FOREWORD

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ABSTRACT

To determine the feasibility of using long tetherlines to aid in orbital docking or to recover astronauts operating outside a vehicle an analysis was made of the behavior of an inert mass on the end of a tetherline as it is reeled in toward a more massive vehicle. Only coplanar motions were considered. Three methods of reeling the mass in toward the vehicle were analyzed: impulsive jerks on a slack tether tetherline, constant line tension, and constant reel-in speed. The first two methods result in complex paths which intersect the vehicle only after long periods of time. The third method results in a spiral path in which both the velocity and centripetal acceleration of the mass approach infinity as the line length approaches zero. Since angular momentum of the mass around the vehicle is very nearly conserved, the magnitude of the retrieval problem depends primarily on the initial length of the tetherline and the initial tangential velocity of the mass. Reeling in a line a few hundred feet in length can result in forces which would be intolerable to an astronaut.

PUBLICATION REVIEW

This technical documentary report has been reviewed and is approved.

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A DIGITAL COMPUTER ANALYSIS OF THE BEHAVIOR

OF LONG TETHERLINES IN SPACE

INTRODUCTION

Future space operations will no doubt involve frequent use of orbital rendezvous and docking techniques. Improvements in extravehicular pressure suits should enable future astronauts to engage in inspection, maintenance, resupply, or rescue tasks outside an orbiting vehicle. A self-maneuvering unit (SMU) has already been designed for the Air Force by Vought Astronautics Division (ref. 2) and a prototype model is currently being evaluated in the zero-gravity research aircraft at Wright-Patterson Air Force Base (figure 1). Many schemes have been suggested for orbital docking, some of which involve the snagging of long ropes or tetherlines by the interceptor vehicle followed by a winching together of the interceptor and target (ref. 2).

In the situation where a man wearing an SMU must travel from his own vehicle to some other object in orbit and return at a later time, the long tetherline has been proposed both as a means of reducing the fuel required to make the return trip and as a safety precaution in the event the SMU should become inoperative. In the latter case, the thinking has been that the man could be reeled in like a fish on the end of a line.

In this paper we show that such schemes may be ill-founded in view of the orbital forces present and could be disastrous to the astronaut or vehicles involved.

An IBM 1620 digital computer was used to compute the trajectories of an inert mass on the end of a tetherline for each of the three methods of retrieval which were investigated.
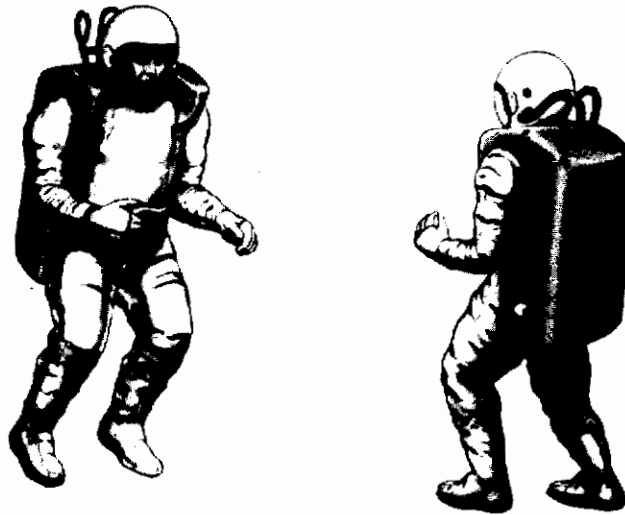
Figure 1. Typical Self-Maneuvering Unit

EQUATIONS OF MOTION

In describing the problem, a rotating xyz-coordinate frame attached to an orbiting vehicle in a circular or near-circular orbit around the earth, moon, or another planet is assumed. As shown in figure 2, the y-axis lies along the local vertical in the orbital plane and the x-axis lies along the local horizontal in the direction of orbital motion. The z-axis is perpendicular to the orbital plane and in a direction consistent with a right-handed coordinate frame.
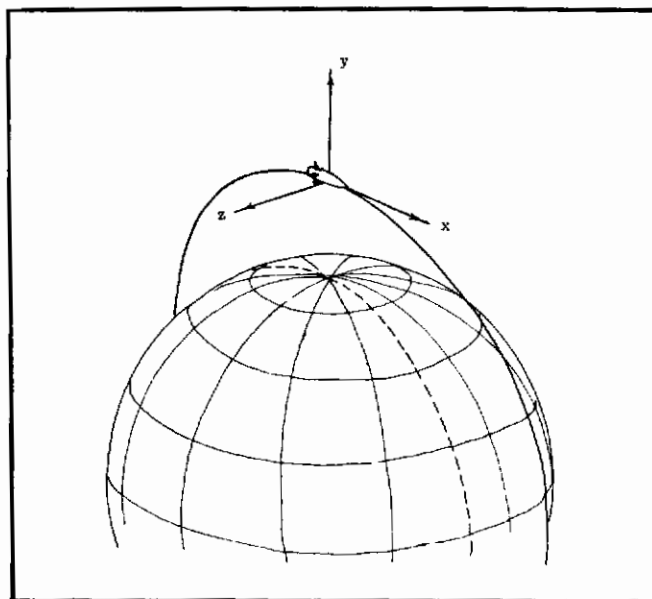

Figure 2. Coordinate Reference Frame

The linearized equations which describe the motion of a mass in this coordinate frame have been derived and discussed in detail by many authors (refs. 1, 3):

$$\ddot{x} = -2\omega\dot{y} + f_x \tag{1}$$

$$\ddot{y} = 2\omega\dot{x} + 3\omega^2 y + f_y \tag{2}$$

$$\ddot{z} = -\omega^2 z + f_z \tag{3}$$

where ω is the angular velocity of the coordinate frame with respect to inertial space about the z-axis and f_x, f_y, and f_z are the x, y, and z components of external force per unit mass acting on the mass in question. In this paper, the only external forces assumed to act are due to the tetherline. We also assume that the mass of the parent vehicle is considerably larger than the mass at the end of the tetherline so that the orbit of the parent vehicle remains unchanged while the orbit of the smaller mass changes in response to tetherline forces. This is not so serious a limitation as one might suppose since it can be shown that, even when the two masses at the opposite ends of the line are equal, the relative motion between them is the same as that described below.

