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# MATHEMATICAL ANALYSIS OF HEAT CONDUCTION IN A SECTOR OF A HOLLOW CIRCULAR CYLINDER

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# FOREWORD

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This report (contractor's report No. OR 8042) was completed in March 1966 and covers the work performed from March 1965 to January 1966. Mathematical analysis and development were conducted by Mr. J. G. Torian under the supervision of Mr. J. I. Gonzalez, Head of the Fluid Dynamics, Heat, and Mass Transfer Section in the Engineering Mechanics Research Laboratory, Martin Company. The digital programming was performed by Mr. V. T. Murphy under the supervision of Mr. C. J. Gentry of the Scientific Computer Applications Department.

This technical report has been reviewed and is approved.

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Chief, Theoretical Mechanics Branch
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# ABSTRACT

This study was undertaken to develop an analytical method of determining temperature distribution in a sector of a hollow circular cylinder, with temperature dependent thermophysical properties, based on an exact mathematical solution. The subject non-linear differential equation is reduced to a linear differential equation by an integral transform and with the assumption that only the thermal conductivity is a function of temperature, whereas the thermal diffusivity remains constant. The solution has been programmed in FORTRAN IV for application to an IBM 7094. In addition, a successive transform applicable to the solution of similar problems involving temperature dependent thermal diffusivity is illustrated and discussed.



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## ILLUSTRATIONS





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TABLES





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# SYMBOLS

| Mathematical | Program Counterpart | Use |
|---|---|---|
| $v$ | | real temperature |
| $T$ | | transform temperature |
| $r$ | | radius |
| $\phi$ | | angle |
| $\theta$ | | time |
| $\lambda$ | | dummy time variable |
| $K(v)$ | | thermal conductivity |
| $\rho$ | | density |
| $C(v)$ | | specific heat |
| $\alpha$ | ALPHØ | thermal diffusivity |
| $r_1$ | ARE(1) | inner radius |
| $r_2$ | ARE(2) | outer radius |
| $\phi_1$ | PHI(1) | lower angular boundary (equal to zero) |
| $\phi_2$ | PHI(2) | upper angular boundary |
| $K_o$ | AZERØ | thermal conductivity at v equal zero |
| $\gamma$ | GAMMØS(i) | angular eigenvalue |
| $\beta$ | BAYTØS(i) | radial eigenvalue |
| $\epsilon$ | EPILØN(i) | radial eigenvalue |
| $A_{\gamma\beta}$ | ARBA(i,j) | arbitrary coefficient |
| $B_\gamma$ | BSUB(i) | arbitrary coefficient |
| $G_{\gamma\beta}$ | ARBG(i,j) | arbitrary coefficient |



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| | | |
|---|---|---|
| $F_\gamma$ | FSUB(i) | arbitrary coefficient |
| $F_\epsilon$ | AF(i) | arbitrary coefficient |
| $M_\epsilon$ | AM(i) | arbitrary coefficient |
| $\ell_{11}$ | AL(1,1) | boundary condition indicator |
| $\ell_{12}$ | AL(1,2) | boundary condition indicator |
| $\ell_{21}$ | AL(2,1) | boundary condition indicator |
| $\ell_{22}$ | AL(2,2) | boundary condition indicator |
| $k_{11}$ | AK(1,1) | boundary condition indicator |
| $k_{12}$ | AK(1,2) | boundary condition indicator |
| $k_{21}$ | AK(2,1) | boundary condition indicator |
| $k_{22}$ | AK(2,2) | boundary condition indicator |
| b | BEE | constant used in definition of K(v) |
| c | SEE | constant used in definition of K(v) |
| $f_{r1}(\phi)$ | FARE1(i) | coordinate varying boundary condition in real temperature at $r_1$ |
| $f_{r2}(\phi)$ | FARE2(i) | coordinate varying boundary condition in real temperature at $r_2$ |
| $f_{\phi 1}(r)$ | FPHI1(i) | coordinate varying boundary condition in real temperature at $\phi_1$ |
| $f_{\phi 2}(r)$ | FPHI2(i) | coordinate varying boundary condition in real temperature at $\phi_2$ |
| $F_{r1}(\phi)$ | FARE1(i) | coordinate varying boundary conditions in transform variable at $r_1$ |
| $F_{r2}(\phi)$ | FARE2(i) | coordinate varying boundary conditions in transform variable at $r_2$ |
| $F_{\phi 1}(r)$ | FPHI1(i) | coordinate varying boundary conditions in transform variable at $\phi_1$ |
| $F_{\phi 2}(r)$ | FPHI2(i) | coordinate varying boundary conditions in transform variable at $\phi_2$ |



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| | | |
|---|---|---|
| $h(r,\phi)$ | DZERØ(i,j) | initial temperature distribution in real temperature |
| $f(r,\phi)$ | STEADY(i,j) | steady state solution |
| $\Upsilon[v]$ | | transform of real temperature |
| $\Lambda[T]$ | | inversion of transform temperature |
| $T_i(\theta,r,\phi)$ | TERMA | initial temperature distribution solution in transform variable |
| $T_s(\theta,r,\phi)$ | -TERMB+ $g(\theta) f(r,\phi)$ | surface effects solution in transform variable |
| $\xi$ | XI | function as defined in text |
| $\Xi(v,\theta,r,\phi)$ | | transform time function |
| $\Omega(v,\psi,r,\phi)$ | | inversion of time function |
| $J_\gamma(\beta r)$ | | Bessel function of first kind and order $\gamma$ |
| $Y_\gamma(\beta r)$ | | Bessel function of second kind and order $\gamma$ |
| $J'_\gamma(\beta r)$ | | $\frac{\partial}{\partial\beta} J_\gamma(\beta r)$ |
| $Y'_\gamma(\beta r)$ | | $\frac{\partial}{\partial\beta} Y_\gamma(\beta r)$ |
| $g(\theta)$ | GTHETA | time function |
| $\psi$ | | transform time |



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# SECTION I

## INTRODUCTION

This study was undertaken to develop an analytical method for determining temperature distribution in a sector of a hollow circular cylinder, with temperature dependent thermophysical properties, based on an exact mathematical solution. The subject non-linear differential equation was reduced to a linear differential equation by an integral transform with the assumption that only the thermal conductivity is a function of temperature, whereas the thermal diffusivity remains constant.

The integral transform method used in linearizing the equations is presented in Reference 1. Carslaw and Jaeger attribute the method, as applied to transient problems, to Van Dusen (1930) and note that the method as applied to steady heat flow dates back to 1894 (Kirchhoff). The Russian authors, Kudryashev and Zhemkov (Reference 2), rediscovered the method in 1963. They present a method of solution to problems involving temperature dependent thermal diffusivity using a successive transform. The formulation of these methods to analysis of two-dimensional transient heat conduction in a segment of a hollow circular cylinder is presented in Section IIA of this report.

The resulting linear differential equation, initial temperature distribution, and coordinate and time varying boundary conditions constitute a Sturm-Liouville System in the transform variable. A series solution of the linear system, as applicable to hollow semicircular cylinders, is presented by Luzzi (Reference 3) and by Torian (Reference 4) as applicable to hollow sectors of cylinders. Section IIB of this report presents a revised solution to the linear system based on Reference 4. The revisions are directed primarily toward greater applicability to calculations on a digital computer.

The solution of the linear system is applicable to either the temperature dependent or constant thermal diffusivity case. A digital computer program was developed based on this system as applicable to the temperature dependent thermal conductivity, constant diffusivity case. Mathematical formulation is given in Section IIB. A discussion of the digital program is given in Section III. The authors' comments on mathematical formulation of the temperature dependent thermal diffusivity case are included in Section IIC and the latter part of Section IIA.



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The utilization of the computer program is given in Section IIIB. A detailed description of the program is provided in Section IIIA, and Section IV shows its application to typical sample problems.



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# SECTION II

# MATHEMATICAL FORMULATION

## A. TRANSFORMATION OF THE NON-LINEAR EQUATION

The differential equation to be solved is

$$\rho C(v) \frac{\partial v}{\partial \theta} = K(v)\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right] + \frac{\partial K(v)}{\partial v}\left[\frac{1}{r^2}\left(\frac{\partial v}{\partial \phi}\right)^2 + \left(\frac{\partial v}{\partial r}\right)^2\right] \quad (1)$$

where the initial condition is

$$v(0,r,\phi) = h(r_1\phi) \quad (2)$$

and the coordinate and time varying surface conditions (Figure 1) are:

1 At $\phi_1$

$$- K(v) \frac{\partial v}{\partial \phi} = r f_{\phi 1}(r)\, g(\theta) \quad (3)$$

(representing a coordinate and time varying heat flux at this surface), or

$$v = f_{\phi 1}(r)\, g(\theta) \quad (4)$$

(representing a coordinate and time varying temperature at this surface);

2 At $\phi_2$

$$K(v) \frac{\partial v}{\partial \phi} = r f_{\phi 2}(r)\, g(\theta) \quad (5)$$

(representing a coordinate and time varying heat flux at this surface), or

$$v = f_{\phi 2}(r)\, g(\theta) \quad (6)$$

(representing a coordinate and time varying temperature at this surface);



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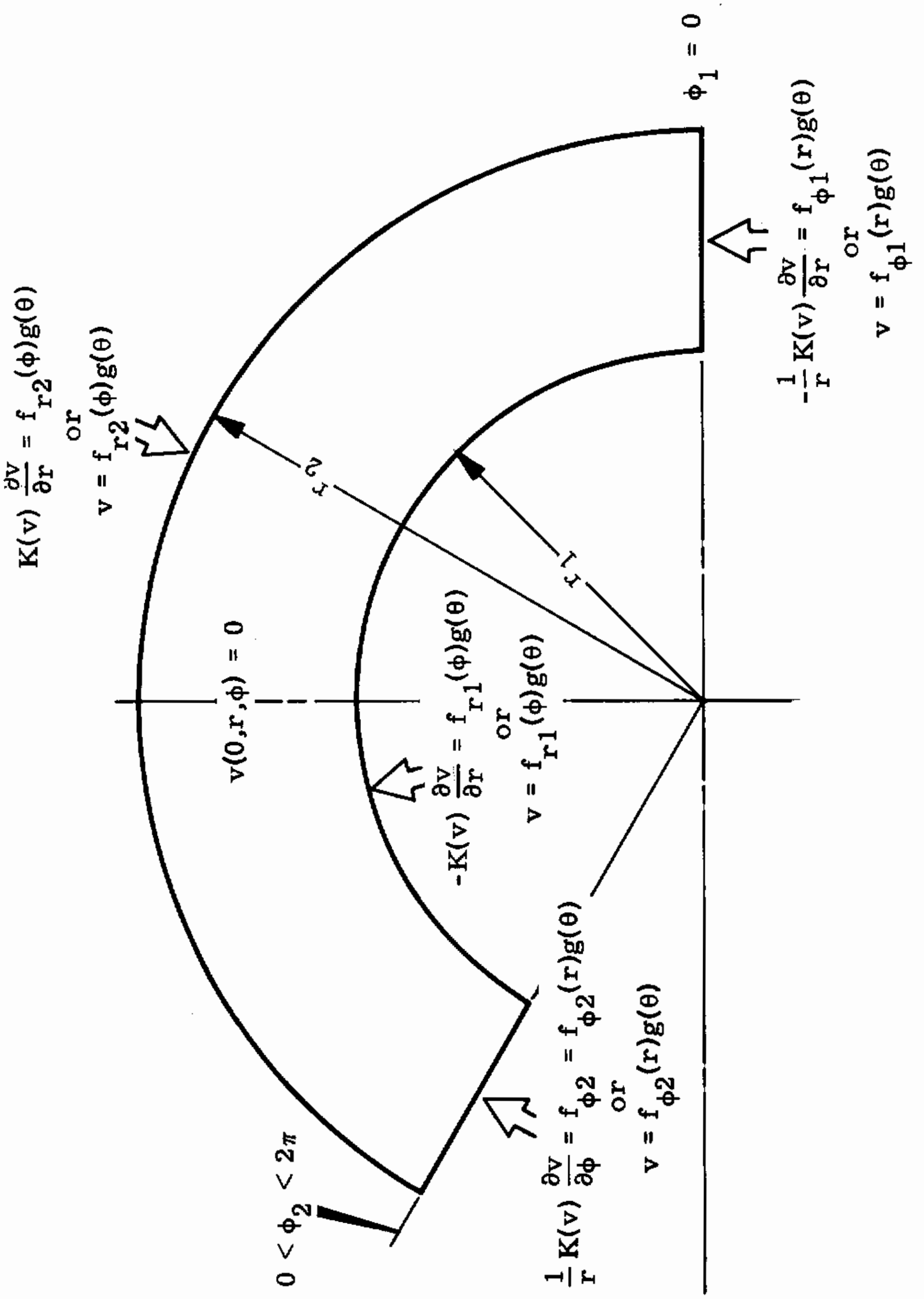


Figure 1. Mathematical Model



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3 At $r_1$

$$-K(v)\frac{\partial v}{\partial r} = f_{r1}(\phi)\, g(\theta) \tag{7}$$

(representing a coordinate and time varying heat flux at this surface), or

$$v = f_{r1}(\phi)\, g(\theta) \tag{8}$$

(representing a coordinate and time varying temperature at this surface);

4 At $r_2$

$$K(v)\frac{\partial v}{\partial r} = f_{r2}(\phi)\, g(\theta) \tag{9}$$

(representing a coordinate and time varying heat flux at this surface), or

$$v = f_{r2}(\phi)\, g(\theta) \tag{10}$$

(representing a coordinate and time varying temperature at this surface).

By the methods of Reference 1, a new variable is introduced:

$$T = \frac{1}{K_o}\int_o^v K(v)\, dv\,, \tag{11}$$

where K(v) is the thermal conductivity expressed as a function of temperature v, and $K_o$ is the value of K(v) at v equal to zero. We write

$$T = \Upsilon\,[v] \tag{12}$$

after integrating Equation (11). The inversion may be written as

$$v = \Lambda\,[T]\,. \tag{13}$$

When Equation (1) is rewritten in terms of the new variable, we obtain:

$$\frac{\rho\, C(v)}{K(v)}\frac{\partial T}{\partial \theta} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r}\frac{\partial T}{\partial r}. \tag{14}$$



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If we restrict the variation of K(v) and C(v) such that

$$\frac{\rho\, C(v)}{K(v)} = \text{constant} = \frac{1}{\alpha},$$

which is indicative of constant thermal diffusivity, Equation (14) becomes

$$\frac{1}{\alpha}\frac{\partial T}{\partial \theta} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r}\frac{\partial T}{\partial r} \tag{15}$$

and the initial condition becomes

$$T(0,r,\phi) = \Upsilon\left[h(r,\phi)\right]; \tag{16}$$

the boundary conditions, Equations (3) through (10), become:

1 At $\phi_1$

$$-K_o \frac{\partial T}{\partial \phi} = r\, f_{\phi 1}(r)\, g(\theta) \tag{17}$$

or

$$T = \Upsilon\left[f_{\phi 1}(r)\, g(\theta)\right]; \tag{18}$$

2 At $\phi_2$

$$K_o \frac{\partial T}{\partial \phi} = r\, f_{\phi 2}(r)\, g(\theta) \tag{19}$$

or

$$T = \Upsilon\left[f_{\phi 2}(r)\, g(\phi)\right]; \tag{20}$$

3 At $r_1$

$$-K_o \frac{\partial T}{\partial r} = f_{r1}(\phi)\, g(\theta) \tag{21}$$

or

$$T = \Upsilon\left[f_{r1}(\phi)\, g(\theta)\right]; \tag{22}$$



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4 At $r_2$

$$K_o \frac{\partial T}{\partial r} = f_{r2}(\phi)\, g(\theta) \tag{23}$$

or

$$T = \mathcal{T}\left[f_{r2}(\phi)\, g(\theta)\right]. \tag{24}$$

A solution of the system of Equations (1) through (10) can be obtained for the temperature dependent thermal conductivity (constant diffusivity) case by transforming these equations to a linear system, Equations (15) through (24). When a solution to the linear system for r, $\phi$, and $\theta$ in the transform temperature (T) is obtained, a solution to the non-linear system can be obtained for r, $\phi$, and $\theta$ in the real temperature (v) by application of the inversion from Equation (13).

To proceed with solution of the temperature dependent thermal diffusivity case, we introduce the new variable

$$\psi = \int_o^\theta \frac{K(v)}{\rho\, C(v)}\, d\theta. \tag{25}$$

Then

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \psi}\frac{\partial \psi}{\partial \theta} = \frac{K(v)}{\rho\, C(v)}\, \frac{\partial T}{\partial \psi} \tag{26}$$

or

$$\frac{\rho\, C(v)}{K(v)}\, \frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \psi}. \tag{27}$$

When Equation (27) is inserted in Equation (14), we obtain

$$\frac{\partial T}{\partial \psi} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r}\,\frac{\partial T}{\partial r} \tag{28}$$

which is a linear differential equation similar to Equation (15) except in a transform time variable ($\psi$) rather than real time ($\theta$). (The method of reducing non-linear diffusion Equation (14) to linear diffusion Equation (28) is based on the work of Kudryashev and Zhemkov in Reference 2.)



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Equation (25) may be written

$$\psi = \int_0^\theta \alpha\ [\Lambda(T)]\ d\theta = \Xi\ (v,\theta,r,\phi) \tag{29}$$

where

$$\frac{K(v)}{\rho\ C(v)} = \alpha(v) = \alpha\,[\Lambda(T)]\ . \tag{30}$$

Then

$$\theta = \int_0^\psi \frac{d\psi}{\alpha[\Lambda(T)]} = \Omega\,(v,\psi,r,\phi) \tag{31}$$

is the inversion to real time in the transform temperature (T).

Although the differential equation in the transform time variable is linear, the real problem can only be transformed into a complete linear system (including the boundary conditions) for certain cases of the boundary conditions. This is a result of the requirements for transformation of the time function g(θ) into a new function in the transform time variable (ψ). For a prescribed coordinate and time varying temperature at a surface, the transformation, Equation (29), can be accomplished since the transform temperature* is known. A coordinate varying heat flux may be prescribed at a surface, but variation with time is disallowed since the surface temperature is not known and the transformation of the boundary conditions in terms of the new time variable cannot be accomplished.

For a typical coordinate and time varying temperature at a surface in the form

$$v_s = f_1(r, \theta), \tag{32}$$

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*The transforms can be made in terms of the real temperature by noting that an equivalent form of Equation (29) is

$$\psi = \int_0^\theta \alpha(v)\ d\theta\ .$$



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the transformation in temperature is

$$T = \Upsilon\left[f_1(r,\theta)\right]. \tag{33}$$

The succeeding transformation in time is

$$T = \Upsilon\left[f_2(r,\psi)\right], \tag{34}$$

where the functions $f_1$ and $f_2$ are related through Equation (29).

From the preceding, it is concluded that a solution to a system of equations similar to Equations (1) through (10) can also be solved by exact methods (excluding prescribed time varying flux at a surface) for the case of temperature dependent thermal diffusivity. The non-linear system is reduced to a linear system by two successive transformations. Obtaining a solution to the linear system at r and in terms of the transform temperature and time variable, $\Upsilon$ and $\psi$, successive inversions are applied to obtain the solution in the real temperature and time.

## B. SOLUTION OF THE TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY, CONSTANT DIFFUSIVITY CASE

### 1. Transformation and Inversion

The solution of the subject problem involving temperature dependent thermal conductivity requires a transformation of the initial and boundary conditions to the transform variable and a corresponding inversion from transform temperature to real temperature. This transformation and inversion, as used in this report, is independent of the coordinate system or the form of the initial and boundary conditions. It depends only on the relationship of thermal conductivity to real temperature. This section discusses various forms of the relationship between thermal conductivity and real temperature and presents the corresponding transformation and inversion.

For various algebraic relations of thermal conductivity to real temperature, Equation (11) can be manipulated to provide an algebraic expression for the transform temperature. This equation can then be solved for real temperature to provide the inversion of Equation (13). Five such cases were prepared for incorporation in the digital computer program. The expression for thermal conductivity and the corresponding transformation and inversion expressions are given in Table I.

Data obtained from a curve of thermal conductivity versus temperature, which cannot be curve fit by one of the expressions on Table I, may be integrated numerically to obtain the relationship between real and transform



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TABLE I

Transform and Inversion Relations

| K(v) | T v | Λ(T) |
|---|---|---|
| Case 1<br>$K(v) = K_o \exp(-bv)$ | $T = \frac{1}{b}\ 1 - \exp(-bv)$ | $v = -\frac{1}{b} \ln [1 - bT]$ |
| Case 2<br>$K(v) = \frac{K_o}{1 + bv + cv^2}$<br>$q = 4c - b^2$ | | |
| A. $q < 0,\ c > 0$ | $T = \frac{1}{\sqrt{-q}} \ln \left[ \frac{\frac{2cv}{b - \sqrt{-q}} + 1}{\frac{2cv}{b + \sqrt{-q}} + 1} \right]$ | $v = \frac{(b + \sqrt{-q})\ X - b + \sqrt{-q}}{2c\ [1 - X]}$<br>where<br>$X = \exp(\sqrt{-q}\ T) \left[ \frac{b - \sqrt{-q}}{b + \sqrt{-q}} \right]$ |
| B. $q = 0\ \ c > 0$ | $T = 2 \left[ \frac{1}{b} - \frac{1}{2cv + b} \right]$ | $v = \frac{bT}{2 \left[ 2 \frac{c}{b} - cT \right]}$ |
| C. $q > 0\ \ c > 0$ | $T = \frac{2}{\sqrt{q}} \left[ \tan^{-1} \frac{2cv + b}{\sqrt{q}} - \tan^{-1} \frac{b}{\sqrt{q}} \right]$ | $v = \frac{\sqrt{q}}{2c}\ Y - \frac{b}{2c}$<br>where<br>$Y = \frac{\tan \frac{\sqrt{q}\,T}{2} + \frac{b}{\sqrt{q}}}{1 - \frac{b}{\sqrt{q}} \tan \frac{\sqrt{q}\,T}{2}}$ |
| Case 3<br>$K(v) = \frac{K_o}{1 + bv}$ | $T = \frac{1}{b} \ln (1 + bv)$ | $v = \frac{1}{b} (\exp (bT) - 1)$ |
| Case 4<br>$K(v) = K_o (1 + bv)$ | $T = v + \frac{b}{2} v^2$ | $v = -\frac{1}{b} + \frac{1}{b} \sqrt{1 + 2bT}$ |
| Case 5<br>$K(v) = K_o$ | $T = v$ | $v = T$ |



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temperature. The resulting data when plotted can be used for both the transformation and inversion process.

A third form of the relationship between thermal conductivity and real temperature, which combines the input simplicity of the algebraic expressions with the flexibility of the numerical integration of data points, may be obtained by writing n exponential terms in the form

$$K(v) = \sum_{i=1}^{n} a_i \, v^{b_i}; \tag{35}$$

then

$$T = \frac{1}{K_o} \left[ \sum_{i=1}^{n} \frac{a_i}{b_i - 1} \, v^{b_i - 1} \right]. \tag{36}$$

At this point a curve should be prepared based on Equation (36) and used for the inversion process, since a general expression for the inversion in Equation (13) cannot be written directly from Equation (36).

Regardless of the form of relationship of thermal conductivity to real temperature, the thermal conductivity must be greater than zero over the range of temperatures involved in the particular problem. It is obvious that no one would suggest a negative or zero thermal conductivity in a practical problem. However, when using algebraic relations of higher orders to curve fit given data on thermal conductivity, care should be taken to ensure that the function is positive over the argumented range of temperatures. Abrupt maxima or minima should also be avoided when using forms other than the algebraic cases of Table I. This is not a mathematical restriction in the strict sense; however, maxima or minima in the thermal conductivity curve produce "flat spots" in the curve of transform versus real temperature with attendant inaccuracies in the numerical inversion process.

In the preceding sections, the solution to the subject problem will be solved by the principle of superposition in two parts, $T_i$ and $T_s$, where $T_i$ is the solution resulting from the initial temperature distribution effects, and $T_s$ is the solution resulting from the coordinate and time varying surface conditions. The inversion process is applied to the sum of $T_i$ and $T_s$. That is

$$V = \Lambda \left[ T_i + T_s \right]. \tag{37}$$



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## 2. Solution of the Transformed Equation

The solution of the linear system, Equations (15) through (24), may (by the principle of superposition) be expressed as the sum of the solution resulting from the initial temperature distribution with homogeneous boundary conditions and the solution resulting from the prescribed boundary condition with zero initial temperature distribution. These two solutions are discussed separately in the following paragraphs.

### a. Initial Temperature Distribution Effects

The system of equations to be solved is

$$\frac{1}{\alpha}\frac{\partial T_i}{\partial \theta} = \frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r}\frac{\partial T}{\partial r} \tag{38}$$

with the initial condition

$$T_i(0,r,\phi) = \Upsilon\left[h(r,\phi)\right] \tag{39}$$

and the boundary conditions

$$k_{11}\frac{\partial T_i}{\partial \phi} + k_{12}\, T_i = 0 \quad \text{at } \phi_1, \tag{40}$$

$$k_{21}\frac{\partial T_i}{\partial \phi} + k_{22}\, T_i = 0 \quad \text{at } \phi_2, \tag{41}$$

$$\ell_{11}\frac{\partial T_i}{\partial r} + \ell_{12}\, T_i = 0 \quad \text{at } r_1, \tag{42}$$

and

$$\ell_{21}\frac{\partial T_i}{\partial r} + \ell_{22}\, T_i = 0 \quad \text{at } r_2. \tag{43}$$

Note that the boundary conditions of Equations (40) through (43) are expressed in a general form involving constants $k_{ij}$ and $\ell_{ij}$ which may be selected as zero or unity to prescribe heat flux or temperature at the subject boundary.



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The solution to the system of Equations (38) through (43) may be written, based on the author's previous work in Reference 4, as

$$T(\theta,r,\phi) = \sum_{\gamma}\sum_{\beta} A_{\gamma\beta}\left[B_{\gamma} \sin \gamma\phi + F_{\gamma} \cos \phi\right]$$

$$x \left[C_{\gamma\beta} J_{\gamma}(\beta r) + Y_{\gamma}(\beta r)\right] e^{-\alpha\beta^2\theta} \tag{44}$$

where $\gamma$'s are the real positive roots of

$$\begin{vmatrix} (k_{11}\gamma \cos \gamma\phi_1 + k_{12} \sin \gamma\phi_1) & (k_{12} \cos \gamma\phi_1 - k_{11}\gamma \sin \gamma\phi_1) \\ (k_{21}\gamma \cos \gamma\phi_2 + k_{22} \sin \gamma\phi_2) & (k_{22} \cos \gamma\phi_2 - k_{21}\gamma \sin \gamma\phi_2) \end{vmatrix} = 0. \tag{45}$$

Equation (45) may be solved to obtain an explicit expression for $\gamma$ under various combinations of boundary conditions.

If $k_{12} = k_{22} \neq 0,$ (46)

then

$$\gamma = \frac{n\pi}{\phi_2 - \phi_1}; \; n = 1, 2, 3 \ldots \tag{47}$$

If $k_{12} = k_{22} = 0,$ (48)

then

$$\gamma = \frac{n\pi}{\phi_2 - \phi_1}; \; n = 0, 1, 2, \ldots \tag{49}$$

If $k_{12} \neq k_{22},$ (50)

then

$$\gamma = \frac{\frac{\pi}{2}(2n + 1)}{\phi_2 - \phi_1}; \; n = 0, 1, 2, \ldots \tag{51}$$



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The $\beta$'s are the real positive roots of

$$\begin{vmatrix} \ell_{11} J'_\gamma(\beta r_1) + \ell_{12} J_\gamma(\beta r_1) & \ell_{11} Y'_\gamma(\beta r_1) + \ell_{12} Y_\gamma(\beta r_1) \\ \ell_{21} J'_\gamma(\beta r_2) + \ell_{22} J_\gamma(\beta r_2) & \ell_{21} Y'_\gamma(\beta r_2) + \ell_{22} Y_\gamma(\beta r_2) \end{vmatrix} = 0. \quad (52)$$

Equation (52) can be solved only by numerical techniques. The numerical technique based on Newton's method requires the derivative of Equation (52) with respect to the argument $\beta$. The derivative can be obtained by re-writing Equation (52) in the form

$$F = \begin{vmatrix} T_1 & T_2 \\ T_3 & T_4 \end{vmatrix}, \quad (53)$$

then

$$\frac{\partial F}{\partial \beta} = \begin{vmatrix} T_1 & T_2 \\ \frac{\partial T_3}{\partial \beta} & \frac{\partial T_4}{\partial \beta} \end{vmatrix} + \begin{vmatrix} \frac{\partial T_1}{\partial \beta} & \frac{\partial T_2}{\partial \beta} \\ T_3 & T_4 \end{vmatrix}, \quad (54)$$

where

$$\frac{\partial T_1}{\partial \beta} = \ell_{11} J_\gamma(\beta r_1)\left[\frac{\gamma^2}{\beta r_1} - \beta r_1\right] + \frac{r_1}{\beta} \ell_{12} J'_\gamma(\beta r_1), \quad (55)$$

$$\frac{\partial T_2}{\partial \beta} = \ell_{11} Y_\gamma(\beta r_1)\left[\frac{\gamma^2}{\beta r_1} - \beta r_1\right] + \frac{r_1}{\beta} \ell_{12} Y'_\gamma(\beta r_1), \quad (56)$$

$$\frac{\partial T_3}{\partial \beta} = \ell_{21} J_\gamma(\beta r_2)\left[\frac{\gamma^2}{\beta r_2} - \beta r_2\right] + \frac{r_2}{\beta} \ell_{22} J_\gamma(\beta r_2), \text{ and} \quad (57)$$

$$\frac{\partial T_4}{\partial \beta} = \ell_{21} Y_\gamma(\beta r_2)\left[\frac{\gamma^2}{\beta r_2} - \beta r_2\right] + \frac{r_2}{\beta} \ell_{22} Y_\gamma(\beta r_2). \quad (58)$$



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For $\gamma$ not equal to zero,

$$F_\gamma = 1.0, \tag{59}$$

$$B_\gamma = \frac{-k_{12}\cos\gamma\phi_1 + k_{11}\gamma\sin\gamma\phi_1}{k_{11}\gamma\cos\gamma\phi_1 + k_{12}\sin\gamma\phi_1}, \tag{60}$$

$$C_{\gamma\beta} = \frac{-\ell_{11} Y'_\gamma(\beta r_1) - \ell_{12} Y_\gamma(\beta r_1)}{\ell_{11} J'_\gamma(\beta r_1) + \ell_{12} J_\gamma(\beta r_1)}, \tag{61}$$

and

$$A_{\gamma\beta} = \frac{\int_{\phi_1}^{\phi_2}\int_{r_1}^{r_2} \Upsilon\left[h(r,\phi)\right]\left[B_\gamma \sin\gamma\phi + F_\gamma\cos\gamma\phi\right]\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right] r\, dr\, d\phi}{\int_{\phi_1}^{\phi_2}\left[B_\gamma\sin\gamma\phi + F_\gamma\cos\gamma\phi\right]^2 d\phi \int_{r_1}^{r_2} r\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]^2 dr}, \tag{62}$$

where

$$\int_{\phi_1}^{\phi_2}\left[B_\gamma\sin\gamma\phi + F_\gamma\cos\gamma\phi\right]^2 d\phi = \left[\frac{\phi}{2}(B_\gamma^2+1) - \frac{1}{4\gamma}(B_\gamma^2-1)\sin 2\gamma\phi + \frac{B_\gamma}{\gamma}\sin^2\gamma\phi\right]_{\phi_1}^{\phi_2} \tag{63}$$

and

$$\int_{r_1}^{r_2} r\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]^2 dr = \left[\frac{1}{2}r^2\left(1-\frac{\gamma^2}{\beta^2 r^2}\right)\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]^2 + \frac{r^2}{2\beta^2}\left[C_{\gamma\beta} J'_\gamma(\beta r) + Y'_\gamma(\beta r)\right]^2\right]_{r_1}^{r_2}. \tag{64}$$

Under certain combinations of boundary conditions, $B_\gamma$ is infinite for particular $\gamma$'s. In this case that particular term in Equation (44) is an indeterminate form. A usable form for this term can be obtained by setting



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$$B_\gamma = 1.0, \tag{65}$$

$$F_\gamma = 0, \tag{66}$$

and

$$A_{\gamma\beta} = \frac{\int_{\phi_1}^{\phi_2} \int_{r_1}^{r_2} T[h(r,\phi)] \sin \gamma\phi \left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right] r \, dr \, d\phi}{\left[\frac{\phi}{2} - \frac{1}{4\gamma} \sin 2\gamma\phi\right]_{\phi_1}^{\phi_2} \int_{r_1}^{r_2} r\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]^2 dr} . \tag{67}$$

The resulting form was obtained by division of the numerator and denominator of Equation (44) by $B_\gamma^2$ and evaluating as $B_\gamma$ approaches infinity.

If $\gamma$ is equal to zero the required term can be obtained directly from Equation (44) by setting

$$B_\gamma = 0, \tag{68}$$

$$F_\gamma = 1.0, \tag{69}$$

and

$$A_{\gamma\beta} = \frac{\int_{\phi_1}^{\phi_2} \int_{r_1}^{r_2} T[h(r,\phi)]\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right] r \, dr \, d\phi}{(\phi_2 - \phi_1) \int_{r_1}^{r_2} r\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]^2 dr} . \tag{70}$$

The series in Equation (44) converges rapidly for values of time greater than zero, since the exponential term involving time ($\theta$) and $\beta^2$ approaches zero quite rapidly for successive terms in $\beta$. However, for time equal to zero, this exponential term is equal to unity for all terms in Equation (44) and does not play its otherwise dominant role in the rapid convergence of the series. For a similar series, Reference 1 suggests

$$\frac{\alpha\theta}{r_2^2} > 0.02 \tag{71}$$

to ensure suitable convergence.



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b. Coordinate and Time Varying Surface Effects

To obtain a solution of the coordinate and time varying surface effect in a form similar to that used in Reference 4, wherein the coordinate and time functions are separable, the following assumption is required. For a typical prescribed temperature at a boundary such as in Equation (18), we assume

$$\Upsilon\left[f_{\phi 1}(r)\, g(\theta)\right] \cong \Upsilon\left[f_{\phi 1}(r)\right] g(\theta) \tag{72}$$

This is indicative of a linear transformation (constant thermal conductivity) over the range of the time function. This does not place any compromise on exactness of the solution for prescribed heat flux, zero surface temperature, or unit time function at a surface. However, Equation (72) is an approximation for a prescribed surface temperature with $g(\theta)$ not equal to unity. (An example of transformations which do not involve this approximation is discussed in Section IIC). The magnitude of the error introduced by such an approximation will be immediately recognizable in the difference between prescribed surface temperature and the resulting solution at that surface. The magnitude of the error will increase with increased deviations in $g(\theta)$ from unity and large variation in thermal conductivity with temperature.

The system of equations to be solved is

$$\frac{1}{\alpha}\frac{\partial T_s}{\partial \theta} = \frac{\partial^2 T_s}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 T_s}{\partial \phi^2} + \frac{1}{r}\frac{\partial T_s}{\partial r} \tag{73}$$

with the initial condition

$$T_s(0,r,\phi) = 0 \tag{74}$$

and the boundary conditions

$$k_{11}\frac{\partial T_s}{\partial \theta} + k_{12} T_s = \left\{ \frac{k_{11} r}{K_o} f_{\phi 1}(r) + k_{12}\, \Upsilon\left[f_{\phi 1}(r)\right] \right\} g(\theta), \tag{75}$$

$$k_{21}\frac{\partial T_s}{\partial \phi} + k_{22} T_s = \left\{ \frac{k_{21} r}{K_o} f_{\phi 2}(r) + k_{22}\, \Upsilon\left[f_{\phi 2}(r)\right] \right\} g(\theta), \tag{76}$$



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$$\ell_{11} \frac{\partial T_s}{\partial r} + \ell_{12} T_s = \left\{ \frac{\ell_{11}}{K_o} f_{r1}(\phi) + \ell_{12} T\left[f_{r1}(\phi)\right] \right\} g(\theta), \tag{77}$$

and

$$\ell_{21} \frac{\partial T_s}{\partial r} + \ell_{22} T_s = \left\{ \frac{\ell_{21}}{K_o} f_{r2}(\phi) + \ell_{22} T\left[f_{r2}(\phi)\right] \right\} g(\theta). \tag{78}$$

As in the previous section, the boundary conditions have been expressed in the general form involving the constants $k_{ij}$ and $\ell_{ij}$ which can be selected alternately as zero or unity for either (j) to obtain Equations (17) through (24).

