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ANALYTICAL AND EXPERIMENTAL CONSIDERATIONS OF THE VELOCITY DISTRIBUTION IN THE WAKE OF A BODY OF REVOLUTION

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FOREWORD

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ABSTRACT

An analytical and experimental investigation of the velocity and pressure distribution in the wake of a body of revolution has been conducted and an equation was derived which presents the velocity and pressure distribution of the turbulent wake as function of an experimental parameter $\mathcal{H}$. The experimental parameter $\mathcal{H}$ was extracted from newly performed measurements.

Satisfactory agreement between theoretical prediction and experimental results was obtained over a range of between four and twelve diameters downstream of the body.

Publication Review

The publication of this report does not constitute approval by the Air Force of the findings or conclusions contained herein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

Warren P. Shepardson
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LIST OF SYMBOLS

A, K, and η, η'	Coefficients of proportionality
C_D	Drag Coefficient
C_P	Pressure Coefficient
C_1, C_2	Constants of Integration
D	Diameter of the Body of Revolution
S	$\pi \frac{D^2}{4}$
P	Static pressure in the wake
P_o	Total pressure
V_o	Velocity of the undisturbed fluid
b	Width of mixing zone of wake
ℓ	"Mischungsweg" - mixing distance
q	Dynamic pressure of the stream
u,v	Components of velocity
x,z	Cylindrical coordinates
x,y	Cartesian coordinates
$\varkappa$	An empirical constant
ρ	Density of flow

SECTION 1

INTRODUCTION

The velocity and pressure distribution in the turbulent wake of two - and three-dimensional bodies has been treated in a number of investigations. The subject is very complicated and a perfect solution does not seem to exist (Ref. 5).

For problems of aerodynamic retardation, in which a drag-producing secondary body is deployed in the wake of a fast moving primary body in order to reduce through aerodynamic drag the speed of both bodies, the knowledge of the velocity and pressure distribution in the wake of the primary body becomes a very important problem. The final objective of problems of this nature is then the determination of the drag of retardation devices in undisturbed flow and in the wake of a primary body, and the drag of the system consisting of primary and secondary body.

The analytical and experimental determination of the velocity and pressure distribution in the turbulent wake was selected to be the first phase in this field of investigation, and the approach and the results of these studies are reported in the following.

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SECTION 2

THE EQUATION OF MOTION

2.1 The Turbulent Wake Behind a Body

It is customary to present the turbulent wake as shown in Figs. 1 and 2.

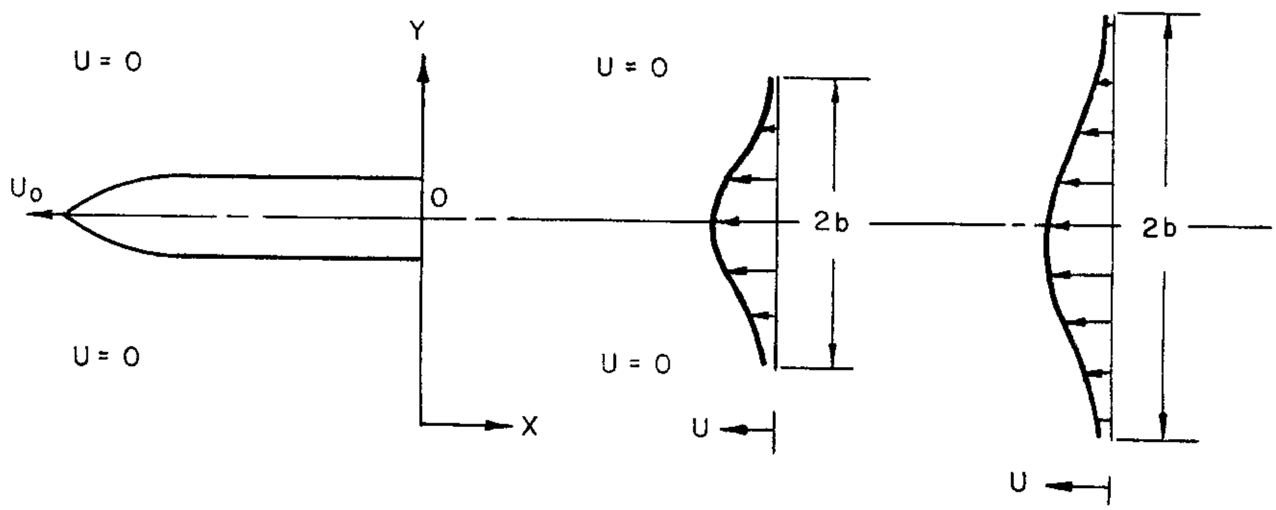

Fig. 1 Velocity Distribution for a Fluid at Rest and a Moving Body

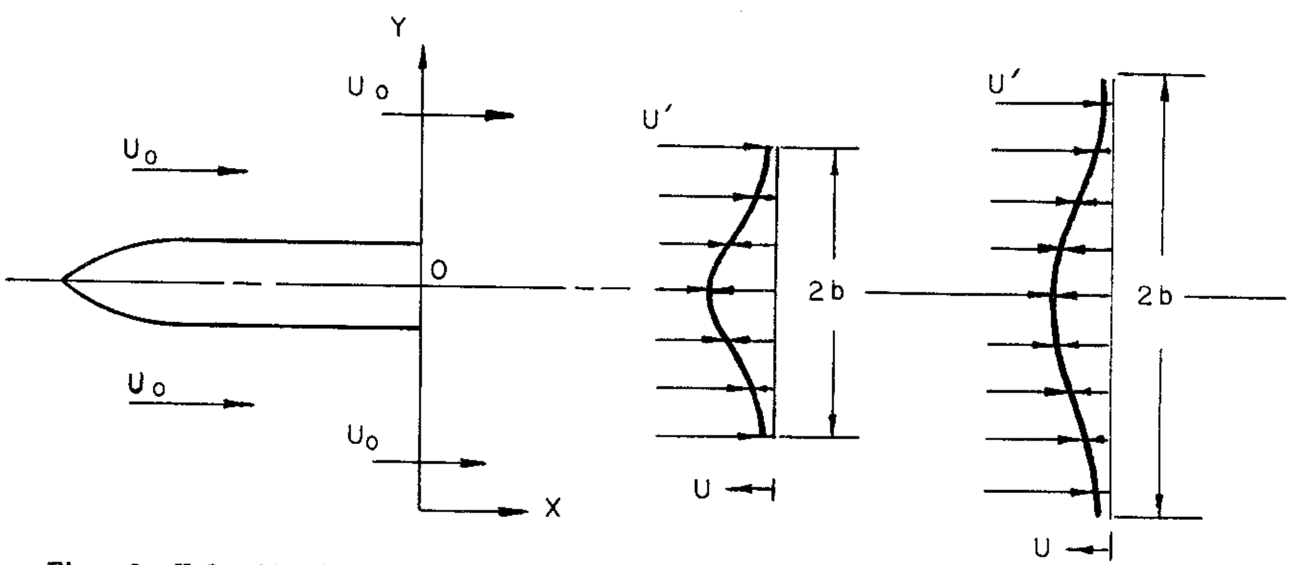

Fig. 2 Velocity Distribution for a Body at Rest and a Moving Fluid

One has to distinguish the case in which the undisturbed flow is at rest and the body moves with the velocity u_o, creating in its wake a turbulent flow with a varying velocity u which is smaller than the velocity of the moving body (Fig. 1). In the other case the body is at rest while the undisturbed fluid moves with the velocity u_0, with a varying velocity u' in the wake of the body (Fig. 2). Experience shows that with increasing distance from the body the area of the disturbed velocity increases while the difference between the free-stream and the disturbed velocity decreases.

