THE USAGE OF ELECTRORHEOLOGICAL MATERIALS IN VISCOELASTIC LAYER DAMPING APPLICATIONS

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ABSTRACT

For many years, viscoelastic material layers have been used as structural components to obtain desired levels of structural damping. In common applications, the viscoelastic material is either placed in a constrained position subjected to shear loading or in a free position subjected to extensional deformation. In such situations, the damping behavior of the overall structure can be related using well developed theories to the viscoelastic material properties of the constrained or free layers. Since most classical viscoelastic damping materials exhibit complex mechanical properties which cannot be controlled after fabrication, once a structure is designed and fabricated the damping characteristics of that structure are unchangeable. On the other hand, if the viscoelastic layer contains an electrorheological material having controllable viscoelastic material properties, then structures can be constructed with controllable damping behavior. This control of structural behavior would be very useful in many situations, examples of which are the vibration of robotic manipulators and space structures. In the present work, the classical viscoelastic theories are extended to constrained layers of electrorheological materials and experimental results demonstrating the controllable structural behavior concept are presented.

NOMENCLATURE

B	flexural rigidity, [N·m]	X	shear parameter
E	tensile modulus, [Pa]	Y	stiffness parameter
f_n	modal frequency, [s^{-1}]	γ_y	shear yield strain
G^*	complex shear modulus (=G' + iG"), [Pa]	η	loss factor
H	lamina thickness, [m]	μ	beam area mass density, [Kg/m^2]
L	beam length, [m]	ν	viscosity, [Pa·s]
n	mode number	ρ	density, [Kg/m^3]
p	wave number, [m^{-1}]	τ_y	shear yield stress, [Pa]

INTRODUCTION

Viscoelastic material layers have been utilized in structural damping applications for many years. In short, there are two classes of viscoelastic damping applications, one in which the viscoelastic material is placed in a state of tension, and the other in which the same material is put in shear. Both classes of structural damping were first conceived in the 1950s. Elongational viscoelastic damping was first reported in France by Lienard in 1951 and shortly thereafter in Germany by Oberst[1,2]. Following this, the concept of shear viscoelastic damping was developed in America and first reported in 1959 by Kerwin[3]. Kerwin and his co-workers developed a theory relating the material properties and geometrical arrangement of panels made up of viscoelastic and elastic layers to the overall flexural behavior of the composite structure. This theory, which has since become well accepted for certain applications and known as the Ross-Kerwin-Ungar (RKU) model, was initially presented in 1959[4]. The RKU model with its limitations is discussed in several contemporary vibration control texts, and is widely used to design for structural vibration behavior in many present day applications ranging from the vibration of robot arms and space structures to the motion of buildings exposed to wind[5,6]. Relatively speaking, high modulus viscoelastic materials are used in extensional damping applications and lower modulus viscoelastic materials are used in shear damping applications. In general, shear damping treatments are the most effective of the two types of damping layers, and only such treatments were considered during the present investigation.

The single most important component in the design of a shear damping treatment is the selection of the viscoelastic material or materials to be used. During the past few decades, the variety and quality of materials designed for such treatments have improved dramatically, but the materials available today still have several shortcomings. The most important of these is the limited temperature range over which any single material is effective as a shear damping layer. This limitation has been partially overcome in many instances by the usage of polymer blends of components with different ranges of temperature applicability and/or by the use of multiple constrained layers of several different damping materials, but an ideal solution to this complexity remains to be found. A second shortcoming of modern day viscoelastic damping materials is the lack of controllability of their material properties once they are installed in a selected application. The majority of design and analysis related to structural behavior control utilizing viscoelastic layers is based on the assumption that the properties of the viscoelastic materials being used remain constant with time, and these materials are placed within structures at strategic locations so as to obtain a single invariable overall structural response to the proposed conditions which the structure is to be exposed. A possible exception to this is the recent use of piezoelectric polymers and ceramics[7-9] to vary structural response in a fully active manner. It would be desirable to have the capability to control structural response by varying the properties of viscoelastic damping materials encased within structural components, for having such a capability would allow for the in service tuning of the structure to avoid selected resonances and possibly accommodate temperature changes. A class of materials which may meet this need are electrorheological (ER) materials which exhibit viscoelastic properties that vary as a function of frequency and applied electric field. The potential usage of ER materials in viscoelastic layer shear damping applications is introduced in the present investigation.

ELECTRORHEOLOGICAL MATERIAL CHARACTERIZATION

Electrorheological materials are suspensions of very fine dielectric particles in insulating mediums which exhibit controllable rheological behavior in the presence of applied electric fields. The typical constitutive behavior of an ER material is shown in figure 1 where shear stress is plotted as a function of shear strain and shear rate respectively. As shown in figure 1a, ER material behavior can be divided into pre-yield and post-yield regions. The electrorheological effect was first observed by Winslow in 1947[10]. At that time, materials which behaved in this manner were labelled electroviscous fluids since from a macroscopic viewpoint the most notable change of a flowing ER suspension is a change in apparent or effective viscosity. Some years later, it was determined that the actual viscosity of the material, ν, remains relatively constant as the applied electric field is varied while the property that changes is τ_y, the yield stress of the Bingham plastic-like suspension. This is shown in figure 1b. The manifestation of this variable yield stress has proved very useful in devices where controllable post yield or flow properties are desired. Examples of electroactive devices in which this behavior has been used effectively include valves and clutches[11].

