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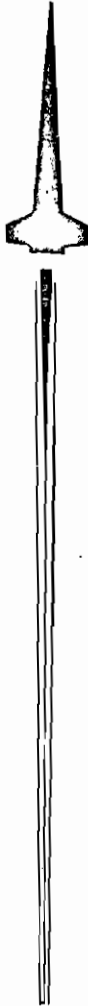
DESIGN OF VHF TRANSISTORIZED CRYSTAL OSCILLATORS
FOR MAXIMUM FREQUENCY STABILITY

by

W. Gary Briscoe

November 1965

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9 Tables (repts)

by

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ABSTRACT

Conditions for maximum oscillator frequency stability when variations occur in the circuit parameters are derived, and a method of designing VHF transistorized crystal oscillators to meet the conditions for maximum frequency stability is given. Two 200-megacycle oscillators were built and tested. The experimental results show that when the derived design conditions are met, the oscillator frequency is least sensitive to small changes in the circuit parameters.

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Section I. INTRODUCTION

The oscillator is a basic component in almost every electronic communication system that has been devised. A large number of oscillator applications occur in Army missile systems, including radar, telemetry, and guidance subsystems, as well as in associated communications systems. The diversity of applications of oscillators is approached by the diversity of oscillator types. These types arise from the different active elements, feedback networks, and frequency controlling elements, which can be used in oscillator circuits. Some well-known oscillator types are Colpitts, Hartley, Clapp, Pierce, and Pierce-Miller. The active element can be a device such as an electronic tube, transistor, tunnel diode, field effect transistor, or a cross field device. The frequency controlling elements may be suitably arranged resistances and capacitances, inductances and capacitances, piezoelectric crystals, or cavity structures.

From this multiplicity of oscillator types the transistorized, piezoelectric crystal controlled oscillator with sinusoidal output at VHF is the subject dealt with in this report. This type of oscillator offers much promise in military and space applications where small size, weight and power, as well as frequency stability and spectral purity under conditions of shock, vibration, temperature change, and supply voltage variations are required.

The oscillator design method is based upon linear network theory. Although more exact equations for the buildup of oscillations to a steady state condition could theoretically be written using nonlinear expressions to describe the active device, the variable terms are not easily determined and finding the solution of the resulting nonlinear differential equations is in practicality prohibitive. Vander Pol¹ and others have derived nonlinear differential equations to describe electrical oscillations which can be solved analytically or with computers, however, accurate quantitative oscillator design information can be obtained from the linear representation where small amplitude steady state oscillations are assumed. (See Reich.²) The effect of incremental variations in all equivalent circuit elements is then determined.

The primary design objective is maximum frequency stability when variations occur in the circuit parameters. As Gartner³ states, in a physical circuit the circuit parameters will vary as functions of a number of environmental factors. New circuit conditions for maximum frequency stability are derived. Other possible design objectives such as maximum power output and efficiency of power conversion are not considered.

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The design method to be described is formulated specifically for frequencies above 30 megacycles. The upper limit is about 200 megacycles, or the highest frequency at which quartz crystals are presently designed for operation. Design methods that are satisfactory for lower frequencies only give a rough qualitative understanding of the requirements for oscillation when applied at VHF. Factors which degrade the usefulness of other design methods and that are benign in the method presented here are:

- 1) Complex values of transistor input and output admittance and forward transfer admittance.
- 2) Stray capacitance, lead inductance, and variation from the nominal value of all circuit elements which cause a considerable change from the low frequency characteristics of a network.
- 3) Quartz crystal characteristics resulting from the use of high overtone crystals.

Section II. GENERAL FEEDBACK OSCILLATOR CIRCUIT

A feedback oscillator may be represented as a closed-loop system having a source of energy within the system. (See Martin.⁴) Thus, the system is capable of being unstable. When functioning properly an oscillator will have a sinusoidal output of reasonably constant amplitude and frequency. "Reasonably" can cover quite a wide range depending on the particular application, but here the design method is to produce oscillators having short-term frequency stabilities of at least $1\text{pp}10^7$ (i. e., within one cycle of the frequency specified at 10 megacycles). Frequency stabilities of this order are required in a number of missile and space systems. The amplitude of oscillation is automatically limited by the decrease of the forward transfer admittance (Y_f) of the active device as the amplitude of oscillations increase. (See Reich.²) Additional amplitude stabilization is desired since amplitude stability and frequency stability are interrelated and is provided in the design method to be discussed.

A generalized representation of a closed-loop system is given in Figure 1.

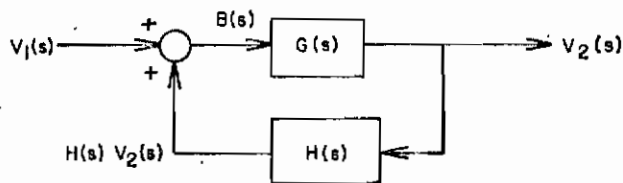


Figure 1. Closed-Loop System

$H(s)$ = feedback-loop transfer function.

$G(s)$ = open-loop transfer function.

The pertinent relations are

$$B(s) = V_1(s) + H(s)V_2(s)$$

$$V_2(s) = G(s)B(s)$$

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It is now possible to relate the input and output voltages in the relation

$$V_2(s) = G(s)V_1(s) + G(s)H(s)V_2(s)$$

from which the closed-loop transfer function may be written.

$$\frac{V_2(s)}{V_1(s)} = \frac{G(s)}{1 - G(s)H(s)} \quad (1)$$

The circuit will oscillate if the poles of the closed-loop transfer function are complex conjugates, and the oscillation will be sustained with constant amplitude if the real parts of these poles are zero. The condition for oscillation is obtained when the denominator of Equation (1) is equated to zero.

$$1 - G(s)H(s) = 0$$

$$G(s)H(s) = 1 \quad (2)$$

It can be seen from Equation (1) that if $V_1(s) = 0$ as in an oscillator and $V_2(s)$ is not zero for a finite gain $G(s)$ it follows that $[1 - G(s)H(s)]$ must be equal to zero.

The condition for oscillation of a closed-loop system is now applied to the equivalent circuit given in Figure 2. This is an almost exact representation of many oscillator circuits if the amplitude of oscillation is sufficiently low and the reverse transfer admittance (Y_r) of the transistor is negligibly small. The requirement of a small amplitude of oscillation is assured by proper design, and the requirement for a negligible Y_r is met by such VHF transistors as the 2N917, 2N918, and T2028. These transistors have reverse transfer admittances of approximately one millimho.

The equivalent circuit shown in Figure 2 was chosen because of its simplicity and its generality. The admittances Y_f , Y_1 , Y_2 , and Y_3 are complex in general. Y_1 includes the output admittance of the active device and the admittance of any external circuitry connected directly across the output. Y_2 includes the input admittance of the active device and the admittance of any external circuitry connected directly across the input. The terminals marked e, b, and c would be emitter, base, and collector respectively if the active device were a grounded emitter transistor amplifier.

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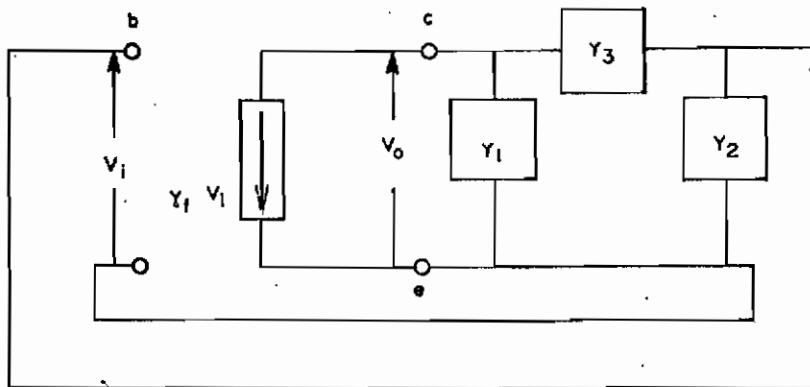


Figure 2. Simplified Oscillator Equivalent Circuit

If the gain and feedback functions, $G(s)$ and $H(s)$, are expressed in terms of the equivalent circuit parameters,

$$H(s) = \frac{V_i}{V_o} = \frac{1/Y_2}{1/Y_3 + 1/Y_2} \quad (3)$$

$$G(s) = \frac{V_o}{V_i} = \frac{-Y_f V_i}{V_i} \frac{1}{Y_1 + \frac{1}{1/Y_3 + 1/Y_2}} \quad (4)$$

Equation (2) may be written as,

$$\left(\frac{-Y_f}{Y_1 + \frac{1}{1/Y_3 + 1/Y_2}} \right) \left(\frac{1/Y_2}{1/Y_3 + 1/Y_2} \right) = 1 \quad (5)$$

Equation (5) may be rearranged in the following steps:

$$\begin{aligned} \frac{-Y_f/Y_2}{Y_1/Y_3 + Y_1/Y_2 + 1} &= 1 \quad , \\ Y_1/Y_3 + Y_1/Y_2 + Y_f/Y_2 + 1 &= 0 \quad , \\ Y_1 Y_2 + Y_1 Y_3 + Y_f Y_3 + Y_2 Y_3 &= 0 \quad . \end{aligned} \quad (6)$$

Equation (6) is the general oscillator equation in terms of the equivalent circuit parameters. The equations for frequency of oscillation, condition for oscillation, and frequency stability will be derived from this expression.

