Statistical and Worst Case Evaluation of Orbital Jitter Reduction Using Passive Damping[1]

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The jitter responses of a damped and undamped precision mounting platform (PMP) are compared. The primary response is due to a platform mounted sensor which slews back and forth as it scans. The sensor torque profile is reduced to Fourier components, and the proximity of the platform resonances to the input frequencies is evaluated. If the resonance is close to a Fourier component, the mode is shifted to the Fourier frequency and the worst case jitter is computed. If the worst case response exceeds the jitter allowable, the limits above and below the Fourier frequency for shifting the mode that produces responses in excess of the allowable is determined. Then, assuming a Gaussian probability density function for the natural frequency, the probability of jitter exceedance is computed. Results show dramatic reductions in orbital jitter from the use of passive damping. Significant weight reduction is also achieved for the PMP analyzed with passive damping.

INTRODUCTION

The purpose of this paper is to present a method for evaluating orbital jitter and to describe the improvement in jitter that can be obtained with passive damping. Improved dynamic stability of remote sensing spacecraft payload mounting platforms (PMPs) is an evolving requirement resulting from improved sensor accuracy and increased satellite disturbances due to size and thermal effects. Jitter causes sensor distortions as a result of small angular motions occurring in a prescribed period of time often measured in seconds. It tends to be a high frequency phenomena that is beyond the control system bandwidth. Because of uncertainties in precisely predicting the orbital resonant frequencies of the spacecraft, "worst case" estimates that consider resonant excitation of critical modes are often used to estimate jitter. This paper describes an alternate statistical method of jitter predication that estimates the probability of exceeding a budgeted value when worst case estimates are excessive.

Passive damping provides a simple, reliable method of jitter reduction which is examined for a representative PMP. A USAF Wright Research and Development Center program, Damping and Metal Matrix Precision Structures (DAMMPS), is being

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performed by GE Astro Space Division to develop and demonstrate PMP performance improvements that can be obtained with passive damping [1]. The PMP is selected from a current GE-ASD spacecraft program, Figure 1, so that a design that considers interdisciplinary constraints are satisfied. The platform is attached to the forward end of the spacecraft through three struts with ball end fittings and a ball fitting shear tie. This provides a kinematic arrangement that prevents PMP distortions due to spacecraft deflections. Nine sensors and instruments weighing 300 pounds and requiring precise alignment and pointing are mounted to this platform. The requirements are to provide a 15.5 Hertz resonant frequency and to limit jitter to 15 arcseconds peak. The benefits of damping are estimated by considering the structural weight required to provide equivalent jitter response to on-board disturbances. In addition, the damping required to achieve a budgeted jitter value is determined. Damped and baseline PMP configurations are used to quantify the benefits of damping.

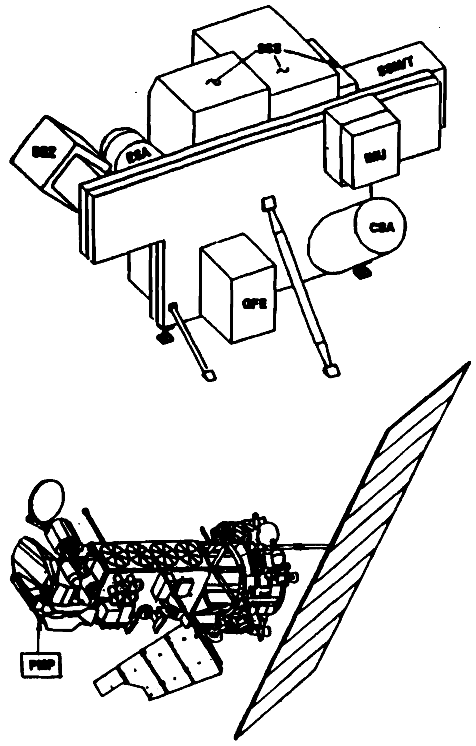

Figure 1. DAMMPS Platform

DAMPED AND UNDAMPED BASELINE PMP CONFIGURATIONS

The DAMMPS design for an advanced instrument mounting platform incorporates a viscoelastic material (VEM) layer for damping and metal matrix composite (MMC) facesheets for high specific stiffness. The configuration is a dual honeycomb sandwich composed of a viscoelastic center layer and honeycomb panels on either side. Each honeycomb panel consists of an aluminum core and MMC facesheets. The undamped baseline design is a traditional dip brazed aluminum eggcrate platform. Cross sections of both designs are shown in Figure 2.

