FUNDAMENTALS OF ROLLING-ELEMENT BEARING LUBRICATION

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ABSTRACT

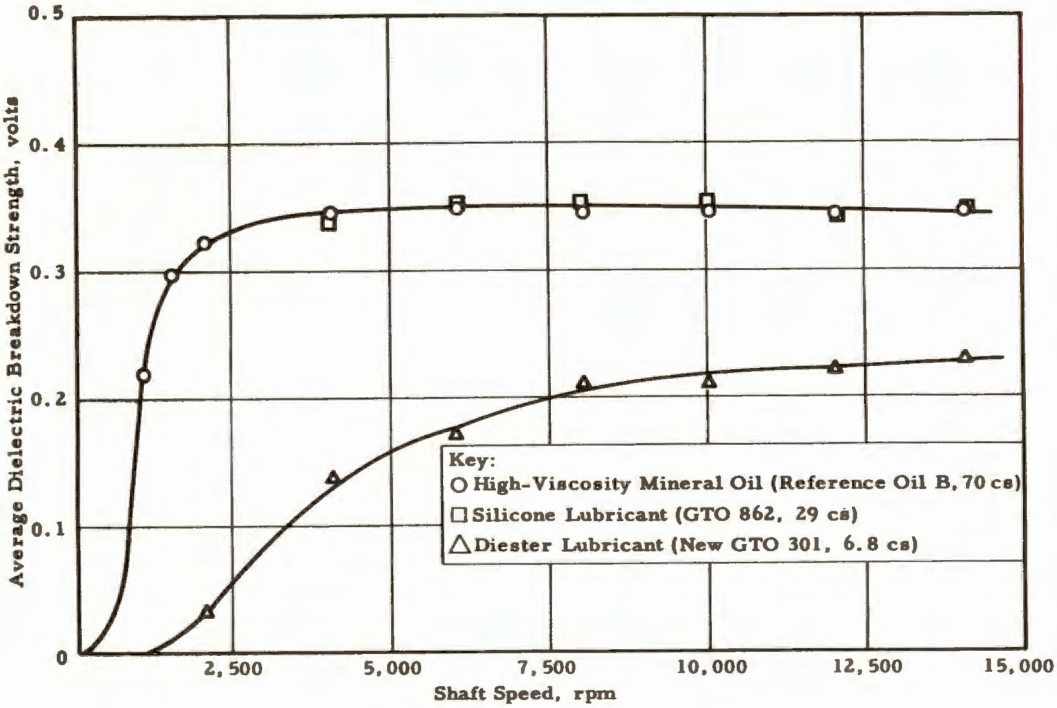
The basic processes of lubrication at the rolling-contact regions between the elements and races in bearings are discussed. Lubricant film-thickness and contact-deformation data with various lubricants in a precision rolling-disk machine, using a special X-ray method, are presented. These data show that lubricant films significantly thicker than molecular dimensions exist at bearing contacts under many practical conditions, and that significant deviations from Hertzian stresses can occur by the action of lubricant pressures, particularly at the edges of the bearing contact regions. Here the lubricant must withstand high shear stresses under rapidly changing pressures to effectively separate bearing surfaces. Even if appreciable breakdown of the lubricant film occurs in the near-contact region, the high-pressure shear behavior of the lubricant at the edges of the contact may still affect bearing contact stresses significantly.

Rolling-element bearings are used in many missile and aircraft applications where operating conditions are so severe that lubrication and reliability are serious problems. In many cases, new synthetic lubricants drastically reduce the life of rolling bearings. Bearing failure by pitting fatigue is known to be affected by the viscosity and type of lubricant as well as by bearing operating conditions. The action of the lubricant in the near-contact regions between the rolling elements and races in rolling bearings is not well understood. Consequently, environmental factors such as stability and fluidity over a wide temperature range, instead of those basic properties that affect rolling-bearing lubrication and performance, have been emphasized in the development of synthetic lubricant materials.

The purpose of a research program in progress at Battelle is to study the important fundamental properties of lubricants in rolling-contact lubrication, so as to guide the development of improved rolling-bearing lubricants. Since the performance of the lubricant at the rolling contacts in bearings is basic to the effect of the lubricant on bearing behavior, this research has been concentrated on the study of the lubrication in these contact regions.

REVIEW OF ROLLING-CONTACT LUBRICATION PHENOMENA

The early work of Hertz and others on contact stresses between elastic solids showed that contacting curved surfaces, such as between the elements and races in rolling bearings, deflect under load to form finite areas of contact. However, early lubrication theory for gears and rolling bearings[1], based on the same assumptions as those used in hydrodynamic journal-bearing theory (sometimes including the increase in viscosity of the lubricant under pressure), neglected this contact deformation. The absurdly small lubricant film thicknesses predicted by such theory, together with the formidable mathematics involved in allowing for deformation under lubricant pressures, seemed to discourage research on mechanical rolling-lubrication phenomena. There existed the widespread notion that at the contacts in rolling bearings and gears the stresses and deflections are entirely Hertzian in nature and, at best, the surfaces are separated by adsorbed lubricant films of only molecular dimensions. However, recent measurements by Cameron and others[2] have indicated a lack of electrical continuity across the contacts in gears and rolling bearings over a wide range of practical operating conditions. Figure 1 illustrates the existence of a significant dielectric breakdown potential across the rings of a 60-mm ball bearing for all operating speeds above about 2000 rpm with mineral oil, diester, and silicone lubricants[3]. Crook[4] has measured significant lubricant film thicknesses between cylindrical rolling disks using capacitance techniques. Archard[5], also using a capacitance method, has detected lubricant films thicker than molecular dimensions between crossed cylinders, in which the contact area under load is circular. Other investigators[6] have used electrical resistance methods to detect lubricant films at concentrated contacts.