By the methods of Reference 4 the solution to Equations (73) through (78) may be written*

$$T_s(\theta,r,\phi) = \sum_\gamma \sum_\beta G_{\gamma\beta}\left[B_\gamma \sin\gamma\phi + F_\gamma \cos\gamma\phi\right]\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]$$

$$\times\left[\alpha\beta^2 \int_0^\theta g(\lambda) e^{-\left[\alpha\beta^2(\theta-\lambda)\right]} d\lambda\right] \tag{79}$$

where

$$G_{\gamma\beta} = \frac{\int_{\phi_1}^{\phi_2}\int_{r_1}^{r_2} f(r,\phi)\left[B_\gamma \sin\gamma\phi + F_\gamma\cos\gamma\phi\right]\left[C_{\gamma\beta}J_\gamma(\beta r) + Y_\gamma(\beta r)\right] r\,dr\,d\phi}{\int_{\phi_1}^{\phi_2}\left[B_\gamma \sin\gamma\phi + F_\gamma\cos\gamma\phi\right]^2 d\phi \int_{r_1}^{r_2} r\left[C_{\gamma\beta}J_\gamma(\beta r) + Y_\gamma(\beta r)\right]^2 dr} \tag{80}$$

and $B_\gamma$, $F_\gamma$, $C_{\gamma\beta}$, $\gamma$ and $\beta$ have been defined in the previous section. Here, $f(r,\phi)$ is the steady state solution resulting from the coordinate varying boundary condition functions in Equations (75) through (78). This steady state solution is given in detail in Appendix A of this report. Equation (79) is subject to the same conditions regarding $B_\gamma$ of infinity and $\gamma$ equal to zero as discussed for Equation (44) in the previous section. Under these conditions $G_{\gamma\beta}$, Equation (80), is expressed in forms similar to Equations (67) and (70).

*A more rapidly converging form will be introduced subsequently.



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By adding and subtracting g(θ) in the latter bracket, Equation (79) may be written

$$T_s(\theta,r,\phi) = \sum_\gamma \sum_\beta G_{\gamma\beta}\left[B_\gamma \sin\gamma\phi + F_\gamma \cos\gamma\phi\right]\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]$$

$$\times\left[g(\theta) - g(\theta) + \alpha\beta^2 \int_0^\theta g(\lambda)\, e^{-\alpha\beta^2(\theta-\lambda)}\, d\lambda\right] \qquad (81)$$

and also noting that

$$f(r,\phi) = \sum_\gamma \sum_\beta G_{\gamma\beta}\left[B_\gamma \sin\gamma\phi + F_\gamma \cos\gamma\phi\right]\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]. \qquad (82)$$

Equation (81) becomes

$$T_s(\theta,r,\phi) = g(\theta)\ f(r,\phi) - \sum_\gamma \sum_\beta G_{\gamma\beta}\left[B_\gamma \sin\gamma\phi + F_\gamma \cos\gamma\phi\right]$$

$$\times\left[C_{\gamma\beta} J_\gamma(\beta r) + Y_\gamma(\beta r)\right]\left[g(\theta) - \alpha\beta^2\int_0^\theta g(\lambda)\, e^{-\alpha\beta^2(\theta-\lambda)}\, d\lambda\right] \qquad (83)$$

Equation (83) offers several advantages in practical computation over Equation (79). The form given by Equation (83) is such that the transient series is converging toward the difference between the steady state solution that results from the boundary condition at that time and the actual temperature. As such, this series has convergence characteristics similar to the initial temperature distribution effects solution in Equation (44) of the previous section, in regards to large values of time. In contrast, the series defined by Equation (79) converges rapidly for small times, but rather slowly for large times. In addition, Equation (83) gives the proper solution at the boundaries whereas Equation (79) characteristically converges to zero at a boundary when temperature is prescribed. The form expressed by Equation (83) is used in the digital computer program.

## C. COMMENT ON SOLUTION OF THE TEMPERATURE DEPENDENT THERMAL DIFFUSIVITY CASE

A successive transformation, Equation (25), which linearizes the temperature dependent thermal diffusivity equations was presented in Section IIA. It covers transformation of the non-linear differential equation associated with temperature dependent thermophysical properties. Since



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there are two successive transforms involved, particular attention should be given to combinations of the arbitrary functions that define the boundary conditions and the thermophysical properties in the application of this scheme to practical problems. Therefore, it is advisable to define the algebraic form of all the relationships involved and manipulate them as a set through the successive transformations.

As an example, we consider a specific set of the various arbitrary functions of the form

$$K(v) = K_o(1 + k_1 v), \tag{84}$$

$$\alpha(v) = \alpha_1 v, \tag{85}$$

$$g(\theta) = g_1 \theta, \text{ and} \tag{86}$$

with a prescribed temperature at the boundary $\phi_1$ of

$$v(\theta, r, \phi_1) = g(\theta) \left[f(r)\right]^2. \tag{87}$$

Then from case IV of Table I

$$T(\theta, r, \phi_1) = \left\{ g_1 \left[f(r)\right]^2 \theta + \frac{k_1}{2} g_1^{\ 2} \left[f(r)\right]^4 \theta^2 \right\} \tag{88}$$

from Equation (25)

$$\psi = \frac{\alpha_1 g_1}{2} \left[f(r)\right]^2 \theta^2 \tag{89}$$

$$\theta = \sqrt{\frac{2\psi}{\alpha_1 g_1}} \frac{1}{f(r)} \tag{90}$$

When Equation (90) is substituted in Equation (88), we obtain the transformed boundary condition

$$T(\psi, r, \phi_1) = \left\{ \sqrt{\frac{2\psi g_1}{\alpha_1}} f(r) + \frac{k_1 \psi g_1}{\alpha_1} \left[f(r)\right]^2 \right\}. \tag{91}$$



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This equation is the complete transformation of the original boundary conditions in terms of both transform temperature and transform time. The two members in Equation (91) are products of a time function and a coordinate function and can be solved by methods similar to the one used in this report. Note that in this development the transform boundary condition is exact. It does not involve an approximation such as in Equation (72)*.

Since the solution to the linear differential equation is in series form, an algebraic expression for the inversion to real time is not generally obtainable and numerical integration must be used. It is, however, under some combinations possible to obtain usable expressions for the inversion process. As an example, consider the relations

$$K(v) = K_o \tag{92}$$

and

$$\alpha(v) = \frac{1}{a_o + a_1 v}. \tag{93}$$

Here the transform temperature is

$$T = v \tag{94}$$

and

$$\alpha(T) = \frac{1}{a_o + a_1 T}. \tag{95}$$

Consider a typical solution in the form

$$T(x,\psi) = \sum_m X(x)\, e^{-m^2 \psi} \tag{96}$$

where X(x) is the coordinate part of the product solution.

---

*In place of Equation (88), the approximation defined by Equation (72) would lead to

$$T(\phi, r, \phi_1) = \left\{ g_1 [f(r)]^2\, \theta + \frac{k_1}{2}\, g_1 [f(r)]^4\, \theta \right\} .$$



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Then from Equation (31), we obtain real time θ in the form

$$\theta = \left| a_o\psi + a_1\left[\sum_m \frac{X(x)e^{-m^2\psi}}{-m^2}\right]\right|_o^\psi . \tag{97}$$

The summation in Equation (97) is in the same form as the solution to Equation (96) except that each term is divided by minus $m^2$. In a practical solution these terms could easily be calculated parallel with the calculation of Equation (96).



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# SECTION III

# PROGRAMMING

## A. PROGRAM DESCRIPTION

### 1. Programming Philosophy

The mathematical formulation presented in the previous section is practical only when solved on a high speed computer. The temperature dependent thermal conductivity, constant thermal diffusivity, case has been developed in FORTRAN IV for application to an IBM 7094.

The computer program was developed with the view toward several objectives not usually considered in such a project. In addition to affording an analytical tool for development and study of thermal structures, the items discussed in the following paragraphs were considered.

Those familiar with mathematical texts on heat conduction such as Reference 1 will recognize that the solutions to diffusion problems have common mathematical expressions and methods of solution independent of the coordinate system. For example, the transformation and inversion used in linearizing the equations, the need for root searching techniques, the application of Duhamel's integral to incorporate time varying boundary conditions, etc., are common to many problems other than the specific problem considered in this task. Accordingly, this program was developed with these anticipated common elements as individual subroutines. This provides the user with building blocks or, as a minimum, specific guidelines for the development of similar computer programs. Detailed discussions of the subroutines are given in Appendix C.

Mathematics required to develop a program that considers temperature dependent thermal diffusivity have been discussed in preceding sections. The solution reduces to a linear system similar to that programmed in this task. Under this effort the solution of the linear system was isolated, thus providing the current program with the revision potential to incorporate temperature dependent thermal diffusivity at a later date.

For those users who are interested in the study of the characteristics of series solutions, either academically or in conjunction with development of solutions to similar problems, two control cards were incorporated as



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part of the input. These control cards, discussed subsequently, provide variation in the number of terms in the series, as well as increments used in numerical integration, and control a unique array of intermediate print options.

2. Basic Program Flow

The basic digital relationship between the MAIN routine and the subroutines of the digital program is given in Figure 2. Detailed discussions of these routines are given in Appendix C. Flow diagrams of the MAIN routine and prime subroutines are given in Appendix D.

3. Output Data and Format

Certain intermediate print options, as discussed previously, are available for detailed study. The printout for these options is described for the individual subroutines in Appendix C. Independent of the use of these options, the standard program output is:

1 The problem title

2 All of the input problem description

3 The steady state solution

4 The times and lattice coordinates where the program is computing temperatures

5 The final temperatures at these coordinates.

An illustration of this output is given with the sample problems in Section IV.

In reference to 3 above, the steady state solution printout also includes a printout of the last terms in the series due to boundary condition contribution at each of the faces. The $T_1$, $T_2$, $T_3$, and $T_4$ refer to the contribution of the boundaries $r_1$, $r_2$, $\phi_1$, and $\phi_2$, respectively. The IR and IP are program subscripts which locate the solution in the r and $\phi$ directions, respectively. The latter column is the partial sum of the four series involved. With the exception of corrections at the corners in cases where temperatures are prescribed at the corresponding adjacent faces (Appendix A) this is the steady state solution $f(r,\phi)$ used in evaluating Equation (80).



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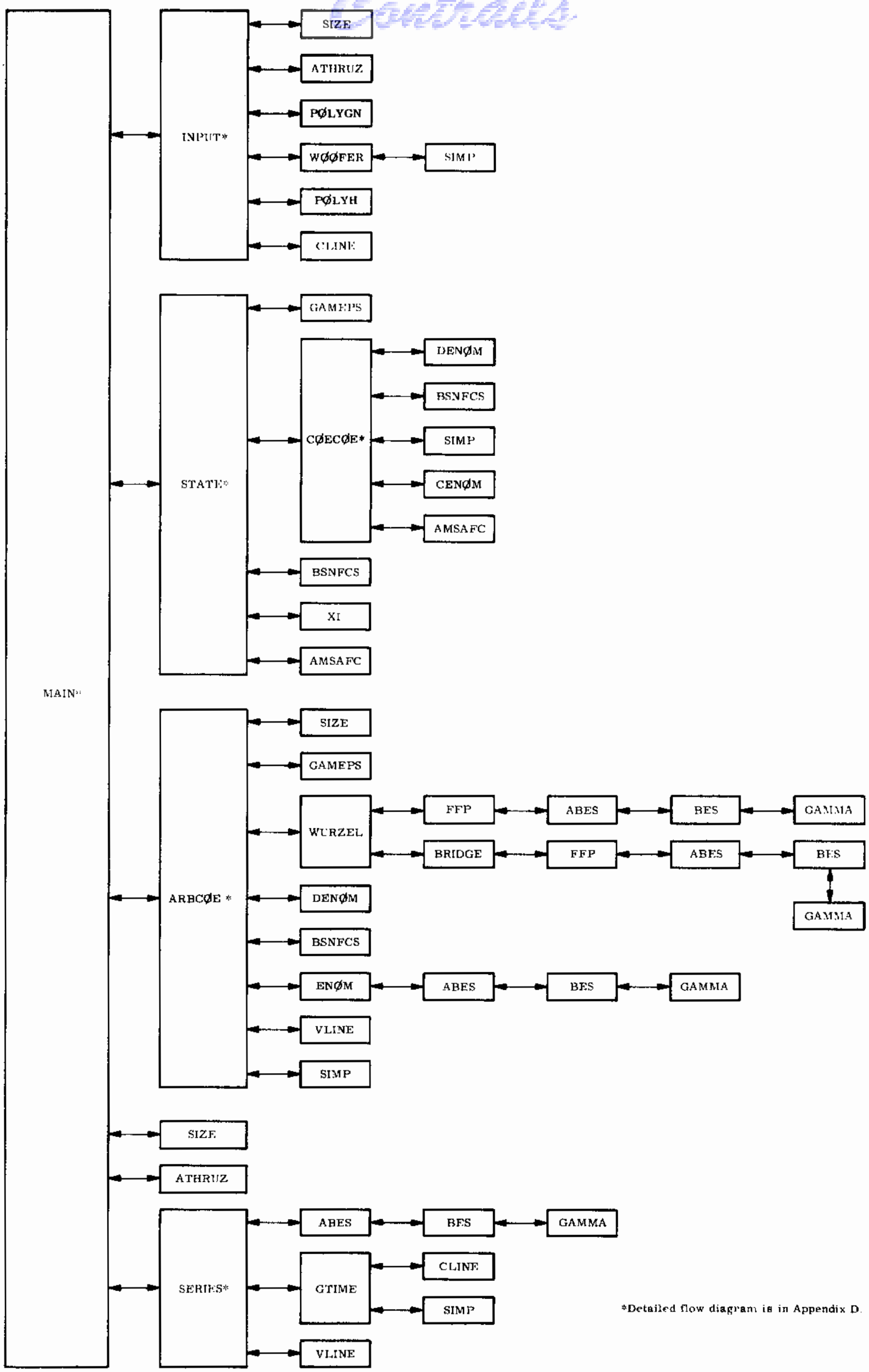


Figure 2. Program Subroutine Relations



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## B. UTILIZATION OF THE COMPUTER PROGRAM

Before reading this section a review of Equations (1) through (10) and Figure 1 in Section IIA is recommended.

This section describes the utilization of the computer program via a general discussion of the input requirements followed by a detailed description of the input and output formats on Table II. Two sample problems are included in Section IV to illustrate utilization of the program.

### 1. Discussion of Input

It is assumed in the calculation that the physical units input for a particular problem are dimensionally compatible. That is, units of heat length, time, mass, and temperature must be consistent in a given problem. Heat flux at a surface is treated as per unit area.

#### a. Control (Cards A and B)

A control system, as previously mentioned, was developed to increase the flexibility of the program. Input the numbers given in the input data format, which follows for normal use of the program. See sequence line numbers 000180 through 000280 of the program listing (Appendix B) and detailed discussion of MAIN (Appendix C) for definitions if alternation to number of terms used in the series or number of increments used in numerical integration is desired.

#### b. Geometry (Cards D and E)

As presently programmed, the system will consider sectors between 0 and 360°, where

$$\phi_1 = 0$$

$$0 < \phi_2 < 360°.$$

The radial boundary restrictions are

$$r_2 > r_1$$

$$r_1 \neq 0$$



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c. Thermophysical Properties (Card(s) F)

The variation of thermal conductivity with temperature may be defined by selecting any of the programmed algebraic expressions given in Table I, inputting tabular data, or defining the coefficients and exponents in an expression of the form:

$$K(v) = \sum_{i=1}^{n} a_i v^{b_i} . \tag{35}$$

The latter expression is referred to as PØLY in the utilization instructions. The thermal diffusivity is input as a constant. Refer to Section IIB.1 (page 9) for a discussion of the associated transformation and inversion.

d. Initial Temperature Distribution (Card(s) G)

The initial temperature distribution may be input as either a lattice array of temperatures or in polynomial form similar to Equation (35). The polynomial form is:

$$h(r,\phi) = \sum_{i=1}^{n} a_i r^{b_i} \phi^{c_i} . \tag{98}$$

e. Boundary Conditions (Card(s) H, I, J, K, and optional L)

The coordinate varying portion of the boundary conditions is incorporated in the solution via a steady state solution (see Section IIB.2.b, page 17). This steady state solution will be calculated in response to definition of the coordinate varying functions and mode (prescribed temperature or heat flux) at each surface. The solution is written such that a positive heat flux represents heat into the surface. The coordinate varying portion of a boundary condition may be input either as tabular data or by defining number of terms, coefficients, and exponents in a form similar to Equation (35).

In particular cases, the steady state solution may be known by the user either from a previous problem or because of the simplicity of the particular problem. In such cases, the steady state temperature distribution in either the real or the transform variable may be input either as a lattice array of temperatures or in polynomial form similar to the initial temperature distribution. The boundary condition mode is still required when using this option.



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f. Time Function (Card(s) M)

The time function $g(\theta)$ may be input as either tabulated or polynomial data. For a discussion of the approximation associated with application of this function refer to Section IIB.2.b, page 17.

g. Output Requested (Card(s) N)

Temperature distribution data at various times may be requested either at specific values of r and $\phi$ or in an evenly distributed lattice. It is advisable to request calculations for times in which

$$\frac{\alpha\theta}{r_2^{\ 2}} > .02. \qquad (71)$$

For smaller times an increased number of terms in the series may be required.

2. Input Data Format

All input data must contain a decimal point unless otherwise specified. If data is specified as not containing a decimal point, it must be right adjusted in its field.

When point data is furnished, the independent variable should always be supplied in monotonically increasing order. Table II presents the input data loading procedure.



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# TABLE II

## Input Data Loading Procedure

(Note: Quotation marks indicate Hollerith terms.)

| Instruction | Card Designation | Column No. | Description |
|---|---|---|---|
| Control | A | 5 | 6 |
| | | 10 | 6 |
| | | 13-15 | 100 |
| | | 18-20 | 100 |
| | | 25 | 6 |
| | | 29-30 | 10 |
| | | 35 | 20 |
| | | 39-40 | 30 |
| | | 44-45 | 30 |
| | B | 1-35 | blank - Intermediate print options |
| | C | 1-72 | Problem title |
| Geometry | | | |
| | D | 1-10 | ARE(1) - the value of $r_1$ |
| | | 11-20 | ARE(2) - the value of $r_2$ |
| | E | 1-10 | PHI(1) - the value of $\phi_1$ which must be zero |
| | | 11-20 | PHI(2) - the value of $\phi_2$ in degrees and fractions |
| Thermophysical Properties | | | |
| • To input point data for K(v) versus v | | | |
| | F | 1-5 | "PØINT" |
| | | 11-20 | N - the number of point pairs to be input. This number is right adjusted with no decimal point ($N \leq 50$). |
| | | 21-30 | ALPHØ - the value of $\alpha$ |
| | | 31-40 | AZERØ - the value of $K_o$ |
| | F1 | 1-60 | The point pairs in ten column fields with the abscissa specified first in each case. This card (F1) is repeated as necessary. |
| • To input K(v) as a polynomial | | | |
| | F | 1-4 | "PØLY" |
| | | 11-20 | N - the number of polynomial terms to be input. This number is right adjusted with no decimal point ($N \leq 10$). |
| | | 21-30 | A - the minimum value at which the polynomial is to be evaluated |
| | | 31-40 | B - the maximum value at which the polynomial is to be evaluated |



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| Instruction | Card Designa-tion | Column No. | Description |
|---|---|---|---|
| | | 41-50 | ALPHØ - the value of $\alpha$ |
| | | 51-60 | AZERØ - the value of $K_o$ |
| | F1 | 1-10 | A coefficient of the polynomial |
| | | 11-20 | The corresponding order of the independent variable. This card (F1) is repeated as necessary. |
| • To input special cases of K(v) (see Table I). | | | |
| | F | 1-4 | "CASE" |
| | | 11-20 | N - the case number of the transform. This value is right adjusted with no decimal point ($N \leq 7$). |
| | | 21-30 | BEE - the value of b in the transform equation |
| | | 31-40 | SEE - the value of c in the transform equation |
| | | 41-50 | ALPHØ - the value of $\alpha$ |
| | | 51-60 | AZERØ - the value of $K_o$ |
| Initial Temperature Distribution<br>• To input $h(r,\phi)$ as point data | | | |
| | G | 1-5 | "PØINT" |
| | | 11-20 | N - the number of points in the r direction. This value is right adjusted with no decimal point ($N \leq 7$). |
| | | 21-30 | M - the number of points in the $\phi$ direction. This value is right adjusted with no decimal point ($M \leq 25$). |
| | G1 | 1-70 | The values of the dependent variable in ten column fields starting with $h(r_1, \phi_2)$ and varying (r) most rapidly, with all the values for one complete cycle (r) on one card. The values of dependent variable must be evenly spaced in any given direction. This card (G1) is repeated as necessary. |
| • To input $h(r,\phi)$ in polynomial form | | | |
| | G | 1-4 | "PØLY" |
| | | 11-20 | N - the number of terms in the polynomial. This value is right adjusted with no decimal point ($N \leq 10$). |



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| Instruction | Card Designa-tion | Column No. | Description |
|---|---|---|---|
| | G1 | 1-10 | A - coefficient of the polynomial |
| | | 11-20 | The corresponding order of the polynomial (r). |
| | | 21-30 | The corresponding order of the polynomial ($\phi$). This card (G1) is repeated as necessary. |
| Boundary Conditions | | | |
| 1 Boundary conditions at $r_1$ | | | |
| • To input $f_{r1}(\phi)$ as point data | | | |
| | H | 1-4 | "TEMP" - for a temperature at the boundary |
| | | 1-4 | "FLUX" - for a flux at the boundary |
| | | 11-15 | "PØINT" |
| | | 21-30 | N - the number of values of the dependent variable to be input. This number is right adjusted with no decimal point ($N \leq 200$). |
| | | 31-32 | These columns blank mean the steady state solution will be worked by the program. "NØ" - means the steady state soulution will be input. (Note: If the NØ option is used, only columns 1 through 4 of this card need to be filled in and card H1 is omitted.) |
| | | 31-34 | "ØNLY" - to work the steady state solution and then terminate. |
| | | 1-70 | Evenly spaced values of the dependent variable from $\phi_1$ to $\phi_2$ in ten column fields. This card (H1) is repeated as necessary. |
| • To input $f_{r1}(\phi)$ as a polynomial | | | |
| | H | 1-4 | "TEMP" - for a temperature at the boundary |
| | | 1-4 | "FLUX" - for a flux at the boundary |
| | | 11-14 | "PØLY" |
| | | 21-30 | N - the number of terms in the polynomial. This value is right adjusted with no decimal point ($N \leq 10$). |
| | | 31-32 | Blank for the program to work the steady state solution. "NØ" - to input the steady state solution. (Note: If the NØ option is used, only columns 1 through 4 need to be filled in and card H1 is omitted.) |
| | | 31-34 | "ØNLY" - to work the steady state solution and then terminate. |



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| Instruction | Card Designation | Column No. | Description |
|---|---|---|---|
| | H1 | 1-10<br>11-20 | A coefficient of the polynomial<br>The corresponding order of the polynomial. This card (H1) is repeated as necessary. |
| 2 Boundary conditions at $r_2$, $\phi_1$, and $\phi_2$ | I, J, K | | Cards I, J, and K follow the same form as cards H (and H1) but for the $r_2$, $\phi_1$, and $\phi_2$ boundaries, respectively. The options of TEMP, FLUX, PØINT, and PØLY available on card H may be used in any combination at the remaining boundaries except that the option to input or solve for the steady state solution has been fixed with card H and cannot be modified thereafter. |
| 3 Steady state solution input directly* | L, L1 | | The steady state solution card L and card(s) L1 are input in exactly the same format as the initial temperature distribution (card G and card(s) G1 above) with the same restrictions and options but withe the following addition. If the steady state solution is input in the real variable (v), write "TRANSFØRM" in columns 31 through 39 of card L. (The program will perform the required transformations before proceeding with the remainder of the solution.) If the steady state solution is input in the transform variable (T) leave columns 31 through 39 blank. |
| Time Functions<br>• To input the time function g(θ) as point data | M | 1-5<br>11-20 | "PØINT"<br>N - the number of point pairs to be input. This value is right adjusted with no decimal point ($N \leq 50$). |
| | M1 | 1-60 | The point pairs in ten column fields with the time value (θ) given first in each case. This card (M1) is repeated as necessary. |

*All L cards are omitted if steady state solution is not input.



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| Instruction | Card Designation | Column No. | Description |
|---|---|---|---|
| • To input the time function g(θ) as a polynomial | | | |
| | M | 1-4 | "PØLY" |
| | | 11-20 | N - the number of terms in the polynomial. This value is right adjusted with no decimal point ($N \le 10$). |
| | | 21-30 | B - the maximum value at which the polynomial is to be evaluated. (The time function will be evaluated over the domain 0 to B). |
| | M1 | 1-10 | A coefficient of the polynomial |
| | | 11-20 | The corresponding order of the polynomial in θ. This card (M1) is repeated as necessary. |
| Output Requested<br>• Individual Points | | | |
| | N | 1-10 | The time at which an answer is desired |
| | | 11-20 | An r at which an answer is desired |
| | | 21-30 | A ϕ in degrees and fractions at which an answer is desired. This card (N) is repeated as necessary. |
| • Lattice Points | | | |
| | N | 1-10 | The time at which an answer is desired |
| | | 31-32 | "ØN" |
| | | 41-50 | N - the number of lattice points in the r direction where answers are desired. This value is right adjusted with no decimal point ($N \le 400$). |
| | | 51-60 | M - the number of lattice points in the ϕ direction where answers are desired. This value is right adjusted with no decimal point ($M \le 400$). |
| | N1 | 1-10 | Additional time at which answers are desired. This card (N1) is repeated as required. (Note: Card N for lattice points may follow card N for individual points.) |



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# SECTION IV
# SAMPLE PROBLEMS

This section presents two typical samples and their solutions as obtained from the digitial computer program. These sample problems are primarily to familiarize the reader with input and output format of the program. A sample test problem is discussed in Appendix E.

## A. SAMPLE PROBLEM 1

### 1. Geometry

$r_1$ = 0.7 inch

$r_2$ = 0.9 inch

$\phi_1$ = 0 degrees

$\phi_2$ = 270 degrees

### 2. Thermophysical Properties

Based on glass reinforced polyester*

$K(v) = K_o [1 + bv]$ Case 4

$K_o$ = 0.925 x $10^6$ Btu/sec inch °R

b = 0.00125

$\alpha$ = 0.000122 $\text{inch}^2$/sec

### 3. Initial Temperature Distribution

$h(r,\phi)$ = 600°R

### 4. Boundary Conditions

at $r_1$ v = 1000°R,

at $r_2$ v = 1000°R,

at $\phi_1$ v = 1000°R,

and at $\phi_2$ v = 1000°R.

* Data per Reference 5.



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In this case the steady state solution can be input directly since it is obviously 1000°R at all points. It can be input in either the real or the transform variable. In this case it will be input in the transform variable. Referring to Table I.

if

$$K(v) = K_o \left[1 + bv\right]$$

then

$$T = v + \frac{bv^2}{2}$$

which for v = 1000°R yields

$$T = 1625°R = f(r,\phi).$$

Since the boundary conditions define a step function, $g(\theta)$ is equal to unity.

The input to obtain the temperature distribution at 40, 60, 80, and 120 seconds for five values in each of the radial and circumferential directions is shown in Table III. Printout is shown in Table IV. Figures 3 through 7 are illustrations of the temperature distribution.

B. SAMPLE PROBLEM 2

1 Geometry

$r_1$ = 0.2 inch

$r_2$ = 0.8 inch

$\phi_1$ = 0 degree

$\phi_2$ = 90 degrees

2 Thermophysical properties

Same as sample problem 1.

3 Boundary Conditions

at $r_1$ $\quad v = (720 - 25.4\phi)g(\theta),$

at $r_2$ $\quad \frac{\partial v}{\partial r} = 0,$



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$$\text{at } \phi_1 \qquad v = (700 + 100r)\, g(\theta),$$

$$\text{at } \phi_2 \qquad v = (700 - 100r)\, g(\theta);$$

where

$$g(\theta) = 0.8 + 0.00333\theta.$$

4. Initial Temperature Distribution

$$h(r,\phi) = 600°R.$$

The input to obtain the temperature distribution at 40, 60, 80, 120 seconds for five points in the radial direction and five points in the circumferential direction is shown in Table V. Printout is shown in Table VI. Figures 8 through 12 illustrate the temperature distribution.



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## TABLE III

## FORTRAN Coding, Sample Problem Number 1

FORM 2172 AU 668 — **GENERAL LOADING FORM** — ☐ FORTRAN

AWO IDENTIFICATION #

| TITLE | ANALYST | PHONE EXT. | CARD NO. (INDICATE CARD NO. OR TITLE IF OTHER THAN FORTRAN.) | DATE | SHEET OF |
|---|---|---|---|---|---|
| SAMPLE PROB 1-2385 | | | | | |

```
    6    6  100  100    6   10   15   30   30
0
SAMPLE PRØBLEM 1
 0.7        0.9
 0.0        270.0
CASE               4  0.00125    0.0    0.000122  .000000925
PØLY               1
 600.0        0.0       0.0
TEMP                          NØ
TEMP
TEMP
TEMP
PØLY               1
 1625.0      0.0        0.0
PØLY               1   120.0
   1.0       0.0
 40.0                         ØN                 5         5
 60.0
 80.0
 120.0
```

NOTE: ADMINISTRATIVE WORK ORDER IDENTIFICATION MUST APPEAR ON EACH SHEET.

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# TABLE IV

## Program Printout, Sample Problem Number 1

```
SAMPLE PROBLEM 1

INPUT
GEOMETRY
R1=  0.70000000E 00       R2=  0.90000000E 00
P1=  0.                   P2=  0.47123883E 01      (RADIANS)

THERMO PHYSICAL PROPERTIES
TRANSFORM ACCORDING TO CASE 4,B=  0.12500000E-02   C=  0.
ALPHA=  0.12200000E-03      KZERO=  0.92500000E-06

INITIAL TEMPERATURE DISTRIBUTION
POLYNOMIAL DATA,   1 TERMS
              (  0.60000000E 03)(R**    0.     )(P**    0.     )

BOUNDRY CONDITIONS
AT R1, TEMP     L11=       0.     L12=      1.00
AT R2, TEMP     L21=       0.     L22=      1.00
AT P1, TEMP     K11=       0.     K12=      1.00
AT P2, TEMP     K21=       0.     K22=      1.00

STEADY STATE SOLUTION IS BEING INPUT
POLYNOMIAL DATA,   1 TERMS
              (  0.16250000E 04)(R**    0.     )(P**    0.     )

TIME FUNCTION
POLYNOMIAL DATA,   1 TERMS, X=DUMMY VARIABLE
              (  0.10000000E 01)(X**    0.     )
EVALUATE POLYNOMIAL FROM    0.                TO      0.12000000E 03



   5 LATTICE POINT ANSWERS IN -R-
 0.70000000E 00  0.75000000E 00  0.80000000E 00  0.84999999E 00  0.90000000E 00

   5 LATTICE POINT ANSWERS IN -PHI-
 0.              0.11780971E 01  0.23561941E 01  0.35342912E 01  0.47123883E 01

TIME=  0.40000000E 02      R=  0.70000000E 00      PHI=  0.              (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.75000000E 00      PHI=  0.              (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.80000000E 00      PHI=  0.              (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.84999999E 00      PHI=  0.              (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.90000000E 00      PHI=  0.              (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.70000000E 00      PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.75000000E 00      PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.89553053E 03
```



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```
TIME=  0.40000000E 02      R=  0.80000000E 00      PHI=  0.11780971E 01  (RADIANS)
        TEMPERATURE=  0.85510616E 03

TIME=  0.40000000E 02      R=  0.84999999E 00      PHI=  0.11780971E 01  (RADIANS)
        TEMPERATURE=  0.90183670E 03

TIME=  0.40000000E 02      R=  0.90000000E 00      PHI=  0.11780971E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.70000000E 00      PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.75000000E 00      PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.88835024E 03

TIME=  0.40000000E 02      R=  0.80000000E 00      PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.84493773E 03

TIME=  0.40000000E 02      R=  0.84999999E 00      PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.89500820E 03

TIME=  0.40000000E 02      R=  0.90000000E 00      PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.70000000E 00      PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.75000000E 00      PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.89539008E 03

TIME=  0.40000000E 02      R=  0.80000000E 00      PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.85490806E 03

TIME=  0.40000000E 02      R=  0.84999999E 00      PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.90170380E 03

TIME=  0.40000000E 02      R=  0.90000000E 00      PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.70000000E 00      PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.40000000E 02      R=  0.75000000E 00      PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999995E 03

TIME=  0.40000000E 02      R=  0.80000000E 00      PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999992E 03

TIME=  0.40000000E 02      R=  0.84999999E 00      PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999995E 03

TIME=  0.40000000E 02      R=  0.90000000E 00      PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02      R=  0.70000000E 00      PHI=  0.                (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02      R=  0.75000000E 00      PHI=  0.                (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02      R=  0.80000000E 00      PHI=  0.                (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02      R=  0.84999999E 00      PHI=  0.                (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02      R=  0.90000000E 00      PHI=  0.                (RADIANS)
        TEMPERATURE=  0.99999999E 03
```



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```
TIME=  0.60000000E 02     R=  0.70000000E 00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02     R=  0.75000000E 00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.94338583E 03

TIME=  0.60000000E 02     R=  0.80000000E 00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.92201011E 03

TIME=  0.60000000E 02     R=  0.84999999E 00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.94686591E 03

TIME=  0.60000000E 02     R=  0.90000000E 00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02     R=  0.70000000E 00     PHI=  0.23561941E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02     R=  0.75000000E 00     PHI=  0.23561941E 01 (RADIANS)
        TEMPERATURE=  0.93992796E 03

TIME=  0.60000000E 02     R=  0.80000000E 00     PHI=  0.23561941E 01 (RADIANS)
        TEMPERATURE=  0.91717767E 03

TIME=  0.60000000E 02     R=  0.84999999E 00     PHI=  0.23561941E 01 (RADIANS)
        TEMPERATURE=  0.94357659E 03

TIME=  0.60000000E 02     R=  0.90000000E 00     PHI=  0.23561941E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02     R=  0.70000000E 00     PHI=  0.35342912E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02     R=  0.75000000E 00     PHI=  0.35342912E 01 (RADIANS)
        TEMPERATURE=  0.94331553E 03

TIME=  0.60000000E 02     R=  0.80000000E 00     PHI=  0.35342912E 01 (RADIANS)
        TEMPERATURE=  0.92191216E 03

TIME=  0.60000000E 02     R=  0.84999999E 00     PHI=  0.35342912E 01 (RADIANS)
        TEMPERATURE=  0.94679937E 03

TIME=  0.60000000E 02     R=  0.90000000E 00     PHI=  0.35342912E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02     R=  0.70000000E 00     PHI=  0.47123883E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.60000000E 02     R=  0.75000000E 00     PHI=  0.47123883E 01 (RADIANS)
        TEMPERATURE=  0.99999997E 03

TIME=  0.60000000E 02     R=  0.80000000E 00     PHI=  0.47123883E 01 (RADIANS)
        TEMPERATURE=  0.99999995E 03

TIME=  0.60000000E 02     R=  0.84999999E 00     PHI=  0.47123883E 01 (RADIANS)
        TEMPERATURE=  0.99999997E 03

TIME=  0.60000000E 02     R=  0.90000000E 00     PHI=  0.47123883E 01 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02     R=  0.70000000E 00     PHI=  0.                 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02     R=  0.75000000E 00     PHI=  0.                 (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02     R=  0.80000000E 00     PHI=  0.                 (RADIANS)
        TEMPERATURE=  0.99999999E 03
```



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```
TIME=  0.80000000E 02      R=  0.84999999E 00     PHI=  0.              (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.90000000E 00     PHI=  0.              (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.70000000E 00     PHI=  0.11780971E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.75000000E 00     PHI=  0.11780971E 01  (RADIANS)
        TEMPERATURE=  0.96917486E 03

TIME=  0.80000000E 02      R=  0.80000000E 00     PHI=  0.11780971E 01  (RADIANS)
        TEMPERATURE=  0.95762274E 03

TIME=  0.80000000E 02      R=  0.84999999E 00     PHI=  0.11780971E 01  (RADIANS)
        TEMPERATURE=  0.97106580E 03

TIME=  0.80000000E 02      R=  0.90000000E 00     PHI=  0.11780971E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.70000000E 00     PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.75000000E 00     PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.96749291E 03

TIME=  0.80000000E 02      R=  0.80000000E 00     PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.95528220E 03

TIME=  0.80000000E 02      R=  0.84999999E 00     PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.96946320E 03

TIME=  0.80000000E 02      R=  0.90000000E 00     PHI=  0.23561941E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.70000000E 00     PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.75000000E 00     PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.96913924E 03

TIME=  0.80000000E 02      R=  0.80000000E 00     PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.95757343E 03

TIME=  0.80000000E 02      R=  0.84999999E 00     PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.97103207E 03

TIME=  0.80000000E 02      R=  0.90000000E 00     PHI=  0.35342912E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.70000000E 00     PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.80000000E 02      R=  0.75000000E 00     PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999997E 03

TIME=  0.80000000E 02      R=  0.80000000E 00     PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999997E 03

TIME=  0.80000000E 02      R=  0.84999999E 00     PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999997E 03