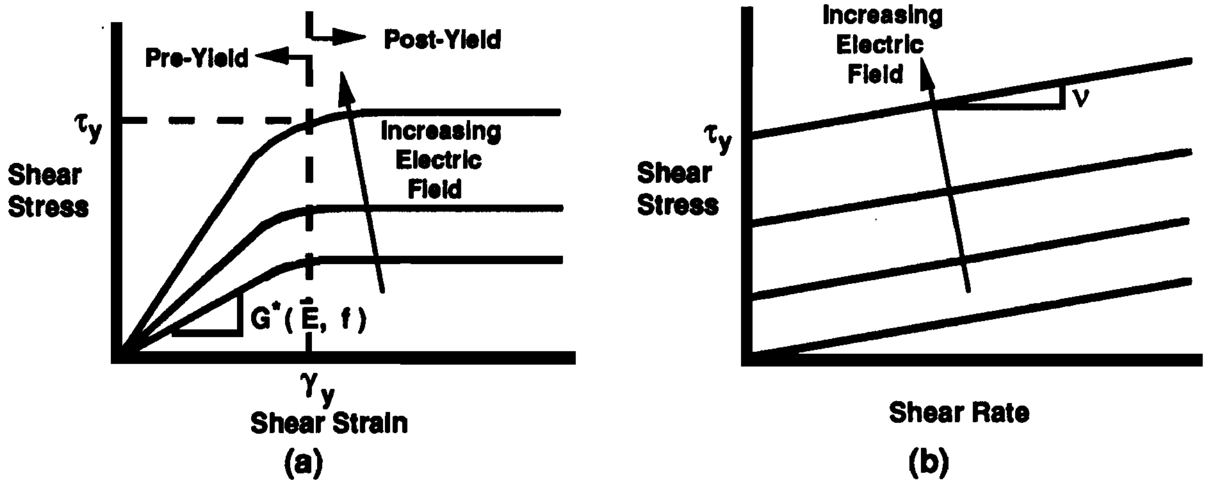


Figure 1: Typical rheological behavior of an ER material denoting the pre- and post-yield material behavior regions.

In comparison to the post-yield behavior, the controllable viscoelastic material behavior that ER materials exhibit while in the pre-yield phase remains virtually uninvestigated. As shown in figure 1a, this region is defined by a yield strain (γ_y), a yield stress (τ_y), and a complex shear modulus (G^*). Since the yield stress in an ER material increases with increasing applied electric field and the yield strain normally remains at approximately 1% for all field strengths, the complex shear modulus increases with increasing electric field. The controllability of this pre-yield complex shear modulus and its usage to obtain controllable structural behavior was the focus of the present investigation.

The complex shear modulus can vary several orders in magnitude as a result of electric field strengths ranging between 0 and 4000 V/mm. A full analytical understanding of this phenomenon has not yet been obtained, but several workers including Adriani and Gast are pursuing this matter[12]. In the absence of this understanding, an experimental characterization of the pre-yield complex shear modulus of ER materials was undertaken. Many techniques exist for measuring the complex shear modulus of viscoelastic materials[13,14]. Of these, the best test method to use with ER materials has not yet been determined. Two methods, however, were selected for initial testing. These included a standardized resonating cantilever sandwich beam method, (ASTM G756-83) and a simplistic but as of yet non-standardized concentric cylinder annular pumping technique. In addition to the fact that it is a well accepted and widely used test method, the vibrating sandwich beam test was chosen for its inherent large frequency range capability and its direct relationship to distributed structural damping. Following some initial testing, however, the applicability of this technique was found to be questionable due to fact that present ER materials were found to have modulus values much lower than the materials commonly characterized using this method. Thus, for the purposes of the present study, the complex shear modulus of the chosen ER materials was determined using the concentric cylinder annular pumping technique. A diagram of the experimental apparatus used to accomplish this is shown in Figure 2.

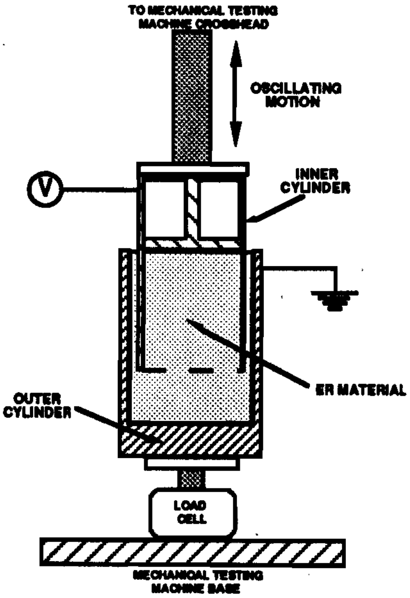


Figure 2: Cylindrical annular pumper apparatus used for complex shear modulus measurement.

A hollow aluminum cylinder having a thickness of 0.8 mm., an outside diameter of 44.5 mm., and a length of 47.6 mm. was connected to the actuator of a hydraulic mechanical testing machine. This cylinder was then lowered into an ER material filled stationary aluminum cup of thickness equal to 3.0 mm., inside diameter equal to 46.5 mm., and inside depth equal to 63.5 mm. This cup was connected to a load cell with a force range of 0-110 N. The depth of the cup was chosen to be much larger than the immersed length of the inner cylinder to minimize end effects and the diameters were chosen as stated to produce an ER material filled annulus with a thickness of 1.0 mm. An electric field was maintained in the gap between the cylinders by using the inner cylinder as a DC high voltage electrode and the outer cylinder as electrical ground. For each selected electric field, the inner cylinder was displaced vertically using band limited white noise with a bandwidth from 0 to 200 Hz. The frequency range was limited below 200 Hz. due to the capabilities of the mechanical testing machine. An amplitude of excitation was chosen so as to produce a maximum shear strain level of 1% within the ER material. The complex shear modulus was then deduced from the force and displacement waveforms resulting for each set of test conditions. All tests were performed at room temperature, and at electric field levels between 0 and 4000 V/mm.