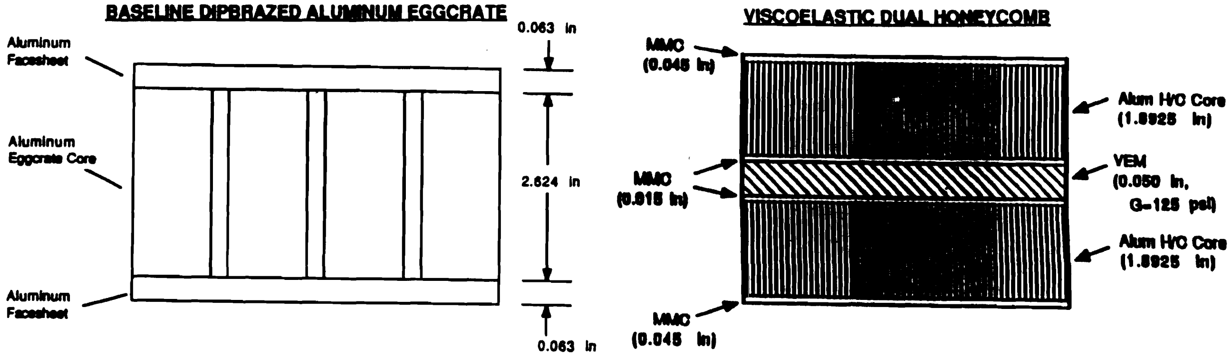

Figure 2. Cross Sectional Configuration

Finite element models were generated for both configurations. The dual honeycomb finite element model is composed of plate elements (CQUAD4) representing the honeycomb layers and solid elements (CHEXA) representing the VEM layer. Lumped masses positioned at the c.g. locations are used to simulate the instruments. The undamped eggcrate model, due to its single panel configuration, consists of only one layer of plate elements. Both models are shown in Figure 3.

