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AERODYNAMIC SHAPE OF THE WING TIPS.

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Dr. Sighard Hoerner

Approved:

    O.J.Ritland, Colonel,
    USAF Chief, Aircraft Lab.

For the commanding General:

    Fred.R. Dent,Jr., Colonel,
    USAF CHIEF, Engineering
    Operations, Eng. Division.

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ABSTRACT

The author studied the aerodynamic characteristics of wing tips, and he did, and had German research institutions do, experimental investigations concerning the mechanism of the tip vortices and the lift/drag ratio of a wing fitted with several differently shaped tip caps.

There are two ways in which the wing tips affect flow and forces on a wing: (a) Considering the tip vortices the effective span of a wing does not coincide with the geometrical span; that is, the geometrical location of the tip vortices depends upon the shape of the wing tips. (b) Some parasite drag originates from the wing tips, its amount also depending upon the shape of the tip caps.

The main results of the tests as mentioned, are presented in this report, and the optimum wing-tip shape (No. 5) is described in Figure 9. This tip cap has a sharp side edge, its plane form shows a corner at the trailing edge and its upper surface extends approximately straight to the end. Employing this kind of wing tip the climbing speed is increased in the order of 1 ft/sec and the range by 1 or 2%, in comparision to disadvantageous tip shapes.

AERODYNAMIC SHAPE OF THE WING TIPS

I. Location of The Wing-Tip Vortices

From each tip of a wing a sharply determined vortex is originating. These tip vortices must not be confused with the vortex sheet originating from the trailing edge of the wing. This sheet transforms itself very soon into the pair of tip vortices. Fig. 1 shows the formation of one of the two tip vortices, made visible when releasing gasoline through an emergency outlet at the left wing tip. The two vortices form a stable system (horseshoe vortex), the inherent momentum of which is the equivalent of the induced drag. Like this drag, the vortices cannot be avoided in practical airplane construction.

Effective Span

The distance b_i between the two vortices is a measure of the effective span of the wing. If measured along the trailing edge of the wing, b_i is usually smaller than the actual span b by 10% or 20% of the wing chord. The larger the value of b_i, the smaller is the induced angle of attack and the induced drag and the greater the lift curve slope. The induced drag is an important property of the wing of an airplane; the slope is of special interest when using the wing as a horizontal tail surface.

Figure 2 shows the geometric location of the tip vortex core for six different tip shapes, as measured in Ref. 3. The tips No. 1, 2 and 6 present the largest effective spans b_i. Evidently the trailing edge of the wing tip is important, directing the vortex as far outward as possible.

Side Edges

The tips 1 in comparison to 2, and 4 in comparison to 3, show larger b_i values. Obviously the sharp side edges are advantageous; on the other hand, well rounded edges facilitate the flow around them, from the lower to the upper wing surface, as illustrated with tip No. 2 in Fig. 4.

Lift Curve Slope

Figure 3 shows the relation between the lift curve slope and the distance b_i of the vortices; for the aspect ratio 3 as used with the experiments in Ref. 3. From the lifting line theory

$$\frac{d\alpha}{dC_L} = \left(\frac{d\alpha}{dC_L}\right)_\infty + \frac{d\alpha_i}{dC_L} = \frac{1}{2\pi} + \frac{1}{\pi A} \qquad (1)$$

Taking into account the influence of the wing chord and converting the angle of attack into degrees, it is derived in Ref. 8:

$$\frac{d\alpha}{dC_L} = 11.2 + \frac{20 + \frac{10}{A_i}}{A_i} \qquad (2)$$

with 11.2 chosen as to match the experimental points in Fig. 3 and A_i indicating the effective aspect ratio = (b^2/S) - ΔA. In the present case $\Delta A = 2y/c$ was taken from Fig. 2 as measured on the prolongation of the wing trailing edge.

End-Plate Effect

The points 1 and 5 lie below the calculated curve. It was learned from the geometrical measurements on the tip vortices, that these two tip shapes have a vortex-core location which is approximately 5% of the wing chord higher than with the other forms. In effect this location means a formation of the vortex sheet leaving the trailing edge, similar to that of a wing fitted with small end plates.

Ref. 4 confirms this effect. A circle wing with the upper surface running straight from side edge to side edge, shows a higher lift/drag ratio than a symmetrically designed wing. That means, the effective wing span is somewhat increased, owing to the dihedral effect.

Rolling Moment

The rolling moment of a wing due to yaw, is affected by a wing tip similar to No. 5 in Fig. ?. With a wing of aspect ratio 6, a thickness ratio t/c - 12% and a trapezodial plan form having c_{tip}/c_{center} = 0.6, the form No. 5 or the tip cap as recommended in Part V, presents an affect equivalent to approximately 1.5° of positive wing dihedral.