2.2 The Simplified Equation of Motion

Let us assume that a body of revolution is immersed in a fluid with the velocity u_0 while the body is held at rest and that a fixed longitudinal axis is arranged which coincides with the direction of the undisturbed flow (Fig. 2).

At a certain distance x behind the body the components of the disturbed velocity in the x- and y-directions are $u' = u_0 - u$ and v respectively. As is usual in considerations concerning boundary layer phenomena, one may assume that the variation of the velocity perpendicular to the x-axis is large compared to that along the x-axis, and that the pressure gradient parallel to the x-axis may be neglected. Under these circumstances one may establish with Swain (Ref. 3) the equation of continuity and the approximate differential equation of motion as follows:

$$\frac{\partial}{\partial x}(yu) + \frac{\partial}{\partial y}(yv) = 0 \tag{1}$$

and

$$(u_0 - u)\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left[\ell^2 y \left(\frac{\partial u}{\partial y}\right)^2\right] \tag{2}$$

The term

$$\rho \, \ell^2 \left(\frac{\partial u}{\partial y}\right)^2$$

in equation (2) is the so-called "apparent stress" or "Reynolds stress," while ℓ represents the "Mischungsweg" (Ref. 1). Using the condition of continuity and the fact that the aerodynamic drag of the body equals the loss of momentum in the turbulent wake, one may write

$$\frac{\rho}{2} c_D \, S \, u_o^2 = 2\pi\rho \int_o^\infty (u^2 + u_o \, u) \, y \, dy, \qquad (3)$$

in which the term

$$\frac{\rho}{2} c_D \, S \, u_o^2$$

is the conventional presentation of the aerodynamic drag of the body. Since the velocities u and v are small compared to u_o, equations (2) and (3) can be written as follows:

$$u_o \frac{\partial u}{\partial x} = \frac{1}{y} \frac{\partial}{\partial y} \left[\ell^2 \, y \left(\frac{\partial u}{\partial y}\right)^2 \right] \qquad (4)$$

and

$$\frac{\rho}{2} c_D \, S \, u_o^2 = 2\pi\rho \int_o^\infty u_o \, u \, y \, dy. \qquad (5)$$

In cylindrical coordinates equation (5) is written as

$$\frac{\rho}{2} c_D \, S \, u_o^2 = 2\pi\rho \int_o^\infty u_o \, u \, r \, dr. \qquad (5a)$$

Replacing the term for the apparent stress in the conventional manner, one obtains (Ref. 5)

$$\ell^2 \left(\frac{\partial u}{\partial y}\right)^2 = \varkappa b \, (u_{max} - u_{min}) \, \frac{\partial u}{\partial y}, \qquad (6)$$

where $\varkappa$ is an empirical constant and b denotes the width of the mixing zone.

With $u_{min} = 0$ and the derivation completed, equation (4) may be written in cylindrical coordinates r and x as

$$u_o \frac{\partial u}{\partial x} = \varkappa\, b\, u_{max} \left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) . \tag{7}$$

Assuming that at a great distance from the body the profiles of u in different cross-sections of the turbulent wake are similar, and using the method of analysis of dimensions and power-laws (Refs. 3 and 5), one obtains

$$r = \eta\ x^{1/3},$$

$$b = K\, x^{1/3},$$

and the velocity at the centerline

$$u_{max} = \frac{A\, u_o}{x^{2/3}} .$$

In these equations η, K, and A are coefficients of proportionality.

The solution of equation (7) may be assumed to be

$$u = \frac{A\, u_o}{x^{2/3}}\, e^{s\eta^2} , \tag{8}$$

in which s needs to be determined.

The value of the individual terms of equation (7) with the assumption under (8) amounts to

$$\frac{\partial u}{\partial x} = -\frac{2}{3}\frac{A\, u_o}{x^{5/3}} (1 + s\eta^2)\, e^{s\eta^2} ,$$

$$\frac{\partial u}{\partial r} = \frac{2\, A\, u_o}{x} s\, \eta\ e^{s\eta^2} ,$$

and

$$\frac{\partial^2 u}{\partial r^2} = \frac{2\, A\, u_o\, s}{x^{4/3}} (1 + 2\, s\eta^2)\, e^{s\eta^2} .$$

Substituting these expressions into equation (7) results in

$$(1 + s\eta^2)\ (1 + 6\varkappa K A s) = 0.$$

which provides

$$s_1 = -\frac{1}{\eta^2}$$

and

$$s_2 = -\frac{1}{6\varkappa K A}$$

Equation (8) may now be written

$$u = \frac{A\,u_o}{x^{2/3}}\left(C_1\, e^{-\frac{\eta^2}{6\varkappa K A}} + C_2\, e^{-1}\right).$$

The boundary conditions of the wake are

$$u = 0 \quad \text{for } r = \infty\,,$$

$$u = u_{max} \quad \text{for } r = 0\,,$$

from which the constants can be evaluated as

$$C_1 = 1,$$

$$C_2 = 0.$$

Therefore, the disturbing velocity amounts to

$$u = u_o\frac{A}{x^{2/3}}\, e^{-\frac{\eta^2}{6\varkappa K A}}\,. \qquad (9)$$

2.3 The Coefficients of A and K

The velocity relationship presented in equation (9) contains two unknown coefficients A and K. For their determination one has so far merely the drag momentum equation (5a). The second condition will be taken from experimental data given in Ref. 10 and those shown in Fig. 4. Furthermore, the analytical treatment by Swain (Ref. 3) can be used to derive another condition for the determination of A and K for the purpose of comparing the validity of the entire treatment.

