DODCO, INC. RESEARCH IN OPTIMUM ADAPTIVE FLIGHT CONTROL

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I.) BACKGROUND

DODCO, INC. is a small analytical research group located near Blawenburg, New Jersey in the countryside between Princeton and the Delaware River. The firm was founded in 1955 by Mr. Daniel O. Dommasch, then an Associate Professor of Aeronautical Engineering at Princeton University. In 1956 Mr. Dommasch left Princeton to devote full time to the rapidly expanding activities of the company. During that period, DODCO gained considerable initial momentum in the exhaustive study of advanced aircraft dynamic performance capabilities, and an energetic research team was created to focus attention on the exploitation and further development of certain dynamic performance concepts evolved by Mr. Dommasch while at Princeton.

As of the present time DODCO has published between 40 and 50 technical reports under contract with the U. S. Air Force, the Navy and private industry. This work has covered virtually the entire spectrum of aircraft dynamic performance and has branched off into a great variety of related problem areas. Primary emphasis has been placed on utilization of the Euler-Lagrange variational calculus and special techniques of dynamic performance (as perfected at DODCO) to predict optimum performance trajectories (paths in space) for high-speed aircraft and missiles. Some of the maneuvers analyzed are:

optimum climbs and descents

programmed-throttle optimum range

minimum-time turns

optimum flight under conditions of very low "q", including "zoom" maneuvers and turns occurring at less than one "g" normal loading

least radiation hazard (or escape) paths

optimum boost of semi-orbital and orbital vehicles

optimum re-entry for controlled orbital equipment

In addition, a number of non-optimum topics have been treated:

dynamic tilt-plane turns

effects of engine control acceleration time on aircraft performance

perturbations on satellite orbits due to earth oblateness and motion of the satellite relative to the sun, viewing the latter as center of inertial coordinates

dispersion of ballistic re-entry vehicles

achievement of optimum supersonic range through program control

On the basis of these methods (for which proven computer programs are available) it has become possible to push aircraft to the very limits of their dynamic performance capability. But, the operational attainment of such performance and successful flight on the performance envelope itself, have posed severe problems in flight-path stabilization and control. For example, an optimum "zoom" maneuver which carries an aircraft to altitudes far above its normal static ceiling poses unusual demands on the controller-airframe combination due to the extremely low dynamic pressure acting on the aerodynamic control surfaces. If we are to gain the significant tactical advantages implicit in operational utilization of optimum flight at altitudes in excess of the static ceiling, we must provide control systems having a high degree of anticipation and adaptability in order to surmount the inherent lag and sluggishness of control surfaces under these conditions.

2.) NATURE OF THE AIRCRAFT CONTROL PROBLEM

Before attempting discussion of optimum adaptive control configurations, it will be worthwhile to consider for a moment the nature of the aircraft control problem. If we restrict our thoughts to airplane motion within a vertical plane, and if we ignore the effects of structural elasticity, then the longitudinal equations of force and moment balance are (see Figure 2:1):

along the flight path

$$F_e \cos\alpha - D - m\dot{V} - mg\sin\gamma = 0 \quad \ldots\ldots 2:1$$

normal to the flight path

$$F_e \sin\alpha + L - mV\dot{\gamma} - mg\cos\gamma = 0 \quad \ldots\ldots 2:2$$

moments about the mass center

$$M_S + M_D + M_C - \frac{d}{dt}(J\dot{\theta}) = 0 \quad \ldots\ldots 2:3$$

where

F_e = effective engine thrust

α = angle of attack (between vehicle body axis and velocity vector)

D = drag = $qS(C_{D_e} + KC_L^{2})$

m = mass = W/g

γ = inclination of velocity vector to local horizontal

L = lift = qSC_L = $qSa\sin\alpha$

M_S = static stability moment = $qSc\alpha C_{m_\alpha}$

M_D = damping moment = $qSc(\dot{\alpha} C_{m_{\dot{\alpha}}} + \dot{\theta} C_{m_{\dot{\theta}}})$

M_C = control moment = $qSc(\delta C_{m_\delta} + \dot{\delta} C_{m_{\dot{\delta}}})$

J = polar moment of inertia in pitch = $f(t)$

θ = angle between vehicle body axis and local horizontal = $\alpha + \gamma$

This, in simplest form, is the complete longitudinal dynamics picture. Should we, however, wish to consider flight at speeds in excess of about Mach 3.5, then additional terms must be incorporated to account for "orbital relief" effects, and at altitudes in excess of approximately 150,000 feet we must include the variation of gravity potential with altitude.

Using coefficient notation, we may write the moment equation (2:3) in the form

$$\ddot{\theta} = \frac{qSc}{J}\left[\alpha C_{m_\alpha} + \dot{\alpha} C_{m_{\dot{\alpha}}} + \dot{\theta}(C_{m_{\dot{\theta}}} - \dot{J}/qSc) + \delta C_{m_\delta} + \dot{\delta} C_{m_{\dot{\delta}}}\right] \quad \text{..........2:4}$$

The $\dot{J}$ term in this expression is of considerable interest; it arose during total differentiation of the product $J\dot{\theta}$ in the basic moment equation, and its appearance in 2:4 indicates that time rate of change of polar moment of inertia may produce an alteration in the apparent damping due to $\dot{\theta}$. The $\dot{J}$ effects may become quite significant during periods of high fuel flow rate, such as occurs with afterburner use.

The control of an aircraft involves a surprisingly long sequence of events, which (essentially in the order of their occurrence) are:

1. The pilot reacts to a combination of stimuli.
2. A control-stick motion or force applied by the pilot is converted to a command signal by the controller.
3. Hydraulic fluid flows in the servo actuator.
4. The control surface undergoes acceleration.
5. Once sufficient time has elapsed, a significant change in control surface displacement has occurred, and thus the aerodynamic flow pattern about the elevator begins to change.
6. After another delay, the new aerodynamic circulation field is obtained, and a new resultant force is produced on the control surface.

7. This control force will, in general, alter $\ddot{\theta}$.

8. The angular acceleration integrates to a value of θ, which occurs, initially, primarily in the form of an α increment.

9. The new value of α changes (in the course of time) the circulation about the wings and the resulting lift.

10. The change in lift produces a change in $\dot{\gamma}$.

11. $\dot{\gamma}$ integrates to a new flight path inclination γ.

With unfavorable conditions, the cumulative effects of the various lags just mentioned may produce a very critical "dead time", which can (in some cases) render aerodynamic controls virtually useless. *

We mention these rather gloomy considerations because they have considerable bearing on the design of optimum configurations for adaptive controllers. Mathematically, we are interested in certain partial derivatives of θ, $\dot{\theta}$ and $\ddot{\theta}$ with respect to control displacement δ; it follows that:

$$\partial\theta/\partial\delta = 0 \quad \text{2:5}$$

$$\partial\dot{\theta}/\partial\delta = 0 \quad \text{2:6}$$

$$\partial\ddot{\theta}/\partial\delta = qScC_{m_\delta}/J \equiv \beta_0 \quad \text{2:7}$$

for any instant of time t.

The problem of aircraft control is fundamentally one of generating appropriate command signals for the control-surface actuator. As we have seen, displacements of the control surface ultimately have an effect on aircraft angle of

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* Professor D. C. Hazen of Princeton University has reported a 180 degree phase difference between the angular position of an airfoil oscillating at 5 c.p.s. and the resultant lift direction. This data was obtained at a low airspeed (30-40 f.p.s.). It is probable that the frequency for 180 degree lag is roughly proportional to airspeed.

attack, which in turn must prescribe simultaneous values of $\dot{V}$ and $\dot{\gamma}$, obtainable (without ambiguity*) from the equations of motion (2:1, 2:2). In the course of its flight the aircraft "integrates" these differential relationships to give values of V and γ which are everywhere compatible with the "path function", expressed either as $\alpha(t)$, $\theta(t)$ or $\delta(t)$ with, of course, appropriate initial conditions where required. Inasmuch as α and θ are directly related through γ, they may be viewed as alternative variables; however, the specification of a $\delta(t)$ demands that we employ the moment balance equation (2:4).

From the adaptive control systems point of view, the aircraft equations of force and moment balance, along with such relations as might exist between servo-actuator command and output, constitute the fundamental system properties in mathematical form. It is important that we recognize the inseparable nature of these relationships, that is to say, that a process of continuous interaction is going on between them and that--therefore--no single equation can possibly express the total dynamic situation. This fact remains just as true in the limit (at a localized point in space and time) as it does for the entire integrated path.

The crux of the adaptive control problem is that we seldom have adequate foreknowledge of the various system coefficients (and there are many) within the pertinent set of equations. Handicapped as we are by this lack of vital information, we are forced to base control system adaptation on the instantaneous values of pilot command and existing system output, along with certain lower-ordered derivatives of these quantities.

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* If we specify $\dot{V}$ rather than α, there are at least two values of $\dot{\gamma}$ which will satisfy the equations of motion. This fact is readily established by eliminating α between equations 2:1 and 2:2.

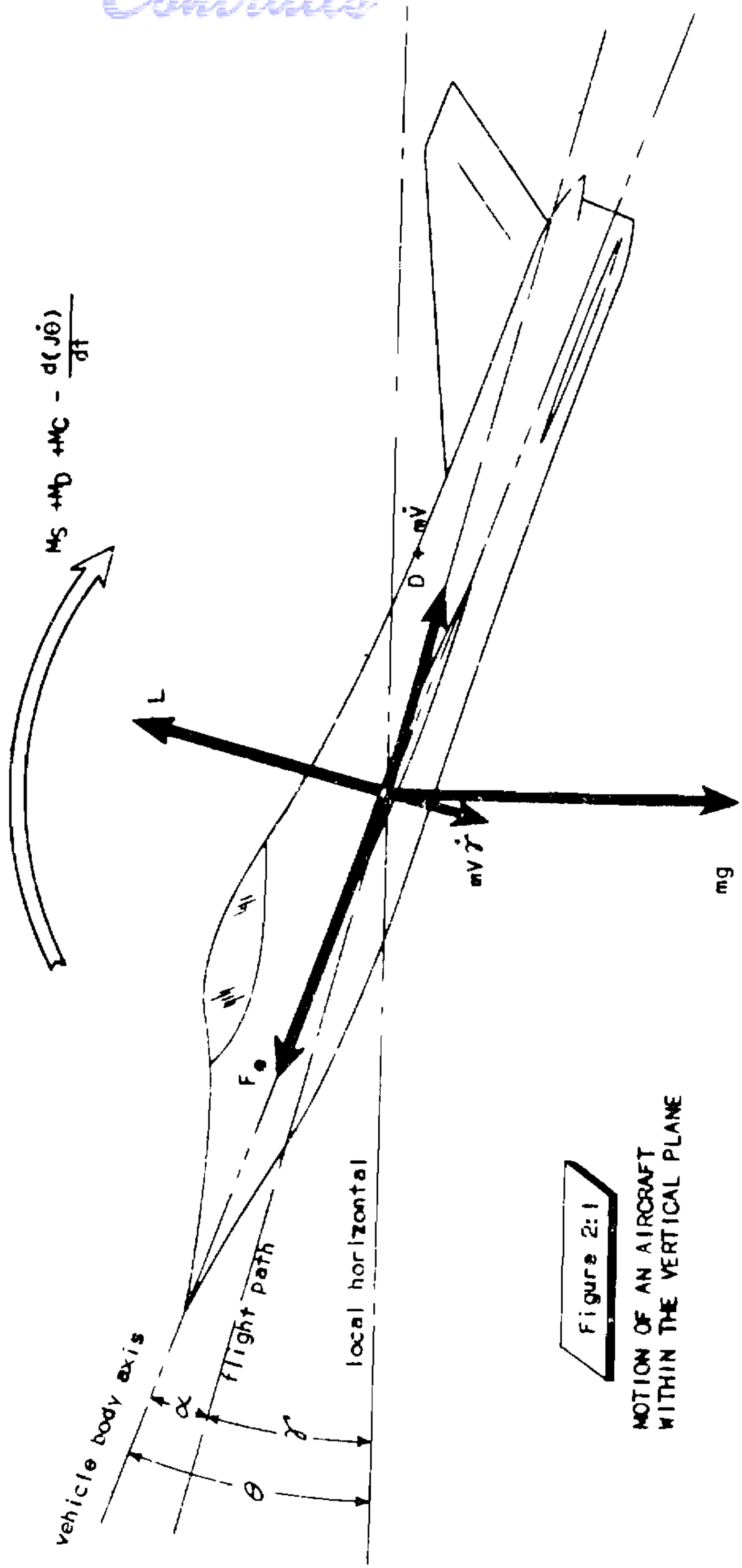


Figure 2:1

MOTION OF AN AIRCRAFT WITHIN THE VERTICAL PLANE

3.) ADAPTIVE CRITERIA AND THEIR VARIATIONAL INTERPRETATION

For the problem we have just defined, namely, control with limited information about the dynamic elements involved, one may postulate a number of plausible adaptive criteria. Much of the adaptive control work undertaken by DODCO has been concerned with the relative merits of various criteria expressible in integral form, and particularly, with the variational calculus interpretation of these integrals.