Only coplanar motions will be considered in this paper, so we need concern ourselves only with equations 1 and 2.

IMPULSIVE JERKS ON A SLACK TETHERLINE

In considering the technique to be employed in retrieving a man on the end of a long tetherline, it is obvious that so long as tension is maintained on the line the man will be accelerated. Since accelerating the man continuously to higher and higher closure velocities would be undesirable, it is instructive to see what happens if the line is impulsively jerked so as to give the man a small initial closure velocity toward the parent vehicle and then to allow the man to coast in toward the vehicle while the line remains slack.

The coasting trajectory of the man will be determined by solving equations 1 and 2 with f_x and f_y equal to zero. If the line becomes taut, however, a rebound will occur which will cause the radial component of velocity to reverse while the tangential component remains constant. The coasting trajectory subsequent to such a bounce will be determined by the same equations with appropriately altered initial conditions.

As a numerical example, we will consider the vehicle to be in a 200-nautical-mile earth orbit ($\omega = 0.00114$ radian per second) with the man initially positioned horizontally ahead of the vehicle in the orbital plane and at rest relative to it in the xyz-coordinate frame.

The motion which ensues depends only on the ratio of the initial distance, x_o, to the initial velocity, $\dot{x}_o$. Figure 3 shows the trajectories which result for four different values of $x_o/\dot{x}_o$. If, for example, we assume that the man is initially 500 feet ahead of the vehicle, figure 3a would represent the case in which the line is jerked so as to give the man an initial coasting velocity of 5 feet per second. Figure 3b would then represent an initial coasting velocity of 2 feet per second; figure 3c, a velocity of 1 foot per second; and figure 3d, a velocity of $\frac{1}{2}$ foot per second. The time in seconds at which each rebound occurs is labeled in figure 3.

The consequence of giving the man a small closure velocity toward the vehicle is to reduce the orbital velocity of the man by this same amount. As a result, the man, in addition to coasting toward the vehicle, drops to a lower orbital altitude as shown. When the line is stretched to its original length, it becomes taut again, and, assuming that the line has a high modulus of elasticity, the man is jerked sharply toward the vehicle. As a result of orbital forces, the man again describes a curved path terminating in a similar rebound, repeating the process indefinitely.
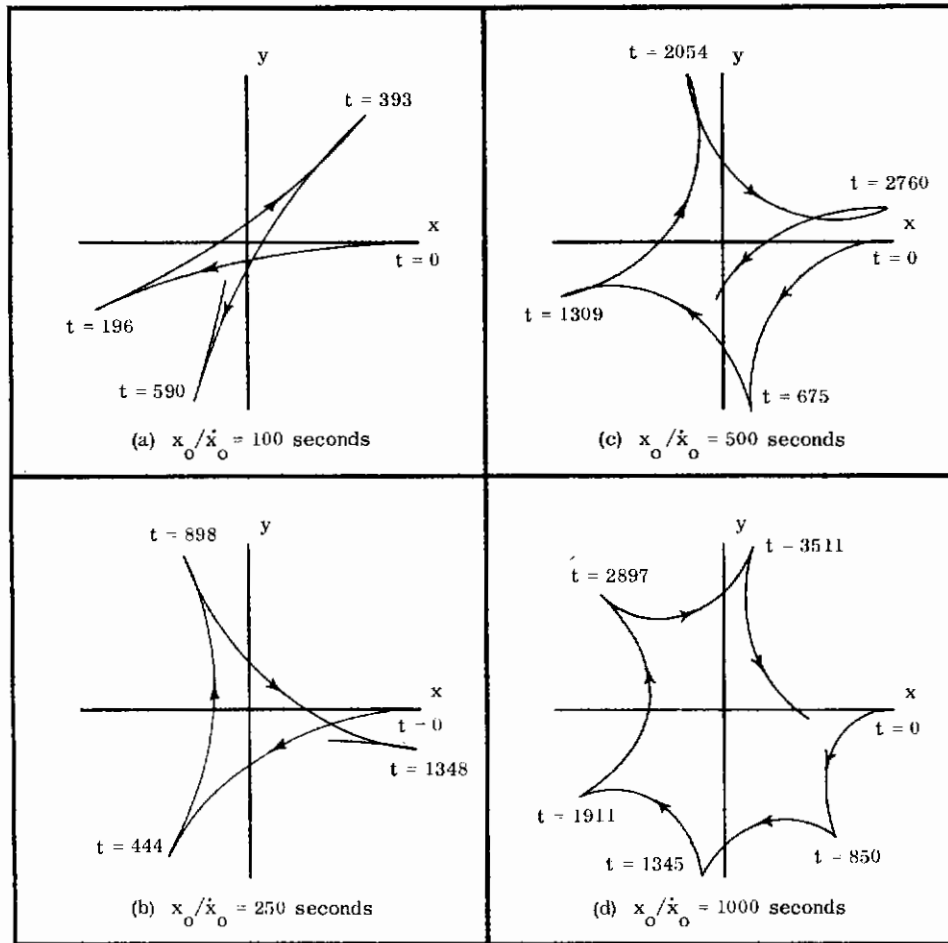
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Figure 3. Path of a Mass on the End of a Slack Tetherline after an Initial Impulse (Mass Initially Ahead of Vehicle)

There is evidence (see figure 3c) that the trajectory approaches the vehicle after many bounces, but the long times involved would seem to preclude this technique as a method for retrieving a disabled astronaut.