For the utilization of the experimental data, equation (9) will be transferred into the form

$$\frac{u}{u_{max}} = e^{-\frac{K\, r'^2}{6\, \varkappa A}} \qquad (10)$$

with the notation

$$r' = \frac{r}{K\, x^{1/3}} = \frac{r}{b}$$

Experiments show that the ratio of u/u_{max} varies very little in the vicinity of the centerline, which qualifies this region particularly for numerical evaluation. Therefore the velocity ratio at a point with $r' = 0.1$ shall be chosen, which amounts in accordance to Ref. 10 and Fig. 4 to approximately $u/u_{max} = 0.96$. Therefore one may set

$$0.96 = e^{-\frac{0.01\, K}{6\, \varkappa A}}$$

which provides

$$K = 24.48\, A\, \varkappa\, . \qquad (11)$$

Expressing the drag-momentum equation (5a) in terms of the velocity as presented in (10), one obtains

$$\frac{C_D S}{4\pi} = A\, K^2 \int_0^\infty e^{-\frac{K\,(r')^2}{6\,\varkappa\, A}}\, r'\, dr'$$

$$\frac{C_D S}{4\pi} = 3\, A^2\, K\, \varkappa$$

Substituting the previously obtained value for K into the last equation gives

$$C_D S = 12\, \pi\, 24.48\, A^3\, \varkappa^2 \qquad (12)$$

or

$$A = 0.103\, \cdot \left(\frac{C_D\, S}{\varkappa^2}\right)^{\frac{1}{3}}$$

From equation (11) and (12) one finds

$$K = 2.52 \, (C_D S \varkappa)^{1/3} \qquad (13)$$

Swain (Ref. 3) derived for the velocity distribution the equation

$$\frac{u}{u_{max}} = (\xi^{3/2} - 1)^2 \qquad (14)$$

in which

$$\xi = \frac{r}{r_o} = \frac{r}{b} ,$$

r_o being the value of r at the edge of the wake.

One may now consider the velocity ratio of the particular point at which $u = \frac{u_{max}}{2}$ and call the related coordinate

$$\xi = \xi_m$$

This coordinate can numerically be determined from the relationship

$$u = \frac{u_{max}}{2} = u_{max} (\xi_m^{3/2} - 1)^2$$

which provides

$$\xi_m = 0.293^{2/3}$$

Equation (10) can be transformed by means of r_m as the coordinate at which $u = \frac{u_{max}}{2}$ in the same manner and one obtains

$$u = \frac{u_{max}}{2} = u_{max} \, e^{-\frac{K r_m^2}{6 \varkappa A}}$$

From this one finds

$$r_m = \left(4.158 \ \frac{\varkappa A}{K} \right)^{1/2} .$$

If one postulates that both systems are correct, the values of ξ_m and r_m must be equal. Consequently,

$$\left(4.158 \ \frac{\varkappa A}{K} \right)^{1/2} = \left(0.293 \right)^{2/3}$$

from which follows

$$A = 0.0468 \ \frac{K}{\varkappa} .$$

Substituting A into (12) provides

$$C_D \ S = 12 \ \pi \ (0.0486)^2 \ \frac{K^3}{\varkappa} ,$$

whence

$$\left.\begin{aligned} K &= 2.3 \ (C_D \ S \ \varkappa)^{1/3} \\ \text{and} \quad A &= 0.107 \ (C_D \ S \ / \varkappa^2)^{1/3} \end{aligned}\right\} \qquad (17)$$

The values of A and K which were obtained from experimental data, shown under (12) and (13), and the related data derived from Swain's equation, presented above, agree very well. In view of the fact the results from the experimental data as well as the ones derived from Swain's equation include certain assumptions, it appears to be justified to establish average values for A and K based on the two independent methods.

In this manner one can establish

$$A = 0.105 \left(C_D S / \varkappa^2 \right)^{1/3}$$

$$K = 2.41 \left(C_D S \varkappa \right)^{1/2} .$$

These values shall be used in the further treatment of the wake problem. The velocity equation (9) presented with $\eta = \frac{r}{x^{1/3}}$ and with A and K from above may now be written in the following form:

$$\frac{u}{u_o} = \frac{0.105}{x^{2/3}} \left(C_D S / \varkappa^2 \right)^{1/3} e^{-\frac{r^2}{1.525 \left(C_D S \varkappa x \right)^{2/3}}} \qquad (18)$$

SECTION 3

NUMERICAL COMPARISON

The equations (14) and (18) represent the local velocity in the turbulent wake. Judging from their structure, it appears that equation (18) might offer an easier numerical process and it appears to be justified to pursue its exploitation. The first point in this study shall be a comparison of the predictions of available theoretical methods with experimental results. For this purpose, one may compare the predictions represented by equation (18) with those given by Swain's equation (14), Goldstein's results obtained by means of "modified vorticity theory" (Ref. 9), and the mean experimental results obtained by Hall and Hislop (Ref. 10). For a convenient comparison one may introduce the quantity r_m or ξ_m as defined before, and derive from equations (14) and (18)

$$\frac{u}{u_{max}} = \left[0.293 \left(\frac{\xi}{\xi_m}\right)^{3/2} - 1\right]^2 \qquad (19)$$

and

$$\frac{u}{u_{max}} = e^{-0.693 \left(\frac{r}{r_m}\right)^2} \qquad (20)$$

respectively. Equations (19) and (20) represent now the velocity ratio as functions of $\frac{r}{r_m} = \frac{\xi}{\xi_m} = z$. The same process can also be applied to the findings given by Goldstein's (Ref. 9) and the experimental results by Hall and Hislop (Ref. 10). In the manner the results of Table 1 were calculated. Figure 3 illustrates the same results and it can be seen that equation (20) agrees with the experimental values best in the central portion, while equation (19) appears to be a better approximation in the outer portion of the wake.

3.1 Revised Equations for the Velocity Distribution behind a Body of Revolution

For practical cases the drag coefficient C_D is usually known and the related area is generally given by $S = \frac{\pi D^2}{4}$, where D is the diameter of the body of revolution. In such a case, it is convenient to present equation (18) in terms of C_D and D and one obtains

$$\frac{u}{u_o} = \frac{0.107}{\left(\frac{x}{D}\right)^{2/3}} \left(\frac{C_D \pi}{4 \varkappa^2}\right)^{1/3} e^{-\frac{0.415 \, (r^*)^2}{\left(\frac{x}{D}\right)^{2/3} (C_D \pi \varkappa)^{2/3}}} \qquad (21)$$

where $r^* = \frac{2r}{D}$.

Thus, the velocity in the wake of the body of revolution which moves with a velocity u_o through the fluid is given by equation (21). If the fluid moves with the velocity u_o past the fixed body, the velocity of the fluid in the wake u' equals

$$u' = u_o - u \ ,$$

and may be expressed by means of equation (21) as follows:

$$\frac{u'}{u_o} = 1 - \frac{0.107}{\left(\frac{x}{D}\right)^{2/3}} \left(\frac{C_D \pi}{4 \varkappa^2}\right)^{1/3} e^{-\frac{0.415 \, (r^*)^2}{\left(\frac{x}{D}\right)^{2/3} (C_D \pi \varkappa)^{2/3}}}$$

EQUATION	(19)	(20)	GOLD-STEIN*	HALL, HISLOP**
Z	U/U$_{MAX.}$	U/U$_{MAX.}$	U/U$_{MAX.}$	U/U$_{MAX.}$
0	1.000	1.000	1.000	1.000
0.2	0.973	0.948	0.940	0.980
0.4	0.895	0.857	0.860	0.900
0.5	0.841	0.802	0.800	0.830
0.6	0.779	0.746	0.740	0.760
0.8	0.642	0.625	0.630	0.630
1.0	0.500	0.500	0.500	0.500
1.2	0.368	0.376	0 400	0.380
1.4	0.256	0.264	0.300	0.260
1.6	0.169	0.165	0.220	0.140
1.8	0.106	0.079	0.140	0.060
2.0	0.063	0.029	0.100	——