The admittances in the equivalent circuit of Figure 2 can be defined in terms of the conductances, inductance, and capacitances of Figure 3.

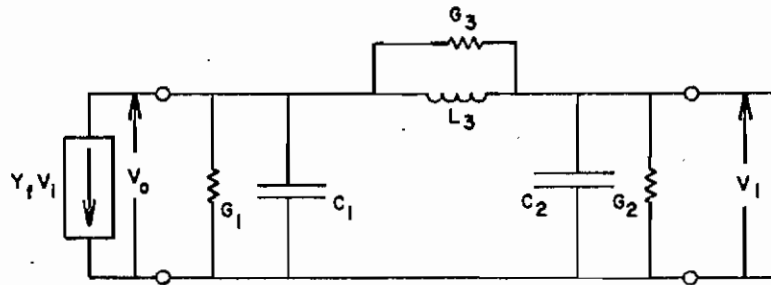


Figure 3. Oscillator Circuit

Using Laplace transform notation, Y_1 , Y_2 , Y_3 , and Y_f are defined as

$$Y_1 = G_1 + sC_1$$

$$Y_2 = G_2 + sC_2$$

$$Y_3 = G_3 + \frac{1}{sL_3}$$

$$Y_f = G_f + \frac{1}{sL_f}$$

If these admittance expressions are substituted into Equation (6),

$$G_f G_3 + \frac{G_f}{sL_3} + \frac{G_3}{sL_f} + \frac{1}{s^2 L_3 L_f} + G_1 G_2 + sG_1 C_2 + sG_2 C_1 + s^2 C_1 C_2 +$$

$$G_1 G_3 + \frac{G_1}{sL_3} + sG_3 C_1 + \frac{C_1}{L_3} + G_2 G_3 + \frac{G_2}{sL_3} + sG_3 C_2 + \frac{C_2}{L_3} = 0$$

The result can be simplified yielding

$$s^4 (C_1 C_2) + s^3 (G_1 C_2 + G_2 C_1 + G_3 C_1 + G_3 C_2) + s^2 (G_f G_3 + G_1 G_2 + G_1 G_3 + \frac{C_1}{L_3} + G_2 G_3 + \frac{C_2}{L_3}) + s (\frac{G_f}{L_3} + \frac{G_3}{L_f} + \frac{G_1}{L_3} + \frac{G_2}{L_3}) + \frac{1}{L_3 L_f} = 0 \quad (7)$$

Equation (7) has the form

$$a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

where

$$a_4 = C_1 C_2$$

$$a_3 = G_1 C_2 + G_2 C_1 + G_3 C_1 + G_3 C_2$$

$$a_2 = G_f G_3 + G_1 G_2 + G_1 G_3 + \frac{C_1}{L_3} + G_2 G_3 + \frac{C_2}{L_3}$$

$$a_1 = \frac{G_f}{L_3} + \frac{G_3}{L_f} + \frac{G_1}{L_3} + \frac{G_2}{L_3}$$

$$a_0 = \frac{1}{L_3 L_f}$$

The roots can be found by application of the Routh-Hurwitz criterion. The Routh-Hurwitz array is constructed below.

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\frac{a_3 a_2 - a_4 a_1}{a_3}$	a_0	0
s^1	$\frac{\frac{a_1}{a_3} (a_3 a_2 - a_4 a_1) - a_3 a_0}{\frac{a_3 a_2 - a_4 a_1}{a_3}}$	0	0
s^0	a_0	0	0

The requirement for steady state oscillation is that every term in a row of the array be zero. This requirement can be met by setting the first term in the fourth row equal to zero.

$$\frac{a_1}{a_3} (a_3 a_2 - a_4 a_1) = a_3 a_0 \quad (8)$$

This is the condition for oscillation. The frequency of oscillation is found from the last nonvanishing row of the Routh-Hurwitz array.

$$s^2 \frac{a_3 a_2 - a_4 a_1}{a_3} + a_0 = 0 \quad (9)$$

The roots of Equation (9) give the frequency of oscillation. Solving for s ,

$$s^2 = \frac{-a_0 a_3}{a_3 a_2 - a_4 a_1}$$

$$s = \pm j \sqrt{\frac{a_0 a_3}{a_3 a_2 - a_4 a_1}} \quad (10)$$

Substituting Equation (8) into Equation (10) yields

$$s = \pm j \sqrt{\frac{a_1}{a_3}} = \pm j\omega$$

thus, the frequency of oscillation is

$$\omega = \sqrt{\frac{a_1}{a_3}} \quad (11)$$

If the coefficients of Equation (7) are substituted into Equation (11), the frequency of oscillation can be obtained in terms of the circuit constants.

$$\omega = \sqrt{\frac{G_f/L_3 + G_3/L_f + G_1/L_3 + G_2/L_3}{C_1(G_3 + G_2) + C_2(G_3 + G_1)}}$$

$$\omega = \sqrt{\frac{G_3 L_3 / L_f + G_f + G_1 + G_2}{L_3 C_1 (G_3 + G_2) + L_3 C_2 (G_3 + G_1)}} \quad (12)$$

In a like manner, the coefficients of Equation (7) can be substituted into Equation (8) to obtain the condition for oscillation in terms of the circuit constants.

$$(G_1 C_2 + G_2 C_1 + G_3 C_1 + G_3 C_2) \left(G_f G_3 + G_1 G_2 + G_1 G_3 + \frac{C_1}{L_3} + G_2 G_3 + \frac{C_2}{L_3} \right) - C_1 C_2 (G_f / L_3 + G_3 / L_f + G_1 / L_3 + G_2 / L_3) = \frac{(G_1 C_2 + G_2 C_1 + G_3 C_1 + G_3 C_2)^2}{L_3 L_f (G_f / L_3 + G_3 / L_f + G_1 / L_3 + G_2 / L_3)}$$

The expression can be simplified slightly to

$$\begin{aligned} & \left[C_1(G_2 + G_3) + C_2(G_1 + G_3) \right] (G_f G_3 + G_1 G_2 + G_1 G_3 + G_2 G_3) + \\ & G_1 \frac{C_2^2}{L_3} + G_2 \frac{C_1^2}{L_3} + \frac{G_3}{L_3} (C_1 + C_2)^2 - G_f \frac{C_1 C_2}{L_3} - G_3 \frac{C_1 C_2}{L_f} = \\ & \frac{(G_1 C_2 + G_2 C_1 + G_3 C_1 + G_3 C_2)^2}{G_f L_f + G_3 L_3 + G_1 L_f + G_2 L_f} \end{aligned} \quad (13)$$

Most oscillator designers resort to a number of simplifying approximations when an abstruse equation such as Equation (13) is encountered. However, it is not necessary to make those approximations or to simplify Equation (13), nor is it necessary to use the condition for oscillation as presented in Equation (13). Instead, it will be shown that the same information which is contained in Equation (6) may be used to define the condition for oscillation in a much simpler form.

The condition for maximum frequency stability can be derived by finding the relationship between incremental changes in the admittances of Equation (6). The validity of the derivation is dependent upon the frequency of oscillation being largely controlled by one of the circuit admittances. In a Pierce oscillator, changes in Y_1 , Y_2 , and Y_f with frequency are negligible compared to changes in Y_3 with frequency because Y_3 includes a very high Q crystal. Y_3 is also assumed to be a function of frequency only. Thus, any changes in the crystal branch Y_3 caused by the environment are not considered.

$$\begin{aligned} Y_3 &= f(\omega) \\ dY_3 &= \frac{dY_3}{d\omega} \Delta\omega \end{aligned} \quad (14)$$

If the condition for oscillation given in Equation (6) is used to define Y_3 ,

$$Y_3 = \frac{-Y_1 Y_2}{Y_f + Y_1 + Y_2} \quad (15)$$

Y_3 is expressed as a function of Y_1 , Y_2 , and Y_f .