II. Parasite Wing-Tip Drag

Figure 4 shows the side component of the flow at the wing tip as derived from visual observations in a water tunnel (Ref 2). A larger or smaller amount of parasite drag is connected with this flow, owing to flow separation. Its magnitude was determined by momentum loss measurements within the vortex core (Ref 5); that is, measurements were conducted in the wake of a wing model, employing a small sphere (2-dimensional yaw head) for determining pressures and angles of the local flow. At zero lift the parasite tip drag depends on the shape of its forward portion. Tips well rounded toward their leading edge present a small negative amount of tip drag, obviously owing to the fact that the 2-dimensional wing flow is changed to a 3-dimensional near the tips.

Parasite Drag Depending Upon Lift

At positive lift coefficients Fig. 5 indicates relatively small drag coefficients for all the sharp edges (Nos. 4, 5 and 6). Evidently they match the natural flow shown on Fig. 4 much better than the square and blunt shapes Nos. 1 and 2. Accordingly the latter show a parasite drag approximately twice as large as the sharp-edged forms.

The parasite tip-drag must not be confused with the induced drag. However, similar to this one, it increases with a high power of C_L. From Fig. 5 it can be derived for a pair of sharp-edged tip caps that

$$C_{DTc} = \frac{\Delta D}{qc^2} \approx 0.01 \, C_L^3$$

(3)

III. Force Measurements on Wings

Experiments on a Wing Series

Fib. 6 shows the lift and drag coefficients for a wing with aspect ratio 3, fitted with the 6 different tip caps as illustrated in Fig. 1. Neglecting No. 1, which naturally is the worst at small lift coefficients, the Nos. 5 and 6 are the best ones; that means they show the highest lift/drag ratios. Applying the knowledge acquired in the Paragraphs I and II, the efficiency of the different tips can be attributed to the following causes:

Tip Shape	Lift/Drag	Result Due to:
1	high	Sharp side edge and complete trailing edge.
2	low	Round side edge, in spite of complete trailing edge.
3	low	Round side edge, cut-away trailing edge.
4	low	Cut-away trailing edge, in spite of sharp side edge.
5	high	Sharp side edge and rather complete trailing edge.
6	high	Sharp side edge, full trailing edge.

Shape No. 5 follows best the natural flow direction, as shown in Fig. 4. This number and No. 1 show in addition a favorable end plate effect as explained before. Evidently those tip forms are the best which employ sharp side edges and complete or full trailing edges. It can also be seen from Fig. 6 that tips 1, 5 and 6 have the highest $C_{L_{max}}$ values, probably owing to the effect of the trailing edge. Considering practical application, tip No. 5 is preferred over No. 6 because of simpler design and construction.

Wing-Tip Tanks

The worst form, as far as lift and drag is concerned, is the No. 2, because of its well rounded edge, which allows the flow to get around it from the lower to the upper side, as shown in Fig. 4. In this connection Fig. 7 furnishes further information, presenting the influence of a wing-tip shape similar to a small drop tank. Owing to this tip form, the lift-curve slope is decreased by 5 and 9% respectively as compared to conventional tip shapes. That means, the induced angle of attack (and accordingly the induced drag) is increased by approximately 11 to 20% in this case. However, different experimental results on a pair of tip tanks are presented in Ref.6. Upon adding the tanks to the wing the lift/drag ratio is somewhat increased above $C_L = 0.4$. Obviously in this case the tanks act like endplates, their dimensions being much larger than those in Figs. 4 and 7; $d = 6\ t_{tip}$ and $l = 3\ l_{tip}$. In fact it is possible to compute a difference of induced drag as measured when taking into account both the endplate effect and the wing span, which is somewhat enlarged by the tanks.

Experimental Data

In Fig. 8 general data are given, indicating the effective aspect ratio of various wing forms. In addition to the systematic measurements of Ref. 3, some more references have been used, indicating especially the values of elliptical and swept wings. It is remarkable that the elliptical plan form

practically does not present the smallest possible amount of induced angle and
induced drag as predicted by theory, especially not if the quarter chord points are
lined up on the lateral wing axis. In this case (see Fig. 8), not only a rear
corner is missing, but the wing looks like a large portion of wing area was cut
off at the outer part of the trailing edge. Such a cutting off results in shifting
the tip vortex inward, as shown in Fig. 2, and in reducing the effective wing aspect-
ratio correspondingly.

Swept Wings

Considering swept wings from various sources (Ref 7), it is found that the
minimum induced drag and the maximum lift curve slope are attained approximately
at an angle of sweep $\varphi = +5^o$; with φ measured on the quarter chord line.
This amount of positive sweep means for the average trapezoidal wing almost a
straight trailing edge. Here again, the importance of the trailing edge is
obvious; according to Fig. 8 an effective aspect ratio is obtained which is
slightly larger than the geometrical one.