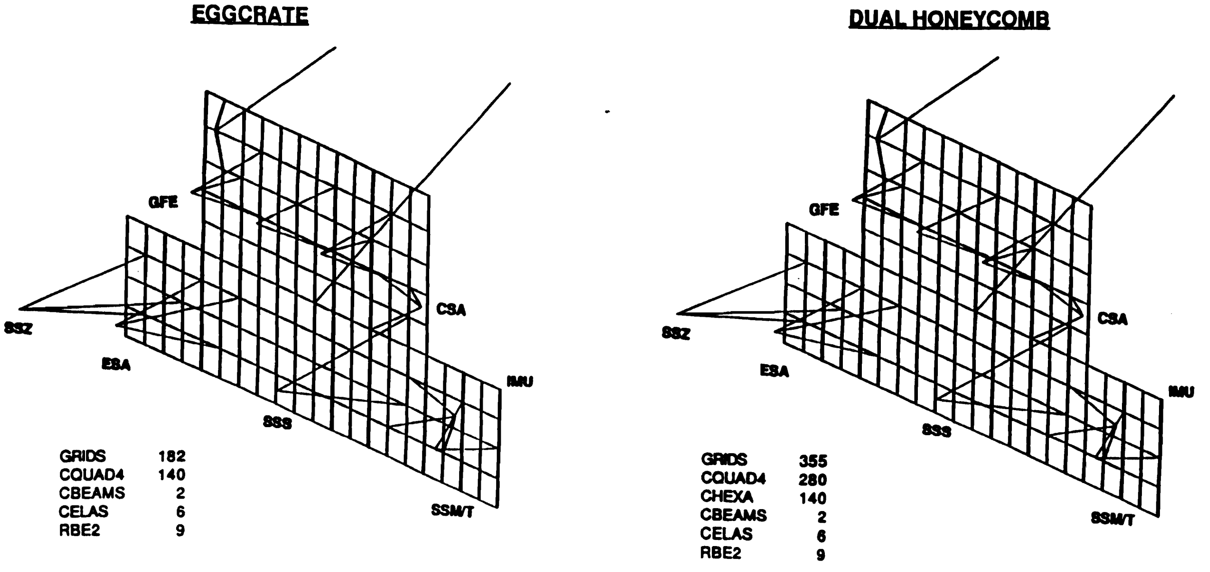

Figure 3. Finite Element Models

Modes and frequencies up to 200 Hz were generated using MSC/NASTRAN. Results, presented in Table 1, show that each design had approximately the same mass and stiffness. The primary difference between the two designs was the extent of damping. An orbital damping value of 0.002 c_C was assumed for the aluminum eggcrate platform. This damping value has been used on a variety programs incorporating precision mounting platforms where alignment tolerances are stringent. The viscoelastic dual honeycomb damping values were based on the modal strain energy method. Since development of low modulus VEM was still in progress, a material loss factor of 1.0 was assumed for the viscoelastic material. Subsequent development work of a low modulus VEM has indicated that this assumption was somewhat conservative [2]. The damping values for the most critical modes were calculated to be approximately 0.055 c_C.

Table 1. Results of FEM Analysis

MODE	EGGCRATE FREQUENCY (Hz)	EGGCRATE DAMPING (ζ)	DUAL HONEYCOMB FREQUENCY (Hz)	DUAL HONEYCOMB DAMPING* (ζ)
1	17.3	.002	16.8	.052
2	29.4	.002	25.2	.068
3	48.5	.002	40.0	.078
4	58.6	.002	48.3	.054
5	63.3	.002	56.0	.072
6	72.6	.002	62.9	.057
7	81.4	.002	66.5	.066
8	93.5	.002	69.1	.060
9	101.7	.002	80.3	.063
10	107.7	.002	86.3	.050
11	114.3	.002	88.0	.060
12	128.4	.002	94.8	.051
13	133.6	.002	102.0	.047
14	142.6	.002	107.2	.049
15	147.7	.002	112.4	.052
16	165.3	.002	116.0	.050
17	172.2	.002	121.6	.053
18	195.3	.002	142.2	.056
19			152.5	.053
20			183.3	.056

*DHC DAMPING VALUE BASED ON MODAL STRAIN ENERGY METHOD, $\eta_{VEM} = 1$

NOMINAL TRANSIENT JITTER ANALYSIS

A preliminary jitter study was performed using the predicted modes and frequencies for both designs. The transient jitter response of each instrument was determined for several input disturbances. The largest disturbance is the input torque to the Scanning Sensor Subsystem (SSS) which is located directly on the platform. The torque pulse, broken into its Fourier components in Figure 4, includes significant excitation as high as 102 Hz.

Results of the nominal jitter study are plotted in Figure 5. For each instrument, the undamped baseline response exceeded that of the damped design by a factor of 3.0 or more. For the SSZ instrument, which had the largest jitter response due to its c.g. offset and its relative location on the platform, the undamped eggcrate peak response exceeded that of the dual honeycomb by a factor of 6.0. Additionally, several instruments on the undamped platform had responses greater than the 15 arcsecond peak budget. The dual honeycomb design, due to its greater damping, experienced no response greater than the 15 arcsecond budget.