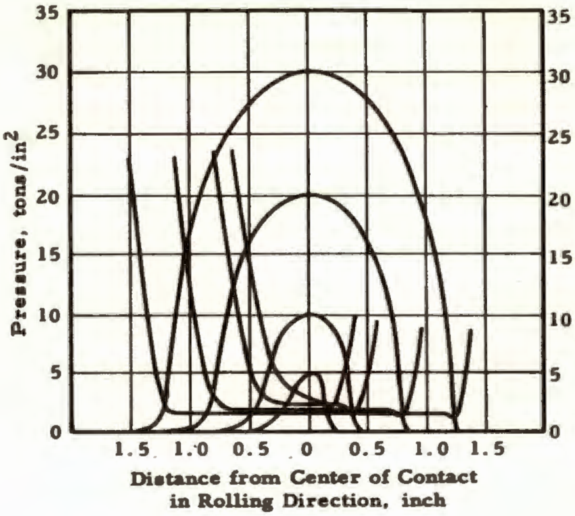


980 lb Thrust Load (Approximately 200,000 psi Maximum Hertz Stress)
150 F Oil-Inlet Temperature

FIGURE 1. DIELECTRIC BREAKDOWN STRENGTH MEASURED ACROSS A 60-MM BALL BEARING

The above studies have shown that significantly thick lubricant films do exist at the concentrated contacts in rolling bearings and gears under many conditions and that the mechanics of such film formation may have important practical implications. Meldahl[7], who published some of the first rolling lubrication theory to account realistically for surface deformation, and more recently Dörr[8], showed that a marked increase in load-carrying capacity occurred if the lubricant pressures were allowed to deform the bearing surfaces elastically, as surely they must do. However, if there are large enough pressures to deflect the steel surfaces, these same pressures must also modify the viscous properties of the lubricant film to a great extent. Pressures of 150,000 psi, certainly a conservative figure for the contact pressures in many bearings, are known to increase the viscosity of typical oils about four or five orders of magnitude[9]. Thus, any realistic theory of mechanics for the lubrication of rolling bearings and gears must account for, at least, the simultaneous elastic deformation of the contact surfaces and the increase in viscosity of the lubricant.

The mathematical problem representing both surface deformation and increase of viscosity with pressure is so complex that only scattered numerical solutions, often utilizing high-speed electronic computers in solving each particular case, are available. Such solutions have been published by Poritsky[10], Petrusevich[11], and more recently by Dowson and Higginson[12], from whose paper Figure 2 is taken. The lubricant pressure profiles and shapes of the deformed contact surfaces between two cylinders shown in Figure 2 illustrate how, as the load is increased, the pressures and deformation approach the Hertzian solution for dry, static contact, except for some separation of the flattened surfaces by the lubricant film. (Note the region of extreme curvature around a protrusion from the surface at the trailing edge of the contact area, which must result in a rather severe surface stress concentration caused by the lubricant.) It was this approach to



Taken from Dowson and Higginson, "A Numerical Solution to the Elasto-Hydrodynamic Problem," J. Mech. Engrg. Sci. 1 (1), 6-15 (1959).

FIGURE 2. PRESSURE DISTRIBUTIONS AND FILM SHAPES UNDER INCREASING LOAD

Hertzian conditions that led Grubin(13) to derive an approximate solution for this elasto-hydrodynamic problem by assuming that the deformation is entirely Hertzian, but the surfaces are merely translated slightly away from each other, and by integrating the hydrodynamic flow equation for the lubricant (assuming, of course, an exponential increase of viscosity with pressure) merely over the converging inlet region before the parallel region of closest approach between the surfaces. By thus determining how much lubricant must be left at the start of the parallel region, and being trapped there, continue to pass through and separate the surfaces, Grubin came up with an approximate formula for the film thickness, as follows:

$$h_o = 1.13 \ (P'/E')^{-0.091} \ [(\mu_o \gamma V)^2 R]^{0.364},$$

where P' is the load per unit width of the contact between two cylinders, μ_o is the viscosity at atmospheric pressure, γ is the pressure-viscosity coefficient defined by the formula, $\mu = \mu_o e^{\gamma p}$, where p is pressure, V is the sum of the velocities of the two contacting surfaces in the direction of rolling, the surface conformity radius $R = 1/(1/R_1 + 1/R_2)$, where R_1 and R_2 are the radii of curvature in the direction of rolling at the contact, and the "reduced" elasticity

$$E' = 1/\left(\frac{1-\nu_1^2}{\pi E_1} + \frac{1-\nu_2^2}{\pi E_2}\right),$$

where E_1 and E_2 are the moduli of elasticity and ν_1 and ν_2 are the Poisson's ratios of the surfaces. Although the assumptions used in deriving this formula might appear drastic, it has the advantage of generality found lacking in the other solutions, and it has recently been found to correlate reasonably well with experimental measurements of film thickness(3, 4)

The assumption of rolling cylinders is used in all the above advanced lubrication theories. Although some early theories were extended to the case of the rolling ball, the additional complexities of a realistic 3-dimensional solution make it impractical at this time. In addition, if the rolling speeds are sufficiently high (as they must be for the mechanical aspects of the lubrication to be significant) and the contact ellipses between the balls and races are sufficiently narrow (as they are in most high-speed ball bearings), a good approximation of the film thickness in ball bearings can be obtained by taking P' in the above equation as the load per 2/3 of the major axis of the contact ellipse calculated from Hertzian formulas.

Other refinements in contact lubrication theory include the introduction of heat generation and dissipation at the contact resulting in a temperature rise in the film, and the assumptions of mathematical models for the lubricant in which relaxation time and other non-Newtonian effects are important. The effect of temperature in reducing the viscosity, and thus the film thickness, was estimated by Grubin[13] (using some rough calculations) to be insignificant for gross sliding velocities less than about 200 fpm at the contact under typical bearing conditions. Thus, temperature effects are probably important at the tips and roots of gear teeth but not in rolling bearings where slip from ball spin, retainer drag, and combined loading normally does not exceed a few per cent of the rolling speed. Apparently, the lubricant films at rolling contacts are so thin, and the high shear rates resulting in viscous heating are so near the surface under near-rolling conditions, that the temperature of the lubricant film very closely approaches the surface temperature of the bearing. In fact, it was found by Crook[4] as well as in our work[3] that the important temperature for determining the lubricant viscosity in rolling lubrication is the surface temperature of the bearing contact path. Lubricant inlet temperature appears to have little direct effect on the lubrication other than its change of the over-all heat transfer to the bearing.