IV. Effect on Flight Performances

From Fig. 1 or 8 it can be seen that the advantageous tip shapes act
approximately as if the aspect ratio of the wing was enlarged by $\Delta A = 0.3$ over
that of some of the disadvantageous wing forms. In the case of an average
military airplane, with $W/S = 40$ lb/ft^2 and $C_L^3/C_D^2 = 150$ and $A = 7$, therefore,
by changing the tip shape from No. 2 or 3 to No. 5, the climbing speed is
increased at the rate of 1 ft/sec. Also concerning range, the shape of the wing
tips has a certain bearing. Corresponding to $\Delta A = 0.3$ the range is increased
by 1 or 2%. These improvements are not large; but they can be accomplished by
proper design without any disadvantages or additional expenditures in construction.
Of course, the effect on the flying qualities (rolling moment), as presented by
a tip shape similar to No. 5 (Fig. 2), has to be considered too.

V. Practical Design of the Wing-Tip Caps

The tip form appearing as the most advantageous in Figs. 5 and 6 is the
No. 5, besides No. 6. If, by imagination, the end of a sufficiently long
trapezodial wing becomes cut off from below in an oblique direction, automatically
the basic form of No. 5 is obtained. This shape has to be rounded in its
forward part, approximately up to the maximum span. Finally, by fairing the
transition from the lower wing surface toward the tip and by replacing the plane
cut surface by a curved one, the shape as shown in Fig. 9 is obtained.

The span of the tip cap is approximately $a = (0.25 \text{ to } 0.30) c_{tip}$. The
aileron might be extended up to the rear corner. The design has to be adjusted
faithfully to thickness ratios and section shapes different from the example
as shown in Fig. 9.

Wing-tip caps designed according to the rules outlined above, have been
used with German airplanes, as the Fieseler 256, the Junkers 290 and with the more

recent versions of the Junkers 87 and 88. Probably there are also American designs incorporating similar principles.

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8. Hoerner, Berechnung des raeumlichen Tragfluegels, Calculation of Wings with Limited Span, German Doct Messerschmitt TB 45/1941.

