INTEGRATED OPTIMIZATION OF COMPOSITE STRUCTURES FOR ADVANCED DAMPED DYNAMIC CHARACTERISTICS

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ABSTRACT

Polymer matrix composites exhibit significantly higher material damping compared to most common metals. The current paper summarizes recent research on the development of design methodologies for optimizing the damping and the damped dynamic performance of composite structures. The optimal tailoring involves multiple material/structural levels, that is, the micromechanics level (fiber/matrix properties, fiber volume ratio), laminate level (ply angles/thicknesses, stacking sequence), and structural level (structural geometry and shape). The dynamic response and the modal damping of the composite structure are simulated with finite element analysis based on a special composite element. A multi-objective constrained optimization scheme is proposed for the best handling of the many competing design criteria involved. Applications on basic structural components (beams and plates) demonstrate that properly tailored composite structures can exhibit significantly improved damped dynamic performance.

Keywords: damping; composite materials; composite structures; optimization; design; dynamic performance.

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INTRODUCTION

Fiber composite materials are widely used in structural applications requiring high stiffness-to-weight and strength-to-weight ratios, as they readily provide high specific moduli, high specific strengths, and tailorable anisotropic elastic properties. Polymer matrix composites may also exhibit significantly higher damping compared to most common metals. The previously stated requirements for advanced light-weight structures virtually restrict the use of many traditional sources of passive damping, therefore, the option to utilize the damping capacity of polymer-matrix composites appears very attractive. Reported research on the damping of unidirectional composites and laminates [1-6] has shown that the damping of composites is highly-tailorable and is primarily controlled by constituent parameters (fiber/matrix properties, fiber volume ratio), and laminate parameters (ply angles/thicknesses, stacking sequence). Additional research work [7] demonstrated that the modal damping of composite structures depends also on the structural geometry and deformation (mode shapes). This work also suggested that properly designed composite structures can provide significant passive damping, and they may further improve the dynamic performance and fatigue endurance by attenuating undesirable elasto-dynamic phenomena such as structural resonances, overshooting, and long settling times. The previous studies have also demonstrated that any increase in damping typically results in decreased stiffness and strength, therefore, any tailoring of the composite material for optimal damped response will be based on trade-offs between damping, stiffness, and strength.

Although the optimization of composite structures for multiple design criteria including damping appears to be worthwhile and its significance has been acknowledged [8], reported research on the subject has been mostly limited to the laminate level [9,10]. Resent research performed by the authors has been focused on the optimal tailoring of composite structures for optimal transient or forced dynamic response [11-13]. This work is summarized herein and involves methodologies for the optimal design of polymer matrix composite structures. The methods are equally applicable to structures subject in steady or transient response, and they further entail: (1) multiple objectives to effectively represent the array of competing design requirements; (2) capability for tailoring of the basic composite materials and/or laminate; (3) capability for concurrent shape optimization; and (4) design criteria based on the global static and dynamic response of the composite structure.

The proposed design objectives are minimization of resonance amplitudes (or maximization of structural damping), minimization of structural weight, and minimization of material cost. Additional performance constraints are imposed on static deflections, dynamic resonance amplitudes, natural frequencies, static ply stresses, and dynamic ply stresses. The analysis involves unified composite mechanics, which entail micromechanics, laminate and structural mechanics theories for the passive damping and other mechanical properties of the composite. The structural damping and the damped dynamic response are simulated with finite element analysis. Applications of the methodology on the optimization of a cantilever composite beams and a cantilever composite plate are presented. The results quantify the importance of structural damping in improving the dynamic performance of composite structures, and illustrate the effectiveness of the proposed design methodology.

DAMPED STRUCTURAL DYNAMIC RESPONSE

To enable the design of general composite structures, a finite element discretization is utilized. In such case, the dynamic response of a structure which is excited by a force P(t) is expressed by the following system of dynamic equations:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{P(t)\} \qquad (1)$$

where $\{u\}$ is the discretized displacement vector. In the case of laminated composite structures, the stiffness, damping, and mass matrices, $[K]$, $[C]$, and $[M]$ respectively, are synthesized utilizing micromechanics, laminate, and structural mechanics theories representing the various material and structural scales in the composite structure.