TIME=  0.80000000E 02      R=  0.90000000E 00     PHI=  0.47123883E 01  (RADIANS)
        TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.70000000E 00     PHI=  0.              (RADIANS)
        TEMPERATURE=  0.99999999E 03
```



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```
TIME=  0.12000000E 03      R=  0.75000000E 00      PHI=  0.               (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.80000000E 00      PHI=  0.               (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.84999999E 00      PHI=  0.               (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.90000000E 00      PHI=  0.               (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.70000000E 00      PHI=  0.11780971E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.75000000E 00      PHI=  0.11780971E 01   (RADIANS)
         TEMPERATURE=  0.99079582E 03

TIME=  0.12000000E 03      R=  0.80000000E 00      PHI=  0.11780971E 01   (RADIANS)
         TEMPERATURE=  0.98734950E 03

TIME=  0.12000000E 03      R=  0.84999999E 00      PHI=  0.11780971E 01   (RADIANS)
         TEMPERATURE=  0.99135558E 03

TIME=  0.12000000E 03      R=  0.90000000E 00      PHI=  0.11780971E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.70000000E 00      PHI=  0.23561941E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.75000000E 00      PHI=  0.23561941E 01   (RADIANS)
         TEMPERATURE=  0.99039390E 03

TIME=  0.12000000E 03      R=  0.80000000E 00      PHI=  0.23561941E 01   (RADIANS)
         TEMPERATURE=  0.98678927E 03

TIME=  0.12000000E 03      R=  0.84999999E 00      PHI=  0.23561941E 01   (RADIANS)
         TEMPERATURE=  0.99097077E 03

TIME=  0.12000000E 03      R=  0.90000000E 00      PHI=  0.23561941E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.70000000E 00      PHI=  0.35342912E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.75000000E 00      PHI=  0.35342912E 01   (RADIANS)
         TEMPERATURE=  0.99078654E 03

TIME=  0.12000000E 03      R=  0.80000000E 00      PHI=  0.35342912E 01   (RADIANS)
         TEMPERATURE=  0.98733665E 03

TIME=  0.12000000E 03      R=  0.84999999E 00      PHI=  0.35342912E 01   (RADIANS)
         TEMPERATURE=  0.99134679E 03

TIME=  0.12000000E 03      R=  0.90000000E 00      PHI=  0.35342912E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.70000000E 00      PHI=  0.47123883E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.75000000E 00      PHI=  0.47123883E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.80000000E 00      PHI=  0.47123883E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03

TIME=  0.12000000E 03      R=  0.84999999E 00      PHI=  0.47123883E 01   (RADIANS)
         TEMPERATURE=  0.99999999E 03
```



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```
TIME=  0.12000000E 03      R=  0.90000000E 00      PHI=  0.47123883E 01 (RADIANS)
       TEMPERATURE=  0.99999999E 03

END-OF-DATA ENCOUNTERED ON SYSTEM INPUT FILE.
```

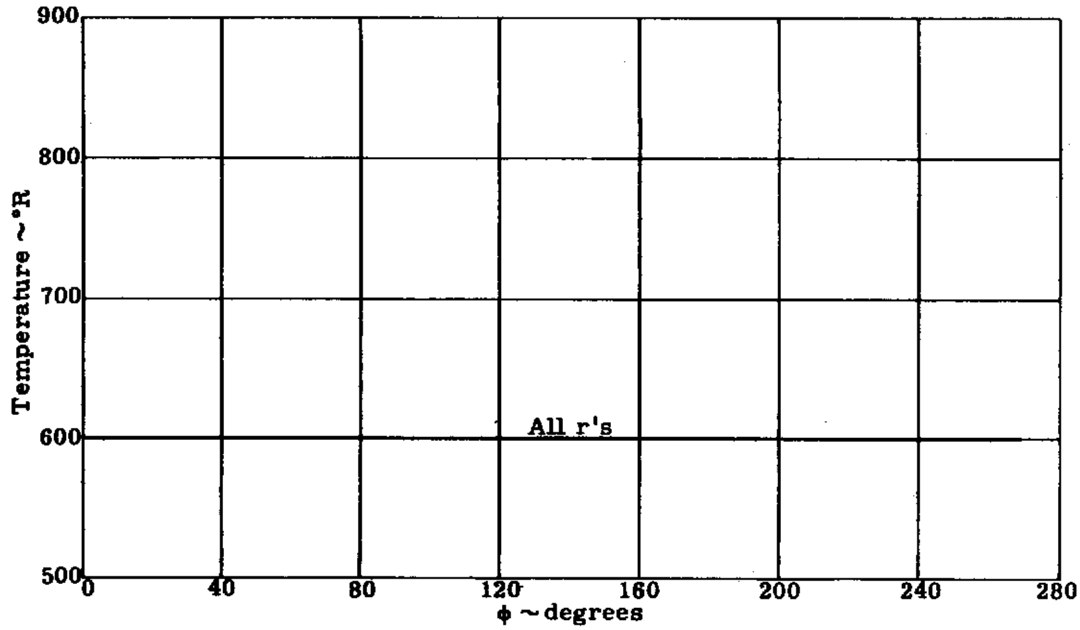


Figure 3. Temperature Distribution, Sample Problem No. 1, θ = 0 Second



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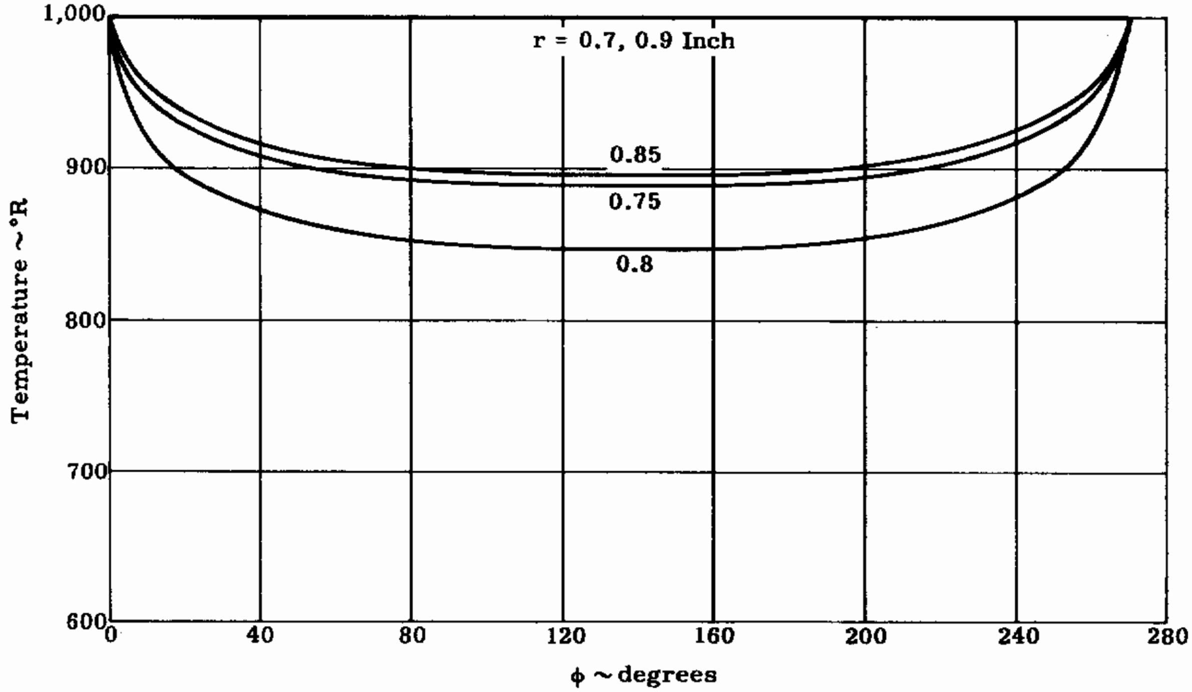


Figure 4. Temperature Distribution, Sample Problem No. 1, θ = 40 Seconds

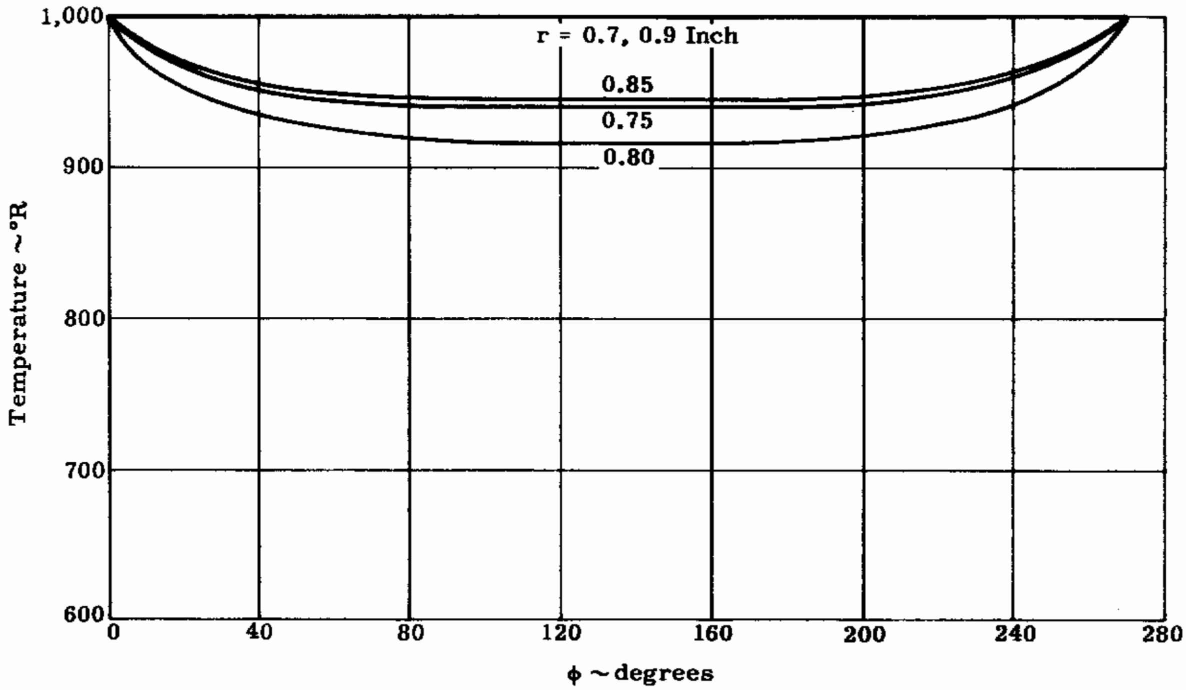


Figure 5. Temperature Distribution, Sample Problem No. 1, θ = 60 Seconds



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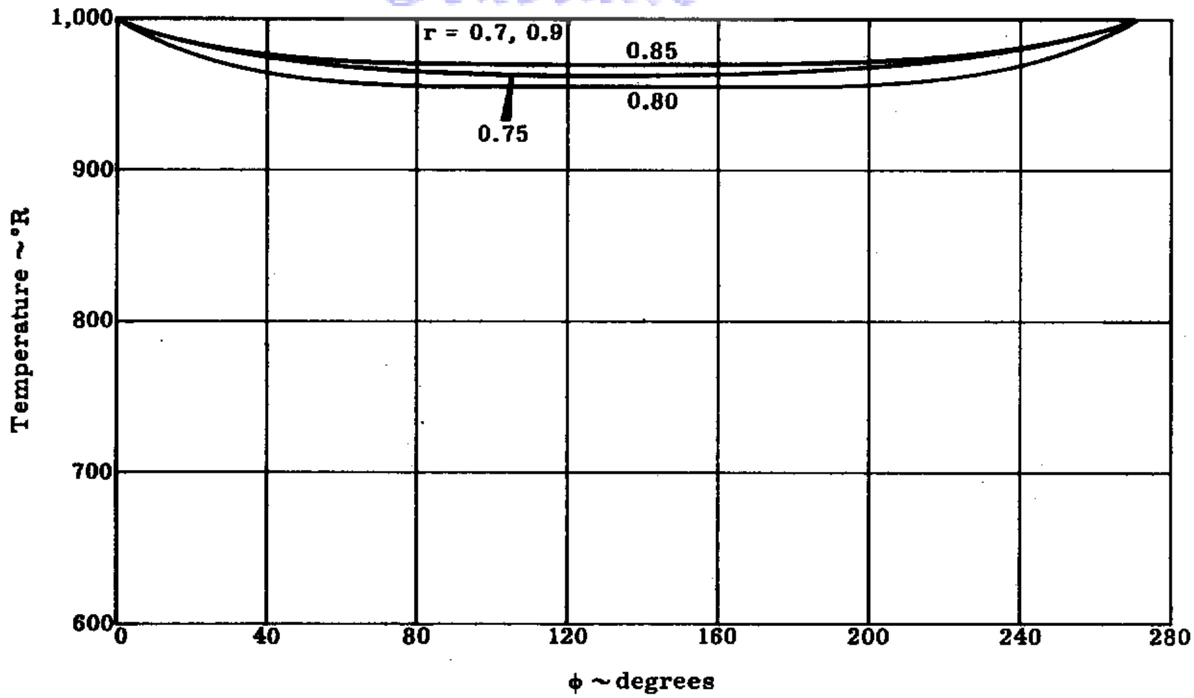


Figure 6. Temperature Distribution, Sample Problem No. 1, $\theta$ = 80 Seconds

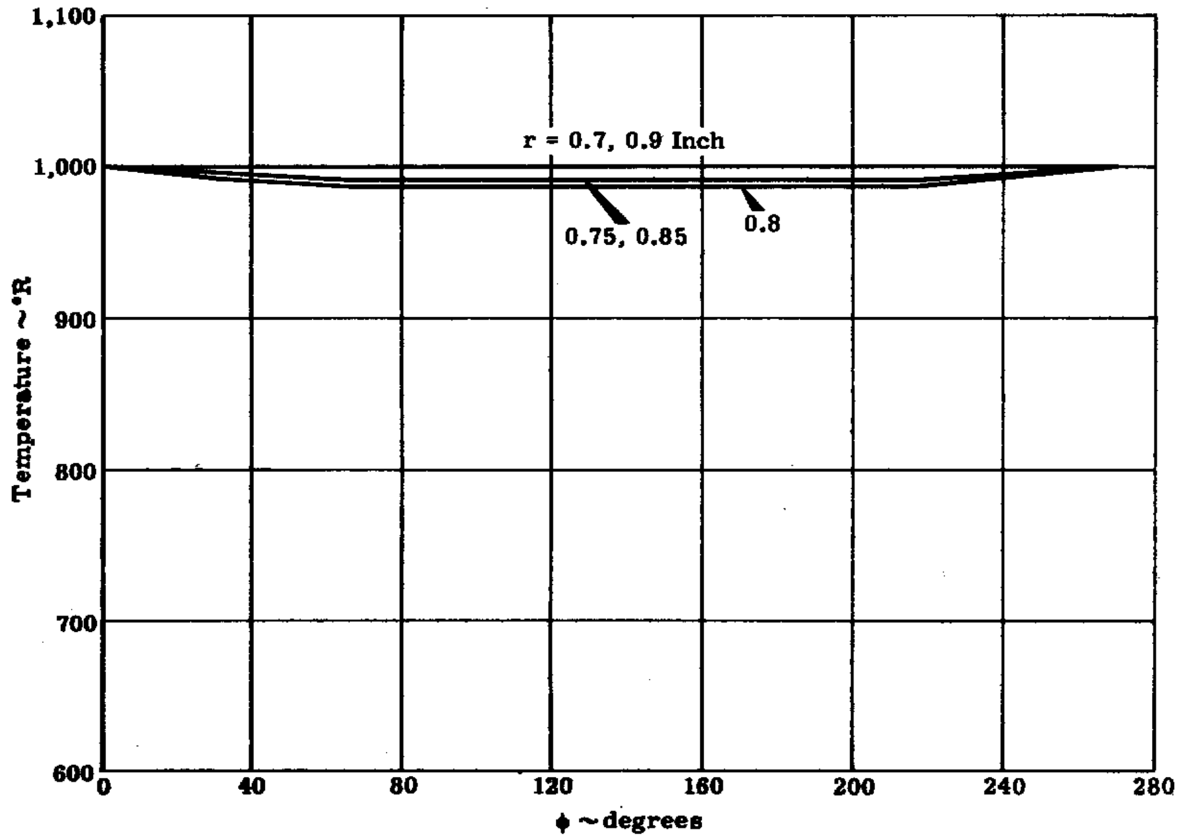


Figure 7. Temperature Distribution, Sample Problem No. 1, $\theta$ = 120 Seconds



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# TABLE V

## FORTRAN Coding, Sample Problem Number 2

FORM 2172 AU 000 — GENERAL LOADING FORM — ☐ FORTRAN

| | | | | | |
|---|---|---|---|---|---|
| TITLE: SAMPLE PROB. Z-2385 | ANALYST | PHONE EXT. | CARD NO. (INDICATE CARD NO. OR TITLE IF OTHER THAN FORTRAN.) | DATE | SHEET OF |

AWO IDENTIFICATION #

```
    6    4  100  100    6   10   25   30   30
0
SAMPLE PROBLEM 2
  0.2       0.8
  0.0       90.0
CASE               4  0.00125   0.0     0.000122  .000000925
POLY               1
  600.0      0.0       0.0
TEMP      POLY                2
   720.0      0.0
   -25.4      1.0
FLUX      POLY                1
   0.0        0.0
TEMP      POLY                2
   700.0      0.0
   100.0      1.0
TEMP      POLY                2
   700.0      0.0
  -100.0      1.0
POLY               2     120.0
  0.8        0.0
 0.00333     1.0
 40.0                          ON                5         5
 60.0
 80.0
 120.0
```

NOTE: ADMINISTRATIVE WORK ORDER IDENTIFICATION MUST APPEAR ON EACH SHEET.

MARTIN



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## TABLE VI

### Program Printout, Sample Problem Number 2

```
SAMPLE PROBLEM 2

INPUT
GEOMETRY
R1=  0.20000000E-00      R2=  0.80000000E 00
P1=  0.                  P2=  0.15707961E 01      (RADIANS)

THERMO PHYSICAL PROPERTIES
TRANSFORM ACCORDING TO CASE 4,B=  0.12500000E-02   C=  0.
ALPHA=  0.12200000E-03     KZERO=  0.92500000E-06

INITIAL TEMPERATURE DISTRIBUTION
POLYNOMIAL DATA,  1 TERMS
                 (  0.60000000E 03)(R**     0.     )(P**     0.     )

BOUNDRY CONDITIONS
AT R1, TEMP      L11=       0.      L12=       1.00
POLYNOMIAL DATA,  2 TERMS, X=DUMMY VARIABLE
                 (  0.72000000E 03)(X**     0.     )
                 ( -0.25400000E 02)(X**     1.0000)
AT R2, FLUX      L21=       1.00    L22=       0.
POLYNOMIAL DATA,  1 TERMS, X=DUMMY VARIABLE
                 (  0.             )(X**     0.     )
AT P1, TEMP      K11=       0.      K12=       1.00
POLYNOMIAL DATA,  2 TERMS, X=DUMMY VARIABLE
                 (  0.70000000E 03)(X**     0.     )
                 (  0.10000000E 03)(X**     1.0000)
AT P2, TEMP      K21=       0.      K22=       1.00
POLYNOMIAL DATA,  2 TERMS, X=DUMMY VARIABLE
                 (  0.70000000E 03)(X**     0.     )
                 ( -0.10000000E 03)(X**     1.0000)

TIME FUNCTION
POLYNOMIAL DATA,  2 TERMS, X=DUMMY VARIABLE
                 (  0.80000000E 00)(X**     0.     )
                 (  0.33300000E-02)(X**     1.0000)
EVALUATE POLYNOMIAL FROM     0.            TO      0.12000000E 03
```

| T1 | T2 | T3 | T4 | STEADY(IR,IP) | IR | IP |
|---|---|---|---|---|---|---|
| 0. | 0. | 0.52713818E-07 | 0. | -0.71517314E-05 | 1 | 1 |
| 0. | 0. | 0.23593566E 02 | 0. | 0.10640674E 04 | 2 | 1 |
| 0. | 0. | -0.58817416F 01 | -0. | 0.10756797E 04 | 3 | 1 |
| 0. | 0. | 0.16640617E 02 | 0. | 0.11162323E 04 | 4 | 1 |
| 0. | 0. | -0.26464082E 02 | -0. | 0.11225780E 04 | 5 | 1 |
| 0. | 0. | 0.28750360E 02 | 0. | 0.11710614E 04 | 6 | 1 |
| 0.32769979E 02 | 0. | 0.32619344E-11 | 0. | 0.11014208E 04 | 1 | 2 |
| 0.20392770E-08 | 0. | 0.14599714E-02 | 0.46899782E-32 | 0.10550722E 04 | 2 | 2 |
| 0.24793645E-15 | 0. | -0.36396254E-03 | -0.11691849E-32 | 0.10728598E 04 | 3 | 2 |
| 0.14373831E-20 | 0. | 0.10297225E-02 | 0.33078565E-32 | 0.10889541E 04 | 4 | 2 |
| 0.87403295E-25 | 0. | -0.16375991E-02 | -0.52605853E-32 | 0.11015144E 04 | 5 | 2 |
| 0.51701507E-28 | 0. | 0.17790741E-02 | 0.57150564E-32 | 0.11065804E 04 | 6 | 2 |
| -0.50206508E 02 | -0. | 0.20184871E-15 | 0.16933679E-36 | 0.99902129E 03 | 1 | 3 |
| -0.31243529E-08 | -0. | 0.90343122E-07 | 0.75791487E-28 | 0.10420061E 04 | 2 | 3 |
| -0.37986058E-15 | -0. | -0.22522026E-07 | -0.18894386E-28 | 0.10544606E 04 | 3 | 3 |
| -0.22021981E-20 | -0. | 0.63719291E-07 | 0.53455977E-28 | 0.10645759E 04 | 4 | 3 |
| -0.13390958E-24 | -0. | -0.10133474E-06 | -0.85012673E-28 | 0.10711369E 04 | 5 | 3 |
| -0.79211284E-28 | -0. | 0.11008922E-06 | 0.92357065E-28 | 0.10732670E 04 | 6 | 3 |
| 0.44150827E 02 | 0. | 0.12490416E-19 | 0.27365293E-32 | 0.10317192E 04 | 1 | 4 |
| 0.27475076E-08 | 0. | 0.55904402E-11 | 0.12248114E-23 | 0.10282341E 04 | 2 | 4 |
| 0.33404352E-15 | 0. | -0.13936649E-11 | -0.33533850E-24 | 0.10356669E 04 | 3 | 4 |
| 0.19365790E-20 | 0. | 0.39429552E-11 | 0.85386336E-24 | 0.10412486E 04 | 4 | 4 |
| 0.11775802E-24 | 0. | -0.62706021E-11 | -0.13738283E-23 | 0.10445328E 04 | 5 | 4 |
| 0.69657179E-28 | 0. | 0.68123303E-11 | 0.14925157E-23 | 0.10455226E 04 | 6 | 4 |



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| | | | | | | |
|---|---|---|---|---|---|---|
| -0.17436476E 02 | -0. | 0.77290743E-24 | 0.44223133E-28 | 0.10074655E 04 | 1 | 5 |
| -0.10850725E-08 | -0. | 0.34593667E-15 | 0.19793319E-19 | 0.10141319E 04 | 2 | 5 |
| -0.13192373E-15 | -0. | -0.86240039E-16 | -0.49343616E-20 | 0.10168617E 04 | 3 | 5 |
| -0.76481266E-21 | -0. | 0.24399023E-15 | 0.13960291E-19 | 0.10188134E 04 | 4 | 5 |
| -0.46506145E-25 | -0. | -0.38802511E-15 | -0.22201477E-19 | 0.10199094E 04 | 5 | 5 |
| -0.27509693E-28 | -0. | 0.42154727E-15 | 0.24119501E-19 | 0.10202305E 04 | 6 | 5 |
| -0.17436610E 02 | -0. | 0.47827567E-28 | 0.71465873E-24 | 0.99694684E 03 | 1 | 6 |
| -0.10850809E-08 | -0. | 0.21406585E-19 | 0.31986580E-15 | 0.99993795E 03 | 2 | 6 |
| -0.13192475E-15 | -0. | -0.53365398E-20 | -0.79740722E-15 | 0.99810152E 03 | 3 | 6 |
| -0.76481858E-21 | -0. | 0.15098133E-19 | 0.22560237E-15 | 0.99673614E 03 | 4 | 6 |
| -0.46506504E-25 | -0. | -0.24011022E-19 | -0.35878234E-15 | 0.99595206E 03 | 5 | 6 |
| -0.27509905E-28 | -0. | 0.26085375E-19 | 0.38977817E-15 | 0.99572010E 03 | 6 | 6 |
| 0.44150918E 02 | 0. | 0.29595746E-32 | 0.11549085E-19 | 0.10081296E 04 | 1 | 7 |
| 0.27475132E-08 | 0. | 0.13246617E-23 | 0.51691210E-11 | 0.98583507E 03 | 2 | 7 |
| 0.33404420E-15 | 0. | -0.33022562E-24 | -0.12886324E-11 | 0.97933614E 03 | 3 | 7 |
| 0.19365829E-20 | 0. | 0.93427398E-24 | 0.36457975E-11 | 0.97442137E 03 | 4 | 7 |
| 0.11775826E-24 | 0. | -0.14858044E-23 | -0.57980231E-11 | 0.97151839E 03 | 5 | 7 |
| 0.69657322E-28 | 0. | 0.16141657E-23 | 0.62989244E-11 | 0.97064191E 03 | 6 | 7 |
| -0.50206479E 02 | -0. | 0.18313869E-36 | 0.18663668E-15 | 0.95803342E 03 | 1 | 8 |
| -0.31243510E-08 | -0. | 0.81968920E-28 | 0.83534543E-07 | 0.97204189E 03 | 2 | 8 |
| -0.37986036E-15 | -0. | -0.20434385E-28 | -0.20824686E-07 | 0.96059722E 03 | 3 | 8 |
| -0.22021968E-20 | -0. | 0.57812939E-28 | 0.58917179E-07 | 0.95132963E 03 | 4 | 8 |
| -0.13390950E-24 | -0. | -0.91941684E-28 | -0.93697789E-07 | 0.94533459E 03 | 5 | 8 |
| -0.79211238E-28 | -0. | 0.99884685E-28 | 0.10179250E-06 | 0.94339016E 03 | 6 | 8 |
| 0.32769869E 02 | 0. | 0. | 0.30161042E-11 | 0.10383348E 04 | 1 | 9 |
| 0.20392702E-08 | 0. | 0.50722378E-32 | 0.13499631E-02 | 0.95889435E 03 | 2 | 9 |
| 0.24793562E-15 | 0. | -0.12644800E-32 | -0.33653312E-03 | 0.94220467E 03 | 3 | 9 |
| 0.14373783E-20 | 0. | 0.35774654E-32 | 0.95211915E-03 | 0.92726015E 03 | 4 | 9 |
| 0.87403001E-25 | 0. | -0.56893526E-32 | -0.15141842E-02 | 0.91570383E 03 | 5 | 9 |
| 0.51701334E-28 | 0. | 0.61808656E-32 | 0.16449972E-02 | 0.91105254E 03 | 6 | 9 |
| 0.11926919E-03 | 0. | -0. | 0.48741141E-07 | 0.18571208E-02 | 1 | 10 |
| 0.74221262E-14 | 0. | -0. | 0.21815482E 02 | 0.94421585E 03 | 2 | 10 |
| 0.90238628E-21 | 0. | 0. | -0.54384753E 01 | 0.91165409E 03 | 3 | 10 |
| 0.52314809E-26 | 0. | -0. | 0.15386529E 02 | 0.90590479E 03 | 4 | 10 |
| 0.31811190E-30 | 0. | 0. | -0.24469666E 02 | 0.86845119E 03 | 5 | 10 |
| 0.18817214E-33 | 0. | -0. | 0.26583643E 02 | 0.87382974E 03 | 6 | 10 |

TIME= 0.40000000E 02 R= 0.20000000E-00 PHI= 0. (RADIANS)
TEMPERATURE= 0.68284091E 03

TIME= 0.40000000E 02 R= 0.35000000E-00 PHI= 0. (RADIANS)
TEMPERATURE= 0.69436102E 03

TIME= 0.40000000E 02 R= 0.50000000E-00 PHI= 0. (RADIANS)
TEMPERATURE= 0.70877225E 03

TIME= 0.40000000E 02 R= 0.64999999E 00 PHI= 0. (RADIANS)
TEMPERATURE= 0.72110973E 03

TIME= 0.40000000E 02 R= 0.80000000E 00 PHI= 0. (RADIANS)
TEMPERATURE= 0.74548862E 03

TIME= 0.40000000E 02 R= 0.20000000E-00 PHI= 0.39269903E-00 (RADIANS)
TEMPERATURE= 0.66419402E 03

TIME= 0.40000000E 02 R= 0.35000000E-00 PHI= 0.39269903E-00 (RADIANS)
TEMPERATURE= 0.60492823E 03

TIME= 0.40000000E 02 R= 0.50000000E-00 PHI= 0.39269903E-00 (RADIANS)
TEMPERATURE= 0.60424066E 03

TIME= 0.40000000E 02 R= 0.64999999E 00 PHI= 0.39269903E-00 (RADIANS)
TEMPERATURE= 0.59773344E 03

TIME= 0.40000000E 02 R= 0.80000000E 00 PHI= 0.39269903E-00 (RADIANS)
TEMPERATURE= 0.60081771E 03

TIME= 0.40000000E 02 R= 0.20000000E-00 PHI= 0.78539805E 00 (RADIANS)
TEMPERATURE= 0.66164771E 03

TIME= 0.40000000E 02 R= 0.35000000E-00 PHI= 0.78539805E 00 (RADIANS)
TEMPERATURE= 0.59783714E 03



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```
TIME=  0.40000000E 02     R=  0.50000000E-00     PHI=  0.78539805E 00 (RADIANS)
   TEMPERATURE=  0.59914182E 03

TIME=  0.40000000E 02     R=  0.64999999E 00     PHI=  0.78539805E 00 (RADIANS)
   TEMPERATURE=  0.59515677E 03

TIME=  0.40000000E 02     R=  0.80000000E 00     PHI=  0.78539805E 00 (RADIANS)
   TEMPERATURE=  0.59714668E 03

TIME=  0.40000000E 02     R=  0.20000000E-00     PHI=  0.11780971E 01 (RADIANS)
   TEMPERATURE=  0.64539223E 03

TIME=  0.40000000E 02     R=  0.35000000E-00     PHI=  0.11780971E 01 (RADIANS)
   TEMPERATURE=  0.59432542E 03

TIME=  0.40000000E 02     R=  0.50000000E-00     PHI=  0.11780971E 01 (RADIANS)
   TEMPERATURE=  0.59956348E 03

TIME=  0.40000000E 02     R=  0.64999999E 00     PHI=  0.11780971E 01 (RADIANS)
   TEMPERATURE=  0.59611794E 03

TIME=  0.40000000E 02     R=  0.80000000E 00     PHI=  0.11780971E 01 (RADIANS)
   TEMPERATURE=  0.59883289E 03

TIME=  0.40000000E 02     R=  0.20000000E-00     PHI=  0.15707961E 01 (RADIANS)
   TEMPERATURE=  0.64463718E 03

TIME=  0.40000000E 02     R=  0.35000000E-00     PHI=  0.15707961E 01 (RADIANS)
   TEMPERATURE=  0.62747079E 03

TIME=  0.40000000E 02     R=  0.50000000E-00     PHI=  0.15707961E 01 (RADIANS)
   TEMPERATURE=  0.61312303E 03

TIME=  0.40000000E 02     R=  0.64999999E 00     PHI=  0.15707961E 01 (RADIANS)
   TEMPERATURE=  0.59666834E 03

TIME=  0.40000000E 02     R=  0.80000000E 00     PHI=  0.15707961E 01 (RADIANS)
   TEMPERATURE=  0.59453669E 03

TIME=  0.60000000E 02     R=  0.20000000E-00     PHI=  0.             (RADIANS)
   TEMPERATURE=  0.71989007E 03

TIME=  0.60000000E 02     R=  0.35000000E-00     PHI=  0.             (RADIANS)
   TEMPERATURE=  0.73193055E 03

TIME=  0.60000000E 02     R=  0.50000000E-00     PHI=  0.             (RADIANS)
   TEMPERATURE=  0.74699023E 03

TIME=  0.60000000E 02     R=  0.64999999E 00     PHI=  0.             (RADIANS)
   TEMPERATURE=  0.75988066E 03

TIME=  0.60000000E 02     R=  0.80000000E 00     PHI=  0.             (RADIANS)
   TEMPERATURE=  0.78534646E 03

TIME=  0.60000000E 02     R=  0.20000000E-00     PHI=  0.39269903E-00 (RADIANS)
   TEMPERATURE=  0.70039690E 03

TIME=  0.60000000E 02     R=  0.35000000E-00     PHI=  0.39269903E-00 (RADIANS)
   TEMPERATURE=  0.61812373E 03

TIME=  0.60000000E 02     R=  0.50000000E-00     PHI=  0.39269903E-00 (RADIANS)
   TEMPERATURE=  0.60948319E 03

TIME=  0.60000000E 02     R=  0.64999999E 00     PHI=  0.39269903E-00 (RADIANS)
   TEMPERATURE=  0.59947894E 03

TIME=  0.60000000E 02     R=  0.80000000E 00     PHI=  0.39269903E-00 (RADIANS)
   TEMPERATURE=  0.60137347E 03

TIME=  0.60000000E 02     R=  0.20000000E-00     PHI=  0.78539805E 00 (RADIANS)
```



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```
          TEMPERATURE=  0.69773465E 03

TIME=  0.60000000E 02      R=  0.35000000E-00      PHI=  0.78539805E 00 (RADIANS)
          TEMPERATURE=  0.60268202E 03

TIME=  0.60000000E 02      R=  0.50000000E-00      PHI=  0.78539805E 00 (RADIANS)
          TEMPERATURE=  0.59929526E 03

TIME=  0.60000000E 02      R=  0.64999999E 00      PHI=  0.78539805E 00 (RADIANS)
          TEMPERATURE=  0.59451863E 03

TIME=  0.60000000E 02      R=  0.80000000E 00      PHI=  0.78539805E 00 (RADIANS)
          TEMPERATURE=  0.59646590E 03

TIME=  0.60000000E 02      R=  0.20000000E-00      PHI=  0.11780971E 01 (RADIANS)
          TEMPERATURE=  0.68073674E 03

TIME=  0.60000000E 02      R=  0.35000000E-00      PHI=  0.11780971E 01 (RADIANS)
          TEMPERATURE=  0.60161132E 03

TIME=  0.60000000E 02      R=  0.50000000E-00      PHI=  0.11780971E 01 (RADIANS)
          TEMPERATURE=  0.59952266E 03

TIME=  0.60000000E 02      R=  0.64999999E 00      PHI=  0.11780971E 01 (RADIANS)
          TEMPERATURE=  0.59479115E 03

TIME=  0.60000000E 02      R=  0.80000000E 00      PHI=  0.11780971E 01 (RADIANS)
          TEMPERATURE=  0.59791932E 03

TIME=  0.60000000E 02      R=  0.20000000E-00      PHI=  0.15707961E 01 (RADIANS)
          TEMPERATURE=  0.67994710E 03

TIME=  0.60000000E 02      R=  0.35000000E-00      PHI=  0.15707961E 01 (RADIANS)
          TEMPERATURE=  0.66199214E 03

TIME=  0.60000000E 02      R=  0.50000000E-00      PHI=  0.15707961E 01 (RADIANS)
          TEMPERATURE=  0.64698175E 03

TIME=  0.60000000E 02      R=  0.64999999E 00      PHI=  0.15707961E 01 (RADIANS)
          TEMPERATURE=  0.62976302E 03

TIME=  0.60000000E 02      R=  0.80000000E 00      PHI=  0.15707961E 01 (RADIANS)
          TEMPERATURE=  0.62753208E 03

TIME=  0.80000000E 02      R=  0.20000000E-00      PHI=  0.             (RADIANS)
          TEMPERATURE=  0.75605733E 03

TIME=  0.80000000E 02      R=  0.35000000E-00      PHI=  0.             (RADIANS)
          TEMPERATURE=  0.76860051E 03

TIME=  0.80000000E 02      R=  0.50000000E-00      PHI=  0.             (RADIANS)
          TEMPERATURE=  0.78428653E 03

TIME=  0.80000000E 02      R=  0.64999999E 00      PHI=  0.             (RADIANS)
          TEMPERATURE=  0.79771102E 03

TIME=  0.80000000E 02      R=  0.80000000E 00      PHI=  0.             (RADIANS)
          TEMPERATURE=  0.82422649E 03

TIME=  0.80000000E 02      R=  0.20000000E-00      PHI=  0.39269903E-00 (RADIANS)
          TEMPERATURE=  0.73574660E 03

TIME=  0.80000000E 02      R=  0.35000000E-00      PHI=  0.39269903E-00 (RADIANS)
          TEMPERATURE=  0.63483462E 03

TIME=  0.80000000E 02      R=  0.50000000E-00      PHI=  0.39269903E-00 (RADIANS)
```



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```
        TEMPERATURE=  0.61723180E 03

TIME=  0.80000000E 02     R=  0.64999999E 00     PHI=  0.39269903E-00 (RADIANS)
        TEMPERATURE=  0.60327910E 03

TIME=  0.80000000E 02     R=  0.80000000E 00     PHI=  0.39269903E-00 (RADIANS)
        TEMPERATURE=  0.60368118E 03

TIME=  0.80000000E 02     R=  0.20000000E-00     PHI=  0.78539805E 00 (RADIANS)
        TEMPERATURE=  0.73297232E 03

TIME=  0.80000000E 02     R=  0.35000000E-00     PHI=  0.78539805E 00 (RADIANS)
        TEMPERATURE=  0.61101406E 03

TIME=  0.80000000E 02     R=  0.50000000E-00     PHI=  0.78539805E 00 (RADIANS)
        TEMPERATURE=  0.59999804E 03

TIME=  0.80000000E 02     R=  0.64999999E 00     PHI=  0.78539805E 00 (RADIANS)
        TEMPERATURE=  0.59432294E 03

TIME=  0.80000000E 02     R=  0.80000000E 00     PHI=  0.78539805E 00 (RADIANS)
        TEMPERATURE=  0.59615901E 03

TIME=  0.80000000E 02     R=  0.20000000E-00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.71525703E 03

TIME=  0.80000000E 02     R=  0.35000000E-00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.61377184E 03

TIME=  0.80000000E 02     R=  0.50000000E-00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.60192450E 03

TIME=  0.80000000E 02     R=  0.64999999E 00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.59438409E 03

TIME=  0.80000000E 02     R=  0.80000000E 00     PHI=  0.11780971E 01 (RADIANS)
        TEMPERATURE=  0.59701450E 03

TIME=  0.80000000E 02     R=  0.20000000E-00     PHI=  0.15707961E 01 (RADIANS)
        TEMPERATURE=  0.71443398E 03

TIME=  0.80000000E 02     R=  0.35000000E-00     PHI=  0.15707961E 01 (RADIANS)
        TEMPERATURE=  0.69571693E 03

TIME=  0.80000000E 02     R=  0.50000000E-00     PHI=  0.15707961E 01 (RADIANS)
        TEMPERATURE=  0.68006610E 03

TIME=  0.80000000E 02     R=  0.64999999E 00     PHI=  0.15707961E 01 (RADIANS)
        TEMPERATURE=  0.66210883E 03

TIME=  0.80000000E 02     R=  0.80000000E 00     PHI=  0.15707961E 01 (RADIANS)
        TEMPERATURE=  0.65978186E 03

TIME=  0.12000000E 03     R=  0.20000000E-00     PHI=  0.             (RADIANS)
        TEMPERATURE=  0.82598023E 03

TIME=  0.12000000E 03     R=  0.35000000E-00     PHI=  0.             (RADIANS)
        TEMPERATURE=  0.83948170E 03

TIME=  0.12000000E 03     R=  0.50000000E-00     PHI=  0.             (RADIANS)
        TEMPERATURE=  0.85636165E 03

TIME=  0.12000000E 03     R=  0.64999999E 00     PHI=  0.             (RADIANS)
        TEMPERATURE=  0.87080405E 03

TIME=  0.12000000E 03     R=  0.80000000E 00     PHI=  0.             (RADIANS)
```



