GENERAL SESSION III

PROPERTIES OF MATERIALS AND STRUCTURAL CONFIGURATIONS WHICH AFFECT ACOUSTICAL FATIGUE LIFE

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FATIGUE OF MATERIALS AND STRUCTURES UNDER RANDOM LOADING

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I. INTRODUCTION

The fact that testing machines, with the aid of which cyclic loads of constant amplitude and frequency can be repeatedly applied to material specimens and structural parts in bending, torsion, or uniform stress, are not particularly difficult to construct has decisively affected the approach to fatigue research since the time when, with the first large-scale use of metals in repeatedly loaded structures, fatigue emerged as a phenomenon of considerable engineering significance. Since it appeared much more difficult to reproduce the actual variable-load service conditions than to apply constant-amplitude load cycles, this variability was conveniently overlooked as long as this seemed practically feasible, that is, as long as design for fatigue could either be completely disregarded or based on the results of constant-amplitude tests under the expected maximum load-amplitudes.

In recent years, the increased use of stronger or lighter, but also more fatigue-sensitive, materials and methods of connection, such as welding, coupled with increased speed and increased severity of operating conditions as well as with increased operating times of most means of transportation, has made the range of variation of loading conditions during the expected long lives so wide that the variability of stress-amplitudes in relation to fatigue can no longer be disregarded in the design of aircraft, ships, motor vehicles, bridges, and various other types of structures. It is obvious that the results of constant-amplitude fatigue tests are not directly related to such conditions; therefore, they provide no information of direct design use, except in tests of full-scale structures and structural parts the principal purpose of which is the effective demonstration and elimination of inadequate local design details for repeated loading (1).

On the other hand, the difficulty and cost of construction of fatigue testing equipment for variable-amplitude testing of any but small material specimens or structural connections of reduced scale precludes the use of such equipment on a large scale. It appears, therefore, to be of considerable importance to attempt a correlation of the results of constant and of variable amplitude fatigue tests, so as to permit at least a partial utilization of the very large volume of constant-amplitude fatigue data, especially those obtained in tests of full-scale structures or structural connections, the complete loss of which for design purposes we could not well afford, as it would take many years of effort to obtain even a fraction of such data under variable-amplitude loading.

Because of the highly statistical character of fatigue test results, direct correlation appears possible only when a sufficient number of tests under both conditions can be performed so as to separate the trend from the chance variations. Since this is obviously impractical with respect to large scale structures or parts, there remains no other way but to establish such correlations systematically for small scale material specimens, with a view of subsequently applying the established relations to large scale structures, considering the necessary modifications arising from known differences in behavior, such as form and size-effects, effects of residual stresses (pre-existing as well as load-induced), uncertainty of stress levels and stress spectra associated with imposed load spectra arising from the complexity of structural action, etc. Such modifications must themselves be based on (existing or future) studies of the correlation between the results of material and of structural fatigue tests under conventional constant load amplitudes. A limited number of variable-amplitude tests must then be performed on large scale structures to check the prediction of expected fatigue lives based on the established small scale correlation and modification for structural action, considering that in individual tests not more than agreement within the same order of magnitude can be expected.

It is frequently claimed that the scatter of fatigue lives in the tests of full scale structures and structural parts is much narrower than that characteristic for tests of unnotched material specimens at a similar nominal stress. While some of this difference is probably real and due to the fact that for similar nominal stress levels higher local stresses exist in the structure, with associated reduction in scatter, a large part of the difference is apparent only and reflects the fact that the range of scatter decreases with the size of the sample. Since the scatter of constant-amplitude fatigue lives of large samples of small material specimens usually exceeds one order of magnitude, unless the stresses are so high as to produce lives of the order of $N<10^5$ cycles, it is unlikely that the fatigue lives of a population of nominally identical full scale structures under nominally identically constant load amplitudes would be more closely reproducible than within one order of magnitude.

It is the main purpose of the present paper to discuss the effect of random loading on fatigue of material specimens and to indicate the trend of the correlation between variable- and constant-amplitude fatigue tests for such specimens.

II. THE FATIGUE MECHANISM UNDER CONSTANT AND UNDER VARIABLE REPEATED STRESSING

The essential features of the micro-mechanism of fatigue-crack initiation have by now been well established by the extensive observations of the micro-structure under repeated constant stress-amplitudes by W. A. Wood, P. J. E. Forsyth, N. Thompson, G. C. Smith, and others (2). It is generally recognized that a distinction must be made between the effect on the micro-structure of reversed high stress-amplitudes and low stress-amplitudes, as well as between low and high frequencies of stress-cycling.

The stress-amplitude is considered high and/or the frequency is low if the mechanism of deformation within the structure is akin to that observable in uni-directional "static" deformation; the associated failure is usually referred to as "high level fatigue", and may be related to a critical amount of total plastic deformation or of strain-hardening due to distortion and break-down of the grain structure. It corresponds to the short life part of the S-N curve which drops very sharply toward the N-axis.

In contrast, rapid cyclic stressing of small amplitudes produces considerably less strain-hardening and no significant distortion, but a multitude of fine slip bands congregated in "striations", the spacing of which is that of the set of slip bands produced by the first stress cycle, and in which the density of dislocations is so high that individual dislocations cannot be resolved even by electron-microscopic magnification. The number of striations in any particular region is therefore proportional to the difference $(\tau - \tau_0)$, where τ is the local shear stress and τ_0, the yield-stress in shear (3). Because of the reduced strain-hardening, the total sum of permanent deformations to failure can attain any value, being clearly unrelated to the fatigue phenomenon, while in "high level" fatigue a limiting amount of total plastic deformation might be associated with a critical limit of strain hardening, as suggested by Orowan (4). This localization of reversed slip into "striations" of sharply grooved contours which, at high frequencies, are not only more pronounced but may also show marks of heat staining, as well as the complete absence of polygonization characterizes the deformation process in the micro-structure associated with "fatigue" proper and produces the deep surface grooves ("persistent striations") that cannot be removed by surface etching, and in which micro-cracks appear rather early in the fatigue life. The relation between the applied stress-amplitude and the long lives characteristic of this type of failure corresponds to the later, long and less steep part of the S-N curve, the total shape of which thus results from the superposition of two distinctly different mechanisms.

The difference between the two mechanisms is also reflected by the energy latently stored in the micro-structure of the cyclically stressed material (5). While in the metal subjected to high level fatigue, this energy is released within two clearly distinct, rather norrow, temperature ranges, that of recovery and that of recrystallization, most of the latent energy of the metal subjected to fatigue proper is released in a much widened recovery range, with practically no release on recrystallization, which starts at a much higher temperature. Thus, the structural changes introduced by low amplitude fatigue stressing appear to be much more stable than those due to high level fatigue, implying that the slip movements responsible for the widening of the striations in low stress fatigue do not build up high micro-residual stresses.

The difference in the mechanism of crack initiation also affects the stage of micro-crack formation; in low stress fatigue, the cracks are concentrated within the striations, while in high level fatigue they are more likely to be of irregular shape and to include grain boundaries, due to their interaction with the micro-residual stress field of the work-hardened structure.