*EXTRACTED FROM REF. 9
**EXTRACTED FROM REF. 10

TABLE I— ANALYTICAL AND EXPERIMENTAL VELOCITY DISTRIBUTION

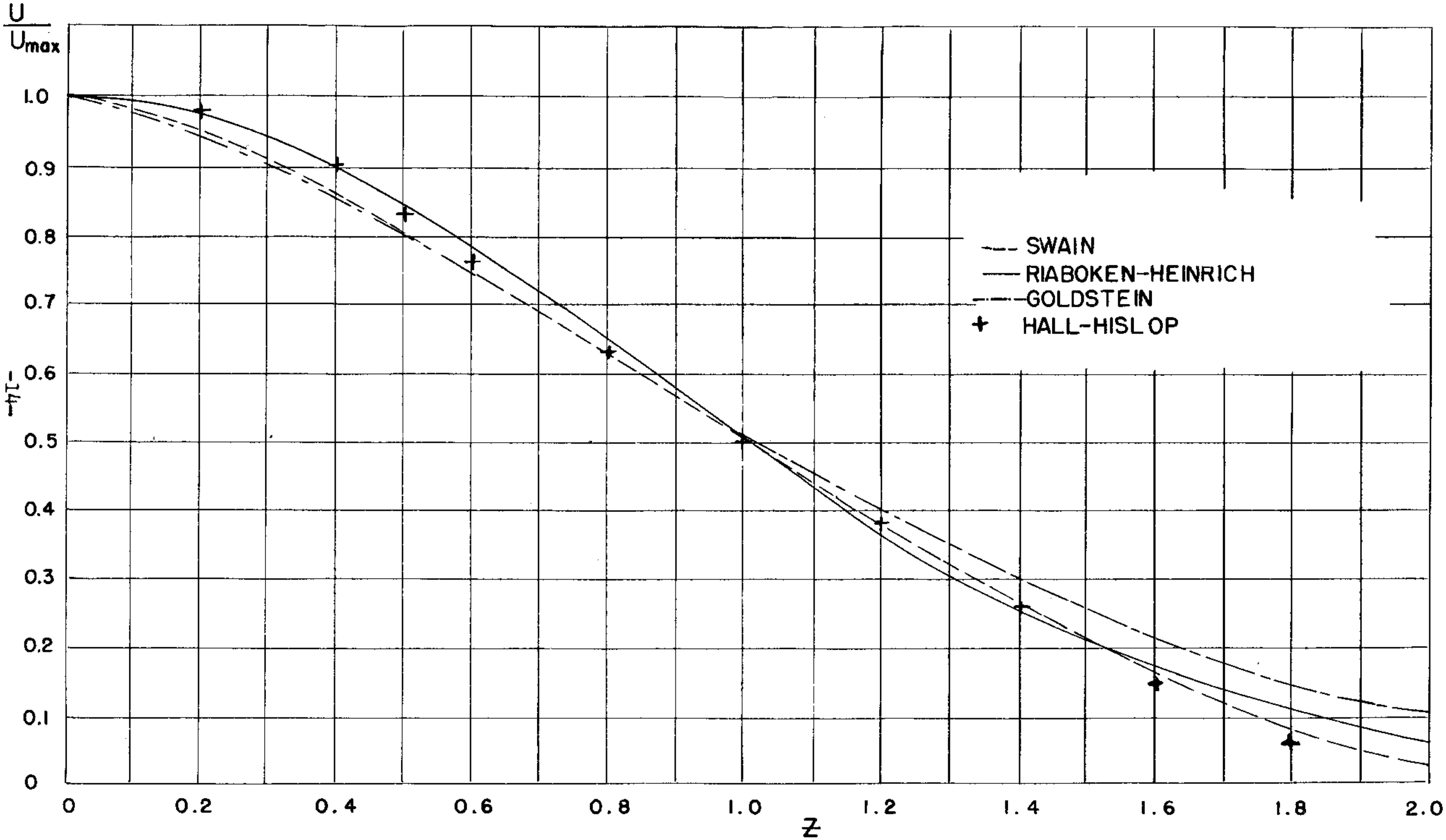

FIG. 3 VELOCITY DISTRIBUTION IN ACCORDANCE WITH ANALYTICAL AND EXPERIMENTAL STUDIES

SECTION 4

THE EXPERIMENTAL PARAMETER

The equations (21) and (22) still contain the empirical parameter $\varkappa$ which for any practical solution needs to be known. For the purpose of this study and for a more experimental approach to the principal problem of aerodynamic retardation, the pressure distribution in the wake of a cylindrical body with an ogive nose and blunt tail was experimentally determined. Some results of this study are presented in Fig. 4 and Table 2, which are abstracts of Ref. 12. An attempt will be made to correlate the experiments with the analytical prediction of equation (21) and various values of the parameter $\varkappa$.

4.1 Experimental Pressure Distribution

The pressure distribution of the wake shall be presented by means of the pressure coefficient, which is defined as the difference between the total and the static pressure in the wake divided by the dynamic pressure of the free stream. For incompressible flow this fraction is identical to the ratio of the square of the velocities in the wake and in the free stream. In view of the derivation above one may set

$$C_p = \frac{\Delta p}{q} = \left(\frac{u'}{u_o}\right)^2 ,$$

which can be expressed in terms of equation (22) and one obtains

$$\frac{\Delta p}{q} = \left[1 - \frac{0.107}{\left(\frac{x}{D}\right)^{2/3}} \left(\frac{C_D \pi}{4 \varkappa^2}\right)^{1/3} e^{\dfrac{0.415\,(r^*)^2}{\left(\frac{x}{D}\right)^{2/3} (C_D \pi \varkappa)^{2/3}}} \right]^2 . \qquad (23)$$

X/D	2	4	6	8	10	12
R^*	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$
0	0.520	0.740	0.790	0.810	0.820	0.830
0.2	0.580	0.742	0.795	0.814	0.822	0.835
0.4	0.712	0.778	0.813	0.824	0.828	0.841
0.6	0.850	0.832	0.838	0.841	0.838	0.841
0.8	0.955	0.900	0.870	0.862	0.852	0.868
1.0	0.992	0.948	0.908	0.890	0.878	0.885
1.2	1.000	0.980	0.948	0.920	0.900	0.905
1.4	1.000	0.998	0.970	0.945	0.925	0.925
1.6	1.000	1.000	0.988	0.965	0.950	0.942
1.8	1.000	1.000	0.995	0.982	0.970	0.958
2.0	1.000	1.000	0.998	0.998	0.980	0.972
2.5	1.000	1.000	1.000	1.000	0.985	0.990

TABLE 2 — RESULTS OF EXPERIMENTAL DETERMINATION OF THE PRESSURE DISTRIBUTION IN THE WAKE OF A BODY OF REVOLUTION ($M \approx 0.2$, $R_e = 2.74 \times 10^5$, $C_p = 0.35$)