Published contact-lubrication theories for non-Newtonian lubricants have been limited to the use of the Maxwell model (for example, Milne[14]) and the Bingham model for greases (for example, Kotova[15]). More recently, Crouch and Cameron[16] compared the results of a Maxwell lubrication theory with experiments on the scuffing of gears and concluded that lubricant relaxation effects probably do not play a significant part in rolling bearing and gear lubrication. Non-Newtonian effects will be discussed in more detail later in this paper.

MEASUREMENTS OF LUBRICANT-FILM THICKNESS AND CONTACT DEFORMATION BETWEEN ROLLERS BY AN X-RAY METHOD

The performance of lubricants at rolling contacts depends greatly on the thickness of the lubricant film that separates the surfaces. Lubricant film formation involves the deformation of the contact surfaces, which is also reflected as contact stress in the bearing. In order to measure lubricant film thickness and contact deformation experimentally, a special precision rolling-disk machine was designed and constructed for use with an X-ray system. Essentially this method of measuring film thickness consists of directing a monochromatic, collimated, square beam of high-energy X-rays between two rolling-disk surfaces. The amount of radiation passing between the disks parallel to the flattened contact regions is thus related to the thickness of the lubricant film separating the surfaces, as shown in the sketch in Figure 3. A particular wave-length X-radiation was selected which penetrated lubricants quite readily but not steel to a significant amount. Figure 4 shows the disk machine set up on a standard X-ray diffraction machine which was used as the initial source of X-rays.

Each disk was made integral with the rotor which was driven by an electric motor from a variable-frequency power supply and was supported in pivoted-pad hydrodynamic bearings on precision-lapped journals for dynamic stability. The disk surfaces were slightly crowned with a transverse radius selected so that the Hertzian contact ellipse would simulate the ball-race contacts in typical ball bearings. By mounting the disk machine on ball and runner tracks, traverses were made across the 30-mil wide X-ray beam at a controlled rate to obtain profiles of the deformed contact surfaces on the disks during operation.

EFFECT OF OPERATING CONDITIONS AND LUBRICANTS ON MINIMUM FILM THICKNESS

The minimum lubricant film thickness has been measured with several mineral oils, a diester lubricant, and a silicone over a wide range of conditions. Using the Grubin formula for film thickness as a guide, the data has been plotted in the form of three dimensionless numbers as follows:

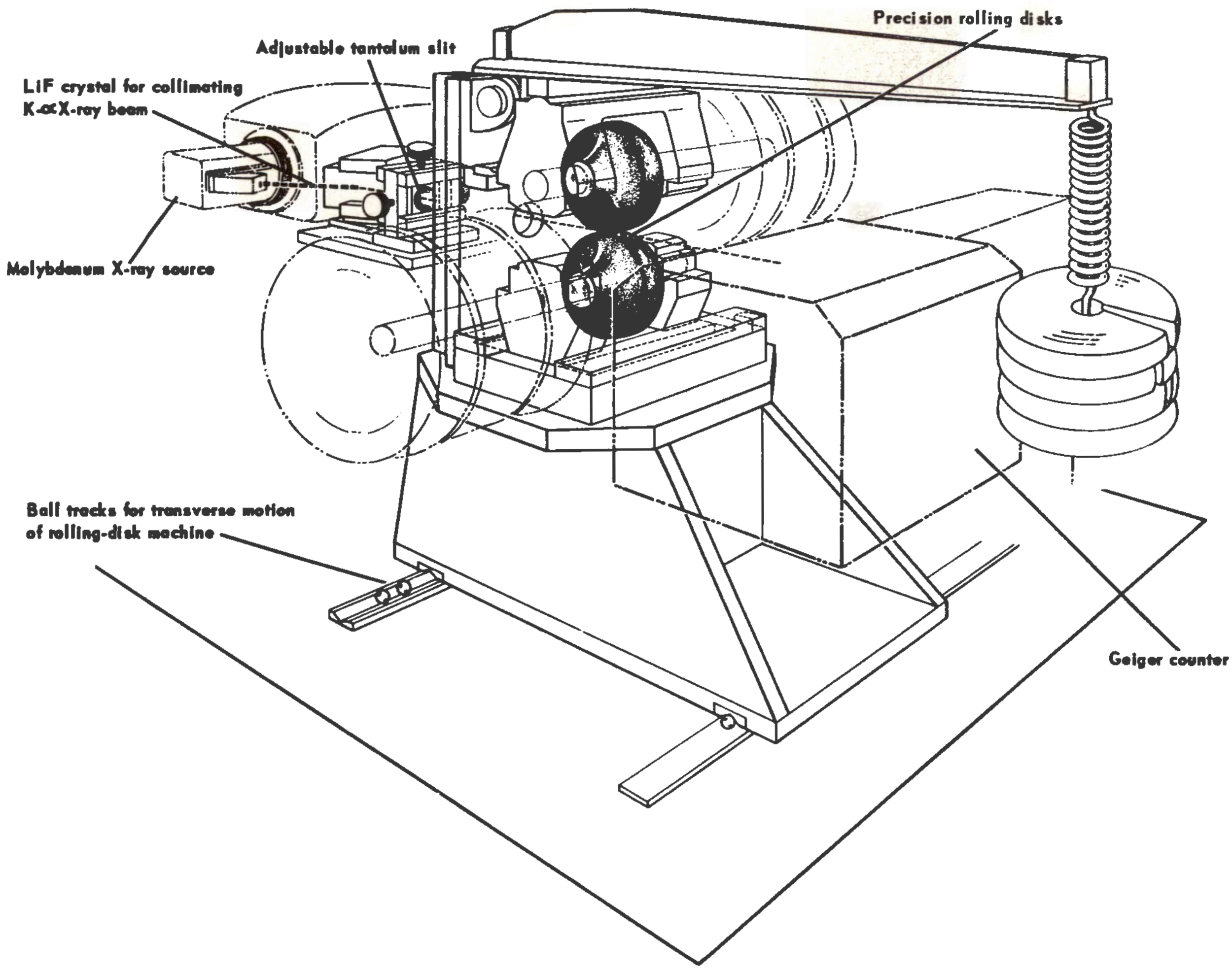
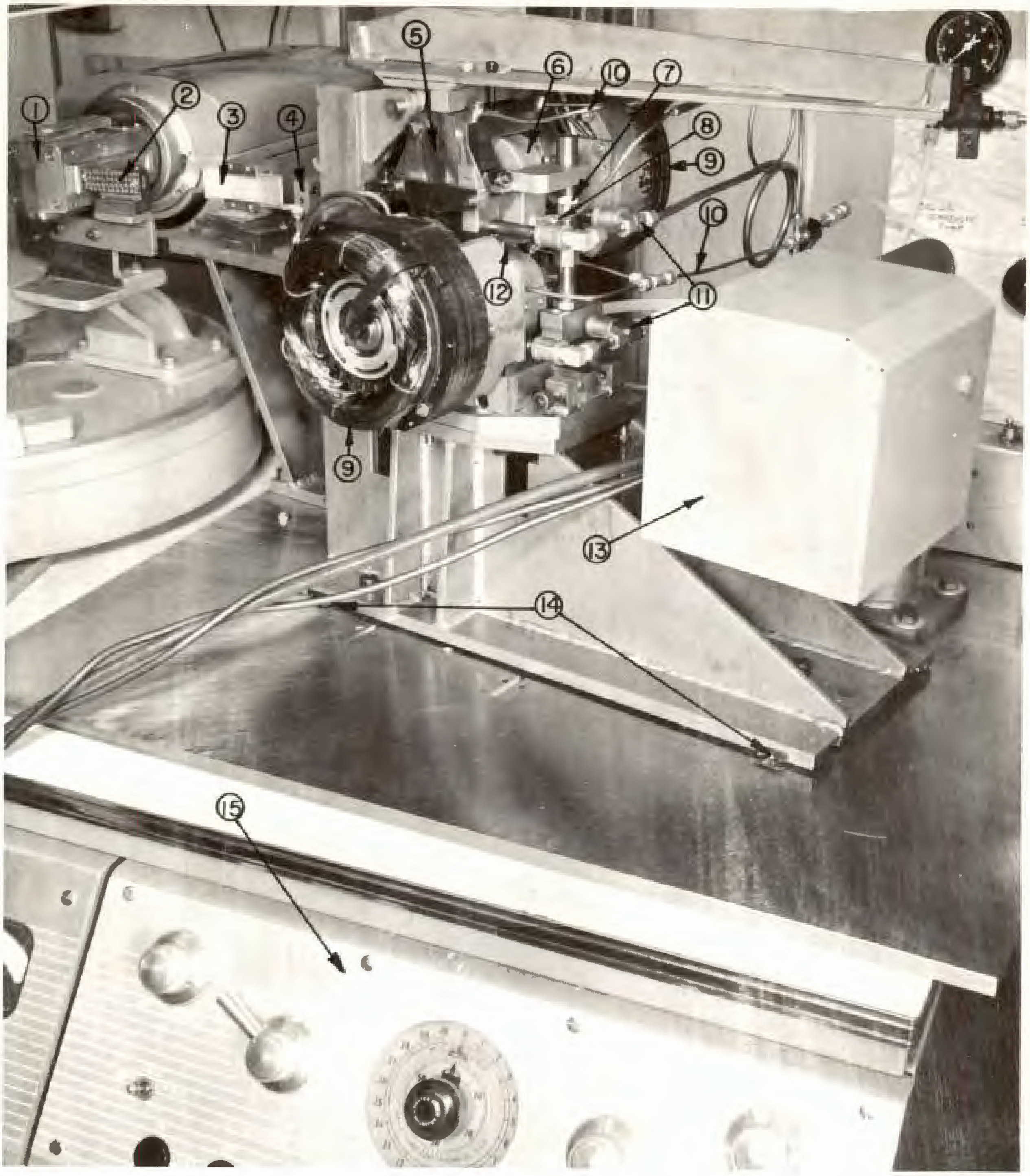