The total differential of Y_3 is by definition,

$$dY_3 = \frac{\partial Y_3}{\partial Y_1} \Delta Y_1 + \frac{\partial Y_3}{\partial Y_2} \Delta Y_2 + \frac{\partial Y_3}{\partial Y_f} \Delta Y_f \quad (16)$$

The partial derivatives can be derived from Equation (15).

$$\frac{\partial Y_3}{\partial Y_1} = \frac{-Y_2(Y_f + Y_1 + Y_2) + Y_1 Y_2}{(Y_f + Y_1 + Y_2)^2}$$

$$\frac{\partial Y_3}{\partial Y_1} = \frac{-Y_2(Y_f + Y_2)}{(Y_f + Y_1 + Y_2)^2} \quad (17)$$

$$\frac{\partial Y_3}{\partial Y_2} = \frac{-Y_1(Y_f + Y_1)}{(Y_f + Y_1 + Y_2)^2} \quad (18)$$

$$\frac{\partial Y_3}{\partial Y_f} = \frac{Y_1 Y_2}{(Y_f + Y_1 + Y_2)^2} \quad (19)$$

If the partial derivatives are substituted into Equation (16),

$$dY_3 = \frac{-Y_2(Y_f + Y_2)}{(Y_f + Y_1 + Y_2)^2} \Delta Y_1 - \frac{Y_1(Y_f + Y_1)}{(Y_f + Y_1 + Y_2)^2} \Delta Y_2 + \frac{Y_1 Y_2}{(Y_f + Y_1 + Y_2)^2} \Delta Y_f$$

Dividing through by Y_3 and using Y_3 as expressed in Equation (15) on the right side of the equation results in

$$\frac{dY_3}{Y_3} = \frac{Y_f + Y_2}{Y_f + Y_1 + Y_2} \left(\frac{\Delta Y_1}{Y_1} \right) + \frac{Y_f + Y_1}{Y_f + Y_1 + Y_2} \left(\frac{\Delta Y_2}{Y_2} \right) - \frac{Y_f}{Y_f + Y_1 + Y_2} \left(\frac{\Delta Y_f}{Y_f} \right) \quad (20)$$

When dY_3 from Equation (14) is substituted into Equation (20), the relationship between small changes in the circuit parameters Y_f , Y_1 , and Y_2 and the resulting change in frequency is obtained. The partial of Y_3 with respect to ω must be evaluated at the quiescent frequency (ω_0), and is assumed to be constant throughout the pertinent frequency change.

$$\frac{\frac{\partial Y_3}{\partial \omega} \Big|_{\omega = \omega_0} (\Delta \omega)}{Y_3} = \frac{Y_f + Y_2}{(Y_f + Y_1 + Y_2)Y_1} (\Delta Y_1) + \frac{Y_f + Y_1}{Y_2(Y_f + Y_1 + Y_2)} (\Delta Y_2) - \frac{1}{Y_f + Y_1 + Y_2} (\Delta Y_f) \quad (21)$$

Hafner⁵ derived an equation similar to Equation (20), and at this point, separated the equation into its real and imaginary parts. He stated that the frequency was determined only by the imaginary part and derived conditions for frequency stability from this equation. The entire equation will be used in this report since the coefficient of $\Delta \omega$ is complex. Furthermore, a change in amplitude of oscillation, which is determined by the real part of the oscillator equation, will cause a change in frequency because of the dependance of Y_f on the amplitude.

For any ΔY_1 , ΔY_2 , or ΔY_f , Equation (21) shows that ω changes by $\Delta \omega$ to maintain the equality. To have the smallest possible $\Delta \omega$, the $\frac{\partial Y_3}{\partial \omega}$ must be a maximum and the coefficients of ΔY_1 , ΔY_2 , and ΔY_f must be minimized. The coefficients of ΔY_1 and ΔY_2 are zero if

$$Y_2 = -Y_f \text{ and } Y_1 = -Y_f \quad (22)$$

These are the conditions for perfect frequency stability with respect to changes in the circuit parameters Y_1 and Y_2 . Unfortunately, none of these conditions can be met and still maintain oscillations.

If the conditions given in Equation (22) are substituted into the condition for oscillation (Equation (15)), Y_3 must be equal to Y_f . This is an impossible requirement since Y_f has a negative real part. The real part of Y_f is necessarily negative if Equation (22) is satisfied because the passive elements Y_1 and Y_2 always have positive real parts.

Although the coefficients of ΔY_1 and ΔY_2 cannot be made equal to zero, they can be made small. The condition for maximum frequency stability then requires that the quantities $(Y_f + Y_1)$ and $(Y_f + Y_2)$ be made as small as possible. This can be accomplished by making $B_1 = B_2 = -B_f$ and G_f , G_1 , and G_2 small. The minimum values for G_f , G_1 , and G_2 are determined by the condition for oscillation given in Equation (15).

The minimum value of G_1 , G_2 , and G_f can be found by first separating Equation (15) into its conductive and susceptive components. Substituting the relations

$$Y_1 = G_1 - jB_f$$

$$Y_2 = G_2 - jB_f$$

$$Y_f = G_f + jB_f$$

into Equation (15) yields

$$Y_3 = \frac{-(G_1 - jB_f)(G_2 - jB_f)}{G_1 + G_2 + G_f - jB_f}$$

If Y_3 is separated into its conductive and susceptive components

$$Y_3 = \frac{(-G_1G_2 + B_f^2)(G_1 + G_2 + G_f) - B_f^2(G_1 + G_2)}{(G_1 + G_2 + G_f)^2 + B_f^2} - j \frac{B_f [G_1G_2 - B_f^2 - (G_1 + G_2)(G_1 + G_2 + G_f)]}{(G_1 + G_2 + G_f)^2 + B_f^2} \quad (23)$$

The admittance of the crystal network Y_3 can be bilinearly transformed to any point in the admittance plane where the conductive component is positive. This will be demonstrated in Section IV. It follows that if the real part of Y_3 required by Equation (23) is positive, the equation can be satisfied by using the proper bilinear transformation. Since $\text{Re}(Y_3) > 0$, Equation (23) requires

$$(-G_1G_2 + B_f^2)(G_1 + G_2 + G_f) - B_f^2(G_1 + G_2) > 0 ,$$

which reduces to

$$B_f^2 G_f > G_1 G_2 (G_1 + G_2 + G_f) \quad (24)$$

Equation (24) gives the condition for oscillation.

It can be seen that to satisfy the inequality for the condition $G_f = 0$ would require the impossible condition that G_1 or G_2 be negative. Unless B_f^2 is considerably less than $G_1 G_2$, the limiting case for G_f is zero. Further, it may be said about the inequality that G_1 and G_2 may be made small without limit. In an actual circuit G_1 and G_2 could not be made zero since G_1 includes the transistor output conductance and G_2 includes the transistor input conductance.

If B_f is zero, the inequality reduces to

$$G_f < -(G_1 + G_2) \quad (25)$$

An increase in B_f from zero would require an increase in the inequality of Equation (25). The needed increase in negative G_f soon becomes physically impossible to achieve. The nebulous limits on B_f when G_f is negative are also dependent on the product $G_1 G_2$.

The conditions for maximum frequency stability are summarized below.

- 1) $B_1 = B_2 = -B_f$,
- 2) G_1 and G_2 small,
- 3) G_f small but larger than zero,

or depending on the transistor parameters

- 1) $B_f \approx B_1 \approx B_2 \approx 0$,
- 2) G_f slightly more negative than $-(G_1 + G_2)$.

Note that the first set of conditions involves a cancellation of susceptances and a minimization of conductance of the terms $(Y_f + Y_1)$ and $(Y_f + Y_2)$, while the second set of conditions involves a cancellation of conductance and a minimization of susceptance. These conditions for maximum frequency stability are for a Pierce oscillator, but the same method could be used for the Pierce-Miller and other crystal circuits.

Conditions for maximum frequency stability for other oscillator equivalent circuits have been derived. Llewellyn⁶ and Fair⁷ derived circuit conditions for minimum oscillator frequency change when r_p , r_g , and μ of a triode vacuum tube vary. Oakes⁸ derived stability conditions for audio transistor oscillators when the transistor parameters are purely resistive. Hafner⁵ assumed a real forward transfer admittance in his work.

Section III. FEEDBACK OSCILLATORS

Feedback oscillators are classified according to the feedback elements used and their position with respect to the terminals of the active device. The Colpitts, Hartley, Clapp, Pierce, and Pierce-Miller circuits are given in simplified form in Figure 4.