The related theories for this multi-level simulation of structural composite damping are described in refs. 1,2, and 7. Analogous theories are utilized for the synthesis of other mechanical properties [14]. At the micromechanics level, the on-axis damping capacities of the basic composite material systems are calculated based on constituent properties, material microstructure, fiber volume ratio (FVR), temperature, and moisture. The off-axis damping capacities of the composite plies are calculated at the laminate level, and the local laminate damping matrices are predicted based on on-axis damping values, ply thicknesses, and laminate configuration. The damping contributions of the interlaminar matrix layers due to in-plane interlaminar shear are also incorporated [2].

The structural modal damping is synthesized by integrating the local laminate damping contributions over the structural volume. The modal specific damping capacity (SDC) of the n-th vibration mode ψ_n is:

$$\psi_n=\frac{\int_A \Delta W_{Ln} dA}{\int_A W_{Ln} dA} \qquad (2)$$

where: A is the structural area; ΔW_{Ln} and W_{Ln} are the dissipated and maximum stored laminate strain energy distributions, respectively, of the n-th mode per unit area per cycle. Utilizing the finite element discretization scheme proposed in ref. 7, the modal

SDC is related to the element damping and stiffness matrices, $[C_e]$ and $[K_e]$ respectively:

$$\psi_n = \frac{\frac{1}{2}\sum_{i=1}^{nel}\{u_{ein}\}^T[C_{ei}]\{u_{ein}\}}{\frac{1}{2}\sum_{i=1}^{nel}\{u_{ein}\}^T[K_{ei}]\{u_{ein}\}} \qquad (3)$$

where, *nel* is the total number of elements and $\{u_{ein}\}$ the nodal displacements of the i-th element corresponding to the n-th vibration mode.

The dynamic response of the structure is simulated based on modal superposition. The dynamic system in eq. (1) is transferred to the $p \times p$ modal space via the linear modal transformation $\{u\} = [\phi]\{q\}$. Assuming proportional damping, then the damping matrix is synthesized from the modal damping values. The frequency response (FRF) of the structure, or the transient dynamic response is subsequently calculated. Typically the resonance amplitudes, or the undamped amplitudes in transient response of most critical vibration modes are used as performance measures.

OPTIMAL DESIGN

Originally, the optimal design of composite structures was conceived as a single-objective constrained optimization problem [11,12]. Although this research demonstrated the advantages of damping tailoring, it indicated that the design of composite structures for optimal dynamic performance is a multi-objective task, and may be best accomplished as the constrained minimization of multiple objective functions. Increases in composite damping may typically result in stiffness/strength reductions and/or mass addition, for this reason, the minimization of weight and material cost was also included in the objectives. The material cost is a crucial factor, restricting in many cases the use of composite materials. Moreover, the distinction between weight minimization and material cost minimization is also stressed, because fiber reinforced composites are nonhomogeneous materials and the minimization of the weight does not also imply the minimization of the material cost. Therefore, the multi-objective formulation is summarized herein, as the more general case.

A constrained multi-objective problem involving minimization of l objective functions is described in the following mathematical form:

$$\min \{F_1(z), F_2(z), \ldots, F_l(z)\} \qquad (4)$$

subject to lower and upper bounds on the design vector z and inequality constraints

$G(z)$:

$$z^L \leq z \leq z^U \tag{5}$$
$$G(z) \leq 0 \tag{6}$$

In the rest of the paper, upper and lower values are represented by superscripts L and U respectively. Individual minimizations of each objective function subject to constraint set (5,6) will result in a set of ideal solutions which define a target point $F^* = (F_1^*, F_2^*, ..., F_l^*)$. A solution of the multi-objective problem is then obtained by finding a feasible point $\{F\} = (F_1, ..., F_l)$ as closely as possible to the target point $\{F^*\}$. This is achieved by minimizing the following scaled objective function:

$$\min \sum_{i=1}^{l} v_i \frac{(F_i - F_i^*)^2}{F_i^{*2}} \tag{7}$$

subject to constraints (5,6). The weighting coefficients are represented with v_i. Other metrics or scaling procedures may be utilized in eq. (7), but in general, they are expected to result in different solutions.