FIGURE 3. PRECISION ROLLING-DISK MACHINE AND X-RAY SYSTEM FOR ROLLING-CONTACT-LUBRICATION EXPERIMENTS



1. Molybdenum X-ray source
2. X-ray beam collimator
3. Lif diffraction crystal
4. Tantalum slit adjustor
5. Upper support bearing block
6. Test oil case around rollers
7. Differential screw for calibration adjustment of gap between rollers
8. Test oil supply jet
9. Roller drive motors
10. Support bearing oil inlet lines
11. Scavenge oil outlet lines
12. Plastic window in test oil case for X-ray bean
13. Geiger counter
14. Ball tracks for transverse motion of rolling-disk machine
15. X-ray control panel

FIGURE 4. PRECISION ROLLING-DISK MACHINE AND X-RAY SYSTEM FOR MEASURING LUBRICANT-FILM THICKNESS

Film thickness number $= h_o/R$,

Load number $= P'/E'R$,

Contact-lubrication flow number $= \mu_o \gamma V/R$,

where h_o, R, P', E', μ_o, γ, and V are defined as before.

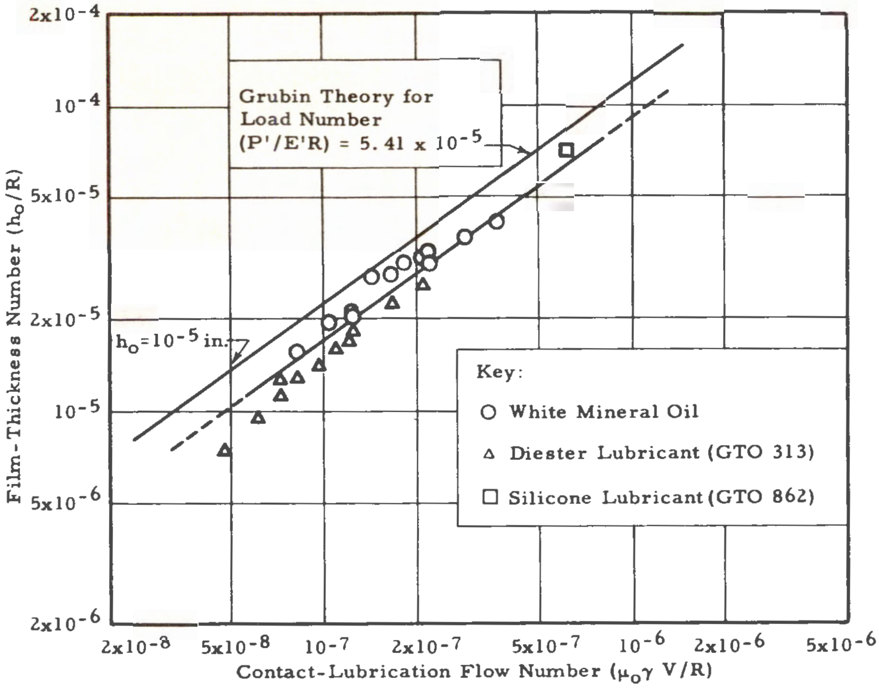
Plots of film thickness number versus contact-lubrication flow number for two load levels are shown in Figures 5 and 6. Lines representing the Grubin formula for these loads are also plotted in Figures 5 and 6. It can be seen that there is some scatter of the experimental points on these plots, but that the data consistently fell below the predicted film thickness from the Grubin theory. This deviation from theory appeared to increase for minimum film thicknesses below 10 millionths of an inch, when the electrical continuity between the surfaces increased appreciably. Figure 7 illustrates the gradual increase in local contact between the surfaces as the film thickness decreased. These observations may well be related, since appreciable amounts of so-called "asperity contact" probably result in increased heat generation at the contact, and the subsequent reduction in effective viscosity in the rest of the lubricant film should reduce the film thickness.