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Fig. 1. — Flow Pattern Behind an Airplane; From Luftwissen 1942 p.306. The Flow was made Visible by Releasing Gasoline through Emergency Outlets. From the Tail the Gasoline Droplets Follow the Downwash. From the Left Wing Tip the Gasoline Concentrates Within the Vortex Core.
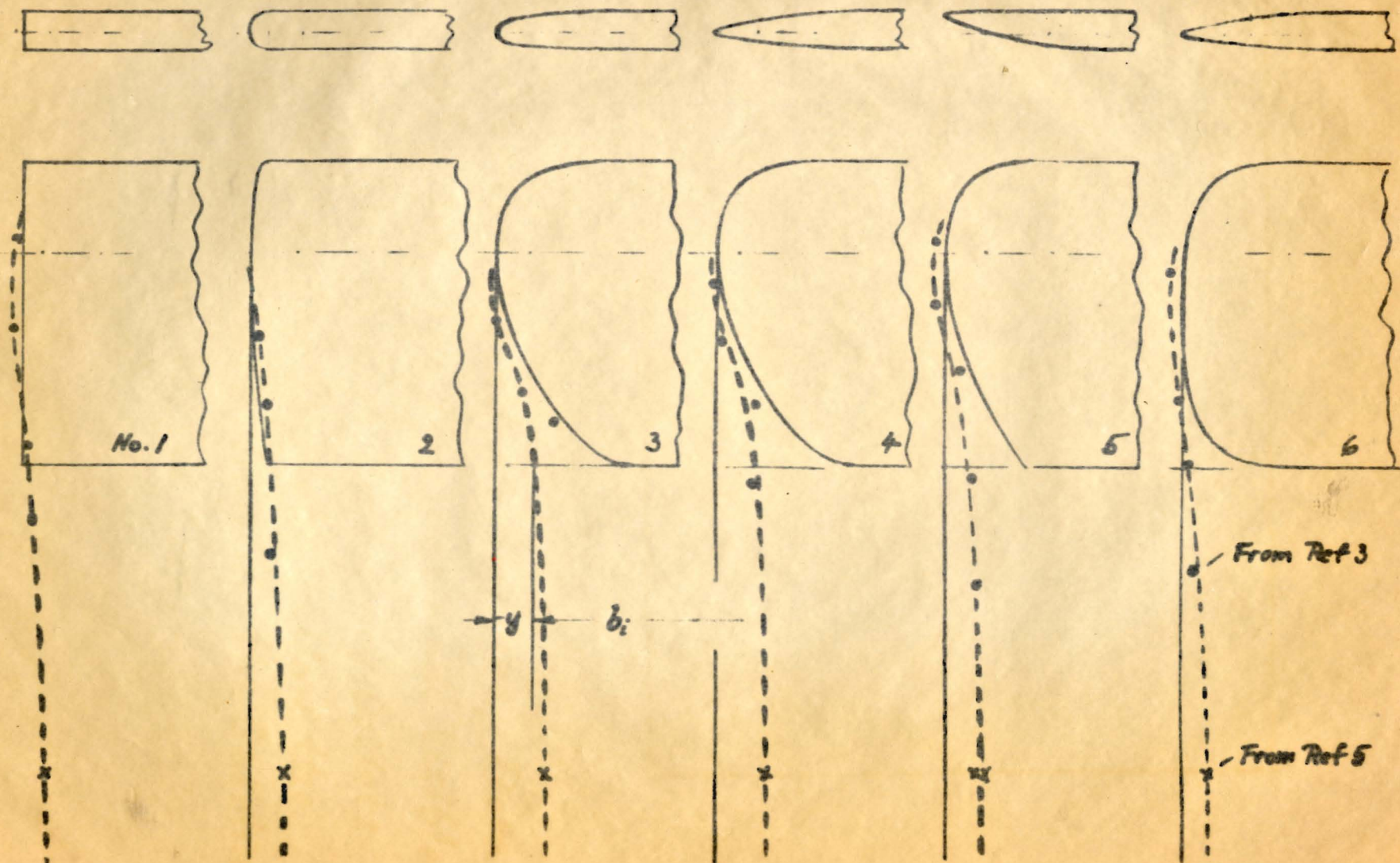
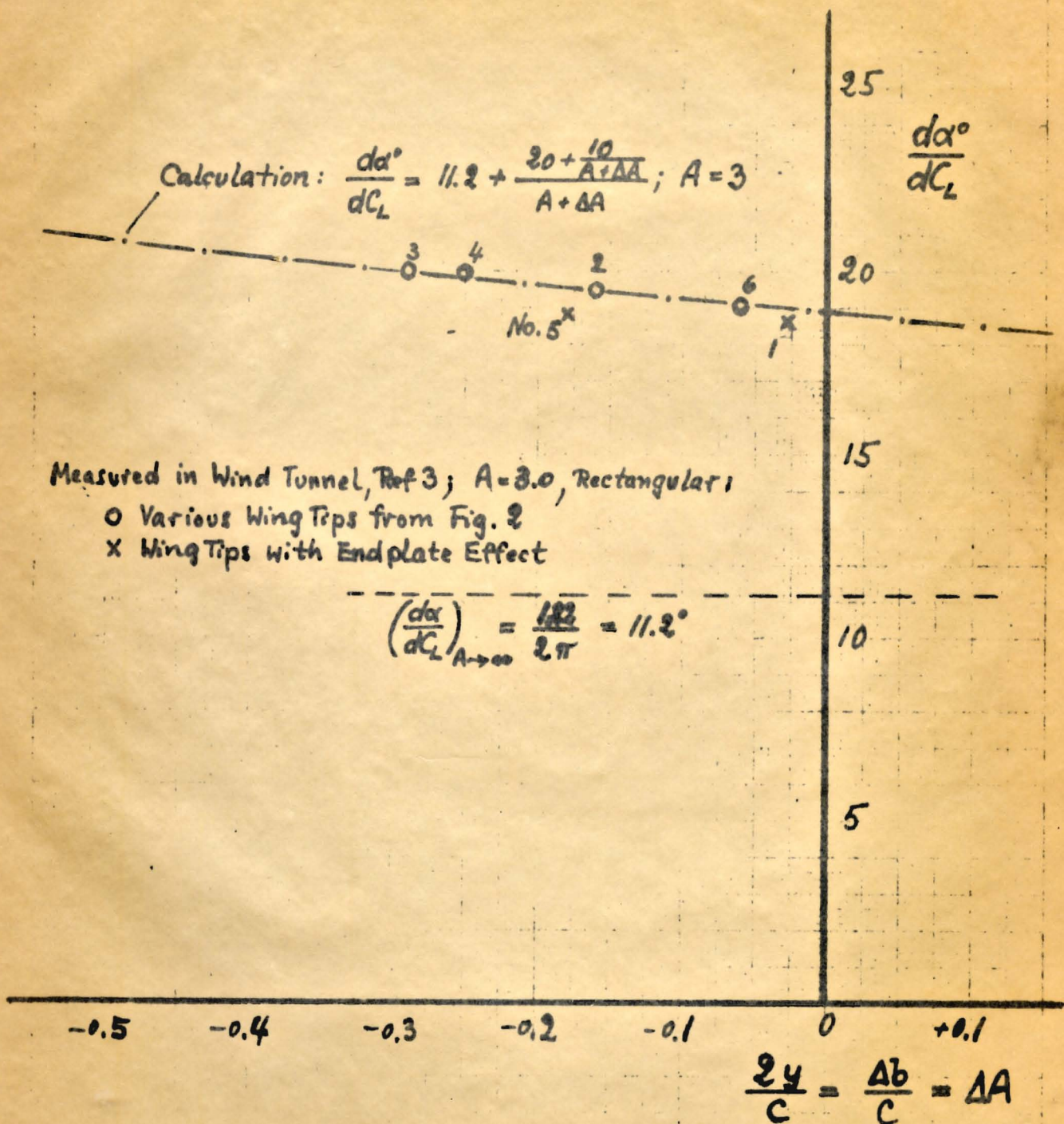Fig. 2.- Shapes of the Wing Tips as Tested in Refs 3 and 5, and Locations of the Tip-Vortex Cores.
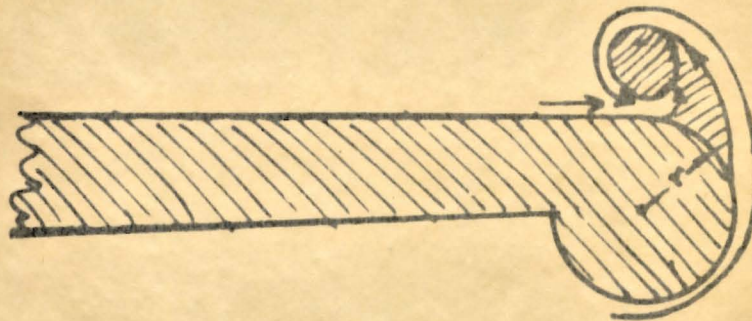
Calculation: $\dfrac{d\alpha^\circ}{dC_L} = 11.2 + \dfrac{20 + \frac{10}{A + \Delta A}}{A + \Delta A}$; $A = 3$