The transition range between the two mechanisms should necessarily be rather sharp; its location is, however, affected by the frequency of the cyclic stressing (at rather high frequencies), by the initial condition of the grain structure (degree of initial work-hardening), by the character of the slip-blocking mechanisms existing in the particular metal or alloy (anchoring of solute atoms, blocking of primary slip-system by dislocations) by the intensity of the mean stress and, probably, by the surface conditions.

The character of the progressive damage, including the range of crack propagation, is necessarily affected by the outlined differences in the micro-crack development and propagation. The character of the final stage of the critical macro-crack formation and propagation, however, is probably much less affected by the nominal stress amplitude than the preceding stages.

The principal effect on fatigue under randomly varying stress amplitudes of the existence of two mechanisms depending primarily on amplitude and frequency of the cyclic stress is the interaction between the progress of damage at the different stress amplitudes. This interaction of high stress amplitudes with the progressive damage at low amplitudes is probably weakest within the range of high level fatigue, dominated by quasi-isotropic crystal fragmentation, distortion, and polygonization.

In fact, interaction with this range may be such that a small number of high stress amplitudes, by hardening the metal, might even reduce the rate of initial fatigue damage at lower stress levels within the same range. As the state of crack propagation is reached, however, the effect of intermittent higher stress cycles will be to accelerate the rate of propagation of cracks produced by lower stress amplitudes. Thus, for instance, a "non-propagating" crack at this lower amplitude is likely to start to propagate, as a result of a few high stress amplitude cycles, having increased beyond the "critical size" delimiting, under a given state of stress, propagating from non-propagating cracks.

It appears reasonable to assume that interaction of stress amplitudes belonging to different fatigue mechanisms as well as of those producing low stress fatigue "striations" is much more pronounced than in the high level fatigue range, as even a few intermittent cycles of high amplitude producing a shear stress intensity, τ_H, may be very effective in accelerating the subsequent damage rate at the lower amplitude associated with shear stress intensity τ_L by multiplying by a factor $(\tau_H-\tau_o)/(\tau_L-\tau_o)$ the number of slip-bands which, under the subsequent low stress cycles widen into "striations" along which micro-cracks form and propagate. A relatively small number of such cycles must therefore be expected to shorten the fatigue life under the low constant stress amplitudes far beyond the immediate damage associated with them, unless they are able to produce residual stresses of sufficient intensity to impede the progress of subsequent damage at the low stress amplitude, as in the case of notched specimens or structural connections of suitable geometry. Stress interaction in notched specimens and structural parts may therefore differ significantly from that observed on plain specimens, at least within the stages of micro-crack initiation and formation.

Obviously, the interaction effect between two stress amplitudes increases with the interval between them. Also, it is strongest when the direct damaging effect of the high stress amplitude is insignificant, the number of cycles applied being only a very small fraction of the number that would produce failure in a constant-amplitude test. As the direct damaging effect becomes significant, the importance of the interaction effect dimishes. Thus, for instance, in a two-level test with stress amplitudes S_H and S_L associated with constant-amplitude lives $N_H = 0.1\ N_L$ and N_L, a repeated stress sequence in which the ratio of the number of high and low stress amplitudes applied is 0.1 implies that without interaction and under the assumption of linear accumulation more than half of the total damage will be produced directly by the high stress amplitude. Clearly, under such testing conditions interaction effects will be rather low; the results of studies of fatigue damage accumulation with the aid of such two-level tests may, therefore, be quite misleading.

In the stage of crack propagation, the interaction effects probably show a similar trend in the ranges of high and low level fatigue, making the existence of non-propagating cracks under variable stress amplitudes rather unlikely in both ranges. Since the "critical length" of a crack is a function of the stress intensity, any crack length may be above the critical length provided the applied stress amplitude is high enough.

An additional effect of stress interaction in variable stress amplitude fatigue is the lowering or elimination of the endurance limit associated with constant amplitude stress cycling, by intermittently applied stress amplitudes exceeding this limit.

In relation to the stress interaction effects the probable non-linearity of fatigue damage accumulation under constant amplitude stress cycling appears less significant. This effect can, however, be considered by approximating the initial stage of very low damage rate by a state of zero damage rate between $N = 1$ and $N = N_{oi}$ where N_{oi} denotes the "minimum life" at constant stress amplitudes S_i, followed by a linear damage rate between $N_{oi} < N < N_i$. The introduction of the concept of "minimum life" reflects the basic fact that "fatigue" implies failure following the repetition of a number of stress or load cycles larger than one. Its true significance can, however, be clearly seen only in the context of the statistical aspect of fatigue (6).

It has been found expedient to consider stress interaction effects phenomenologically by the introduction of "fictitious" S-N' diagrams which, for a given randomly applied stress spectrum, present the associated "interaction factors" ω_i at all stress amplitudes S_i in the form of simple ratios between the constant amplitude fatigue life N_i at this level and the life N_i' obtained when the stress amplitudes of the spectrum above S_i are interspersed with S_i in the overall ratio defined by the spectrum. As long as the ratio of "over-stress" cycles $> S_i$ is very small, their direct damaging effect may be neglected in comparison to the interaction effects.

Reducing N_i by the "interaction factor" $\omega_i = N_i/N_i'$ the fictitious $S_i - N_i'$ diagram is obtained (Figure 1) with the aid of which the linear concept of damage accumulation can be reinstated as a reasonable approximation, implying a damage rate (ω_i/N_i) per cycle under the assumption of zero minimum life $N_{oi}' = 0$, or of $[\omega_i/(N_i - N_{oi})]$ for finite minimum life N_{oi}. As schematically indicated in Figure 1, the S_i-N_i and S_i-N_i' diagrams differ significantly only within the range of the true (low stress) fatigue mechanism. Recent results of random fatigue tests with exponential spectra show interaction factors of the order of magnitude $\omega_i \sim 10^2$ for constant stress amplitudes producing fatigue lives of the order of $N_i = 10^7$ cycles (7).

Considering the conventional form of the trend of the S-N relation at some central probability level

$$(N_i/\bar{N}) = (S_i/\bar{S})^{-\nu} \qquad \text{Equation (1)}$$

where $(\bar{N},\bar{S})$ defines a reference point on the diagram, which plots as a straight line in double-logarithmic representation, the stress interaction effect is expediently introduced by changing the exponent ν to ρ and considering the point $\bar{N},\bar{S})$ as the boundary between high stress and low stress fatigue, and thus between slight and strong interaction. For plain material specimens in general $\rho < \nu$ (life reducing interaction), as established in the recent random fatigue tests with exponential spectra. However, for notched specimens, structural parts, and full scale structures, in which a stable field of compressive residual stresses can be locally built up in the regions of potential critical fatigue damage, values $\rho > \nu$ seem possible. Such values will obviously be restricted to conditions under which the intensity of the residual stresses is sufficiently above the range of most of the stress amplitudes that might be expected during the fatigue life of the specimen or part, so as not be affected by them. However, before exponents $\rho > \nu$ may be accepted as real and admissible in design, the complete stability in time of the residual stress field that could justify such values of ρ would have to be reliably established, a condition that might be difficult to fulfill.