The design objectives typically include minimization of: (1) the maximum resonance amplitude (min F_1); (2) the total structural weight (min F_2); and (3) the material cost represented by the average cost of fibers (min F_3). Alternatively, F_1 may represent the maximization of select modal damping values. The explicit maximization of modal damping may be preferred in the case of transient or a-priori unknown dynamic excitations. The fiber cost is used as a measure of the total material cost due to the high cost of fibers compared to the cost of matrix. The design vector includes fiber volume ratios (FVRs), ply angles, and shape parameters.

Performance constraints are imposed on static deflections u_s,

$$\{u^s\} \leq \{u^{sU}\} \tag{8}$$

dynamic amplitudes,

$$\{U^d\} \leq \{U^{dU}\} \tag{9}$$

natural frequencies $\{f_n\}$,

$$\{f^L\} \leq \{f_n\} \leq \{f^U\} \tag{10}$$

and the static and dynamic stresses of each ply σ_l in the form of the modified distortion energy criterion [14],

$$f(\sigma_i, S_i) - 1 \leq 0 \tag{11}$$

The constrained optimizations mentioned above are solved with the modified feasible directions non-linear programming method [15,16]. The feasible directions algorithm performs a direct search in the design space involving a series of iterations. In each iteration, a search direction is calculated based on first order derivatives of the objective function and active constraints. A line search is subsequently performed along the search direction and a suboptimum along the search direction is calculated. The iterations are repeated until convergence to a local optimum is achieved.

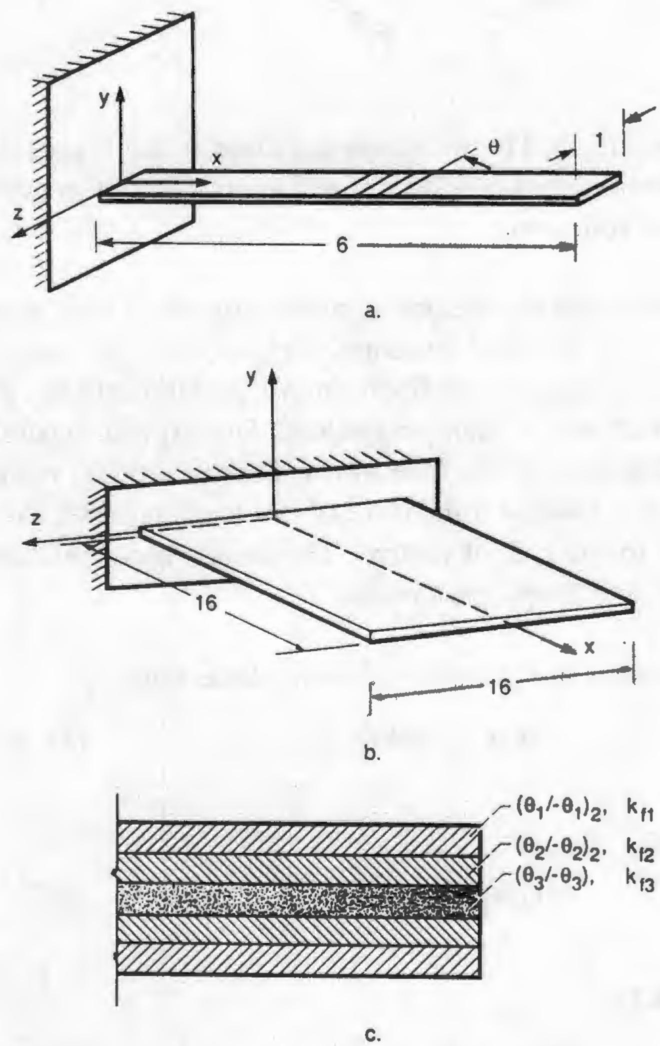

Fig. 1 Candidate composite structures. (a) Initial beam geometry; (b) Initial plate geometry; (c) Laminate configuration. Dimensions are in inches.

APPLICATIONS

Selected evaluations on the method on the optimal tailoring of a cantilever graphite/epoxy composite beam and a cantilever graphite/epoxy composite plate are presented (Fig. 1). The assumed laminate configuration in both structures is symmetric consisting of angle-ply sublaminates 1, 2, and 3 in each side. All sublaminates had plies of equal thickness (0.01 in). The ply angles θ_i and FVRs k_{fi} of each sublaminate, and the thicknesses h_j at 0%, 30%, 60%, and 100% (tip) of the span were optimized. The thickness at other sections was interpolated using a cubic spline fit. A unidirectional ply configuration was selected as the initial baseline composite design for both cases, because it provides high axial bending rigidity.