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```
         TEMPERATURE=   0.89931993E 03

TIME=   0.12000000E 03      R=  0.20000000E-00      PHI=  0.39269903E-00 (RADIANS)
         TEMPERATURE=   0.80411071E 03

TIME=   0.12000000E 03      R=  0.35000000E-00      PHI=  0.39269903E-00 (RADIANS)
         TEMPERATURE=   0.67477295E 03

TIME=   0.12000000E 03      R=  0.50000000E-00      PHI=  0.39269903E-00 (RADIANS)
         TEMPERATURE=   0.63892967E 03

TIME=   0.12000000E 03      R=  0.64999999E 00      PHI=  0.39269903E-00 (RADIANS)
         TEMPERATURE=   0.61598204E 03

TIME=   0.12000000E 03      R=  0.80000000E 00      PHI=  0.39269903E-00 (RADIANS)
         TEMPERATURE=   0.61299543E 03

TIME=   0.12000000E 03      R=  0.20000000E-00      PHI=  0.78539805E 00 (RADIANS)
         TEMPERATURE=   0.80112283E 03

TIME=   0.12000000E 03      R=  0.35000000E-00      PHI=  0.78539805E 00 (RADIANS)
         TEMPERATURE=   0.63573340E 03

TIME=   0.12000000E 03      R=  0.50000000E-00      PHI=  0.78539805E 00 (RADIANS)
         TEMPERATURE=   0.60465853E 03

TIME=   0.12000000E 03      R=  0.64999999E 00      PHI=  0.78539805E 00 (RADIANS)
         TEMPERATURE=   0.59474177E 03

TIME=   0.12000000E 03      R=  0.80000000E 00      PHI=  0.78539805E 00 (RADIANS)
         TEMPERATURE=   0.59607594E 03

TIME=   0.12000000E 03      R=  0.20000000E-00      PHI=  0.11780971E 01 (RADIANS)
         TEMPERATURE=   0.78203952E 03

TIME=   0.12000000E 03      R=  0.35000000E-00      PHI=  0.11780971E 01 (RADIANS)
         TEMPERATURE=   0.64688766E 03

TIME=   0.12000000E 03      R=  0.50000000E-00      PHI=  0.11780971E 01 (RADIANS)
         TEMPERATURE=   0.61398914E 03

TIME=   0.12000000E 03      R=  0.64999999E 00      PHI=  0.11780971E 01 (RADIANS)
         TEMPERATURE=   0.59765728E 03

TIME=   0.12000000E 03      R=  0.80000000E 00      PHI=  0.11780971E 01 (RADIANS)
         TEMPERATURE=   0.59766819E 03

TIME=   0.12000000E 03      R=  0.20000000E-00      PHI=  0.15707961E 01 (RADIANS)
         TEMPERATURE=   0.78115273E 03

TIME=   0.12000000E 03      R=  0.35000000E-00      PHI=  0.15707961E 01 (RADIANS)
         TEMPERATURE=   0.76098222E 03

TIME=   0.12000000E 03      R=  0.50000000E-00      PHI=  0.15707961E 01 (RADIANS)
         TEMPERATURE=   0.74410966E 03

TIME=   0.12000000E 03      R=  0.64999999E 00      PHI=  0.15707961E 01 (RADIANS)
         TEMPERATURE=   0.72474324F 03

TIME=   0.12000000E 03      R=  0.80000000E 00      PHI=  0.15707961E 01 (RADIANS)
         TEMPERATURE=   0.72223310E 03

END-OF-DATA ENCOUNTERED ON SYSTEM INPUT FILE.

    PP,F0081 ,008180,181-26203

E N D   O F   J O B  * F0081  *     START TIME=171347  STOP TIME=172148
```



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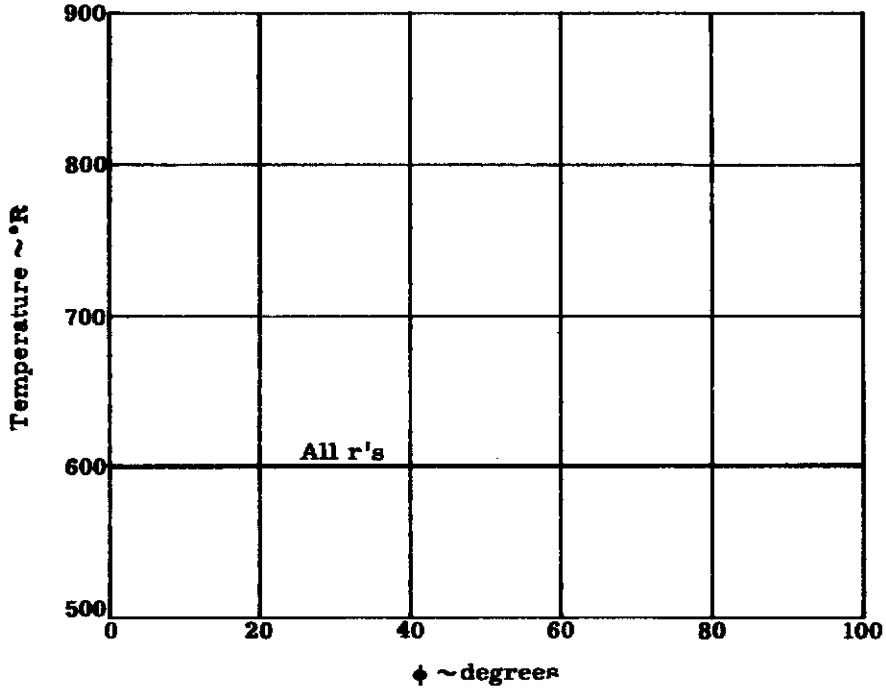


Figure 8. Temperature Distribution, Sample Problem No. 2, θ = 0 Second

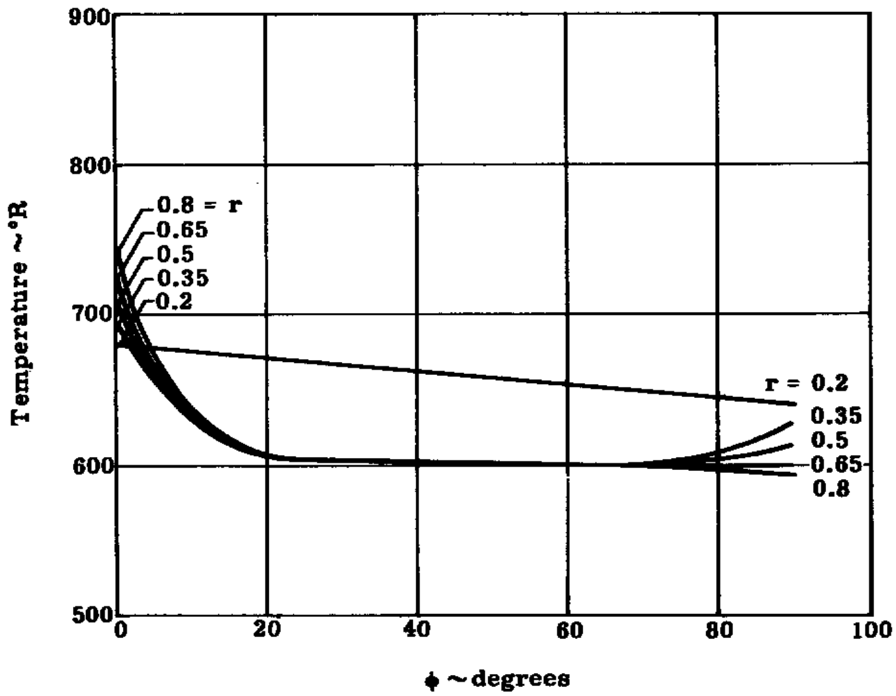


Figure 9. Temperature Distribution, Sample Problem No. 2, θ = 40 Seconds



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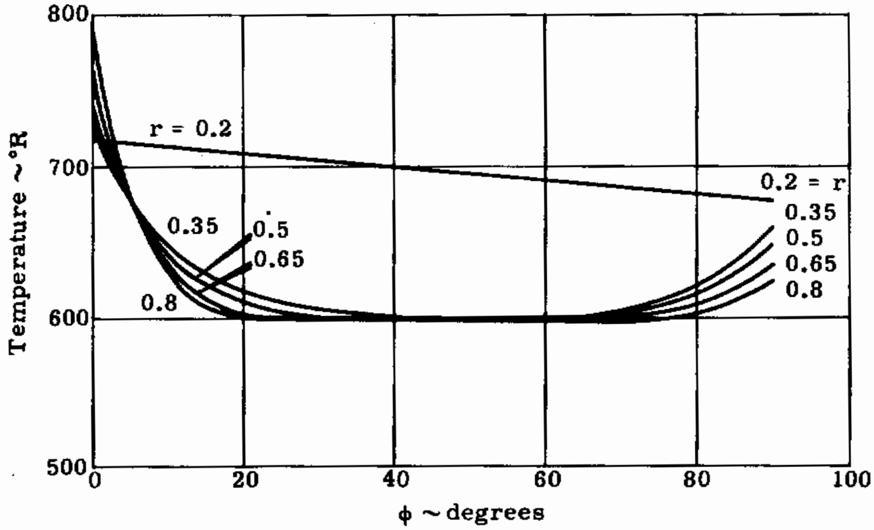


Figure 10. Temperature Distribution, Sample Problem No. 2, θ = 60 Seconds

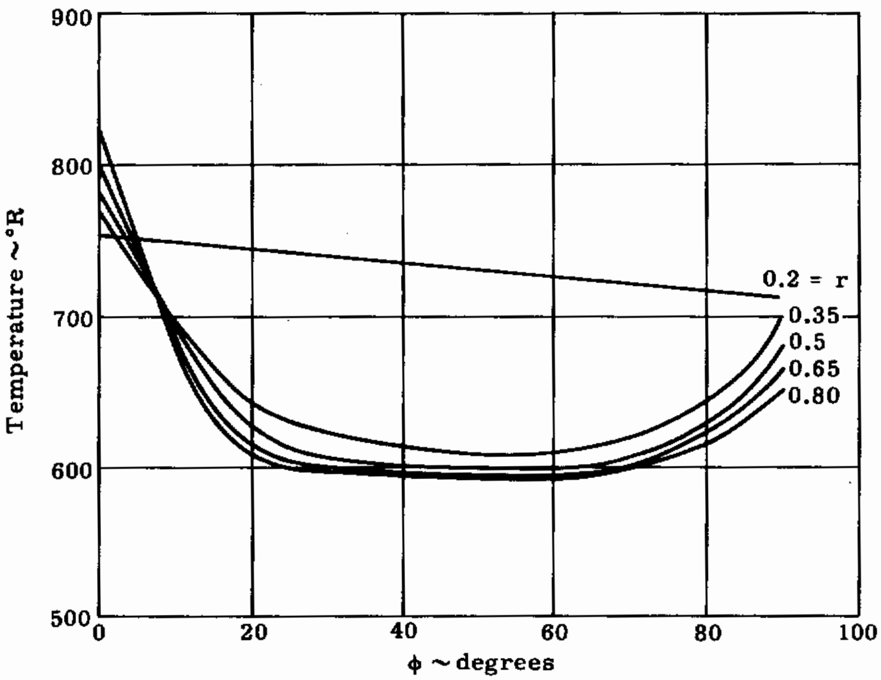


Figure 11. Temperature Distribution, Sample Problem No. 2, θ = 80 Seconds



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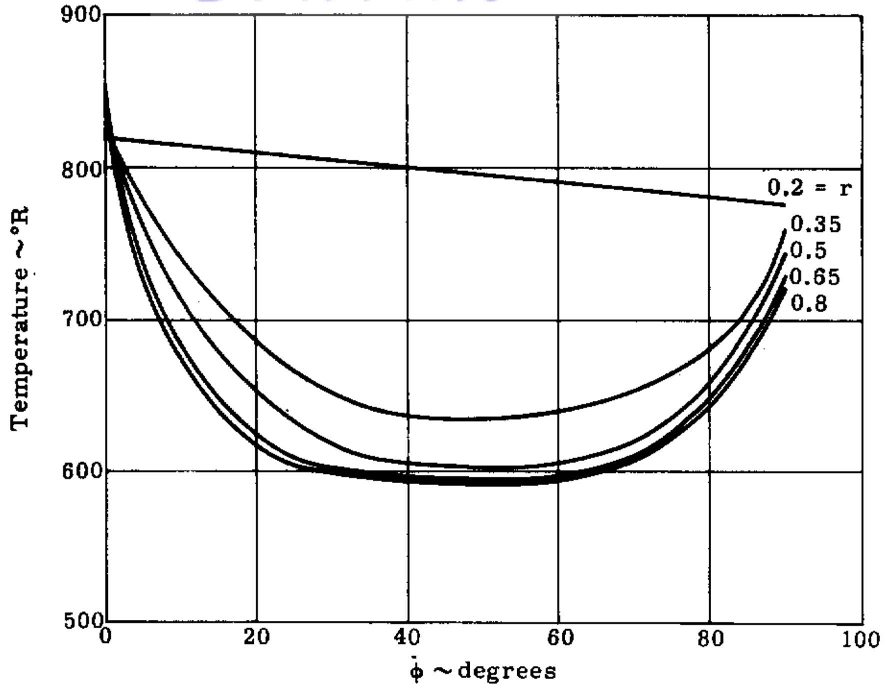


Figure 12. Temperature Distribution, Sample Problem No. 2, $\theta$ = 120 Seconds



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# APPENDIX A

# STEADY STATE SOLUTION



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## APPENDIX A

## STEADY STATE SOLUTION

The steady state solution resulting from the coordinate varying boundary condition functions in Equations (75) through (78) is required to obtain $f(r,\phi)$ as discussed in Section II. This steady state solution can be written

$$f(r,\phi) = T_{r1}(r,\phi) + T_{r2}(r,\phi) + T_{\phi 1}(r,\phi) + T_{\phi 2}(r,\phi) \tag{99}$$

where the individual functions are the result of the prescribed boundary condition at the particular surface noted by the subscript. A solution similar to that used in Reference 4 is presented in this Appendix. The solution primarily differs from that used in Reference 4 in that basic exponentials are used rather than hyperbolic functions.

The use of hyperbolic functions in the solution of problems in heat transfer is so universal that this departure should be explained. The use of basic exponentials offers several advantages over hyperbolic functions in preparing mathematical solutions applicable to digital computers. A prime advantage is that the solution in basic exponentials can be manipulated to avoid machine overflow by keeping arguments raised to positive or negative powers less than or greater than unity, respectively. Hyperbolic functions, as calculated by machine, necessarily contain both a positive and negative exponent of the argument and cannot be so manipulated. Secondly, the basic exponential form is easier to evaluate in certain indeterminate forms and special cases. The solution used in the digital program is as follows*:

1. $T_{r1}(r,\phi)$

$$T_{r1}(r,\phi) = \sum_{\gamma} P_{\gamma}\left[B_{\gamma}\sin\gamma\phi + F_{\gamma}\cos\gamma\phi\right]\xi_{r1} \tag{100}$$

where in general ($\gamma \neq 0$)

$$\xi_{r1} = \frac{(-1)^{\ell_{22}}\left(\frac{r}{r_1}\right)^{-\gamma} + \left(\frac{r_1}{r_2}\right)^{\gamma}\left(\frac{r}{r_2}\right)^{\gamma}}{\left(\frac{\gamma}{r_1}\right)^{-\ell_{11}}\left[(-1)^{\ell_{22}}(-1)^{-\ell_{11}} + \left(\frac{r_1}{r_2}\right)^{2\gamma}\right]} \quad \text{and} \tag{101}$$

*Valid for $\phi_1 = 0$.



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$$P_\gamma = \frac{\int_{\phi_1}^{\phi_2} F_{r1}(\phi) \left[B_\gamma \sin \gamma\phi + F_\gamma \cos \gamma\phi\right] d\phi}{\int_{\phi_1}^{\phi_2} \left[B_\gamma \sin \gamma\phi + F_\gamma \cos \gamma\phi\right]^2 d\phi} . \tag{102}$$

If $B_\gamma$ is infinite, that particular term in the series of Equation (100) is indeterminate and must be replaced by

$$P' \sin \gamma\phi \, \xi_{r1} \tag{103}$$

where

$$P' = \frac{\int_{\phi_1}^{\phi_2} F_{r1}(\phi) \sin \gamma\phi}{\left[\frac{\phi}{2} - \frac{1}{4\gamma} \sin 2\gamma\phi\right]_{\phi_1}^{\phi_2}} . \tag{104}$$

The arbitrary function in Equation (104) is defined in relation to the boundary condition at the surface as follows:

1 If $\ell_{12}$ equals unity

$$F_{r1}(\phi) = T\left[f_{r1}(\phi)\right] \qquad \text{(transform)} . \tag{105}$$

2 If $\ell_{12}$ equals zero

$$F_{r1}(\phi) = \frac{1}{k_o} f_{r1}(\phi) . \tag{106}$$

If $\gamma$ equals zero, i.e., both $k_{11}$ and $k_{21}$ equal zero, an additional term must be added to Equation (110) as follows:

1 If $(\ell_{12}, \ell_{22})$ equals (1, 0), the term is

$$\frac{1}{\phi_2 - \phi_1} \int_{\phi_1}^{\phi_2} F_{r1}(\phi) \, d\phi . \tag{107}$$



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2 If $(\ell_{12}, \ell_{22})$ equals (1, 1) the term is

$$\frac{\ell n \frac{r}{r_2}}{(\phi_2 - \phi_1)\ell n \frac{r_1}{r_2}} \int_{\phi_1}^{\phi_2} F_{r1}(\phi)\, d\phi. \tag{108}$$

3 If $(\ell_{12}, \ell_{22})$ equals (0, 1) the term is

$$\frac{r_1 \ \ell n \frac{r}{r_2}}{\phi_2 - \phi_1} \int_{\phi_1}^{\phi_2} F_{r1}(\phi)\, d\phi \ . \tag{109}$$

4 If $(\ell_{12}, \ell_{22})$ equals (0, 0) the term is infinite.

2. $T_{r2}\,(r, \phi)$

$$T_{r2}\,(r, \phi) = \sum_{\gamma} L_{\gamma} \left[ B_{\gamma} \sin \gamma\phi + F_{\gamma} \cos \gamma\phi \right] \xi_{r2} \tag{110}$$

where, in general $(\gamma \neq 0)$

$$\xi_{r2} = \frac{(-1)^{\ell_{12}} \left(\frac{r_1}{r_2}\right)^{\gamma} \left(\frac{r}{r_1}\right)^{-\gamma} + \left(\frac{r}{r_2}\right)^{\gamma}}{\frac{\gamma}{r_2}^{\cdot \ell_{21}} \left[ (-1)^{\ell_{21}} (-1)^{\ell_{12}} \frac{r_1}{r_2}^{2\gamma} + 1 \right]} \tag{111}$$

$$L_{\gamma} = \frac{\int_{\phi_1}^{\phi_2} F_{r2}(\phi) \left[ B_{\gamma} \sin \gamma\phi + F_{\gamma} \cos \gamma\phi \right] d\phi}{\int_{\phi_1}^{\phi_2} \left[ B_{\gamma} \sin \gamma\phi + F_{\gamma} \cos \gamma\phi \right]^2 d\phi} \ . \tag{112}$$

If $B_{\gamma}$ is infinite, that particular term in the series (Equation (109) is replaced by



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$$L'_{\gamma} \sin \gamma\phi \, \xi_{r2} \tag{113}$$

where

$$L'_{\gamma} = \frac{\displaystyle\int_{\phi_1}^{\phi_2} F_{r2}(\phi) \sin \gamma\phi \, d\phi}{\left[\frac{\phi}{2} - \frac{1}{4\gamma} \sin 2\gamma\phi\right]_{\phi_1}^{\phi_2}} . \tag{114}$$

The arbitrary function in Equation (114) is defined in relation to the boundary condition at the surface as follows:

1 If $\ell_{22}$ equals unity

$$F_{r2}(\phi) = \Upsilon \left[f_{r2}(\phi)\right] \quad \text{(transform)}. \tag{115}$$

2 If $\ell_{22}$ equals zero

$$F_{r2}(\phi) = \frac{1}{K_o} f_{r2}(\phi). \tag{116}$$

If $\gamma$ is equal to zero, an additional term must be added to Equation (109), as follows:

1 If $(\ell_{12}, \ell_{22})$ equals (0, 1) the term is

$$\frac{1}{\phi_2 - \phi_1} \int_{\phi_1}^{\phi_2} F_{r2}(\phi) \, d\phi \quad . \tag{117}$$

2 If $(\ell_{12}, \ell_{22})$ equals (1, 1) the term is

$$\frac{\ell n \dfrac{r}{r_2}}{(\phi_2 - \phi_1) \, \ell n \dfrac{r_2}{r_1}} \int_{\phi_1}^{\phi_2} F_{r2}(\phi) \, d\phi . \tag{118}$$



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3 If $(\ell_{12}, \ell_{22})$ equals (1, 0) the term is

$$\frac{r_2 \; \ell n \frac{r}{r_2}}{(\phi_2 - \phi_1)} \int_{\phi_1}^{\phi_2} F_{r2}(\phi) \, d\phi. \tag{119}$$

4 If $(\ell_{12}, \ell_{22})$ equals (0, 0) the term is infinite.

3. $T_{\phi 1}(r,\phi)$

$$T_{\phi 1}(r,\phi) = \sum_{\epsilon} Q_\epsilon \left[M_\epsilon \sin(\epsilon \, \ell n r) + F_\ell \cos(\epsilon \, \ell n r_1)\right] \xi_{\phi 1} \tag{120}$$

where $\epsilon$ are the real positive roots of

$$\begin{vmatrix} \frac{\ell_{11}\epsilon}{r_1} \cos(\epsilon \ell n r_1) + \ell_{12} \sin(\epsilon \ell n r_1) & \ell_{12} \cos(\epsilon \ell n r_1) - \frac{\ell_{11}\epsilon}{r_1} \sin(\epsilon \ell n r_1) \\ \frac{\ell_{21}\epsilon}{r_2} \cos(\epsilon \ell n r_2) + \ell_{22} \sin(\epsilon \ell n r_2) & \ell_{22} \cos(\epsilon \ell n r_2) - \frac{\ell_{21}\epsilon}{r_1} \sin(\epsilon \ell n r_2) \end{vmatrix} = 0 \tag{121}$$

Equation (121) can be solved explicitly similar to Equation (45).

In general $(\epsilon \neq 0)$ the terms in Equation (120) are defined as:

$$\xi_{\phi 1} = \frac{(-1)^{k_{22}} e^{-\epsilon\phi} + e^{-\epsilon(2\phi_2 - \phi)}}{\left[(-1)^{k_{11}} (-1)^{k_{22}} + e^{-2\epsilon\phi_2}\right] \epsilon^{-k_{11}}} \tag{122}$$

$$Q_\epsilon = \frac{\int_{r_1}^{r_2} F_{\phi 1}(r) \left[M_\epsilon \sin(\epsilon \, \ell n r) + F_\epsilon \cos(\epsilon \ell n r)\right] \frac{dr}{r}}{\int_{r_1}^{r_2} \left[M_\epsilon \sin(\epsilon \ell n r) + F_\epsilon \cos(\epsilon \ell n r)\right]^2 \frac{dr}{r}} \tag{123}$$



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$$M_\epsilon = \frac{\frac{\ell_{11}\epsilon}{r_1} \sin(\epsilon \ell n r_1) - \ell_{12} \cos(\epsilon \ell n r_1)}{\frac{\ell_{11}\epsilon}{r_1} \cos(\epsilon \ell n r_1) + \ell_{12} \sin(\epsilon \ell n r_1)} . \tag{124}$$

If $M_\epsilon$ is infinite, that particular term in the series from Equation (120) is replaced by

$$Q' \sin(\epsilon \ell n r) \xi_{\phi 1} \tag{125}$$

where

$$Q' = \frac{\int_{r_1}^{r_2} F_{\phi 1}(r) \sin(\epsilon \ell n r) \frac{dr}{r}}{\left[\frac{\ell n r}{2} - \frac{1}{4\epsilon} \sin(2\epsilon \ell n r)\right]_{r_1}^{r_2}} . \tag{126}$$

The arbitrary function in Equation (126) is defined in relation to the boundary condition at the surface as follows:

1 If $k_{12}$ is equal to unity

$$F_{\phi 1}(r) = \Upsilon \left[f_{\phi 1}(r)\right] . \tag{127}$$

2 If $k_{12}$ is equal to zero

$$F_{\phi 1}(r) = \frac{r}{K_o} f_{\phi 1}(r). \tag{128}$$

If $\epsilon$ is equal to zero, i.e., both $\ell_{12}$ and $\ell_{22}$ equal zero, an additional term must be added to Equation (120) as follows

1 If $(k_{12}, k_{22})$ equals (1,0) the term is

$$\frac{1}{\ell n \frac{r_2}{r_1}} \int_{r_1}^{r_2} F_{\phi 1}(r) \frac{dr}{r} . \tag{129}$$



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2 If $(k_{12}, k_{22})$ equals (1,1) the term is

$$\frac{\phi_2 - \phi}{\phi_2 \ \ell n \frac{r_2}{r_1}} \int_{r_1}^{r_2} F_{\phi 1}(r) \frac{dr}{r} \ . \tag{130}$$

3 If $(k_{12}, k_{22})$ equals (0,1) the term is

$$\frac{\phi - \phi_2}{\ell n \frac{r_2}{r_1}} \int_{r_1}^{r_2} F_{\phi 1}(r) \frac{dr}{r} \ . \tag{131}$$

4 If $(k_{12}, k_{22})$ equals (0, 0) the term is infinite.

4. $T_{\phi 2}(r, \phi)$

$$T_{\phi 2}(r, \phi) = \sum_{\epsilon} \left[ M_{\epsilon} \sin (\epsilon \ell nr) + F_{\epsilon} \cos (\epsilon \ell nr) \right] \xi_{\phi 2} \tag{132}$$

where in general

$$\xi_{\phi 2} = \frac{(-1)^{k_{12}} e^{-\epsilon(\phi_2 + \phi)} + e^{\epsilon(\phi - \phi_2)}}{\left[ (-1)^{k_{21}} (-1)^{k_{12}} e^{-2\epsilon\phi_2} + 1 \right] (\epsilon)^{k_{21}}} . \tag{133}$$

$$W_{\epsilon} = \frac{\int_{r_1}^{r_2} F_{\phi 2}(r) \left[ M_{\epsilon} \sin (\epsilon \ell nr) + F_{\epsilon} \cos (\epsilon \ell nr) \right] \frac{dr}{r}}{\int_{r_1}^{r_2} \left[ M_{\epsilon} \sin (\epsilon \ell nr) + F_{\epsilon} \cos (\epsilon \ell nr) \right] \frac{dr}{r}} \ . \tag{134}$$

If $M_{\epsilon}$ is infinite, that particular term in the series from Equation (132) is replaced by

$$W' \sin (\epsilon \ell nr) \ \xi_{\phi 2} \tag{135}$$



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where

$$W' = \frac{\displaystyle\int_{r_1}^{r_2} F_{\phi 2}(r) \sin(\epsilon \ell n r) \frac{dr}{r}}{\left[\frac{\ell n r}{2} - \frac{1}{4\epsilon} \sin(2\epsilon \ell n r)\right]_{r_1}^{r_2}} . \tag{136}$$

The arbitrary function in Equation (136) is defined in relation to the boundary condition at the surface as follows:

1 If $k_{22}$ is equal to unity

$$F_{\phi 2}(r) = \Upsilon\left[f_{\phi 2}(r)\right] \quad \text{(transform)} . \tag{137}$$

2 If $k_{22}$ is equal to zero

$$F_{\phi 2}(r) = \frac{r}{K_o} f_{\phi 2}(r) . \tag{138}$$

If $\epsilon$ is equal to zero an additional term must be added to Equation (132), as follows

1 If $(k_{12}, k_{22})$ equals (0, 1) the term is

$$\frac{1}{\ell n \frac{r_2}{r_1}} \int_{r_1}^{r_2} F_{\phi 2}(r) \frac{dr}{r} . \tag{139}$$

2 If $(k_{12}, k_{22})$ equals (1,1) the term is

$$\frac{\phi}{\phi_2 \ell n \frac{r_2}{r_1}} \int_{r_1}^{r_2} F_{\phi 2}(r) \frac{dr}{r} . \tag{140}$$



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3 If $(k_{12}, k_{22})$ equals (1, 0) the term is

$$\frac{\phi}{\ln \frac{r_2}{r_1}} \int_{r_1}^{r_2} F_{\phi 2}(r) \frac{dr}{r} \tag{141}$$

4 If $(k_{12}, k_{22})$ equals (0, 0) the term is infinite.

The preceding solution characteristically converges to zero at a corner if both adjacent faces have prescribed temperature as the boundary condition. Consequently, a dummy term equal to the sum of one-half of the adjacent transform temperatures prescribed at that corner must be added before $f(r,\phi)$ is used in Equation (80). The average of the prescribed temperatures, rather than either of the temperatures, is used to null out possible mismatch in defining arbitrary functions at a corner.



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# APPENDIX B

## FORTRAN LANGUAGE OF THE PROGRAM



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```
$IBFTC MAIN
      COMMON/CKOUT/NANS(10)
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/SPACE1/RR(400),PP(400)
      COMMON/TORIAN/STOOD,ONLY,SNOW
      COMMON/ZO1/JPRINT(35)
    1 FORMAT(3F10.0,A2,8X,2I10,A6)
    2 FORMAT(1H0, 5HTIME=E16.8,5X,2HR=E16.8,5X,4HPHI=E16.8,1X,
     1  9H(RADIANS))
    3 FORMAT(F10.0,50X,A6)
    5 FORMAT(70I1)
   10 FORMAT(1H0I5,1X,29HLATTICE POINT ANSWERS IN -R-    /(8E16.8))
   11 FORMAT(1H0I5,1X,31HLATTICE POINT ANSWERS IN -PHI-  /(8E16.8))
   20 FORMAT(16I5)
   30 FORMAT( 44H0ERROR IN INPUT, PROBLEM UNABLE TO CONTINUE  /
     1    52H0ALTERATION OF FIXED INPUT SHOULD BE DONE WITH CARE     )
C
      CALL ATHRUZ(ONN,6HON     )
      CALL ATHRUZ(ALASS,6HLAST T)
  100 CONTINUE
      OVER=0.0
      READ(5,20) NANS
C  NANS(1)= THE NUMBER OF GAMMAS USED IN MAIN
C  NANS(2)= THE NUMBER OF BETAS USED IN MAIN
C  NANS(3)= THE NUMBER OF INTERVALS FOR PHI INTEGRATION IN STATE
C  NANS(4)= THE NUMBER OF INTERVALS FOR R INTEGRATION IN STATE
C  NANS(5)= THE NUMBER OF LATTICE POINTS IN THE R DIRECTION IN STATE
C  NANS(6)= THE NUMBER OF LATTICE POINTS IN THE PHI DIRECTION IN STATE
C  NANS(7)= THE NUMBER OF GAMMAS / EPSILONS USED IN STATE
C  NANS(8)= THE NUMBER OF INTERVALS FOR PHI INTEGRATION IN MAIN
C  NANS(9)= THE NUMBER OF INTERVALS FOR R INTEGRATION IN MAIN
C
      IF(NANS(1).GT.30.OR.NANS(2).GT.30.OR.NANS(3).GT.200.OR.
     1 NANS(4).GT.200.OR.NANS(5).GT.7.OR.NANS(6).GT.25.OR.
     2 NANS(7).GT.30.OR.NANS(8).GT.200.OR.NANS(9).GT.200.OR.
     3 NANS(10).GT.3)     GO TO 9000
      READ(5,5) JPRINT
C JPRINT IS THE INTERMEDIATE PRINT INDICATOR FOR INDVIDUAL SUBROUTINES
      IF(JPRINT(1).EQ.1) WRITE(6,20) NANS
      CALL INPUT
      IF(STOOD.NE.SNOW) CALL STATE
      IF(STOOD.EQ.ONLY) GO TO 100
      CALL ARBCOE
  200 CONTINUE
C     READ(5,1) TIME,R,P,OPTION,N,M
      READ(5,1) TIME,R,P,OPTION,N,M,OVER
      IF(OPTION.EQ.ONN) GO TO 300
      P=P*.01745329
      WRITE(6,2) TIME,R,P
      CALL SERIES(TIME,R,P,NG,NB)
      IF(OVER.EQ.ALASS) GO TO 100
      GO TO 200
  300 CALL SIZE(RR,N,ARE(1),ARE(2))
      CALL SIZE(PP,M,PHI(1),PHI(2))
      IF(JPRINT(1).NE.1) GO TO 400
      WRITE(6,10) N,(RR(I),I=1,N)
      WRITE(6,11) M,(PP(I),I=1,M)
  400 DO 500 IP=1,M
      DO 500 IR=1,N
      WRITE(6,2) TIME,RR(IR),PP(IP)
  500 CALL SERIES(TIME,RR(IR),PP(IP))
      IF(OVER.EQ.ALASS) GO TO 100
      READ(5,3) TIME,OVER
      GO TO 400
 9000 WRITE(6,30)
      STOP
      END
```