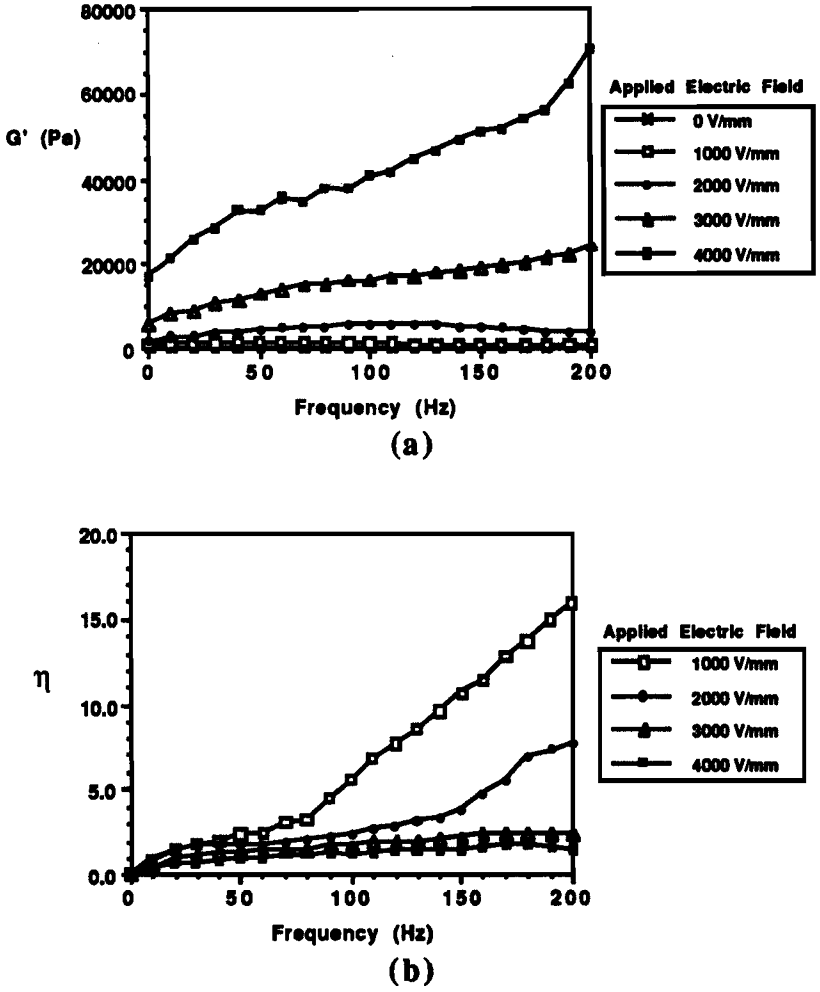
The ER materials chosen for use in the present study were two proprietary materials developed over the past three years by Lord Corporation. The formulations are known as materials ERX-03-145 and 5501-21C respectively. The resultant pre-yield complex shear moduli for these two materials are presented as functions of frequency and applied electric field in figures 3 and 4. These results are presented in the form of shear storage modulus, G', and shear loss factor, η. The shear loss factor is related to the real and imaginary components of the complex shear modulus, G' and G" respectively by the relation

$$\eta = \frac{G''}{G'} \qquad (1)$$

and was chosen for presentation due to its importance in viscoelastic damping applications. The shear modulus for material ERX-03-145 is presented in figure 3 and that for material 5503-21C in figure 4. Both materials exhibited storage modulus values that increased with applied electric field and frequency. For ERX-03-145 the magnitude of the storage modulus ranged from 0 up to 70 KPa. at the highest applied electric field and frequency tested while the corresponding range for 5503-21C only reached a level of 21 KPa. The shear loss factor for the two materials also increased with frequency but was generally found to decrease with applied electric field. This decrease, however, was less evident at the higher levels of electric field than it was at the lower levels where the material is more fluid-like in nature. Thus, for the 2000 to 4000 V/mm electric fields commonly applied in ER material applications, the shear loss factor of the two materials generally stayed between 0 and 4.0.

When comparing the properties of the ER materials discussed above with those of standard viscoelastic damping materials, it is immediately obvious that while the loss factor ranges are comparable, the storage modulus values of ER materials are several orders of magnitude less than those of present day shear damping treatment materials. The continued development of stronger ER materials along with their unique behavior, however, appropriates further discussion of their use in damping applications. The unique feature referred to is the controllability of the material properties within certain ranges. Examples of this are shown in figures 3 and 4. For each frequency value, the complex shear modulus can be controlled within the minimum and maximum values by the adjustment of applied electric field. This control can be adjusted over time increments of several milliseconds, which is the response time of present day ER materials to changes in electric field levels. The capability for such control would be very useful to the structural damping industry.

Aside from the material property behavior discussed above, several other parameters are commonly considered when discussing ER materials. Two such parameters are current density and temperature dependence. The current density must be kept low to avoid large electrical power requirements at the high voltage levels required. For the materials presented, the current density is on the order of 0.1 μA/cm^2 at an electric field of 4000 V/mm The useful temperature range of both ERX-03-145 and 5501-21C is approximately 10-85°C. Within this range the pre-yield mechanical properties of the materials are relatively independent of temperature. The current density, however, does increase with temperature. Other factors worth mentioning are the stability and long term behavior of the materials. Some formulations of ER materials have shown a tendency of particle/carrier fluid separation over time. ERX-03-145 has exhibited this behavior in the past, while 5503-21C is much more stable. The long term behavior of both materials is presently being studied.



(a)

(b)

Figure 3: Complex shear modulus of electrorheological material ERX-03-145. (a) Storage modulus, (b) Loss factor.