Composite Beam in Impulsive Excitation: Typical improvements in the predicted impulse response (y-axis) of an optimized composite beam design are shown in Fig. 2. In this particular case, a single objective function was implemented, such that, the modal damping corresponding to the mode with the higher undamped dynamic amplitude was maximized [11]. The optimization variables involved only composite parameters, that is, FVRs and fiber orientation angles. The baseline and resultant optimum design is shown in Table 1. Clearly, the free response of the optimized beam has been drastically improved, although the undamped dynamic amplitude was increased.

Table 1. Optimum design for Composite Beam in Impulsive Excitation

	Baseline Design	Optimum Design
Ply Angles, (degrees)		
θ_1	0.0	30.24
θ_2	0.0	30.49
θ_3	0.0	29.76
Fiber volume ratios		
k_{f1}	0.50	0.69
k_{f2}	0.50	0.53
k_{f3}	0.50	0.50

Multi-Objective Design of the Composite Beam in Forced Excitation: As a next application, a case of optimal design of the composite beam, involving the three objective functions mentioned above, is presented [13]. The assumed loading conditions involved a

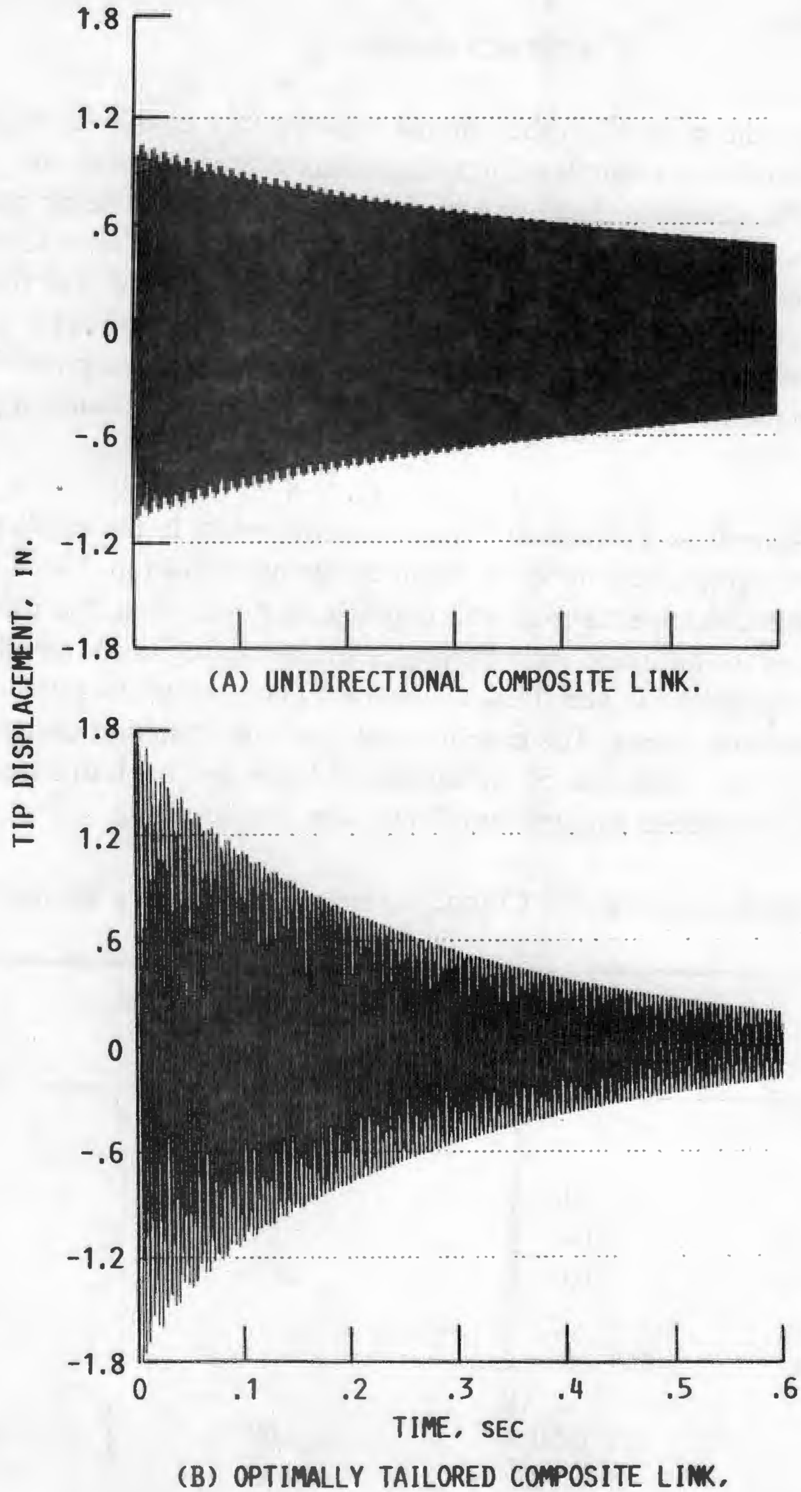

Fig. 2 **Impulse response of the optimum and baseline (initial) composite beams.**

combination of uniform static transverse out-of-plane (y-axis) forces (50 lbs/in) and transverse out-of-plane harmonic forces (0.1 lbs/in amplitude) applied at the tip of the beam. The design variables included both composite parameters (FVRs and ply angles) and shape parameters (cross-sectional thicknesses). In addition to constraints (8) and (11), constraints included upper bounds on the transverse static deflections at the free end lower bounds on the first two natural frequencies, and upper bounds on the transverse resonance amplitudes at the tip, for each of the first four modes.

Table 2. Multi-objective optimum designs: Composite beam

	Baseline	Single-Objective Designs			Multi-Objective
		min F1	min F2	min F3	
Ply Angles, (degrees)					
θ_1	0.0	24.68	13.55	4.306	24.46
θ_2	0.0	24.05	-41.19	41.150	53.53
θ_3	0.0	-50.33	-65.56	44.863	90.00
Fiber volume ratios					
k_{f1}	0.50	0.637	0.630	0.294	0.512
k_{f2}	0.50	0.700	0.021	0.010	0.010
k_{f3}	0.50	0.010	0.010	0.010	0.010

