EFFECTS OF MULTIPLE BURSTS ON STRUCTURAL RESPONSE

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INTRODUCTION

In performing casualty/survivability studies in the civil defense area, it has been customary to consider the hazards produced by a single, 1-MT size weapon. A vast majority of the casualty estimates available today are based on the 1-MT weapon assumption. In real cases, however, there are situations where a given personnel shelter may be subjected to the effects of multiple bursts. This is likely to occur when the target area containing the shelter is subjected to more than one attack. This may also occur when the given shelter is located between several potential targets. In such a case, when each of the targets is attacked at different times, then the shelter will experience as many blast loadings as the combined number of attacks.

Generally, a shelter structure is designed to withstand a predetermined "design" blast load. The structure will experience damage to the extent that a given blast load is more intense than the design blast load. The extent of additional damage from subsequent loadings will be in direct proportion to the "available" strength of the structure, i.e., to the extent to which its strength has been degraded due to previous blast loadings. The collapse will take place once the available strength is below the limit determined by the designer.

This paper describes the problem of structural failure (collapse) as a result of a multiple blast load condition. Due to the non-deterministic nature of the problem, the method described herein considers the failure probability of the structure after each blast. The structure is modeled as a single degree of freedom dynamic system with a resistance function which provides for degradation of strength. The method considers uncertainties in both structural and blast load parameters. Probability of structural collapse is determined for a series of attack conditions separated in time for which the ratio of blast load to resistance of the structure is greater than 1.0. Practical applications of the approach are illustrated along with further recommended applications.

GENERAL ASSUMPTIONS

(i) The structure is modeled as a single-degree of freedom system

(ii) The applied load is assumed to consist of a series of step loads (see Fig. 1) of different peak intensities, F_i.

(iii) The resistance capacity of the structure is represented by means of an elasto-plastic resistance shown in Fig. 2. The yield and maximum displacements are represented respectively, by X_y and X_m. The stiffness of the elastic part is $k = R(X_y)^{-1}$ in which R is the resistance capacity.

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(iv) The applied blast load will leave the structure undamaged if the ratio of the load to resistance is less than 1/2, i.e. $F_i/R \leq 1/2$.

BASIC FORMULATION

The resistance function shown in Fig. 2 is further idealized by means of an "effective" linear resistance function shown in Fig. 3. The effective displacement X_e is found by equating the energy corresponding to elasto-plastic case and that of the corresponding linear curve (1). Such linearization yields

$$X_e^2 = X_y^2 \, (2 \, X_m/X_y - 1) \tag{1}$$

Introducing the ductility ratio $Z_i = X_m/X_y$, Eq. (1) may be written as

$$X_e^2 = X_y^2 \, (2 \, Z_i - 1) \tag{2}$$

or

$$Z_i = X_e^2/2 \, X_y^2 + 1/2 \tag{3}$$

Given the step load shown in Fig. 1, the maximum response of the linear system is (1).

$$X_e = 2 \, F_i/k = 2 \, F_i \, X_y/R \tag{4}$$

In the light of Eq. (4), Eq. (3) becomes

$$Z_i = (2F_i \, X_y/R)^2/2X_y{}^2 + 1/2 = 2/(R/F_i)^2 + 1/2 = 2/\theta_i^2 + 1/2 \tag{5}$$

where $\theta_i = R/F_i$.

Damage is likely to occur if $F_i \geq R/2$. This corresponds to $Z_i \geq 1$. Thus the probability of damage P(D) is:

$$P(D) = P(Z_i > 1) \tag{6}$$

Using arbitrarily a lognormal probability distribution for θ_i (2), the probability of damage is:

$$P(D) = \Phi\left(\frac{\ln \bar{Z}_i}{\Omega_{Z_i}}\right) \tag{7}$$

Where Φ (.) = the standard normal probability function, $\bar{Z}_i$ = the mean of Z_i and Ω_{Z_i} = the coefficient of variation (C.O.V.) of Z_i representing the uncertainty in Z_i. If $\bar{\theta}_i$ and Ω_{θ_i} are respectively the mean and C.O.V. of θ_i, $\bar{Z}_i$ and Ω_{Z_i} are calculated as (3):

$$\bar{Z}_i = 2/\bar{\theta}_i^2 + 1/2 \quad (8)$$

$$\Omega_{Z_i} = 8\ \Omega_{\theta_i}/(4 + \bar{\theta}_i^2) \quad (9)$$

in which (Ref. 2)

$$\bar{\theta}_i = \bar{R}/\bar{F}_i \quad (10)$$

and $$\Omega_{\theta_i} = (\Omega_R^2 + \Omega_i^2)^{1/2} \quad (11)$$

where $\bar{R}$ and $\bar{F}$ are, respectively the means of R and F_i and Ω_R and Ω_{F_i} are the respective C.O.V.'s.