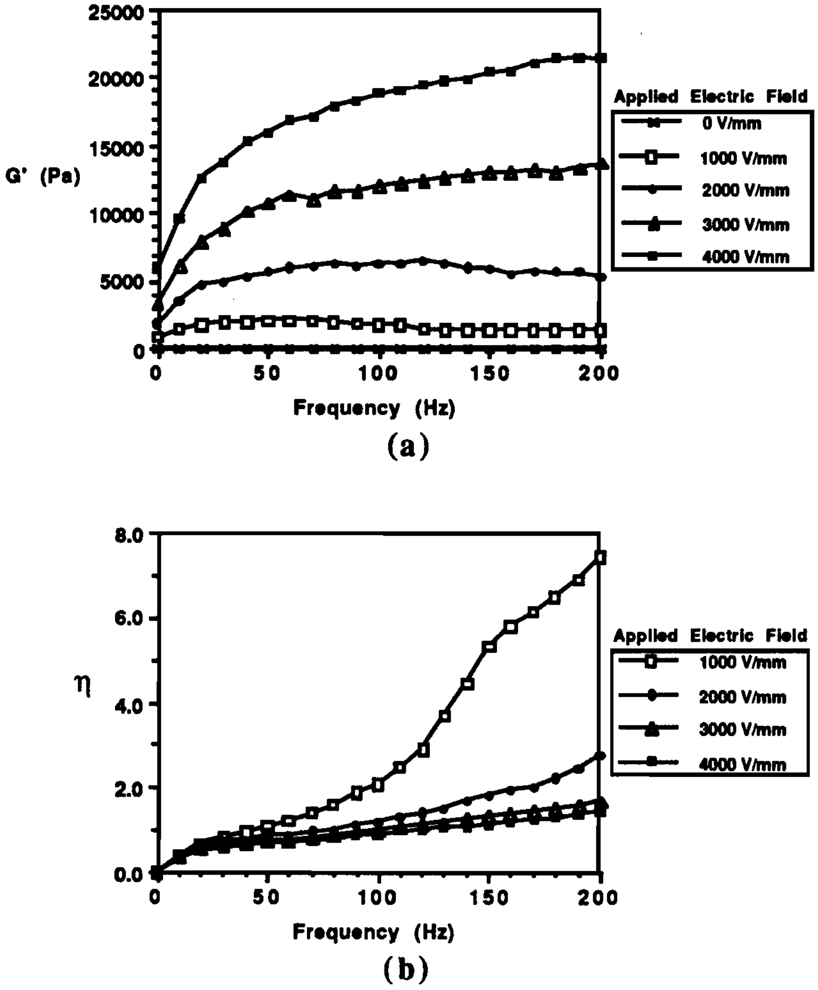
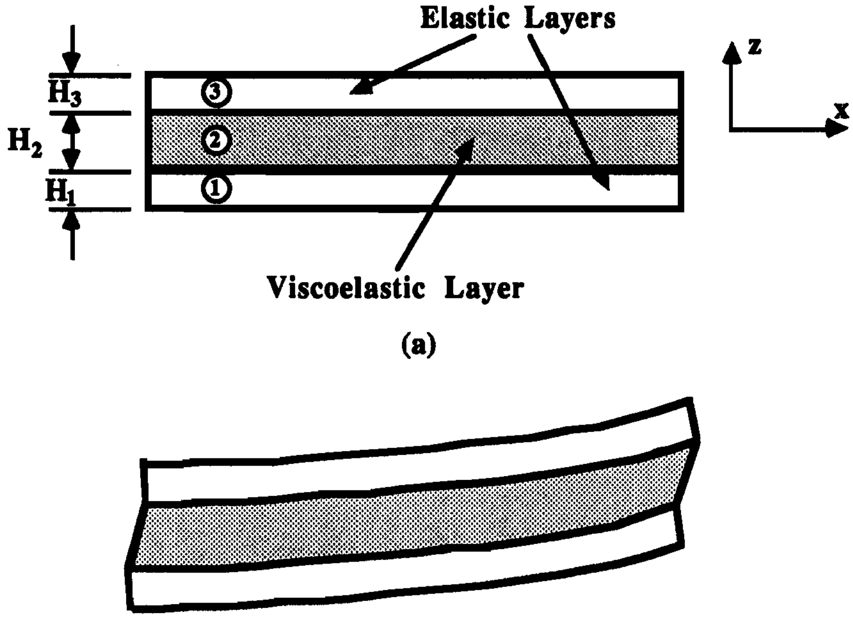


Figure 4: Complex shear modulus of electrorheological material 5503-21C.
(a) Storage modulus, (b) Loss factor.

THEORETICAL BACKGROUND

As mentioned previously, the obtainment of structural damping using constrained viscoelastic layers in shear is common practice in the vibration control industry today. In many instances the Ross-Kerwin-Ungar model (RKU) is used in the design of structures. One of the purposes of the present study was to investigate whether or not this theory is applicable to structures containing ER materials as viscoelastic damping layers. A brief summary of the RKU model applied to shear damping treatments follows.

A sample shear damping configuration is shown in figure 5a. In such configurations, a viscoelastic material, denoted as material 2 in the figure, is sandwiched between two elastic material layers, which are shown in the figure as materials 1 and 3. Flexural deformation of the structure produces not only bending and extension but also shear, which occurs primarily in the viscoelastic layer as shown in figure 5b. The strain energy associated with this shear tends to dominate the damping behavior of the overall structure.



(a)

(b)

Figure 5: Typical section of a panel with a constrained viscoelastic layer. (a) Undeformed, (b) Deformed.

Assuming that the extensional stiffness of the viscoelastic layer is small compared to that of the elastic materials, the loss factor of the entire structure is

$$\eta = \frac{\eta_2 \, Y X}{1 + (2+Y)X + (1+Y)(1+\eta_2^2)X^2} \quad . \tag{2}$$

In equation (2), η_2 is the loss factor in shear of the viscoelastic layer, ($\eta_2 = G'' / G'$), Y is a stiffness parameter, and X is a shear parameter. In its full form, the stiffness parameter, Y, is defined as

$$Y = \left[\frac{E_1 H_1^3 + E_3 H_3^3}{12 H_{31}^2} \left(\frac{1}{E_1 H_1} + \frac{1}{E_3 H_3} \right) \right]^{-1} \tag{3}$$

where E_i and H_i are the tensile modulus and thickness of the ith layer respectively and H_{31} is the distance between the mid-planes of the elastic constraining layers, ($H_{31} = H_2 + (H_1 + H_3) / 2$). For the present discussion, the elastic layers will be considered to be made of the same material and to be of equal thickness, thus $E_1 = E_3$ and $H_1 = H_3$. With these assumptions, the stiffness parameter becomes simply a geometric parameter, which is

$$Y = 3\left(\frac{H_{31}}{H_1}\right)^2 . \tag{4}$$

The shear parameter, X, is defined as

$$X = \frac{G_2'}{p^2 H_2}\left(\frac{1}{E_1 H_1} + \frac{1}{E_3 H_3}\right) \tag{5}$$

where p is the wavenumber , which for simply supported beams is related to the length of the beam, L, and mode of flexural vibration of the composite panel, n, by the relation $p = n\pi / L$. Thus, by choosing a viscoelastic material with a given complex shear modulus and designing a layer of this material into an otherwise elastic composite panel, one can determine the effective loss factor, and thereby the damping, of the overall structure.