Several alternatives are, however, still open to us. We could, for example, reel in the slack line up to the point of closest approach at which time the line would become taut. If nothing else were done the man would circle endlessly about the vehicle. By now we can see the real nature of the problem: the initial velocity of the man which was originally radial has been transformed into a tangential velocity with a corresponding build-up of angular momentum about the vehicle. Since we are assuming that the man has lost his means of propulsion, there is nothing that we can do by pulling radially on the line that will remove this tangential velocity. Furthermore, the tendency for angular momentum to be conserved means that, if the line length is decreased to zero, the tangential velocity will tend toward infinity.

If the man started from some other initial position, the situation would be similar. Figure 4 shows the trajectories which result if the man is initially positioned directly above the orbiting vehicle and at rest relative to it.* Note in figures 4c and 4d that the gravity gradient effect (which tends to keep a dumbbell-shaped satellite oriented along the local vertical) causes the path of the man to be more sharply curved and tends to keep him above the vehicle's altitude.

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*Trajectories for starting positions behind or below the parent vehicle are identical in shape to those shown in figures 3 and 4 and may be visualized by rotating these figures 180 degrees.
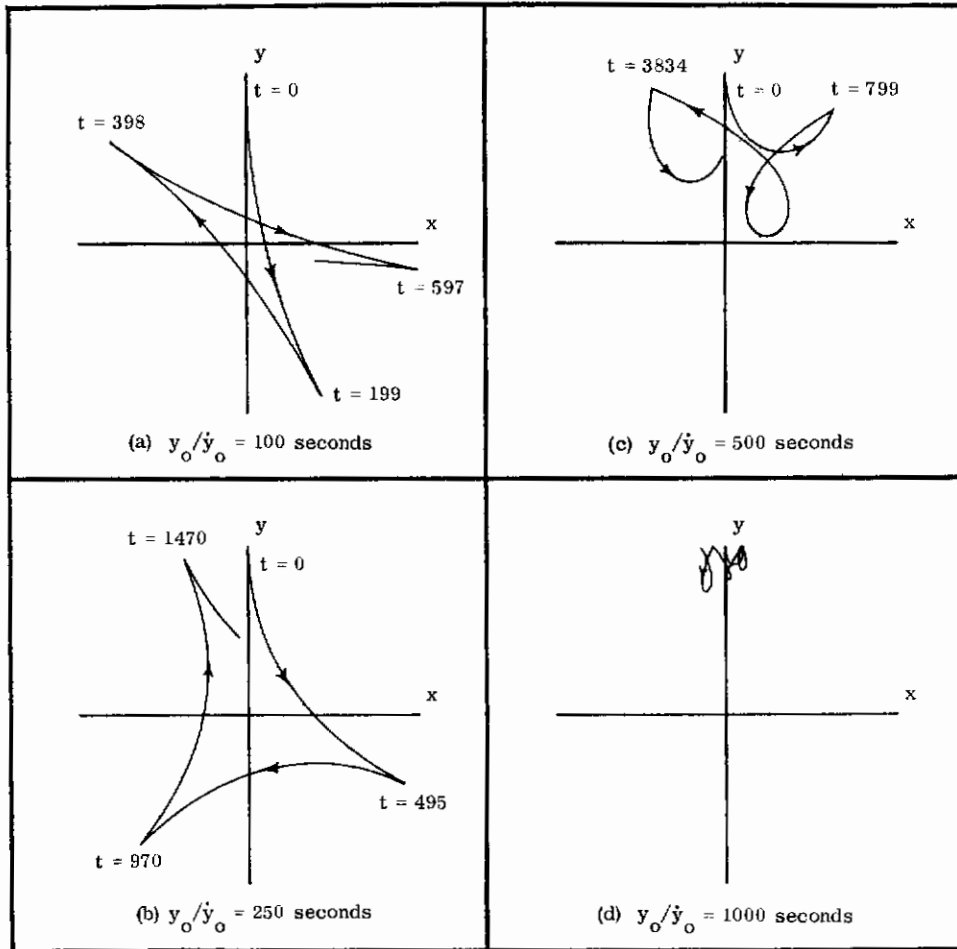
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Figure 4. Path of a Mass on the End of a Slack Tetherline after an Initial Impulse (Mass Initially Above Vehicle)

In figure 4c, the path is so sharply curved that, by the time the first bounce occurs at 799 seconds, the man has acquired a counterclockwise component of tangential velocity. Any time a counterclockwise component exists, it is possible to add to it a radial component by means of the tetherline which will cause the man to impact the vehicle. In the particular case of figure 4c, a small pull on the line after the first bounce could have straightened the path so as to intersect the vehicle.

In all cases, the successful retrieval of an object on the end of a tetherline involves the generation of a favorable component of tangential velocity followed by the addition of a radial component, by means of the line, which will cause impact with the vehicle. Unfortunately, the generation of a favorable component of tangential velocity where none exists initially can take hundreds of seconds, and is feasible only when relatively long times are available for the recovery of the object at the end of the tetherline.