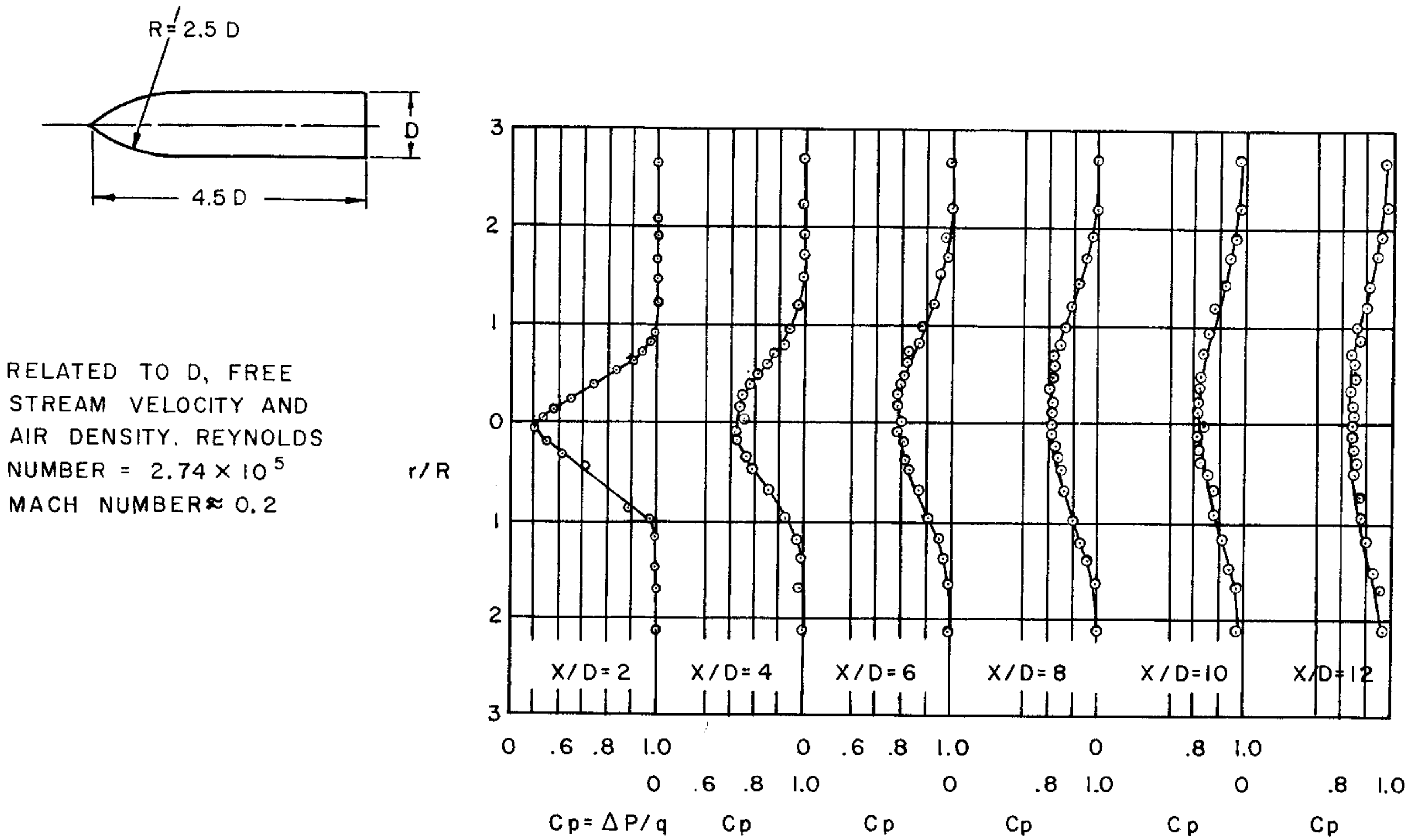

FIG. 4 PRESSURE DISTRIBUTION IN THE WAKE OF A BODY OF REVOLUTION (C_D = 0.35)

In the equations (21), (22), and (23) the velocity ratio $\frac{u}{u_o}$ depends, for given C_D, $\frac{x}{D}$, and r* values, only upon the parameter $\varkappa$, which can be determined from any point at which $\frac{u}{u_o}$ is known. From Fig. 4, it is evident that $\varkappa$ will vary depending on the location of the control section. Otherwise it is convenient to choose points at the longitudinal axis as reference points. With r* = 0 and C_D = 0.35, equation (21) becomes

$$\frac{u}{u_o} = \frac{0.107}{\left(\frac{x}{D}\right)^{2/3}} \left(\frac{C_D \pi}{4\varkappa^2}\right)^{1/3},$$

from which $\varkappa$ is found to be

$$\varkappa = \frac{0.0176}{\frac{x}{D}\left(\frac{u}{u_o}\right)^{3/2}} . \qquad (24)$$

With equation (24) and the experimental data given in Fig. 4 one can determine for each $\frac{x}{D}$ value a related value of $\varkappa$ as shown in Table 3 and Fig. 5.

TABLE 3

$\frac{x}{D}$	2	4	6	8	10	12
$\frac{\Delta P}{q}$ (EXPERIMENTAL)	0.525	0.725	0.775	0.800	0.815	0.830
$\frac{u}{u_o}$ (CALCULATED)	0.275	0.149	0.120	0.106	0.097	0.089
$\varkappa^2$ (CALCULATED)	0.00372	0.00584	0.00497	0.00406	0.00339	0.00305
$\varkappa$	0.0610	0.0764	0.0705	0.0633	0.0582	0.0552