In writing integral criteria, we observe that, although various coordinate transformations may be possible, time remains the fundamental independent variable for all dynamic systems. Thus it is appropriate to write generalized integral criteria in the form

$$\psi = \int_o^t G \, dt \quad \text{.......} 3{:}1$$

where G might be referred to as the "variational integrand" and the specification of the upper limit t signifies that this is a <u>definite</u> integral with which we are dealing (in practice, the value of t need not be known in advance).

For purposes of direct comparison with existing (arbitrary) adaptive flight controllers, we may define system "error" as the difference between the output of a linear model (which is continuously responding to the pilot's commands) and a measure of the aircraft pitch rate, $\dot{\theta}$. Hence

$$e = \dot{\theta}_m - \dot{\theta} \quad \text{.......} 3{:}2$$

where $\dot{\theta}_m$ is the model value. In terms of this definition, several variational integrands suggest themselves immediately, for example

$G = e^2$

$G = t/e/$

These functions (and a number of other related forms) have received considerable attention in linear analyses, and we might therefore expect them to yield useful results when subjected to variational treatment (by which we mean other than the statistical approach). If we set aside for the moment any considerations of fundamental system restraints (which may require introduction of one or more Lagrange multipliers) we may rapidly examine integrands of the type just given to ascertain the nature of their variational extremums, if any.

Since we are dealing with an aircraft control problem, the elevator angular displacement δ represents (to a large extent) the basic Euler "freedom" within the system, that is, the primary dependent variable in the Euler sense. The secondary dependent quantities (governed by δ acting within the framework of the system equations) are, presumably, θ, $\dot{\theta}$ and $\ddot{\theta}$.

Now if we recall the conclusions summarized in the partial derivative relations 2:5, 2:6 and 2:7 it is (perhaps rather painfully) evident that $\partial\ddot{\theta}/\partial\delta$ is the only such parameter which does not vanish explicitly from any formulation. The blunt interpretation of this fact is that δ cannot in any way alter the value of e (equation 3:2) in less than a finite time interval, and hence that no valid "time now" variational formulation exists which can handle a G = e type integrand.

This may seem somewhat disappointing, and in fact, we have considered going to a small-interval prediction scheme so as to forcefully introduce θ and $\dot{\theta}$ dependence on elevator deflection. But this, however, proves unnecessary, for the real fault may be shown to lie within the postulated criterion. In the course of our study of the problem at DODCO, we have arrived at the following general approach to the specification of adaptive criteria:

1. It is the task of the control system used to "control with authority" and therefore the unrestrained integrand must be based on a term which demands this accomplishment.

2. To prevent excessive actuation levels, and most particularly, to provide constraints on the error and error derivatives built up along the path, a subsidiary condition must be introduced which demands simultaneous satisfaction of the error requirements peculiar to the given system.

To be specific, the first item mentioned leads to the following term within the integrand G

$$\frac{1}{\beta_o \Delta\delta}$$

where β_o is defined as the value of $\partial\ddot{\theta}/\partial\delta$ (see 2:7) and $\Delta\delta$ is defined as the hypothetical instantaneous jump in the displacement of the elevator which we should <u>like</u> to have occur at the moment of time in question.

The second item discussed may lead variously to restraints of the form

$$\frac{\lambda}{e + K\dot{e}} \quad \text{.......... A}$$

$$\frac{\lambda}{e + K_1\dot{e} + K_2\ddot{e}} \quad \text{.......... B}$$

$$\frac{\lambda}{(1+t/\tau)e + K\dot{e}} \quad \text{.......... C}$$

$$\frac{\lambda}{e_c + e + K\dot{e}} \quad \text{.......... D}$$

and

$$\frac{\lambda}{\frac{e}{\dot{e}^2} + \frac{K_1\dot{e}}{e^2} + A_1 e + A_2\dot{e}} \quad \text{.......... E}$$

in which λ is the Lagrange multiplier (a constant due to the definite integral form of the subsidiary condition); A_1, A_2, K, K_1, K_2 and τ are positive constants (other than zero) and e_c is what we might best term the "feed-forward error", given by

$$e_c = \dot{\theta}_c - \dot{\theta} \quad \text{......} 3:3$$

where the subscript "c" denotes the value of the pilot's command.

Let us consider restraint "A", the simplest of those listed. Using it, the complete integrand becomes

$$G = 1/\beta_0 \Delta\delta + \lambda/(e + K\dot{e}) \quad \text{......} 3:4$$

It should be emphasized that variational integrands of the type given in 3:4 are not necessarily the most appropriate when almost everything is known in advance about system dynamic properties, or when reasonably accurate measurements may be readily made which will yield the needed information. Functions of this type do, however, appear to work out very well in terms of the strict adaptive control problem, for which there exists a definite paucity of either advance or in-flight measured data regarding characteristics of the aircraft.

Variationally, we seek to find extremums of the integral of G as expressed in equation 3:4. It is possible to show that the extremums obtained by an application of the pertinent Euler relation are those which involve maximums of the product $\beta_0 \Delta\delta$, subject of course to the stipulated condition on e and $\dot{e}$. Since the quantity β_0 is precisely the instantaneous control effectiveness, it is evident that the extremums will produce greater elevator deflection changes $\Delta\delta$ when the control effectiveness is small, yet will be "satisfied" with lesser deflections when the effectiveness is large.

As for the subsidiary condition, it follows from 3:4 that we are demanding that the integral values of e and $\dot{e}$ (between the limits 0 and t) possess the particular values fore-ordained by our choice of λ. Thus we are, mathematically, insisting that the system produce a certain amount of

$$\int_0^t (e + K\dot{e})\, dt$$

No system can ever be perfectly free of error in its response to commands and disturbances, so from the practical point of view this restraint is tantamount to a stringent limitation on the build-up of error. Furthermore, the restraint prevents the variationally-correct but unwanted and trivial solution which says, "set $\Delta\delta$ to infinity."

4.) APPLICATION OF THE VARIATIONAL METHODS

The integrand given in equation 3:4 may be readily analyzed to obtain an expression for the optimum $\Delta\delta$ as a function of time. Treating $\Delta\delta$ as the aircraft system freedom, we ask the question: "What is the optimum way in which to vary $\Delta\delta$ with time so that the adaptive performance index (integral criterion) is rendered a minimum?"

The pertinent Euler variational relation is

$$\frac{d}{dt}(\partial G/\partial \Delta\dot{\delta}) - \partial G/\Delta\delta = 0 \quad \text{.......... 4:1}$$

which constitutes the necessary condition which the controller must satisfy. Evaluating the various derivative terms, we obtain:

$$\partial G/\Delta\dot{\delta} = 0 \quad \text{(ignoring the very small } C_{m_{\dot{\delta}}} \text{ moment contribution)} \quad \text{... 4:2}$$

$$\partial G/\Delta\delta = -1/\beta_o(\Delta\delta)^2 \;\; -\lambda(K\partial\dot{e}/\partial\Delta\delta)/(e + K\dot{e})^2 \quad \text{.......... 4:3}$$

But

$$\partial\dot{e}/\partial\Delta\delta = -\partial\ddot{\theta}/\partial\Delta\delta = -\beta_o \quad \text{.......... 4:4}$$

We note in this connection that β_o is independent of $\Delta\delta$, although saturation (control-surface stall) effects cause β_o to vary with δ. Combining the four equations just set forth, and solving for $\beta_o\Delta\delta$ gives

$$\beta_o\Delta\delta = (e + K\dot{e})/\sqrt{\lambda K} \quad \text{.......... 4:5}$$

Were the instantaneous value of β_o known as a function of time, equation 4:5 would yield an immediate answer in terms of the optimum $\Delta\delta$. Although measurement of the control effectiveness does not present an impossible air-data problem, it is desirable to free ourselves as much as we can from the necessity of measuring or computing such a parameter during flight.

To circumvent the need for measurement or calculation of β_O, it is only necessary that we recognize the relationship between increments in angular acceleration and increments in elevator position; thus

$$\beta_o \Delta\delta = \Delta\ddot{\theta} \qquad \text{4:6}$$

This expression is especially useful because it takes into account all the various lags which are involved in producing a $\ddot{\theta}$ response to elevator deflection.

Presuming the existence of a small airborne analog or digital computer capable of generating desired values of $\Delta\ddot{\theta}$, all that remains for us to accomplish within the system is the algebraic addition of this desired increment to the actual angular acceleration existing at the point in question, and then the excitation of an acceleration control loop which causes the aircraft to follow the desired total $\ddot{\theta}$ with a reasonable degree of precision. This inner acceleration loop (which encompasses the servo-actuator and the aircraft) might also contain some form of optimum adaptive compensation, but for our present purposes it is sufficient to think of it as simply a fairly high-gain conventional loop driven by an error signal defined as

$$\epsilon = \ddot{\theta}_d - \ddot{\theta} \qquad \text{4:7}$$

where $\ddot{\theta}_d$ is the desired total $\ddot{\theta}$ obtained on a sampled basis from the control computer, in accordance with the relation

$$\ddot{\theta}_d = \ddot{\theta}_{actual} + \Delta\ddot{\theta} \qquad \text{4:8}$$

The sampling interval should be of the order of 0.005 to 0.010 second.

Perhaps a few words regarding the general character of equation 4:5 (the optimum governing expression) are in order. It might have been anticipated that the results applicable to the limited-information control problem would not be especially complex, yet it is refreshing to find the variational methods leading to so straightforward a solution.

It is, further, of great interest that the variational calculus has led to a configuration which (from past experience) we know to be highly workable. Thus, the use of proportional-error and proportional-error-rate control is known to produce very good response, with the possible exception being the steady-state condition, in which some integral error feedback may be of value.