In addition to dominating structural damping, the existence of a constrained viscoelastic material in a composite panel as shown in figure 5 also modifies the flexural rigidity of the structure. For the configuration shown, the real component of the effective complex flexural rigidity per unit depth of the structure is

$$B = (B_1 + B_3)\left\{1 + \left[\frac{XY + X^2 Y(1+\eta_2^2)}{1 + 2X + X^2(1+\eta_2^2)}\right]\right\} \tag{6}$$

where

$$B_i = \frac{E_i H_i^3}{12} \tag{7}$$

This change in effective stiffness of composite panels due to the existence of a viscoelastic layer brings about a corresponding change in the resultant frequency of flexural vibration, f_n. For simple beam panels, this is evident through the relation

$$f_n = \frac{p^2}{2\pi}\sqrt{\frac{B}{\mu}} \tag{8}$$

in which μ is the mass per unit area of the panel. For three layer laminated composite beams, μ can be determined using the expression:

$$\mu = \sum_{i=1}^{3} \rho_i H_i \tag{9}$$

in which ρ_i is the mass density of the material of layer i. Thus, like the loss factor, the frequency of vibration of composite panel structures encompassing a viscoelastic layer changes with corresponding changes in the complex shear modulus of the viscoelastic material.

DESCRIPTION OF VIBRATION EXPERIMENTS

In order to test the usage of ER materials in distributed viscoelastic layer damping applications, small amplitude vibration experiments were performed with ER material filled symmetric sandwich beam specimens. A diagram of a specimen is shown in figure 6. As shown in the figure, sandwich beams were constructed with 0.53 mm thick aluminum face layers and 2.0 mm thick ER material filled cores. The total length and width of each beam was 0.254 m and 0.0254 m respectively. A constant core thickness throughout each beam was obtained by positioning small silicone rubber face plate separators of the desired thickness at the ends and center of each side of the beam. At the center of the beams, these separators were adhered to one side only while at the ends of each beam, the silicone was bonded to both the upper and lower face plates. The sides of the beams were sealed by the loose application of a 0.25 mm thick latex sheet material bonded to the outside surfaces of the face plates to create a sealed core. For the purpose of clarity, only one latex side skin is shown in figure 6. After the appropriate ER material was placed in the core, the ends of each beam were sealed with silicone rubber sealant. The application of an electric potential to each face plate was allowed for by the inclusion of small end tabs at one of the corners of each plate. Two such beams were fabricated, each containing one of the two ER materials previously discussed.

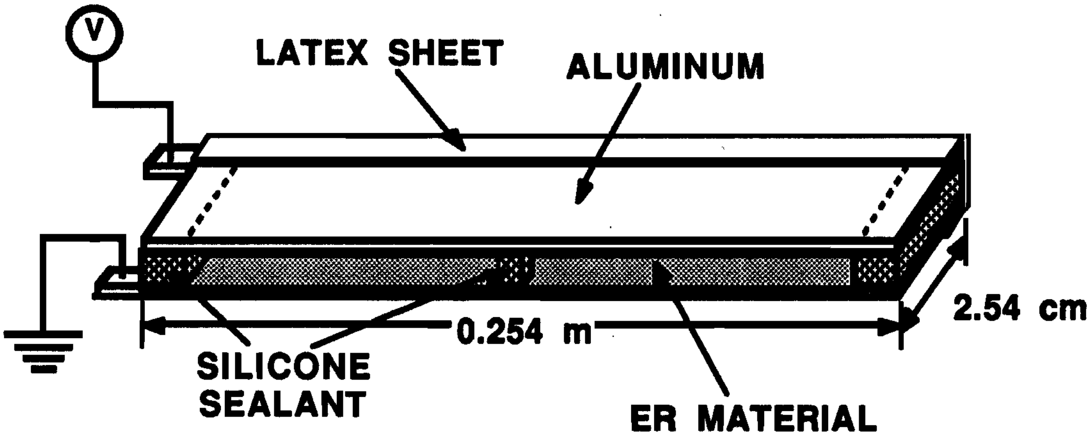


Figure 6: Configuration of ER material filled composite sandwich beam specimens.

During testing, each beam was positioned in a horizontal simply supported configuration as shown in figure 7. A non-contacting electromagnetic actuator was used to input a 0-200 Hz random vertical oscillating force on the beam at an axial position equal to one fourth of the length of the beam from one of the supports. Likewise, a non-contacting electromagnetic proximity probe was used at the other quarter length position to record the vertical displacement at that location as a function of time. Small steel targets were attached to the beam at the actuator and sensor locations to permit their use. The maximum amplitude of oscillation at any point along the beam was maintained at a level less than 0.5 mm. Controllable high voltage was applied by grounding one face plate of the beam and applying a DC voltage to the other face plate using a high voltage power supply. Voltage levels between 0 and 5000 volts were applied to the beam in 1000 volt increments. This resulted in electric field levels between 0 and 2500 V/mm. At each level of applied electric field, the natural frequency for each mode of vibration below 200 Hz was determined from the spectral response of displacement at the proximity probe location. Corresponding structural loss factors were determined for each mode from the half power bandwidth of the frequency response.