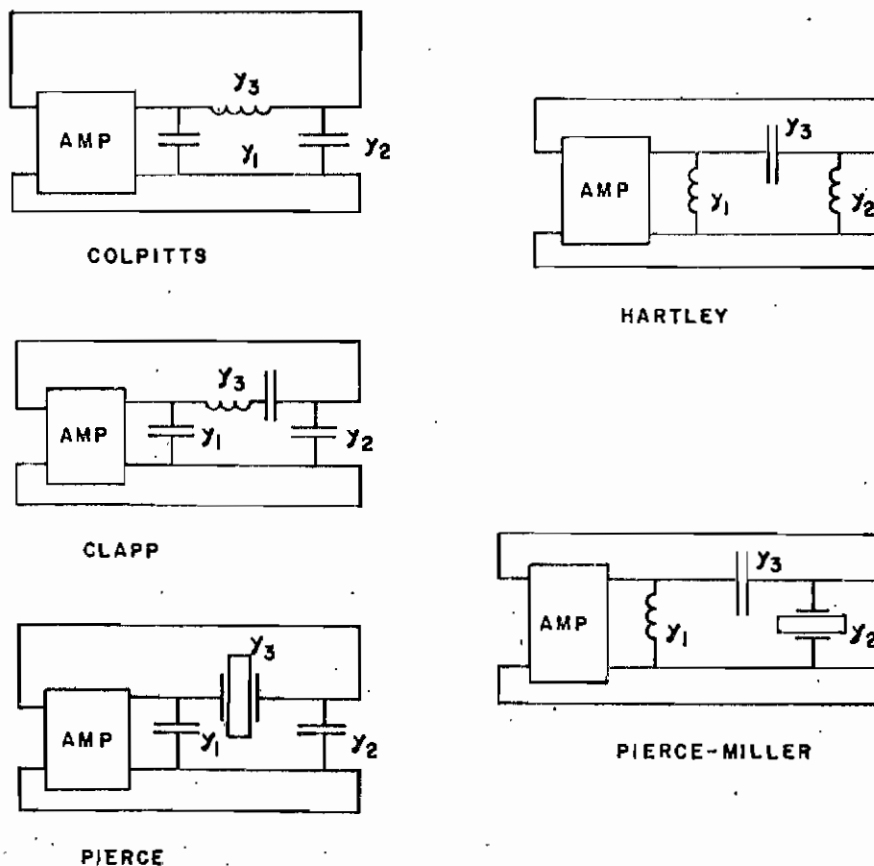


Figure 4. Feedback Oscillators

In the original form of these oscillators the amplifier utilized a triode vacuum tube. However, the oscillator circuits of Figure 4 may be considered to have any type of transistor or tube amplifier. The following discussion will be limited to the case of a single transistor amplifier.

A transistor amplifier may be represented by the equivalent circuit in Figure 5 if y_{12} is negligibly small. At VHF y_{11} , y_{22} , and y_{21} are complex admittances.

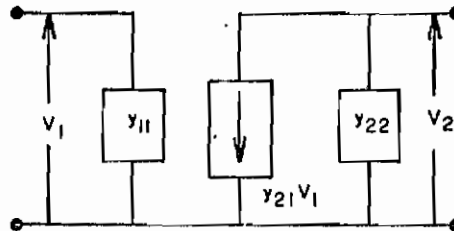


Figure 5. Transistor Amplifier Equivalent Circuit

The circuits given in Figure 4 may be redrawn using the amplifier equivalent circuit. The resulting circuit is given in Figure 6. The admittances y_1 , y_2 , and y_3 are either capacitive, inductive, or a quartz crystal, depending upon the circuit under consideration.

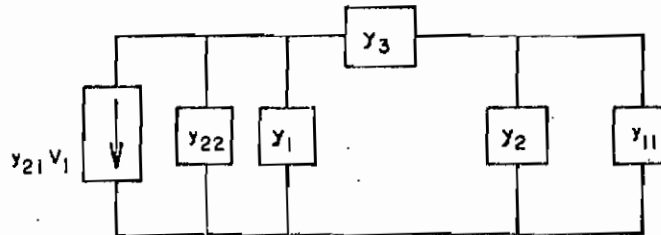


Figure 6. Transistor Oscillator Equivalent Circuit

If Y_1 is defined as the sum of y_1 and y_{22} , and Y_2 is defined as the sum of y_2 and y_{11} , all the oscillator circuits reduce to the basic equivalent circuit given in Figure 2.

When this basic equivalent circuit is used, the five oscillator circuits can be classified as basically a Colpitts or a Hartley type. A given oscillator circuit is then said to be a Colpitts circuit if the susceptive components of Y_1 and Y_2 are positive and Y_3 has a negative susceptance. The Clapp and Pierce circuits are, from this viewpoint, the same as the Colpitts circuit. In the Hartley and Pierce-Miller

circuits, Y_1 and Y_2 have negative susceptances and Y_3 has a positive susceptance. Cady,⁹ Martin,⁴ and others have classified oscillators in this manner.

A wide range of values for Y_1 , Y_2 , Y_3 , and Y_f may be used in any of the oscillator circuits. There are obviously many different combinations of circuit values which satisfy the condition for oscillation as expressed in Equation (13). An equal number of combinations satisfies Equation (15). Thus, one is free to adjust the circuit for maximum frequency stability within a wide domain of possible circuit designs.

Section IV. CRYSTAL OSCILLATORS

The unique feature of the oscillators devised by Pierce and Miller is the utilization of the piezoelectric effect. The piezoelectric effect is the slight deformation of crystals when voltage gradients are established in particular directions, and, conversely, a voltage gradient is created by a deforming force. Quartz crystals are used almost exclusively as the piezoelectric element in crystal oscillator circuits. By properly cutting the quartz crystal, it can be made to have certain resonant frequencies. The resonance phenomenon is both mechanical and electrical because of the piezoelectric effect. The resonant quality factor of a quartz crystal, Q , is typically of the order of 10^4 to 10^5 at VHF.

In 1918, A. M. Nicolson was the first to use a piezoelectric crystal with electrodes attached in an oscillating vacuum tube circuit. Professor W. G. Cady independently experimented with similar circuits in 1921. Cady discovered that the frequency stability of a quartz crystal controlled oscillator was much greater than any other type of oscillator. Two years later a two-electrode crystal was connected between plate and grid of a triode by Pierce.¹⁰ Stable oscillations were observed. Both Pierce¹¹ and Miller¹² invented oscillators having a two-electrode crystal connected between grid and cathode. These two oscillators are now called the Pierce and Pierce-Miller oscillators.

Figures 7, 8, 9, and 10 show the Nicolson, Cady, Pierce, and Pierce-Miller oscillator circuits respectively.

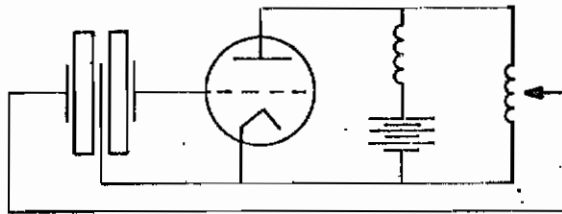


Figure 7. Nicolson's Crystal Oscillator Circuit

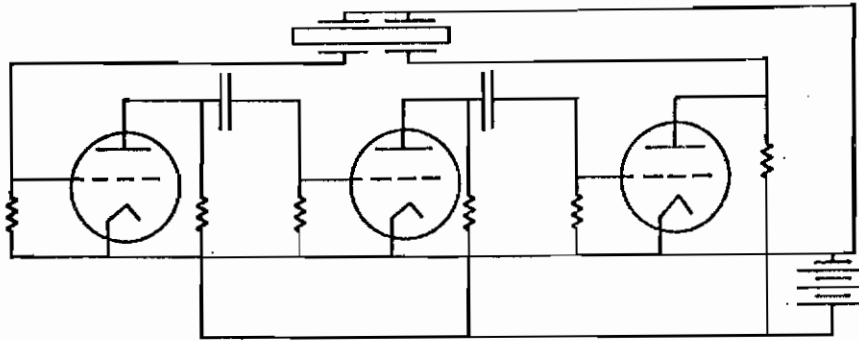


Figure 8. Cady's Oscillator Circuit

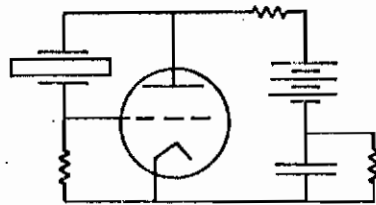


Figure 9. Pierce's Oscillator Circuit

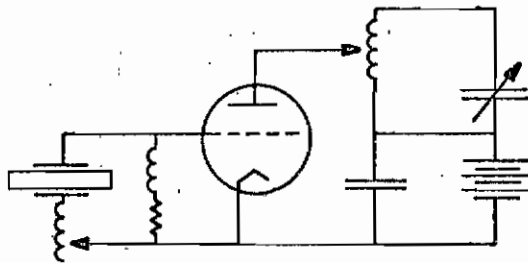


Figure 10. Pierce-Miller Oscillator Circuit

In 1952, transistorized crystal oscillators were briefly described by Oser, Enders, and Moore.¹³ All their oscillator circuits had a parallel or series resonant circuit in series with the crystal. In 1953, Sulzer¹⁴ proposed two oscillators, one having a crystal connected between collector and base (see Figure 11) and one having a crystal connected between collector and emitter. None of these early

investigators classified their circuits as Pierce or Pierce-Miller oscillators although they were, no doubt, aware of the analogy between the vacuum tube and transistor circuits.