It is clearly possible to specify alternate criteria which place more exacting demands on the control system and therefore, in general, exhibit superior response characteristics. One such restraint is that labeled "E" in the list above. In addition to the usual restriction on e and $\dot{e}$ we have introduced the terms $e/\dot{e}^2$ and $K_1\dot{e}/e^2$. The first of these places an exceptionally strong limitation on the build-up of steady-state-error; when $\dot{e}$ becomes small (as in the steady state), e is greatly emphasized and the controller is forced to reduce it vigorously. The term $K_1\dot{e}/e^2$ limits transient overshoot type errors, since when e becomes small in the presence of a large $\dot{e}$ (overshoot conditions) the $\dot{e}$ is strongly emphasized and therefore restrained. The use of this restraint condition leads to the relations

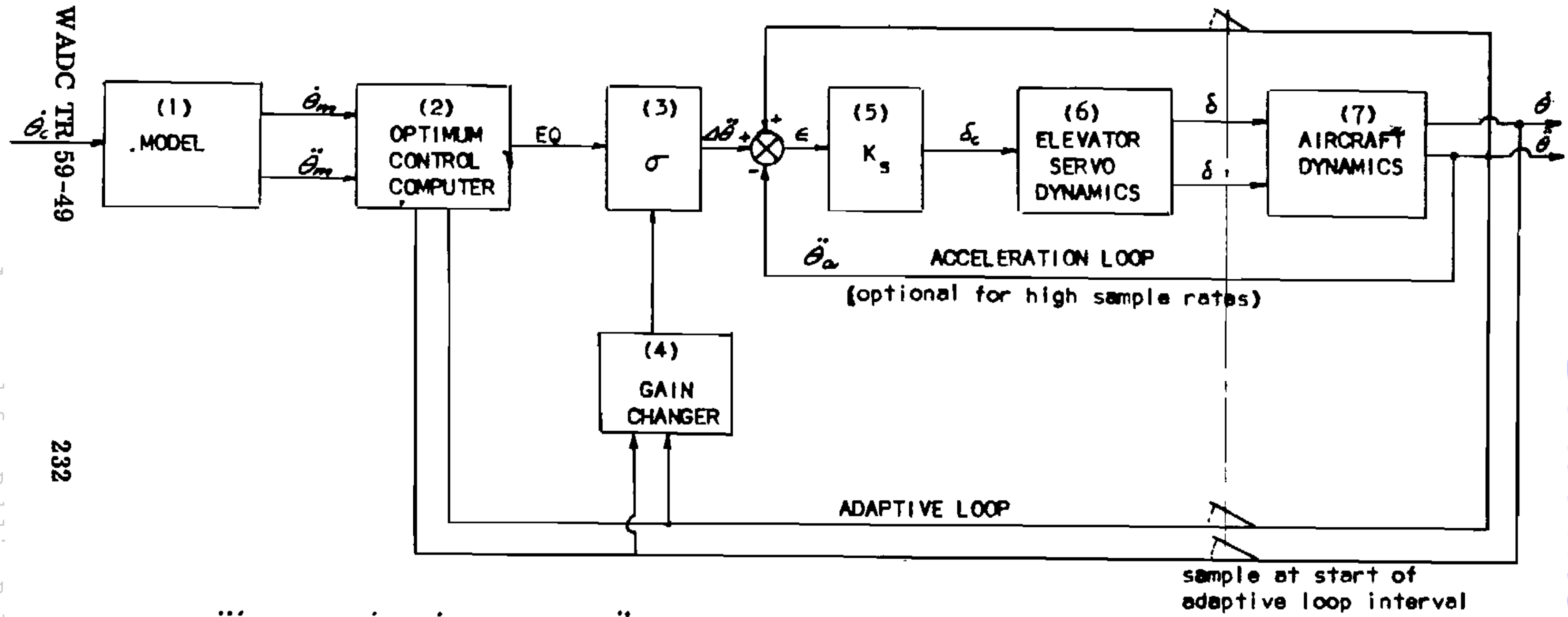
$$\Delta\ddot{\theta} = \beta_o\Delta\delta = \sigma \left[\frac{e}{\dot{e}^2} + K_1 \frac{\dot{e}}{e^2} + A_1 e + A_2\dot{e} \right] \quad \text{.....................4:9}$$

$$\sigma = 1 \Big/ \sqrt{\lambda \left[A_2 + \frac{K_1}{e^2} - \frac{2e}{\dot{e}^3} \right]} \quad \text{.................................4:10}$$

The factor σ may be considered as a variable optimum gain, since it multiplies the error quantity of the restraint "E" to produce the actuation signal, $\Delta\ddot{\theta}$. This is in accord with conventional servomechanism terminology regarding gain. A preliminary analysis of numerical results obtained using this controller indicates that the optimum gain plays a very important role.

Perhaps the greatest significance of these results lies in the clear-cut connection between them and the relevant limited-information integral criteria. For instance, it has been demonstrated that proportional-error plus proportional-error-rate control is actually the best that can be done in terms of the criterion of equations 3:1 and 3:4. Likewise, a configuration more sensitive to transient overshoot and steady state error conditions is derived from an integrand involving the restraint "E".

Figure 4:1 presents the optimum adaptive control configuration described by equations 4:9 and 4:10. In keeping with the previous discussion an inner acceleration control loop has been incorporated in the system.



(1) $\dddot\theta_m = \omega_{n_m}^2 (\dot\theta_c - \dot\theta_m) - 2\zeta_m \omega_{n_m} \ddot\theta_m$

(2) $(EQ) = e/\dot e^2 + K_1 \dot e/e^2 + A_1 e + A_2 \dot e$

(3) $\Delta\ddot\theta_d = \sigma \cdot (EQ)$

(4) $\sigma = 1 \Big/ \sqrt{(\lambda)(e/\dot e^2 + A_2 - 2e/\dot e^3)}$

(5) $\delta_c = K_S \quad \epsilon$

(6) $\ddot\delta = \omega_{n_S}^2 \delta_c - 2\zeta_S \omega_{n_S} \dot\delta - \omega_{n_S}^2 \delta$

(7) $\ddot\theta = q\frac{S_c}{J} (\alpha C_{m_\alpha} + \dot\alpha C_{m_{\dot\alpha}} + \dot\theta C_{m_{\dot\theta}} + \dot\delta C_{m_{\dot\delta}} + \delta C_{m_\delta}) - \dot\theta (\dot J/J)$

Figure 4:1
OPTIMUM ADAPTIVE CONTROL SYSTEM BLOCK DIAGRAM

5.) DIGITAL COMPUTER RESULTS AND CONCLUDING REMARKS

Figures 2:2 through 2:9 on the next three pages present curves of the various aircraft coefficients and engine thrust values as functions of Mach number and (in some cases) Mach number plus altitude or elevator deflection. These coefficient values have been arranged in tabular form suitable for rapid table-lookup routines prepared especially for the LGP-30 stored program computer. The actual numbers assigned to the coefficients represent a purely hypothetical aircraft, and no aircraft with these exact characteristics is known to exist.

These data have been utilized in the ABLE computer program (Adaptive Behavior in Longitudinal Environment); the equation flow within this program follows essentially the pattern given in the Appendix to this paper.

The figures on the succeeding pages of this section present preliminary numerical results obtained with the ABLE routine, employing two different optimum control relations and several different initial conditions, as well as acceleration-loop gain constants. Figure 5:1 is devoted exclusively to the response characteristics of the elevator-servo plus aircraft combination, and illustrates the acceleration-control servo dynamic properties as well as the nature of the aircraft $\ddot{\theta}$ buildup (assuming no circulation buildup lag, which is essentially true at these altitudes and Mach numbers). Figures 5:2 -- 5:4 are detailed plots of the aircraft response, elevator displacement, and parameter variations for a ramp input command from the pilot. The controller used was a linear optimum configuration derived from a combination of restraints C and D, and thus involved both time-weighted and feed-forward errors. The final four figures in this section represent data obtained using the optimum variable-gain controller described by equations 4:9 and 4:10, as compared with an arbitrary configuration developed from empirical considerations.

The first two figures (5:5 and 5:6) show the response to a step input command; it can be seen that the optimum variable-gain configuration response follows the second order model characteristics almost identically. It is doubtful that following errors below this level could be sensed. Figures 5:7 and 5:8 show the response to a large amplitude, high frequency sinusoidal input obtained with the variable gain and arbitrary controllers, respectively.

A great number of problem areas still remain to be investigated as a part of the contractual effort in which DODCO is engaged, however, we feel that we have gained some degree of initial orientation in the adaptive control subject. The power of the variational calculus in the design of high performance control configurations for the case of limited system information has been demonstrated, and there is every reason to believe that even higher levels of sophistication may be attained, both as understanding of integral criteria increases and as the methods are applied to problems for which we possess somewhat more advance or measurable data regarding the system dynamic properties. The preliminary results of a complete digital-computer simulation of aircraft response would seem to indicate that therein exists a very fruitful avenue for future exploitation.

ABLE-VEHICLE CHARACTERISTICS

(TRIMMED FLIGHT)

