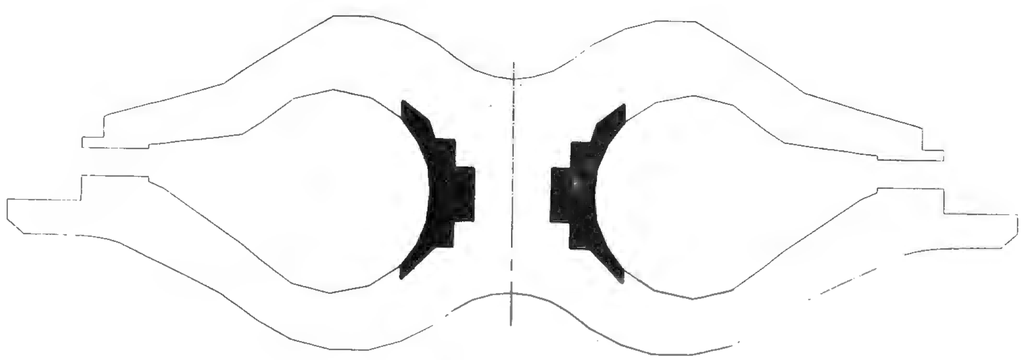


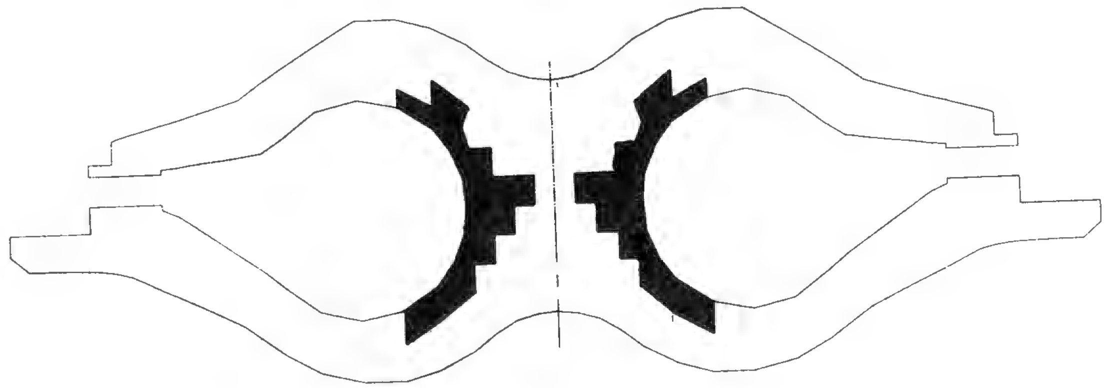
STRESS ANALYSIS
(PRESSURE;THERMAL)
TURBINE NOZZLE BOX

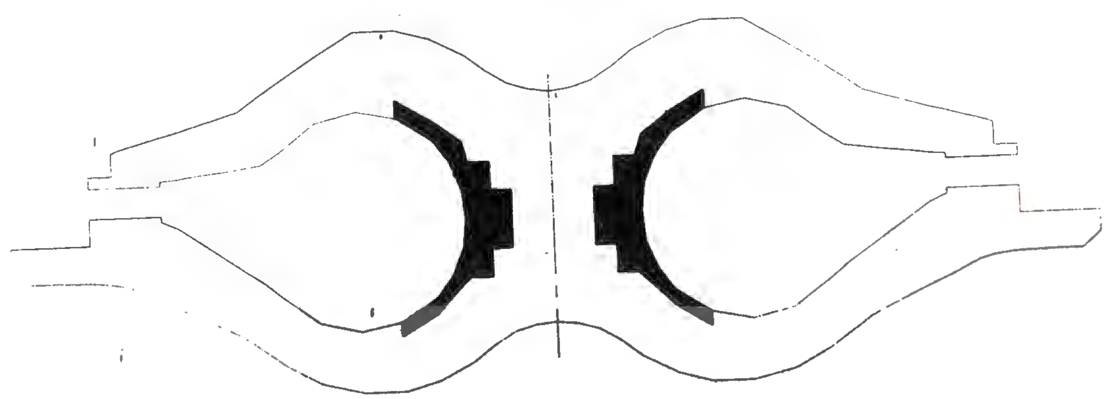
④ Plastic Thermal Stress Analysis

The black area exceeds the yield point.









6. CONCLUSION

Some kinds of stress analyses are expected when we design a nuclear power plant. These are the thermal stress, the stress due to pressure, and the stress due to the external force including its own weight, earthquake force and so on. Normally, the value of the thermal stress is the highest one among these kinds. Therefore, a precise thermal stress analysis should be expected.

The remarkable advantage of the finite element method lies in the fact that this method has complete generality with respect to geometry, material properties, and boundary conditions. Besides, the non-linear heat problem and the material non-linear stress analysis can be solved by the same meshes.

However, this method has two disadvantages. One is that the order of the matrix is very large. Another is that the input data specifying the geometry are large in quantity. The author uses the conjugate gradient method to treat the large matrix and develops the automatic mesh generation routine in order to avoid the large quantity of the input data.

Finally, it should be emphasized that it is not better to divide the model into more elements, even though the large matrix can be treated easily. We should select the point where the error of the geometrical shape and that of the numerical calculation balance. And also, we should select the most reasonable displacement function for each problem.

7. REFERENCES

(1) Wilson, E.L., "Non-linear Heat Transfer Analysis of Axisymmetric Solids" *Report of Structural Engineering Laboratory,* Univ. of California, Berkeley (1971)

(2) Zienkiewicz, O.C., and Cheung, Y.K., "The Finite Element Method in Structural and Continuum Mechanics" *McGraw Hill* (1967)

(3) "Japan-U.S. Seminar on Matrix Methods of Structural Analysis and Design" *Society of Steel Construction of Japan* (1969)

- 1 Clough, R.W., "Analysis of Structural Vibrations and Dynamic Response"
- 2 Marcal, P.V., "Finite Element Analysis with Material Non-Linearities"
- 3 Melosh, R.J., "Manipulation Errors in Finite Element Analyses"
- 4 Kawai, T., "Finite Element Analysis of Shell Structures"
- 5 Yamada, Y., "Recent Japanese Developments in Displacement Method for Elastic-Plastic Problems"

(4) Miyamoto, H., and Miyoshi, T., "Elastic and Plastic Analysis by Finite Element Method" *Society of Steel Construction of Japan* (1968)

(5) Akyuz, F.C., "Natural Co-ordinate Systems. An Automatic Input Data Generation Scheme for a Finite Element Method" *Nuclear Engineering and Design* (1970)

(6) Mizumachi, W., "Application of a Finite Element Method to the Atomic Power Plant" *Report of Research Institute for Mathematical Sciences,* Univ. of Kyoto (1971)

(7) Mizumachi, W., "How to Solve the Large Matrix" *Journal of the Computer Sciences & Information Processing,* Japan Computer Society (1971)

(8) Mizumachi, W., "Analyses for Atomic Power Plant by Finite Element Method" *Society of Steel Construction of Japan* (1971)

(9) Mizumachi, W., "Application of Finite Element Method to the Three-Dimensional Stress and Vibration Analysis of BWR Primary Plant Systems" *Structural Mechanics in Reactor Technology,* First International Conference, Bundesanstalt für Materialprüfung (BAM), Berlin (1971)