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```
$IBFTC BES
      SUBROUTINE BES(A,X,Y,B,BP)
    2 FORMAT(1H ,4E20.8)
      DOUBLE PRECISION  P        , BJO      , BJ1      , ARG      , BR
      P=0.
      BJO=0.
      BJ1=0.
      ARG=0.
      PI=3.1415927
      INEG=0
      P=Y
      ARG =.5*A*X
      IF(ARG-.5)90,91,91
   90 IF(P)91,11,11
   91 CONTINUE
      IF(P)5,11,4
    5 INEG=-1
      I=P
      PINT=I
      NIT=1-I
      PF=1.+P-PINT
      P=PF
      GO TO 11
    4 IF(ABS (P-1.)-1.)11,11,6
    6 INEG=1
      I=P
      PINT=I
      NIT=I-1
      PF=P-PINT
      P=PF
   11 CONTINUE
      P=P
      ARG=.5*A*X
      IF(ARG+P)15,10,15
   10 B=1.
      BP=0.
      GO TO 50
   15 IF(ARG)17,16,17
   16 ER=0.
      GO TO 25
   17 IF(ARG-4.) 20,30,30
   20 ER=EXP (P*ALOG(ARG))
   25 AD=P+2.
      AP=P+1.
      A1=ARG**2
      AK=1.
      CALL GAMMA(AP,G)
      T1=ER/G
      B=T1
      BP=T1*P
      DO 28 I=1,100
      TN=-T1/AK*A1/AP
      B=B+TN
      TNP=TN*AD
      BP=BP+TNP
      IF(ABS (TN)-.1E-07)26,26,27
   26 IF(ABS (TNP)-.1E-07)29,29,27
   27 T1=TN
      AK=AK+1.
      AD=AD+2.
   28 AP=AP+1.
   29 BP=BP/X
      IF(INEG)60,50,60
   60 BJO=B
      BJO=BJO
      BJ1=(B*P-X*BP)/(A*X)
      BJ1=BJ1
      GO TO 50
```



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```
30 F1=SQRT (PI*ARG)
   IDER=0
   A1=-(A*X)**2
   R=4.*P**2
   F2=2.*ARG-(2.*P+1.)*PI/4.
31 CF2=COS (F2)
   SF2=SIN (F2)
   T1= (R-1.)*(R-9.)/(128.*A1)
   T1M=T1
   T2=(1.-R)/(8.*A*X)
   T2M=T2
   S1=1.+T1
   S2=T2
   Z=2.
   J=1
   DO 36 I=1,100
   ICNT=I
   GO TO(40,45,40),J
40 TM1=(R-(4.*Z-3.)**2)*(R-(4.*Z-1.)**2)/((2.*Z-1.)*Z*128.*A1)
   IF(ABS (TM1)-ABS (T1M))43,51,51
51 IF(ABS (TM1)-1.)43,41,41
41 GO TO(42,42,37),J
42 J=2
   GO TO 44
43 T1=T1*TM1
   T1M=TM1
   S1=S1+T1
   IF(ABS (T1)-.1E-07)41,41,44
44 GO TO(45,45,36),J
45 TM2=(R-(4.*Z-3.)**2)*(R-(4.*Z-5.)**2)/((2.*Z-1.)*(Z-1.)*128.*A1)
   IF(ABS (TM2)-ABS (T2M))48,49,49
49 IF(ABS (TM2)-1.)48,46,46
46 GO TO(47,37,47),J
47 J=3
   GO TO 36
48 T2=T2*TM2
   T2M=TM2
   S2=S2+T2
   IF(ABS (T2)-.1E-07)46,46,36
36 Z=Z+1.
37 IF(IDER)39,38,39
38 B=(CF2*S1+SF2*S2)/F1
   F2=F2-PI/2.
   R=4.*(P+1.)**2
   IDER=1
   GO TO 31
39 BD=(CF2*S1+SF2*S2)/F1
   BP=P*B/X-A*BD
   BJO=B
   BJO=BJO
   BJ1=BD
   BJ1=BJ1
50 IF(INEG)65,70,80
65 DO 61 I=1,NIT
   BR=P*BJO/ARG-BJ1
   BJ1=BJO
   BJO=BR
61 P=P-1.
   B=BJO
   BP=P*B/X-A*BJ1
   GO TO 70
80 P=P+1.
   DO 81 I=1,NIT
   BR=(P*BJ1)/ARG-BJO
   BJO=BJ1
   BJ1=BR
81 P=P+1.
   B=BJ1
```



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```
      BP=A*BJO-P*B/X
   70 RETURN
      END


$IBFTC POLYH
      SUBROUTINE POLYH(Z,X,Y,NX,NY,NTERMS)
C
C ACCEPTS NO MORE THAN 10 TERMS
C
      COMMON/Z01/JPRINT(35)
      DIMENSION A(2),V(2),E(20),COE(10)
      DIMENSION X(1),Y(1),Z(7,1)
    1 FORMAT(3F10.0)
    2 FORMAT(35HOPOLYNOMIAL EXCEEDS PROGRAM DESIGNS)
    3 FORMAT( 3H X=E16.8,5X,2HY=E16.8,5X, 2HZ(I1,1H,I2,2H)=E16.8)
    4 FORMAT(15X,1H(E16.8,5H)(R**F10.4,5H)(P**F10.4,1H))
 2000 FORMAT(1X,17HPOLYNOMIAL DATA, ,I2,1X,5HTERMS)
      WRITE(6,2000) NTERMS
      IF(NTERMS.LE.10) GO TO 8
      WRITE(6,2)
      STOP
    8 CONTINUE
      READ(5,1)(COE(I),E(2*I-1),E(2*I),I=1,NTERMS)
      WRITE(6,4) (COE(I),E(2*I-1),E(2*I),I=1,NTERMS)
      DO 200 IY=1,NY
      V(2)=Y(IY)
      DO 200 IX=1,NX
      V(1)=X(IX)
      Z(IX,IY)=0.
      DO 60 NT=1,NTERMS
      IF(COE(NT).EQ.0.) GO TO 60
      DO 40 L=1,2
      K=NT+L-1
      IF(E(K).EQ.0.) GO TO 35
      IF(V(L).EQ.0.) GO TO 60
C INDETERMINANT FORM  0**0   IS SET TO UNITY
      A(L)=V(L)**E(K)
      GO TO 40
   35 A(L)=1.0
   40 CONTINUE
      Z(IX,IY)=Z(IX,IY)+COE(NT)*A(1)*A(2)
   60 CONTINUE
      IF(JPRINT(3).EQ.1) WRITE(6,3) V,IX,IY,Z(IX,IY)
  200 CONTINUE
      RETURN
      END


$IBFTC VLINE
      SUBROUTINE VLINE(NX,NY,XIN,YIN,XARRAY,YARRAY,ZARRAY,L,M,ZOUT)
      DIMENSION XARRAY(1),YARRAY(1),ZARRAY(L,M),A(2)
C
      DO 100 I=2,NX
      K=I
  100 IF(XIN.LE.XARRAY(I)) GO TO 110
  110 DO 200 I=2,NY
      J=I
  200 IF(YIN.LE.YARRAY(I)) GO TO 210
  210 REG=(YIN-YARRAY(J-1))/(YARRAY(J)-YARRAY(J-1))
      DO 220 I=1,2
      A(I)=ZARRAY(K-1,J-1)+(ZARRAY(K-1,J)-ZARRAY(K-1,J-1))*REG
  220 K=K+1
      ZOUT=A(1)+(A(2)-A(1))*(XIN-XARRAY(K-3))/(XARRAY(K-2)-XARRAY(K-3))
      RETURN
      END
```



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```
$IBFTC LEFT
      SUBROUTINE LEFT(A,B,C,X,Y,Z)
      A=X
      B=Y
      C=Z
      RETURN
      END


$IBFTC SIZE
      SUBROUTINE SIZE(A,MAX,AMIN,AMAX)
      DIMENSION A(1)
      A(1)=AMIN
      A(MAX)=AMAX
      L=MAX-1
      AL=L
      DX=(AMAX-AMIN)/AL
      DO 10 I=2,L
      AI=I-1
   10 A(I)=AI*DX+A(1)
      RETURN
      END


$IBFTC WOOFER
      SUBROUTINE WOOFER
      COMMON/SPACE/YDUM(200),Y(200)
      COMMON/XCAB/XTRAN(50),YTRAN(50),MAXMAX
      COMMON/ZCCAB/AZERO
      COMMON/Z01/JPRINT(35)
    1 FORMAT(40HOTRANSFORMED THERMO PHYSICAL PROPERTIES      )
    2 FORMAT(10X, 9HXTRAN(I)=E16.8,5X, 9HYTRAN(I)=E16.8,5X,3HI= ,I5)
      IF(JPRINT(7).EQ.1) WRITE(6,1)
      YTRAN(1)=0.
      DO 40 I=2,MAXMAX
      Y(1)=YDUM(I-1)
      Y(50)=YDUM(I)
      XDIFF=XTRAN(I)-XTRAN(I-1)
      YDIFF=YDUM(I)-YDUM(I-1)
      DX=XDIFF/49.
      DO 30 J=2,49
      AJ=J-1
      X=XTRAN(I-1)+AJ*DX
      Y(J)=YDUM(I-1)+(YDIFF*(X-XTRAN(I-1)))/XDIFF
   30 CONTINUE
      CALL SIMP(50,DX,Y,YTRAN(I))
      YTRAN(I)=YTRAN(I)/AZERO +YTRAN(I-1)
   40 CONTINUE
      IF(JPRINT(7).EQ.1) WRITE(6,2) (XTRAN(I),YTRAN(I),I,I=1,MAXMAX)
      RETURN
      END


$IBFTC POLYGN
      SUBROUTINE POLYGN(X,Y,NY,NTERMS)
C
C ACCEPTS NO MORE THAN 10 TERMS
      COMMON/Z01/JPRINT(35)
      DIMENSION X(1),Y(1),COE(10),ORDER(10)
    1 FORMAT(3F10.0)
    2 FORMAT(35HOPOLYNOMIAL EXCEEDS PROGRAM DESIGNS)
    3 FORMAT(15X,1H(E16.8,5H)(X**F10.4,1H))
    4 FORMAT(18H POLYNOMIAL DATA, ,I2,1X,23HTERMS, X=DUMMY VARIABLE)
    5 FORMAT(3H X=E16.8,5X,2HY(I3,2H)=E16.8)
```



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```
C
      WRITE(6,4) NTERMS
      IF(NTERMS.LE.10) GO TO 7
      WRITE(6,2)
      STOP
    7 DO 8 I=1,NTERMS
      READ(5,1) COE(I),ORDER(I)
      WRITE(6,3) COE(I),ORDER(I)
    8 CONTINUE
      DO 50 I=1,NY
      Y(I)=0.
      DO 30 J=1,NTERMS
      IF(ORDER(J).EQ.0.) GO TO 10
C    INDETERMINANT FORM    0**0    IS SET TO UNITY
      IF(X(I).EQ.0.) GO TO 30
      A=X(I)**ORDER(J)
      GO TO 20
   10 A=1.0
   20 Y(I)=Y(I)+COE(J)*A
   30 CONTINUE
   50 IF(JPRINT(8).EQ.1) WRITE(6,5) X(I),I,Y(I)
      RETURN
      END


$IBFTC ENOM
      SUBROUTINE ENOM(IG,IB,EHOM)
      DIMENSION A(2)
      COMMON/CCAB/CSUB(30,30)
      COMMON/CKOUT/NANS(10)
      COMMON/ECAB/GAMMOS(30),BAYTOS(30,30)
      COMMON/GCAB/ARE(2),AREARE(200)
      COMMON/RCAB/AL(2,2),AK(2,2)
      COMMON/SPACE/DUM(200),STO2(200)
      COMMON/Z01/JPRINT(35)
    1 FORMAT(6H EHOM=E16.8,5X,12HCSUB(IG,IB)=E16.8)
C REF. P135 (11), THEORY OF BESSEL FUNCTIONS *G.N.WATSON* CAMBRIDGE 1958
C
      NR=NANS(9)
      REG1=1.0/ BAYTOS(IG,IB)**2
      DO 500 IR=1,NR
      CALL ABES(BAYTOS(IG,IB),AREARE(IR),GAMMOS(IG),U,UP,W,WP)
      IF(IR.GT.1) GO TO 40
      REG2=AREARE(IR)**2
      CSUB(IG,IB)=(-AL(1,1)*WP-AL(1,2)*W)/(AL(1,1)*UP+AL(1,2)*U)
      A(1)=.5*REG2*((1.-REG1*GAMMOS(IG)**2/REG2)*(CSUB(IG,IB)*U+W)**2
     1     +REG1*(CSUB(IG,IB)*UP+WP)**2)
   40 STO2(IR)=(CSUB(IG,IB)*U+W)*AREARE(IR)
      IF(IR.NE.NR) GO TO 500
      REG2=AREARE(IR)**2
      A(2)=.5*REG2*((1.-REG1*GAMMOS(IG)**2/REG2)*(CSUB(IG,IB)*U+W)**2
     1     +REG1*(CSUB(IG,IB)*UP+WP)**2)
  500 CONTINUE
      EHOM=A(2)-A(1)
      IF(JPRINT(9).EQ.1) WRITE(6,1) EHOM,CSUB(IG,IB)
      RETURN
      END


$IBFTC GAMEPS
      SUBROUTINE GAMEPS(ISEEK,MANY,VALUES)
C SELECT ISEEK=1 OR 2  FOR GAMMAS-SIGMAS,OR EPSILONS
C MANY= THE NUMBER OF EIGEN VALUES DESIRED
      DIMENSION VALUES(1)
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/RCAB/AL(2,2),AK(2,2)
```



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```
      COMMON/Z01/JPRINT(35)
    1 FORMAT(1X,32H*GAMEPS* ILLEGAL SELECTION ERROR,5X,7HISEEK= I5)
  500 FORMAT(1X,32H*GAMEPS* ERROR IN TAKING THE LOG)
  950 FORMAT(1X,17H*GAMEPS*  ISEEK= I3,3X, 5HI12= I5,3X, 5HI22=  I5,3X,
     1 6HMANY= I5,5X,5HREG1=E16.8,5X,7HARE(1)=E16.8/1X,7HARE(2)=E16.8,
     2  5X, 7HPHI(1)=E16.8,5X, 7HPHI(2)=E16.8,5X, 8HAL(1,2)=E16.8/ 1X,
     3  8HAL(2,2)=E16.8,5X, 8HAK(1,2)=E16.8,5X, 8HAK(2,2)=E16.8/ 1X,
     4  7HVALUES=/ (1X,6E20.8))
      PI=3.1415926
      IF(ISEEK.EQ.1.OR.ISEEK.EQ.2) GO TO 100
      WRITE(6,1) ISEEK
      STOP
  100 IF(ISEEK.NE.1) GO TO 2
C GAMMAS OR SIGMAS
      REG1=PHI(2)-PHI(1)
      REG2=AK(1,2)
      REG3=AK(2,2)
      GO TO 3
C EPSILONS
    2 REG1=ARE(2)/ARE(1)
      IF(REG1.LE.0.) GO TO 9997
      REG1=ALOG(REG1)
      REG2=AL(1,2)
      REG3=AL(2,2)
    3 I12=SIGN(ABS(REG2)+.005,REG2)
      I22=SIGN(ABS(REG3)+.005,REG3)
      IF(I12.NE.I22) GO TO 20
C SPECIAL CASE OF BOUNDARY CONDITIONS
C REG2=REG3=0.
      IF(I12.NE.0) GO TO 8
      VALUES(1)=0.
      DO 5 M=2,MANY
      AM=M-1
    5 VALUES(M)=AM*PI/REG1
      GO TO 900
C REG2=REG3
    8 DO 10 M=1,MANY
      AM=M
   10 VALUES(M)=AM*PI/REG1
      GO TO 900
   20 PIO2=1.5707963
C SPECIAL CASE OF BOUNDARY CONDITIONS
      DO 15 M=1,MANY
      AM=2*M-1
   15 VALUES(M)=PIO2*AM/REG1
      GO TO 900
 9997 WRITE(6,500)
      STOP
  900 IF(JPRINT(10).EQ.1) WRITE(6,950) ISEEK,I12,I22,
     1 MANY,REG1,ARE(1),ARE(2),PHI(1),PHI(2),AL(1,2),AL(2,2),AK(1,2),
     2 AK(2,2),(VALUES(I),I=1,MANY)
      RETURN
      END


$IBFTC GTIME
      SUBROUTINE GTIME(TIME,BETA,GSIMP)
      COMMON/COMTIM/XLAMDA(50),GLAMDA(50),NLAMDA
      COMMON/JOHN/GTHETA,SAMETM
      COMMON/SPACE/X(100),Y(100),SX(20),SY(20),DUM(160)
      COMMON/VCAB/ALPHO
    1 FORMAT(1H0,40H*GTIME* ERROR, TIME OUT OF BOUNDS,  MIN=E16.8,
     15X,4HMAX=E16.8,5X,5HTIME=E16.8)
C
      IF(TIME.LT.XLAMDA(1).OR.TIME.GT.XLAMDA(NLAMDA))GO TO 55
      ABQ=-ALPHO*BETA**2
      IF(TIME.EQ.SAMETM) GO TO 25
```



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```
      SAMETM=TIME
      DX=TIME/19.0
      DO 18 I=1,20
      AI=I-1
      SX(I)=DX*AI
   18 CALL CLINE(NLAMDA,SX(I),XLAMDA,GLAMDA,SY(I))
      GTHETA=SY(20)
C
   25 DO 30 I=2,20
      A=SY(I)*EXP(ABQ*(TIME-SX(I)))
   30 IF(ABS(A).GE..0001) GO TO 35
      I=20
   35 BEGIN=SX(I-1)
      DX=(TIME-BEGIN)/99.0
      DO 45 I=1,100
      AI=I-1
      X(I)=AI*DX+BEGIN
      CALL CLINE(NLAMDA,X(I),XLAMDA,GLAMDA,Y(I))
   45 Y(I)=Y(I)*EXP(ABQ*(TIME-X(I)))
C
      CALL SIMP(100,DX,Y,REG)
      GSIMP=-ABQ*REG
      GO TO 60
C
   55 WRITE(6,1) XLAMDA(1),XLAMDA(NLAMDA),TIME
      STOP
C
   60 RETURN
      END


$IBFTC WURZEL
      SUBROUTINE WURZEL
      COMMON/CKOUT/NANS(10)
      COMMON/ECAB/GAMMOS(30),BAYTOS(30,30)
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/ZOI/JPRINT(35)
    3 FORMAT( 14H0FIRST GUESS=     E16.8,5X, 10HINTERVAL=       E16.8)
    5 FORMAT(1H1)
      NB=NANS(2)
      NG=NANS(1)
C
      DO 4000 IG=1,NG
C INITIAL GUESS MUST NOT EXCEED FIRST MAX OR MIN
      G=GAMMOS(IG)
      IF(G.EQ.0.0) GO TO 350
      IF(IG.LT.3) GO TO 100
      X= 2.* BAYTOS(IG-1,1) - BAYTOS(IG-2,1)
      GO TO 370
  100 X= (.25*G) / ARE(1)
      GO TO 370
  350 X=1.0/(ARE(2)-ARE(1))
  370 DX=0.1/(ARE(2)-ARE(1))
C
C
      IF(JPRINT(12).GE.1) WRITE(6,3) X,DX
  400 CALL FFP(IG,X,Y,YP)
  500 IF(Y.EQ.0.0) GO TO 2100
      IF(SIGN(1.0,YP).NE.SIGN(1.0,Y)) GO TO 650
C
C  THE FIRST ROOT IS BEHIND YOUR GUESS
  610 CALL LEFT(XH,YH,SH,X,Y,YP)
      X=XH-.1*DX
      IF(X.LE.0.0) GO TO 640
      CALL FFP(IG,X,Y,YP)
      IF(SIGN(1.0,YH).NE.SIGN(1.0,Y)) GO TO 630
      IF(ABS(Y).GE.ABS(YH)) GO TO 640
```



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```
      GO TO 610
  630 CALL LEFT(XL,YL,SL,X,Y,YP)
      GO TO 1000
  640 CALL LEFT(XL,YL,SL,XH,YH,SH)
      DX=2.0*DX
      GO TO 660
C
C  THE FIRST ROOT IS FOWARD OF YOUR GUESS,(OR YOU ARE PAST THE FIRST
C  MAX OR MIN)
  650 CALL LEFT(XL,YL,SL,X,Y,YP)
  660 X=XL+DX
      CALL FFP(IG,X,Y,YP)
      IF(Y.EQ.0.) GO TO 2100
      IF(SIGN(1.0,YL).NE.SIGN(1.0,Y)) GO TO 800
      IF(ABS(Y).GE.ABS(YL)) DX=2.0*DX
      GO TO 650
  800 CALL LEFT(XH,YH,SH,X,Y,YP)
C
 1000 IB=1
      CALL BRIDGE(IB,XH,YH,SH,XL,YL,SL,IG)
      IF(NB.LE.1) GO TO 4000
      DX=2.5*DX
      GO TO 2120
C
C Y=0 TRAP
 2100 BAYTOS(IG,1)=X
      IF(NB.LE.1) GO TO 4000
      XH=X+DX
      CALL FFP(IG,XH,YH,SH)
C
C SECOND AND REMAINING ROOTS
 2120 DO 3980 IB=2,NB
      CALL LEFT(XL,YL,SL,XH,YH,SH)
      IF(IB.LT.3) GO TO 2200
      DX=BAYTOS(IG,IB-1)-BAYTOS(IG,IB-2)
      IF(DX.LE.0.) DX=.25
 2200 X=XL+DX
      CALL FFP(IG,X,Y,YP)
      IF(Y.EQ.0.) GO TO 3970
      IF(SIGN(1.0,YL).NE.SIGN(1.0,Y)) GO TO 2500
      CALL LEFT(XL,YL,SL,X,Y,YP)
      GO TO 2200
 2500 CONTINUE
      CALL LEFT(XH,YH,SH,X,Y,YP)
      CALL BRIDGE(IB,XH,YH,SH,XL,YL,SL,IG)
      GO TO 3980
C
C Y=0 TRAP
 3970 BAYTOS(IG,IB)=X
      IF(IB.GE.NB) GO TO 3980
      XH=X+DX
      CALL FFP(IG,XH,YH,SH)
 3980 CONTINUE
      IF(JPRINT(12).GE.1) WRITE(6,5)
 4000 CONTINUE
      RETURN
      END
```

```
$IBFTC BRIDGE
      SUBROUTINE BRIDGE(IB,XH,YH,SH,XL,YL,SL,IG)
      COMMON/ECAB/GAMMOS(30),BAYTOS(30,30)
      COMMON/Z01/JPRINT(35)
      DOUBLE PRECISION D1,D2,D3
    1 FORMAT(34H0ATTEMPT AT ROOT WAS NOT JUSTIFIED /20H XH,YH,SH,XL,YL,S
     1L, 6E16.8)
    3 FORMAT( 6H0BETA(,I2,1H,,I2,5H) =  E16.8/)
    4 FORMAT(23H0FINE SEARCH PHASE IG= I2,3X,4HIB= I2)
```



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```
C
C
C (XH,YH) AND (XL,YL) BOARDER A ROOT
C
      J=1
  100 CONTINUE
      IF(SIGN(1.0,YH).EQ.SIGN(1.0,YL)) GO TO 9000
C
C COURSE SEARCH PHASE
  200 SLINE=(YH-YL)/(XH-XL)
      A=XH-YH/SLINE
      CALL FFP(IG,A,B,BP)
C
C NEIGHBORHOOD CRITERION
      IF(SIGN(1.0,SL).EQ.SIGN(1.0,SH).AND.SIGN(1.0,SL).EQ.SIGN(1.0,BP))
     1                              J=J+1
      IF(J.GE.3) GO TO 7500
C
C ITERATIVE CONTRACTION
      DEL=(XH-XL)/3.0
      DO 220 I=1,3
      X=XL+DEL
      CALL FFP(IG,X,Y,YP)
      IF(Y.EQ.0.) GO TO 8050
      IF(SIGN(1.0,YL).NE.SIGN(1.0,Y)) GO TO 225
      CALL LEFT(XL,YL,SL,X,Y,YP)
  220 CONTINUE
      GO TO 100
  225 CONTINUE
      CALL LEFT(XH,YH,SH,X,Y,YP)
      GO TO 100
C
C FINE SEARCH PHASE
 7500 CONTINUE
      DO 8000 I=1,10
      IF(JPRINT(13).GE.2) WRITE(6,4) IG,IB
      IF(ABS(B).LE.1.E-6) GO TO 8040
      D1=A
      D2=B
      D3=BP
      D1=D1-D2/D3
      A=D1
      CALL FFP(IG,A,B,BP)
 8000 CONTINUE
      WRITE(6,4) IG,IB
C
 8040 BAYTOS(IG,IB)=A
      GO TO 8080
C
 8050 BAYTOS(IG,IB)=X
C
 8080 IF(JPRINT(13).GE.1) WRITE(6,3) IG,IB,BAYTOS(IG,IB)
      RETURN
 9000 WRITE(6,1) XH,YH,SH,XL,YL,SL
      STOP
      END
```

```
$IBFTC ABES
      SUBROUTINE ABES(B6,R6,G6,A6,AP6,Y6,YP6)
C     WRITE(6,10001) B6,R6,G6
C
C INPUT BETA,R,GAMMA,  OUTPUT J-GAMMA,J-PRIME-GAMMA,Y-GAMMA,Y-PRIME GAMM
C
      DIMENSION Y(2)
      DOUBLE PRECISION Y
      DOUBLE PRECISION D1,D2,D3,D4,D5
```



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Contrails

```
      DOUBLE PRECISION XO2S,PI,ONEOPI
      DOUBLE PRECISION PIO2
      DOUBLE PRECISION PIO4
      DOUBLE PRECISION WOOD
      DOUBLE PRECISION BRO2
      DOUBLE PRECISION XSQRD
      DOUBLE PRECISION BLOCKA
      DOUBLE PRECISION BLOCKB
      DOUBLE PRECISION BLOCKC
      DOUBLE PRECISION BLOCKD
      DOUBLE PRECISION BLOCKE
      DOUBLE PRECISION SUM1
      DOUBLE PRECISION SUM2
      DOUBLE PRECISION SUM3
      DOUBLE PRECISION BFACT
      DOUBLE PRECISION BFACT2
      DOUBLE PRECISION ADENOM
      DOUBLE PRECISION BDENOM
      DOUBLE PRECISION HANK1
      DOUBLE PRECISION HANK2
      DOUBLE PRECISION HANK3
      DOUBLE PRECISION HANK4
      DOUBLE PRECISION TERM
   11 FORMAT(1X, 6HGAMMA=1PE16.8,5X,28HNEGATIVE ORDERS ARE ILLEGAL ,
     1  7HIN ABES)
   12 FORMAT(1X,48HNEGATIVE AND ZERO ARGUMENTS ARE ILLEGAL IN ABES,, 5X,
     1  4HARG= 3E16.8)
10000 FORMAT(4H N= I5,5X,5HY(N)=E16.8,5X,E16.8)
10001 FORMAT(17H B,R,G,J,JP,Y,YP,            7E16.8)
C
C REF.A = APPLIED MATH FOR ENGR AND PHY *L.A.PIPES* MCGRAW-HILL 1958
C (NOTE DISREPANCY IN PIPES TEXT, LOG SHOULD BE LN)
C REF.B = THEORY OF BESSEL FUNCTIONS *G.N.WATSON* CAMBRIDGE UNIV. 1958
C
      BR=B6*R6
      IF(BR.LE.0.) GO TO 9001
      IF(G6.LT.0.) GO TO 9002
C
      PI=3.14159265D0
      ONEOPI=.31830989D0
      PIO2=1.57079633D0
      PIO4=.78539816D0
      BRO2=BR/2.0
      XO2S=-(BRO2*BRO2)
      TWOBR=2.*BR
      XSQRD=-(TWOBR*TWOBR)
      AAA=8.
      TOL=1.E-09
C
      CALL BES(B6,R6,G6,A6,AP6)
      NG=G6+.00001
      GMIX=NG
      FRACT=G6-GMIX
      IF(FRACT.LE..0001) GO TO 50
C
C GAMMA IS NOT AN INTEGER
      CALL BES(B6,R6,-G6,A5,AP5)
      D1=G6
      D2=A6
      D3=A5
      D4=DCOS(PI*D1)
      D5=DSIN(PI*D1)
C (4.2) P.350 REF.A
      Y6=(D4*D2-D3)/D5
      D2=AP6
      D3=AP5
      YP6=(D4*D2-D3)/D5
C     WRITE(6,10001) B6,R6,G6,A6,AP6,Y6,YP6
      RETURN
```



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```
   50 IF(BR.GE.AAA)GO TO 100
C
C GAMMA IS AN INTEGER, ARGUMENT IS LESS THAN THE CROSS OVER POINT
      IF(NG.NE.1) GO TO 55
      AONE=A6
      APONE=AP6
      GO TO 56
   55 CONTINUE
      CALL BES(B6,R6,1.,AONE,APONE)
   56 CONTINUE
      WOOD=2.0D0*(DLOG(BRO2)+.5772157D0)
      D1=AONE
      DO 85 N=1,2
      AN=N
      BLOCKA=D1*WOOD
      BLOCKB=0.0D0
      SUM1=BRO2
      IF(N.EQ.2) SUM1=BRO2*BRO2/2.0D0
      SUM2=1.0D0
      IF(N.EQ.2) SUM2=1.5D0
      DO 75 K=1,15
      NR=K-1
      R=NR
      IF(NR.LT.1)GO TO 60
      D2=R*(AN+R)
      SUM1=SUM1*(XO2S/D2)
      D3=1.0/(AN+R)+1.0/R
      SUM2=SUM2+D3
   60 SUM3=SUM1*SUM2
      BLOCKB=BLOCKB+SUM3
      XYZ=SUM3
      IF(ABS(XYZ).LE.TOL) GO TO 78
   75 CONTINUE
   78 BLOCKD=1.0D0/BRO2
      IF(N.EQ.2) BLOCKD=1.0D0/(BRO2*BRO2)+1.0D0
C (4.7) P.351 REF.A
      Y(N)=ONEDPI*(BLOCKA-BLOCKB-BLOCKD)
      D2=BR
      D3=APONE
      D4=B6
      D1=D1/D2-D3/D4
   85 CONTINUE
      GO TO 200
C
C GAMMA IS AN INTEGER, ARGUMENT IS GREATER THAN THE CROSS OVER POINT
  100 CONTINUE
      D5=BR
      BLOCKA=DSQRT((2.0D0*PI)/D5)/PI
      DO 115 N=1,2
      AN=N
      D2=AN
      D1=D5-PIO2*D2-PIO4
      BLOCKB=DSIN(D1)
      BLOCKC=DCOS(D1)
      BLOCKD=0.0D0
      BLOCKE=0.0D0
      BFACT=1.0D0
      ADENOM=1.0D0
      J=1
C     J=CONVERGENCE INDICATOR
      BFACT2=1.0D0
      BDENOM=TWOBR
      IF(N.EQ.2)GO TO 101
      HANK1=.886226925D0
      HANK3=1.32934039D0
      HANK4=1.77245385D0
      GO TO 102
  101 HANK1=1.32934039D0
      HANK4=.886226925D0
      HANK3=2.5*HANK1
```



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```
  102 HANK2=HANK1
      DO 110 K=1,7
      M=K-1
      AM=M
      D1=2*M
      GO TO (104,105,104),J
  104 IF(M.EQ.0)GO TO 103
      HANK1=HANK1*(D2+D1-.5D0)*(D2+D1-1.5D0)
      BFACT=BFACT*D1*(D1-1.0D0)
      HANK2=HANK2/((D2-D1+.5D0)*(D2-D1+1.5D0))
      ADENOM=ADENOM*XSQRD
  103 TERM=(HANK1/(BFACT*HANK2))/ADENOM
      BLOCKD=BLOCKD+TERM
      XYZ=TERM
      IF(ABS(XYZ).LE.TOL) J=J+1
  105 GO TO (109,109,110,110),J
  109 IF(M.EQ.0)GO TO 108
      HANK3=HANK3*(D2+D1+.5D0)*(D2*D1-.5D0)
      BFACT2=BFACT2*(D1+1.0D0)*D1
      HANK4=HANK4/((D2-D1-.5D0)*(D2-D1+.5D0))
      BDENOM=BDENOM*XSQRD
  108 TERMA=(HANK3/(BFACT2*HANK4))/BDENOM
      BLOCKE=BLOCKE+TERMA
      XYZ=TERMA
      IF(ABS(XYZ).LE.TOL) J=J+2
  110 IF(J.GT.3) GO TO 114
C (5) P.199 REF.B
  114 Y(N)=BLOCKA*(BLOCKB*BLOCKD+BLOCKC*BLOCKE)
C     WRITE(6,10000) N,Y(N)
  115 CONTINUE
C
C RECURRENCE FORMULAS
  200 IF(NG.NE.0)GO TO 210
C
C GAMMA ZERO
      D1=B6
      YP6=-Y(1)*D1
      D1=BR
      Y6=(2.0D0*Y(1))/D1-Y(2)
      RETURN
C
  210 IF(NG.NE.1)GO TO 220
C
C GAMMA ONE
      Y6=Y(1)
      D1=GMIX
      D2=R6
      D3=B6
      YP6=(D1*Y(1))/D2-D3*Y(2)
      RETURN
C
  220 IF(NG.NE.2)GO TO 230
C
C GAMMA TWO
      D3=Y(1)
      D4=Y(2)
      Y6=Y(2)
      GO TO 270
C
  230 D4=Y(2)
      D2=Y(1)
      DO 240 N=3,NG
      AN=N-1
      D5= BR
      D1=AN
      D1=(2.0D0*D1)/D5
      D3=D4
      D4=D1*D3-D2
C     WRITE(6,10000) N,D4,D3
```



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```
      D2=D3
  240 CONTINUE
      Y6=D4
C
  270 CONTINUE
      D1=B6
      D2=GMIX
      D5=R6
      YP6=D1*D3-(D2*D4)/D5
C     WRITE(6,10001) B6,R6,G6,A6,AP6,Y6,YP6
      RETURN
C
 9001 CONTINUE
      WRITE(6,12) BR,B6,R6
      STOP
 9002 CONTINUE
      WRITE(6,11) G6
      STOP
      END
```

```
$IBFTC GAMMA
      SUBROUTINE GAMMA(X,Y)
      DIMENSION A(10)
      A(1) = .83333333E-01
      A(2) =-.27777778E-02
      A(3) = .79365079E-03
      A(4) =-.59523810E-03
      A(5) = .84175084E-03
      A(6) =-.19175269E-02
      A(7) = .64102564E-02
      A(8) =-.29550654E-01
      A(9) = .17964437E 00
      A(10)=-.13924322E 01
      IF(X)10,13,30
   10 K000FX=X
      XINT=K000FX
      XF=XINT-X
      AX=X
      XM=1.0
      J=3-K000FX
      DO 15 I=1,J
      XM=XM*AX
   15 AX=AX+1.0
      IF(ABS(XM)-1.0E-30)11,11,29
   11 IE=K000FX/2
      IF(K000FX-2*IE)13,12,13
   12 Y=-1.0E 30
      GO TO 50
   13 Y=1.0E 30
      GO TO 50
   29 AX=3.0-XF
      GO TO 38
   30 IF(X-1.0)34,37,36
   34 IF(X-1.0E-30)13,13,37
   37 AX=X+2.0
      XM=X*(X+1.0)
      GO TO 38
   36 IF(X-2.0)39,39,14
   39 AX=X+1.0
      XM=X
      GO TO 38
   14 AX=X
      XM=1.0
      IF(X-30.0)38,13,13
   38 C=1.0/AX**2
      T1=A(1)/AX
```



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```
      F=T1
      DO 35 I=2,10
      TN=T1*A(I)/A(I-1)*C
      IF(ABS(TN)-ABS(T1))32,32,33
   32 F=F+TN
      T1=TN
      IF(ABS(TN)-.2E-08)33,33,35
   35 CONTINUE
   33 EX=(AX-.5)*ALOG(AX)-AX+F+.91893853
      Y=EXP(EX)
      Y=Y/XM
   50 RETURN
      END


$IBFTC AMSAFC
      FUNCTION AMSAFC(IG,V)
      COMMON/ECAB/DUM(30),EPILON(30),DUMY(870)
      COMMON/WCAB/AM(30),AF(30)
      COMMON/Z01/JPRINT(35)
    1 FORMAT(1X,7HAMSAFC=E16.8)
C
      A=ALOG(V)
C
      A=A*EPILON(IG)
      IF(AM(IG).EQ.0.) GO TO 110
      B=AM(IG)*SIN(A)
      GO TO 120
  110 B=0.0
  120 IF(AF(IG).EQ.0.) GO TO 130
      C=COS(A)
      GO TO 140
  130 C=0.0
  140 AMSAFC=B+C
      IF(JPRINT(16).EQ.1) WRITE(6,1) AMSAFC
 8800 RETURN
      END


$IBFTC SERIES
      SUBROUTINE SERIES(TIME,R,P)
C
      DOUBLE PRECISION SERIA,SERIB,TERMA,TERMB
      COMMON/ACAB/ARBA(30,30),ARBG(30,30)
      COMMON/CCAB/CSUB(30,30)
      COMMON/CKOUT/NANS(10)
      COMMON/ECAB/GAMMOS(30),BAYTOS(30,30)
      COMMON/JOHN/GTHETA,SAMETM
      COMMON/OCAB/STEADY(7,25),XR(7),YP(25),KR,KP
      COMMON/VCAB/ALPHO
      COMMON/Z01/JPRINT(35)
    3 FORMAT(3X,6HTERM A,10X,6HTERM B,10X,8HSERIES A,8X,8HSERIES B,
     1      8X,5HGAMMA,11X,4HBETA,12X,6HGTHETA,10X,5HGSIMP)
    4 FORMAT(1X,8E16.8)
    5 FORMAT(9X,12HTEMPERATURE=E16.8,10X,E16.8)
C
      NG=NANS(1)
      NB=NANS(2)
C
      IF(JPRINT(17).GE.2) WRITE(6,3)
C
      SERIA=0.0D0
      SERIB=0.0D0
      DO 400 IG=1,NG
      DO 400 IB=1,NB
      CALL ABES(BAYTOS(IG,IB),R,GAMMOS(IG),U,UP,W,WP)
      B2=CSUB(IG,IB)*U+W
```



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```
      REG1=B2*BSNFCS(IG,P)
C
      TERMA=ARBA(IG,IB)*REG1*EXP(-ALPHO*BAYTOS(IG,IB)**2*TIME)
      CALL GTIME(TIME,BAYTOS(IG,IB),GSIMP)
      TERMB=          ARBG(IG,IB)*REG1*(-GTHETA+GSIMP)
C
      SERIA=SERIA+TERMA
      SERIB=SERIB+TERMB
C
      IF(JPRINT(17).GE.2)
     1  WRITE(6,4) TERMA,TERMB,SERIA,SERIB,GAMMOS(IG),BAYTOS(IG,IB),
     2  GTHETA,GSIMP
  400 CONTINUE
C
      CALL VLINE(KR,KP,R,P,XR,YP,STEADY,7,25,S)
      HEAT=SERIA+SERIB +GTHETA*S
      IF(JPRINT(17).GE.1) WRITE(6,5) HEAT,S
      HEAT=ATRAN(HEAT)
  500 WRITE(6,5) HEAT
      RETURN
      END



$IBFTC STATE
      SUBROUTINE STATE
      DOUBLE PRECISION T1,T2,T3,T4
      COMMON/ACAB/FARE1(200),FARE2(200),FPHI1(200),FPHI2(200),DUME(1000)
      COMMON/CCAB/STO1(25),STO2(7),STO3(7),STO4(7),STO5(25),STO6(25),
     1 DUMY(804)
      COMMON/CKOUT/NANS(10)
      COMMON/ECAB/GAMMOS(30),EPILON(30),DUM(870)
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/OCAB/STEADY(7,25),XR(7),YP(25),KR,KP
      COMMON/RCAB/AL(2,2),AK(2,2)
      COMMON/Z01/JPRINT(35)
      EQUIVALENCE (KR,NR)
      EQUIVALENCE (KP,NP)
   13 FORMAT(6H0    T1,13X,2HT2,14X,2HT3,14X,2HT4,14X,13HSTEADY(IR,IP),
     1    10X,2HIR,8X,2HIP)
   14 FORMAT(1X,5E16.8,2I10)
C
      NG=NANS(7)
      NP=NANS(6)
      NR=NANS(5)
      DO 100 IP=1,NP
      DO 100 IR=1,NR
      STEADY(IR,IP)=0.0
  100 CONTINUE
C
      N=1
      CALL GAMEPS(N,NANS(7),GAMMOS)
      N=2
      CALL GAMEPS(N,NANS(7),EPILON)
C
      DO 4000 IG=1,NG
      CALL COECOE(IG,CP,CL,CQ,CW)
      DO 300 IP=1,NP
      STO1(IP)=BSNFCS(IG,YP(IP))
      N=3
      STO5(IP)=XI(IG,YP(IP),N)
      N=4
      STO6(IP)=XI(IG,YP(IP),N)
  300 CONTINUE
C
      DO 400 IR=1,NR
      N=1
      STO2(IR)=XI(IG,XR(IR),N)
      N=2
```