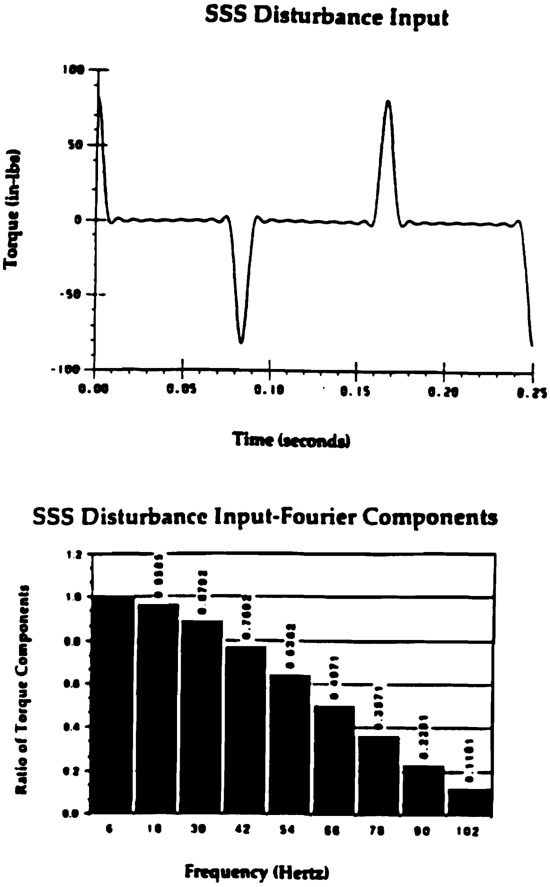

Figure 4. SSS Disturbance Input

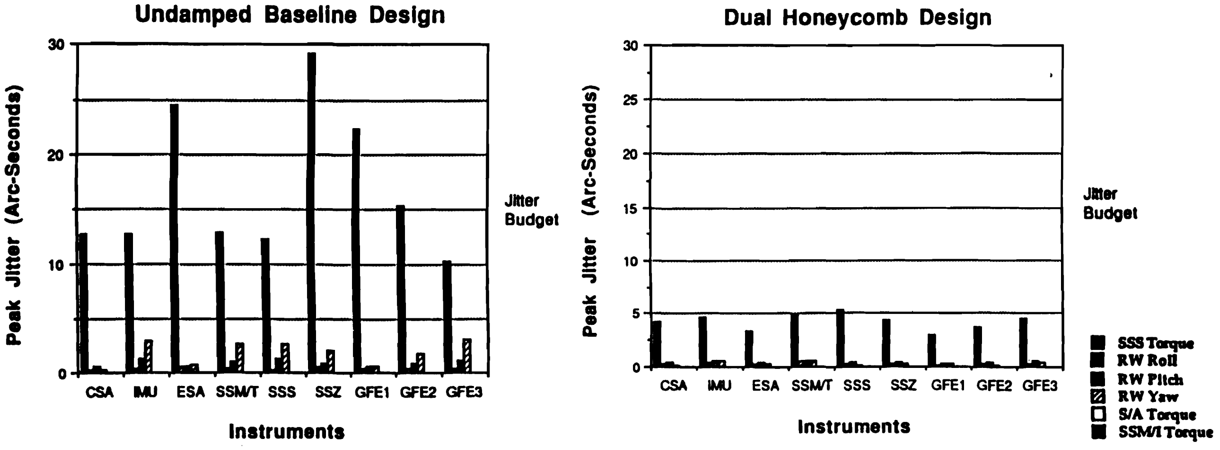

Figure 5. Nominal Jitter Response

In Figure 6, a plot of the transfer function for the DAMMPS dual honeycomb SSZ θ_x response due to a SSS torque input compares the relative location of the platform resonances and the Fourier frequencies of the SSS torque input. The magnitude of the torque pulse coupled with the proximity of the SSS Fourier frequencies to the fundamental modes of the structure account for the SSS input being the dominant disturbance. Because of this dominance, only the SSS input was considered in the worst case and statistical solutions.