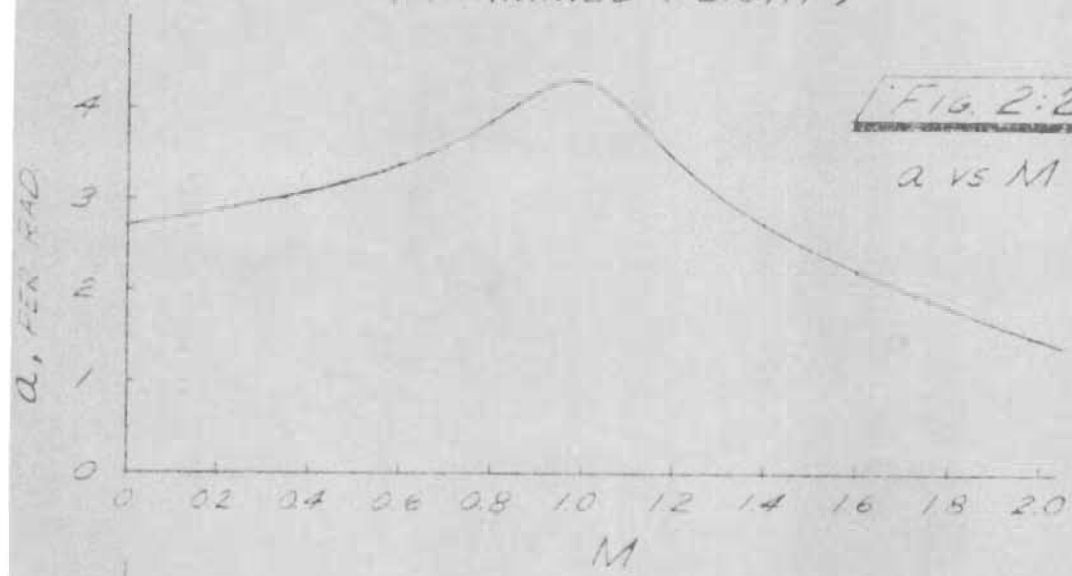


FIG. 2:2

a VS M

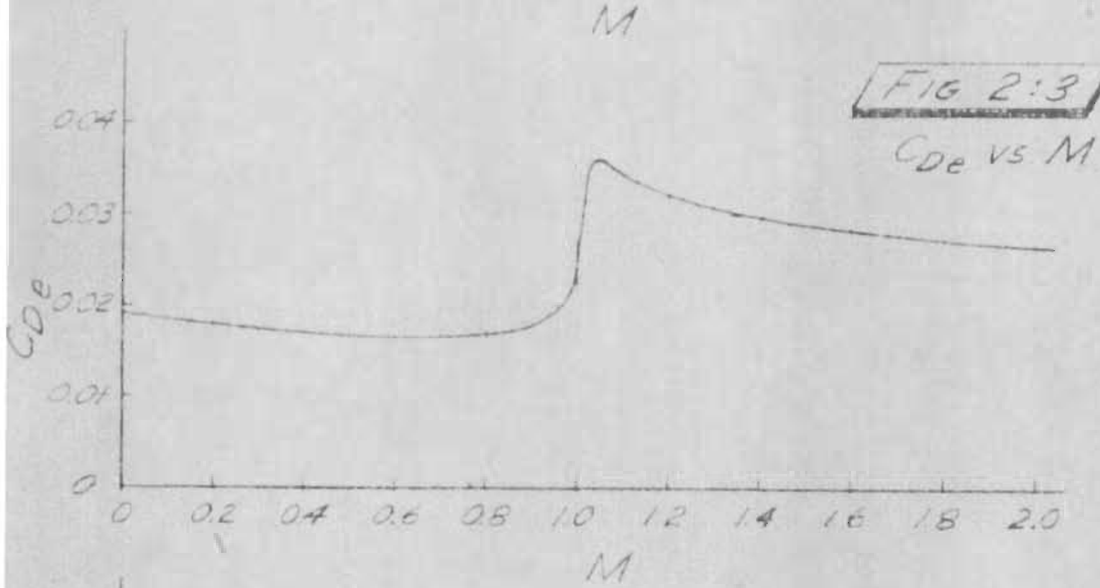


FIG 2:3

C_{De} VS M

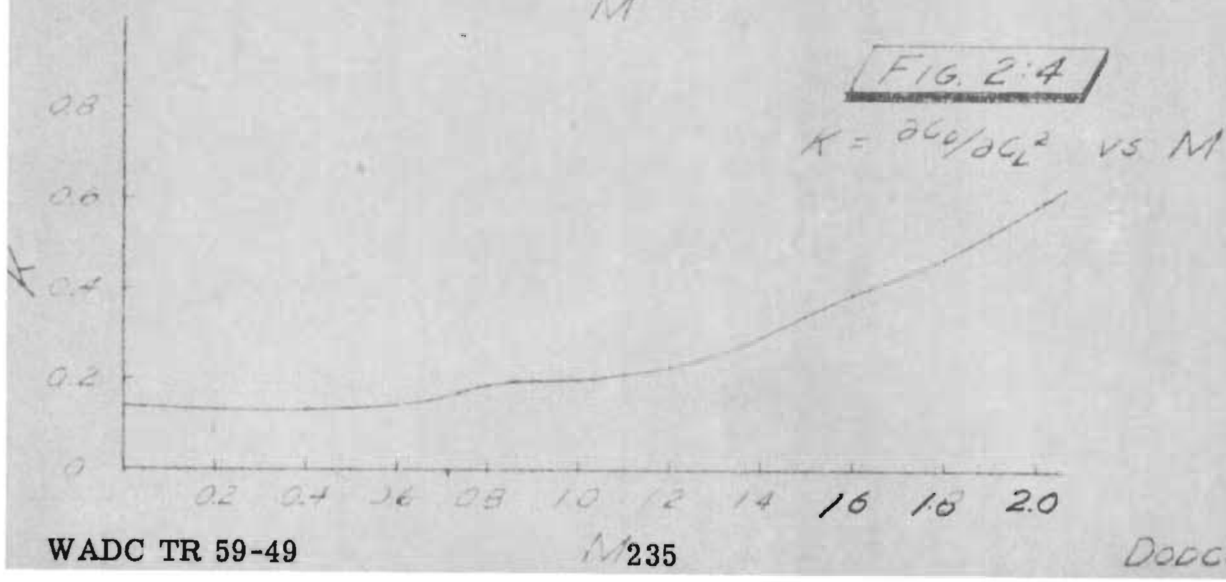


FIG. 2:4

$K = \partial C_D / \partial C_L^2$ VS M

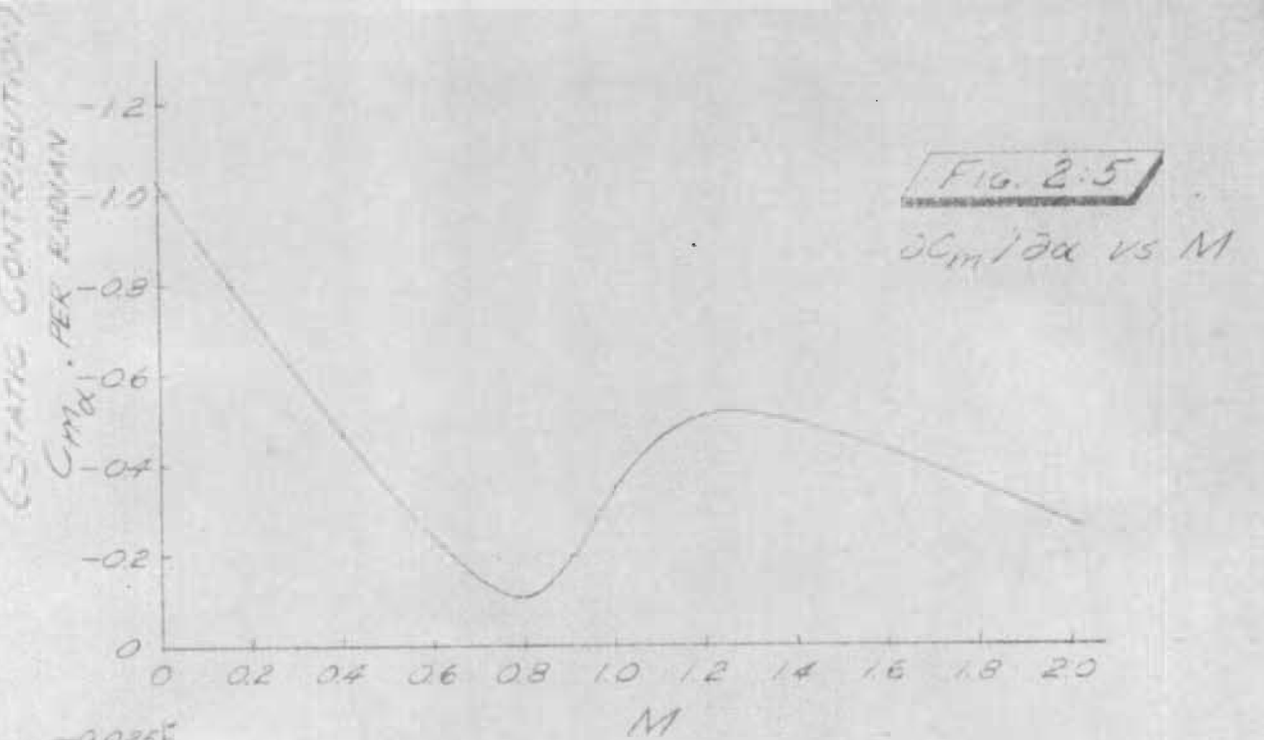


FIG. 2:5
∂Cm/∂α vs M
(STATIC CONTRIBUTION) Cmα, PER RADIAN
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
M

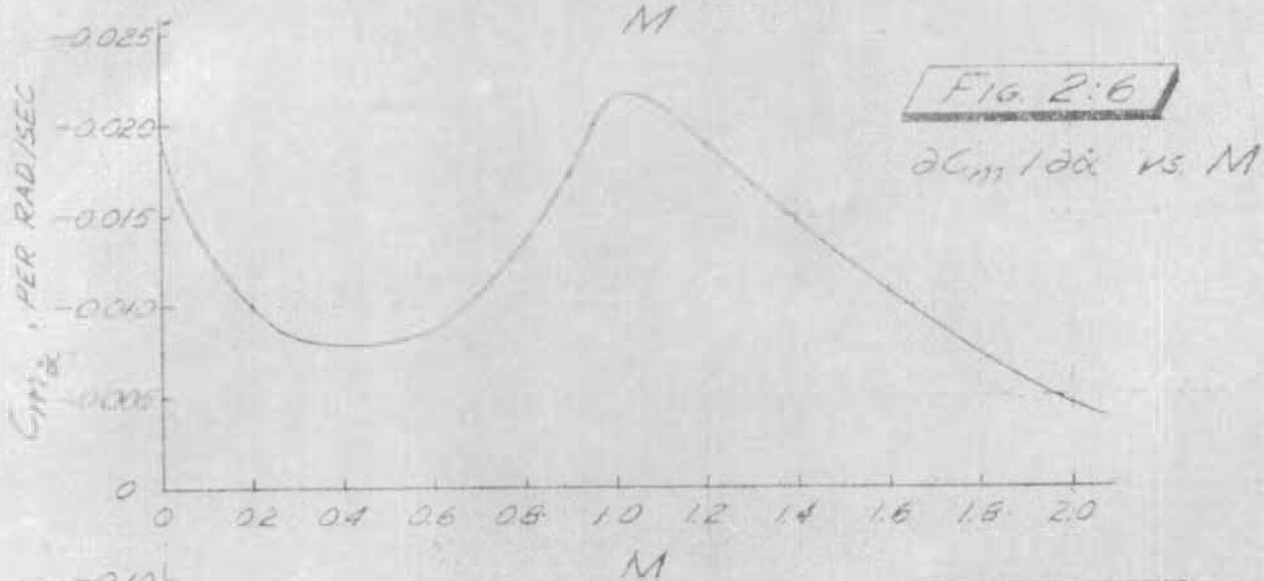


FIG. 2:6
∂Cm/∂α̇ vs M
Cmα̇, PER RAD/SEC
-0.025
-0.020
-0.015
-0.010
-0.005
0
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
M

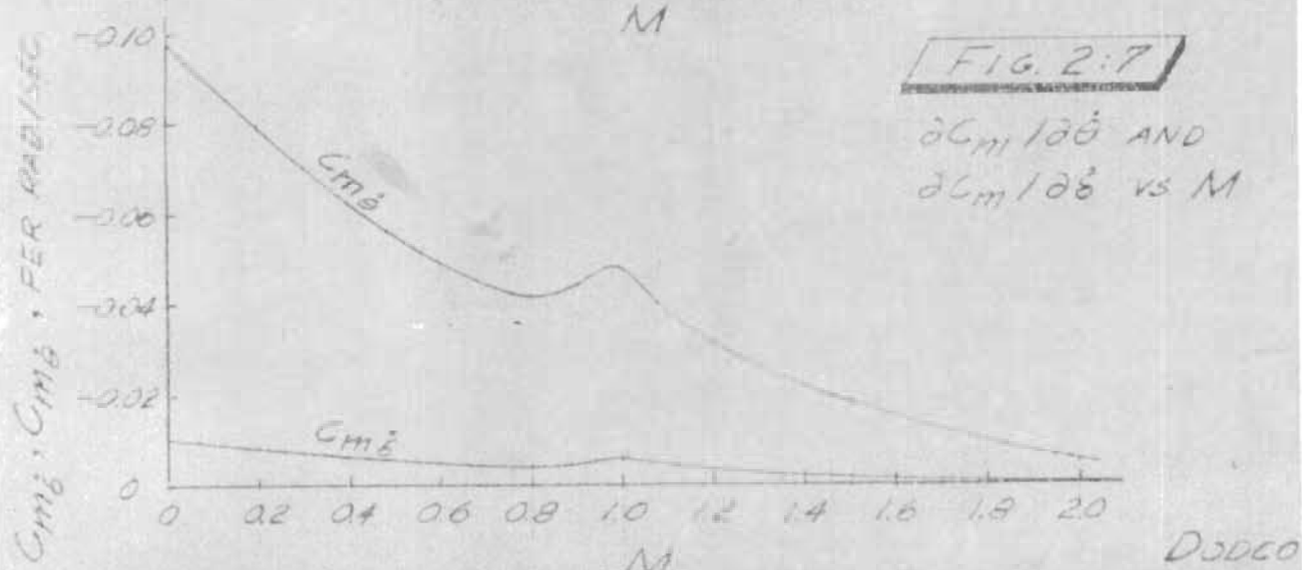


FIG. 2:7
∂Cm/∂θ̇ AND
∂Cm/∂δ̇ vs M
Cmθ̇, Cmδ̇, PER RAD/SEC
-0.10
-0.08
-0.06
-0.04
-0.02
0
Cmθ̇
Cmδ̇
0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
M
DODCO