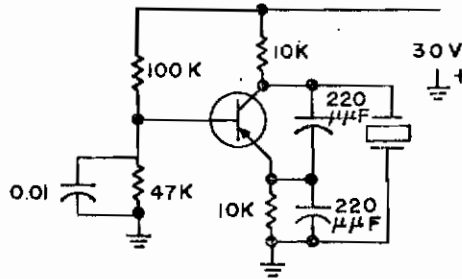


Figure 11. Transistorized Pierce Oscillator

By 1960, transistorized crystal oscillators had been developed which had frequency stabilities comparable to the best vacuum tube oscillators at frequencies below 10 megacycles. A precision oscillator described by Smith¹⁵ employed a five-megacycle AT cut quartz crystal. The circuit temperature was held constant to within $\pm 0.1^\circ \text{C}$, and the crystal temperature was stabilized to better than 0.01°C by a crystal oven. Supply voltage regulation and automatic gain control were included. Short-time stability of Smith's oscillator was $2\text{pp}10^{10}$ per second (a change in frequency of 0.001 cycle at five megacycles).

Transistorized crystal oscillators at VHF, up to 200 megacycles, have been investigated recently because of their possible applications in single sideband systems and solid state microwave signal sources. (See Layden¹⁶ and Hines.¹⁷) Moderate precision oscillators at lower frequencies without temperature control or supply voltage regulation have been studied by several investigators. Kconjian¹⁸ built a one-megacycle transistorized crystal oscillator which had a frequency stability of $1\text{pp}10^7$ per degree centigrade and $1\text{pp}10^6$ per ± 10 -percent supply voltage change. In 1957, Witt¹⁹ reported the design of a one-megacycle oscillator with $.4\text{pp}10^8$ frequency change when the supply voltage changed one percent. Knapp²⁰ and Boyle²¹ developed oscillators at 5 megacycles and 10 megacycles respectively. Short-time stabilities of their oscillators were typically $1\text{pp}10^8$.

Sherman²² described several transistor oscillator circuits using quartz crystals. Sherman and others of the General Electric Company built oscillators which operated at frequencies up to 175 megacycles.

In 1962, non-temperature controlled quartz crystal oscillator design procedures were developed by Firth and Yope.²³ Their investigation covered the frequency range from 1 kilocycle to 200 megacycles. Test data given on three of their VHF transistorized oscillators are $\pm 2.5\text{pp}10^6$, $\pm 1.4\text{pp}10^6$, and $\pm 2.2\text{pp}10^6$ frequency change at 120, 150, and 193 megacycles respectively for a ± 10 -percent supply voltage change. Total frequency change over the temperature range -10°C to $+80^\circ\text{C}$ was about $70\text{pp}10^6$ for the 193- and 150-megacycle oscillators.

The quartz crystal resonators commonly used in oscillator circuits consist of a quartz crystal disk from 0.18 to 1.50 inches in diameter and about 0.005 inch thick with a circular electrode attached to each side of the crystal disk. The crystal is mounted inside an evacuated enclosure by clips attached to the outer circumference of the crystal disk. Figure 12 shows a 2.5-megacycle quartz crystal unit developed by Bell Telephone Laboratories.

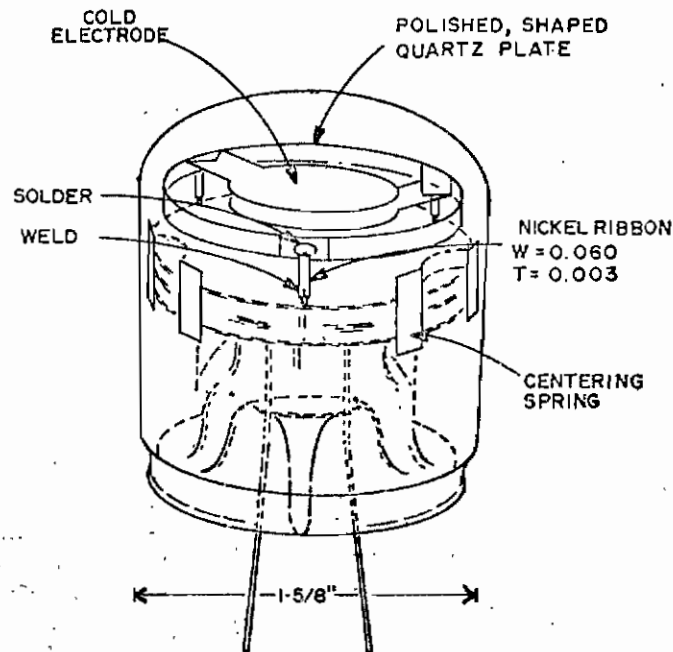


Figure 12. 2.5-Megacycle Quartz Crystal Unit Developed by Bell Telephone Laboratories

A crystal resonator always has a fundamental and overtone responses which are integral multiples of the fundamental, as well as spurious responses. A typical spectrum of the spurious responses is shown in Figure 13. The magnitude and frequency of the spurious responses are dependent upon the diameter and thickness of the electrodes. The crystal frequency specified by the manufacturer is either the fundamental or one of the overtone frequencies.

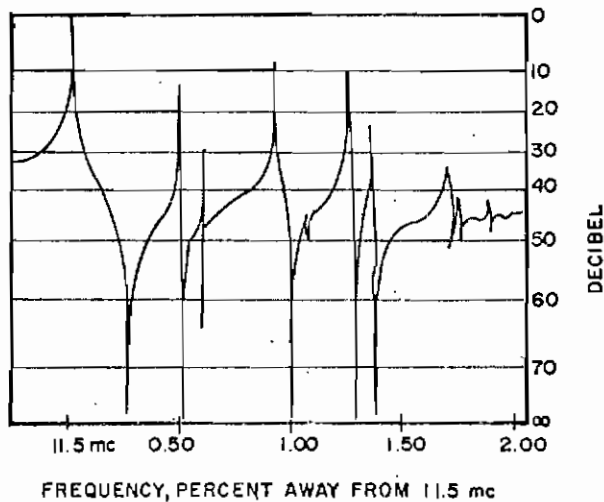


Figure 13. Spurious Responses of an Oscillator Crystal

Van Dyke²⁴ has shown that the electrical equivalent circuit of a quartz crystal resonator consists of an RLC series branch in parallel with a capacitance (C_0) due mainly to the electrodes attached to the quartz. The RLC series branch elements are called the motional impedance elements. The equivalent circuit given in Figure 14 describes the crystal resonator in the vicinity of the resonant frequency $\frac{1}{2\pi\sqrt{L_1C_1}}$. This equivalent circuit also applies at or near the fundamental or overtone frequencies, but the values of the elements in the equivalent circuit are different for each frequency.

The calculations below show that the impedance of the crystal, when plotted in the rectangular impedance plane, is a circle. The circular locus is traced out as frequency is varied near the series resonant frequency of the crystal.

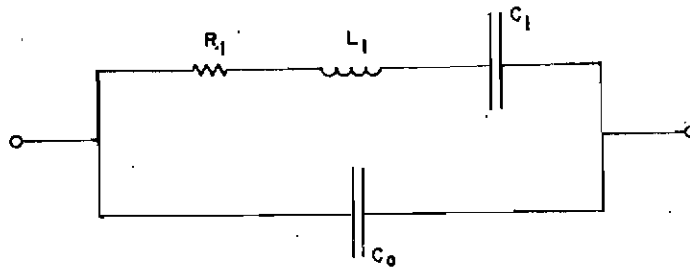


Figure 14. Quartz Crystal Equivalent Circuit

Let $x = \omega L_1 - \frac{1}{\omega C_1}$, and using this relation the crystal impedance (Z) can be expressed as

$$Z = \frac{1}{j\omega C_0 + \frac{1}{R_1 + jx}} = R + jX \quad (26)$$

Z may be divided into its real and imaginary parts

$$Z = \frac{R_1}{(1 - \omega C_0 x)^2 + (\omega C_0 R_1)^2} + \frac{j(x - \omega C_0 x^2 - \omega C_0 R_1^2)}{(1 - \omega C_0 x)^2 + (\omega C_0 R_1)^2}$$

Solving the real part for x

$$(1 - \omega C_0 x)^2 + (\omega C_0 R_1)^2 = \frac{R_1}{R}$$

$$(1 - \omega C_0 x) = \left[\frac{R_1}{R} - (\omega C_0 R_1)^2 \right]^{1/2}$$

$$x = \frac{1}{\omega C_0} \left[1 - \left[\frac{R_1}{R} - (\omega C_0 R_1)^2 \right]^{1/2} \right] \quad (27)$$

where R is the real part of Z ($Z = R + jX$).