COLLAPSE OF THE SYSTEM

The collapse of the structure may be defined as a ductility level above which the system suffers extensive damage so that failure is certain. If M represents this ductility level, collapse is represented by $\mu_i > M$ where μ_i is the overall ductility of the system at time of the ith blast load, whereas Z_i is the ductility because of ith blast only. The value of μ_i depends on the previous ductilities μ_1, μ_2, ... μ_{i-1}. The probability of collapse, $P(C_i)$ at the ith blast load depends on whether or not $Z_i > 1$. From the total probability theorem (Ref. 2), the probability of collapse is:

$$P(C_i) = P(C_i \mid Z_i > 1)\ P(Z_i > 1) + P(C_i \mid Z_i \leq 1)\ P(Z_i \leq 1) \quad (12)$$

where $P(C_i \mid Z_i > 1) = P(\mu > M)$; whereas $P(C_i \mid Z_i < 1)$ depends on the ductility at (i-1)th blast. This can be postulated as $P(C_i \mid Z_i \leq 1)$ $P(\mu_{i-1} > M)$. Eq. (12), therefore, becomes:

$$P(C_i) = P(\mu_i > M)\ P(Z_i > 1) + P(\mu_{i-1} > M)\ P(Z_i \geq 1) \quad (13)$$

The probability $P(\mu_i > M)$ may be calculated as follows.

After application of load F_{i-1} as part of a series of loads F_1, F_2, ... F_n if $Z_{i-1} > 1$, a permanent displacement $X_{p_{i-1}}$ will be produced. This displacement will be added to the displacement produced by load F_i (see Fig. 3).

For an equivalent linear system, under the action of F_i the system starts from rest with a permanent displacement $X_{p_{i-1}}$, and the total displacement X_{e_i} (see Fig. 4) is

$$X_{e_i} = X_{p_{i-1}} + 2F_i /k \tag{14}$$

If $\mu_i = X_{m_i}/X_y$ (see Fig. 4), a relationship between μ_i and μ_{i-1} may then be derived based on equalizing the energy of the elasto-plastic system and that of the linear one, i.e.

$$\mu_i = \mu_{i-1} + 2/\theta_i^2 - 1/2 \tag{15}$$

For a special condition of i=1, there is no previous permanent displacement. This condition will lead to $\bar{\mu}_o = 1$ so that Eq. (15) may still be used for i=1. Assuming lognormal distributions for μ_i and Z_i the collapse probability at ith blast may then be calculated in terms of $\bar{\mu}_i$ and Ω_{μ_i} the C.O.V. of μ_i

$$\begin{aligned} P(C_i) = & \{1-\Phi[(1/\Omega_{\mu_i}) \ln(M/\bar{\mu}_i)]\} \Phi[(1/\Omega_{Z_i}) \ln(Z_i)] + \\ & \{1-\Phi[(1/\Omega_{\mu_{i-1}}) \ln(M/\bar{\mu}_{i-1})]\} \{[1-\Phi(\ln Z_i)/\Omega_{Z_i}]\} \end{aligned} \tag{16}$$

SPECIAL CASES

For a large $\bar{\theta}_i$, $\bar{\mu}_i$ may be smaller than $\bar{\mu}_{i-1}$. This is, of course, not possible. It is, therefore, more appropriate to set $\bar{\mu}_i \geq \bar{\mu}_{i-1}$ as a necessary condition in this formulation.

If for every blast, $\bar{\theta}_i > 2$. $\bar{\mu}_i$ remains constant and equal to 1. Although Ref. (4) specifies this condition as a no failure case, the present formulation still yields a value for failure probability. This is because of the uncertainties associated with F_i and R.

NUMERICAL ILLUSTRATION

For a shelter under repeated identical loads, the collapse probabilities for different $\bar{\theta}_i$ ranging from 1.0 to 2.0 were obtained using the above formulations. The uncertainties associated with F_i and R are taken as 20%. Furthermore a ductility level M=2 is assumed for defining the borderline between failure and no failure. The results (see Fig. 5) show that even for relatively large $\bar{\theta}_i$ (i.e. $\bar{\theta}_i$ = 2.0) the collapse probability may become significant after the 3rd or fourth attack.

CONCLUSIONS

A method was formulated for studying the probability of failure of structures when subjected to repeated blast loads. It was applied to the analysis of a structure subjected to a series of identical blast loads and several different ranges from ground zero. Results indicate that even as few as three repeated blast loads can significantly increase the probability of failure even for cases with a relatively high R/F. (The R/F ratio can be looked at as indicating the relative strength of the structure or as an indication of its range from the point of detonation.)

This method can be extended to consider a variety of different loading and resistance functions and attack conditions. For the civil defender, this method is a potentially useful tool for evaluating the effectiveness of different shelter mixes. For the targeteer it is a useful tool for evaluating the effectiveness of different attack conditions.