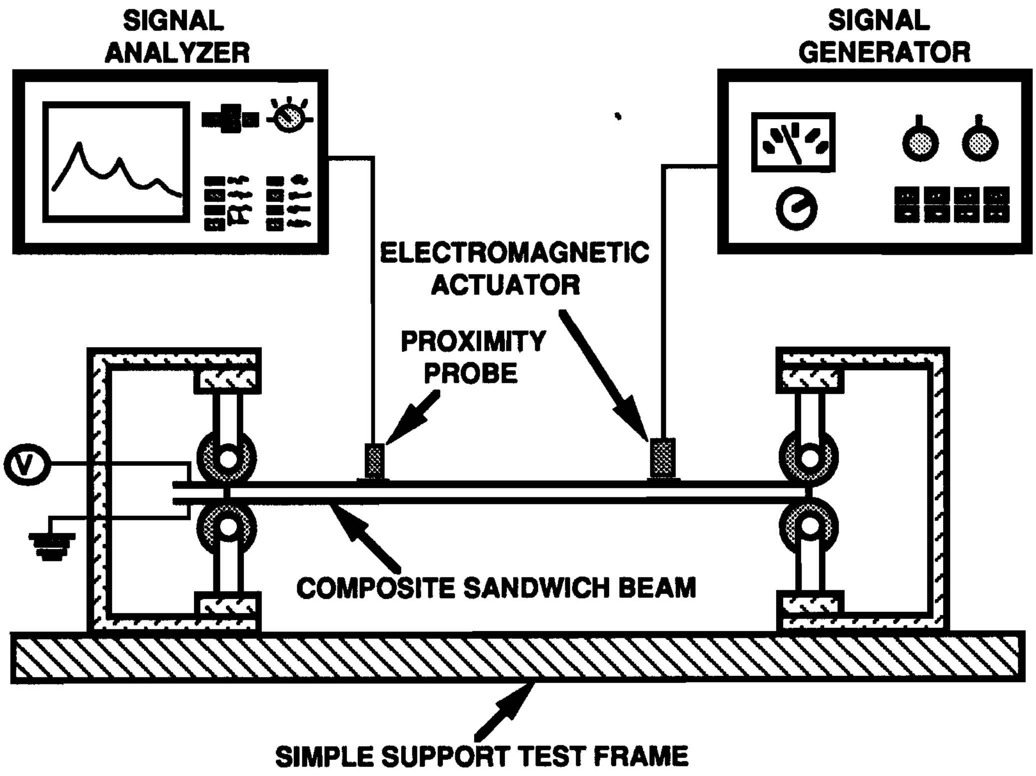


Figure 7: Experimental apparatus used during vibrational testing.

RESULTS AND DISCUSSION

For all but the lowest voltages applied during testing, only the first three flexural modes of vibration existed within the 0-200 Hz. range chosen. The resultant modal frequencies and corresponding loss factors observed for the two sandwich beams are presented as functions of electric field applied across the ER material in figures 8 and 9. As evident from figure 8a , the natural frequency of each of the three modes of the beam containing ER material ERX-03-145 increased significantly with applied electric field. Over the 0-2500 V/mm range tested, this increase was nearly linear, and the lines resulting from least squares analyses of the data are included in the figure. The change in modal frequency with applied electric field for modes 1 through 3, in units of Hz/(KV/mm), were 12.1, 13.0, and 18.6 respectively. All the curve fits yielded correlation coefficient values exceeding 0.97. The observed behavior in modal loss factor with applied electric field for the same beam, shown in figure 8b, was not as orderly as the frequency behavior. It can be concluded, however, that the modal loss factor was found to decrease with mode number and generally increase with applied electric field. Overall structural loss factor values ranging between 0.03 and 0.11 resulted.

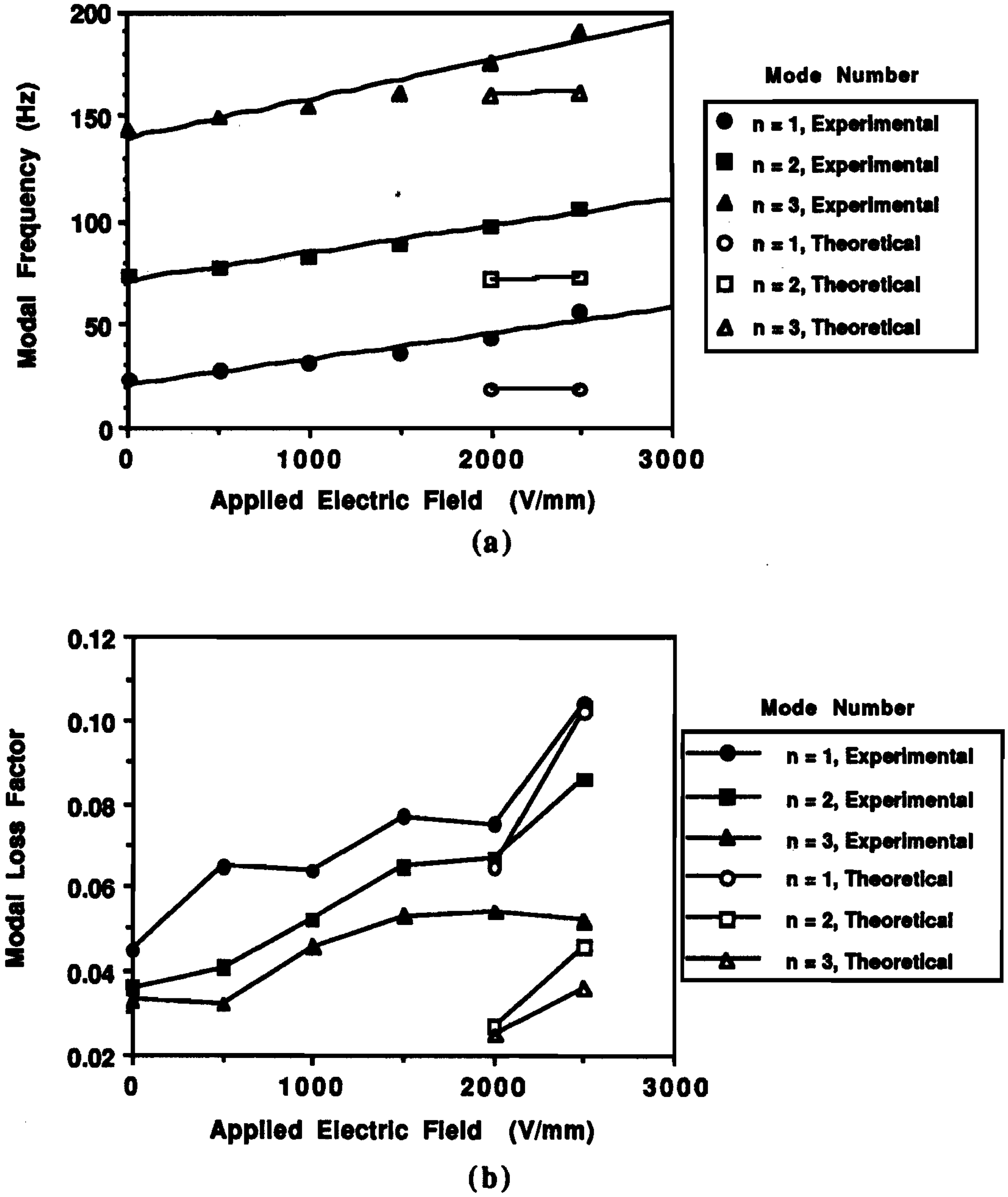
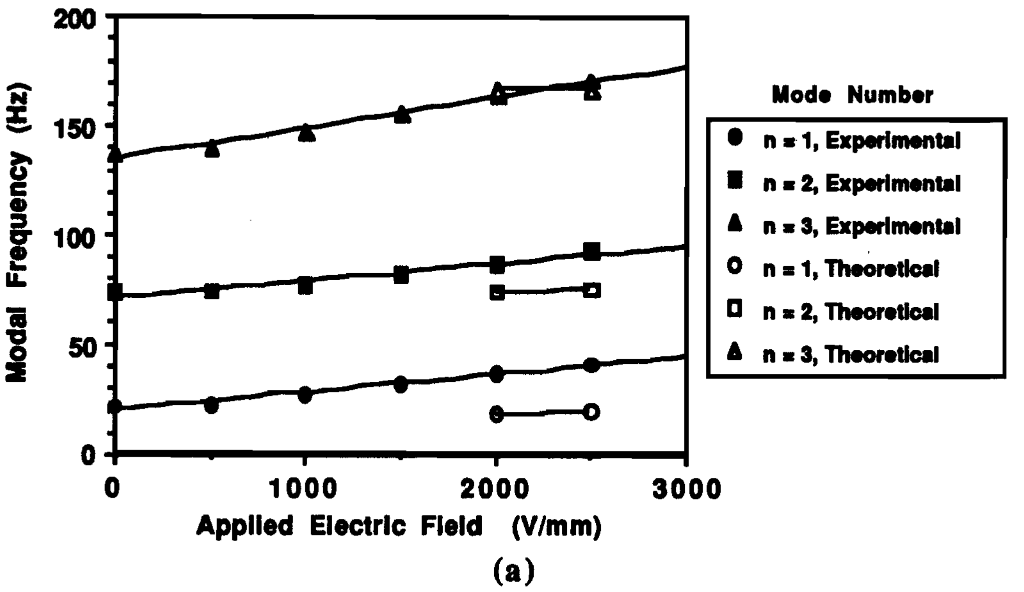
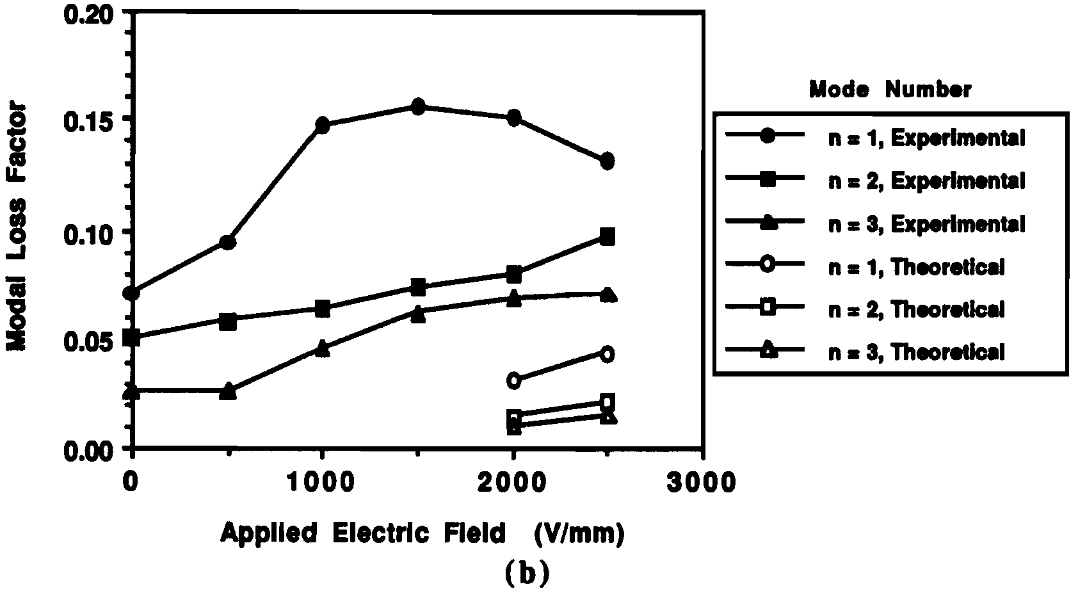