The consistently lower measured film thicknesses than predicted by the Grubin formula, even for such thick films that no significant electrical continuity was detected, may well be related to deviation of the contact-deformation profile from the Hertzian, on which Grubin based his theory. Any rounding of the flattened contact region, particularly at the inlet edge, will result in more lubricant being squeezed out before reaching the high-pressure region, thus permitting the surfaces to approach each other more closely. These changes in deformation profile are best illustrated by the traversing measurements described previously.

EFFECT OF OPERATING CONDITIONS AND LUBRICANTS ON CONTACT DEFORMATION

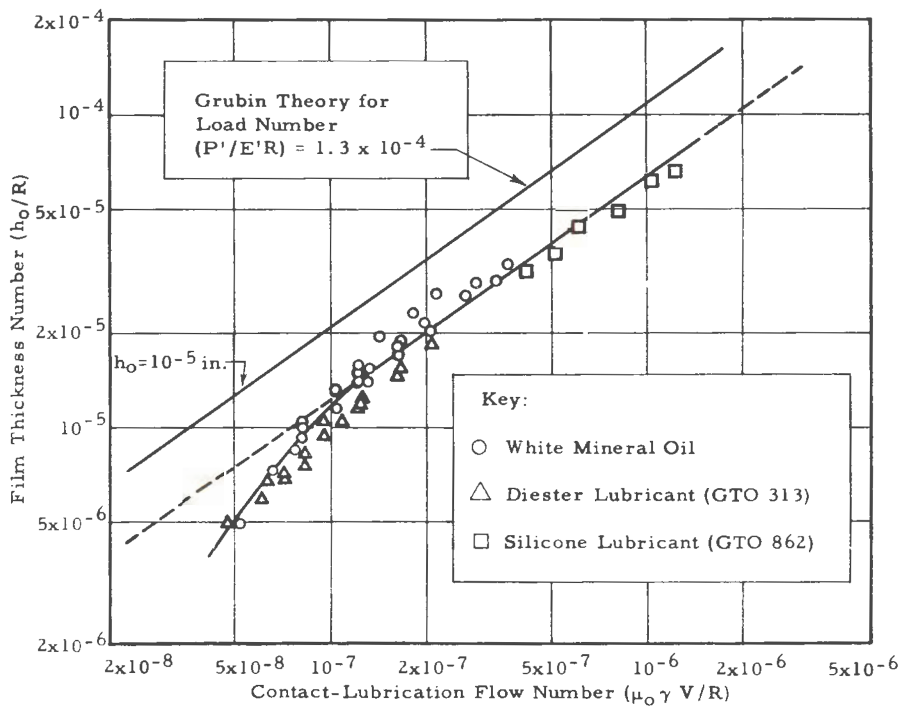
Figure 8 is an example of a profile of the contact region under no load showing the sharp parabolic trace corresponding to the 36-inch transverse radius to which the disk surfaces were lapped. As the load was increased (using a 15-cs white mineral oil at 2600 fpm rolling speed), the contact region flattened noticeably while the minimum film thickness decreased only slightly, as shown in Figure 9. The curves in Figure 9 are qualitatively similar to the theoretical film shapes in Figure 2, although, of course, the experimental profiles were taken in the transverse direction rather than in the rolling direction as in Figure 2. In Figure 10 the effect of rolling speed with the white mineral oil is shown, and Figure 11 shows the effect of the viscosity of an engine oil (Reference Oil B) on contact-deformation profile. Rolling speed and viscosity (at the disk surface temperature) seemed to have similar effects; they both markedly increased film thickness and changed the shape of the contact-deformation profile more toward the rounded, or unloaded shape, and further from the flattened, Hertzian profile. Even different type lubricants of essentially the same viscosity produced slightly different contact-deformation profiles under the same operating conditions, as shown in Figure 12.

The changes in the shapes of the contact-deformation profiles under different conditions show that low load, high speed, high viscosity, and certain lubricant types tend to make the contact deformation more rounded at rolling contacts, particularly at the edges of the contact regions. That is, the load is less concentrated in the "Hertz" region of nearest approach under these conditions, the lubricant pressures are distributed over a wider area of the surface, and the contact stresses thus are less severe. To a certain extent these trends are predicted by elasto-hydrodynamic lubrication theory. For example, in Figure 2 the more rounded film shapes for the lower loads resulted from spreading pressure profiles that deviated considerably from the Hertzian ellipse. These general trends should apply as well to the transverse direction, in which the contact-deformation profiles were measured.