CONSTANT LINE TENSION

An alternative to jerking the line and then leaving it slack would be to maintain a constant tension on the tetherline. This would produce a constant acceleration toward the parent vehicle at all times. The forces per unit mass acting on the mass in this case would be:

$$f_x = -T/m \cos \theta \qquad (4)$$

$$f_y = -T/m \sin \theta \qquad (5)$$

where T is the line tension, m is the mass at the end of the tetherline, and θ is the angle defined in figure 5.
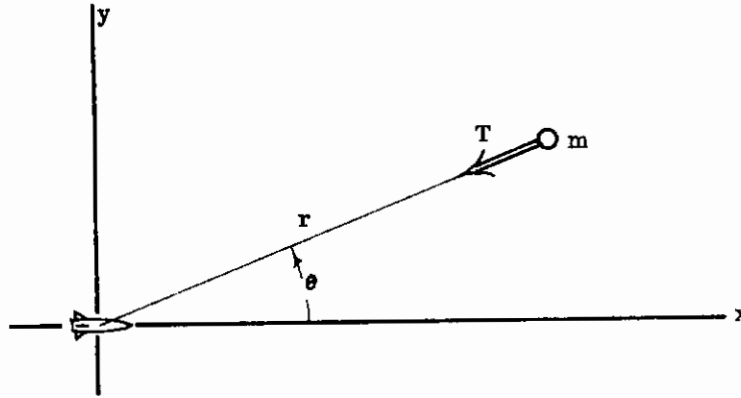

Figure 5. Line Tension Force Acting on Mass

If we again take as a numerical example a mass 500 feet horizontally ahead of a vehicle in a 200-nautical-mile earth orbit, and assume that T/m is equal to 0.01 foot per second[2], the result is similar in many respects to the previous examples.

Figure 6 illustrates the effect of reducing the line tension. Note the greater loss in altitude and more gradual turn-around as the value of T/m is decreased. The trajectory for T/m = 0.005 foot per second[2] is particularly interesting since it indicates that the path intersects the vehicle. The impact occurs after 1450 seconds at a speed of 2 feet per second.

Clearly, then, an object on the end of a tetherline can be retrieved by exerting a constant line tension; but, where no initial tangential velocity exists, the process is time consuming.

Figure 6 also illustrates a curious phenomenon which occurs if the value of T/m is reduced to 0.0025 foot per second[2]. The mass turns around and proceeds ahead of the vehicle, losing altitude steadily. At the end of 5800 seconds the mass is 30,888 feet ahead of the vehicle and 14,321 feet below it with a velocity of 30 feet per second relative to the vehicle. Because the line is continuously reducing the orbital velocity of the mass, the mass is constantly forced into a lower orbit with a shorter period. This behavior is similar to the decay of a satellite orbit due to atmospheric drag. While this particular case is of no interest to us if we are seeking to retrieve the mass, it does suggest that a re-entry from a low earth orbit can be initiated by means of a very long cable without the expenditure of energy. The principle behind such a re-entry is very simple. If we consider the parent vehicle and an escape capsule as a system, we can say that the system possesses a certain total energy by virtue of the fact that it is in orbit. It is impossible to change the total energy of the system without applying an external force and thereby expending energy. But it is possible, by means of an internal force (the line tension in this case), to transfer energy from one part of the system (the escape capsule) to the other (the parent vehicle), thereby causing the escape capsule to re-enter the earth's atmosphere while the parent vehicle rises to a slightly higher orbit. The very low tensions required in such a scheme may make it entirely feasible to carry several hundred miles* of lightweight line as an emergency re-entry system in the event of retro-rocket failure. For the example given above, a line tension of less than 1 pound would be sufficient to cause a 1000-pound capsule to re-enter.

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* Two hundred nautical miles of 15-pound test nylon line, 0.013 inch in diameter and 100 pounds in weight, can be wrapped on a spool 1 foot in diameter and 1 foot wide.
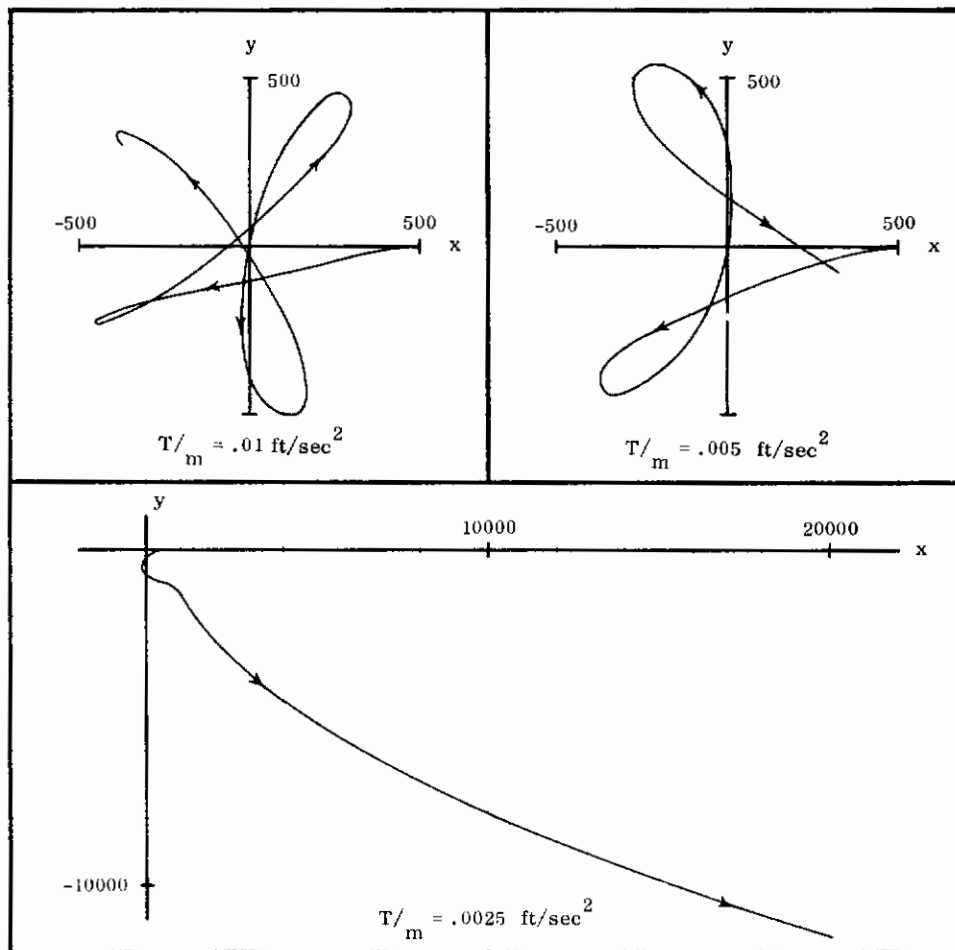
Figure 6. *Path of a Mass on the End of a Tetherline if Line Tension Is Kept Constant*