Figure 8: Observed dependence of beam modal frequency and loss factor on applied electric field; ER material ERX-03-145. (a) Modal frequency, (b) Modal loss factor.

The modal frequencies of the beam filled with ER material 5503-21C were also found to increase linearly with applied electric field. This is shown in figure 9a, and the resultant gradients of frequency with applied electric field were 8.2, 7.9, and 14.1 Hz/(KV/mm) for modes 1 through 3 respectively. The correlation coefficients of the curves all exceeded 0.98. The dependence of modal loss factor on applied electric field for this case, as shown in figure 9b, was again somewhat disordered. For modes 2 and 3 the loss factor followed the previously mentioned pattern of generally increasing with applied electric field. For mode 1 however it was observed in this case that the loss factor increased with applied electric field for fields below 1500 V/mm and then decreased with electric field for field levels above this point. This behavior was seen consistently during repeated testing.



(a)



(b)

Figure 9: Observed dependence of beam modal frequency and loss factor on applied electric field; ER material 5503-21C. (a) Modal frequency, (b) Modal loss factor.

Following the obtainment of experimental results, an attempt was made to theoretically predict the observed behavior using the classical RKU theory. This was done by first obtaining appropriate ER material shear storage modulus and loss factor values at the observed frequencies of vibration and applied electric field levels using the data presented in figures 3 and 4. These values were used along with the density of the ER material, the properties of aluminum, and the geometry of the beams constructed to determine predicted structural modal frequencies and loss factors using equations (2) through (9). Initially, no effects of the silicone rubber or the latex were considered. The predicted modal frequencies and loss factors resulting from these calculations for the highest two electric field levels tested are included in figures 8 and 9. From figures 8a and 9a it can be concluded that the theory as applied generally underpredicted the vibrational modal frequencies observed. The degree of deviation from experimental observation ranged from practically none for mode three of the beam filled with ER material 5503-21C to as much as 63% for mode one of the beam filled with ER material ERX-03-145. The loss factor data shown in figures 8b and 9b reveal that although the loss factor was predicted to increase with applied electric field and mode number, the actual magnitudes of the theoretical loss factors calculated were much less than those observed. The only exception to this is the loss factor for the first mode of vibration of the beam filled with ERX-03-145.