Under experimental conditions under which the interaction is damaging $(\rho < \nu)$ it has been found that for structural metals such as 2024 and 7075 aluminum and 4340 steel with $8 < \nu < 16$, exponential spectra of various slopes (in semi-logarithmic representation) produced fictitious S-N' curves reflecting interaction with roughly $4 < \rho < 8$, the lower values being characteristic of the more severe spectra and shorter random fatigue lives; the difference between $\rho = 4$ and $\rho = 8$ is associated with a difference in life of the order of 10^4 at the lower stress amplitudes (7).

Use of the constant stress amplitude S-N diagram in conjunction with a linear damage accumulation rule considering interaction, but with $N_{oi} = 0$, seems to imply the possibility of fatigue life prediction based on critical values of the conventional sum of cycle ratios $\sum \frac{P_i N}{N_i} = (\frac{1}{\bar{\omega}}) < 1$. This is illustrated by Figures 2, 3 and 4, showing that this relation can be considered as a low (safe) limit enclosing all test results for a suitable selected $\bar{\omega}$.

It should, however, be pointed out that the value of this average interaction parameter $\bar{\omega}$ is not a very effective measure for the prediction of fatigue lives under random loading. This is illustrated by Figure 5 in which the relations between the (negative) slope h of the stress spectrum in semi-logarithmic representation, the difference $(\nu - \rho)$ of the slopes of the conventional and fictitious S-N and S-N' diagrams in double-logarithmic representation and $\bar{\omega}$ are schematically illustrated, considering spectra with the same minimum stress amplitude. Figure 5a shows the same value of $\bar{\omega}$ can arise from various combinations of h and $(\nu-\rho)$, while Figure 5b illustrates the variability of $\bar{\omega}$ as a function of h for a constant difference $(\nu-\rho)$.

Thus the fictitious S_i-N_i' diagram with (negative) slope ρ in double-logarithmic representation, producing sums of fictitious cycle ratios $\sum_i \frac{n}{N_i} = 1$, should be used in preference to the conventional S_i-N_i diagram with sums of cycle ratios $(1/\bar{\omega}) < 1$, as there is no basis on which a value of $(1/\bar{\omega})$ could be selected, while the specification of some minimum value of ρ could be justified by a combination of physical reasoning with the results of a relatively small number of fatigue tests under random loading. Thus, reported values of $(1/\bar{\omega})$ vary between $0.1 < (1/\bar{\omega}) < 10$, while on the basis of the evaluation of the test results on small material specimens of structural metals (7) a range of values $5 < \rho < 7$ would seem to enclose most of those results. Considering that fatigue tests of sharply notched ($K_t = 4.0$) specimens and structural parts of aluminum show central trends of the constant stress amplitude S_i-N_i diagrams steeper than those of small material specimens ($4 < \nu < 8$), as well as that interaction effects tend to become smaller for the short lives associated with the very steep slopes of the S_i-N_i diagrams, it would appear that for these metals $\rho \sim 6$ for small materials specimens and $\rho \sim 4$ for notched specimens or parts represent tentative reasonable lower limits for the evaluation of the trend of expected minimum lives under exponential or nearly exponential load spectra, with a value of $\bar{N}$ of the order of $\bar{N} = 10^4$ to 10^5 load cycles delimiting high level and low level fatigue. For fatigue lives $N < \bar{N}$, ρ is replaced by ν.

III. THE STATISTICAL ASPECT OF FATIGUE UNDER CONSTANT AMPLITUDE AND RANDOM LOADING

The fact that the highly localized initiation of fatigue damage and formation of micro-cracks, with its dependence on initial microstructure and subsequent change under repeated stressing, produces a characteristically wide scatter of fatigue lives under nominally identical conditions of testing and service makes the statistical aspect of fatigue of particular significance with respect to fatigue design. Its significance is comparable to that of the physical aspect and the two aspects are, in fact, closely related, particularly with respect to cumulative damage under random loading.

Extensive studies of the scatter of fatigue lives under constant and under variable stress amplitudes have shown the asymptotic distribution of extreme (smallest) values with a lower limit N_o (minimum life) to provide a satisfactory representation of the test results and to be of sufficient physical relevance to the phenomenon to justify extrapolation on its basis beyond the range of practically possible test results (8). The cumulative probability function associated with this distribution representing the probabilities of survival $\ell(N)$ at lives N is of the form

$$\ell(N) = \exp\left[-\left(\frac{N-N_o}{V-N_o}\right)^{\alpha}\right] \qquad \text{Equation (2)}$$

where $N = N_i$ may refer to constant amplitude tests at stress levels S_i, or $N = N_R$ to lives under random loading. Similarly, the minimum life N_o may denote $N_o = N_{oi}$ or $N_o = N_{oR}$, and the central value $N = V$ at $\ell(V) = 1/e$ ("characteristic value") $V = V_i$ or $V = V_R$ therefore may refer to constant or variable stress amplitudes, respectively.

As pointed out in the discussion of the physical mechanism, the existence of values $N_o \gg 1$ reflects the fact that extrapolation of the trend of the S-N diagram towards $N = 1$ is physically meaningless, as the definition of "fatigue" presupposes a damaging effect arising from repetition of the load cycle. "Progressive damage" thus implies that the crack initiation and subsequent propagation to a critical size requires a finite number of load repetitions to produce failure by separation. Failure after a single load cycle, reflecting a different failure mechanism, can therefore not be considered as a lower limit of life associated with a very low probability of failure; physically only values $N_o \gg 1$ are acceptable. Statistical considerations may, however, justify the approximation $N_o = 0$, since the minimum life can only be found by extrapolation to $\ell(N_o) = 1.0$ from an $\ell(N)$ diagram fitted to a series of observed fatigue lives, with the considerable error of estimation associated with this procedure. Thus, the simplifying assumption $N_o = 0$ (which is physically meaningless) may be justified as an estimate of a relatively small and highly uncertain value of $N_o > 1$.

The exponent α is a scale factor which, for $N_o = 0$, is inversely proportional to the standard deviation and defines the (negative slope) of the straight line representation of Equation (2) with $N_o = 0$ on extreme value probability paper. For values $N_o \gg 1$ the joint effect of α and N_o on the form of the distribution is rather complicated and can be better perceived if the influence of V is eliminated from the function (Equation 2). Introducing the "relative life" v = N/V, Equation (2) is transformed into

$$\ell(v) = \exp\left[-\left(\frac{v-\epsilon}{1-\epsilon}\right)^{\alpha}\right] \qquad \text{Equation (3)}$$

with $\epsilon = N_o/V$; for this form the interrelation of ϵ and α has been studied (9), and the fact established that the appearance on probability paper of a stright line representation of Equation (2) is not by itself an indication of $N_o = 1$, but may also arise from various combinations of ϵ and α, particularly within the range $1/\alpha$ (10).