Substituting x from Equation (27) into the imaginary part of Z results in

$$X = \frac{\frac{1}{\omega C_0} \left[1 - \left[\frac{R_1}{R} - (\omega C_0 R_1)^2 \right]^{1/2} \right] - \frac{1}{\omega C_0} \left[1 - 2 \left[\frac{R_1}{R} - (\omega C_0 R_1)^2 \right]^{1/2} + \frac{R_1}{R} - (\omega C_0 R_1)^2 \right] - \omega C_0 R_1^2}{\frac{R_1}{R} - (\omega C_0 R_1)^2 + (\omega C_0 R_1)^2}$$

Simplification of Equation (28) yields

$$X = \frac{R}{R_1 \omega C_o} \left[\frac{R_1}{R} - (\omega C_o R_1)^2 \right]^{1/2} - \frac{1}{\omega C_o} \quad (29)$$

Equation (29) can be rearranged to give the desired form,

$$\left(X + \frac{1}{\omega C_o} \right)^2 = \left(\frac{R}{R_1 \omega C_o} \right)^2 \left[\frac{R_1}{R} - (\omega C_o R_1)^2 \right]$$

$$\left(X + \frac{1}{\omega C_o} \right)^2 = -R^2 + R \left(\frac{1}{R_1 \omega^2 C_o^2} \right)$$

Completing the square gives

$$\left(X + \frac{1}{\omega C_o} \right)^2 + \left(R - \frac{1}{2R_1 \omega^2 C_o^2} \right)^2 = \left(\frac{1}{2R_1 \omega^2 C_o^2} \right)^2 \quad (30)$$

This is the equation of a circle in the impedance plane. A plot of Equation (30) is shown in Figure 15. The quantity ωC_o is assumed to be constant for small frequency changes. (See Gerber.²⁵) Because the crystal Q is very high, most of the circular plot is obtained by changing the frequency less than 0.01 percent.

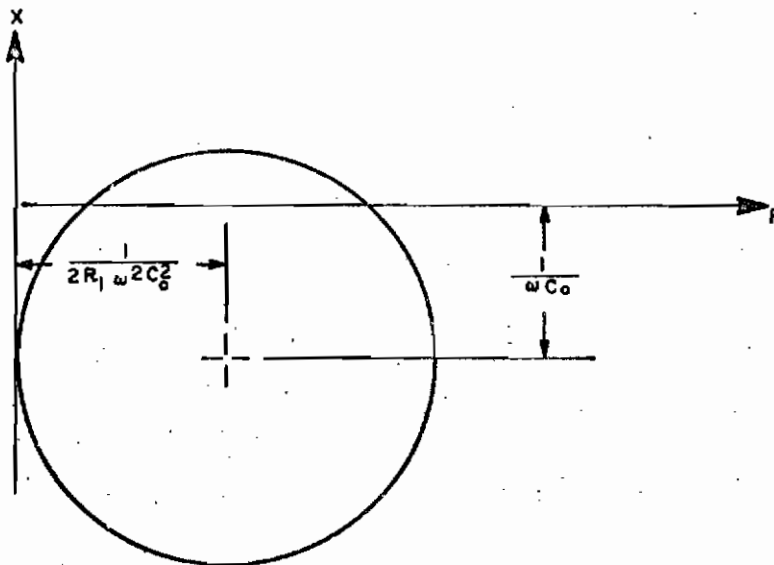


Figure 15. Crystal Impedance Plot

A Smith chart presentation of the crystal impedance locus is convenient for measurement and interpretation at VHF. The impedance locus of a crystal is also a circle in the reflection coefficient plane as depicted on a Smith chart. (See Witt.¹⁹) This is true because the impedance points on the Smith chart can be obtained using a bilinear transformation to transform the rectangular impedance plane into the circular reflection coefficient plane. The equation of the transformation is

$$k = \frac{Z - Z_0}{Z + Z_0} \quad (31)$$

where k is the reflection coefficient and Z_0 is the characteristic impedance of the transmission line.

The impedance circle of the crystal is tangent to the $k = 1$ circle at approximately the point $Z = 0 + jX_{CO}$. Since the impedance of the RLC series branch is large at frequencies a few kilocycles from resonance, the series branch is essentially an open circuit and only the reactance (X_{CO}) of C_0 remains.

Quartz crystal measurements at frequencies from 150 to 400 megacycles were investigated by Witt.¹⁹ He plotted measured values of the crystal impedance at a number of frequencies near the resonant frequencies of the crystal on a Smith chart and obtained a circular curve at frequencies near each of the crystal overtone responses as shown in Figure 16.

A method of plotting the admittance of crystals as frequency is varied near the crystal resonant frequency was devised. (See the appendix.) The admittance locus can be plotted directly on an X-Y recorder by this method. The recorder is calibrated to plot the locus on a circular admittance chart which is a slight distortion of the Smith chart. Figure 17 gives the admittance locus of a 200-megacycle crystal which was plotted by this method.

Hafner²⁶ found that experimental results of Q measurements on quartz crystals at a number of frequencies indicate Q times frequency is a constant in perfect crystals. Figure 18 shows the measured Q 's of the best AT cut crystals available. At frequencies above 100 megacycles, the Q is not sensitive to lattice imperfections and the measured Q 's fall on the ideal straight line. However, at lower frequencies lattice imperfections cause measured Q 's to be below the straight-line values predicted for perfect crystals. An important result of this relationship between frequency and Q , as pertains to crystal oscillators,

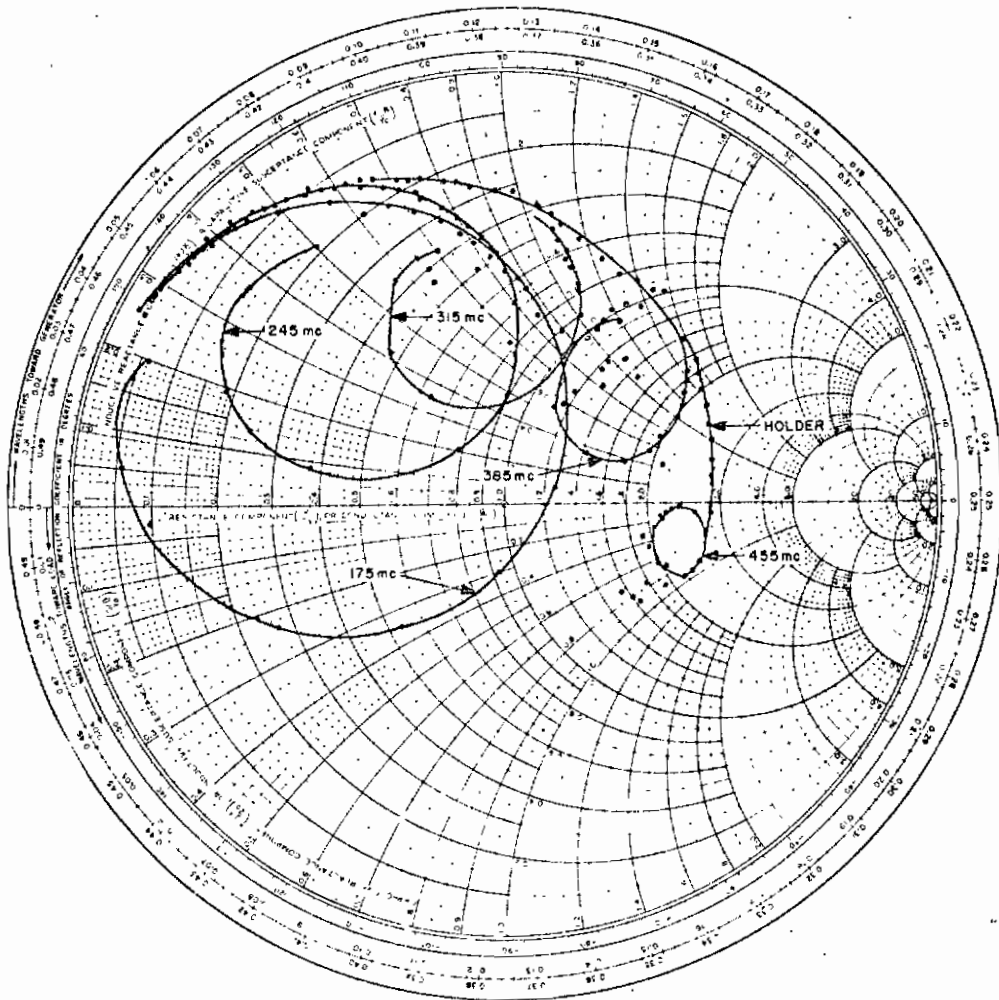
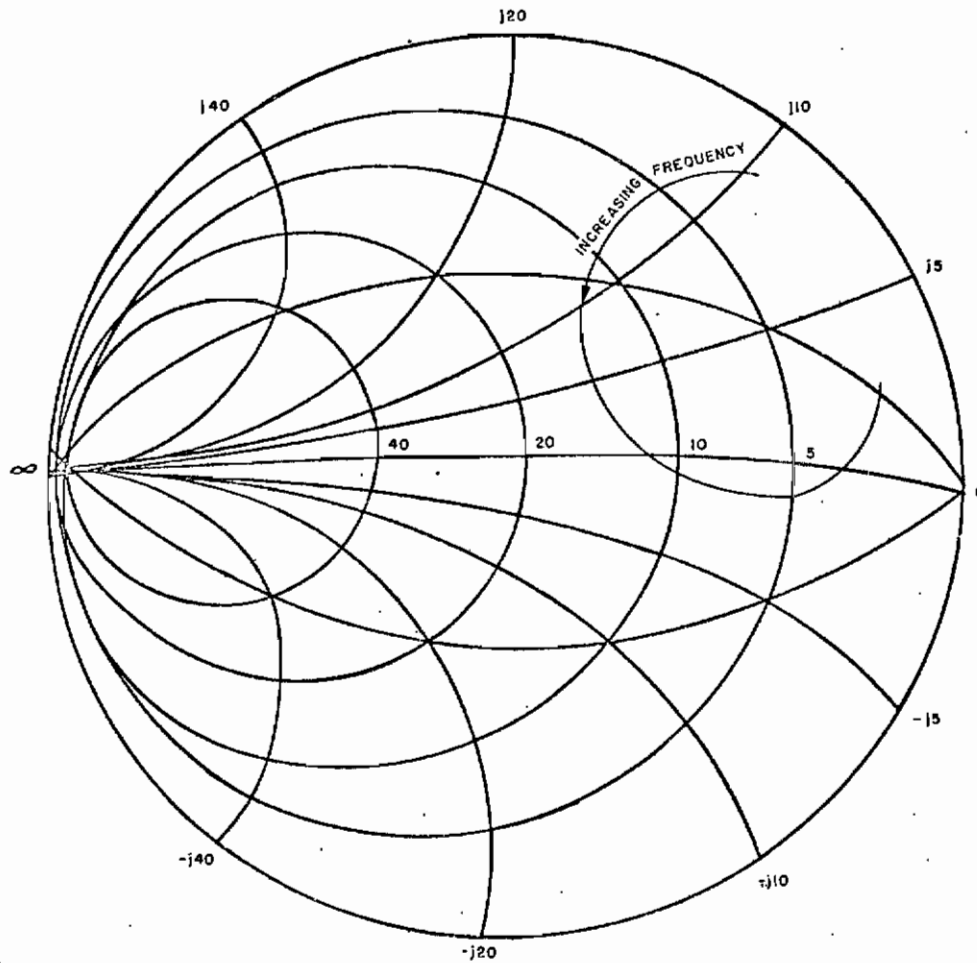