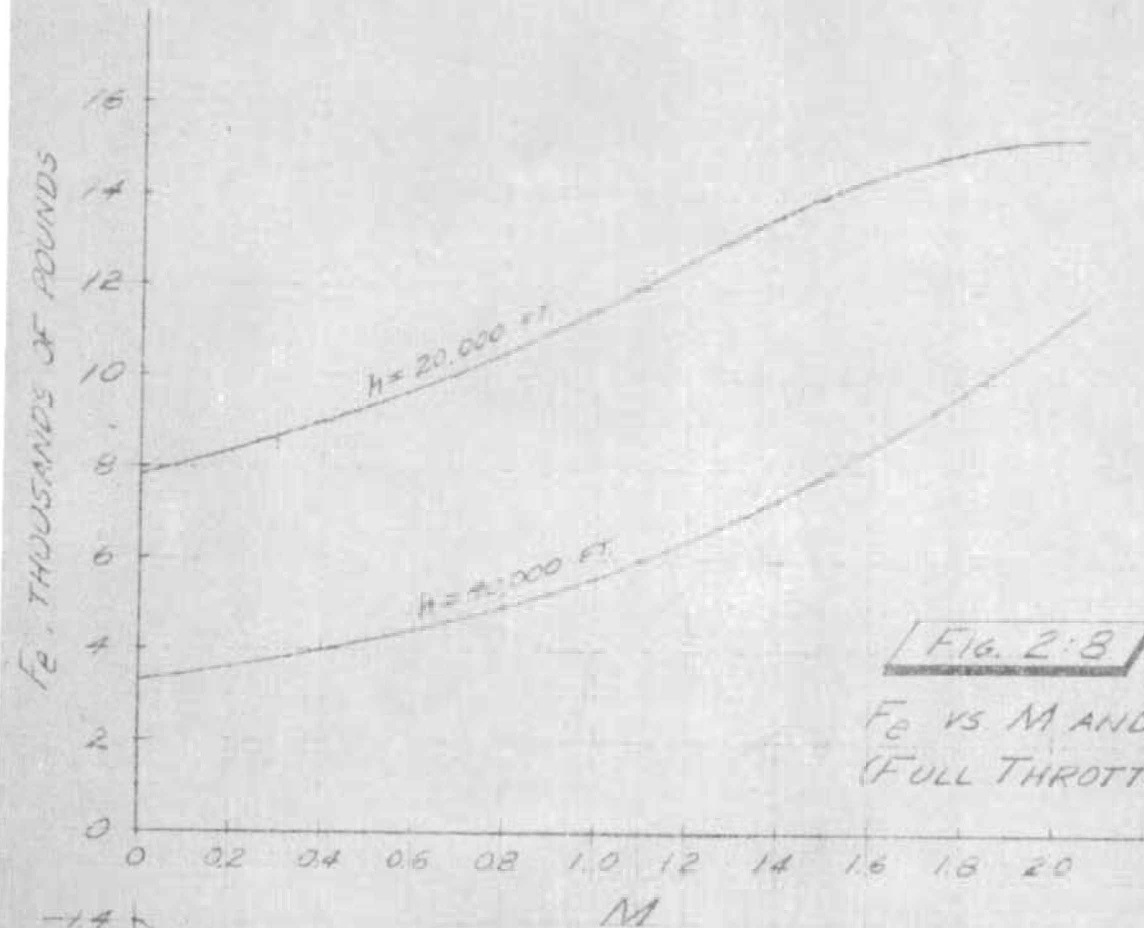


FIG. 2:8

F_e VS. M AND h
(FULL THROTTLE)

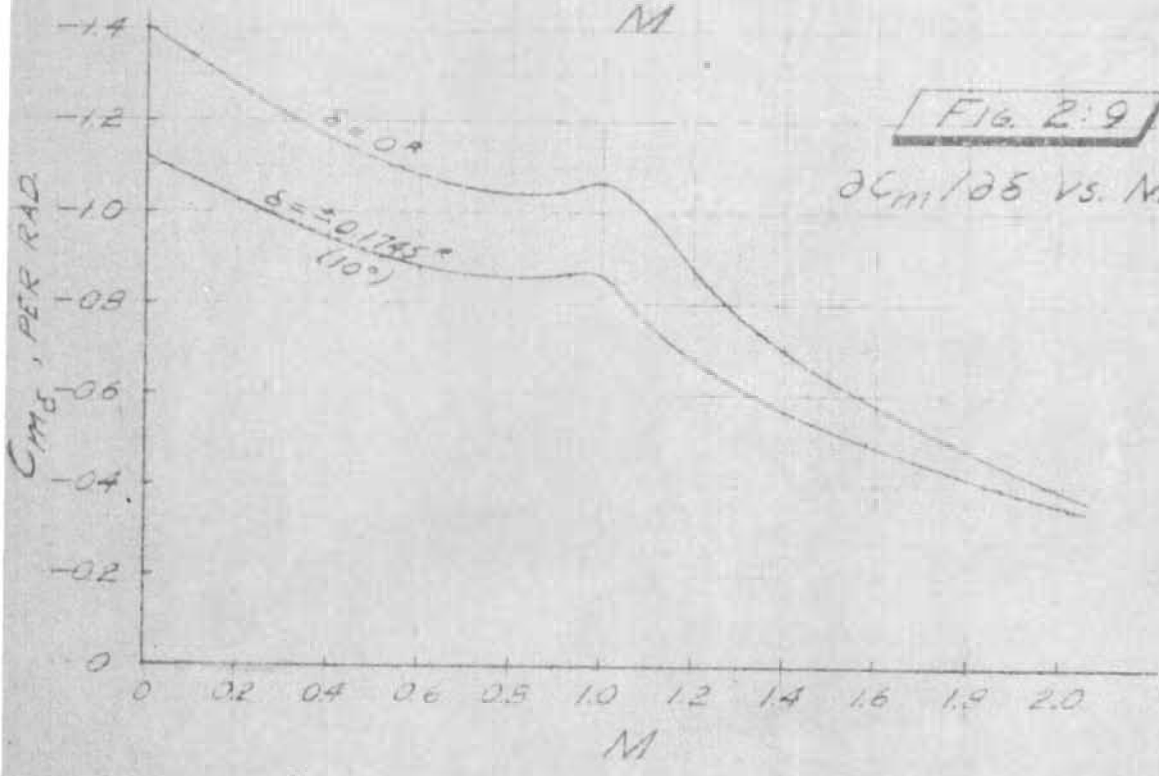


FIG. 2:9

$\partial C_m / \partial \delta$ VS. M AND δ

DODCO

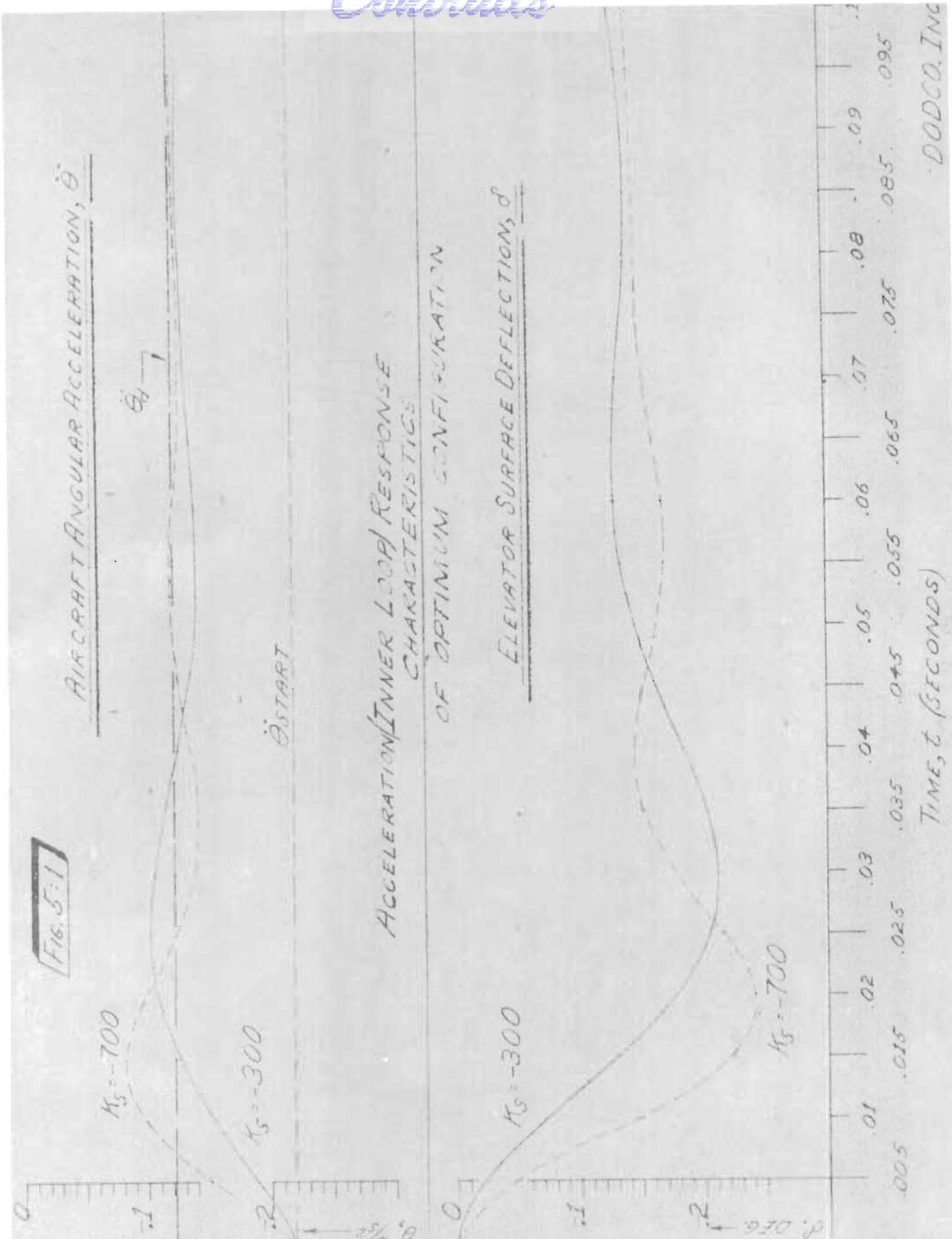


FIG. 5:1
AIRCRAFT ANGULAR ACCELERATION, θ̈
ACCELERATION (INNER LOOP) RESPONSE CHARACTERISTICS OF OPTIMUM CONFIGURATION
ELEVATOR SURFACE DEFLECTION, δ
K_S = -700
K_S = -300
θ̈ START
TIME, t (SECONDS)
.005 .01 .015 .02 .025 .03 .035 .04 .045 .05 .055 .06 .065 .07 .075 .08 .085 .09 .095
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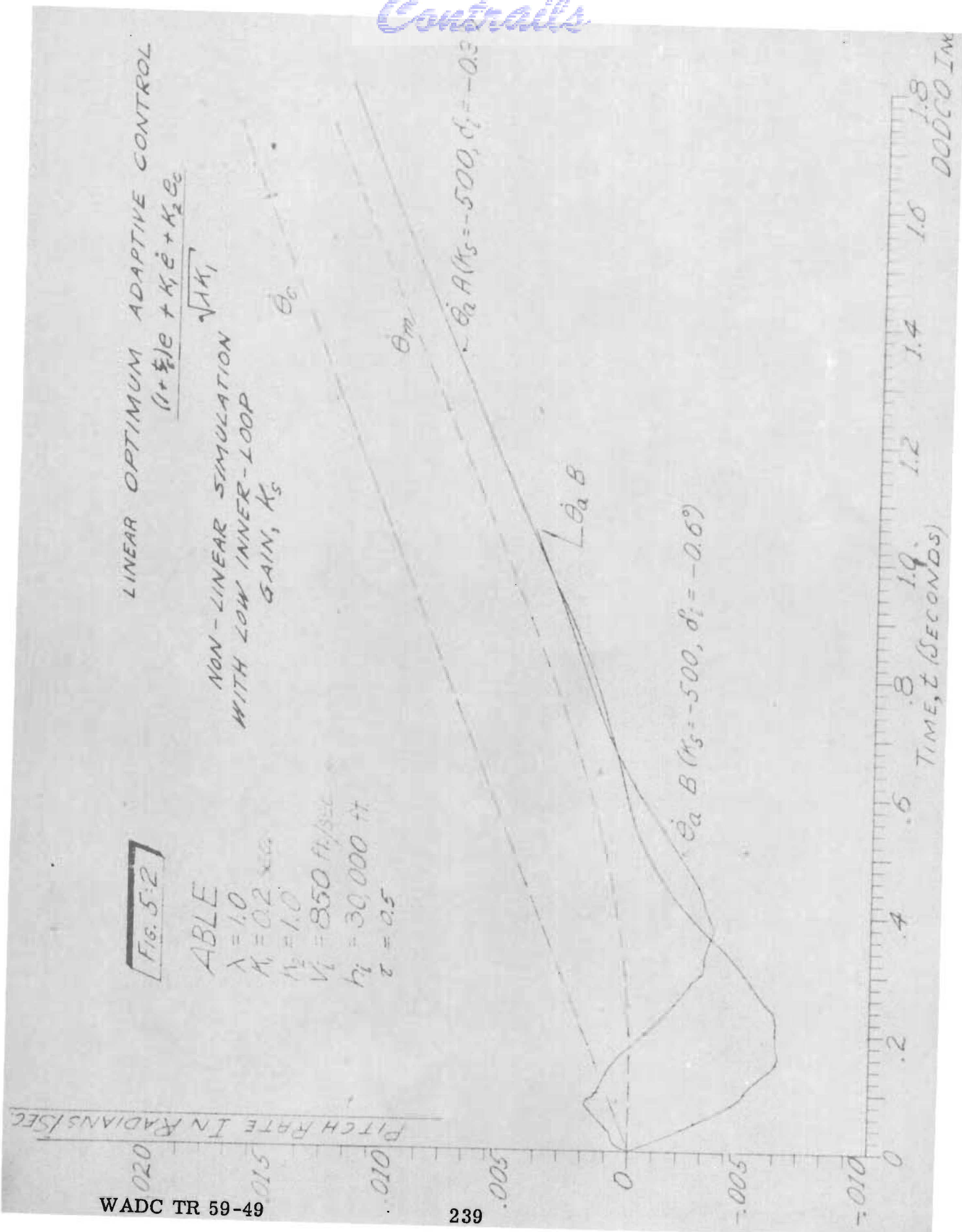
LINEAR OPTIMUM ADAPTIVE CONTROL
NON-LINEAR SIMULATION WITH LOW INNER-LOOP GAIN, K_S
FIG. 5.2
ABLE
λ = 1.0
K_1 = 0.2 sec
K_2 = 1.0
V_i = 850 ft/sec
h_i = 30,000 ft
τ = 0.5
$\dot{\theta}_c$
$\dot{\theta}_m$
$\dot{\theta}_a$ A (K_S = -500, δ_i = -0.3°)
$\dot{\theta}_a$ B
$\dot{\theta}_a$ B (K_S = -500, δ_i = -0.6°)
$\dot{\theta}_a$
PITCH RATE IN RADIANS/SEC
.020
.015
.010
.005
0
-.005
-.010
TIME, t (SECONDS)
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
DODCO INC