The form of the distribution function of fatigue lives can be closely related to the risk of failure at a particular cycle N, that is to the probability that the specimen or structure which has not failed before N cycles will fail at the N^{th} cycle. This risk or "rate" of failure r(N) is therefore the ratio of f(N) = P(N)-P(N-1), where P(N) = $[1 - \ell(N-1)]$ denotes the (cumulative) probability of failure up to and including N cycles, - by $\ell(N-1)$, the probability of survival to (N-1) cycles. Hence

$$r(N) = \frac{P(N) - P(N-1)}{\ell(N-1)} = \frac{f(N)}{1 - P(N)} \qquad \text{Equation (4)}$$

where for assumed continuity of the variable

$$f(N) = -\frac{d}{dN}\left[1 - F(N)\right]$$

Hence

$$r(N) = -\frac{d}{dN} \ln \ell(N) \qquad \text{Equation (5)}$$

and

$$\ell(N) = \exp\left[-\int_0^N r(u)du\right] \qquad \text{Equation (6)}$$

Various assumptions concerning the form of $r(N)$ will lead to various survivorship functions $\ell(N)$. Conversely, various observed forms of $\ell(N)$ imply functions of the failure rate $r(N)$ associated with different physical aspects of the failure mechanism.

The simplest assumption is that of a failure rate $r(N)$ independent of N; hence $r(N) = \text{const} = 1/V$ or $1/\bar{N}$, based either on the characteristic life V at $\ell(V) = 1/e$ or on the mean life $\bar{N}$ (or any other measure of central tendency). This assumption leads to the simple exponential survivorship function associated with chance-failures and a consequent complete randomness of fatigue lives, which implies the absence of a process of progressive damage:

$$\ell(N) = \exp\left[-(N/V)\right] \quad \text{or} \quad \exp\left[-(N/\bar{N})\right] \qquad \text{Equation (7)}$$

It is the same function that is obtained from a Poisson process based on the simple assumption that fatigue failure is a rare event governed by pure chance.

Considering that the risk of failure in fatigue necessarily increases with the life N, a failure rate function of the form

$$r(N) = \alpha k^{\alpha} N^{\alpha-1} \qquad \text{Equation (8)}$$

may be introduced, the exponent α indicating the steepness of the increase of the risk of fatigue with N. From Equation (6) with $r(N)$ according to Equation (8) the survivorship function is obtained

$$\ell(N) = \exp\left[-(kN)\right] \qquad \text{Equation (9)}$$

which, with $k = 1/V$, is transformed into the extreme value distribution Equation (2) with $N_o = 0$. Equation (2) with $N_o > 0$ is obtained by setting

$$r(N) = 0 \quad \text{for} \quad N < N_o \quad \text{and} \quad r(N) = \alpha k^{\alpha} (N - N_o)^{\alpha-1}$$

$$\text{for } N > N_o \qquad \text{Equation (10)}$$

with $k = (V - N_o)^{-1}$. The extreme value distribution (Equation 2) therefore reflects the assumption of an increase of the failure rate according to a power of N, the exponent α being inversely related to the standard deviation of the distribution. In the limiting case $\alpha = 1$, $r(N) = k$ and thus independent of N, Equation (9) degenerates into the exponential survivorship function (Equation 7) with $k = 1/V$. The larger α the narrower the distribution and the less skew its appearance (Figure 6). For $3.2 < \alpha < 3.7$ the differences between mode, mean, and median are so small as to produce an appearance of symmetry and thus of "normality" (10).

Since fatigue implies an increasing chance of failure with increasing life, it appears that Equation (8) represents a reasonable, physically relevant expression for the failure rate; the resulting survivorship functions (Equation 9 or 2) can therefore be used for extrapolation beyond the range of observed lives.

It follows from Equation (5) that the logarithmic normal as well as the normal distributions of fatigue lives are also associated with a risk of failure monotonically increasing with life. However, the risk functions cannot be represented in closed form, nor is the relation between the parameters of the risk function and of the distribution function of lives as simple as in the case of the extremal distributions.

If

$$\ell(N)_S = \exp\left[-(N/V_S)^{\alpha_S}\right] \qquad \text{Equation (11)}$$

is the survivorship function for constant stress amplitude S, V_S being the characteristic life (mode) and α_S the scale factor of the distribution, the interaction effects at this stress level resulting from a small number of stress amplitudes $> S$ can be considered by applying the interaction factor ω to the characteristic life selected as a measure of central tendency. Hence

$$\ell(N')_S = \exp\left[-(N\omega/V_S)^{\alpha'_S}\right] \qquad \text{Equation (12)}$$

where $V'_S = (V_S/\omega_S)$ is the mode of the distribution of (fictitious) lives N' and α_S the scale factor associated with lives of the reduced order of magnitude

The probability of surviving N_R cycles under a stress-spectrum of i stress amplitudes S_i, each applied randomly a total of $p_i N_R$ times, where p_i are the relative frequencies such that $\sum p_i = 1$, is the product of the probabilities of surviving $p_i N_R$ cycles at each stress-amplitude S_i. Hence (11)

$$\ell(N_R) = \prod_i \ell(p_i N_R)_{S_i} = \prod_i \exp\left[-\left(\frac{p_i N_R \omega_{S_i}}{V_{S_i}}\right)^{\alpha'_{S_i}}\right] \quad \text{Equation (13)}$$

assuming a similar distribution of N_R, Equation (13) can be written in the form

$$\exp\left[-(N_R/V_R)^{\alpha_R}\right] = \exp\left[-\sum_i \left(\frac{p_i N_R \omega_{S_i}}{V_{S_i}}\right)^{\alpha'_{S_i}}\right] \quad \text{Equation (14)}$$

Therefore, at each probability level $\ell(N_R)$ determined by the ratio N_R/V_R, the relation holds

$$(N_R/V_R)^{\alpha_R} = \sum_i \left(\frac{p_i N_R \omega_{S_i}}{V_{S_i}}\right)^{\alpha'_{S_i} \alpha'_{S_i}} \quad \text{Equation (15)}$$

For the central trend at the level $\ell = 1/e$ for which $N_R = V_R$, Equation (15) is transformed into the non-linear damage-accumulation rule

$$\sum_i \left(\frac{p_i V_R \omega_{S_i}}{V_{S_i}}\right)^{\alpha_{S_i} s_i} = 1 \quad \text{Equation (16)}$$

which can be solved by trial and error provided ω_{S_i} and α_{S_i} are known or can be roughly estimated. It follows that even the improved linear damage accumulation rule considering stress-interaction can be justified only on the basis of the assumption $\alpha_{S_i} = 1$, which implies pure chance failure and a life-dependent failure rate $r = const = 1/V_R$, an implication contradicted by all fatigue observations. It appears therefore, that Equation (14) cannot be simplified and should be used with estimated values $\alpha'_S > 1$ rather than with one value $\alpha_{S_i}=1$ known to be incompatible with the reality of fatigue damage. Observed values of α_S in constant amplitude tests for technical metals $2 < \alpha_S < 6$, the higher values associated with higher stress

$2 < \alpha_S < 6$, the higher values being associated with higher stress levels. The strong stress interaction at low levels may increase the low values of α'_S without significantly affecting the high values. Hence a range of $4 < \alpha'_S < 6$ appears reasonable as a first rough approximation, in the absence of reliable observations.