Load Number $(P'/E'R)$ for 220 lbs Load = 5.41×10^{-5}

FIGURE 5. PLOT OF DIMENSIONLESS PARAMETERS FOR FILM THICKNESS MEASUREMENTS AT 220 LBS LOAD



Load Number $(P'/E'R)$ for 820 lbs Load = 1.3×10^{-4}

FIGURE 6. PLOT OF DIMENSIONLESS PARAMETERS FOR FILM THICKNESS MEASUREMENTS AT 820 LBS LOAD

Rolling Speed, fpm	Film Thickness, 10^{-6} in.
4370	13.5
3480	11.0
2600	9.0

Rolling Speed, fpm	Film Thickness, 10^{-6} in.
2600	9.0
2200	7.5
1735	5.5

Effect of Rolling Speed With Diester Lubricant at 130 F Disk Temperature and 820-Lb Load

Disk Load, lb	Film Thickness, 10^{-6} in.
20	25
120	18
220	14

Disk Load, lb	Film Thickness, 10^{-6} in.
320	12.0
420	10.5
520	9.0

Effect of Load With White Mineral Oil at 150 F Oil-Inlet Temperature (Variable-Disk Temperature) and 2600 Fpm Rolling Speed

13.5 kc Imposed Voltage Across Oil Film Presented Vertically and about 100 cps Sweep Voltage Presented Horizontally on Oscilloscope

FIGURE 7. OSCILLOSCOPE TRACES OF THE DIELECTRIC OIL-FILM BREAKDOWN VOLTAGE IN THE ROLLING-DISK MACHINE

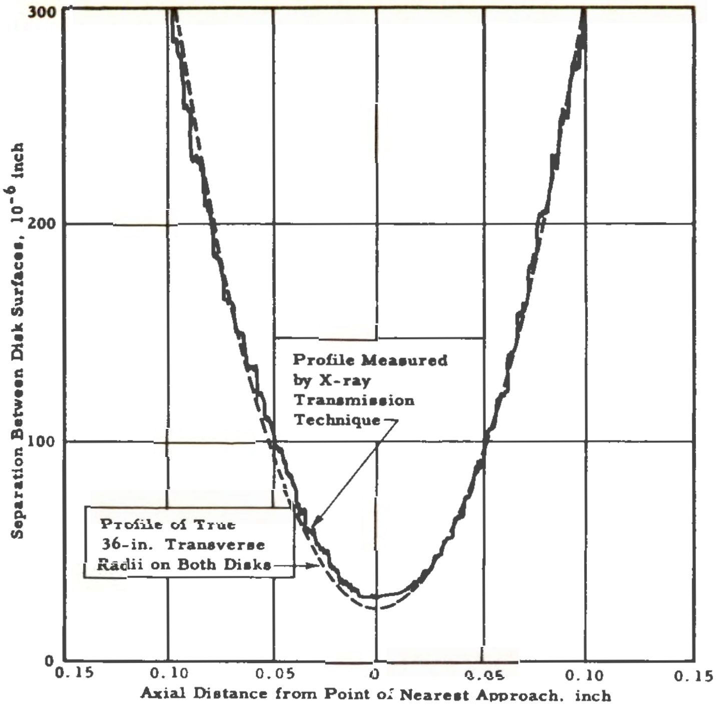
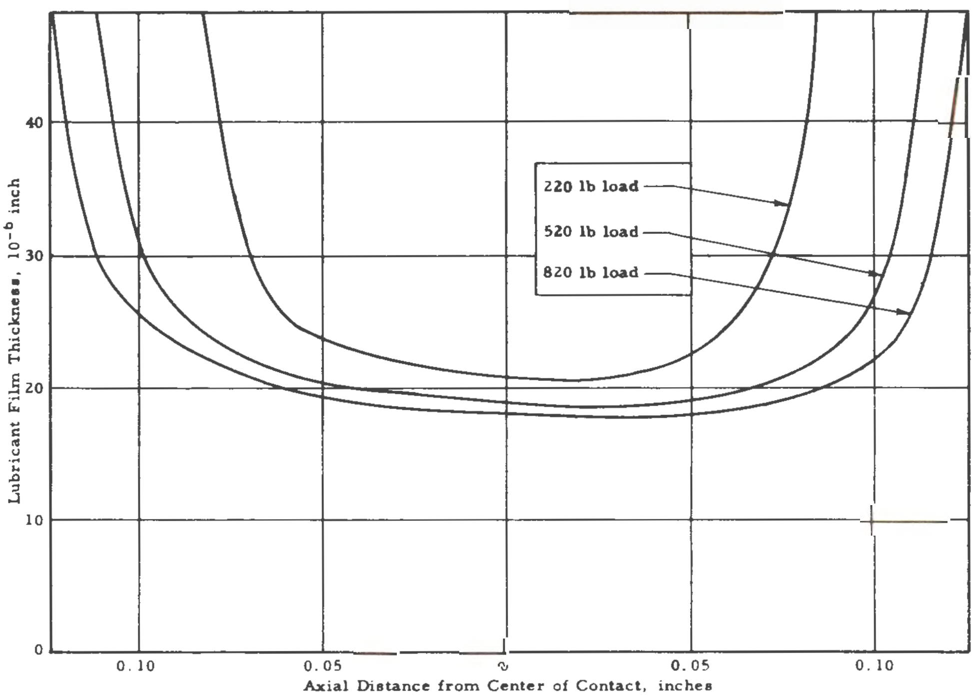


FIGURE 8. TRANSVERSE PROFILE OF SEPARATION BETWEEN ROLLERS UNDER NO LOAD



15 cs White Mineral Oil (128 F Roller Temperature), 2600 fpm Rolling Speed Smooth Curves Have Been Drawn Through Profile Traces