Figure 16. Overtone and Holder Responses of a Quartz Crystal Measured by Witt



Note: All admittance values in millimhos.

Figure 17. Admittance Locus of a 200-Megacycle Crystal

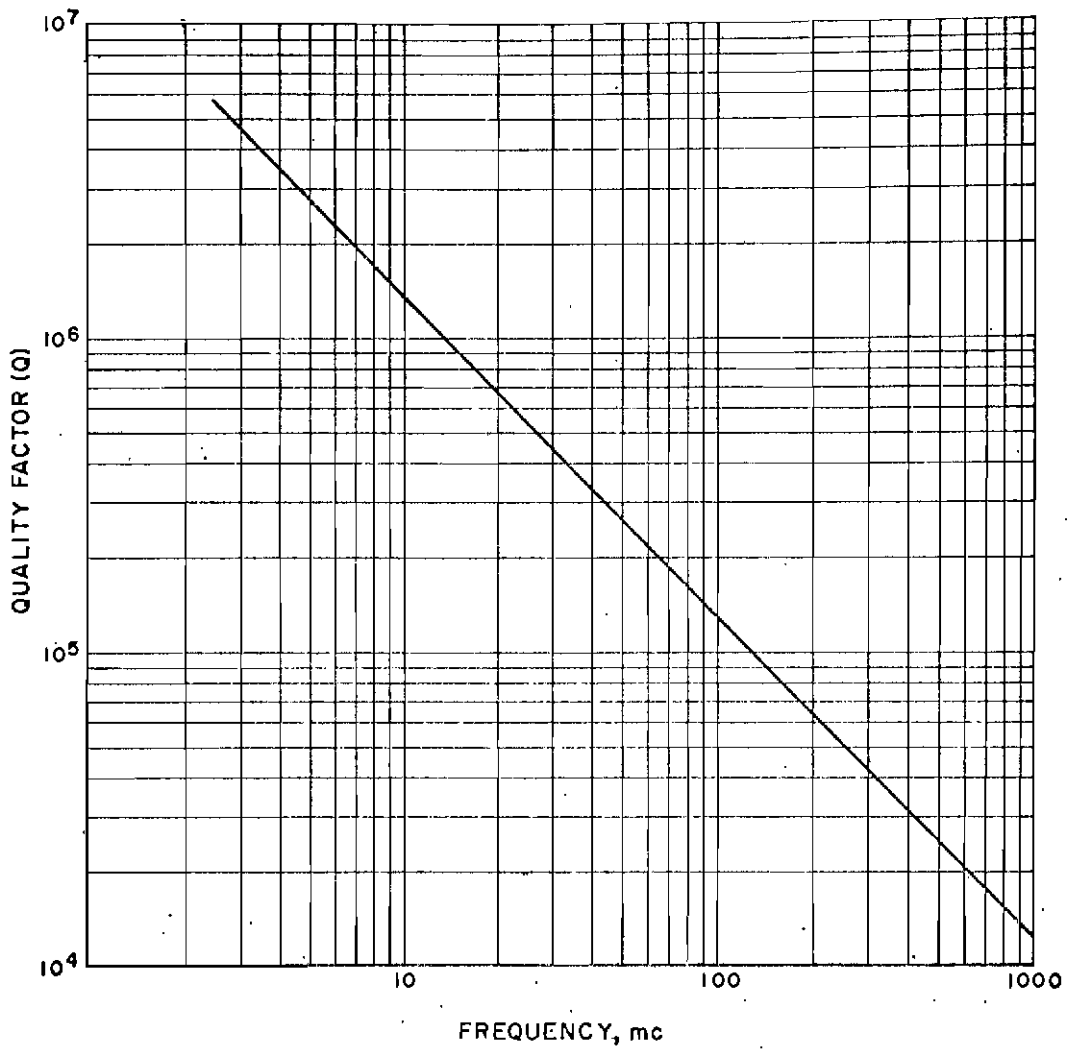


Figure 18. Q of AT Cut Quartz Crystals as a Function of Frequency

is that better frequency stability can be obtained at frequencies from 1 to 30 megacycles than at VHF because of the higher Q's available.

The resonant frequency of a crystal is a function of the temperature as shown in Figure 19 for four crystal cuts. The oscillators described in Section VI use AT optimum angle crystals. Because of

the change in the crystal parameters with temperature, the frequency of oscillation of crystal oscillators is temperature dependent.

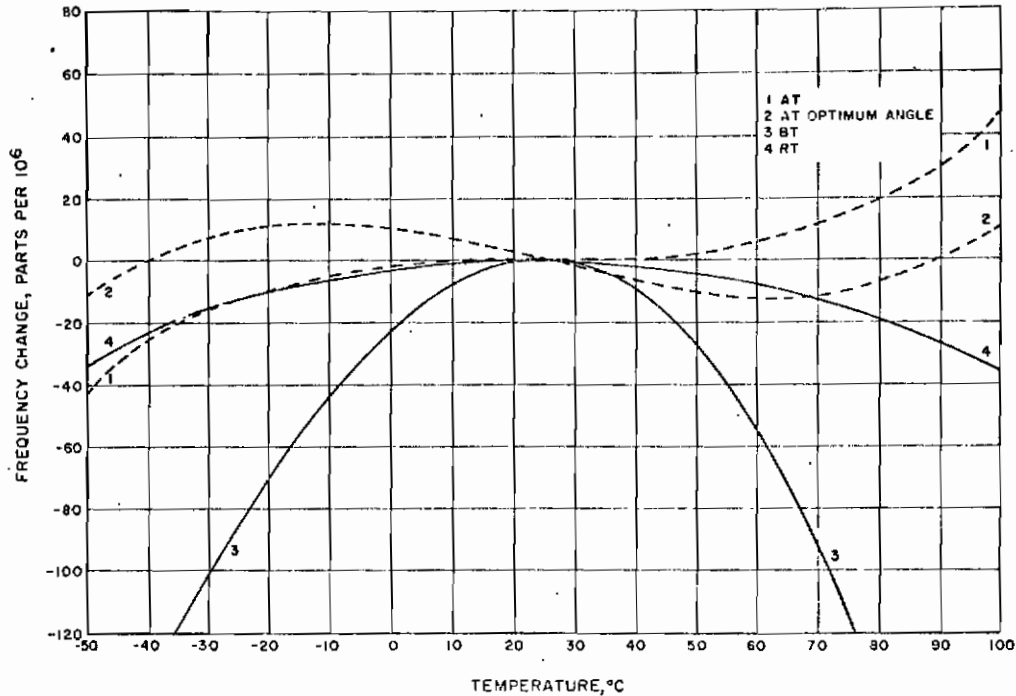


Figure 19. Variation of Crystal Resonant Frequency with Temperature

Three methods have been used to prevent changes of frequency with temperature. The oldest method is to place the crystal in an oven which keeps the crystal temperature relatively constant. An oven is used today in frequency standard oscillators. In applications where the additional power, weight, and volume of an oven are prohibitive, temperature compensation is provided by mechanical forces applied to the crystal or by a varicap-thermistor circuit. Newell²⁷ and Rennick²⁸ have designed temperature compensated crystal oscillators using varicap-thermistor circuits. Newell achieved an oscillator frequency stability of $2.5 \text{ pp}10^7$ over the temperature range -40° C to $+70^\circ \text{ C}$.