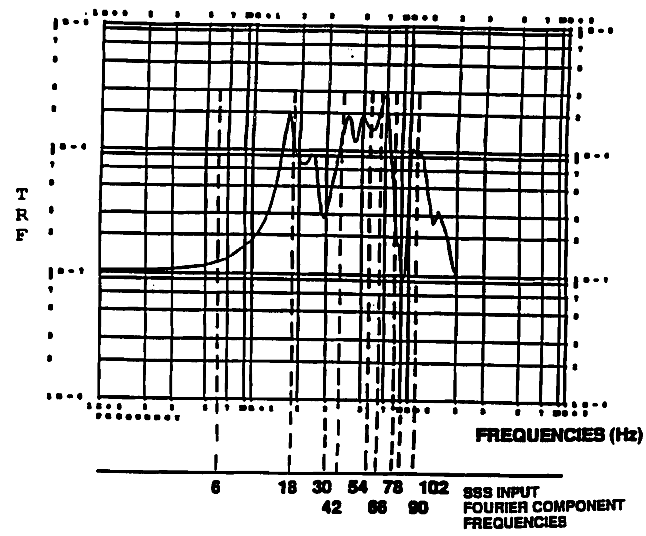

Figure 6. DHC SSZ θ_x Transfer Function to SSS Input

WORST CASE STEADY STATE JITTER ANALYSIS

Uncertainty in predicted structural natural frequencies causes an uncertainty in the estimated jitter response. If actual structural modes are closer to input disturbances than predicted, jitter responses can be significantly higher than the nominal (no shifted modes) estimates would indicate. Therefore, an approach that accounts for resonance response is required.

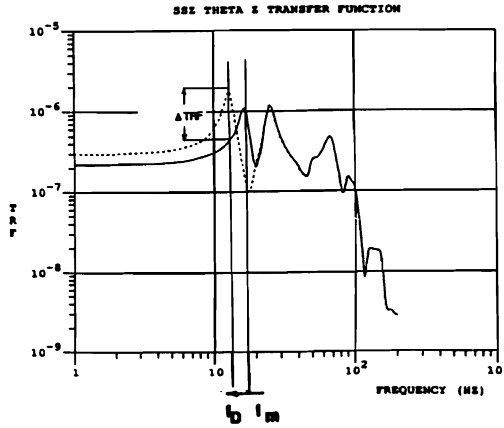

Figure 7. Mode Frequency Shift

Worst case jitter response is calculated by shifting modal frequencies to local input driving frequencies. For a mode with natural frequency f_M within 15 percent of a driving frequency f_D, f_M is shifted to f_D (see Figure 7) and the worst case jitter for a single mode shift is calculated. The mode is returned to its nominal position and another mode is shifted to its local driving frequency and peak jitter determined. The overall worst case peak jitter can be determined once each of the dominant structure modes have been shifted.

Worst case steady state results for each design are compared against their nominal values in Table 2. Note that the nominal results are steady state values and are thus slightly less than the transient values shown earlier. Only the θ_y and θ_z responses for the SSZ, ESA, and SSM/T instruments are presented. These responses were considered most critical based on results from the nominal transient analysis.

The undamped baseline design, which is extremely sensitive to the location of the mode relative to the forcing function, has worst case responses that far exceed the jitter budget. The dual honeycomb design, with its higher damping values, is far less sensitive to resonant coupling. Worst case results were only slightly higher than their nominal values and none of the responses exceed the jitter budget.

Table 2. Results of Worst Case Jitter Analysis

RESPONSE	EGGCRATE		DUAL HONEYCOMB	
	NOMINAL (ARCSECS)	WORST CASE (ARCSECS)	NOMINAL (ARCSECS)	WORST CASE (ARCSECS)
SSZ θ Y	16.1	127.3	4.1	5.2
SSZ θ Z	13.9	121.9	4.5	6.7
ESA θ Y	13.3	99.6	3.2	3.3
ESA θ Z	10.2	92.3	2.9	4.6
SSM/T θ Y	6.6	105.2	4.5	6.1
SSM/T θ Z	13.3	87.1	4.1	6.0
JITTER BUDGET 15 ARCSECS				

A study was conducted to determine how much damping is required to limit the worst case response to less than the jitter budget. Assuming constant damping in each of the dual honeycomb modes, worst case jitter was calculated for several damping values. Results, shown in Figure 8, indicate that a small amount of damping can significantly reduce worst case jitter. Only 1.5 percent damping (0.015 c_c) is required in the critical modes to limit the worst case response to within the jitter budget. At higher damping values, the jitter response approaches the static solution and further damping does not significantly affect the total response.