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```
      STO3(IR)=XI(IG,XR(IR),N)
      STO4(IR)=AMSAFC(IG,XR(IR))
  400 CONTINUE
C
      IF(IG.EQ.NG.OR.JPRINT(18).GE.1) WRITE(6,13)
      DO 3000 IP=1,NP
      DO 3000 IR=1,NR
      T1=CP*STO1(IP)*STO2(IR)
      T2=CL*STO1(IP)*STO3(IR)
      T3=CQ*STO4(IR)*STO5(IP)
      T4=CW*STO4(IR)*STO6(IP)
      STEADY(IR,IP)=STEADY(IR,IP)+T1+T2+T3+T4
      IF(IG.EQ.NG.OR.JPRINT(18).GE.1) WRITE(6,14) T1,T2,T3,T4,
     1   STEADY(IR,IP),IR,IP
 3000 CONTINUE
 4000 CONTINUE
C
      NPI=NANS(3)
      NRI=NANS(4)
      IF(AL(1,2)+AK(1,2).EQ.2.0) STEADY(1,1)=STEADY(1,1)+.5*FARE1(1)+
     1                                                  .5*FPHI1(1)
      IF(AL(1,2)+AK(2,2).EQ.2.0) STEADY(1,NP)=STEADY(1,NP)+.5*FARE1(NPI)
     1                                                  +.5*FPHI2(1)
      IF(AL(2,2)+AK(1,2).EQ.2.0) STEADY(NR,1)=STEADY(NR,1)+.5*FARE2(1)
     1                                                  +.5*FPHI1(NRI)
      IF(AL(2,2)+AK(2,2).EQ.2.0) STEADY(NR,NP)=STEADY(NR,NP)+
     1                            .5*FARE2(NPI)+.5*FPHI2(NRI)
C
      RETURN
      END


$IBFTC XI
      FUNCTION XI(IG,V,NBOUND)
      COMMON/ECAB/GAMMOS(30),EPILON(30),DUM(870)
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/RCAB/AL(2,2),AK(2,2)   ,
      COMMON/UTILTY/R1OR2,R2OR1,AR1OR2,AR2OR1,AR1,AR2
      COMMON/Z01/JPRINT(35)
      DOUBLE PRECISION D1,D2,D3,D4
    1 FORMAT(49H1JOB TERMINATED, DO NOT PUT FLUX AT ALL BOUNDRIES
     1  (8E16.8))
    2 FORMAT(1X,3HXI=E16.8,5X, 8HBOUNDARY,2X,I1)
C
      GO TO(100,2000,3000,4000),NBOUND
C
C
C AT BOUNDARY R1
C
  100 CONTINUE
      IF(GAMMOS(IG).NE.0.) GO TO 1000
      IF(AL(1,2).EQ.0.) GO TO  200
      IF(AL(2,2).EQ.0.) GO TO 8800
      XI=ALOG(V/ARE(2))/AR1OR2
      GO TO 8900
  200 IF(AL(2,2).EQ.0.) GO TO 9000
      XI=ARE(1)*ALOG(V/ARE(2))
      GO TO 8900
C
 1000 CONTINUE
      D1=(V/ARE(1))**(-GAMMOS(IG))
      D2=(R1OR2**GAMMOS(IG))*((V/ARE(2))**GAMMOS(IG))
      D4= R1OR2**(2.0*GAMMOS(IG))
      IF(AL(1,1).EQ.0.) GO TO 1050
      D3=-(GAMMOS(IG)/ARE(1))
      D4=D4*(-D3)
      GO TO 1100
 1050 CONTINUE
```



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```
      D3=1.0D0
 1100 CONTINUE
      IF(AL(2,2).EQ.0.0) GO TO 8700
      D3=-D3
      D1=-D1
      GO TO 8700
C
C AT BOUNDARY R2
C
 2000 CONTINUE
      IF(GAMMOS(IG).NE.0.) GO TO 2800
      IF(AL(1,2).EQ.0.) GO TO 2030
      IF(AL(2,2).EQ.0.) GO TO 2020
      XI=ALOG(V/ARE(1))/AR2OR1
      GO TO 8900
 2020 CONTINUE
      XI=ARE(2)*ALOG(V/ARE(1))
      GO TO 8900
 2030 CONTINUE
      IF(AL(2,2).EQ.0.) GO TO 9000
      GO TO 8800
C
 2800 CONTINUE
      D1=(R1OR2**GAMMOS(IG))*(V/ARE(1))**(-GAMMOS(IG))
      D2=(V/ARE(2))**GAMMOS(IG)
      D3=R1OR2**(2.*GAMMOS(IG))
      IF(AL(2,1).EQ.0.) GO TO 2835
      D3=-(GAMMOS(IG)/ARE(2))*D3
      D4=GAMMOS(IG)/ARE(2)
      GO TO 2880
 2835 CONTINUE
      D4=1.0D0
 2880 CONTINUE
      IF(AL(1,2).EQ.0.) GO TO 8700
      D1=-D1
      D3=-D3
      GO TO 8700
C
C AT BOUNDRY P1
C
 3000 CONTINUE
      IF(EPILON(IG).NE.0.) GO TO 3705
      IF(AK(1,2).EQ.0.) GO TO 3010
      IF(AK(2,2).EQ.0.) GO TO 8800
      XI=PHI(2)-V
      GO TO 8900
 3010 CONTINUE
      IF(AL(2,2).EQ.0.) GO TO 9000
      XI=V-PHI(2)
      GO TO 8900
C
 3705 CONTINUE
      D1=EXP(-EPILON(IG)*V)
      D2=EXP(-EPILON(IG)*(2.*PHI(2)-V))
      D4=EXP(-2.*EPILON(IG)*PHI(2))
      IF(AK(1,1).NE.0.) GO TO 3815
      D3=1.0D0
      GO TO 3860
 3815 CONTINUE
      D3=    -EPILON(IG)
      D4=D4*EPILON(IG)
 3860 CONTINUE
      IF(AK(2,2).EQ.0.) GO TO 8700
      D1=-D1
      D3=-D3
      GO TO 8700
C
C AT BOUNDARY P2
C
```



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```
 4000 CONTINUE
      IF(EPILON(IG).NE.0.) GO TO 5000
      IF(AK(1,2).EQ.0.) GO TO 4025
      IF(AK(2,2).EQ.0.) GO TO 4016
      XI=V/PHI(2)
      GO TO 8900
 4016 CONTINUE
      XI=V
      GO TO 8900
 4025 IF(AK(2,2).EQ.0.) GO TO 9000
      GO TO 8800
C
 5000 CONTINUE
      D1=EXP(-EPILON(IG)*(PHI(2)+V))
      D2=EXP(EPILON(IG)*(V-PHI(2)))
      D3= EXP(-2.*EPILON(IG)*PHI(2))
      IF(AK(2,1).EQ.0.) GO TO 5046
      D3=-D3*EPILON(IG)
      D4=EPILON(IG)
      GO TO 6000
 5046 CONTINUE
      D4=1.0D0
 6000 CONTINUE
      IF(AK(1,2).EQ.0.) GO TO 8700
      D1=-D1
      D3=-D3
C
 8700 XI=(D1+D2)/(D3+D4)
      GO TO 8900
C
 8800 XI=1.0
C
 8900 CONTINUE
      IF(JPRINT(19).EQ.1) WRITE(6,2) XI,NBOUND
      RETURN
C
C
 9000 CONTINUE
      WRITE(6,1) AL,AK
      STOP
      END


$IBFTC FFP
      SUBROUTINE FFP(IG,B,F,FP)
      DOUBLE PRECISION D1,D2
      DOUBLE PRECISION T,TP
      DIMENSION T(4),TP(4),C(4),P(4)
      COMMON/ECAB/GAMMOS(30),BAYTOS(30,30)
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/RCAB/AL(2,2),AK(2,2)
      COMMON/ZO1/JPRINT(35)
   14 FORMAT(1X,2HK=I2,5X,5HC(K)=E16.8,5X,5HP(K)=E16.8,5X,5HT(K)=E16.8,
     1 5X,6HTP(K)=E16.8,E16.8)
   20 FORMAT(3H X=E16.8,3X,2HY=E16.8,3X,3HYP=E16.8,3X,6HGAMMA I2)
      BR=B*ARE(1)
      REG1= ((GAMMOS(IG)**2)/BR)   -BR
      A=AL(1,1)
      D=AL(1,2)
      R=ARE(1)
      CALL ABES(B,ARE(1),GAMMOS(IG),C(1),P(1),C(2),P(2))
      CALL ABES(B,ARE(2),GAMMOS(IG),C(3),P(3),C(4),P(4))
      DO 13 K=1,4
      D1=A*P(K)
      D2=D*C(K)
      T(K)=D1+D2
      D1=A*C(K)*REG1
      D2=R*D*P(K)/B
```



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```
      TP(K)=D1+D2
      IF(K.EQ.1) GO TO 13
      A=AL(2,1)
      D=AL(2,2)
      BR=B*ARE(2)
      REG1= ((GAMMOS(IG)**2)/BR)  -BR
      R=ARE(2)
   13 IF(JPRINT(20).GE.2) WRITE(6,14) K,C(K),P(K),T(K),TP(K),REG1
      F=T(1)*T(4)-T(2)*T(3)
      FP=T(1)*TP(4)-T(2)*TP(3)+TP(1)*T(4)-TP(2)*T(3)
      IF(JPRINT(20).GE.1) WRITE(6,20) B,F,FP,IG
      RETURN
      END


$IBFTC ARBCOE
      SUBROUTINE ARBCOE
      COMMON/ACAB/ARBA(30,30),ARBG(30,30)
      COMMON/CKOUT/NANS(10)
      COMMON/ECAB/GAMMOS(30),BAYTOS(30,30)
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/OCAB/STEADY(7,25),XR(7),YP(25),KR,KP
      COMMON/PCAB/DZERO(7,25),XRD(7),YPD(25),LR,LP
      COMMON/SPACE/STO1(200),STO2(200)
      COMMON/SPACE1/A1(200),A2(200),A3(200),A4(200)
      COMMON/ZO1/JPRINT(35)
    1 FORMAT( 8H A(I,J)=E16.8,5X, 7HG(I,J)=E16.8,5X,3HI= I3,3X,3HJ= I3)
C
      NG=NANS(1)
      NB=NANS(2)
      NR=NANS(9)
      NP=NANS(8)
C
      CALL SIZE(PHIPHI,NP,PHI(1),PHI(2))
      CALL SIZE(AREARE,NR,ARE(1),ARE(2))
C
      DXR=AREARE(2)-AREARE(1)
      DXP=PHIPHI(2)-PHIPHI(1)
C
      CALL GAMEPS(1,NANS(1),GAMMOS)
      CALL WURZEL
C
      DO 500 IG=1,NG
      CALL DENOM(IG,DENOM)
C
      DO 91 I=1,NP
      STO1(I)=BSNFCS(IG,PHIPHI(I))
   91 CONTINUE
C
      DO 500 IB=1,NB
      CALL ENOM(IG,IB,EHOM)
C     STO2 IS FILLED IN ENOM
      DO 300 IP=1,NP
      DO 200 IR=1,NR
      CALL VLINE(LR,LP,AREARE(IR),PHIPHI(IP),XRD,YPD,DZERO,7,25,D)
      D=TRAN(D)
      CALL VLINE(KR,KP,AREARE(IR),PHIPHI(IP),XR,YP,STEADY,7,25,S)
      REG=STO1(IP)*STO2(IR)
      A1(IR)=D*REG
      A2(IR)=S*REG
  200 CONTINUE
      CALL SIMP(NR,DXR,A1,A)
      A3(IP)=A
      CALL SIMP(NR,DXR,A2,B)
      A4(IP)=B
  300 CONTINUE
      CALL SIMP(NP,DXP,A3,V1)
      CALL SIMP(NP,DXP,A4,V2)
```



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```
C
      REG=EHOM*DEHOM
      ARBA(IG,IB)=V1/REG
      ARBG(IG,IB)=V2/REG
C
      IF(JPRINT(21).EQ.1) WRITE(6,1) ARBA(IG,IB),ARBG(IG,IB),IG,IB
  500 CONTINUE
      RETURN
      END
```

```
$IBFTC TRAN
      FUNCTION TRAN(V)
      COMMON/XCAB/XTRAN(50),YTRAN(50),MAXMAX
      COMMON/YCAB/BEE,SEE/ZBCAB/ICASE
      COMMON/Z01/JPRINT(35)
   50 FORMAT(1X,17H*TRANSFORM ERROR*)
   51 FORMAT(1X,14H*TRAN*  CASE= I3,3X,4HBEE=E16.8,5X,4HSEE=E16.8,5X,
     1 2HV=E16.8 / 1X,2HQ=E16.8,5X,5HREG1=E16.8,5X,5HREG2=E16.8,5X,
     2 5HREG3=E16.8 / 1X,5HREG4=E16.8,5X,5HREG5=E16.8,5X,5HTRAN=E16.8)
C SET ICASE EQUAL TO THE CASE NUMBER OF THE DESIRED TRANSFORM
      REG1=0.
      REG2=0.
      REG3=0.
      REG4=0.
      REG5=0.
      Q=0.
      GO TO (1,2,3,4,5,6,7),ICASE
    1 TRAN=1.0/BEE-EXP(-BEE*V)/BEE
C CASE 1
      GO TO 100
    2 IF(SEE.LE.0.) GO TO 9997
C CASE 2
      REG1=4.*SEE-BEE*BEE
      Q=REG1
      REG2=2.*SEE*V
      IF(REG1)11,12,13
   11 REG1=SQRT(-REG1)
      REG3=REG2/(BEE-REG1)
      REG4=REG2/(BEE+REG1)
      REG5=(REG3+1.)/(REG4+1.)
      IF(REG5.LE.0.) GO TO 9997
      TRAN=ALOG(REG5)/REG1
      GO TO 100
   12 REG3=REG2+BEE
      TRAN=2./BEE-2./REG3
      GO TO 100
   13 REG1=SQRT(REG1)
      REG3=REG2+BEE
      REG4=ATAN2(REG3,REG1)
      REG5=ATAN2(BEE,REG1)
      TRAN=2.*(REG4-REG5)/REG1
      GO TO 100
    3 REG1=1.+BEE*V
C CASE 3
      IF(REG1.LE.0..OR.BEE.EQ.0.) GO TO 9997
      TRAN=ALOG(REG1)/BEE
      GO TO 100
    4 TRAN=V+BEE*V*V/2.
C CASE 4
      GO TO 100
    5 CONTINUE
C CASE 5
    6 TRAN=V
C CASE 6
      GO TO 100
    7 CALL CLINE(MAXMAX,V,XTRAN(1),YTRAN(1),TRAN)
```



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```
C CASE 7
      GO TO 100
 9997 WRITE(6,50)
      STOP
  100 IF(JPRINT(22).EQ.1) WRITE(6,51) ICASE,BEE,SEE,V,
     1 Q,REG1,REG2,REG3,REG4,REG5,TRAN
      RETURN
      END


$IBFTC BSNFCS
      FUNCTION BSNFCS(IG,P)
      COMMON/ECAB/GAMMOS(30),DUM(900)
      COMMON/ZECAB/BSUB(30),FSUB(30)
      COMMON/Z01/JPRINT(35)
    1 FORMAT(1X,7HBSNFCS=E16.8)
      A=GAMMOS(IG)*P
      IF(BSUB(IG).EQ.0.) GO TO 100
      R1=BSUB(IG)*SIN(A)
      GO TO 200
  100 R1=0.
  200 IF(FSUB(IG).EQ.0.) GO TO 300
      R2=FSUB(IG)*COS(A)
      GO TO 400
  300 R2=0.
  400 BSNFCS=R1+R2
      IF(JPRINT(23).EQ.1) WRITE(6,1) BSNFCS
      RETURN
      END


$IBFTC COECOE
      SUBROUTINE COECOE(IG,CP,CL,CQ,CW)
      COMMON/ACAB/FARE1(200),FARE2(200),FPHI1(200),FPHI2(200),A1(200),
     1 A2(200),DUMY(600)
      COMMON/CKOUT/NANS(10)
      COMMON/ECAB/GAMMOS(30),EPILON(30),DUM(870)
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/Z01/JPRINT(35)
    1 FORMAT(1X,6HCP(I)=E16.8,5X,6HCL(I)=E16.8,5X,6HCQ(I)=E16.8,5X,
     1 6HCW(I)=E16.8,5X,3HI= I5)
C
      CALL DENOM(IG,DEHOM)
      NTERVL=NANS(3)
      DO 500 INN=1,NTERVL
      B=BSNFCS(IG,PHIPHI(INN))
      A1(INN)=FARE1(INN)*B
      A2(INN)=FARE2(INN)*B
  500 CONTINUE
      CALL SIMP(NTERVL,PHIPHI(2)-PHIPHI(1),A2,AREA2)
      CALL SIMP(NTERVL,PHIPHI(2)-PHIPHI(1),A1,AREA1)
      CP=AREA1/DEHOM
      CL=AREA2/DEHOM
C
C
      NTERVL=NANS(4)
      CALL CENOM(IG,CEHOM)
      DO 1000 INN=1,NTERVL
      B=AMSAFC(IG,AREARE(INN))/AREARE(INN)
      A1(INN)=FPHI1(INN)*B
      A2(INN)=FPHI2(INN)*B
 1000 CONTINUE
      CALL SIMP(NTERVL,AREARE(2)-AREARE(1),A1,AREA1)
      CALL SIMP(NTERVL,AREARE(2)-AREARE(1),A2,AREA2)
      CQ=AREA1/CEHOM
      CW=AREA2/CEHOM
```



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```
C
      IF(JPRINT(24).EQ.1) WRITE(6,1) CP,CL,CQ,CW,IG
      RETURN
      END


$IBFTC CENOM
      SUBROUTINE CENOM(IG,CEHOM)
      DIMENSION A(2),X(2)
      COMMON/ECAB/DUM(30),EPILON(30),DUMY(870)
      COMMON/GCAB/ARE(2),AREARE(200)
      COMMON/RCAB/AL(2,2),AK(2,2)
      COMMON/UTILTY/R1OR2,R2OR1,AR1OR2,AR2OR1,AR1,AR2
      COMMON/WCAB/AM(30),AF(30)
      COMMON/Z01/JPRINT(35)
    1 FORMAT( 7H CEHOM=E16.8,5X,7HAM(IG)=E16.8,5X,4HIG= I5)
C
      T1=0.0
      T2=0.0
      T3=0.0
C
      IF(EPILON(IG).EQ.0.) GO TO 7600
      REG1=SIN(EPILON(IG)*AR1)
      REG2=COS(EPILON(IG)*AR1)
      UNDER=AL(1,1)*EPILON(IG)*REG2/ARE(1)+AL(1,2)*REG1
      IF(UNDER.EQ.0.) GO TO 4000
      AM(IG)=(AL(1,1)*EPILON(IG)*REG1/ARE(1) -AL(1,2)*REG2)/UNDER
      AF(IG)=1.0
      GO TO 4065
 4000 CONTINUE
      AM(IG)=1.0
      AF(IG)=0.0
C
 4065 CONTINUE
      X(1)=EPILON(IG)*AR1
      X(2)=EPILON(IG)*AR2
C
      IF(AM(IG).EQ.0.) GO TO 4800
      IF(AF(IG).EQ.0.) GO TO 5200
C
      DO 4300 I=1,2
      A(I)=.5*SIN(X(I))**2
 4300 CONTINUE
      T3=(2.0*AM(IG)/EPILON(IG))*(A(2)-A(1))
C
 4800 CONTINUE
      DO 4850 I=1,2
      A(I)=X(I)/2.0+SIN(2.0*X(I))/4.0
 4850 CONTINUE
      T2=(A(2)-A(1))/EPILON(IG)
C
      IF(AM(IG).EQ.0.) GO TO 6123
 5200 CONTINUE
      DO 5256 I=1,2
      A(I)=X(I)/2.0-SIN(2.0*X(I))/4.0
 5256 CONTINUE
      T1=(AM(IG)**2)*(A(2)-A(1))/EPILON(IG)
C
 6123 CONTINUE
      CEHOM=T1+T2+T3
      GO TO 8653
C
 7600 CONTINUE
      AM(IG)=0.0
      AF(IG)=1.0
      CEHOM=AR2OR1
C
 8653 CONTINUE
      IF(JPRINT(25).EQ.1) WRITE(6,1) CEHOM,AM(IG),IG
      RETURN
      END
```



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```
$IBFTC ATRAN
      FUNCTION ATRAN(T)
      COMMON/XCAB/XTRAN(50),YTRAN(50),MAXMAX
      COMMON/YCAB/BEE,SEE/ZBCAB/ICASE
      COMMON/ZO1/JPRINT(35)
   50 FORMAT(1X,17H*INVERSION ERROR*)
   51 FORMAT(1X,14H*ATRAN* CASE= I3,3X,4HBEE=E16.8,5X,4HSEE=E16.8,5X,
     1 2HT=E16.8 / 1X,2HQ=E16.8,5X,5HREG1=E16.8,5X,5HREG2=E16.8,5X,
     2 5HREG3=E16.8 / 1X,5HREG4=E16.8,30X,6HATRAN=E16.8)
C SET ICASE EQUAL TO THE CASE NUMBER OF THE DESIRED INVERSION
      REG1=0.
      REG2=0.
      REG3=0.
      REG4=0.
      Q=0.
      GO TO (1,2,3,4,5,6,7),ICASE
    1 REG1=1.-T*BEE
C CASE 1
      IF(REG1.LE.0.) GO TO 9997
      ATRAN=-ALOG(REG1)/BEE
      GO TO 100
    2 IF(SEE.EQ.0.) GO TO 9997
C CASE 2
      REG1=4.*SEE-BEE*BEE
      Q=REG1
      IF(REG1)11,12,13
   11 REG1=SQRT(-REG1)
      REG2=EXP(REG1*T)*(BEE-REG1)/(BEE+REG1)
      ATRAN=((BEE+REG1)*REG2-BEE+REG1)/(2.*SEE*(1.-REG2))
      GO TO 100
   12 ATRAN=BEE*T/(4.*SEE/BEE-2.*SEE*T)
      GO TO 100
   13 REG1=SQRT(REG1)
      REG2=BEE/REG1
      REG3=REG1*T/2.
      REG3=SIN(REG3)/COS(REG3)
      REG4=REG3*REG2
      TWOSEE=2.*SEE
      ATRAN=REG1*(REG2+REG3)/(TWOSEE-TWOSEE*REG4)-BEE/TWOSEE
      GO TO 100
    3 IF(BEE.EQ.0.) GO TO 9997
C CASE 3
      ATRAN=(EXP(BEE*T)-1.)/BEE
      GO TO 100
    4 REG1=1.+2.*BEE*T
C CASE 4
      IF(REG1.LT.0.) GO TO 9997
      ATRAN=(-1.+SQRT(REG1))/BEE
      GO TO 100
    5 CONTINUE
C CASE 5
    6 ATRAN=T
C CASE 6
      GO TO 100
    7 CALL CLINE(MAXMAX,T,YTRAN(1),XTRAN(1),ATRAN)
C CASE 7
      IF(ATRAN.GT.XTRAN(MAXMAX)) GO TO 9997
      GO TO 100
 9997 WRITE(6,50)
      STOP
  100 IF(JPRINT(26).EQ.1) WRITE(6,51) ICASE,BEE,SEE,T,
     1 Q,REG1,REG2,REG3,REG4,ATRAN
      RETURN
      END
```



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```
$IBFTC DENOM
      SUBROUTINE DENOM(IG,DEHOM)
C
      DIMENSION A(2),X(2)
      COMMON/ECAB/GAMMOS(30),DUM(900)
      COMMON/HCAB/PHI(2),PHIPHI(200)
      COMMON/RCAB/AL(2,2),AK(2,2)
      COMMON/ZECAB/BSUB(30),FSUB(30)
      COMMON/Z01/JPRINT(35)
    1 FORMAT(7H DEHOM=E16.8,5X,9HBSUB(IG)=E16.8,5X,4HIG= I5)
C
      IF(GAMMOS(IG).EQ.0.) GO TO 8011
C
      T3=0.0
      T2=0.0
      T1=0.0
C
      X(1)=GAMMOS(IG)*PHI(1)
      X(2)=GAMMOS(IG)*PHI(2)
      REG1=COS(X(1))
      REG2=SIN(X(1))
      UNDER=AK(1,1)*GAMMOS(IG)*REG1+AK(1,2)*REG2
      IF(UNDER.EQ.0.) GO TO 106
      BSUB(IG)=(-AK(1,2)*REG1+AK(1,1)*GAMMOS(IG)*REG2)/UNDER
      FSUB(IG)=1.0
      GO TO 150
  106 CONTINUE
      BSUB(IG)=1.0
      FSUB(IG)=0.0
  150 CONTINUE
      IF(BSUB(IG).EQ.0.) GO TO 3000
      IF(FSUB(IG).EQ.0.) GO TO 1000
      DO 400 I=1,2
      A(I)=BSUB(IG)*(SIN(X(I)))**2/GAMMOS(IG)
  400 CONTINUE
      T2=A(2)-A(1)
 1000 CONTINUE
      B=BSUB(IG)**2/2.0
      G=2.0*GAMMOS(IG)
      DO 1002 I=1,2
      A(I)=B*PHI(I)-B*SIN(2.0*X(I))/G
 1002 CONTINUE
      T1=A(2)-A(1)
      IF(FSUB(IG).EQ.0.) GO TO 7000
 3000 CONTINUE
      G=4.0*GAMMOS(IG)
      DO 3800 I=1,2
      A(I)=PHI(I)/2.0 +SIN(2.0*X(I))/G
 3800 CONTINUE
      T3=A(2)-A(1)
 7000 CONTINUE
      DEHOM=T1+T2+T3
      GO TO 8900
C
 8011 CONTINUE
      BSUB(IG)=0.0
      FSUB(IG)=1.0
      DEHOM=PHI(2)-PHI(1)
C
 8900 CONTINUE
      IF(JPRINT(27).EQ.1) WRITE(6,1) DEHOM,BSUB(IG),IG
      RETURN
      END
```



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```
$IBFTC INPUT
      SUBROUTINE INPUT
C
      COMMON/ACAB/FARE1(200),FARE2(200),FPHI1(200),FPHI2(200),DUME(1000)
      COMMON/CKOUT/NANS(10)
      COMMON/COMTIM/XLAMDA(50),GLAMDA(50),NLAMDA
      COMMON/GCAB/ARE(2),AREARE(200)/HCAB/PHI(2),PHIPHI(200)
      COMMON/OCAB/STEADY(7,25),XR(7),YP(25),KR,KP
      COMMON/PCAB/DZERO(7,25),XRD(7),YPD(25),LR,LP
      COMMON/RCAB/AL(2,2),AK(2,2)
      COMMON/SPACE/DUM(200),R(200)
      COMMON/TORIAN/STOOD,ONLY,SNOW
      COMMON/UTILTY/R1OR2,R2OR1,AR1OR2,AR2OR1,AR1,AR2
      COMMON/VCAB/ALPHO
      COMMON/XCAB/XTRAN(50),YTRAN(50),MAXMAX
      COMMON/YCAB/BEE,SEE
      COMMON/ZBCAB/ICASE
      COMMON/ZCCAB/AZERO
      EQUIVALENCE (R,P)
C
    1 FORMAT(6F10.0)
    2 FORMAT(A5,5X,I10,4F10.0)
    3 FORMAT(A5,5X,2I10,A6)
    4 FORMAT(A5,5X,A5,5X,I10,A2)
    5 FORMAT(12A6)
    6 FORMAT(1H112A6)
    8 FORMAT( 6H0INPUT/ 9H GEOMETRY/1X,3HR1=E16.8,5X,3HR2=E16.8/ 1X,
     1  3HP1=E16.8,5X,3HP2=E16.8, 5X,9H(RADIANS))
    9 FORMAT(27H0THERMO PHYSICAL PROPERTIES)
   10 FORMAT(1X,11HPOINT DATA,3X,I2,12H POINT PAIRS  / (4E20.8))
   11 FORMAT(1X, 6HALPHA=E16.8,5X, 6HKZERO=E16.8)
   12 FORMAT(1X,26HEVALUATE POLYNOMIAL FROM  E16.8,3X,5HTO   E16.8)
   13 FORMAT(29H TRANSFORM ACCORDING TO CASE I1,3H,B=E16.8,3X,2HC=E16.8)
   14 FORMAT(34H0INITIAL TEMPERATURE DISTRIBUTION )
   15 FORMAT(1X,12HPOINT DATA, I2,1HX,I2,18H, VALUES OF F(R,P))
   16 FORMAT(3X,5E20.8)
 1000 FORMAT(44H0ERROR IN INPUT, PROBLEM UNABLE TO CONTINUE ,10X,I12)
 4000 FORMAT(1H0,20HBOUNDRY CONDITIONS   )
 4001 FORMAT(1X,7HAT R1, ,A6,3X,4HL11=F10.2,3X,4HL12=F10.2)
 4002 FORMAT(1X,7HAT R2, ,A6,3X,4HL21=F10.2,3X,4HL22=F10.2)
 4003 FORMAT(1X,7HAT P1, ,A6,3X,4HK11=F10.2,3X,4HK12=F10.2)
 4004 FORMAT(1X,7HAT P2, ,A6,3X,4HK21=F10.2,3X,4HK22=F10.2)
 4005 FORMAT(1X,11HPOINT DATA,3X,I2,43H EVENLY SPACED ORDINATES BETWEEN
     1R1 AND R2    / (6E20.8))
 4006 FORMAT(1X,11HPOINT DATA,3X,I2,43H EVENLY SPACED ORDINATES BETWEEN
     1P1 AND P2    / (6E20.8))
 4007 FORMAT(14H0TIME FUNCTION)
 4009 FORMAT(44H0STEADY STATE SOLUTION IS BEING INPUT        )
 4010 FORMAT(1H1)
10000 FORMAT(7F10.0)
      CALL ATHRUZ(PLY,6HPOLY  )
      CALL ATHRUZ(CACE,6HCASE  )
      CALL ATHRUZ(PONT,6HPOINT )
      CALL ATHRUZ(SNOW,6HNO    )
      CALL ATHRUZ(PHLUX,6HFLUX  )
      CALL ATHRUZ(FEVER,6HTEMP  )
      CALL ATHRUZ(ONLY,6HON    )
      CALL ATHRUZ(SSOP,6HTRANSF)
      J=0
C TITLE
      READ(5,5) (R(I),I=1,12)
      WRITE(6,6) (R(I),I=1,12)
C
C GEOMETRY, AREARE
      READ(5,1) ARE
      R1OR2=ARE(1)/ARE(2)
      R2OR1=ARE(2)/ARE(1)
      AR1OR2=ALOG(R1OR2)
      AR2OR1=ALOG(R2OR1)
```



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```
      AR1=ALOG(ARE(1))
      AR2=ALOG(ARE(2))
C PHIPHI
      READ(5,1) PHI
C PHI IS INPUT IN DEGREES AND FRACTIONS
      DO 18 I=1,2
   18 PHI(I)=PHI(I)*.01745329
      WRITE(6,8) ARE,PHI
      IF(ARE(1).LE.0.0.OR.ARE(2).LE.0.0.OR.PHI(1).NE.0.0.OR.PHI(2).LE.0.
     10) GO TO 900
C
C THERMO PHYSICAL PROPERTIES, TRAN
      WRITE(6,9)
      READ(5,2) WHAT,N,A,B,C,D
      IF(N.GT.50) GO TO 19
      IF(WHAT.EQ.PONT) GO TO 20
      IF(WHAT.EQ.PLY) GO TO 30
      IF(WHAT.EQ.CACE) GO TO 40
   19 J=1
      GO TO 900
   20 ALPHO=A
      AZERO=B
      READ(5,1) (XTRAN(I),DUM(I),I=1,N)
      MAXMAX=N
      WRITE(6,10) MAXMAX,(XTRAN(I),DUM(I),I=1,MAXMAX)
      WRITE(6,11) A,B
      GO TO 35
   30 ALPHO=C
      AZERO=D
      MAXMAX=50
      CALL SIZE(XTRAN,MAXMAX,A,B)
      CALL POLYGN(XTRAN,DUM,MAXMAX,N)
      WRITE(6,12) A,B
      WRITE(6,11) C,D
   35 ICASE=7
C
      CALL WOOFER
C
      GO TO 60
   40 ICASE=N
      BEE=A
      SEE=B
      ALPHO=C
      AZERO=D
      WRITE(6,13) N,BEE,SEE
      IF(ICASE.GT.7) GO TO 19
      WRITE(6,11) C,D
C
C INITIAL TEMPERATURE DISTRIBUTION, DZERO(IR,IP)
   60 WRITE(6,14)
      READ(5,3) WHAT,N,M
      IF(WHAT.EQ.PLY) GO TO 90
      IF(N.GT.7.OR.M.GT.25) GO TO 68
      IF(WHAT.EQ.PONT) GO TO 70
   68 J=2
      GO TO 900
   70 CALL SIZE(XRD,N,ARE(1),ARE(2))
      CALL SIZE(YPD,M,PHI(1),PHI(2))
      WRITE(6,15) N,M
      DO 75 J=1,M
      L=M-J+1
      READ(5,10000) (DZERO(I,L),I=1,N)
   75 WRITE(6,16) (DZERO(I,L),I=1,N)
      LR=N
      LP=M
      GO TO 95
   90 NTERMS=N
      LR=7
      LP=25
```



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```
      CALL SIZE(XRD,LR,ARE(1),ARE(2))
      CALL SIZE(YPD,LP,PHI(1),PHI(2))
      CALL POLYH(DZERO,XRD,YPD,LR,LP,NTERMS)
C
C BOUNDARY CONDITIONS, FARE1(PHI),AL(1,1),AL(1,2)
   95 WRITE(6,4000)
  100 READ(5,4) X,WHAT,N,STOOD
      IF(X.EQ.FEVER) GO TO 110
      IF(X.EQ.PHLUX) GO TO 120
      J=3
      GO TO 900
  110 AL(1,1)=0.
      AL(1,2)=1.
      GO TO 130
  120 AL(1,1)=-1
      AL(1,2)=0.
  130 WRITE(6,4001) X,AL(1,1),AL(1,2)
      IF(STOOD.EQ.SNOW) GO TO 160
      IF(N.GT.200) GO TO 135
      CALL SIZE(PHIPHI,NANS(3),PHI(1),PHI(2))
      CALL SIZE(AREARE,NANS(4),ARE(1),ARE(2))
      CALL SIZE(XR,NANS(5),ARE(1),ARE(2))
      CALL SIZE(YP,NANS(6),PHI(1),PHI(2))
      NP=NANS(3)
      NR=NANS(4)
      IF(WHAT.EQ.PONT) GO TO 140
      IF(WHAT.EQ.PLY) GO TO 150
  135 J=4
      GO TO 900
  140 READ(5,10000) (DUM(I),I=1,N)
      WRITE(6,4006) N,(DUM(I),I=1,N)
      CALL SIZE(P,N,PHI(1),PHI(2))
      DO 145 IP=1,NP
  145 CALL CLINE(N,PHIPHI(IP),P,DUM,FARE1(IP))
      GO TO 155
  150 CALL POLYGN(PHIPHI,FARE1,NANS(3),N)
  155 IF(AL(1,2).EQ.1.0) GO TO 157
      DO 156 IP=1,NP
      FARE1(IP)= FARE1(IP)/AZERO
  156 CONTINUE
      GO TO 160
  157 CONTINUE
      DO 158 IP=1,NP
      FARE1(IP)=TRAN(FARE1(IP))
  158 CONTINUE
C
C FARE2(PHI),AL(2,1),AL(2,2)
  160 READ(5,4) X,WHAT,N
      IF(X.EQ.FEVER) GO TO 170
      IF(X.EQ.PHLUX) GO TO 180
      J=5
      GO TO 900
  170 AL(2,1)=0.
      AL(2,2)=1.
      GO TO 190
  180 AL(2,1)=1.
      AL(2,2)=0.
  190 WRITE(6,4002) X,AL(2,1),AL(2,2)
      IF(STOOD.EQ.SNOW) GO TO 220
      IF(N.GT.200) GO TO 195
      IF(WHAT.EQ.PONT) GO TO 200
      IF(WHAT.EQ.PLY) GO TO 210
  195 J=6
      GO TO 900
  200 READ(5,10000) (DUM(I),I=1,N)
      WRITE(6,4006) N,(DUM(I),I=1,N)
      CALL SIZE(P,N,PHI(1),PHI(2))
      DO 205 IP=1,NP
  205 CALL CLINE(N,PHIPHI(IP),P,DUM,FARE2(IP))
```