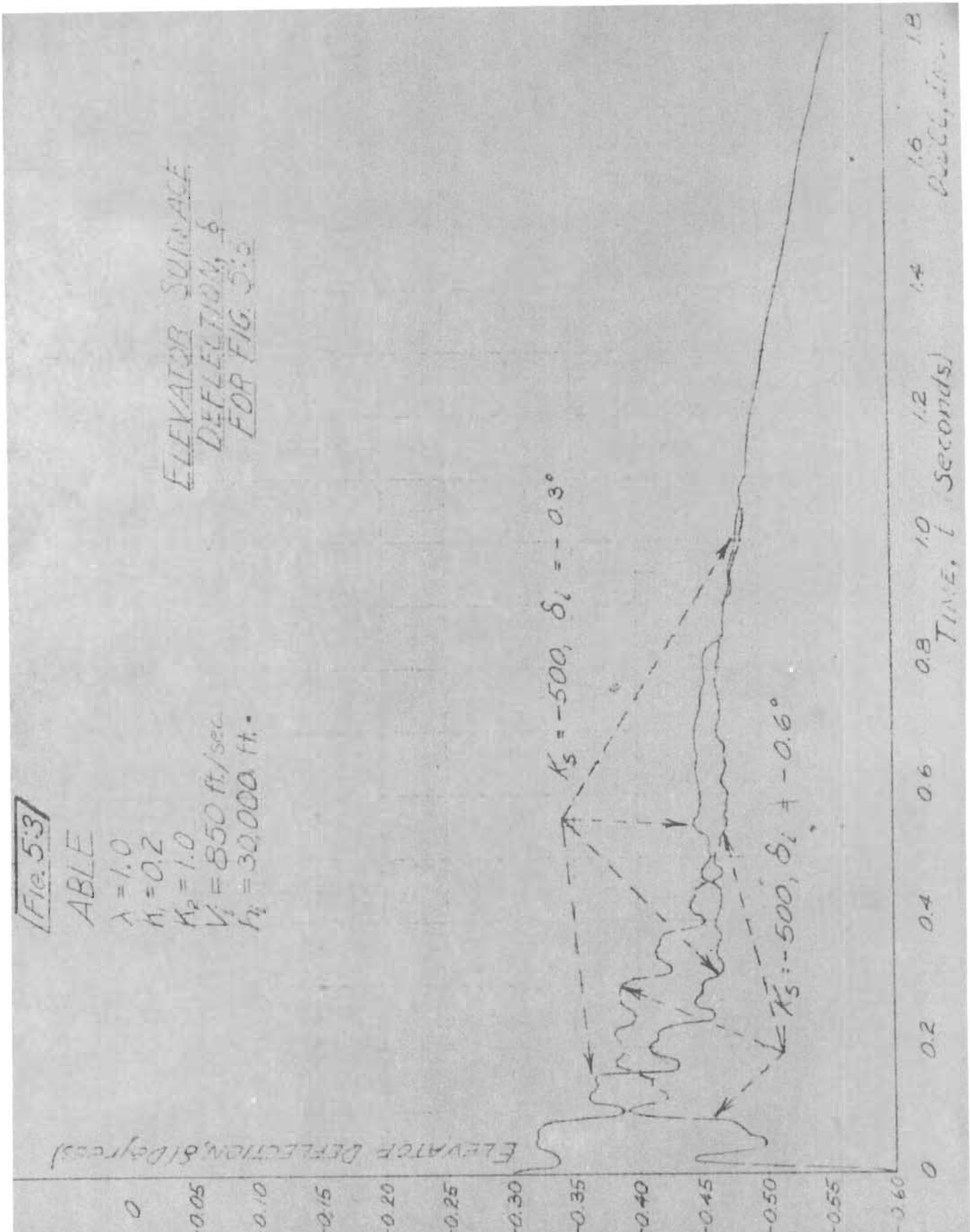
Fig. 5.3
ABLE
λ = 1.0
h1 = 0.2
K2 = 1.0
V1 = 850 ft./sec
h1 = 30,000 ft.
ELEVATOR SURFACE
DEFLECTION, δ
FOR FIG. 5.2
Ks = -500, δi = -0.3°
Ks = -500, δi = -0.6°
ELEVATOR DEFLECTION, δ (Degrees)
TIME, t (Seconds)
0
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
-0.45
-0.50
-0.55
-0.60
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