The form of Equation (16) implies that the terms with the largest interaction factors α_{S_i} will be disproportionally magnified by the powers α'_{S_i}. The effect of the damage at the low stress amplitudes is therefore further increased in relation to that at the high amplitudes, provided failure is not due primarily to high level fatigue. This fact may be of considerable significance in the estimation of the critical stress level of maximum damage.

IV. CONCLUDING REMARKS

An attempt has been presented to develop a rational approach to the problem of fatigue damage accumulation under randomly applied variable stress amplitudes. It has been shown that such an approach must embody combined physical and statistical considerations, the statistical problem in fatigue being in fact an important part of the physical problem.

It should be realized, however, that the actual experimental information on which the approach is based has been derived from tests of plain small material specimens. Thus, the effect of notches and residual stresses associated with them, which seems to be of particular importance with respect to the fatigue behavior of structures, is not included. While there is no reason to believe that the basis of the statistical approach does not remain valid for structures and structural parts, the character and magnitude of stress interaction may be quite different from that found for plain material specimens. It appears therefore that the principal research effort in this field should now be concentrated on the study of stress interaction effects under random loading in the presence of initially well-defined residual stress fields, as well as in the presence of residual stress fields resulting from the high stress amplitudes of the spectrum themselves.

With respect to the application of the developed concepts to acoustic fatigue, it appears that a distinction must be made between conditions of "narrow band" acoustic excitation characteristic of resonant response of a panel in a single dominant mode, which are essentially conditions of short "high level" fatigue, and conditions more representative of "low level" fatigue with mixed stress amplitudes and long lives. Without such distinction there is the danger that conclusions reached on the basis of results of the most frequent "narrow band" tests will be uncritically applied to acoustic fatigue performance of structural parts excited along a wide spectrum of partly damped and coupled modes, with strong interaction effects between the associated stress levels. Moreover, in high level fatigue the energy input appears to provide a failure criterion, while in low level fatigue the total energy input is unrelated to failure which is governed by the stress level.

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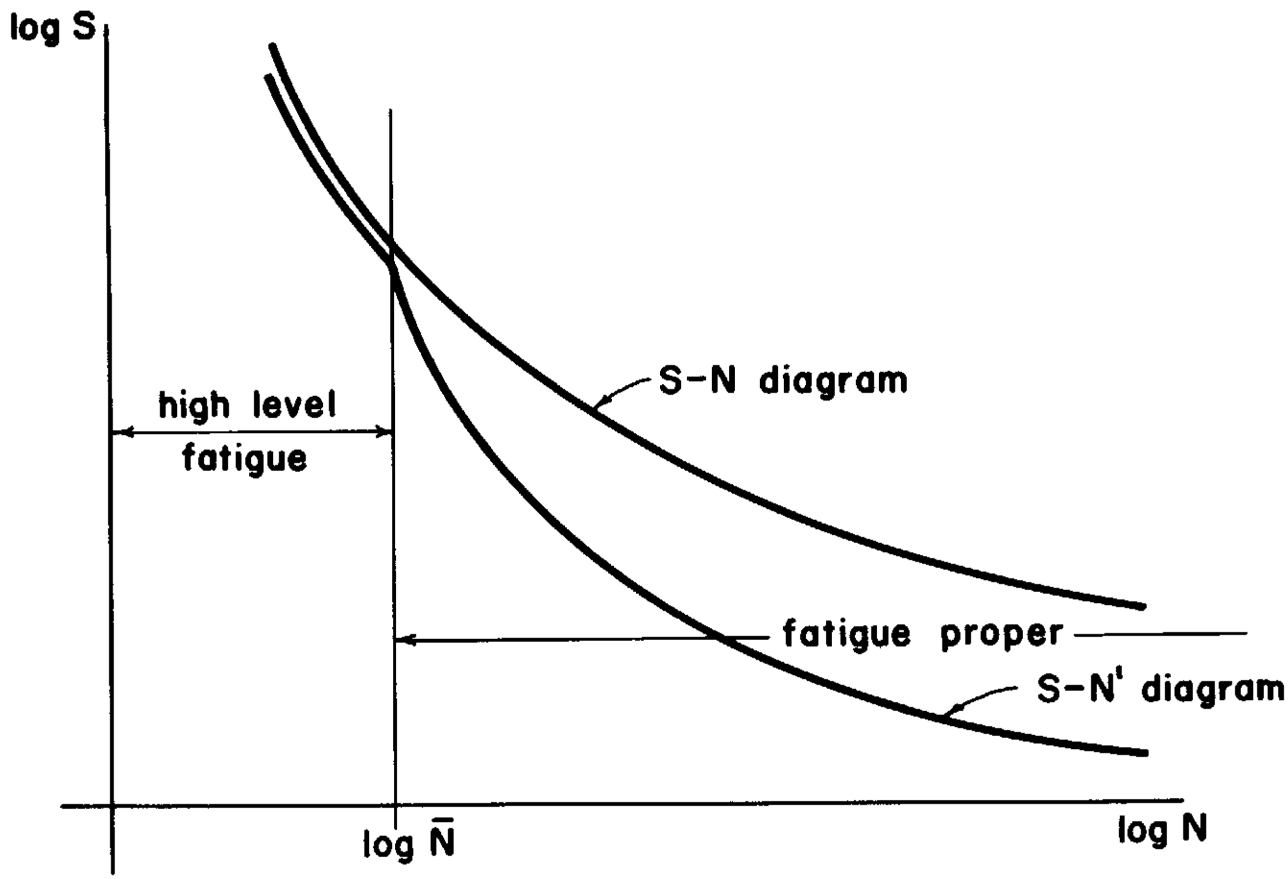