Table 2 also shows the baseline design, the three single-objective optimal designs (each objective function individually optimized), and the resultant multi-objective optimal design. All optimized designs have non-uniform thickness, being thicker at the proximal end and thinner at the distal end. The apparent differences among the optimal shapes demonstrate the significance of shape optimization. The relative improvements of each objective function with respect to the baseline design are plotted in Fig. 3. As seen in Fig. 3, the single-objective optimizations have failed to reduce all objective functions. Only the multi-objective optimal design produced simultaneous improvements in all design objectives.

The frequency response functions at the mid-point of the free-edge of the initial and optimized beams are shown in Fig. 4. The multi-objective optimum design has a better FRF than the minimum cost and minimum weight designs. This suggests that the incorporation of composite damping was crucial in obtaining these significant improvements in all objective functions illustrating, in this manner, the significance of composite damping in the design of high dynamic performance, light-weight, and low-cost composite

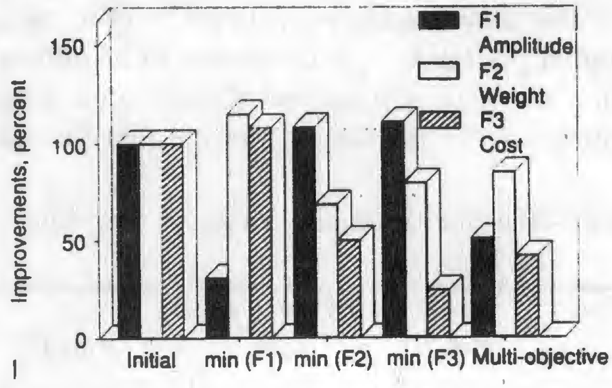

Fig. 3 Relative changes in the objective functions (composite beam).

structures.

Multi-Objective Design of the Composite Plate in Forced Excitation: The optimization of the composite plate involves additional structural complexity, therefore, the present application provides additional insight in the optimal design composite structures [13]. In this case, the loading conditions included combinations of a uniform static transverse out-of-plane (y-axis) force (3.12 lbs/in) at the free end, a uniform transverse out-of-plane harmonic force (0.0063 lbs/in amplitude) at the free end, and a harmonic moment (0.0313 lb-in/in amplitude) also applied along the free-edge of the plate. Under this type of dynamic loading, the maximum resonance amplitude at the tip typically occurs either at the first mode (first out-of-plane bending in the baseline design) or at the second mode (first torsion in the baseline design). Both composite parameters and shape variables were optimized. In addition to constraints (8), and (11), constrains included upper bounds on the transverse static deflections of the free-end, lower bounds on the first four natural frequencies, and upper bounds on the transverse resonance amplitudes of the free-end for the first four modes (Table 3).