CONSTANT REEL-IN SPEED

So far we have seen that a mass at the end of a long tetherline can be retrieved by impulsive jerks or by a constant line tension, but only with the expenditure of considerable time. Another quicker and more direct method is available: simply to reel in the line at a constant speed. Thus, if the mass is initially 500 feet from the vehicle and we reel in the line at 1 foot per second, the mass will arrive at the vehicle at the end of 500 seconds, regardless of the path it follows. The equations of motion which describe this situation are:

$$\ddot{x} = -2\omega\dot{y} - (T/m)\frac{x}{r} \tag{6}$$

$$\ddot{y} = 2\omega\dot{x} + 3\omega^2 y - (T/m)\frac{y}{r} \tag{7}$$

$$(T/m) = \frac{(x\dot{y} - y\dot{x})^2}{r^3} + \frac{y}{r}(2\omega\dot{x} + 3\omega^2 y) - \frac{x}{r}(2\omega\dot{y}) \tag{8}$$

where the symbols are as defined by figure 5.

If we again look at the case where the mass is initially horizontally ahead of the vehicle and at rest relative to it, we find that the shape of the resulting trajectory depends only on the time

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required to reel in the line. The reel-in time, t_r, is simply the initial length of the tetherline divided by the rate at which the line is reeled in.

Figure 7 shows the trajectories which result for several different values of reel-in time. Note that angular momentum is built up and that the mass follows a spiral path in toward the vehicle.
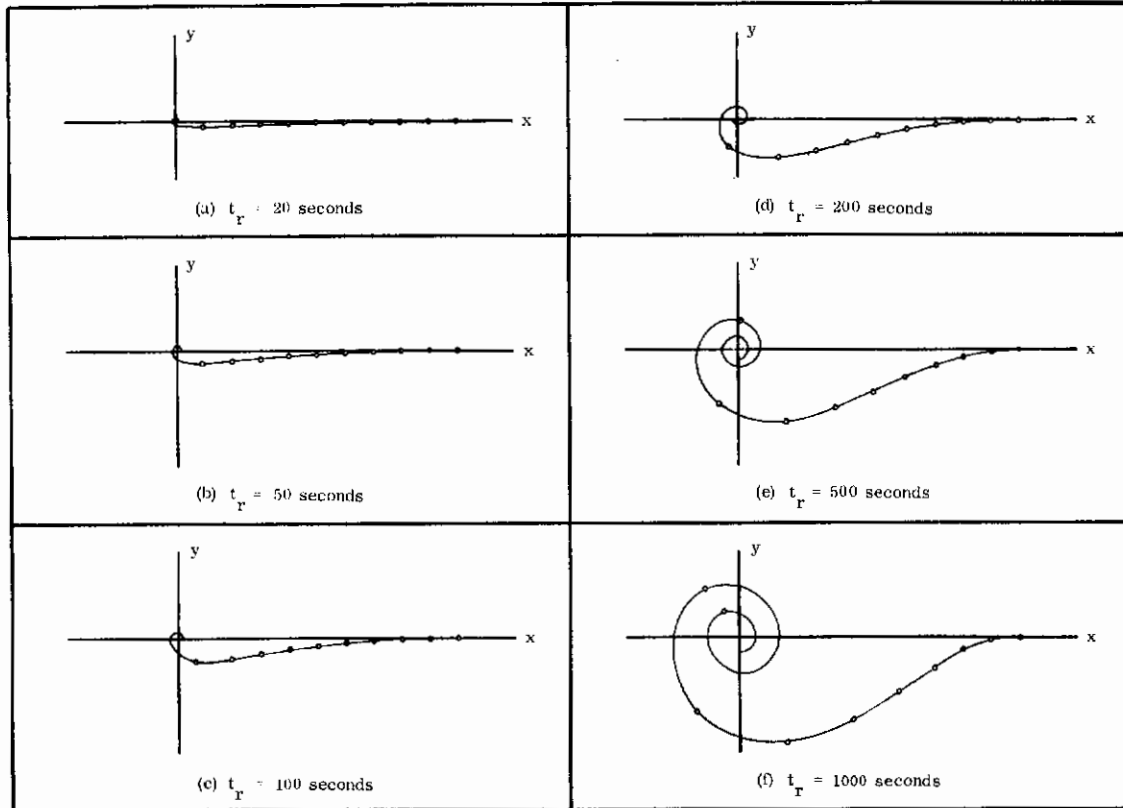