FIGURE 9. EFFECT OF LOAD ON FILM SHAPE BETWEEN ROLLERS

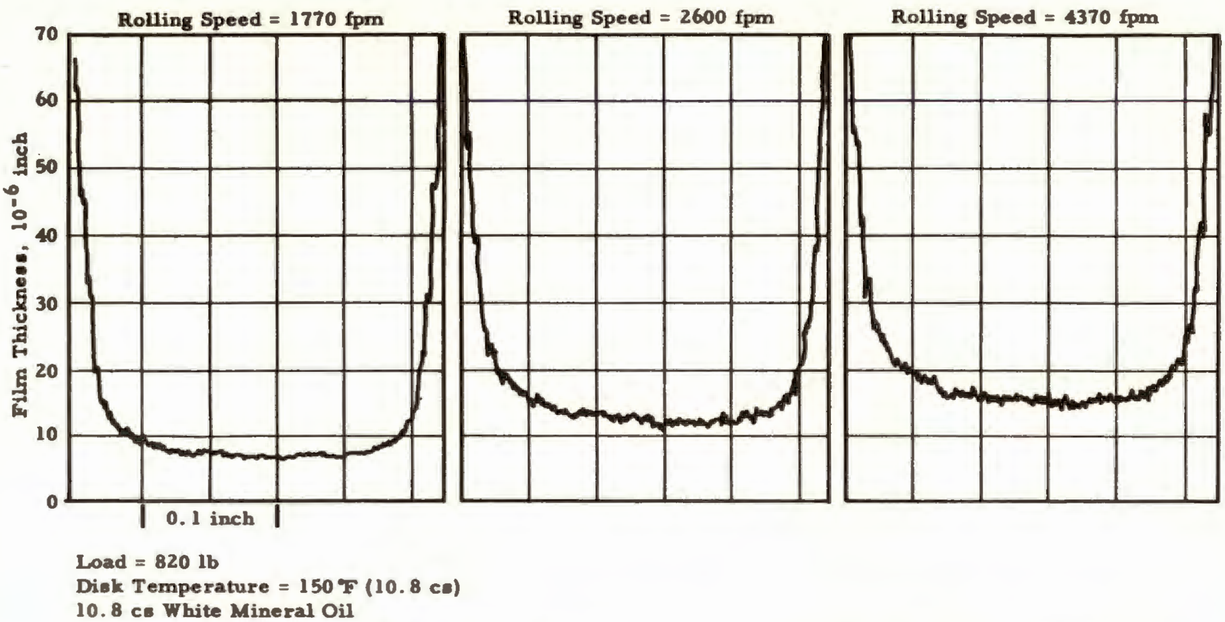


FIGURE 10. EFFECT OF ROLLING SPEED ON FILM SHAPE BETWEEN ROLLERS

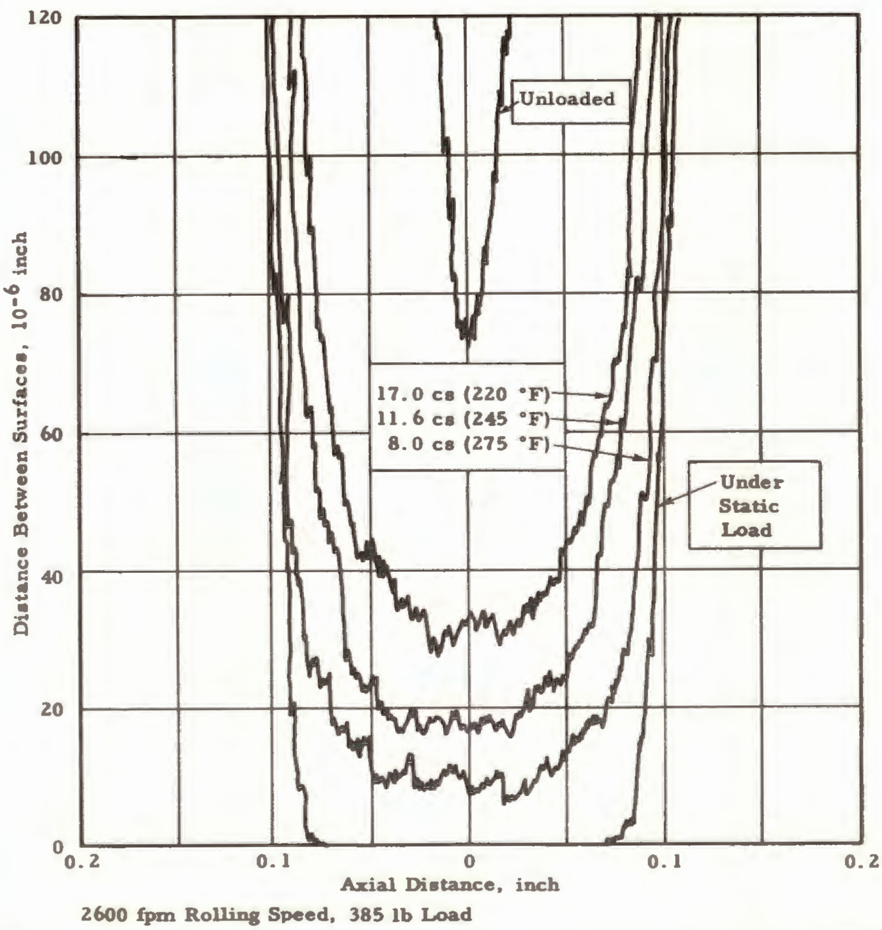
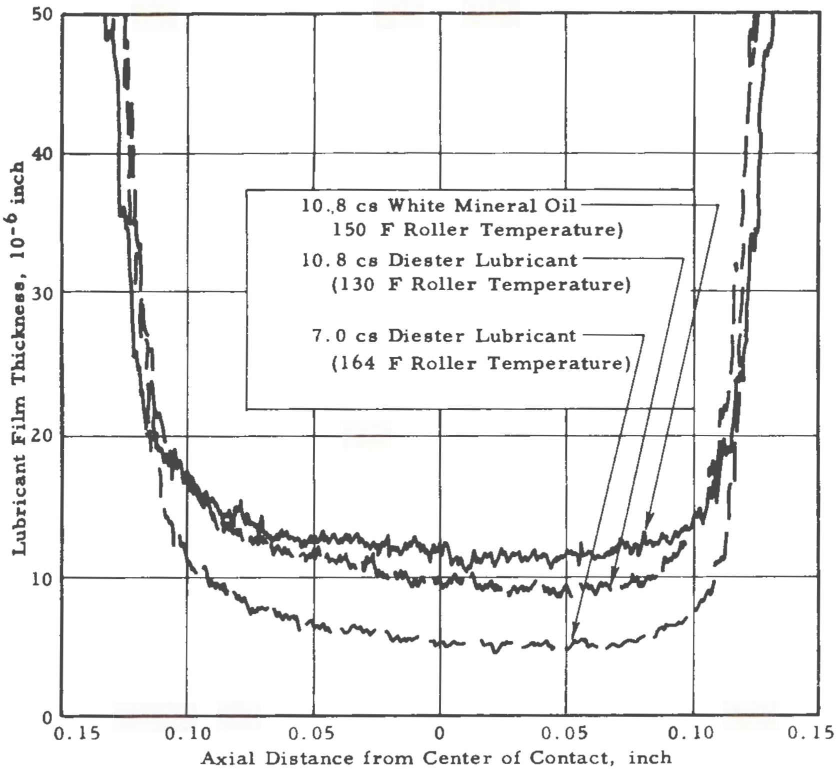