$\dfrac{d\alpha^\circ}{dC_L}$

25

20

Measured in Wind Tunnel, Ref 3 ; $A = 3.0$, Rectangular ;
  O Various Wing Tips from Fig. 2
  X Wing Tips with Endplate Effect

$$\left(\dfrac{d\alpha}{dC_L}\right)_{A\to\infty} = \dfrac{180}{2\pi} = 11.2^\circ$$

15

10

5

3 4 No.5 2 6 1

−0.5   −0.4   −0.3   −0.2   −0.1   0   +0.1

$\dfrac{2y}{C} = \dfrac{\Delta b}{C} = \Delta A$

Fig. 3.- The Variation of the Lift Curve Slope with the Effective Aspect Ratio $A_i = A + \Delta A$; According to Theory, and from Experiments.
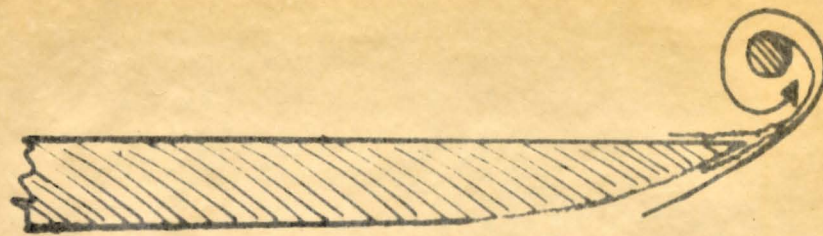
Similar No.5
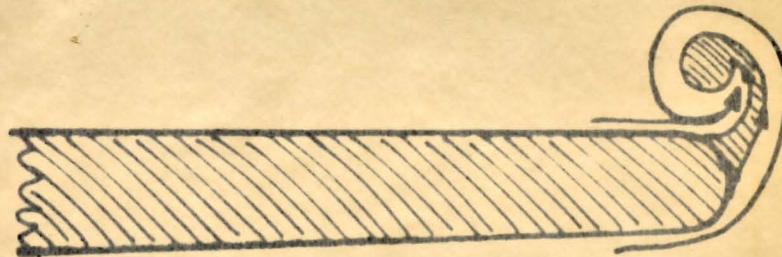
Like No. 2

"Wing Tip Tank" with $r = t$ and $l = c_{tip}$

Fig. 4. - Flow Pattern Around the Wing Tip; from Observations in a Water Tunnel (Ref 2). Note the Location of the Vortex Core.