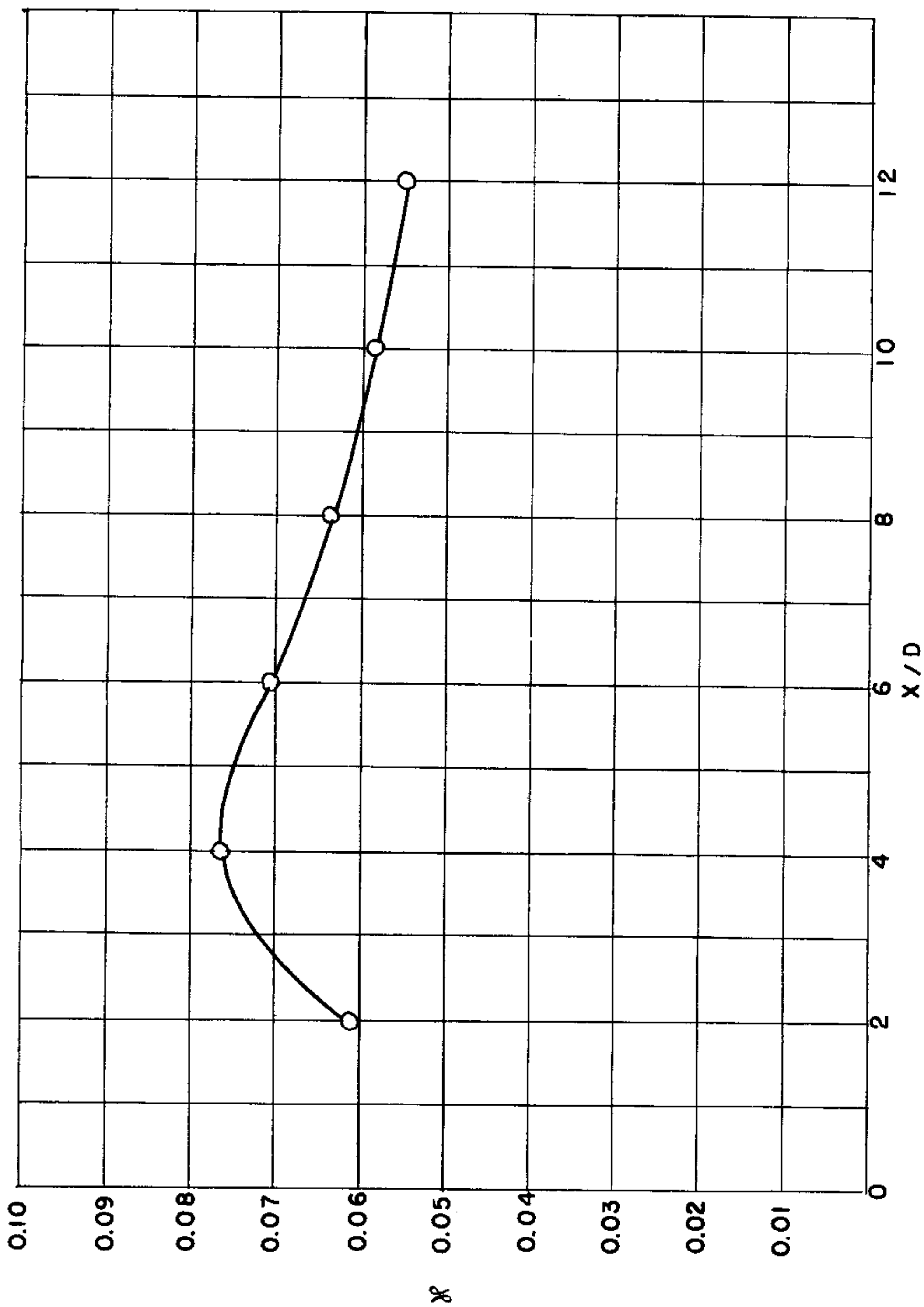

FIG. 5 - ϰ VS. X/D AS DETERMINED FROM EXPERIMENTAL DATA

The average value of $\varkappa$ from Table 3 amounts to $\varkappa = 0.0633$, which corresponds to C_{P_o} at $\frac{x}{D} \approx 8$. Substituting $\varkappa = 0.0633$ and $C_D = 0.35$ into equation (21) provides

$$\frac{u}{u_o} = \frac{0.425}{\left(\frac{x}{D}\right)^{2/3}} e^{-\frac{2.44\,(r^*)^2}{\left(\frac{x}{D}\right)^{2/3}}} \tag{25}$$

Corresponding to this relationship, the pressure distribution was calculated and the results are shown in Table 4 and Fig. 6.

An inspection indicates good agreement between experimental and analytical data at distances of $\frac{x}{D} \geqq 6$.

In an attempt to obtain a better agreement between analytical prediction and experimental results at shorter distances from the body, the velocity and pressure distribution was calculated from equations (22) and (23) using $\varkappa = 0.0764$ corresponding to an $\frac{x}{D}$ value of 4 (See Fig. 5 or Table 3).

The numerical forms of equations (21) and (23) amount to

$$\left(\frac{u}{u_o}\right) = \frac{0.375}{\left(\frac{x}{D}\right)^{2/3}} e^{-\frac{2.15\,(r^*)^2}{\left(\frac{x}{D}\right)^{2/3}}}$$

and

$$\frac{\Delta P}{q} = \left[1 - \frac{0.375}{\left(\frac{x}{D}\right)^{2/3}} e^{-\frac{2.15\,(r^*)^2}{\left(\frac{x}{D}\right)^{2/3}}}\right]^2 .$$

Table 5 and Fig. 7 show the results of these calculations. One observes

X/D	2	4	6	8	10	12
r^*	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$
0	0.536	0.691	0.759	0.799	0.824	0.844
0.2	0.560	0.702	0.766	0.800	0.828	0.847
0.4	0.626	0.732	0.785	0.817	0.839	0.855
0.5	0.669	0.753	0.797	0.826	0.844	0.861
0.6	0.716	0.776	0.810	0.837	0.854	0.869
0.8	0.810	0.827	0.846	0.861	0.872	0.884
1.0	0.888	0.876	0.880	0.887	0.895	0.900
1.25	0.947	0.927	0.922	0.920	0.922	0.923
1.5	0.983	0.962	0.953	0.947	0.945	0.945
1.75	0.995	0.983	0.974	0.968	0.964	0.962
2.0	0.999	0.993	0.986	0.982	0.978	0.974

TABLE 4 — RESULTS OF ANALYTICAL DETERMINATION OF THE PRESSURE DISTRIBUTION IN THE WAKE OF A BODY OF REVOLUTION (BASED ON $\varkappa$ = 0.0633 RELATED TO C_p AT X/D = 8)