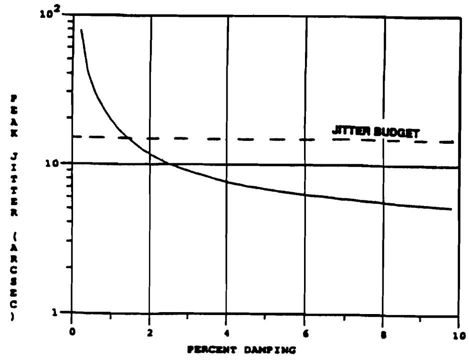

Figure 8. Worst Case Peak Jitter Versus Damping

STATISTICAL JITTER ANALYSIS

For the undamped eggcrate, nominal results indicate that the jitter response is less than the budget for most instruments. Worst case results show high jitter response. A statistical approach was implemented to ascertain the likelihood of any worst case occurrence and to quantify the design's dependability. The statistical method is based on the sensitivity of the jitter response relative to the modal frequencies.

Two assumptions were made in the statistical solution. First, it was assumed that 90 percent of the modes can be predicted to within 10 percent accuracy. This assumption, based on past correlation of modal tests and analysis, was used to generate a Gaussian distribution curve with the predicted modal frequency f_M being the mean value and the area under the curve within a 10 percent bandwidth of f_M being equal to 0.90 (see Figure 9). By assuming a Gaussian distribution, this translates into predicting 98.6 percent of the modes to within 15 percent of the

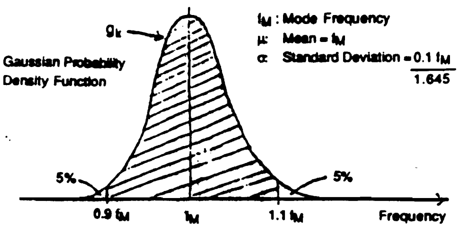

Figure 9. Gaussian Probability Density Function for Modal Frequency

measured test frequencies, 90 percent of the modes to within 10 percent, or 60 percent of the modes to within 5 percent. The second assumption states that the modal frequency predictions are statistically independent. The meaning of this assumption is that the probability of predicting one mode accurately is independent of how another is predicted, just in the same way that the probability of predicting what is thrown on a second roll of a die is independent of what was thrown on the first.

The purpose of the statistical solution is to account for the sensitivity of the jitter response relative to the modal frequencies. The assumption of statistically independent modal frequencies enables the sensitivities of several different modes to be combined to obtain a final statistical solution. If one mode dominates the response, the results of that modes sensitivity analysis will dominate the statistical solution thus making the statistically independent assumption meaningless. The DAMMPS platform response was dominated by the first two modes. Thus, the assumption, although important, was not a critical factor in the solution.

To begin the statistical solution, the selected mode's worst case solution was first determined. As described in the worst case analysis, a mode with natural frequency f_M within 15 percent of a driving frequency f_D is shifted to f_D and the worst case jitter for a single mode shift is calculated. If the worst case results exceed the jitter budget while the nominal value is less than the jitter budget, then there can be found an upper (f_U) and lower (f_L) shifted frequency value which results in jitter equal to the allowable. As an example, peak jitter as a function of the fundamental modal frequency for an instrument on the undamped platform is shown in Figure 10. The nominal response is shown to be less than the allowable. When the modal frequency is shifted to the driving frequency, the worst case response exceeds the allowable. The frequency limits for shifting the modes that produce responses in excess of the allowable are denoted by f_L and f_U. Assuming the Gaussian probability function shown in Figure 11 for the natural frequency f_M, the area under the density function bounded by f_L and f_U is the probability of jitter exceedance for that particular mode.