Table 3 also presents the initial baseline design, the three single-objective optimal designs, and the resultant multi-objective optimal design. The relative changes in the objective function values with respect to the initial unidirectional plate are shown in Fig. 5. A strong tendency was observed in the optimum designs to result in "sandwich" type laminate configurations with a constrained thick matrix core (sublaminates 2, 3) and angle-ply composite skins (sublaminate 1) that provided stiffness and strength. The same tendency was also observed with the beam design but was less predominant. This inter-

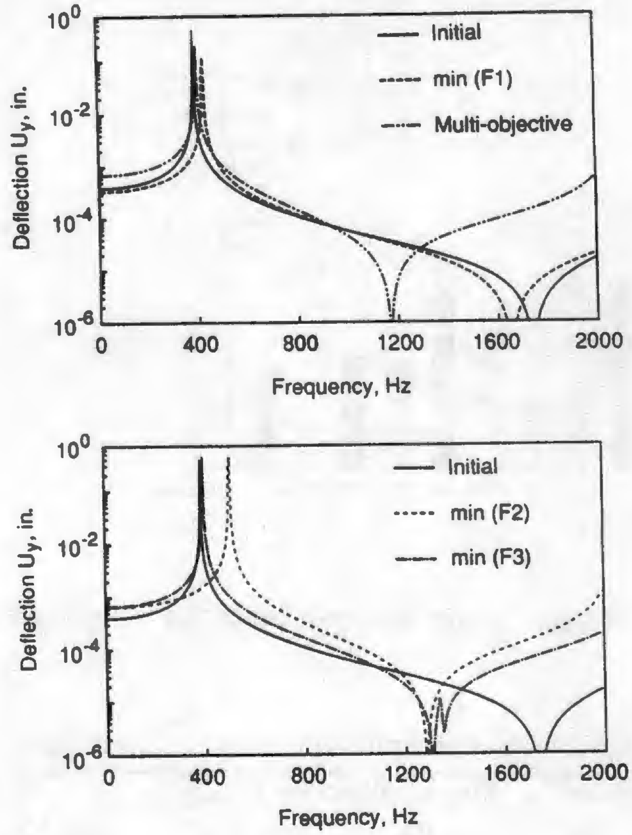

Fig. 4 Frequency response functions of the initial and optimal designs of the composite beam

esting result was the direct benefit of introducing unified micromechanics into the analysis, and consequently, the FVRs into the design parameters. The optimal designs varied drastically in optimal thickness shapes, ply angles, and FVRs, which demonstrated the inherent tendency of composite structures to get overdesigned.

The resultant frequency response functions of the transverse y-axis deflection at the foremost corner of the plate (x=16 in, z=8 in), where the maximum dynamic deflection was observed for almost all optimal designs, are plotted in Fig. 6. Interestingly, the minimum weight design has the higher resonance amplitudes, even than the baseline plate, illustrating the unsuitability of the minimum weight design for improving the dynamic performance. As both case studies illustrated, optimal design methodologies neglecting the damping capacity of composite materials and its controllable anisotropy may lead to structures with inferior dynamic performance near the resonance regimes, hence, they appear unsuitable for optimizing the dynamic performance of composite structures.

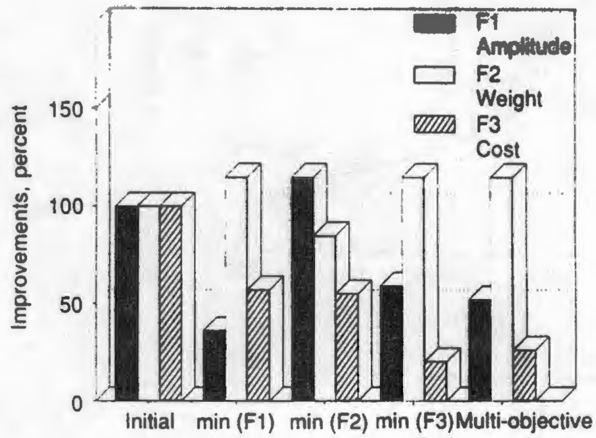

Fig. 5 Relative changes in the objective functions (composite plate).