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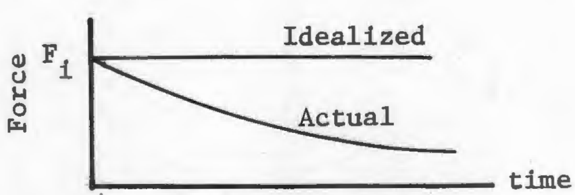

Fig. 1 Force Function

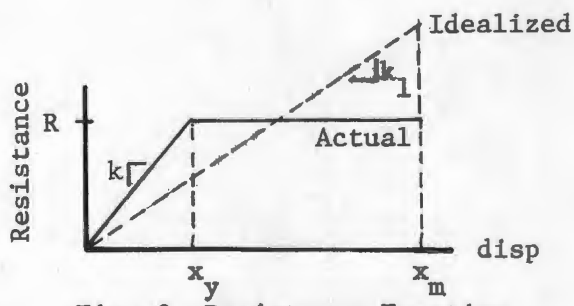

Fig. 2 Resistance Function

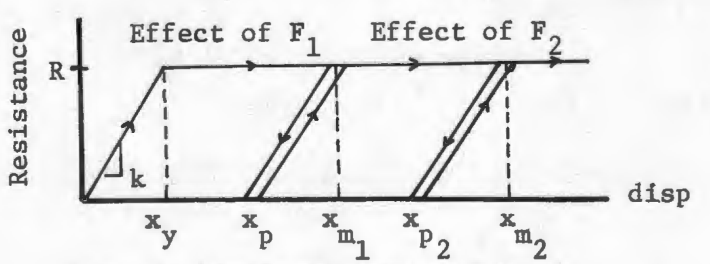

Fig. 3 Action of Repeated Loads

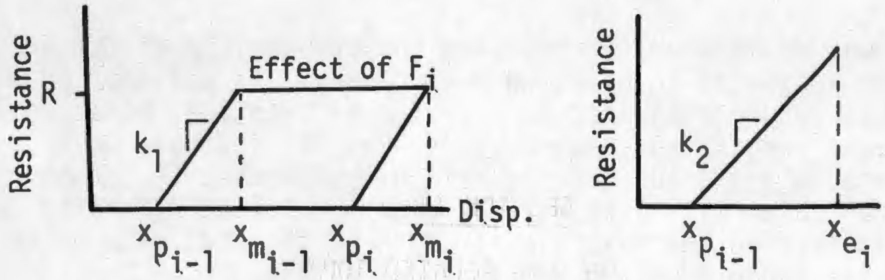

Fig. 4 Elasto-Plastic and Equivalent Linear System

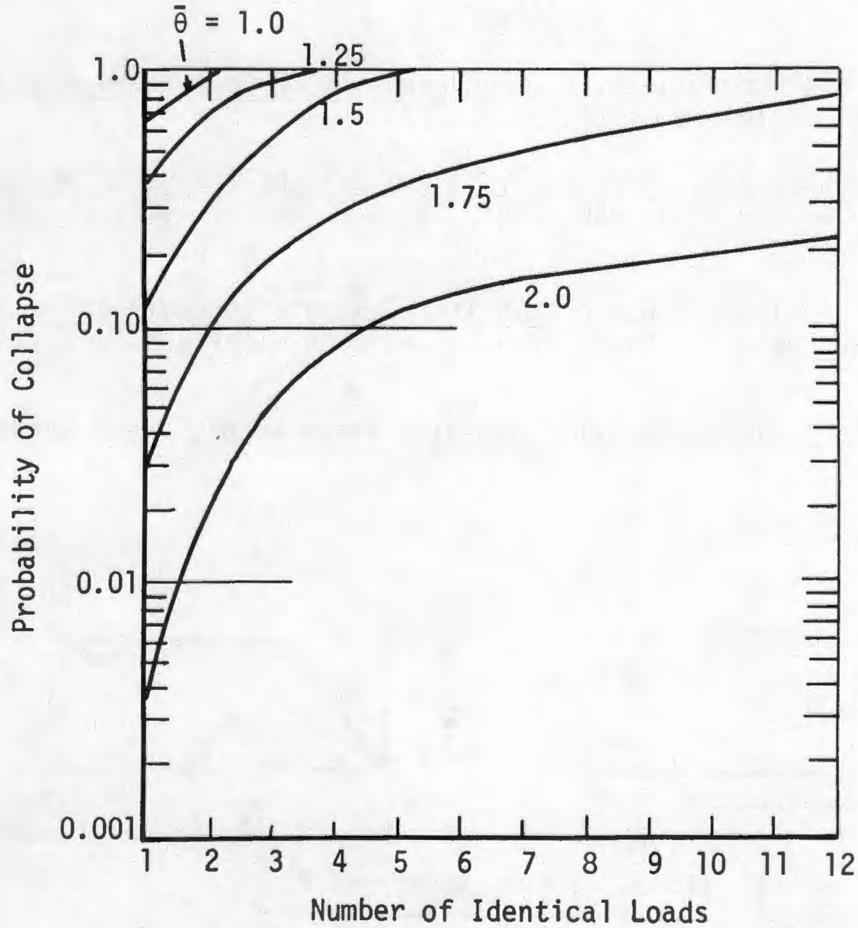

Fig. 5 Numerical Illustration