Fig. 1 - Conventional and "Fictitious"(Interaction) S - N(N') Diagrams

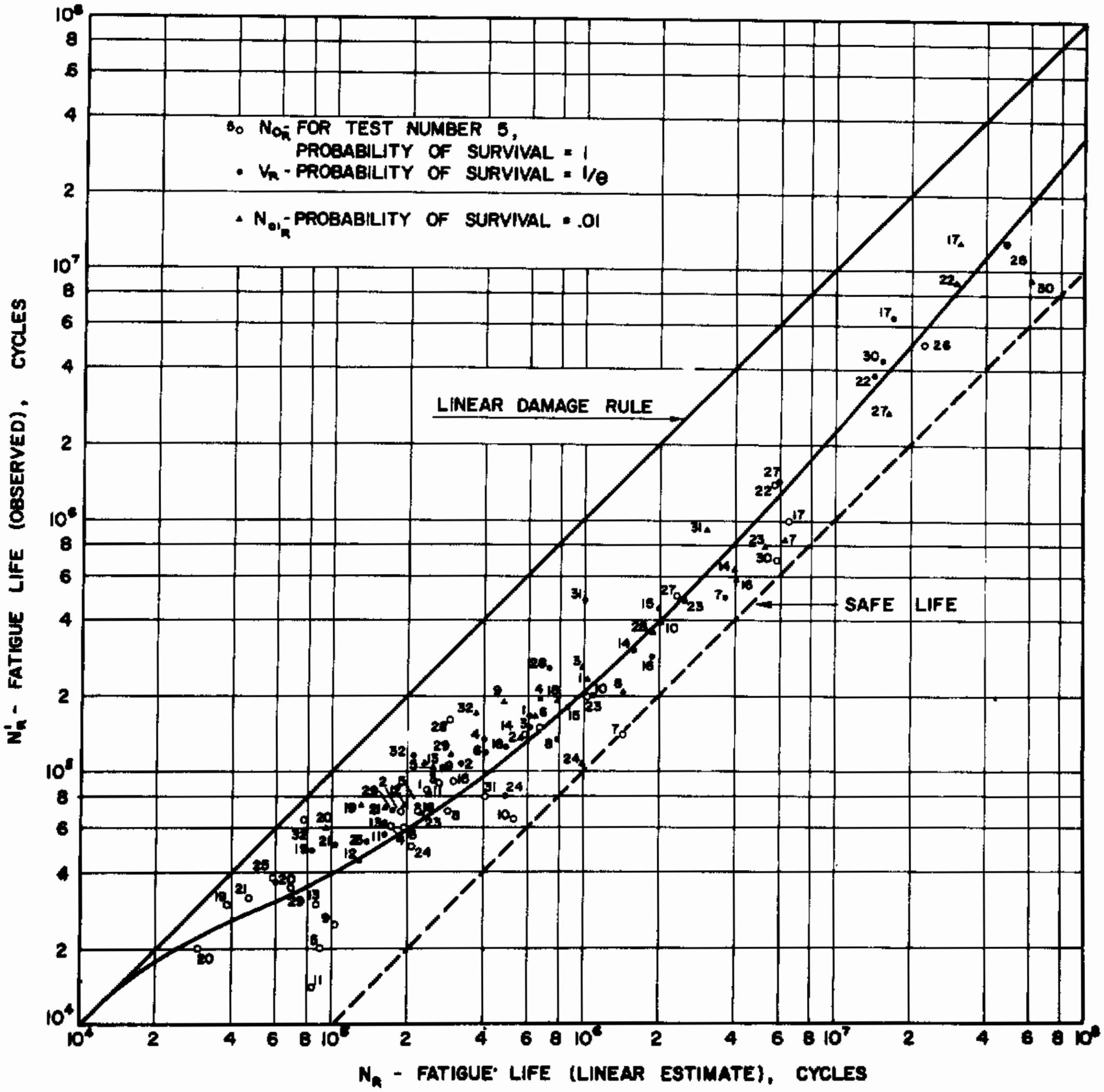

Fig. 2 - Comparison of Spectrum-Fatigue-Test Results With Linear Damage Accumulation Rules (2024 Aluminum Alloy)

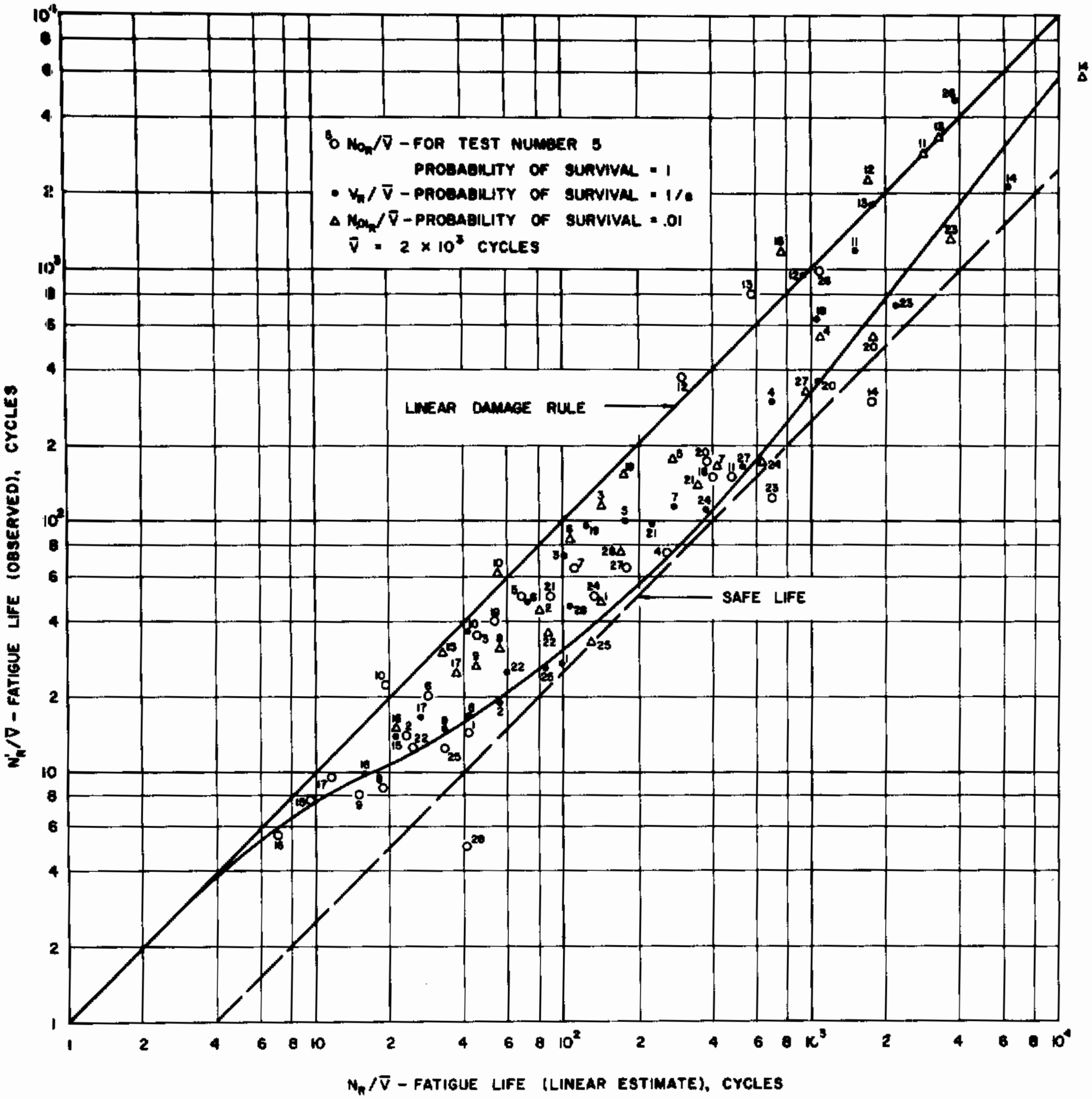

Fig. 3 - Comparison of Spectrum-Fatigue-Test Results With Linear Damage Accumulation Rules (7075 Aluminum Alloy)

