THREE DIMENSIONAL ERROR PREDICTION

George M. Hahn and Edmund J. Pinney*
Dalmo Victor Company, San Carlos, California

I. INTRODUCTION

In a previous paper[1] presented by members of this organization, a number of methods for theoretically predicting the change in boresight due to the presence of the radome were described. One of the methods, referred to in the reference as "Scattering Technique", will be discussed in some detail in the present paper.

The basic idea of this method is to regard the radome as a perturbation source, determine the electromagnetic fields due to this source and then combine them with the fields due to the antenna without the presence of the radome. It is then possible to determine the boresight shift by comparing the two field configurations. The electromagnetic formulation is of necessity fully three-dimensional; none of the usual optic or plane wave approximations are made, and flat panel transmission and reflection coefficients are not employed.

II. DERIVATION OF THE BASIC INTEGRAL EQUATION

Maxwell's Equations for electromagnetic fields in a dielectric of volume V, and whose time variation is of the form $e^{-j\omega t}$, are:

$$\begin{aligned} &\bar{D} = \varepsilon \bar{E} && \bar{B} = \mu \bar{H} \\ &\nabla \times \bar{E} - j\omega\mu \bar{H} = 0 && \nabla \times \bar{H} + j\omega\varepsilon \bar{E} = 0 \qquad (1) \\ &\nabla \cdot \bar{E} = 0 && \nabla \cdot \bar{H} = 0 \end{aligned}$$

where all symbols have their usual meaning.

Similar equations hold in free space, if ε and μ are replaced by ε_0 and μ_0 respectively.

According to Stratton[2], the electromagnetic fields in this problem are the same as those that would be generated if no dielectric were present at all, but instead a distribution of virtual currents and charges. Let J be the virtual current distribution inside V, σ the virtual surface charge on S, the surface bounding the volume V and K the virtual surface current distribution on S. Here

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* Department of Mathematics, University of California, Berkeley, Calif.

1. Damonte, Hahn and Gunter, "Error Prediction Methods", Proceedings of the Radome Symposium, Vol. III, June 1955

2. J.A. Stratton, "Electromagnetic Theory", McGraw Hill Co., 1941

$$J = -j\omega\left(\frac{\varepsilon\mu}{\mu_0} - \varepsilon_0\right)\bar{E} \tag{2}$$

while σ and $\bar{K}$ are determined from the boundary conditions on S. The scalar and vector potentials can then be written:

$$\phi_p(P) = \frac{1}{\varepsilon_0}\int_S \sigma(Q)\,\frac{e^{jk_0R}}{4\pi R}\,dS \tag{3}$$

$$\bar{A}_p(P) = \mu_0\int_V J(Q)\,\frac{e^{jk_0R}}{4\pi R}\,dV + \mu_0\int_S \bar{K}(Q)\,\frac{e^{jk_0R}}{4\pi R}\,dS$$

where P and Q represent field and source points respectively and

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0} \tag{4}$$

The subscript p indicates that we are referring to the perturbation fields; these can be calculated from the perturbation potentials in the usual manner.

For the case $\mu = \mu_0$, one finds for the electric field:

$$\bar{E}_p(P) = (K-1)\left[k_0^2\int_V \bar{E}(Q)\,\frac{e^{jk_0R}}{4\pi R}\,dV - \int_S \bar{n}\cdot\bar{E}(Q)\,\nabla\frac{e^{jk_0R}}{4\pi R}\,dS\right] \tag{5}$$

with $K = \varepsilon/\varepsilon_0$, $\bar{E}(Q)$ the total electric field, and $\bar{n}$ the outward normal to S. Equation (5) is an integral equation relating the perturbation fields to the antenna fields and the physical properties of the radome. Since $\bar{E}(Q) = \bar{E}_p(Q) + \bar{E}_0(Q)$ the equation must be used for two separate calculations:

a) determination of $\bar{E}(Q)$, i.e. the effect of the presence of the radome on the field in the volume occupied by the radome itself,

b) calculation of $\bar{E}_p(P)$, i.e. the far field effects of the perturbation sources.

For the latter calculation, the following approximation is usually made:

Let O be some convenient origin in or near V and let $\bar{r} = \bar{m}r$ be the vector from O to P, $\bar{m}$ being a unit vector. If $\bar{q}$ denotes the vector from O to the point of integration, Q, then:

$$R = r - \bar{m}\cdot\bar{q} + O(1/R) \tag{6}$$

Retaining only the first two terms, equation (5) reduces to:

$$E_p(P) \cong -(K^2-K_0^2)\frac{e^{jk_0 r}}{4\pi r}\,\bar{m}\times\left(\bar{m}\times\int_V \bar{E}(Q)\,e^{-jk_0\bar{m}\cdot\bar{q}}\,dV\right) \tag{7}$$

III. TECHNIQUES OF SOLVING THE INTEGRAL EQUATION

It is clear that one cannot hope to obtain solutions in closed form to equation (5) or to equation (7). The starting data are the measured fields $(\bar{E}_0)$ in numerical form. While it would be possible to fit these data in some polynomial approximation, the complicated geometry of most radomes would still prevent one from exactly solving the equations.