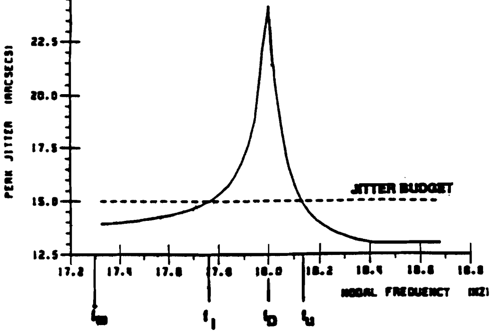

Figure 10. Peak Jitter Versus Fundamental Modal Frequency

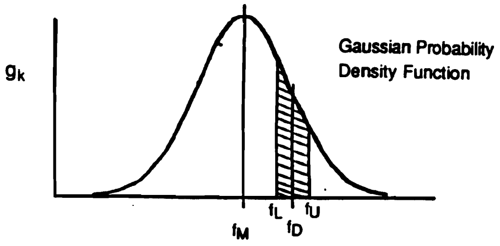

Figure 11. Probability of Jitter Exceedance Due to Single Mode Shift

Now, let $P[A_i]$ be the probability of the i^{th} event that one mode when shifted towards a specific harmonic results in jitter greater than the allowable. The complement of $P[A_i]$ is defined as

$$P[A_i]^C = 1 - P[A_i]$$

where $P[A_i]^C$ represents the probability that event A_i will not occur. Then, assuming that each A_i event is statistically independent, the probability that none of the A_i's will occur, $P[A]^C$, is given by the product of all the $P[A_i]^C$'s:

$$P[A]^C = \Pi_i \, P[A_i]^C$$

The probability of at least one worst case occurring, $P[A]$, is given by the complement of $P[A]^C$:

$$P[A] = 1. - P[A]^C$$

$P[A]$ represents the probability of exceeding the jitter budget based on all the individual mode frequency shifts. Note, that in the higher frequency ranges (above 50 Hz), the modal frequency error causes modes to be shifted to forcing frequencies both above and below its nominal value. If any single mode results in jitter values greater than the allowable in more than one harmonic, they are counted as distinct events for the statistical solution.

A plot showing jitter allowance versus the probability of exceeding the allowance is shown in Figure 12 for the undamped eggcrate and damped dual honeycomb (DHC) SSZ θ_z response. The undamped eggcrate, due to its sensitivity to resonance coupling, has a fairly wide range of possible jitter responses. The worst case response was 120 arcseconds, however the probability of this response is less than 1 percent. There is a 45 percent probability of exceeding the 15 arcsecond peak allowance, a 10 percent probability of exceeding 30 arcseconds peak, and a 5 percent probability of exceeding 60 arcseconds peak. Considering the high degree of reliability required for a spacecraft, the undamped eggcrate is not considered an adequate design. The damped dual honeycomb, which was much less sensitive to resonant coupling than the undamped eggcrate, has a small range of possible jitter responses which accounts for the nearly vertical DHC line when plotted on the same scale as the eggcrate. The worst case response was 6.7 arcseconds, significantly less than the 15 arcsecond allowance, making the dual honeycomb platform a viable concept with regard to orbital jitter.