Fig. 5. Parasite Wing Tip Drag; Momentum-Loss Measurements from Ref. 5.

$$C_L = \frac{L}{q \cdot S}$$

$0.01\,C_L^3$

$C_{DC} = 0.02\,C_L^3$

$0.03 \cdot C_L^3$

For Comparison: 10% of the Induced Drag Coefficient
$$C_{Dic} = D_i/q\,c^2 = C_L^2/\pi$$

Experimental Points; Nos. from Fig. 2:

□ 1 Square Edge
○ 2 Half Round Edge
● 3 Round Tip Cap
▼ 4 Sharp Tip Cap
△ 5 Oblique Tip Cap
◇ 6 Full Plane Form

$$C_{DE} = \frac{D_{TIP}}{q \cdot c^2}$$

Parasite Wing Tip Drag for 2 Tips
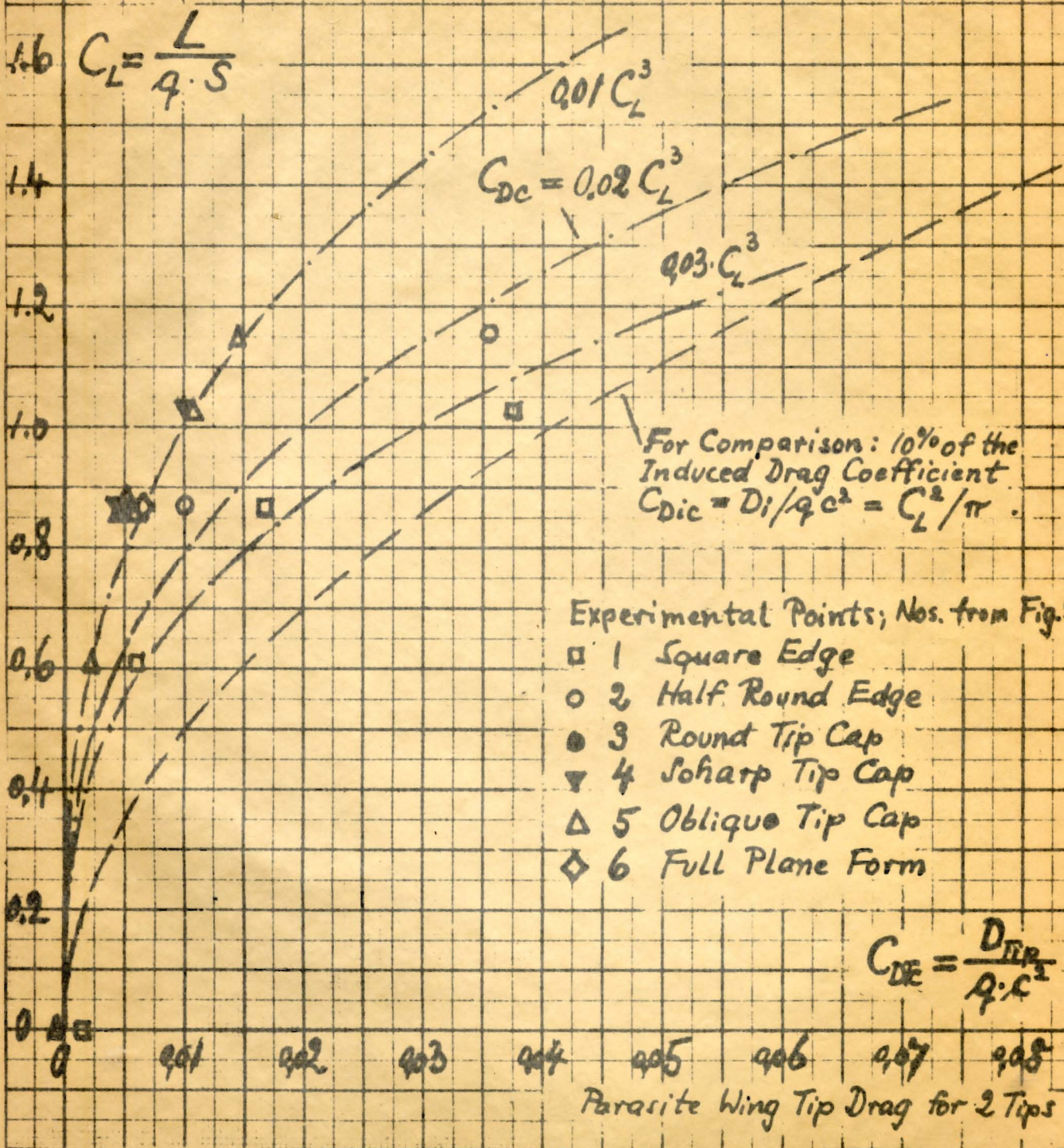
Shape of the Wing Tips According Fig. 2:

+ No. 1
o , 2
△ , 3
□ , 4
● , 5
× , 6
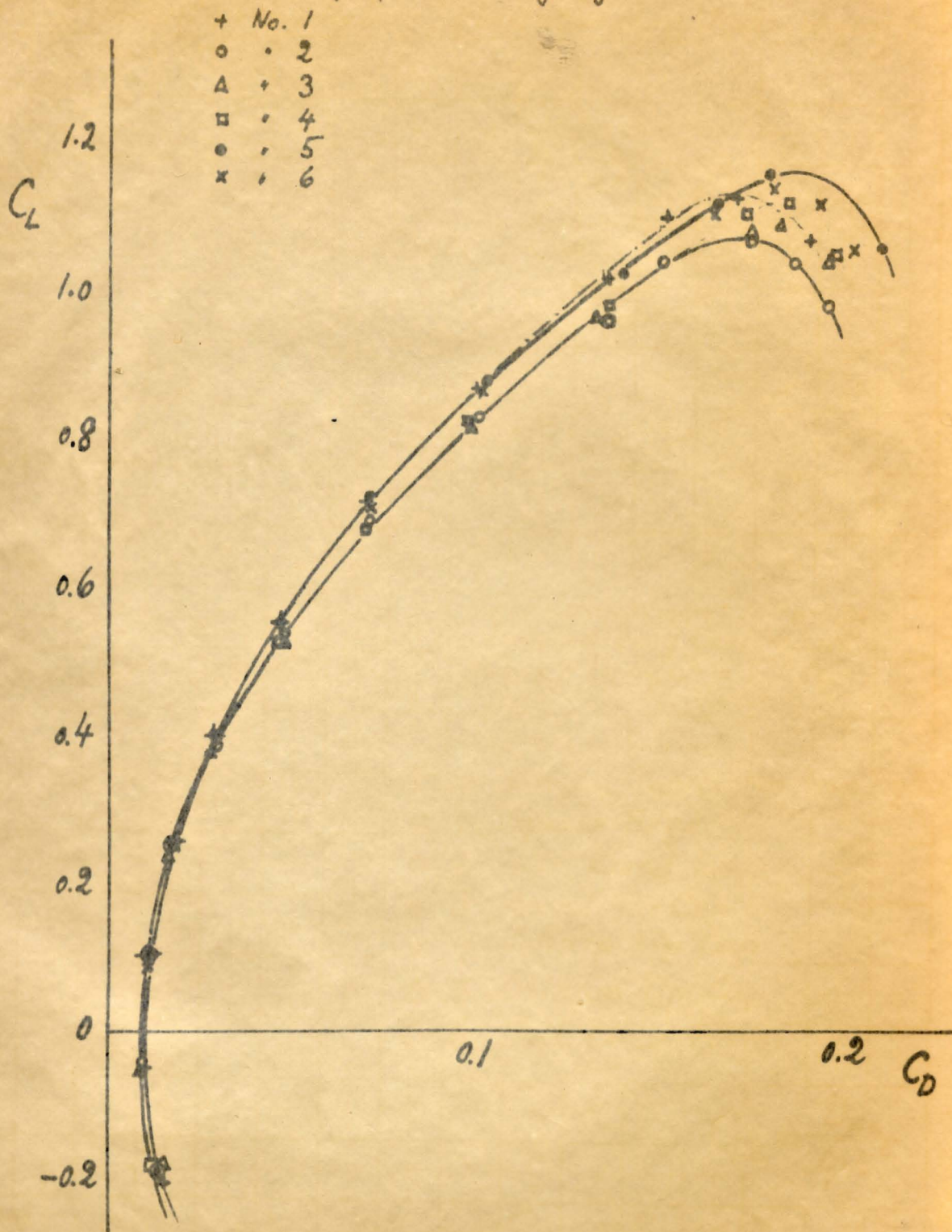
Fig.6.- Wind Tunnel Measurements on a Rectangular 2412 Wing, Equipped with 6 Different Tip Caps as Shown in Fig.2. Aspect Ratio A = 3.0; R = 2.10⁶; Ref.3.