(a) $t_r = 20$ seconds

(b) $t_r = 50$ seconds

(c) $t_r = 100$ seconds

(d) $t_r = 200$ seconds

(e) $t_r = 500$ seconds

(f) $t_r = 1000$ seconds

Figure 7. Path of a Mass on the End of a Tetherline if Line Is Reeled in at Constant Speed

The reason for the build-up in angular momentum is that initially the mass, although it is at rest in the rotating xyz-coordinate frame, has angular momentum around the vehicle (with respect to inertial space) equal to $m\omega x_0^2$. In the absence of gravity gradient forces this angular momentum would be conserved and the mass would possess angular momentum per unit mass as measured in the xyz frame given by the expression:

$$h = \omega(r_0^2 - r^2) \tag{9}$$

where r_0 is the initial line length and r is the instantaneous line length.

The effect of the gravity gradient term ($3\omega^2 y$) in equation 7 is to modify the angular momentum slightly from the values predicted by equation 9. The digital computer data indicated that the deviation from equation 9 increases with reel-in time, approaching 10 percent for reel-in times of 1000 seconds. The actual values of angular momentum were slightly greater than predicted by equation 9 for the case where the mass was initially ahead of or behind the vehicle and slightly less than predicted by equation 9 for the case where the mass was initially above or below the vehicle.

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Figure 8 shows a plot of angular momentum as measured in the rotating xyz frame versus instantaneous line length as predicted by equation 9 for several values of initial line length. The dashed lines in figure 8 show the actual computed values of angular momentum for the case where the mass is initially 500 feet above and also for the case where the mass starts 500 feet ahead of the vehicle. The discrepancy between actual values and those predicted by equation 9 is small and illustrates how nearly angular momentum as measured in a nonrotating frame is conserved.
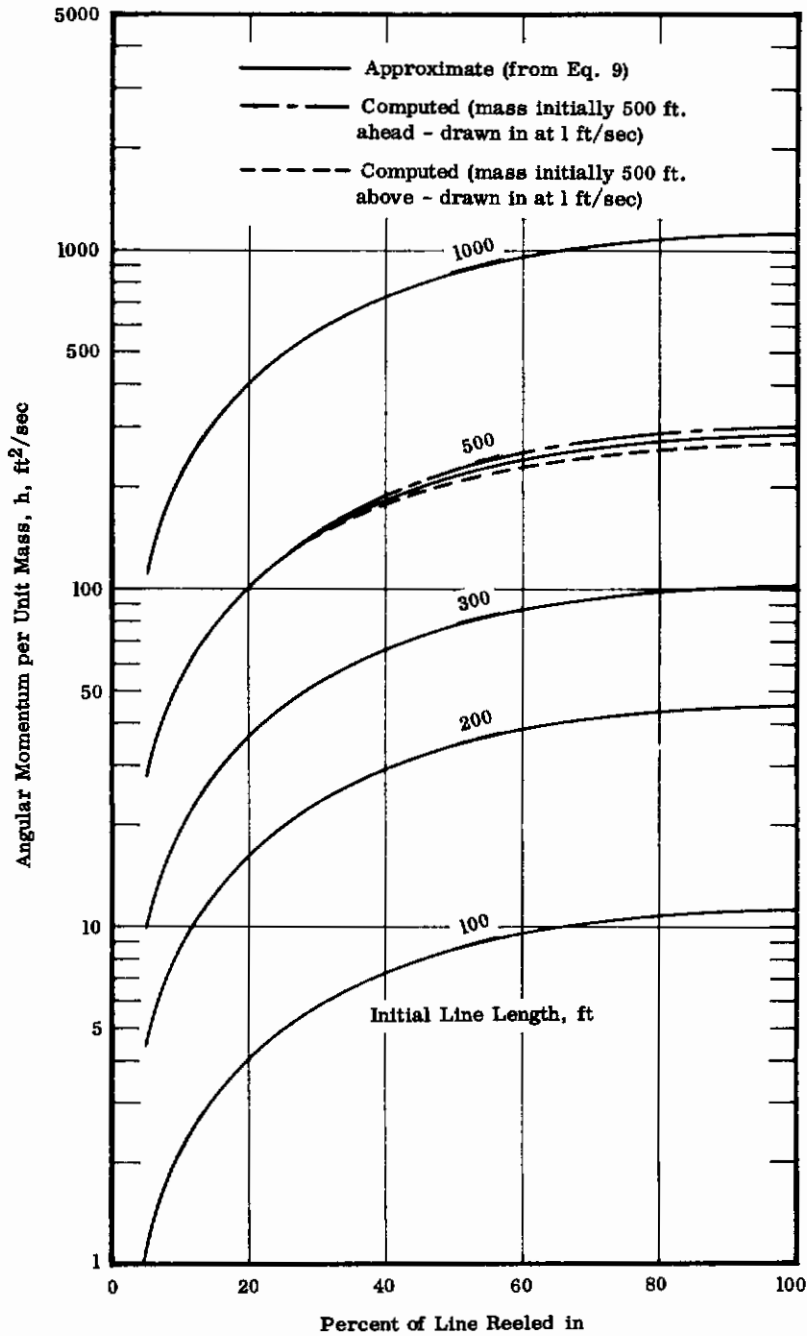