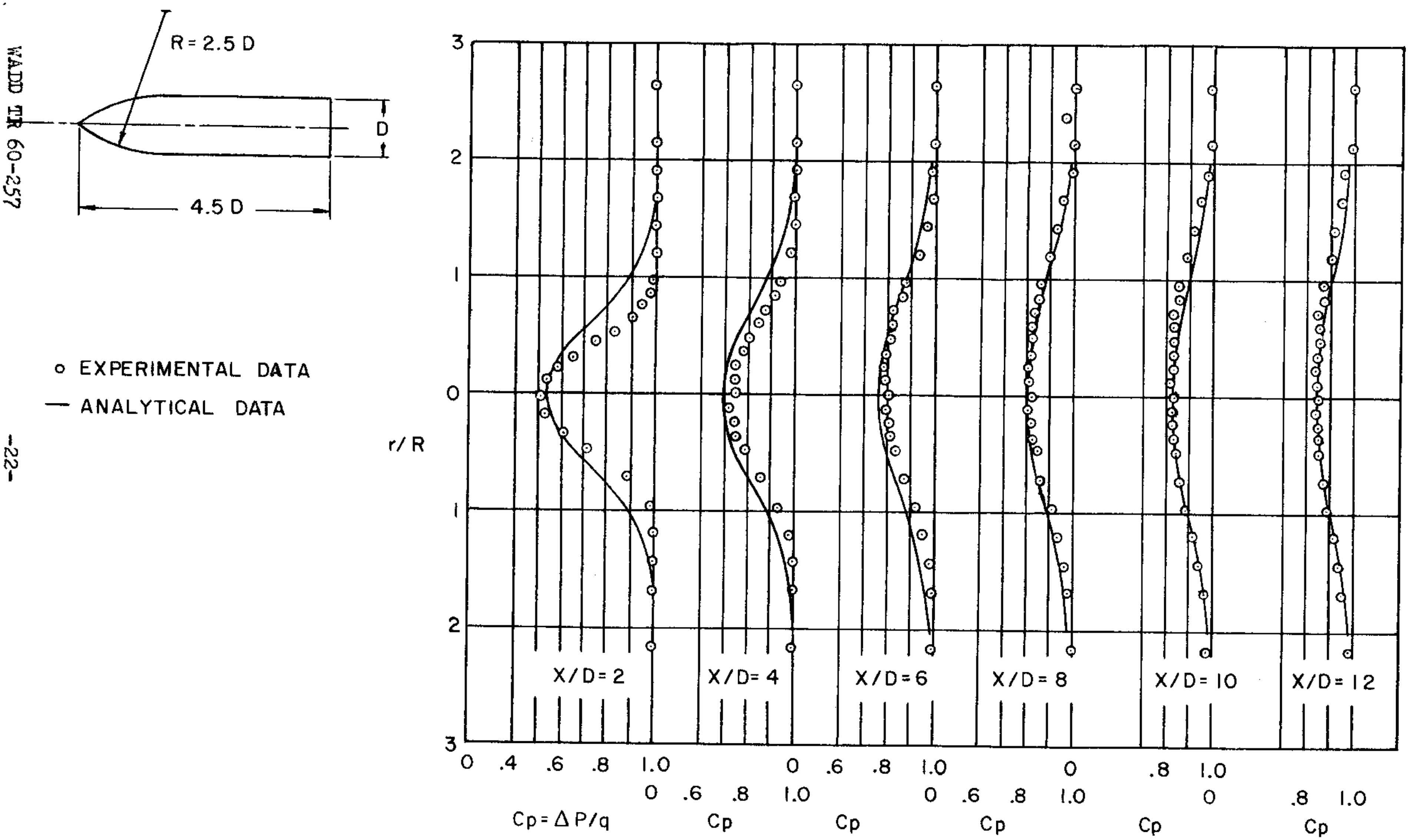

FIG. 6 EXPERIMENTAL AND ANALYTICAL DATA FOR A BODY OF REVOLUTION (BASED ON $\mathcal{H}$ = 0.0633 RELATED TO Cp AT X/D = 8)

X/D	2	4	6	8	10	12
r^*	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$
0	0.584	0.724	0.786	0.821	0.845	0.861
0.2	0.603	0.733	0.791	0.825	0.848	0.865
0.4	0.656	0.755	0.806	0.835	0.856	0.869
0.5	0.692	0.774	0.816	0.843	0.861	0.875
0.6	0.731	0.792	0.828	0.851	0.866	0.880
0.8	0.811	0.835	0.856	0.872	0.883	0.893
1.0	0.882	0.876	0.885	0.893	0.901	0.907
1.25	0.944	0.924	0.919	0.921	0.923	0.926
1.5	0.978	0.956	0.948	0.945	0.944	0.944
1.75	0.993	0.978	0.968	0.964	0.961	0.960
2.0	0.998	0.990	0.983	0.978	0.975	0.972

TABLE 5 — RESULTS OF ANALYTICAL DETERMINATION OF THE PRESSURE DISTRIBUTION IN THE WAKE OF A BODY OF REVOLUTION (BASED ON $\varkappa = 0.0764$ RELATED TO C_P AT $X/D = 4$)

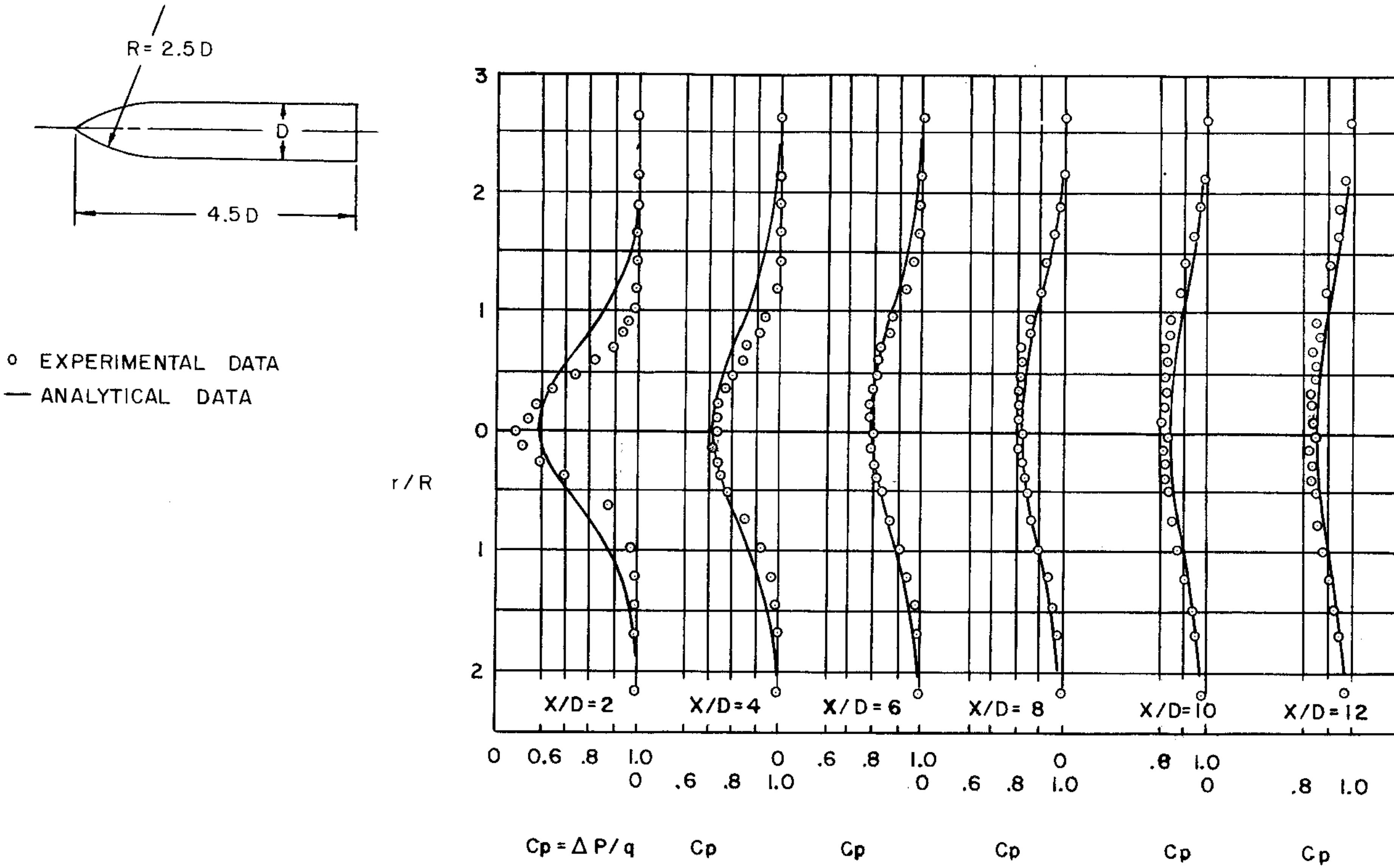

FIG.7 EXPERIMENTAL AND ANALYTICAL DATA FOR A BODY OF REVOLUTION
(BASED ON $\varkappa$ = 0.0764 RELATED TO Cp AT X/D = 4)

now an improved agreement between theory and experiment in the central portion of the wake in the range of $x/D = 4$ to $x/D = 8$, whereas the results in the more outward portion differ as before. Also, the results for $x/D > 8$ do not agree as well as before.