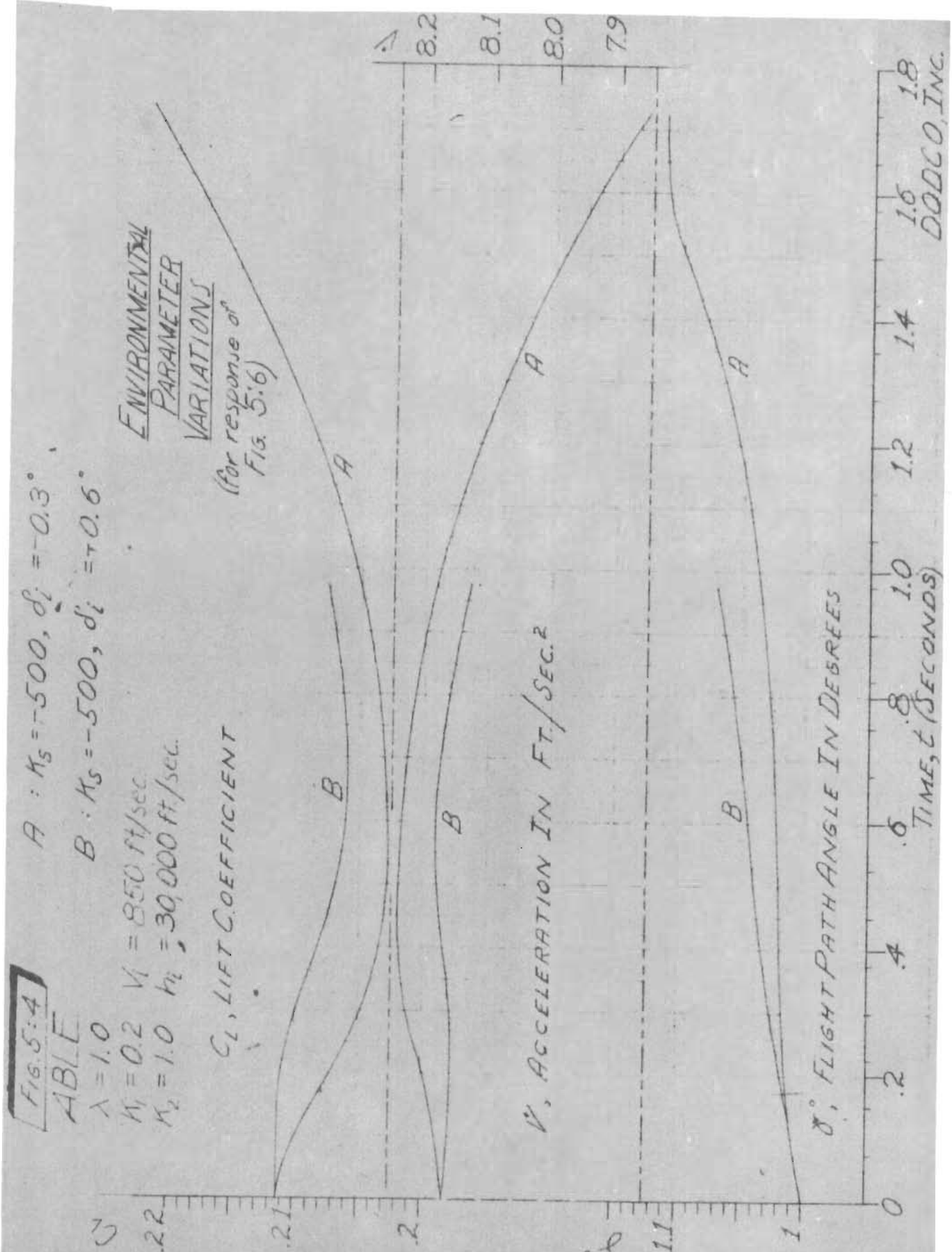
FIG. 5:4
ABLE
λ = 1.0
K1 = 0.2 V1 = 850 ft/sec.
K2 = 1.0 h1 = 30,000 ft/sec.
A : Ks = -500, δi = -0.3°
B : Ks = -500, δi = +0.6°
ENVIRONMENTAL PARAMETER VARIATIONS
(for response of FIG. 5:6)
CL, LIFT COEFFICIENT
V̇, ACCELERATION IN FT./SEC.²
γ°, FLIGHT PATH ANGLE IN DEGREES
TIME, t (SECONDS)
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
A
B
.22
.21
.2
8.2
8.1
8.0
7.9
1.1
1
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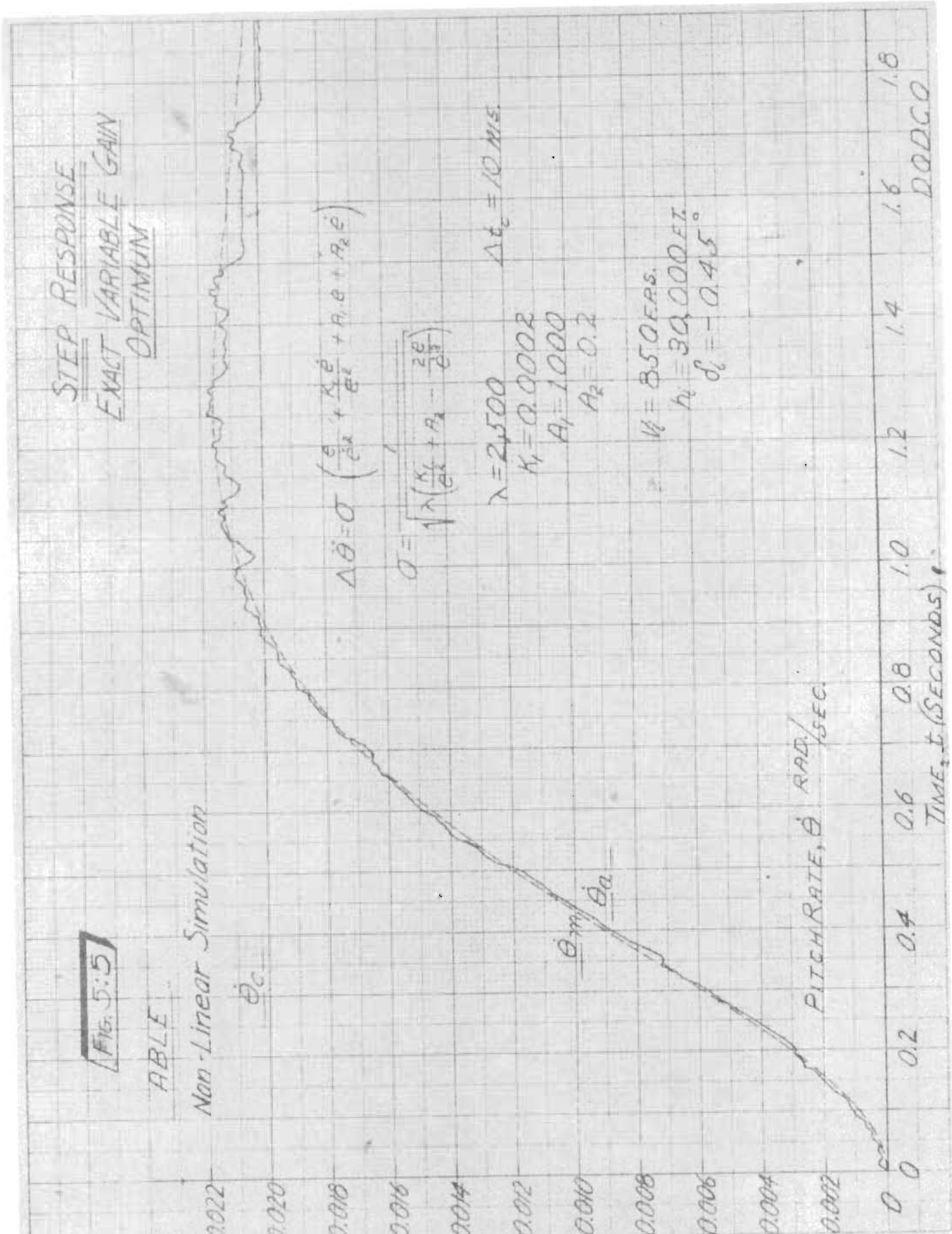
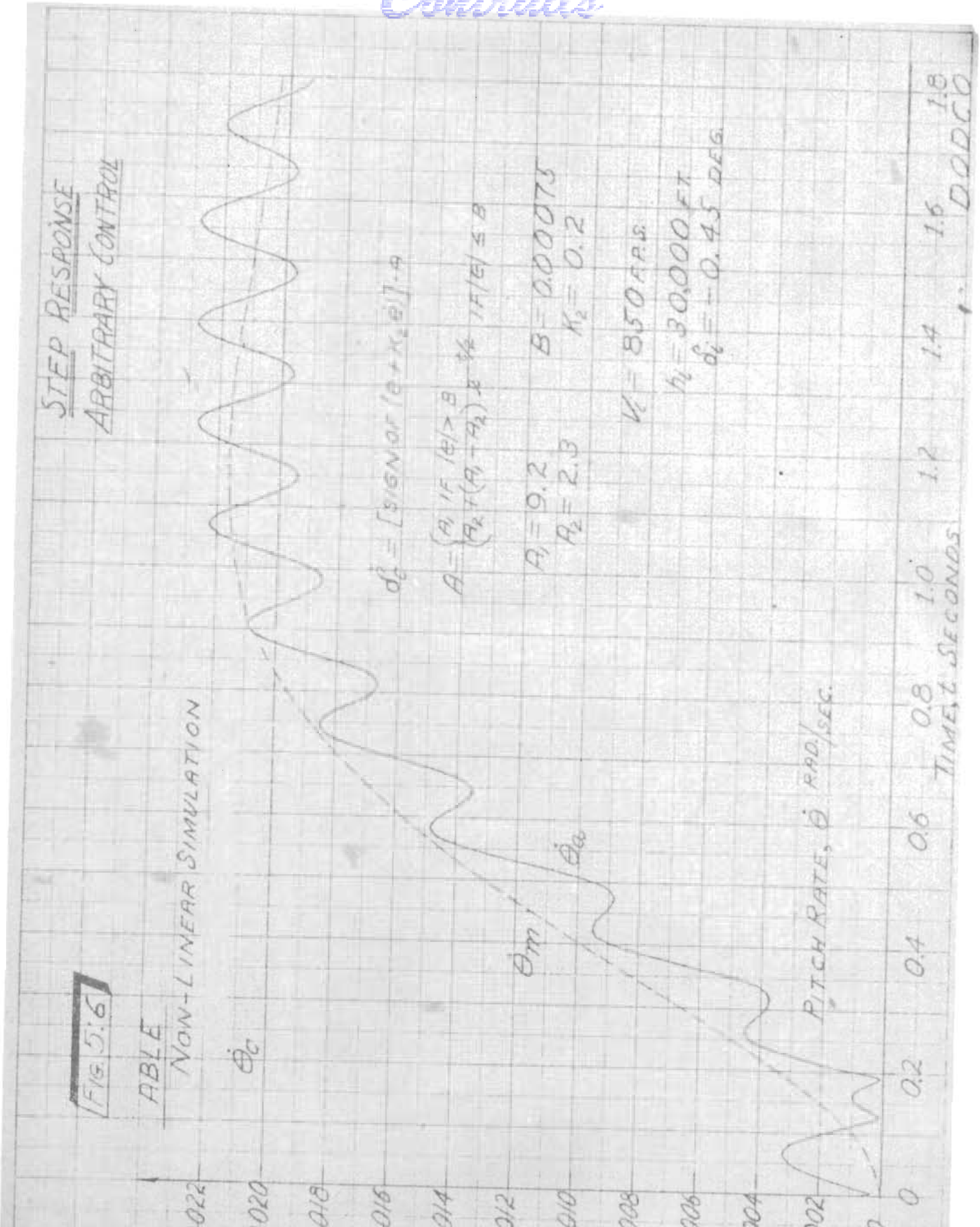
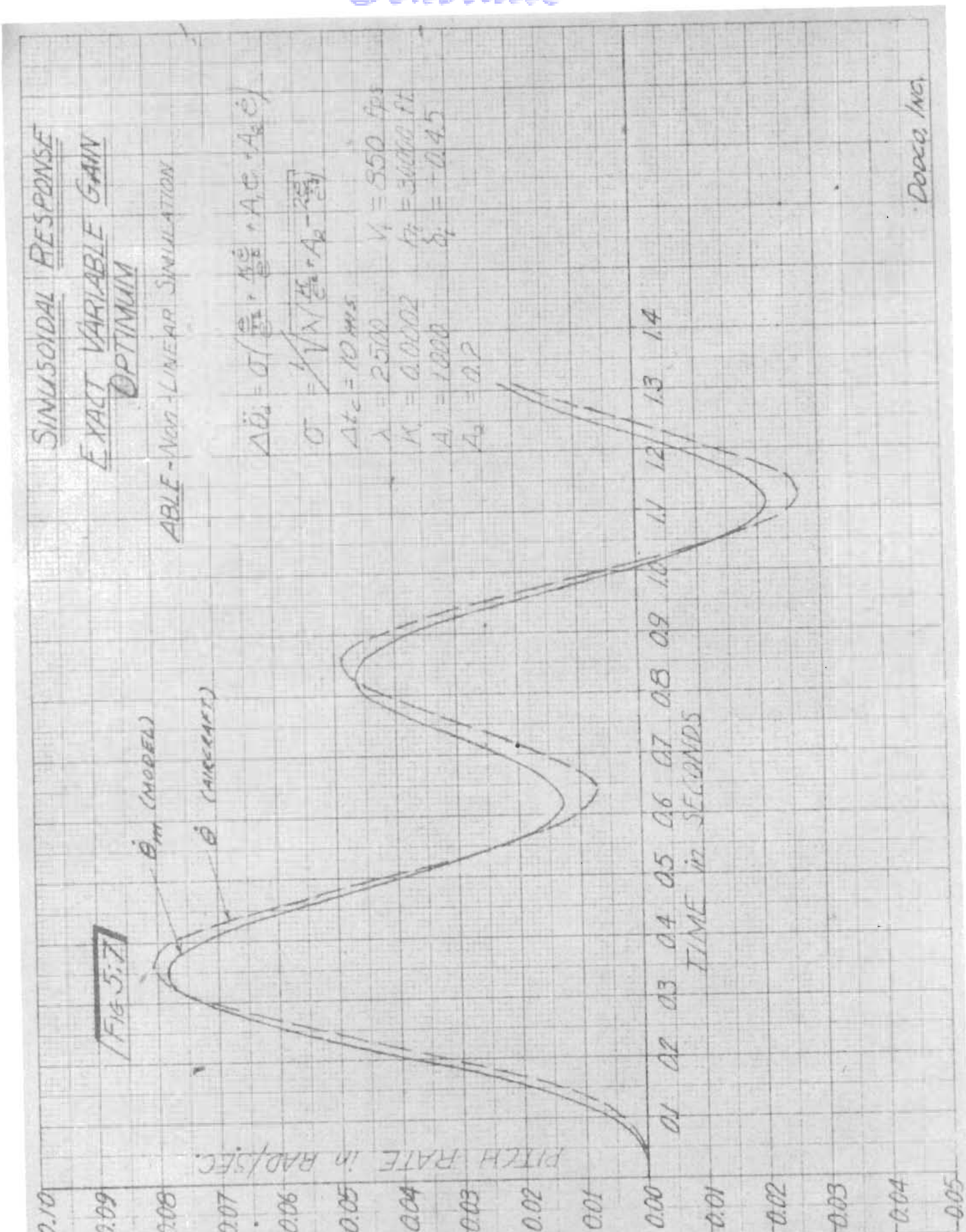


FIG. 5:5
STEP RESPONSE
EXACT VARIABLE GAIN
OPTIMUM
ABLE
Non-Linear Simulation
$\dot{\theta}_c$
$\dot{\theta}_m$
$\dot{\theta}_a$
$\Delta\dot{\theta} = \sigma\left(\frac{e}{\dot{e}^2} + \frac{K_1 e^2}{\dot{e}^2} + A_1 e + A_2 \dot{e}\right)$
$\sigma = \frac{1}{\sqrt{\lambda\left(\frac{K_1}{\dot{e}^2} + A_2 - \frac{2e}{\dot{e}^3}\right)}}$
λ = 2,500
K_1 = 0.0002
A_1 = 1000
A_2 = 0.2
Δt_c = 10 MS.
V_i = 850 FPS.
h_i = 30,000 FT.
δ_i = -0.45°
PITCH RATE, $\dot{\theta}$ RAD./SEC.
0
0.002
0.004
0.006
0.008
0.010
0.012
0.014
0.016
0.018
0.020
0.022
TIME, t (SECONDS)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
DODCO


STEP RESPONSE
ARBITRARY CONTROL
FIG. 5.6
ABLE
NON-LINEAR SIMULATION
$\dot{\theta}_c$
$\dot{\theta}_m$
$\dot{\theta}_a$
$\dot{\delta}_c = [\text{SIGN OF } (e + K_2 \dot{e})] \cdot A$
$A = A_1$ IF $|e| > B$
IF $|e| \le B$
$A_1 = 9.2$
$A_2 = 2.3$
$B = 0.00075$
$K_2 = 0.2$
$V_t = 850$ FPS
$h = 30,000$ FT
$\delta_t = -0.45$ DEG
PITCH RATE, $\dot{\theta}$ RAD/SEC
0
0.002
0.004
0.006
0.008
0.010
0.012
0.014
0.016
0.018
0.020
0.022
TIME, t SECONDS
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
DODCO