Table 3. Multi-objective optimum designs: Composite Plate

	Baseline	Single-Objective Designs min F1	min F2	min F3	Multi-Objective
Ply Angles, (degrees)					
θ_1	0.0	11.74	24.91	33.97	24.23
θ_2	0.0	-83.10	50.38	68.88	49.92
θ_3	0.0	-4.06	56.22	-47.84	-52.70
Fiber volume ratios					
k_{f1}	0.50	0.700	0.698	0.225	0.301
k_{f2}	0.50	0.010	0.010	0.010	0.010
k_{f3}	0.50	0.010	0.010	0.010	0.010

SUMMARY

Research work at NASA-Lewis Research Center on the development of optimal design methodologies for optimizing the damping of composite structures and their dynamic performance was summarized. The design methodologies provide the option of multiple objective functions, and may tailor composite parameters at multiple scale levels of a

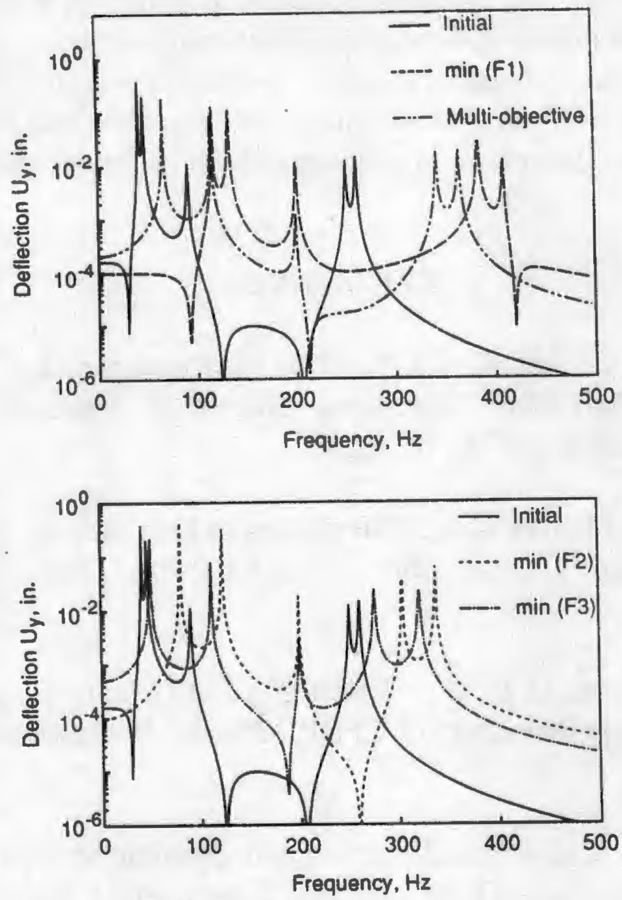

Fig. 6 Frequency response functions of the initial and optimal designs of the composite plate

composite structure. The structural dynamic analysis included the effects of composite passive damping on the dynamic response of composite structures via integrated micromechanics, laminate, and structural damping theories. Performance constraints were imposed on static displacements, static stresses, dynamic resonance amplitudes, natural frequencies, and dynamic stresses. The described method has been integrated into an in-house research code [16].

Basic application cases illustrating the optimal design of a cantilever composite beam and a cantilever plate were reviewed. All cases illustrated that optimal tailoring may significantly improve the damping capacity of composite structures and result in superior dynamic performance. It was also demonstrated that the damping capacity of composites is an important factor in designing light-weight, low-cost composite structures of improved dynamic performance. The multi-objective optimization was proved superior in minimizing the competing requirements involved. The optimizations with single-objective functions have shown a strong tendency to overdesign the structure and did not improve all objectives. The resultant optimal designs illustrated that both material (fiber orientation angles, fiber volume ratios) and shape parameters contributed to the obtained improvements. Overall, the applications of the method appeared very encouraging. Additional studies on more complex structural configurations and dynamic excitations may well worth the effort, therefore, are recommended as future research topics.

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