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```
      GO TO 211
  210 CALL POLYGN(PHIPHI,FARE2,NP,N)
  211 IF(AL(2,2).EQ.1.0) GO TO 213
      DO 212 IP=1,NP
      FARE2(IP)=FARE2(IP)/AZERO
  212 CONTINUE
      GO TO 220
  213 CONTINUE
      DO 214 IP=1,NP
      FARE2(IP)=TRAN(FARE2(IP))
  214 CONTINUE
C
C FPHI1(ARE),AK(1,1),AK(1,2)
  220 READ(5,4) X,WHAT,N
      IF(X.EQ.FEVER) GO TO 230
      IF(X.EQ.PHLUX) GO TO 240
      J=7
      GO TO 900
  230 AK(1,1)=0.
      AK(1,2)=1.
      GO TO 250
  240 AK(1,1)=-1
      AK(1,2)=0.
  250 WRITE(6,4003) X,AK(1,1),AK(1,2)
      IF(STOOD.EQ.SNOW) GO TO 280
      IF(N.GT.200) GO TO 255
      IF(WHAT.EQ.PONT) GO TO 260
      IF(WHAT.EQ.PLY) GO TO 270
  255 J=8
      GO TO 900
  260 READ(5,10000) (DUM(I),I=1,N)
      WRITE(6,4005) N,(DUM(I),I=1,N)
      CALL SIZE(R,N,ARE(1),ARE(2))
      DO 265 IR=1,NR
  265 CALL CLINE(N,AREARE(IR),R,DUM,FPHI1(IR))
      GO TO 271
  270 CALL POLYGN(AREARE,FPHI1,NR,N)
  271 IF(AK(1,2).EQ.1.0) GO TO 273
      DO 272 IR=1,NR
      FPHI1(IR)= AREARE(IR)*FPHI1(IR)/AZERO
  272 CONTINUE
      GO TO 280
  273 CONTINUE
      DO 274 IR=1,NR
      FPHI1(IR)=TRAN(FPHI1(IR))
  274 CONTINUE
C
C FPHI2(ARE),AK(2,1),AK(2,2)
  280 READ(5,4) X,WHAT,N
      IF(X.EQ.FEVER) GO TO 290
      IF(X.EQ.PHLUX) GO TO 300
      J=9
      GO TO 900
  290 AK(2,1)=0.
      AK(2,2)=1.
      GO TO 310
  300 AK(2,1)=1.
      AK(2,2)=0.
  310 WRITE(6,4004) X,AK(2,1),AK(2,2)
      IF(STOOD.EQ.SNOW) GO TO 360
      IF(N.GT.200) GO TO 315
      IF(WHAT.EQ.PONT) GO TO 320
      IF(WHAT.EQ.PLY) GO TO 330
  315 J=10
      GO TO 900
  320 READ(5,10000) (DUM(I),I=1,N)
      WRITE(6,4005) N,(DUM(I),I=1,N)
      CALL SIZE(R,N,ARE(1),ARE(2))
      DO 325 IR=1,NR
```



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```
  325 CALL CLINE(N,AREARE(IR),R,DUM,FPHI2(IR))
      GO TO 331
  330 CALL POLYGN(AREARE,FPHI2,NR,N)
  331 IF(AK(2,2).EQ.1.0) GO TO 333
      DO 332 IR=1,NR
      FPHI2(IR)= AREARE(IR)*FPHI2(IR)/AZERO
  332 CONTINUE
      GO TO 400
  333 CONTINUE
      DO 334 IR=1,NR
      FPHI2(IR)=TRAN(FPHI2(IR))
  334 CONTINUE
      GO TO 400
C
C STEADY STATE INPUT OPTION
  360 WRITE(6,4009)
      READ(5,3) WHAT,N,M,X
      IF(WHAT.EQ.PLY) GO TO 380
      IF(N.GT.7.OR.M.GT.25) GO TO 365
      IF(WHAT.EQ.PONT) GO TO 370
  365 J=11
      GO TO 900
  370 WRITE(6,15) N,M
      DO 375 J=1,M
      L=M-J+1
      READ(5,10000) (STEADY(I,L),I=1,N)
  375 WRITE(6,16) (STEADY(I,L),I=1,N)
      CALL SIZE(XR,N,ARE(1),ARE(2))
      CALL SIZE(YP,M,PHI(1),PHI(2))
      KR=N
      KP=M
      GO TO 390
  380 KR=7
      KP=25
      CALL SIZE(XR,KR,ARE(1),ARE(2))
      CALL SIZE(YP,KP,PHI(1),PHI(2))
      NTERMS=N
      CALL POLYH(STEADY,XR,YR,KR,KP,NTERMS)
  390 IF(X.NE.SSOP) GO TO 400
      DO 395 IR=1,KR
      DO 395 IP=1,KP
  395 STEADY(IR,IP)=TRAN(STEADY(IR,IP))
C
C TIME FUNCTION
  400 READ(5,2) WHAT,N,B
      A=0.0
      WRITE(6,4007)
      IF(WHAT.EQ.PONT) GO TO 420
      IF(WHAT.EQ.PLY) GO TO 430
  410 J=12
      GO TO 900
  420 NLAMDA=N
      IF(NLAMDA.GT.50) GO TO 410
      READ(5,1) (XLAMDA(I),GLAMDA(I),I=1,N)
      WRITE(6,10) N,(XLAMDA(I),GLAMDA(I),I=1,N)
      GO TO 440
  430 NLAMDA=50
      CALL SIZE(XLAMDA,NLAMDA,A,B)
      CALL POLYGN(XLAMDA,GLAMDA,NLAMDA,N)
      WRITE(6,12) A,B
  440 WRITE(6,4010)
      RETURN
C ERRORS
  900 WRITE(6,1000) J
      STOP
      RETURN
      END
```



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```
$IBFTC SIMP
      SUBROUTINE SIMP(NT,DX,V,AREA)
C     EQUALLY SPACED POINTS
C     EVEN NO. OF INTERVALS---SIMPSONS RULE
C     ODD NO.--- FIRST N-1 INTERVALS-SIMPSONS LAST INTERVAL-TRAPEZOID
      DIMENSION  V(1)
C     DIMENSION  V(1)
      AREA=0.0
      IF(NT-1)47,50,47
   47 IF((NT/2)*2-NT)28,29,28
   29 NOD=1
      N1=NT-1
      GO TO 32
   28 N1=NT
      NOD=0
   32 AREA1=0.0
      IF(N1-1)35,35,445
  445 J=N1-1
      DO 33 I=2,J,2
   33 AREA=V(I)+AREA
      IF(N1-3)446,446,7447
 7447 J=N1-2
      DO 34 I=3,J,2
   34 AREA1=V(I)+AREA1
  446 AREA =.33333333*(V(1)+V(N1)+2.0*AREA1+4.0*AREA)
      IF(NOD)35,36,35
   35 AREA= AREA +.5*(V(N1)+V(N1+1))
   36 AREA= DX*AREA
   50 RETURN
      END
```

```
$IBFTC ATHRUZ
       SUBROUTINE ATHRUZ(A, B)
       DIMENSION A(1), B(1)
       DO 25 I = 1, 22
       A(I) = B(I)
       C = COMPL(B(I+1))
       IF( C .EQ. 0.)      RETURN
   25 CONTINUE
       RETURN
       END
```

```
$IBFTC CLINE
      SUBROUTINE CLINE(N,X,A,B,Y)
      DIMENSION A(1),B(1)
C     DIMENSION A(1),B(1)
      IF (N) 100,100,101
  101 DO 102 J=2,N
      I=J
      IF (X-A(J)) 103,103,102
  102 CONTINUE
  103 Y=B(I-1) + (X-A(I-1)) *(B(I)-B(I-1))/(A(I)-A(I-1))
      RETURN
  100 Y=0.0
      RETURN
      END
```



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# APPENDIX C

# DESCRIPTION OF SUBROUTINES



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# APPENDIX C

## DESCRIPTION OF SUBROUTINE

Independent intermediate print is available for many of the subroutines and subprograms. Each routine has been assigned a sequence number. This sequence number corresponds to a column number of card B in the input data (see Section IIIB). Hence, a particular column i of card B controls the intermediate print of the routine or subprogram which has been assigned the sequence number i. A description of this resulting intermediate print for a particular routine is given in the discussion of that routine.

The routines and their associated sequence numbers are as follows:

| Routine | Sequence No. | Routine | Sequence No. |
|---|---|---|---|
| MAIN | 1 | SERIES | 17 |
| BES | 2 | STATE | 18 |
| PØLYH | 3 | XI (function) | 19 |
| VLINE | 4 | FFP | 20 |
| LEFT | 5 | ARBCØE | 21 |
| SIZE | 6 | TRAN (function | 22 |
| WØØFER | 7 | BSNFCS (function) | 23 |
| PØLYGN | 8 | CØECØE | 24 |
| ENØM | 9 | CENØM | 25 |
| GAMEPS | 10 | ATRAN (function) | 26 |
| GTIME | 11 | DENØM | 27 |
| WURZEL | 12 | INPUT | 28 |
| BRIDGE | 13 | | |
| ABES | 14 | ATHRUZ ) | |
| GAMMA | 15 | SIMP ) System routines | |
| AMSAFC (function) | 16 | CLINE ) | |

1. MAIN (Sequence Number 1)

MAIN is the control point for the program. It sets up the order in which the program will execute.

MAIN reads in the fixed input and in the times and locations at which answers are requested. The remainder of the input data is handled in subroutine INPUT.



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The fixed input in MAIN consists of the two leading cards A and B of the input data. Card A contains the mathematical options for the program which are stored in an array called NANS. Card B contains the intermediate print options for the program which are stored in an array called JPRINT. The following are the mathematical options:

NANS(1) = the number of gammas used for a solution (30 max)
NANS(2) = the number of betas used for a solution (30 max)
NANS(3) = the number of intervals for phi integration in STATE (200 max)
NANS(4) = the number of intervals for r integration in STATE (200 max)
NANS(5) = the number of lattice points in the r direction in STATE (7 max)
NANS(6) = the number of lattice points in the phi direction in STATE (25 max)
NANS(7) = the number of gammas/epsilons used in STATE (30 max)
NANS(8) = the number of intervals for phi integration in ARBCØE (200 max)
NANS(9) = the number of intervals for r integration in ARBCØE (200 max)

Each subscript of NANS is a multiple of a five column span on card A. Thus, the number of $\beta$'s, NANS(2), would appear in columns 6 through 10, and it must be right adjusted in its field with no decimal point, as must all of the entries on cards A and B.

Intermediate print for MAIN consists of a printout of the image of card A, and the lattice network in r and $\phi$, if applicable.

A detailed flow diagram of MAIN is given in Appendix D.

2. BES (Sequence Number 2)

Subroutine BES is one of the two subroutines used in a previous program (Reference 4). This subroutine calculates Bessel functions of the form $J_\gamma(\beta r)$ and $J'_\gamma(\beta r)$. The arguments for this routine are A, X, Y, B, and BP, which correspond to $\beta$, r, $\gamma$, $J_\gamma(\beta r)$ and $J'_\gamma(\beta r)$.

This subroutine uses series solution and recursion formulas similar to those used in ABES. (See References 6 and 7).

3. PØLYH (Sequence Number 3)

Subroutine PØLYH is a utility subroutine of subroutine Input. PØLYH reads in an evaluates a polynominal of ten or less terms where each term is of the form:

$$AX^BY^C.$$



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The restrictions on A, B, and C are those imposed by the computer in raising a number to a power. In general it should be noted that a computer error usually occurs when

$$|B| \ \ln X > 88.02$$

or

$$|C| \ \ln Y > 88.02$$

(A negative X or Y also causes a computer error, but the X's and Y's used in this program correspond to r and $\phi$, and these cannot be negative without voiding the solution).

PØLYH automatically prints out the polynominal that was input.

The argument list for PØLYH is Z, X, Y, NX, NY, and NTERMS where:

Z = the name of the array where the evaluations of the polynomial will be stored
X = an independent variable array
Y = an independent variable array
NX = the number of points in X where the polynomial is to be evaluated
NY = the number of points in Y where the polynomial is to be evaluated
NTERMS = the number of terms comprising the polynomial

A 1 placed in column 3 of card B will produce intermediate print of the arrays X, Y, and Z.

4. VLINE (Sequence Number 4)

Subroutine VLINE linearly interpolates on a doubly subscripted array Z(X,Y). The arguments for VLINE are NX, NY, XIN, YIN, XARRAY, YARRAY, L, M, and ZØUT, where:

NX = the number of points in the X direction
NY = the number of points in the Y direction
XIN = the value X at which the value for Z(X,Y) is desired
YIN = the value Y at which a value for Z(X,Y) is desired
XARRAY = the name of the singularly subscripted array containing the X independent variable
YARRAY = the name of the singularly subscripted array containing the Y independent variable
ZARRAY = the name of the doubly subscripted array containing the Z dependent variable



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L and M = the dimensions on the Z array, where the Z array is of the form Z(L,M)
Note: L and M must agree with the dimensions given for the Z array in the calling program.
ZØUT = the interpolated value of the dependent variable.

VLINE was especially designed for the lattice networks of the steady state solution and the initial temperature distribution which are stored as double subscripted variables in the program.

5. LEFT (Sequence Number 5)

Subroutine LEFT was created to avoid unnecessary rewriting of a short set of instructions. It simply moves an X, Y, and Z prime into their appropriate storage areas. LEFT is used in subroutines WURZEL and BRIDGE.

6. SIZE (Sequence Number 6)

Subroutine SIZE divides an interval into a given number of constant size increments. The arguments for size are:

A, MAX, AMIN, and AMAX

where

A = the array name where the increments are to be stored
MAX = the number of increments plus one
AMIN = the lower limit of the interval
AMAX = the upper limit of the interval

7. WØØFER (Sequence Number 7)

Subroutine WØØFER cumulatively integrates numerical input data of the thermophysical properties to obtain the required transformation and inversion. This subroutine performs a numerical integration of Equation (11) and stores an array of real temperature and transform temperature.

A 1 placed in column 7 of card 8 will produce intermediate print of the arrays XTRAN (real temperature) and YTRAN (transform temperature). The input thermophysical property data is always printed out.



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8. PØLYGN (Sequence Number 8)

Subroutine PØLYGN performs the same basic function as subroutine PØLYH but for one independent variable instead of two; i.e., PØLYGN reads and evaluates a polynominal whose terms are of the form

$$Y = AX^{B}$$

(See Subroutine PØLYGH).

The arguments for PØLYGN are X, Y, NY, and NTERMS, where:

X = the independent variable
Y = the dependent variable array
NY = the number of points at which the polynomial is to be evaluated,
NTERMS = the number of terms comprising the polynominal.

The polynomial evaluated by PØLYGN is automatically printed out.

A 1 placed in column 8 of card B will produce intermediate print of the arrays X and Y.

9. ENØM (Sequence Number 9)

Subroutine ENØM solves three related expressions:

$$C_{\gamma\beta} = \frac{-\ell_{11} Y'_{\gamma}(\beta r_1) - \ell_{12} Y_{\gamma}(\beta r_1)}{\ell_{11} J'_{\gamma}(\beta r_1) + \ell_{12} J_{\gamma}(\beta r_1)}$$

$$EH\O M = \int_{r_1}^{r_2} r \left[C_{\gamma\beta} J_{\gamma}(\beta r) + Y_{\gamma}(\beta r)\right]^2 dr$$

$$ST\O 2 = \left[C_{\gamma\beta} J_{\gamma}(\beta r) + Y_{\gamma}(\beta r)\right] r.$$

STØ2 is actually an array of expressions. It is evaluated at a number of r's and stored in common for use in subroutine ARBCØE.

The arguments for ENOM are IG, IB, and EHØM where:



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IG = the index number of an angular eigenvalue ($\gamma$)
IB = the index number of a radial eigenvalue ($\beta$)
EHØM = the name of the location where the value of the integral EHØM is stored

A 1 in column 9 of card B produces intermediate print of EHØM and $C_{\gamma\beta}$.

10. GAMEPS (Sequence Number 10)

Subroutine GAMEPS solves for the eigenvalues $\gamma$ and $\epsilon$. The subroutine evaluates Equations (46) through (51) to obtain $\gamma$'s and a similar set to obtain $\epsilon$'s.

The arguments for GAMEPS are ISEEK, MANY, and VALUES, where:

ISEEK = the selection indicator for $\gamma$'s or $\epsilon$'s
MANY = the number of eigenvalues desired
VALUES = the array name where the eigenvalues will be stored.

A 1 in column 10 of card B (above) produces intermediate print in GAMEPS of the arguments, the eigenvalues, and some of the indicators and constants used by GAMEPS.

11. GTIME (Sequence Number 11)

Subroutine GTIME solves the expression

$$GSIMP = \alpha\beta^2 \int_0^{\theta} g(\lambda)\ e^{-\alpha\beta^2 (\theta-\lambda)}\ d\lambda$$

and also obtains

$$GTHETA = g(\theta)$$

where $g(\theta)$ is the time function which is input by the program user, and $\theta$ is a time at which program answers are being computed.

The arguments for GTIME are: TIME, BETA, and GSIMP, where:

TIME = $\theta$
BETA = $\beta$
GSIMP = the name of the location where the evaluation of the integral GSIMP (above) will be stored.



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Intermediate print is not furnished for GTIME since GSIMP and GTHETA are part of the intermediate print of subroutine SERIES.

12. WURZEL (Sequence Number 12)

Subroutine WURZEL is a root searching routine which works in conjunction with subroutines FFP and BRIDGE. Its method, one of trial and error, is based on limiting root spacing of Equation (52)†.

WURZEL makes continuous guesses at an abscissa value and calls subroutine FFP to get a corresponding ordinate value of Equation (53) and slope of Equation (54). The relationship of the signs of Equations (53) and (54) on the first guess, determines whether the direction of the continued search should be forward or backward. This scheme is illustrated in Figure 13. A first guess at point A would initiate a forward search. A first guess at point B would initiate a backward search. The scheme, however, does not provide recovery from a first guess past the first finite maximum or minimum such as point C, and would subsequently miss the first root. Accordingly, an educated first guess is required. As programmed, a first guess of $0.25\alpha/r_1$* is used as the first guess for the first two $\gamma$'s. The first guesses for the remainder of the given set of $\gamma$'s is based on the spacing of the previously determined first roots.

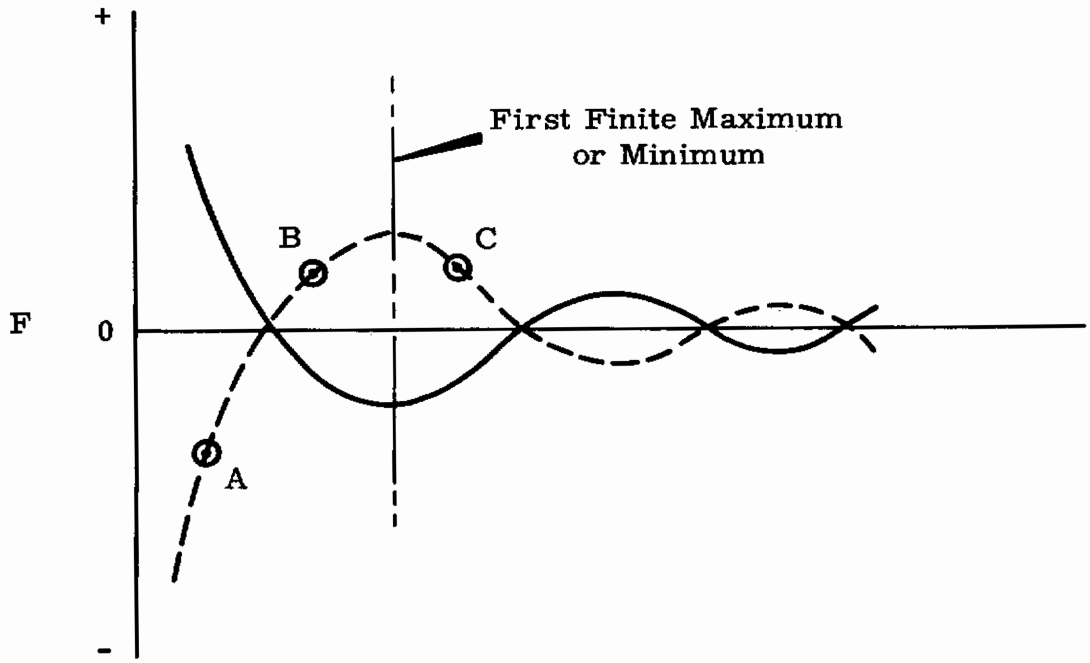


Figure 13. "First Guess" System

* For $\gamma$ equals zero, the first guess is $1/r_2 - r_1$.
† The root spacing approaches $\pi/r_2 - r_1$ as $\beta$ approaches infinity.



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13. BRIDGE (Sequence Number 13)

Subroutine BRIDGE is a direct application of Newton's method of successive approximations (Reference 11) and the regula falsi method (Reference 2) used in root searching. The regula falsi method is applied in the coarse search phase, until it obtains two points in the immediate vicinity of, and boardering a root. Newton's method of successive approximations is then applied in the fine search phase to obtain the root within a desired accuracy.

The arguments for BRIDGE are:

IB, XH, YH, SH, XL, YL, SL, and IG

where

IB = the index number of the desired radial eigenvalue ($\beta$).

(XH, YH and SH) = the coordinates and slope respectively of a point forward of the $\beta$.

(XL, YL and SL) = the coordinates and slope respectively of a point behind the $\beta$.

IG = the index number of an angular eigenvalue ($\gamma$)

14. ABES (Sequence Number 14)

Subroutine ABES solves for Bessel functions of the second kind of the form $Y_\gamma(\beta r)$ and $Y'_\gamma(\beta r)$. It also obtains Bessel functions of the first kind of the form $J_\gamma(\beta r)$ and $J'_\gamma(\beta r)$ by a call to subroutine BES.

The arguments for ABES are B6, R6, C6, A6, AP6, Y6, and YP6, which correspond to

$$\beta,\ r,\ \gamma,\ J_\gamma(\beta r),\ J'_\gamma(\beta r),\ Y_\gamma(\beta r),\ \text{and}\ Y'_\gamma(\beta r).$$

If the desired functions are of non-integer order, an explicit expression for Bessel functions of the second kind, in terms of Bessel functions of the first kind, is used. This expression is

$$Y_\gamma(\beta r) = \frac{1}{\sin \gamma \pi}\left[\cos(\gamma\pi) J_\gamma(\beta r) - J_{-\gamma}(\beta r)\right]$$

Integer order functions are obtained by calculating $Y_1$ and $Y_2$ from the following equations and using recurrence formulas (see Reference 6 or 7)



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to obtain $Y_n$ and $Y'_n$. The use of recurrence formulas avoids the need of special expressions for high orders and provides a scheme for obtaining the derivatives. The derivative is required by WURZEL and ENOM.

For small arguments

$$\pi Y_\gamma(\beta r) = 2J_\gamma(\beta r)\left[\ln\frac{\beta r}{2}+\sigma\right] - \sum_{r=0}^{\infty}(-1)^r\frac{(\beta r/2)^{\gamma+2r}}{r!(\gamma+r)!}$$

$$\times\left[\sum_{m=1}^{\gamma+r} m^{-1} + \sum_{m=1}^{r} m^{-1}\right]$$

$$-\sum_{r=0}^{\gamma-1}(\beta r/2)^{-\gamma+2r}\frac{(\gamma-r-1)!}{r!}$$

where $\sigma$ is Euler's constants.

For larger arguments $(\beta r)$

$$\pi Y_\gamma(\beta r) \sim \left(\frac{2\pi}{\beta r}\right)^{\frac{1}{2}} \sin\left(\beta r - \frac{1}{2}\gamma\pi - \frac{1}{4}\pi\right)\sum_{m=0}^{\infty}\frac{(-1)^m f_\gamma(\gamma,2m)}{(2\beta r)^{2m}}$$

$$+\cos\left(\beta r - \frac{1}{2}\gamma\pi - \frac{1}{4}\pi\right)\sum_{m=0}^{\infty}\frac{(-1)^m f_\gamma(\gamma,2m+1)}{(2\beta r)^{2m+1}}$$

where

$$f_\gamma(a,b) = \frac{\Gamma(a+b+\frac{1}{2})}{b!\,\Gamma(a-b+\frac{1}{2})} .$$

15. GAMMA (Sequence Number 15)

Subroutine GAMMA is the other of the two routines used in Reference 4. GAMMA calculates GAMMA functions for use in Subroutine BES.

The arguments for GAMMA are X and Y where

$$Y = \Gamma(X).$$

16. AMSAFC (Sequence Number 16)

Function subprogram AMSAFC solves the expression



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$$AMSAFC = M_\epsilon \sin(\epsilon \,\ell n\, r) + F_\epsilon \cos(\epsilon \,\ell n\, r)$$

The arguments for AMSAFC are IG and V,

where,

IG = the index number of the $\epsilon$

V = r

the values of $M_\epsilon$, $F_\epsilon$, and $\epsilon$ are stored in common. A 1 placed in column 16 of card B will print the expression AMSAFC as intermediate print.

17. SERIES (Sequence Number 17)

Subroutine SERIES solves the equation

$$T(\theta, r, \phi) = \text{TERMA} + \text{TERMB} + g(\theta) \times f(r,\phi)$$

where TERMA is defined by the double summation of Equation (44) and TERMB is defined by the double summation of Equation (83).

The arguments for SERIES are: TIME, R, and P, which correspond to $\theta$, r, and $\phi$.

The intermediate print for SERIES is in two forms. A 1 in column 17 of card B prints $T(\theta, r, \phi)$, before it is inverted, and the corresponding steady state solution $f(r, \phi)$.

A 2 placed in the same column prints the same information as for a 1, as well as the terms in the series, the time function, the eigenvalues, and the partial sums.

18. STATE (Sequence Number 18)

Subroutine STATE solves the steady state portion of the problem. In particular, it solves the equation

$$f(r, \phi) = \Sigma T_1 + \Sigma T_2 + \Sigma T_3 + \Sigma T_4$$

where

$T_1$ = term defined in (100)

$T_2$ = term defined in (110)



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$T_3$ = term defined in (120)

$T_4$ = term defined in (132).

The intermediate print in STATE is obtained by placing a 1 in column 18 of card B.

The printout consists of $T_1$, $T_2$, $T_3$, $T_4$ and the partial sum of the four series for each eignevalue.

19. XI (Sequence Number 19)

Function subprogram XI solves for the $\xi$ expressions in Equations (101), (111), (122), and (133) in the steady state solution. For example, at boundary $r_1$, XI solves Equation (101). However, special cases in Equation (107), (108), and (109) can be obtained by setting

$$\xi_{r1} = 1.0,$$

$$\xi_{r1} = \ell n\,(r/r_2)\ /\ \ell n\, r_1/r_2\ ,$$

or

$$\xi_{r1} = r_1\ \ell n\ (r/r_2).$$

depending upon the boundary condition indicators.

The arguments for XI are: IG, V, and NBØUND,

where

IG = the index number of an eigenvalue

V = either r or $\phi$ depending upon which boundary is being requested

NBOUND = the number of the boundary being requested (1, 2, 3, or 4 for boundaries $r_1$, $r_2$, $\phi_1$, or $\phi_2$, respectively).

A 1 in column 19 of card B will produce intermediate print of the value of $\xi$ and the boundary which has been evaluated.



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## 20. FFP (Sequence Number 20)

Subroutine FFP calculates the functions

$$F = \begin{vmatrix} T_1 & T_2 \\ T_3 & T_4 \end{vmatrix} \tag{53}$$

and

$$\frac{\partial F}{\partial \beta} = FP = \begin{vmatrix} T_1 & T_2 \\ \frac{\partial T_3}{\partial \beta} & \frac{\partial T_4}{\partial \beta} \end{vmatrix} + \begin{vmatrix} \frac{\partial T_1}{\partial \beta} & \frac{\partial T_2}{\partial \beta} \\ T_3 & T_4 \end{vmatrix} \tag{54}$$

for a particular value of the independent variable $\beta$ for a given $\gamma$. Subroutine WURZEL and BRIDGE use this subroutine (FFP) in searching for roots of equation (52).

The arguments for FFP are: IG, B, F, and FP

where:

IG = the index number of the angular eigenvalue ($\gamma$).

B = the independent variable in equation (53) and (54).

F ~ defined above in equation (53)

FP ~ defined above in equation (54)

Intermediate print in FFP is of two forms:

1 A 1 in column 20 of card B (above) prints out B, F, FP, and IG.

2 A 2 in column 20 of card B prints out the elements of the determinant F, and their derivatives and the Bessel functions necessary for these computations as well as B, F, FP, and IG.



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21. ARBCØE (Sequence Number 21)

Subroutine ARBCØE solves the equations for $A_{\gamma\beta}$, Equation (62), and $G_{\gamma\beta}$, Equation (80). $A_{\gamma\beta}$ and $G_{\gamma\beta}$ are the arbitrary coefficients of the transient solution. Special cases of the arbitrary coefficients emerge automatically in ARBCØE due to the evaluation of composite expressions in supporting subroutines and subprograms.

A 1 in column 21 of card B produces intermediate print of the arbitrary coefficients.

22. TRAN (Sequence Number 22)

Function subprogram TRAN transforms a variable according to the special cases of Table I.

The single argument of TRAN is V which is the variable to be transformed according to Equation (11).

A 1 in column 22 of card B produces intermediate print of the variable V, the transform variable T, and the constants and intermediate expressions used in the transform calculations.

23. BSNFCS (Sequence Number 23)

Function subprogram BSNFCS solves the expression

$$BSNFCS = B_\gamma \sin(\gamma\,\phi) + F_\gamma \cos(\gamma\,\phi)$$

The arguments of BSNFCS are IG and P

where

IG = the index number of an eigenvalue ($\gamma$)

P = $\phi$

$B_\gamma$ and $F_\gamma$ are obtained through common.

A 1 in column 23 of card B produces intermediate print of the expression BSNFCS.



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24. CØECØE (Sequence Number 24)

Subroutine CØECØE solves Equations (102), (112), (123) and (134) for the arbitrary coefficients of the steady state solution, $P_\gamma$, $L_\gamma$, $Q_\epsilon$+ and $W_\epsilon$, respectively. Special cases of the arbitrary coefficients resulting from $B_\gamma$ or $M_\epsilon$ being infinite, or $\gamma$ or $\epsilon$ being zero, are incorporated in CØECØE through supporting subroutines and subprograms.

The arguments for CØECØE are: IG, CP, CL, CQ, and CW where: IG = the index number of an eigenvalue, and CP, CL, CQ, and CW correspond to $P_\gamma$, $L_\gamma$, $Q_\epsilon$ and $W_\epsilon$ respectively.

Intermediate print for CØECØE consists of printout of the arbitrary coefficients calculated in each case, and it is obtained by placing a 1 in column 24 of card B.

25. CENØM (Sequence Number 25)

Subroutine CENØM solves the integral:

$$\text{CEHØM} = \int_{r_1}^{r_2} \left[ M_\epsilon \sin(\epsilon \, \ln r) + F_\epsilon \cos(\epsilon \, \ln r) \right]^2 \frac{dr}{r}$$

which has special cases of

$$\text{CEHØM} = \left[ \frac{\ln r}{2} - \frac{1}{4\epsilon} \sin(2\epsilon \, \ln r) \right]_{r_1}^{r_2}$$

for an infinite $M_\epsilon$, and

$$\text{CEHØM} = \ln(r_2/r_1)$$

when $\epsilon$ equals zero.

Subroutine CENØM also solves Equation (124) for $M_\epsilon$ and sets $F_\epsilon$ equal to zero for special cases.

The arguments to CENØM are: IG, and CEHØM

where

IG = the index number of an eigenvalue and



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CEHØM = the name of the location where the value of the integral CEHØM above will be stored.

The values of $M_\epsilon$ and $F_\epsilon$ are stored in common.

A 1 in column 25 of card B produces intermediate print of IG, CEHØM, and $M_\epsilon$.

## 26. ATRAN (Sequence Number 26)

Function subprogram ATRAN is the inversion routine which corresponds to the transformation routine TRAN. (See function subprogram TRAN.)

The single argument of ATRAN is T which is the transform variable to be inverted to real temperature.

A 1 in column 26 of card B produces print similar to the intermediate print of function subprogram TRAN.

## 27. DENØM (Sequence Number 27)

Subroutine DENØM solves the integral:

$$\text{DEHØM} = \int_{\phi_1}^{\phi_2} \left[B_\gamma \sin(\gamma\phi) + F_\gamma \cos(\gamma\phi)\right]^2 d\phi$$

which has special cases of:

$$\text{DEHØM} = \left[\frac{\phi}{2} - \frac{1}{4\gamma}\sin(2\gamma\phi)\right]_{\phi_1}^{\phi_2}$$

for $B_\gamma$ of infinity and

$$\text{DEHØM} = \phi_2 - \phi_1$$

for $\gamma$ equal to zero

DENØM also solves Equation (60) for $B_\gamma$ and sets $F_\gamma$ equal to zero for special cases. $F_\gamma$ is similar to $F_\epsilon$ in subroutine CENØM.



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The arguments for DENØM are IG and DEHØM

where:

IG = the index number of an eigenvalue and

DEHØM = the name of the location where the value of the integral DEHØM above will be stored.

The values of $B_\gamma$ and $F_\gamma$ are stored in common. Intermediate print in DENØM consists of the values of DEHØM, IG, and $B_\gamma$. To obtain this print a 1 is placed in column 27 of card B.

28. INPUT (Sequence Number 28)

Subroutine INPUT reads in and prints out the problem description and the necessary supporting data for a problem solution. It also serves the needs of the other subroutines by putting raw data in a form usable to the transformation of the thermophysical properties, and each of the boundary condition functions is done by subroutine INPUT. Communication from INPUT to the remainder of the program is accomplished through COMMON.

Subroutine INPUT checks all of the input data for such things as sequence, mode, form, and limits. Together with the computer FORTRAN monitor, this provides a good level of assurance to the program user that computer time will not be expended on an erroneous problem. However, this is by no means complete protection.



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# APPENDIX D

# FLOW DIAGRAMS



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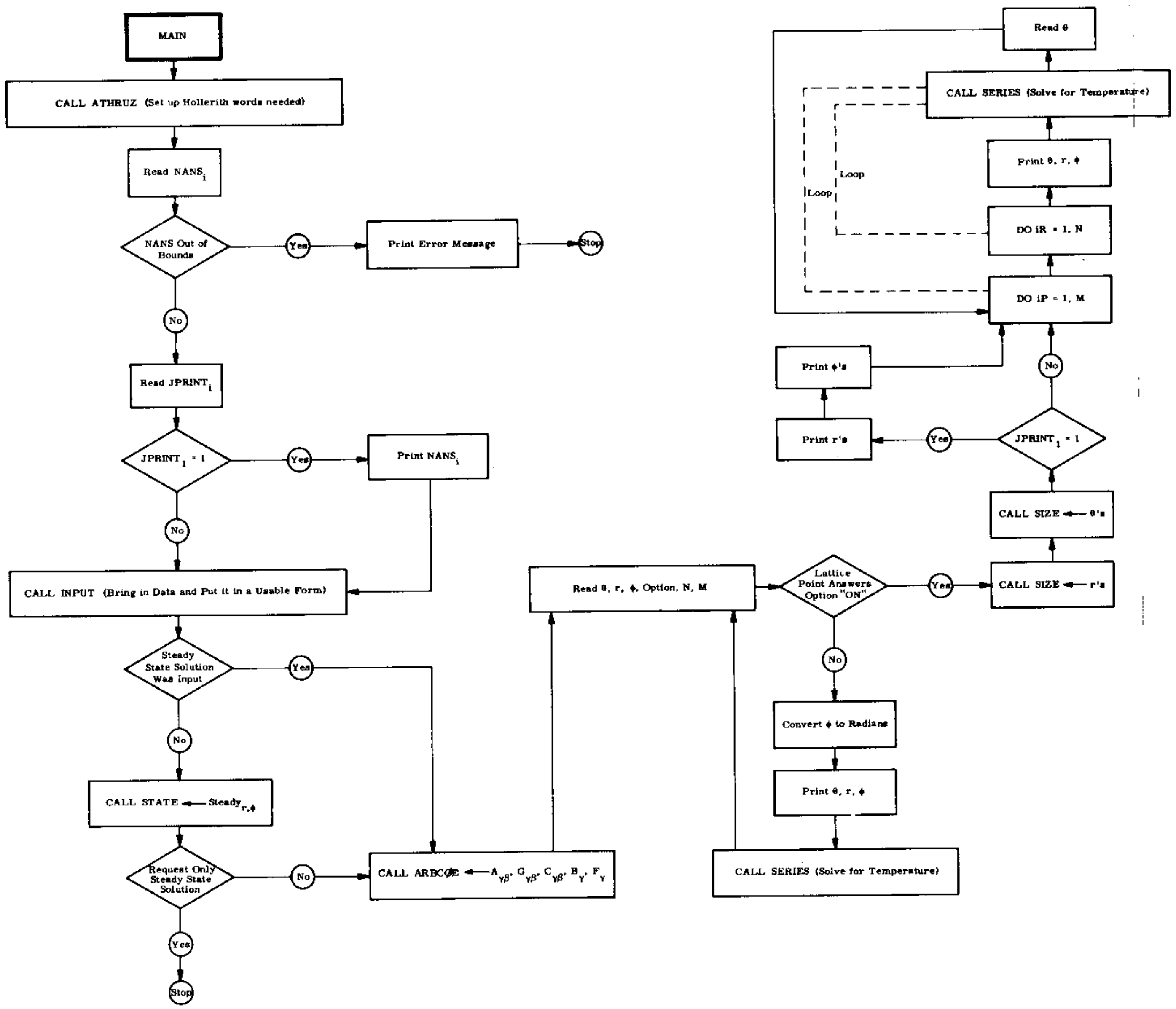


Figure 14. Flow Diagram, MAIN Routine



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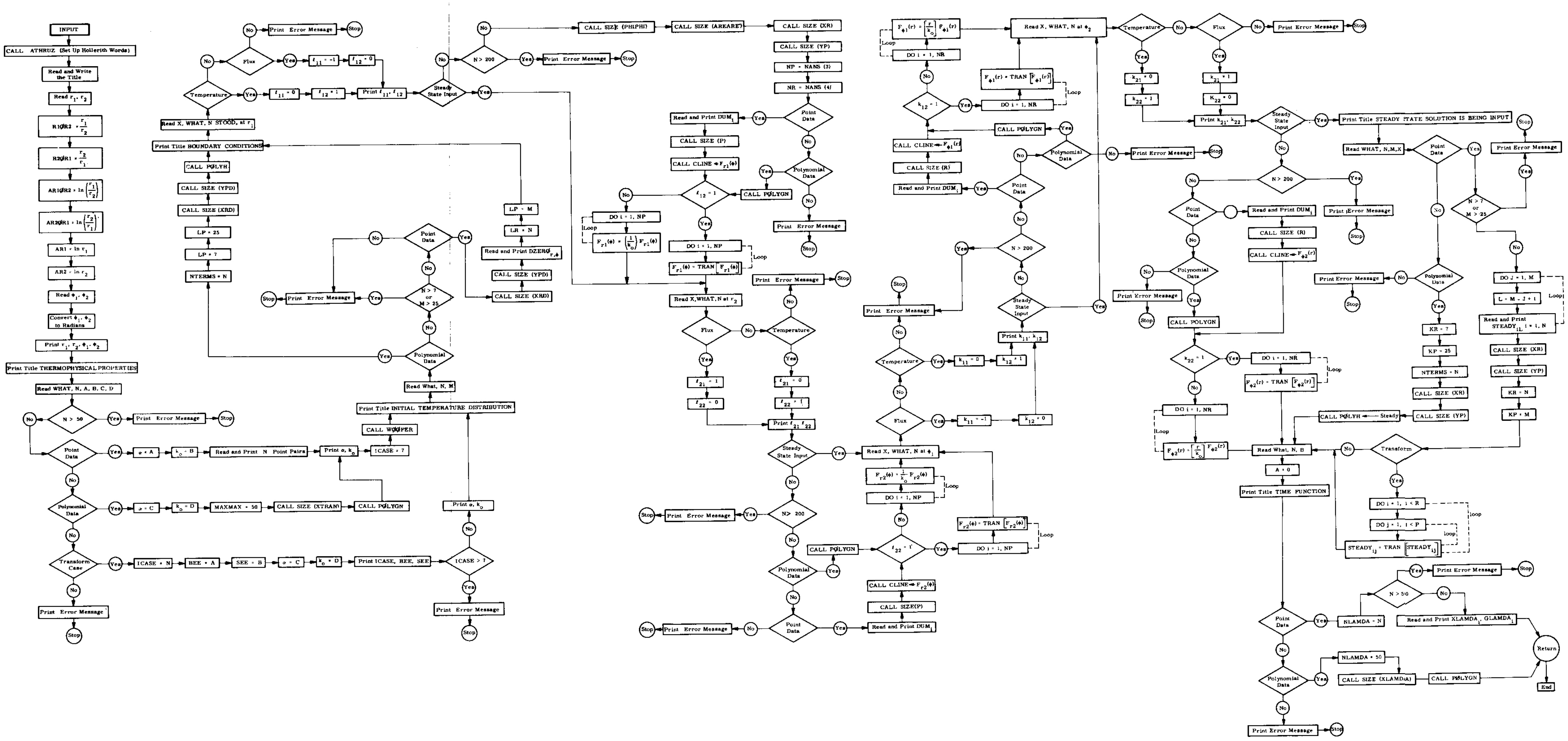

Figure 15. Flow Diagram, Subroutine INPUT



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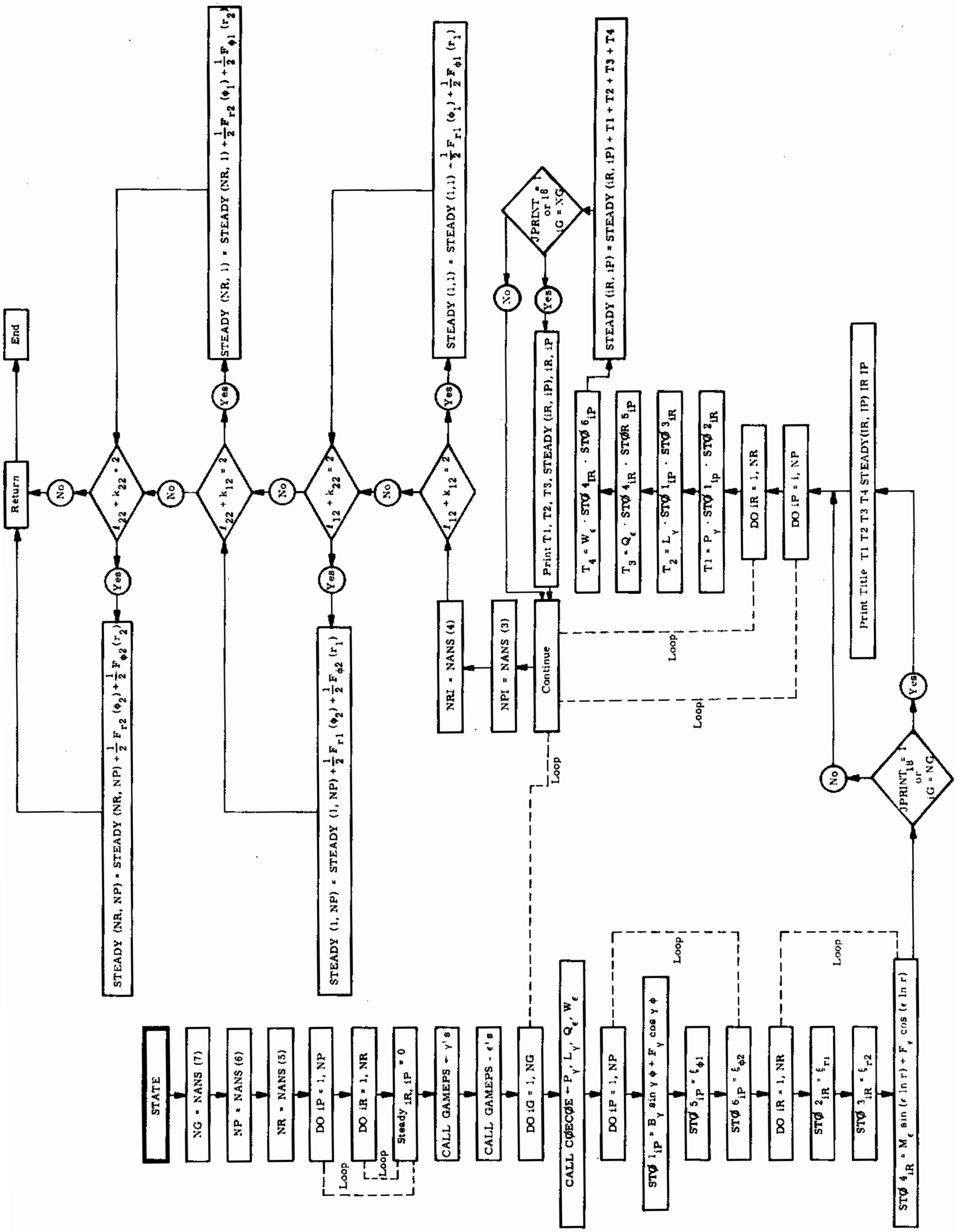


Figure 16. Flow Diagram, Subroutine STATE



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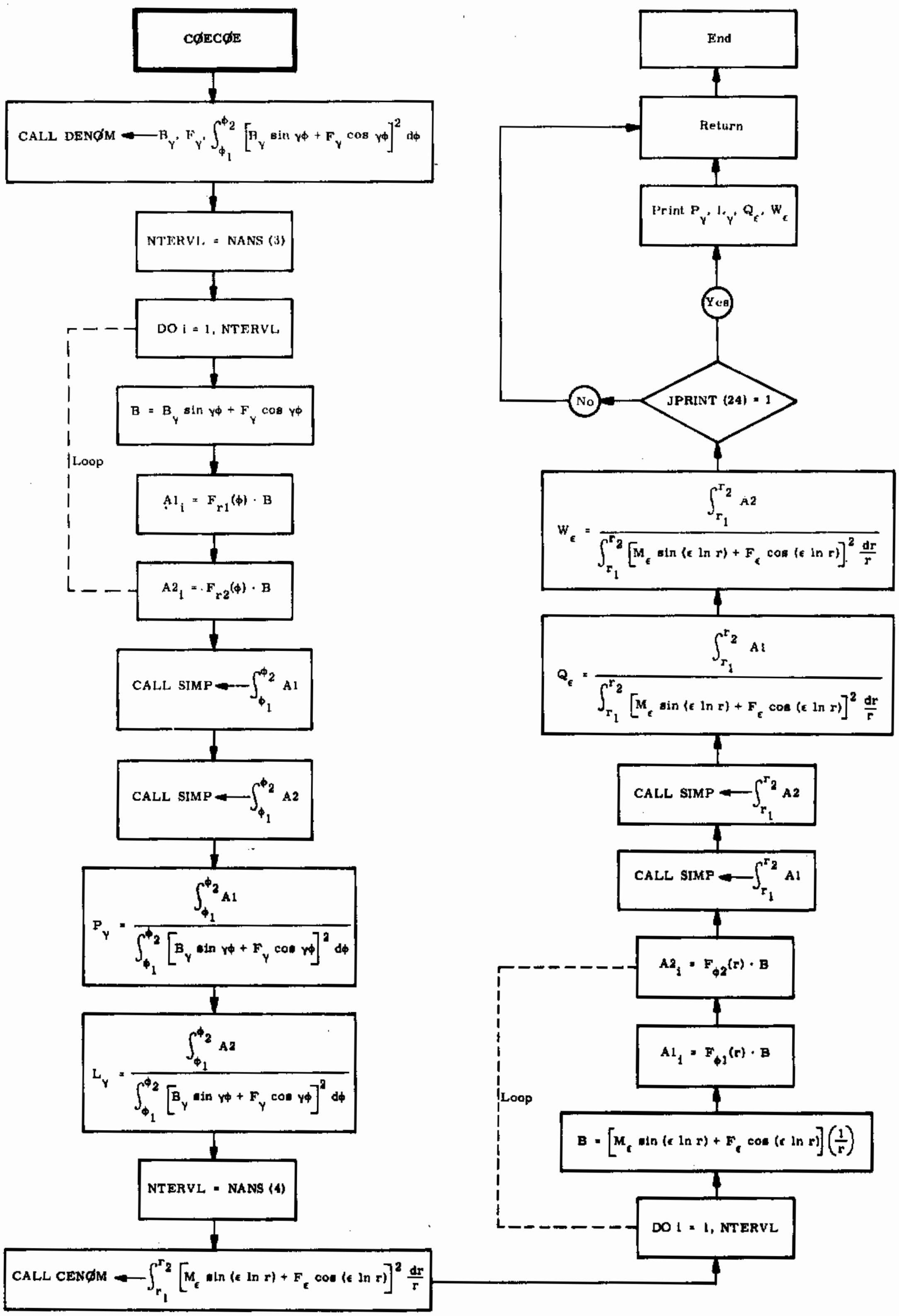


Figure 17. Flow Diagram, Subroutine CØECØE



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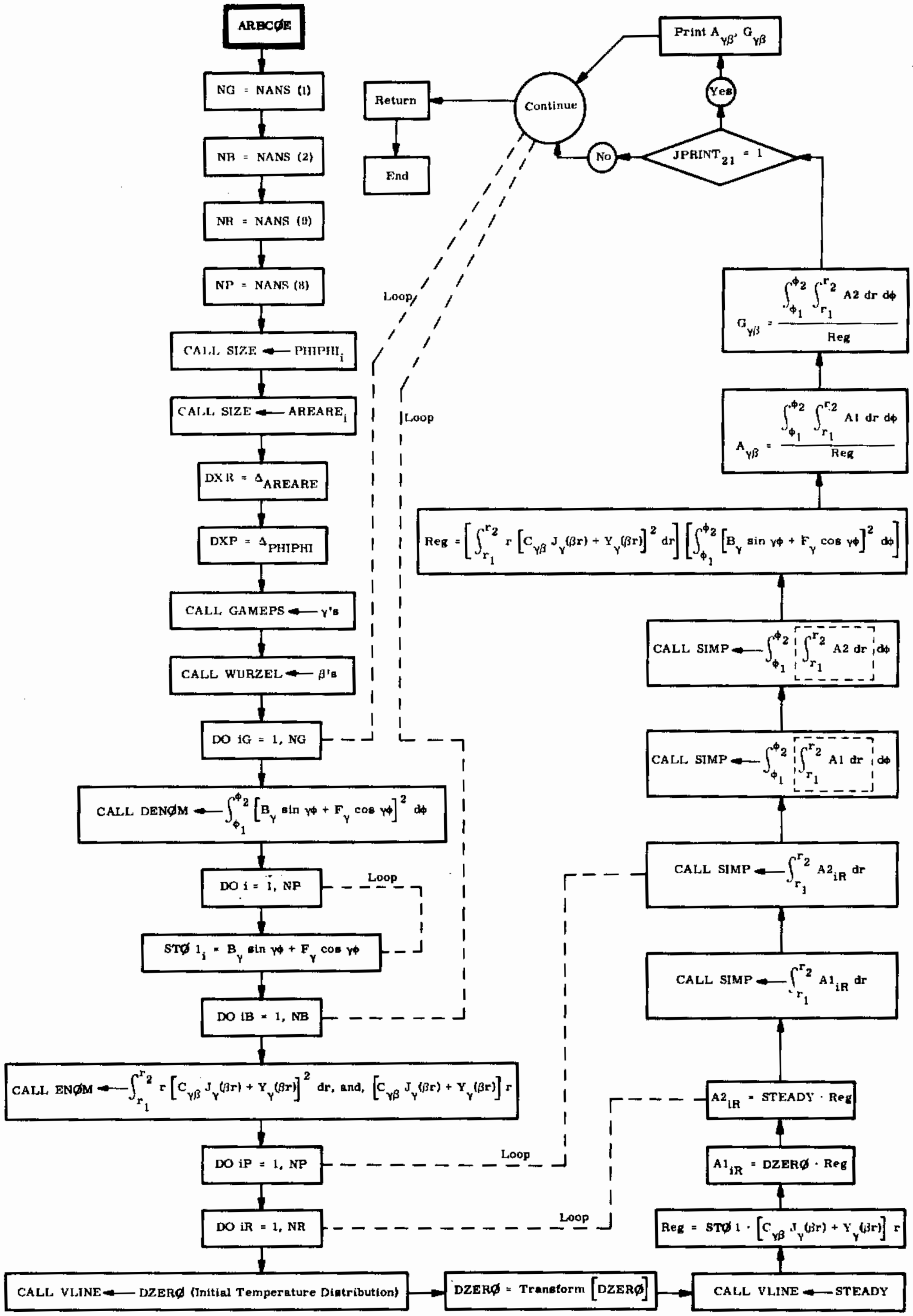


Figure 18. Flow Diagram, Subroutine ARBCØE



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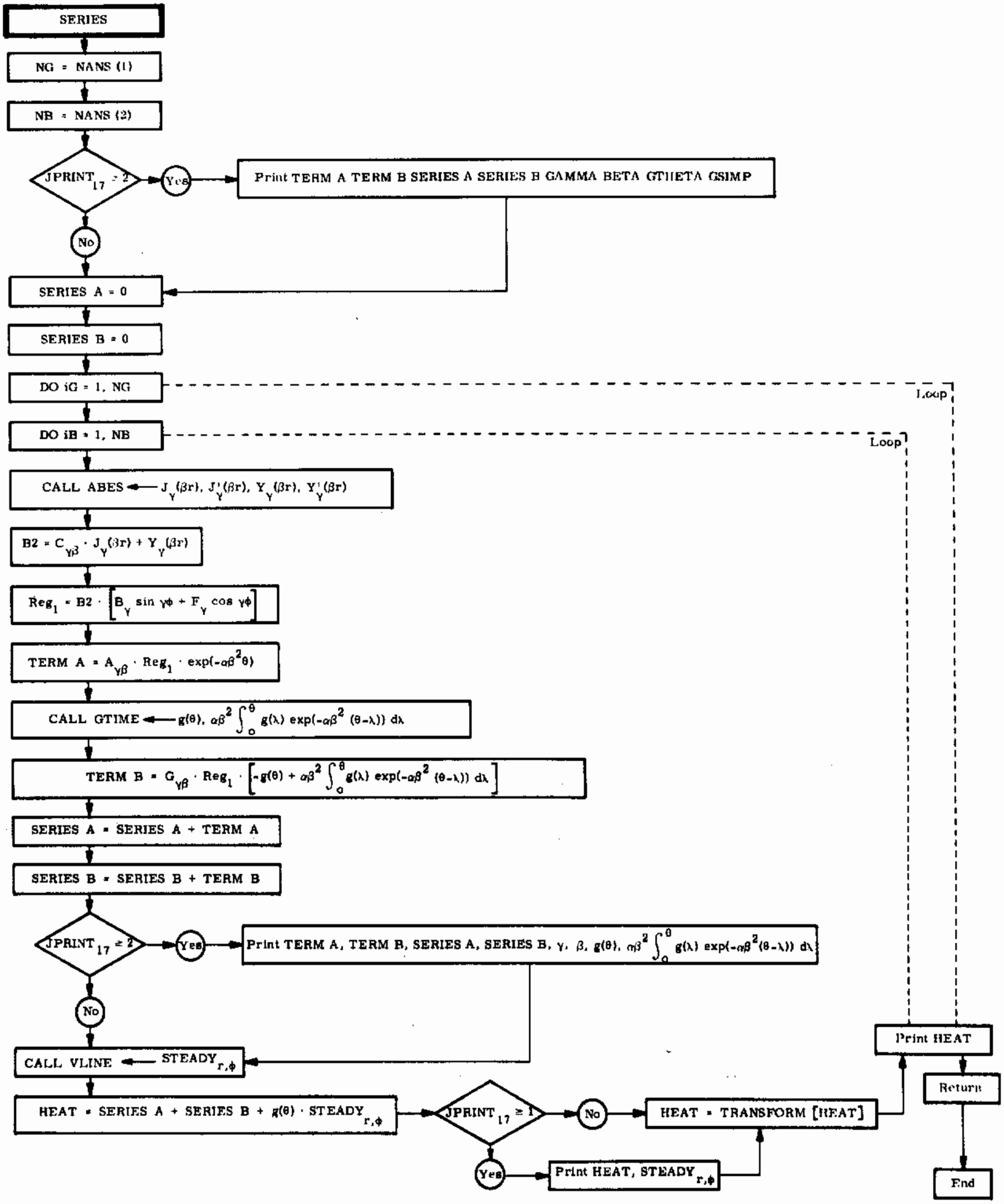


Figure 19. Flow Diagram, Subroutine SERIES



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# APPENDIX E

# SAMPLE TEST PROBLEM



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# APPENDIX E

## SAMPLE TEST PROBLEM

In addition to the sample problems presented in preceding sections, a series of 12 test problems were run on the computer at Wright-Patterson Air Force Base. Similar problems were calculated for comparison using the Thermal Analyzer Computer program (Reference 13).

This appendix presents a typical test problem and a comparison between the two programs. The specific problem is the determination of the temperature response of the center radial line in a 60 degree sector with zero initial temperature distribution, a temperature of 100 sin 6$\theta$ at the surface $r_2$, and the remaining surfaces insulated. Print out of the input and solution obtained with the computer program developed in this report is shown in Table VII. Data from both programs is shown in Figure 20.



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# TABLE VII

## Program Printout, Sample Test Problem