In response to the above mentioned discrepancies in predicted and observed behavior, the possibility of accounting for the influence of the silicone sealant was investigated. The influence of the latex material was assumed to be minimal due to the loose manner in which it was purposefully installed. Since the silicone is believed to have a higher shear storage modulus than the ER materials, the beams constructed were probably stiffer than would be predicted by only considering the ER material between the face plates. This in turn would lead to the observation of higher than predicted frequencies of flexural vibration. To test this hypothesis, a volume weighted rule of mixtures component was added to the theoretical analysis and an effective complex shear modulus of the material occupying the core region of the beams was determined using the relationships

$$G'_{eff} = \frac{G'_{Si} V_{Si} + G'_{ER} V_{ER}}{V_{TOT}} \tag{10}$$

and

$$G''_{eff} = \frac{G''_{Si} V_{Si} + G''_{ER} V_{ER}}{V_{TOT}} \tag{11}$$

where the subscripts Si and ER refer to the silicone rubber and ER material respectively and V represents to volume of viscoelastic material in the beam. The resultant effective complex shear modulus values were then used throughout the remainder of the calculations to determine new structural modal frequency and loss factor values. Since the silicone occupied less than 10% of the viscoelastic material volume in the beams, however, this revised analysis did not lead to significant changes in the theoretical predictions. The possibility remains that the properties used for the silicone rubber were inaccurate since they were taken from the literature to be G'_{Si} = 69,000 Pa and G''_{Si} = 6,900 Pa. The actual properties of the silicone sealant used remain to be measured.

Another potential source of error in the calculations is inaccuracy in the values of the ER material complex shear modulus. During the material characterization phase of the study, the properties of the ER materials tested varied with time of continuous exposure to an electric field and whether field levels were being increased or decreased. Although care was taken to obtain similar material property conditions during material characterization and structural testing, it is likely that the two situations did not exactly coincide. Further work is in progress to better quantify the rheological behavior of ER materials, and as this supporting work progresses it is likely that more accurate structural behavior predictions will also result.

Accepting the fact that further work is needed to more closely correlate theory and experimentation in this area, the controllable vibration observed remains a truly remarkable phenomenon which may be of significant utility in the vibration control industry. This can be seen by simply examining the linear behavior of structural modal frequency with applied electric field, as shown in figures 8a and 9a. A potential use of this technology is in the tuning of structural properties to avoid vibrational resonances. An example of this is shown in figure 10, where the observed structural response amplitude of one of the beams tested is plotted as a function of frequency for the two cases of no electric field and an electric field of 2500 V/mm. If one desired to avoid resonances over a given frequency range, the structure could be "tuned" by either applying or removing the electric field so that the response to a given frequency of excitation was minimized. If the excitation frequency for one reason or another changed, the structure could be adjusted in a matter of milliseconds to again produce a minimized response at the new frequency. This would result in a capability of obtaining a response amplitude spectrum equivalent to the shaded area in figure 10. Such a capability has not been obtainable using any other means of controllable passive damping to date.

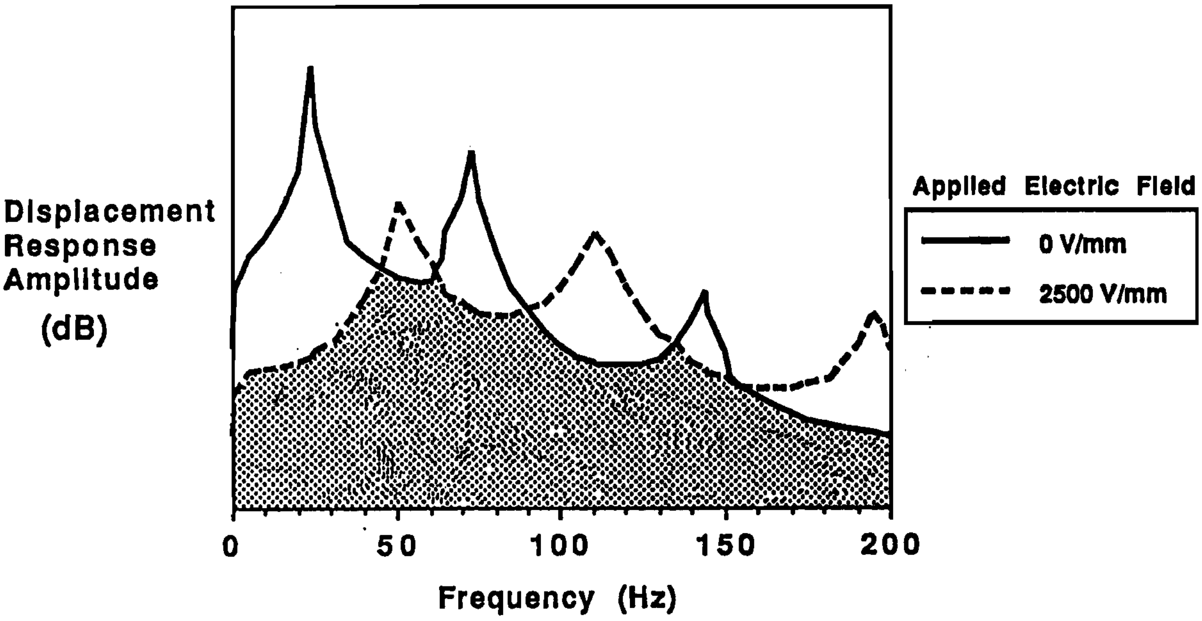


Figure 10: Structural response amplitude as a function of frequency for two states of an ER material filled composite beam.

CONCLUSIONS

The present study was among the first in which the controllable pre-yield properties of electrorheological materials were investigated as to their utility in distributed structural control applications. Although the pre-yield shear storage modulus values of present day ER materials were found to be much lower than those of conventional viscoelastic shear damping materials, composite sandwich beams were constructed encasing ER materials and demonstrated to exhibit controllable response to flexural vibration excitation. Most notably, a linear relationship was found to exist between modal response frequency and applied electric field for the beams tested. An attempt was made to predict this behavior using a classical constrained layer damping analysis, and it was concluded that further work was needed to more closely correlate theory and experimentation. Nevertheless, it was suggested that composite structures containing ER materials would have the unique capability of tunable vibrational response. This tuned passive approach to structural behavior is likely to be very useful in the vibration control industry in the years to come.

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