Figure 8. Build-up of Angular Momentum in xyz Frame due to Conservation of Angular Momentum in Inertial Space

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Since tangential velocity, v_t, is related to angular momentum and instantaneous line length by the expression, $h = v_t r$, the build-up in angular momentum consequently causes the tangential velocity to tend toward infinity as the line length approaches zero. At the same time the centripetal acceleration acting on the mass and the tension in the tetherline also tend toward infinity as the line length is reduced to zero. The centripetal acceleration is related to angular momentum and line length by equation 10:

$$a_r = h^3/r^3 \qquad\qquad (10)$$

where a_r is the centripetal acceleration, and h is the angular momentum per unit mass.

The relationship of tangential velocity and centripetal acceleration to angular momentum and instantaneous line length is plotted in figures 9 and 10.

We may now apply the results of equations 9 and 10 to the specific problem of reeling in an astronaut who has lost his means of propulsion. If we assume that the man is initially 100 feet ahead of the vehicle and at rest relative to it, and that we reel the tetherline in at the rate of 1 foot per second, the reel-in time will be 100 seconds and the trajectory will be as shown in figure 7c. From equation 9 we know that the angular momentum per unit mass will approach a maximum of ωr_o^3 or, in this case, 11.40 feet^3 per second as the line length approaches zero. We may, however, consider the man as retrieved when he comes within arm's reach or about 4 feet of the vehicle. At this distance the angular momentum per unit mass would be 11.38 feet^3 per second. The man would be rotating around the vehicle at the rate of 0.71 radian per second or about 6.8 rpm, and he would be experiencing a centripetal acceleration of approximately $\frac{1}{16}$ of a G.
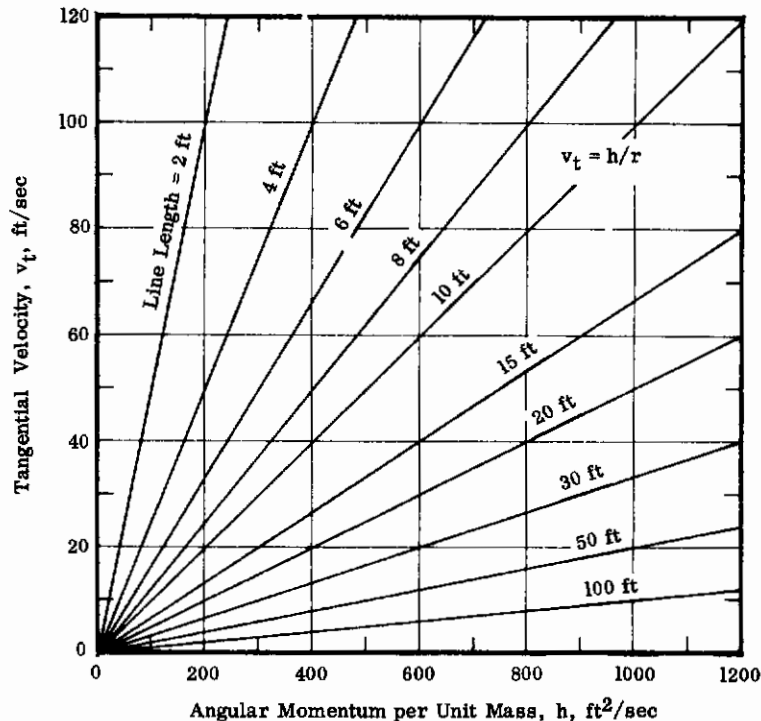