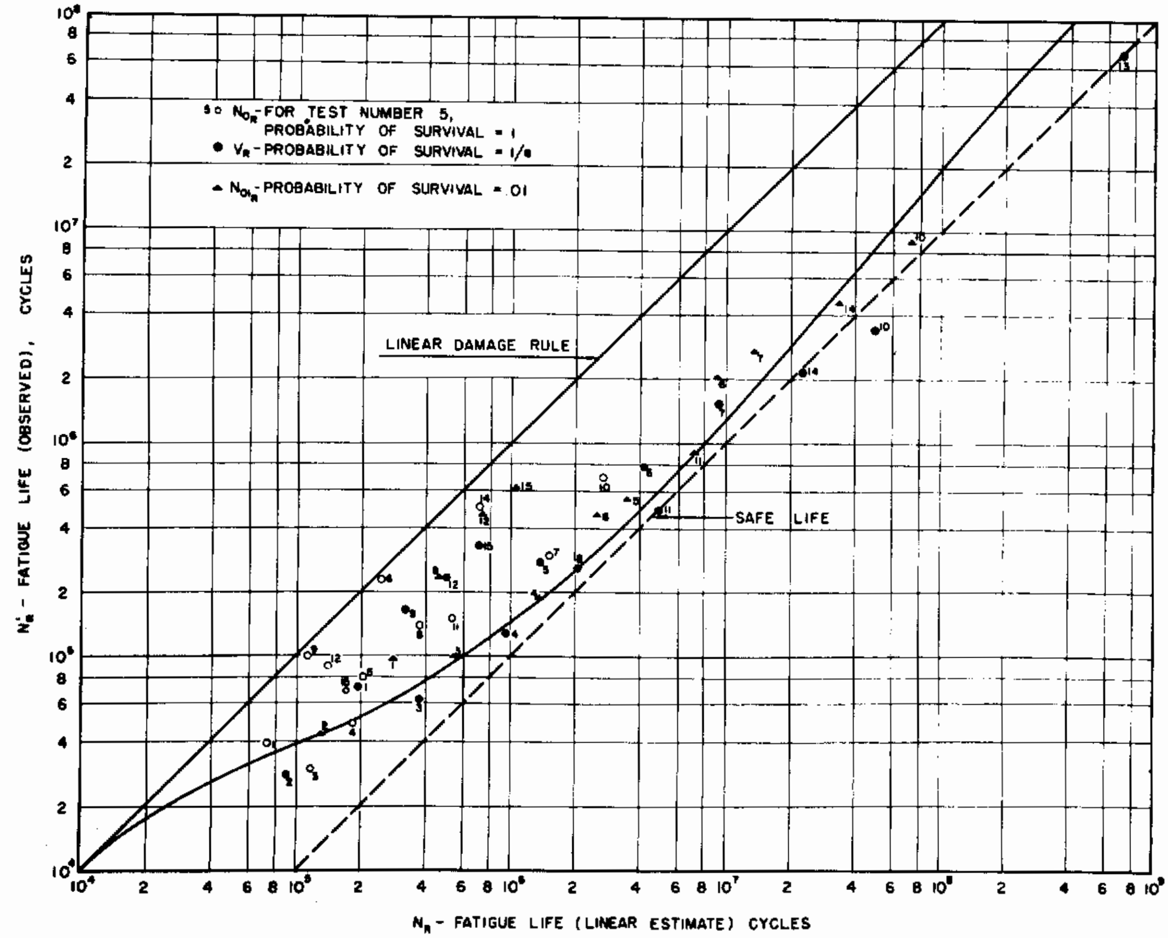

Fig. 4 - Comparison of Spectrum-Fatigue-Test Results With Linear Damage Accumulation Rules (SAE 4340 Steel)