SINUSOIDAL RESPONSE
EXACT VARIABLE GAIN
OPTIMUM
ABLE - Non-Linear Simulation
Fig 5.7
(MODEL)
(AIRCRAFT)
TIME in SECONDS
PITCH RATE in RAD/SEC
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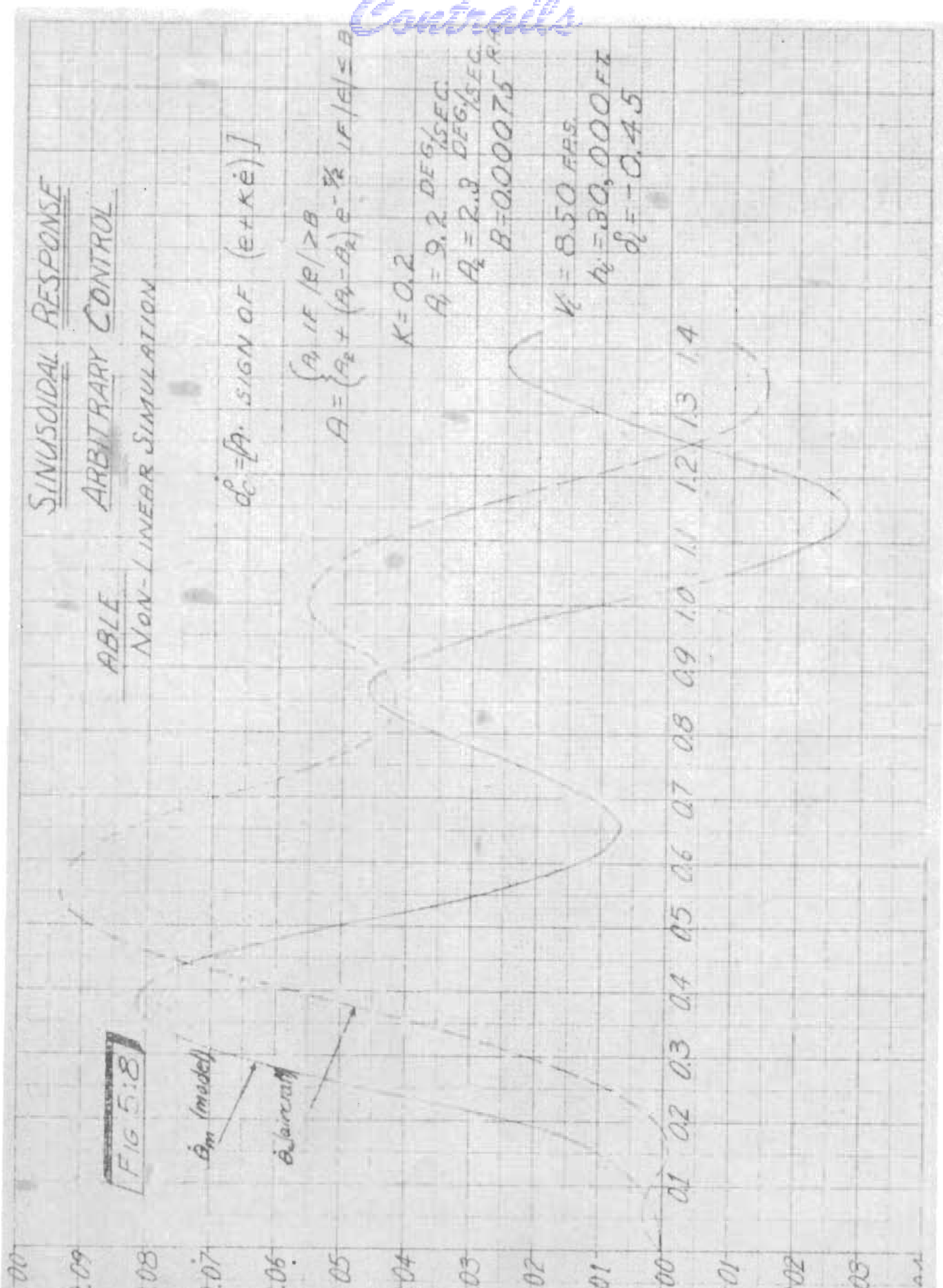
SINUSOIDAL RESPONSE
ARBITRARY CONTROL
ABLE
NON-LINEAR SIMULATION
K=0.2
FIG 5:8
θ̇m (model)
θ̇a (aircraft)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03

APPENDIX

THE EQUATIONS FOR DIGITAL COMPUTER SIMULATION OF AIRCRAFT DYNAMIC RESPONSE AND OPTIMUM ADAPTIVE CONTROL

ABLE -- SUMMARY OF EQUATIONS

Integration Block 1:

$$t_n = t_{n-1} + \Delta t$$

$$V_n = V_{n-1} + \dot{V}_{n-1}\,\Delta t + \ddot{V}_{n-1}\,\overline{\Delta t}^2/2$$

$$\gamma_n = \gamma_{n-1} + \dot{\gamma}_{n-1}\,\Delta t + \ddot{\gamma}_{n-1}\,\overline{\Delta t}^2/2$$

$$h_n = h_{n-1} + \dot{h}_{n-1}\,\Delta t + \ddot{h}_{n-1}\,\overline{\Delta t}^2/2$$

$$W_n = W_{n-1} + \dot{W}_{n-1}\,\Delta t + \ddot{W}_{n-1}\,\overline{\Delta t}^2/2$$

$$m = W/g$$

$$J = K_J m$$

Atmosphere Block:

(a) Gradient Regions:

$$T = T_o + a_j h$$

$$\eta = 1/a_j R$$

$$\partial\rho/\partial h/\rho = -\,a_j(\eta+1)/T$$

$$\partial T/\partial h/T = a_j/T$$

$$\epsilon = -\,\eta \mathrm{LOG}_e\,(T/T_1)$$

$$c_a = 49.04135\sqrt{T}$$

(b) Isothermal Regions:

$$T = T_1$$

$$\partial\rho/\partial h/\rho = -1/RT$$

$$\partial T/\partial h/T = 0$$

$$\epsilon = -\frac{1}{RT}(h-h_1)$$

$$c_a = c_{a_1}$$

(c) All Regions:

$$P/p_o = \delta_1 e^{\epsilon}$$

$$\rho = \rho_o \frac{T_o}{T} \cdot \frac{P}{P_o}$$

Parameter Block 1:

$$M = V/c_a$$

$$q = \rho V^2/2$$

$$\dot{h} = V\sin\gamma$$

$$\alpha = \theta - \gamma$$

$$C_L = a\sin\alpha$$

$$C_D = C_{D_e} + KC_L{}^2$$

$$D = qSC_D$$

$$\dot{\gamma} = \frac{1}{V}\left[\frac{(F_e + aqS)\sin\alpha}{m} - g\cos\gamma\right]$$

$$\dot{V} = \frac{F_e \cos\alpha}{m} - g\sin\gamma - \frac{D}{m}$$

$$\dot{W} = c_T F_e$$

$$\dot{m} = \dot{W}/g$$

$$\dot{q}/q = 2\left[\frac{\dot{V}}{V} + (\partial\rho/\partial h/\rho)\,\frac{\dot{h}}{2}\right]$$

$$\frac{\dot{M}}{M} = \frac{\dot{V}}{V} - (\partial T/\partial h/T)\,\frac{\dot{h}}{2}$$

$$\dot{a} = (\partial a/\partial M)\dot{M}$$

$$\dot{C}_{D_e} = (\partial C_{D_e}/\partial M)\dot{M}$$

$$\dot{K} = (\partial K/\partial M)\dot{M}$$

$$\dot{F}_e = (\partial F_e/\partial M)\dot{M} + (\partial F_e/\partial h)\dot{h}$$

<u>$\ddot{V}$, $\ddot{\gamma}$ Block:</u>

$$\dot{\alpha} = \dot{\theta} - \dot{\gamma}$$

$$\dot{D} = D\left(\frac{\dot{q}}{q}\right) + qS\left[\dot{C}_{D_e} + KC_L{}^2\left(\frac{\dot{K}}{K} + \frac{2\dot{a}}{a} + \frac{2\dot{\alpha}}{\tan\alpha}\right)\right]$$

$$\ddot{V} = \frac{F_e\cos\alpha}{m}\left(\frac{\dot{F}_e}{F_e} - \dot{\alpha}\tan\alpha - \frac{\dot{m}}{m}\right) - \dot{\gamma}g\cos\gamma - \frac{\dot{D}}{m} + \frac{D}{m}\left(\frac{\dot{m}}{m}\right)$$

$$\ddot{\gamma} = \left\{g\cos\gamma\,\left(\dot{\gamma}\tan\gamma + \frac{\dot{V}}{V}\right) + \frac{\sin\alpha}{m}\left[(F_e + aqS)\left(\frac{\dot{\alpha}}{\tan\alpha} - \frac{\dot{m}}{m} - \frac{\dot{V}}{V}\right) + \dot{F}_e + aqS\left(\frac{\dot{a}}{a} + \frac{\dot{q}}{q}\right)\right]\right\}\frac{1}{V}$$

$$\ddot{h} = \dot{h}\left(\frac{\dot{\gamma}}{\tan\gamma} + \frac{\dot{V}}{V}\right)$$

$$\ddot{W} = c_T\,\dot{F}_e$$

"Model" Block:

$$\dot{\theta}_{m_n} = \dot{\theta}_{m_{n-1}} + \ddot{\theta}_{m_{n-1}} \Delta t + \dddot{\theta}_{m_{n-1}} \overline{\Delta t}^2/2$$

$$\ddot{\theta}_{m_n} = \ddot{\theta}_{m_{n-1}} + \dddot{\theta}_{m_{n-1}} \Delta t$$

$$\dot{\theta}_c = a_o + a_1 t$$

$$\dddot{\theta}_m = \omega_{n_m}{}^2 (\dot{\theta}_c - \dot{\theta}_m) - 2 \zeta_m \omega_{n_m} \ddot{\theta}_m$$

$\ddot{\theta}$, $\ddot{\delta}$ and Optimum Control Block:

$$\ddot{\theta} = \frac{qSc}{J}\left[\alpha C_{m_\alpha} + \dot{\alpha} C_{m_{\dot{\alpha}}} + \dot{\theta} C_{m_{\dot{\theta}}} + \dot{\delta} C_{m_{\dot{\delta}}} + \delta C_{m_\delta}\right] - \dot{\theta}\left(\frac{\dot{J}}{J}\right)$$

$$\dot{e} = \ddot{\theta}_m - \ddot{\theta}$$

$$e_c = \dot{\theta}_c - \dot{\theta}$$

$$e = \dot{\theta}_m - \dot{\theta}$$

$$\Delta\ddot{\theta}_d = \frac{e + K_1 \dot{e} + K_2 e_c}{\sqrt{\lambda K_1}}$$

$$\ddot{\delta} = K_S \Delta\ddot{\theta}_d - 2 \zeta_s \omega_{n_s} \dot{\delta} - \omega_{n_s}{}^2 \delta$$

$$\ddot{\theta}_d = \ddot{\theta} + \Delta\ddot{\theta}_d$$

Inner Loop:

(a) Integration Block II:

$$t_j = t_{j-1} + \Delta t_1$$

$$\theta_j = \theta_{j-1} + \dot{\theta}_{j-1} \Delta t + \ddot{\theta}_{j-1} \overline{\Delta t}^2/2$$

$$\dot{\theta}_j = \dot{\theta}_{j-1} + \ddot{\theta}_{j-1} \Delta t$$

$$\delta_j = \delta_{j-1} + \dot{\delta}_{j-1}\,\Delta t + \ddot{\delta}_{j-1}\,\overline{\Delta t}^2/2$$

$$\dot{\delta}_j = \dot{\delta}_{j-1} + \ddot{\delta}_{j-1}\,\Delta t$$

test on t_j $\begin{cases} t_j \geq (t_n + \Delta t) \text{ ..... transfer to Integration Block I} \\ t_j < (t_n + \Delta t) \text{ ..... transfer to Inner Loop } \ddot{\theta},\ \ddot{\delta} \end{cases}$

(b) Inner Loop $\ddot{\theta}$, $\ddot{\delta}$

$$\alpha = \theta - \gamma$$

$$\dot{\alpha} = \dot{\theta} - \dot{\gamma}$$

$$\ddot{\theta} = \frac{qSc}{J}\left[\alpha C_{m_\alpha} + \dot{\alpha} C_{m_{\dot{\alpha}}} + \dot{\theta} C_{m_{\dot{\theta}}} + \dot{\delta} C_{m_{\dot{\delta}}} + \dot{\delta} C_{m_{\delta}}\right] - \dot{\theta}\,(\dot{J}/J)$$

$$\Delta\ddot{\theta} = \ddot{\theta}_d - \ddot{\theta}$$

$$\ddot{\delta} = K_S \Delta\ddot{\theta} - 2\zeta_s \omega_{n_s}\dot{\delta} - \omega_{n_s}^2 \delta$$