FIGURE 11. EFFECT OF VISCOSITY (OR TEMPERATURE) OF ENGINE OIL (REFERENCE OIL B) ON FILM SHAPE BETWEEN ROLLERS



Viscosities Determined at Roller Temperatures 2600 fpm
Rolling Speed, 820 lb Load

FIGURE 12. EFFECT OF LUBRICANT TYPE AND VISCOSITY ON FILM SHAPE BETWEEN ROLLERS

DISCUSSION

The experimental results show that there are certain anomalies in the lubrication at rolling contacts, such as between the elements and races in rolling bearings or between gear teeth, during operation with very thin films, namely, the increasing prevalence of electrical breakdown and the decrease in minimum film thickness below the theoretical. This behavior is consistent with the idea that there are two distinct regimes which exist in varying degrees of importance. These are the relatively thick film elasto-hydrodynamic regime, in which the load is supported by hydrodynamic pressures in the lubricant film balanced against the elastic contact stresses on the bearing surfaces, and the very thin adsorbed molecular film regime, in which local areas in the contact region are pressed so close together that only absorbed lubricant films remain to carry that part of the load. At least some local plastic deformation of the bearing surfaces usually occurs in the thin adsorbed-film regime.

Separation of the lubrication mechanisms into these two regimes does not imply that the operation of any particular machine element is in either one or the other regime; in the majority of practical cases lubrication of both types exists in varying relative degrees. In fact, the elasto-hydrodynamic film may be squeezed so thin in the regions of closest approach between the surfaces that most of the load in those regions is carried on adsorbed surface films but that an appreciable portion of the total load is still supported elasto-hydrodynamically in the regions around the edges of the contact where the lubricant film is still quite thick. Such edge support can have important effects on the contact stresses, particularly the reversing shear stresses in the metal surface which are thought to be important in rolling-contact fatigue and which occur near the edges of the contact region.

The important lubricant properties in elasto-hydrodynamic lubrication are the high-pressure viscosity or rheology of the lubricant material. These properties determine when the lubricant film becomes so thin that a significant amount of local contact on adsorbed films occurs, with the attendant consequences, and they determine the over-all level of the contact stresses, apparently even when there is appreciable contact on adsorbed films. The measured contact-deformation profiles show how different the contact stresses can be with different lubricants and under different operating conditions. These different profiles, resulting in different contact stresses, are caused by the different pressures generated in the lubricant films near the edges of the contact regions. In these regions the lubricant pressures are increasing (or decreasing) very rapidly, thus subjecting the lubricant film to very high shear rates at elevated pressures. The shear stresses that a lubricant can support under these simultaneously increasing pressures and shear rates determine how much the bearing surfaces are deformed in these regions and thus how large shear stresses are developed in the bearing metal.

The flow characteristics of lubricants under high pressure and shear are probably affected by the composition and molecular structure of the major constituents. As the pressure squeezes out the free volume from between the molecules, they tend to approach each other more closely. If the closer approach of the molecules results in new types of intermolecular interactions under pressure, for example, hydrogen bonding in some of the synthetic lubricants(18, 19), such weak interactions may not be stable under high shear and would result in decreasing shear stress at high shear rate. Molecular alignment effects may also be more important at higher pressures as the freedom of molecular motion is reduced. Non-Newtonian effects such as this were detected by Norton, Knott, and Muenger(17) with several fluids including a high-viscosity motor oil. According to the Ree-Eyring theory of viscosity, the decrease of shear stress with increasing shear rate under high pressure can be described in terms of the shear modulus and relaxation time of the lubricant(18). The Ree-Eyring model for the lubricant has been adapted for solution of the elasto-hydrodynamic problem, and film thicknesses much less than with the Newtonian model are predicted(3). Substitution of a simplified Ree-Eyring lubricant into a theory similar to Grubin's has resulted in a formula for film thickness with broad generality. This theory indicates that appreciable deviation from the Grubin formula, in which a Newtonian lubricant was used, occurs when the parameter

$$\frac{\beta V}{h_o} > 1,$$

where β is proportional to, and of the same order of magnitude as, the relaxation time, V is the sum of the contact surface velocities, and h_o is the minimum film thickness. For typical values of V and h_o ($V = 2600$ fpm, $h_o = 10^{-5}$ inch), significant deviations from the Grubin formula occur at a value of β of about 10^{-8} seconds. In other words, for lubricants which have a relaxation time longer than 10^{-8} seconds in the inlet section of the rolling-contact regions the stresses developed in the bearing metal will be less severe than those calculated from an elasto-hydrodynamic theory like that shown in Figure 2, but the minimum lubricant film thickness will be thinner.

Even though it is desirable, of course, to minimize the breakdown of the elasto-hydrodynamic films in rolling-element mechanisms, appreciable contact on adsorbed lubricant films is inevitable under some operating conditions. The ability of lubricants to form desirable adsorbed films on bearing surfaces can still prevent actual metal-to-metal contact and galling under these conditions. The right kind of adsorbed film should also reduce local heat generation and temperature flashes, which result in local lubricant decomposition and thermal stresses on the bearing surfaces. The mechanical properties of the material composing these adsorbed films may be most important in effecting these results. However, other chemical factors are no doubt important in bearing failure mechanisms and they, too, require further study. One such mechanism that has been proposed is the mobility of the adsorbed lubricant film to migrate into surface cracks in the bearing metal, thus influencing the propagation of these cracks into fatigue spalls.

A special apparatus for measuring the high-pressure rheological characteristics of lubricants has been developed. By selecting certain pure lubricant materials for study, we hope to explore the composition and structural aspects of these rheological characteristics that are important in rolling-bearing lubrication. Studies of both the mechanical and chemical aspects of adsorbed surface films, including the effect of different additive types, will help develop further understanding of rolling lubrication and failure mechanisms and thus guide the development of lubricants for future aircraft and missile systems.

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