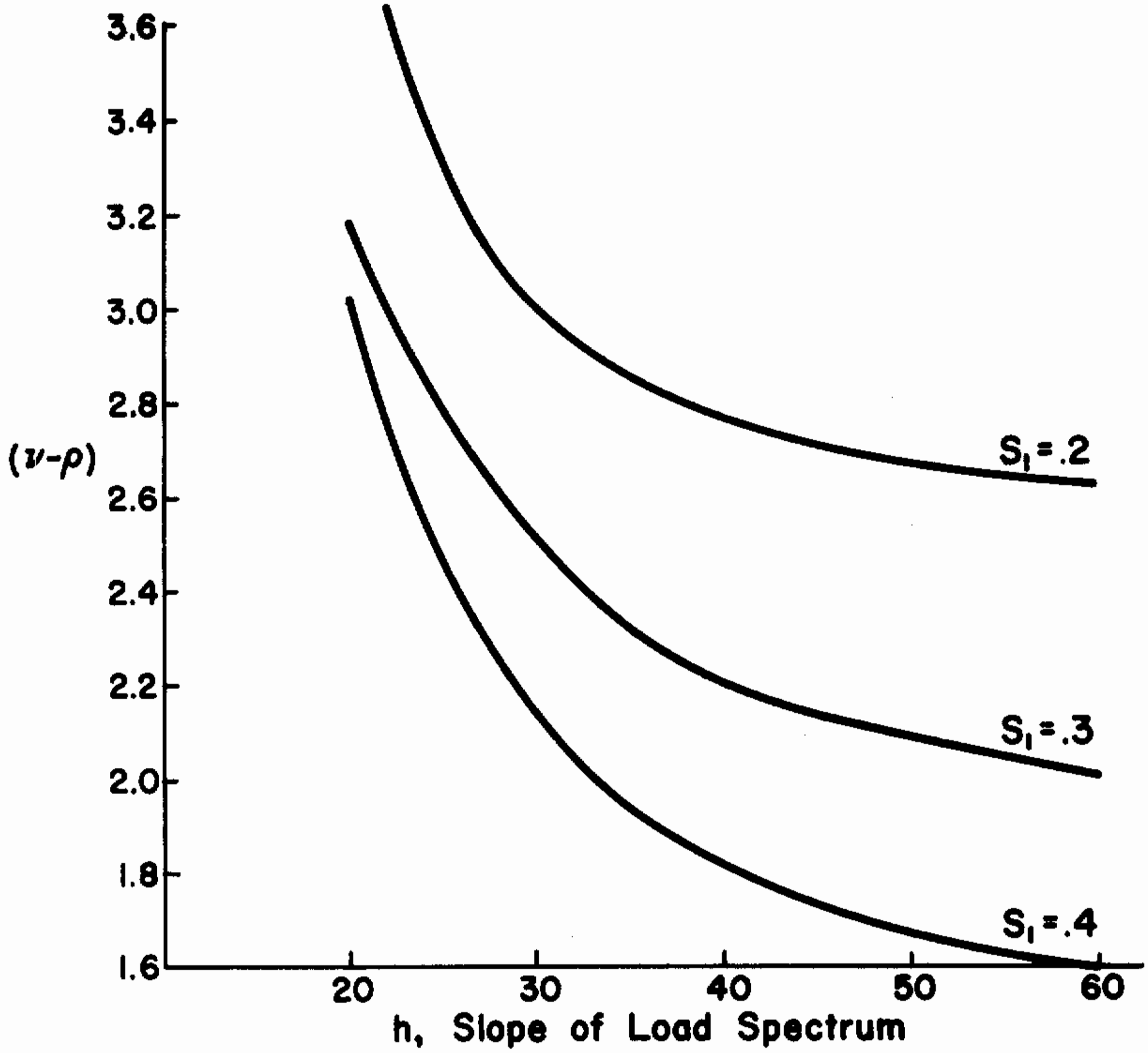

Figure 5a - Relationship Between h and
(ν - ρ) for Constant ϖ

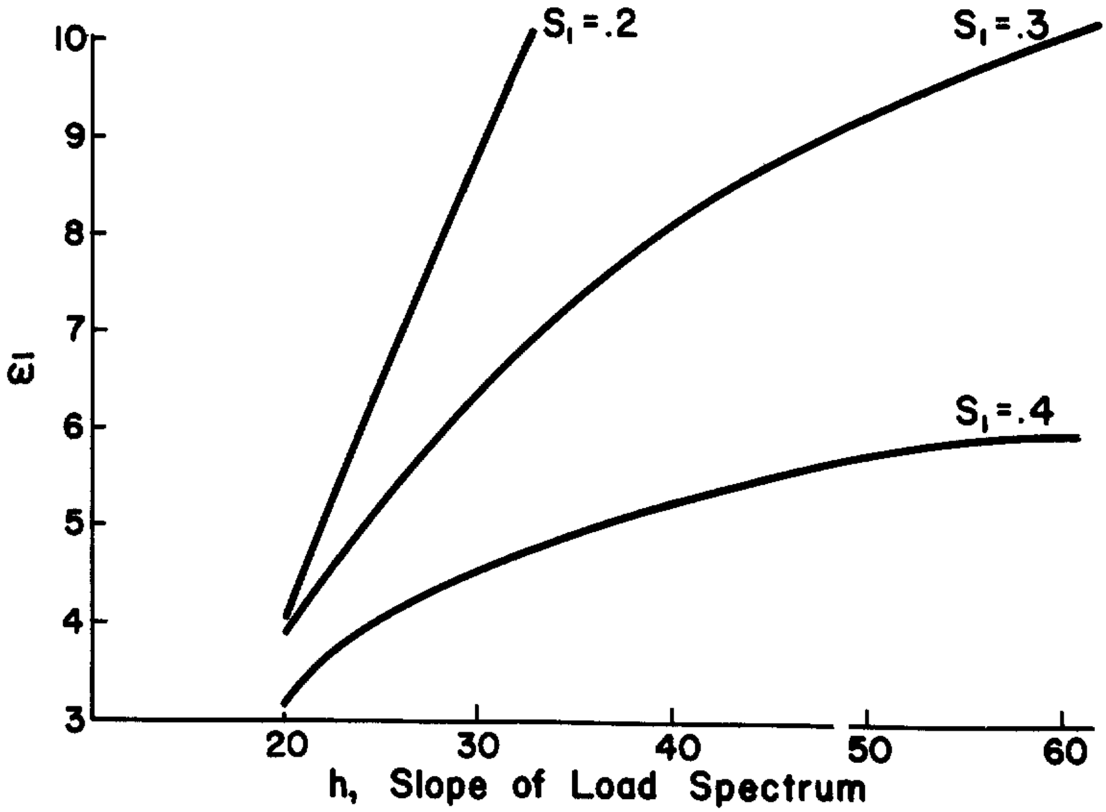

Figure 5b - Variability of ϖ as a Function of h for a Constant Difference $(\nu - \rho)$

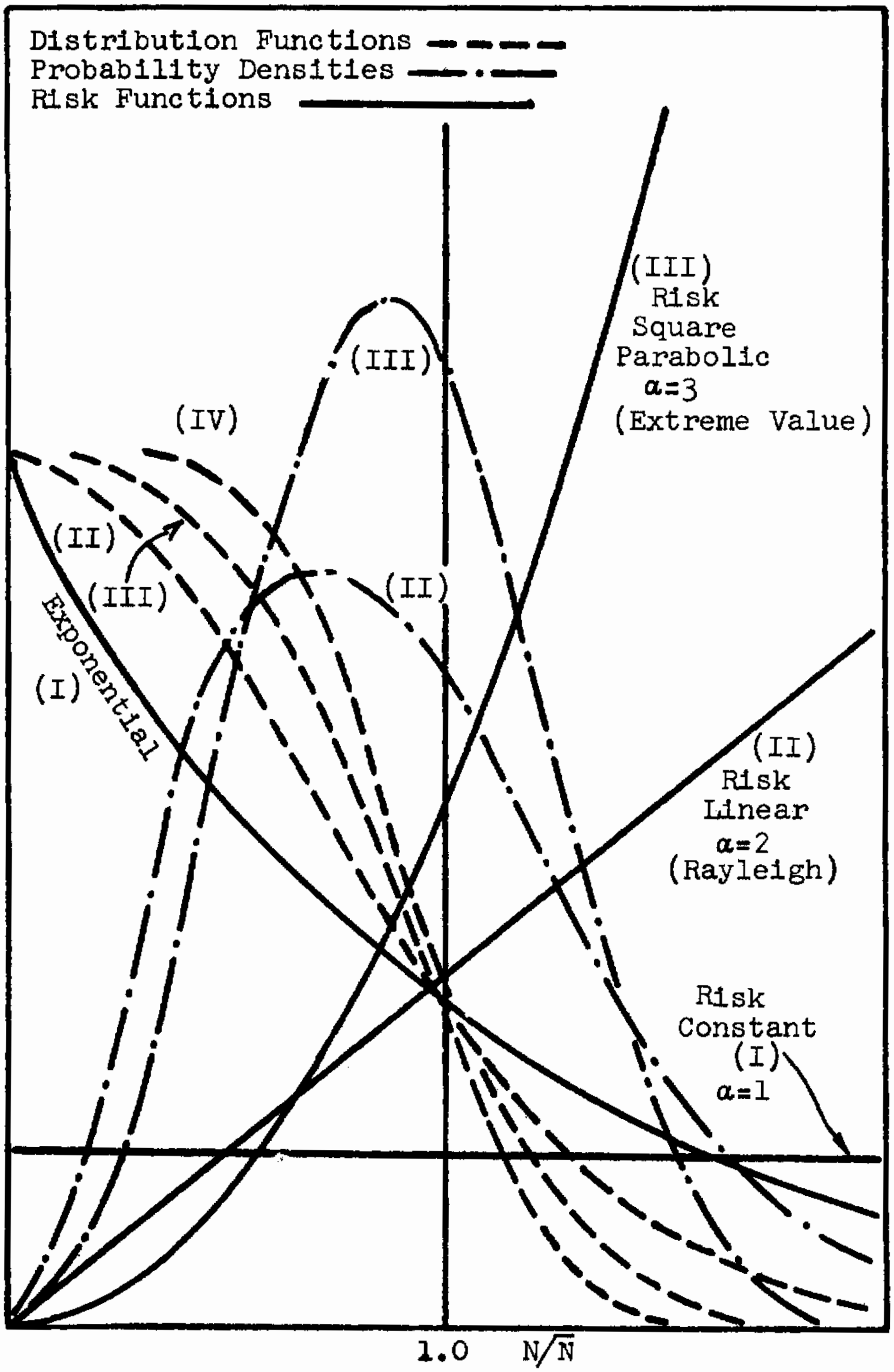

Figure 6 - Distribution Functions, Probability Densities, and Risk Functions for Extremal Distribution