Numerical methods must therefore be resorted to, and three different methods were considered by the authors.

a) The Grid Method:

Suppose $\bar{F}_p(P)$ denotes an approximation to $\bar{E}_p(P)$ then $\bar{F}(P)=\bar{E}_0 P+\bar{F}_p(P)$ is an approximation to $\bar{E}(P)$

It is then possible to define an error term

$$\varepsilon=\int_V \bar{R}(Q)\cdot\bar{R}^*(Q)\,dV \tag{8}$$

where the $\bar{R}$'s are defined in such a manner that they vanish when $\bar{F}(P)=\bar{E}(P)$. The radome volume is now covered by a grid of N points sufficiently close together; the approximation $F_p(P)$ is assumed as due to contribution from the N points:

$$\bar{F}_p(P)=\sum_{n=1}^{N} a_n\,\zeta(P) \tag{9}$$

The error term then becomes

$$\varepsilon=\sum_{m=1}^{N}\sum_{n=1}^{N} R_{mn}\,a_m\,a_n^* \tag{10}$$

which is minimized by equating the $\frac{\partial\varepsilon}{\partial a_n}$ and $\frac{\partial\varepsilon}{\partial a_n^*}$ to zero for $n=1\cdots N$. This gives N linear equations in the N complex a_n's; solving these the results are then substituted into (9) and $\bar{F}_p(P)$ is obtained.

b) Stationary Phase Approximation:

The method to be outlined here depends on the fact that the integrals in the integral equation have integrands involving the product of $\frac{e^{jk_0R}}{4\pi R}$ and the internal electric field. Now if the internal electric field behaves locally like a plane wave, it contains a factor e^{jkx}, where x is the coordinate in the direction of the wave. The two exponential factors are responsible for the change in phase as the point of integration moves about in the dielectric shell.

Now consider a straight line drawn outward from the point of observation $R=0$. Suppose for simplicity that $x=0$ when $R=0$. Then on this line the ratio R/x remains constant, and the exponential argument jk_oR+jkx increases linearly as R increases. However on some lines the factor of proportionality is zero, so the phase remains constant. These are all the lines lying on a cone (which might be called the "cone of stationary phase") with axis parallel to the direction of propagation of the interior electric field, and elements inclined to the axis at an angle equal to the compliment of the critical angle for the dielectric. On one side of this cone the phase increases as R increases, and on the other side the phase decreases as R increases. The contributions to the integral from the region near the cone add together and reinforce each other. The contributions from the region well away from the cone have a tendency toward mutual cancellation.

The idea then is to approximate the dielectric sheet locally by a flat sheet which fits as closely as possible to the curve of intersection of the cone of stationary phase and the surface of the dielectric sheet, and to evaluate the integrals in the integral equation for that flat sheet, and on the basis of the assumption that the electric field is essentially plane in the region of importance in the integral. This is probably not a bad assumption because the cone of stationary phase occupies only a limited region near the point of observation.

This gives rise to what may be considered merely a flat plate theory. However, it is not the usual sort of flat plate theory. In the usual flat plate theory the dielectric shell is approximated by a flat plate whose two sides are tangent to the shell in the local region of interest. In that theory the plate is held fixed in the determination of the field at all points of the local region. In the present theory, on the other hand, the orientation and thickness of the fitting flat plate change from point to point, and depend on other properties of the shell than its thickness alone. This may be expected to give a more accurate theory than the usual flat plate theory.

c) Lumped Fields Method:

Another approach is to divide up the radome into a number of circular disks of the order of a wavelength in diameter. The point is to "lump" the local behavior of the electric field in each disk in a way one can handle, and then to use the basic integral equation to tie together the fields in the different disks. To do this one must first express the electric field throughout each disk in terms of the field at the center. Certain assumptions are required to do this. One may assume that the internal field in the disk is approximated by two plane waves whose directions of propagation may be set at will, but may, in particular, be given the directions that the internal waves in a flat sheet would take when acted upon by the local magnetic field considered as a plane wave. If that is done, one can apply a good deal of plane sheet theory to simplify calculations of the local fields.

It is then possible to calculate the total field at the center of each disk by integrating over each disk and then summing over all the disks. The problem then reduces to a set of linear equations in the unknown fields at the centers of the disks. Rather than attempt to solve these directly, one can again resort to a least squares and successive approximation theory, as described in Paragraph III a).

The three methods just discussed are all of considerable complexity. The grid method, to provide sufficient accuracy, requires an excessively large number of points taken. As a result, the storage requirements exceed the capacities of available computing machines. The stationary phase method is hampered by the difficulty of fitting the sheets to the cones of stationary phase. This process proved so difficult that the approach was abandoned completely. The lumped field technique looks the most promising of the three.

It should be pointed out that the existence of a unique, bounded solution to equations (5) and (7) is by no means obvious. The existence of the singularity in the term $\frac{e^{jk_0R}}{R}$ makes the question non-elementary, from a mathematical point of view. However, the results mentioned in the following paragraph tend to show that such a solution does exist, a condition which one would expect from the physics of the situation.

IV. RESULTS

To test the possibility of a successive approximations scheme as suggested in paragraphs III a) and c) above, the following experiment was carried out. A cylindrical polystirene $(\epsilon = 2.5)$ ring, large compared to the wave length was placed in front of a parabolic reflector transmitting at X-band. The resulting pattern was measured in the E plane of the antenna. Three points of the pattern were then computed using the iterative scheme outlined in III c), on the basis of one plane wave only and then for the next two higher approximations. In each case the higher order approximations were successively closer to the measured values than the low order approximations.

At the present time computations are being carried out at WADC to test the method on a radome; unfortunately no results have become available as of the date of this paper.

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