The above procedure was again repeated using the value $\varkappa = 0.0552$, related to $x/D = 12$. Table 6 and Fig 8 show the results of these calculations. One observes that the analytical and experimental results agree well only for $x/D \geqq 10$.

SECTION 5

CONCLUSION.

An equation was derived which presents the velocity and pressure distribution of the turbulent wake as function of an empirical parameter $\varkappa$

A study revealed that the predictions resulting from the newly derived formula agree satisfactorily with those from known theories and experimental data.

The empirical parameter $\varkappa$ was extracted from newly performed experiments and it was found that $\varkappa$ depends strongly upon the location of the control section in relation to the rear end of the wake producing body.

Fairly good agreement between theoretical prediction and experimental results was obtained by means of an average value of $\varkappa = 0.0633$ over a range of $\frac{x}{D} \geqq 6$.

For better agreement at a particular location, especially closer to the primary body or at great distance from it, specific $\varkappa$-values are recommended which are presented as function of the dimensionless distance $\frac{x}{D}$

The numerical values of $\varkappa$ presented in this study are related to a wake producing body having drag coefficients of $C_D = 0.35$. It may be expected that the $\varkappa$-values as function of x/D depend slightly on C_D of the primary body.

X/D	2	4	6	8	10	12
r^*	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$	$\Delta P/q$
0	0.498	0.664	0.737	0.780	0.809	0.830
0.2	0.526	0.675	0.745	0.786	0.813	0.833
0.4	0.602	0.705	0.767	0.801	0.825	0.843
0.5	0.651	0.736	0.783	0.813	0.834	0.850
0.6	0.705	0.764	0.800	0.825	0.843	0.857
0.8	0.810	0.821	0.839	0.854	0.866	0.876
1.0	0.914	0.876	0.879	0.884	0.891	0.896
1.25	0.960	0.931	0.922	0.920	0.920	0.923
1.5	0.986	0.966	0.955	0.949	0.946	0.945
2.0	0.994	0.994	0.989	0.984	0.980	0.977

TABLE 6— RESULTS OF ANALYTICAL DETERMINATION OF THE PRESSURE DISTRIBUTION IN THE WAKE OF A BODY OF REVOLUTION (BASED ON $\varkappa = 0.0552$ RELATED TO C_p AT X/D = 12)

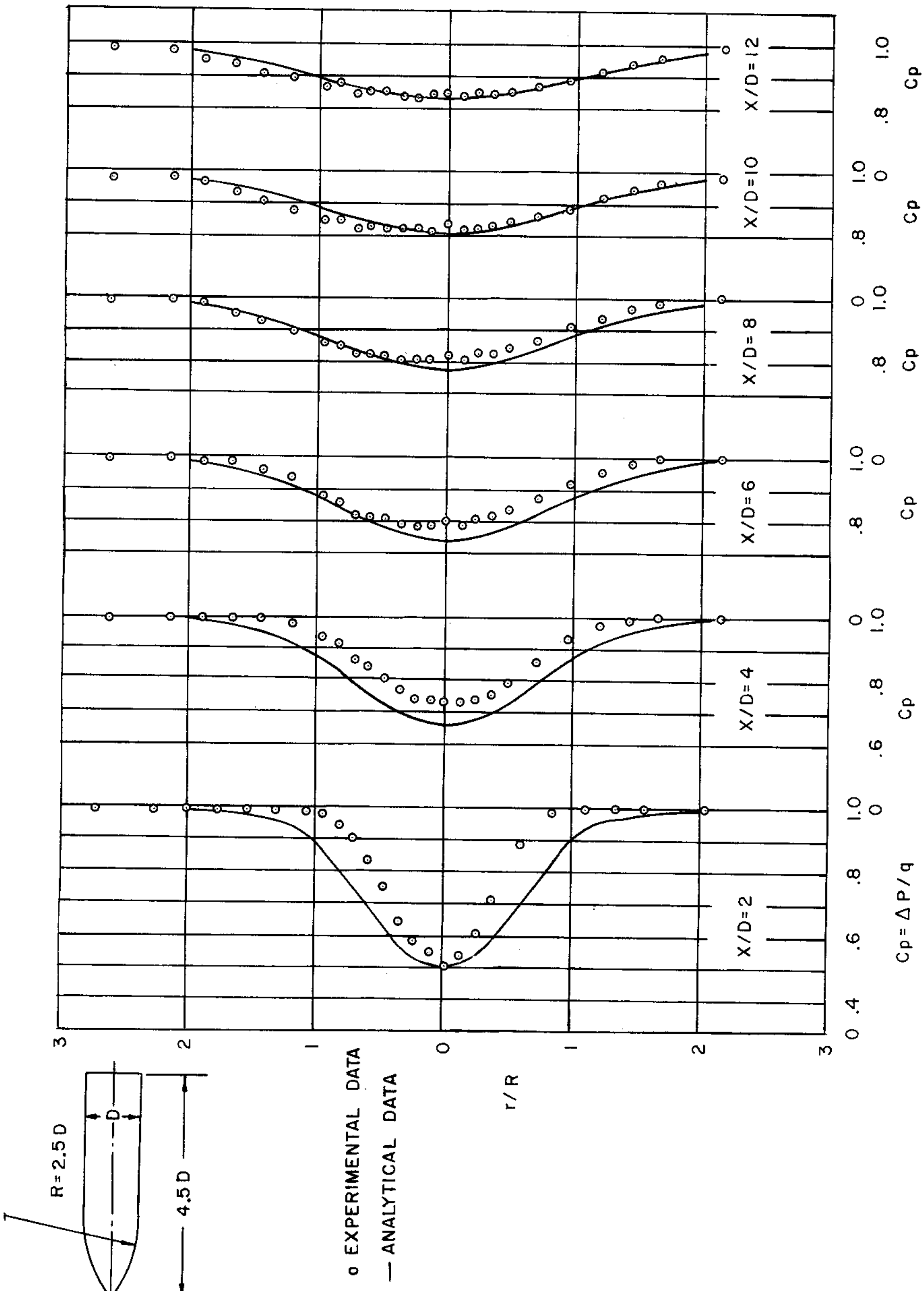

FIG. 8 EXPERIMENTAL AND ANALYTICAL DATA FOR A BODY OF REVOLUTION (BASED ON $\mathcal{K}$ = 0.0552 RELATED TO Cp AT X/D = 12)

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