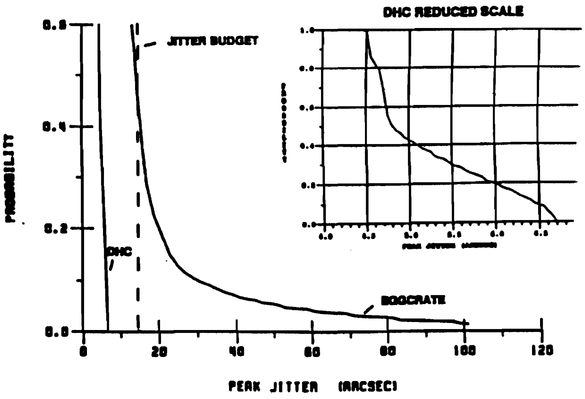

Figure 12. Probability of Exceeding Jitter Allowance

JITTER WEIGHT STUDY

The traditional method of reducing orbital jitter is to stiffen the structure. The approach taken in the DAMMPS program is to add passive damping. A weight study was conducted to determine which method, stiffening the structure or adding passive damping, is a more weight effective method for reducing orbital jitter.

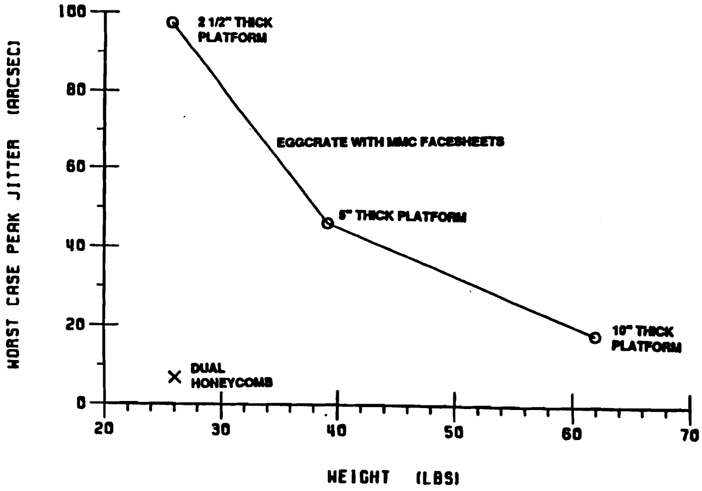

Figure 13. Worst Case Jitter Versus Platform Weight

The weight study was performed on an undamped eggcrate design with MMC facesheets. MMC facesheets were added to the eggcrate in order to remove any benefits the DAMMPS designs had due to their MMC facesheets. The eggcrate core thickness was increased stiffening the structure in the most weight effective manner. Worst case jitter was determined and plotted against the structure weight. The plot, shown in Figure 13, compares the stiffened eggcrate to the dual honeycomb design. Stiffening the eggcrate does reduce worst case orbital jitter, however, when compared to adding passive damping as done on the dual honeycomb design, there is a much greater weight penalty.

CONCLUSIONS

A worst case and statistical jitter analysis were performed on a viscoelastic dual honeycomb and an undamped baseline design. The results of the analysis clearly show the superiority of the damped design with regard to orbital jitter. The response of the instruments as a result of the input disturbances is significantly less for the damped designs. The damped structures are insensitive to resonant coupling, creating a robust design in that the structure modes do not have to be designed around the input disturbances.

A weight study conducted on the undamped eggcrate clearly demonstrated that adding passive damping is a much more weight effective method of reducing orbital jitter as compared to the traditional method of increasing stiffness. In addition, a small amount of damping (only 1.5 percent for the DAMMPS platform) may be adequate to limit even the worst case response to within an acceptable value.

Finally, using the statistical approach to describe jitter results provides additional insight for evaluating worst case responses. It allows us to assess jitter as a function of modal frequency and its relative location near a forcing frequency. The closer a frequency is to a forcing harmonic, the more likely a worst case will occur. At the same time, the magnitude of the jitter response for a particular mode and the overlapping shifts at higher frequencies are considered.

REFERENCES

1. Hill, D., Stahle, C., and Staley, J., "Evaluation of Damping Concepts for Precision Mounting Structures", Damping '91 Conference Proceedings, Feb., 1991.

2. Kirshenbaum, Dr. S., Hill, D., and Stahle, C., "Development of Low Modulus Damping Material for Precision Mounting Platforms", Damping '91 Conference Proceedings, Feb. , 1991.