C_L

1.2

1.0

0.8

0.6

0.4

0.2

-20   -10   10   20   30   40

$\alpha°$

-0.2

-0.4

-0.6
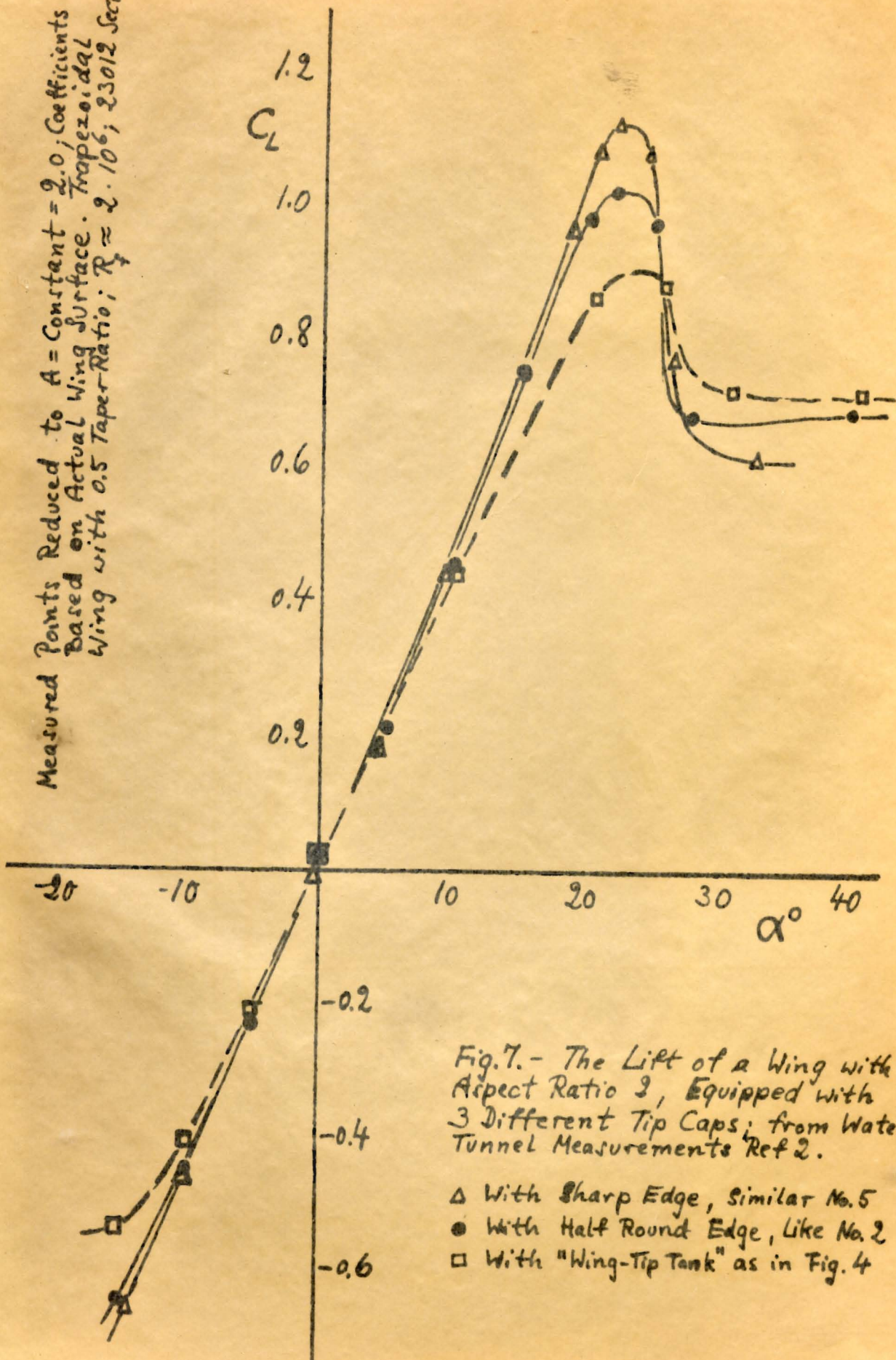
Fig. 7. - The Lift of a Wing with
Aspect Ratio 3, Equipped with
3 Different Tip Caps; from Water-
Tunnel Measurements Ref 2.

△ With Sharp Edge, Similar No. 5
● With Half Round Edge, Like No. 2
□ With "Wing-Tip Tank" as in Fig. 4
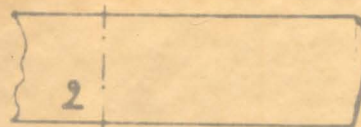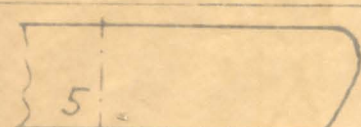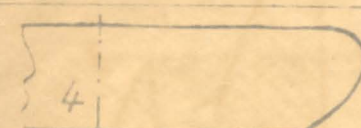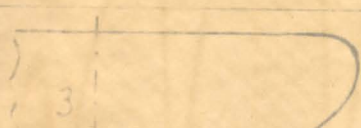
Plan Form	Side Edge	$\Delta A = \dfrac{2y}{c}$	
1	square	0	1
2	half round	-0.22	1, 2, 3
5	sharp edge	-0.13	3
4	sharp edge	-0.32	3
3	round edge	-0.40	3 and 7
$c/4$	round (or sharp)	-0.40	7
$\dfrac{c}{2}$	sharp (or round)	-0.20	7 .
$c/4$	square	-0.05	2
$c/4$	half round	-0.25	2
straight trailing edge	sharp edge	$+0.05$	7

Fig. 8. - Data Indicating the Effective Aspect Ratio $A_i = A + \Delta A$, Depending upon Plan Form and Tip Shape.

Fig. 9.- Design of a Wing Tip Cap as Recommended from the Foregoing Experiments

Cross Section II

$x = 0.35\ c_{tip}$

$a = (0.25\ to\ 0.30)\ c_{tip}$

Cross Section I, Airfoil Section with Chord c_{tip}

Section / 25 10 - 0.225 40

Aileron

Straight Upper Wing Surface