Figure 9. Relationship of Tangential Velocity and Line Length
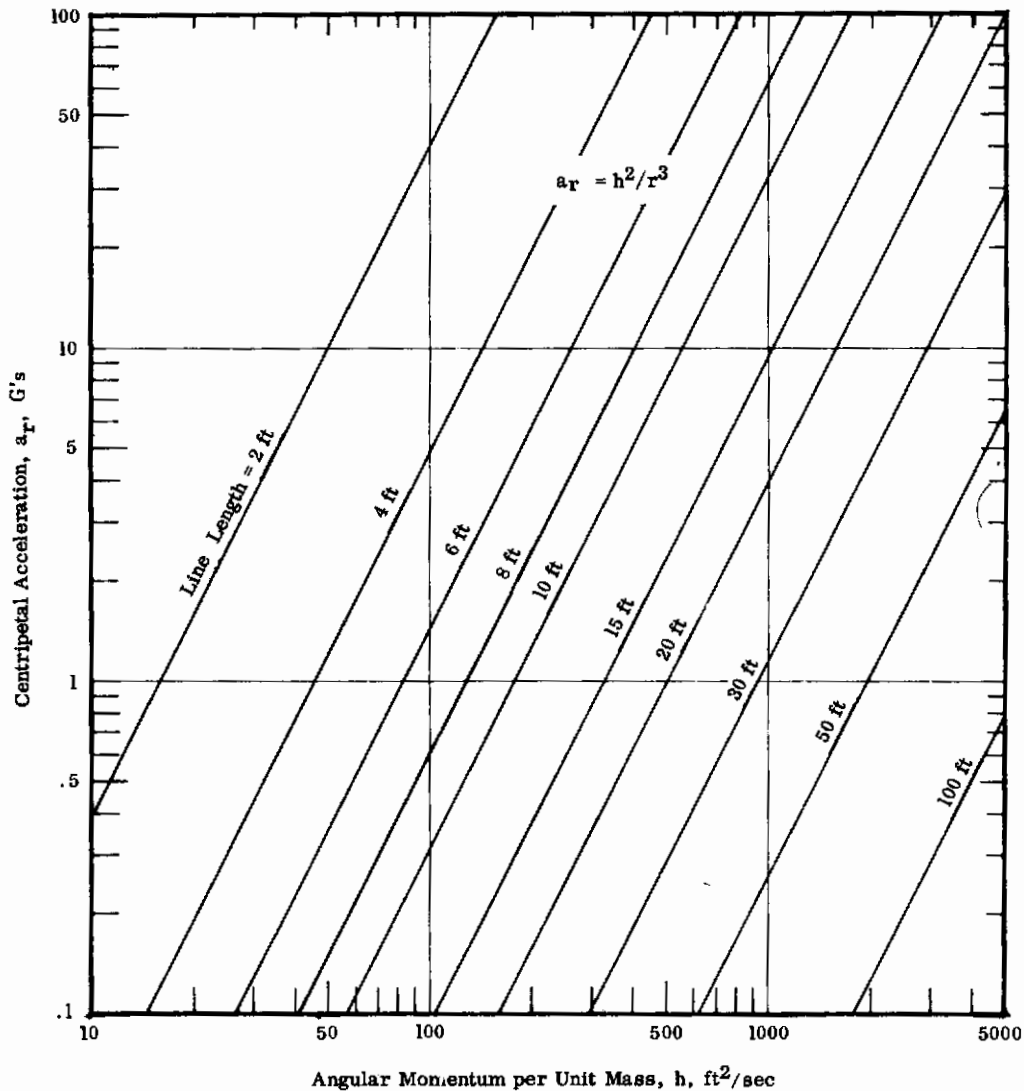for Different Values of Angular Momentum

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Figure 10. *Relationship of Centripetal Acceleration and Line Length for Different Values of Angular Momentum*

If, however, the man were initially 500 feet ahead of the vehicle instead of 100 feet, the angular momentum per unit mass would built to a maximum of 285 feet[2] per second. By the time the line was reeled in to 4 feet the man would be traveling at 72 feet per second and rotating around the vehicle at 170 rpm. He would, if still alive, be experiencing a centripetal acceleration of nearly 40 G's.

It is interesting to note that the speed at which the line is reeled in does not affect the maximum value of angular momentum. The maximum value of h depends only on the initial length of the line. Changing the reel-in speed does change the time required to get the man in and, hence, affects the shape of the trajectory as seen in figure 7. But the angular velocity and centripetal acceleration are a function only of the instantaneous line length and the original line length.

The results given above are for the case where the man is initially at rest with respect to the vehicle. If the man has any tangential component of velocity initially, the situation could be better or worse, depending on the direction of the tangential component. If, for example, the man had a counterclockwise component of exactly ωr_o, he would have exactly zero angular momentum with respect to inertial space and would not spiral around the vehicle as he was drawn in. The difficulty

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of retrieving a man by means of a tetherline, therefore, depends on both the initial line length and the initial component of tangential velocity. Or, more simply, the initial angular momentum as measured in a nonrotating coordinate frame is very nearly conserved and its magnitude determines the difficulty of retrieving an object by this method. Since, in an emergency retrieval of an astronaut, one cannot count on having a low value of angular momentum, the forces generated by reeling him in may be intolerable, particularly if the line is very long.

SUMMARY AND CONCLUSIONS

Three methods for retrieving an inert mass on the end of a long tetherline have been investigated in this paper. The first, in which the line is jerked to start the mass coasting in toward the vehicle while the line is left slack, results in a series of bounces as the mass coasts in along a curved path and is jerked back toward the vehicle each time the line becomes taut. The second method, in which a constant line tension is maintained as the line is reeled in or out, results in a complex, looping path which can be made to intersect the vehicle only after a fairly long time. Both of these methods require accurate control over the line tension or the impulse imparted to the mass and neither seems attractive as a means of retrieving an astronaut whose self-maneuvering unit has failed.

The third method, which involves reeling in the line at a constant rate, has the advantage of being direct and uncomplicated. However, it results in a spiral path which could wrap the line around the vehicle and which causes a rapid build-up of tangential velocity and centripetal acceleration.

The problem of retrieving a mass on the end of a tetherline reduces to one of conservation of angular momentum in inertial space. The only way in which the mass can be reeled in safely is to reduce this angular momentum to zero. Since we are assuming that the mass is inert (without propulsion), one way to accomplish this is to maneuver the vehicle to kill the apparent drift of the mass against the star background. If the line of sight between the vehicle and the mass is not rotating with respect to inertial space (as defined by the stars), the problems associated with all three methods of retrieval are minimized.

In view of the dubious safety value which a long lifeline offers to an astronaut operating outside his vehicle, the astronaut would probably be better off without the encumbrance of such a tetherline. If his self-maneuvering unit fails, it would be simpler for the vehicle to maneuver toward the drifting astronaut.

Schemes for using tetherlines as an aid in the orbital docking of two vehicles must take into account the problems associated with the conservation of angular momentum. This should present no great problem since one or both of the vehicles would have propulsion.

The possibility of using long tetherlines as a means of effecting re-entry from low earth orbits has also been presented in this paper. Further study to determine the feasibility of such a scheme is required.

LIST OF REFERENCES

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