```
SP  8

INPUT
GEOMETRY
R1=  0.50000000E 00      R2=  0.80000000E 00
P1=  0.                  P2=  0.10471974E 01     (RADIANS)

THERMO PHYSICAL PROPERTIES
TRANSFORM ACCORDING TO CASE 4,B=  0.99999999E-02   C=  0.
ALPHA=  0.40000000E-04      KZERO=  0.23100000E-04

INITIAL TEMPERATURE DISTRIBUTION
POLYNOMIAL DATA,  1 TERMS
                (  0.              )(R**     0.     )(P**     0.     )

BOUNDRY CONDITIONS
AT R1, FLUX      L11=      -1.00    L12=        0.
POLYNOMIAL DATA,  1 TERMS, X=DUMMY VARIABLE
                (  0.              )(X**     0.     )
AT R2, TEMP      L21=       0.      L22=        1.00
POLYNOMIAL DATA,  5 TERMS, X=DUMMY VARIABLE
                (  0.30000000E 03)(X**     1.0000)
                ( -0.45000000E 03)(X**     3.0000)
                (  0.20250000E 03)(X**     5.0000)
                ( -0.43499999E 02)(X**     7.0000)
                (  0.53999999E 01)(X**     9.0000)
AT P1, FLUX      K11=      -1.00    K12=        0.
POLYNOMIAL DATA,  1 TERMS, X=DUMMY VARIABLE
                (  0.              )(X**     0.     )
AT P2, FLUX      K21=       1.00    K22=        0.
POLYNOMIAL DATA,  1 TERMS, X=DUMMY VARIABLE
                (  0.              )(X**     0.     )

TIME FUNCTION
POLYNOMIAL DATA,  1 TERMS, X=DUMMY VARIABLE
                (  0.09999999E 01)(X**     0.     )
EVALUATE POLYNOMIAL FROM     0.                    TO      0.30000000E 03
UNDRFLOW AT 32300 IN AC AND MQ.
UNDRFLOW AT 32300 IN AC AND MQ.
```

| T1 | T2 | T3 | T4 | STEADY(IR,IP) | IR | IP |
|---|---|---|---|---|---|---|
| -0. | -0.34710202E-08 | -0. | 0. | 0.80600750E 02 | 1 | 1 |
| -0. | -0.20258909E-06 | 0. | -0. | 0.78605650E 02 | 2 | 1 |
| -0. | -0.14559225E-04 | 0. | -0. | C.72564872E 02 | 3 | 1 |
| -0. | -0.70479304E-03 | 0. | -0. | 0.61232409E 02 | 4 | 1 |
| -0. | -0.24570825E-01 | -0. | 0. | 0.41449883E 02 | 5 | 1 |
| -0. | -0.64933278E 00 | -0. | 0. | 0.42467164E 01 | 6 | 1 |
| -0. | -0.60273565E-09 | -0. | 0. | C.82528167E 02 | 1 | 2 |
| -0. | -0.35179187E-07 | 0. | -0. | C.81044332E 02 | 2 | 2 |
| -0. | -0.25281800E-05 | 0. | -0. | 0.76626717E 02 | 3 | 2 |
| -0. | -0.12238588E-03 | 0. | -0. | 0.68704513E 02 | 4 | 2 |
| -0. | -0.42666741E-02 | -0. | 0. | C.56426386E 02 | 5 | 2 |
| -0. | -0.11275532E-00 | -0. | 0. | C.40783387E 02 | 6 | 2 |
| 0. | 0.32616926E-08 | -0. | 0. | 0.87343854E 02 | 1 | 3 |
| 0. | 0.19037150E-06 | 0. | -0. | 0.87071547E 02 | 2 | 3 |
| 0. | 0.13681198E-04 | 0. | -0. | 0.86345201E 02 | 3 | 3 |
| 0. | 0.66228892E-03 | 0. | -0. | 0.85354899E 02 | 4 | 3 |
| 0. | 0.23089027E-01 | -0. | 0. | 0.84583668E 02 | 5 | 3 |
| 0. | 0.61017331E 00 | -0. | 0. | 0.85270080E 02 | 6 | 3 |
| 0. | 0.17355084E-08 | -0. | 0. | 0.92716964E 02 | 1 | 4 |
| 0. | 0.10129444E-06 | 0. | -0. | C.93705983E 02 | 2 | 4 |
| 0. | 0.72796056E-05 | 0. | -0. | 0.96675146E 02 | 3 | 4 |
| 0. | 0.35239618E-03 | 0. | -0. | 0.10209414E 03 | 4 | 4 |
| 0. | 0.12285401E-01 | -0. | 0. | 0.11085305E 03 | 5 | 4 |
| 0. | 0.32466608E-00 | -0. | 0. | 0.12407698E 03 | 6 | 4 |
| -0. | -0.26589575E-08 | -0. | 0. | 0.96173427E 02 | 1 | 5 |
| -0. | -0.15519235E-06 | 0. | -0. | 0.97932601E 02 | 2 | 5 |
| -0. | -0.11153021E-04 | 0. | -0. | C.10311360E 03 | 3 | 5 |
| -0. | -0.53990316E-03 | 0. | -0. | 0.11225347E 03 | 4 | 5 |
| -0. | -0.18822357E-01 | -0. | 0. | 0.12641241E 03 | 5 | 5 |
| -0. | -0.49741811E-00 | -0. | 0. | 0.14672273E 03 | 6 | 5 |
| -0. | -0.26589534E-08 | -0. | 0. | C.96185058E 02 | 1 | 6 |
| -0. | -0.15519211E-06 | 0. | -0. | C.97944140E 02 | 2 | 6 |
| -0. | -0.11153004E-04 | 0. | -0. | 0.10312445E 03 | 3 | 6 |
| -0. | -0.53990231E-03 | 0. | -0. | 0.11226215E 03 | 4 | 6 |
| -0. | -0.18822327E-01 | -0. | 0. | C.12641680E 03 | 5 | 6 |



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| | | | | | | |
|---|---|---|---|---|---|---|
| -0. | -0.49741732E-00 | -0. | 0. | 0.14672258E 03 | 6 | 6 |
| 0. | 0.17355134E-08 | -0. | 0. | 0.92751843E 02 | 1 | 7 |
| 0. | 0.10129473E-06 | 0. | -0. | 0.93741185E 02 | 2 | 7 |
| 0. | 0.72796263E-05 | 0. | -0. | C.96709935E 02 | 3 | 7 |
| 0. | 0.35239718E-03 | 0. | -0. | 0.10212471E 03 | 4 | 7 |
| 0. | 0.12285435E-01 | -0. | 0. | 0.11087208E 03 | 5 | 7 |
| 0. | 0.32466700E-00 | -0. | 0. | C.12407804E 03 | 6 | 7 |
| 0. | 0.32616902E-08 | -0. | 0. | C.87400795E 02 | 1 | 8 |
| 0. | 0.19037136E-06 | 0. | -0. | C.87130707E 02 | 2 | 8 |
| 0. | 0.13681188E-04 | 0. | -0. | C.86409051E 02 | 3 | 8 |
| 0. | 0.66228844E-03 | 0. | -0. | C.85421223E 02 | 4 | 8 |
| 0. | 0.23089010E-01 | -0. | 0. | C.84641580E 02 | 5 | 8 |
| 0. | 0.61017287E 00 | -0. | 0. | C.85295413E 02 | 6 | 8 |
| -0. | -0.60274187E-09 | -0. | 0. | C.82602119E 02 | 1 | 9 |
| -0. | -0.35179549E-07 | 0. | -0. | 0.81123461E 02 | 2 | 9 |
| -0. | -0.25282060E-05 | 0. | -C. | C.76720396E 02 | 3 | 9 |
| -0. | -0.12238714E-03 | 0. | -0. | C.68821730E 02 | 4 | 9 |
| -0. | -0.42667181E-02 | -0. | 0. | 0.56572578E 02 | 5 | 9 |
| -0. | -0.11275648E-00 | -0. | 0. | 0.40928707E 02 | 6 | 9 |
| -0. | -0.34710202E-08 | -0. | 0. | C.80681277E 02 | 1 | 10 |
| -0. | -0.20258909E-06 | 0. | -0. | C.78692923E 02 | 2 | 10 |
| -0. | -0.14559225E-04 | 0. | -0. | C.72672613E 02 | 3 | 10 |
| -0. | -0.70479304E-03 | 0. | -0. | C.61380392E 02 | 4 | 10 |
| -0. | -0.24570825E-01 | -0. | 0. | C.41679476E 02 | 5 | 10 |
| -0. | -0.64933278E 00 | -0. | 0. | 0.46839292E 01 | 6 | 10 |

TIME= 0.30000000E 02 R= 0.52500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.73015690E-02

TIME= 0.59999999E 02 R= 0.5250C000E 00 PHI= 0.5235987CE 00 (RADIANS)
TEMPERATURE= 0.31241775E-01

TIME= 0.12000000E 03 R= 0.52500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.99881590E 00

TIME= 0.20000000E 03 R= 0.5250C000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.59490845E 01

TIME= 0.30000000E 03 R= 0.52500000E 00 PHI= 0.5235987CE 00 (RADIANS)
TEMPERATURE= 0.14803585E 02

TIME= 0.30000000E 02 R= 0.57500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= -0.12274235E-00

TIME= 0.59999999E 02 R= 0.57500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.56388974E-01

TIME= 0.12000000E 03 R= 0.57500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.32958269E 01

TIME= 0.20000000E 03 R= 0.57500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.11141579E 02

TIME= 0.30000000E 03 R= 0.57500000E 00 PHI= 0.5235987CE 00 (RADIANS)
TEMPERATURE= 0.20894477E 02

TIME= 0.30000000E 02 R= 0.62500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= -0.39322674E-00

TIME= 0.59999999E 02 R= 0.62500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.14172167E 01

TIME= 0.12000000E 03 R= 0.62500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.10520564E 02

TIME= 0.20000000E 03 R= 0.62500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.22130169E 02

TIME= 0.30000000E 03 R= 0.62500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.32333116E 02

TIME= 0.30000000E 02 R= 0.67500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.11884510E 01

TIME= 0.59999999E 02 R= 0.67500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.10009040E 02

TIME= 0.12000000E 03 R= 0.67500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.26531497E 02

TIME= 0.20000000E 03 R= 0.67500000E 00 PHI= 0.52359870E 00 (RADIANS)
TEMPERATURE= 0.39367791E 02



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```
TIME=  0.30000000E 03      R=  0.67500000E 00      PHI=  0.52359870E 00 (RADIANS)
        TEMPERATURE=  0.48302764E 02

TIME=  0.30000000E 02      R=  0.72499999E 00      PHI=  0.52359870E 00 (RADIANS)
        TEMPERATURE=  0.17429347E 02

TIME=  0.59999999E 02      R=  0.72499999E 00      PHI=  0.52359870E 00 (RADIANS)
        TEMPERATURE=  0.35588296E 02

TIME=  0.12000000E 03      R=  0.72499999E 00      PHI=  0.52359870E 00 (RADIANS)
        TEMPERATURE=  0.52197002E 02

TIME=  0.20000000E 03      R=  0.72499999E 00      PHI=  0.52359870E 00 (RADIANS)
        TEMPERATURE=  0.61719963E 02

TIME=  0.30000000E 03      R=  0.72499999E 00      PHI=  0.52359870E 00 (RADIANS)
        TEMPERATURE=  0.67554465E 02

TIME=  0.30000000E 02      R=  0.77500000E 00      PHI=  0.52359870E 00 (RADIANS)
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        TEMPERATURE=  0.76943004E 02

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TIME=  0.20000000E 03      R=  0.77500000E 00      PHI=  0.52359870E 00 (RADIANS)
        TEMPERATURE=  0.86429211E 02

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        TEMPERATURE=  0.88313639E 02
```



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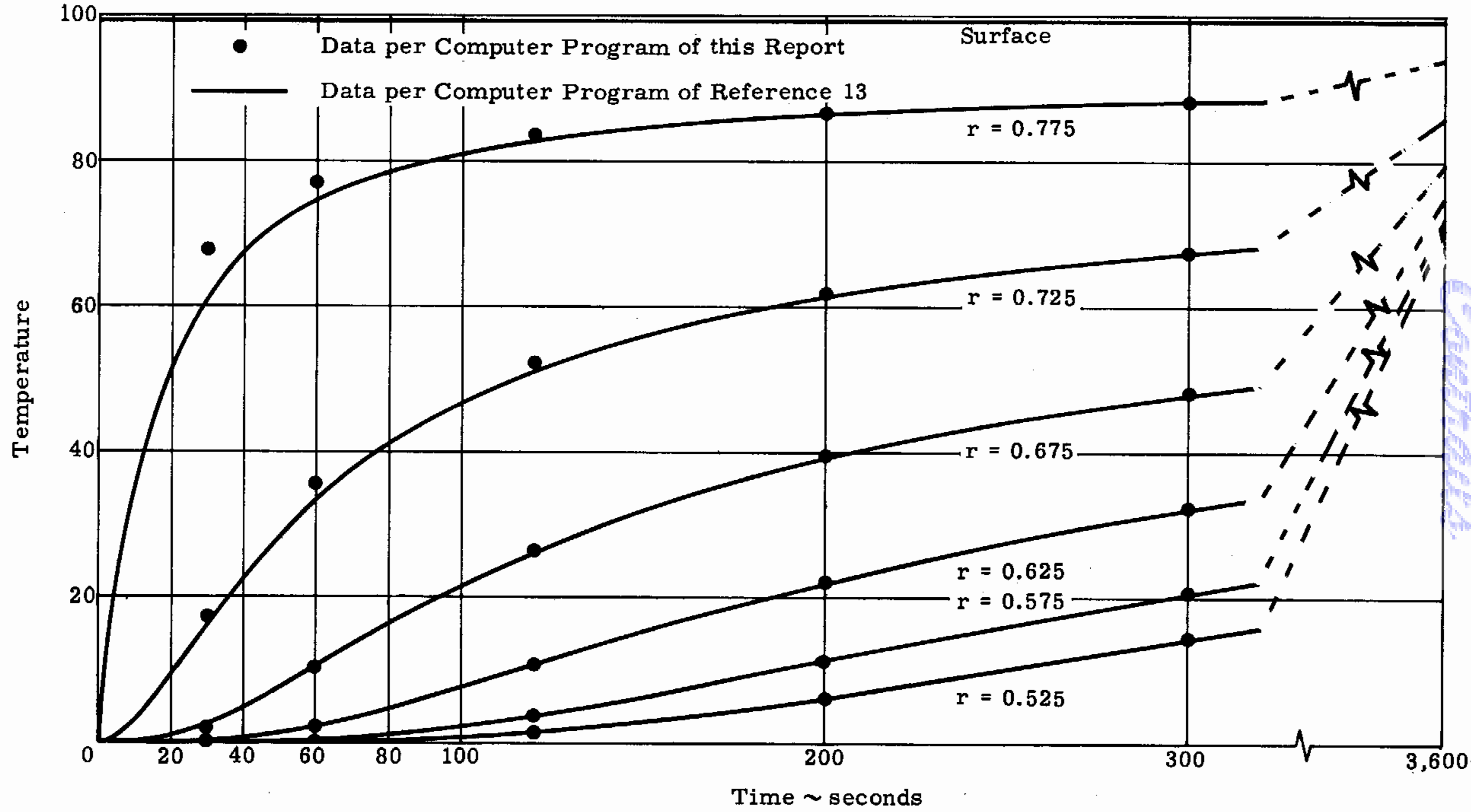


Figure 20. Temperature History, Sample Test Problem



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13. ABSTRACT

This study was undertaken to develop an analytical method of determining temperature distribution in a sector of a hollow circular cylinder with temperature dependent thermophysical properties based on exact mathematical solution. The subject nonlinear differential equation is reduced to a linear differential equation by an integral transform and with the assumption that only the thermal conductivity is a function of temperature whereas the thermal diffusivity remains constant. The solution has been programmed in FORTRAN IV for application to an IBM 7094. In addition, a successive transform applicable to the solution of similar problems involving temperature dependent thermal diffusivity is illustrated and discussed.

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| 14. KEY WORDS | LINK A ROLE | LINK A WT | LINK B ROLE | LINK B WT | LINK C ROLE | LINK C WT |
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| Nonlinear Differential Equation | | | | | | |
| Variable Thermophysical Properties | | | | | | |
| Integral Transform | | | | | | |
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| Computer Program | | | | | | |
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