It was shown above that when frequency near resonance is varied a plot of the admittance of a quartz crystal on the Smith chart is a circle. If other circuit elements are not included in the crystal branch,

one would be limited, in the Pierce oscillator for example, to a condition where the value of Y_3 could lie on the crystal admittance circle and could satisfy the oscillator equation. This handicap of being restricted to values of Y_3 that fall on the crystal admittance circle can be removed by including series and parallel reactances in the crystal branch to transform the admittance to any desired value. Since the transformation is bilinear the transformed admittance curve is also a circle.

Figures 20 through 23 give some of the transformations that can be obtained. These curves were plotted on the admittance plotter described in the appendix. Points marked 0 kc are at 200 megacycles. Points marked +1 kc are one kilocycle above 200 megacycles, and points marked -1 kc are one kilocycle below 200 megacycles.

Figure 20 gives the admittance circles for several different values of susceptance in parallel with the crystal. Curves one through five were traced on the admittance plotter by varying frequency near the resonant frequency of the crystal. Curve one was made with a capacitive susceptance in parallel with the crystal. Curve two was made with only the crystal. Curve three was plotted with an inductive susceptance in parallel with the crystal which almost canceled the effect of C_0 . Curves four and five were made with the same circuit as curve three but with more inductive susceptance in parallel with the crystal.

Figure 21 gives the admittance circles for five different values of susceptance in series with the crystal.

The curves in Figure 22 were plotted with a fixed parallel susceptance and five different values of series susceptance.

The curves in Figure 23 were plotted with a larger fixed parallel susceptance than that used in Figure 22 and five different values of series susceptance.

When the frequency was constant at the crystal series resonant frequency, the major effect of a change in parallel susceptance was to move the admittance point around the chart parallel to the constant conductance circles. When the frequency was constant at the crystal series resonant frequency, the major effect of the change in series susceptance was to move the admittance point along a constant resistance curve. This curve is shown in Figures 21, 22, and 23.

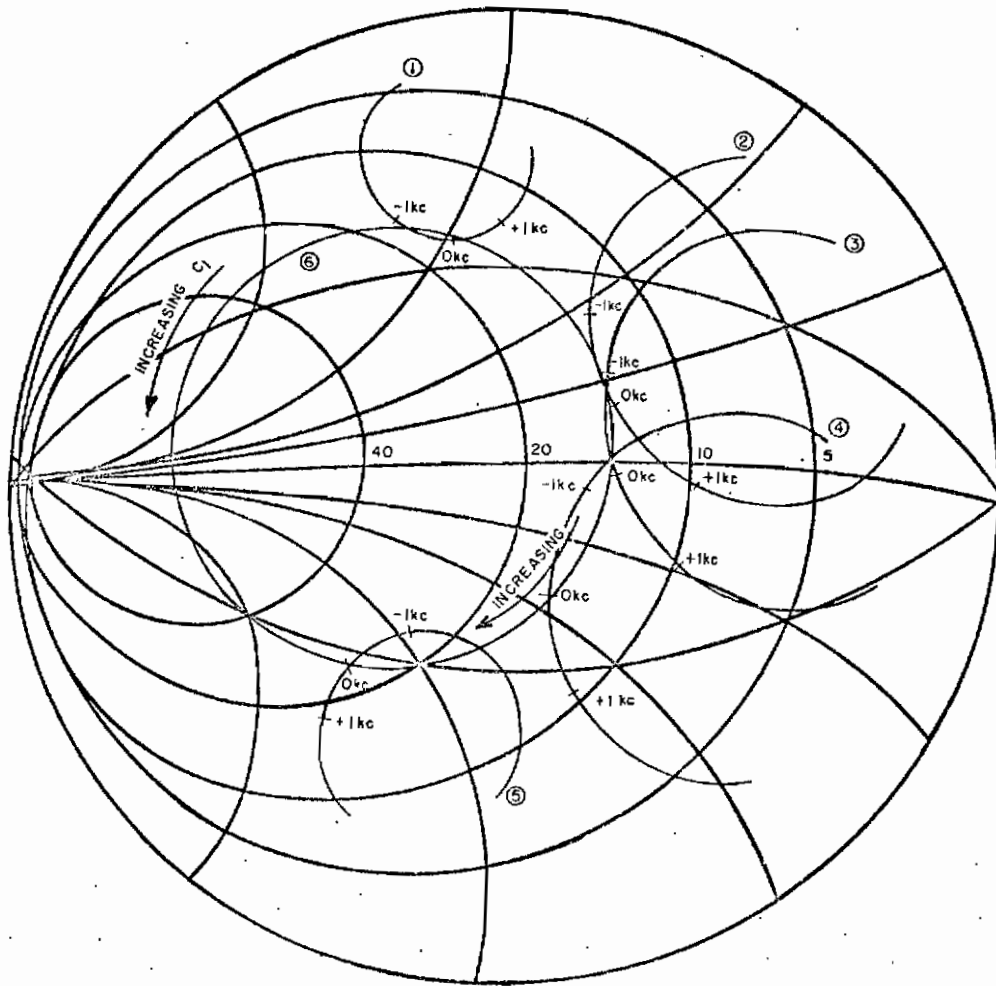


Figure 20. Admittance Circles for Five Different Susceptances in Parallel with a 200-Megacycle Crystal

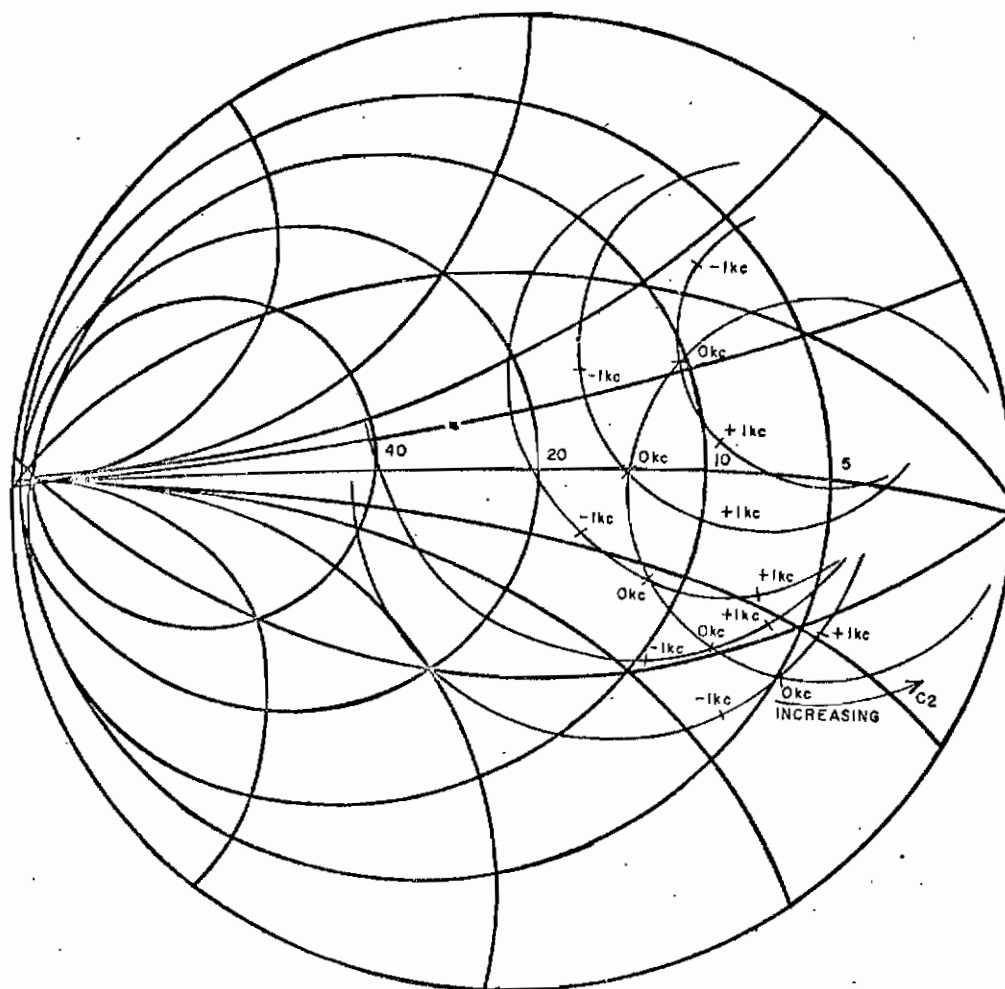


Figure 21. Admittance Circles for Five Different Susceptances in Series with a 200-Megacycle Crystal

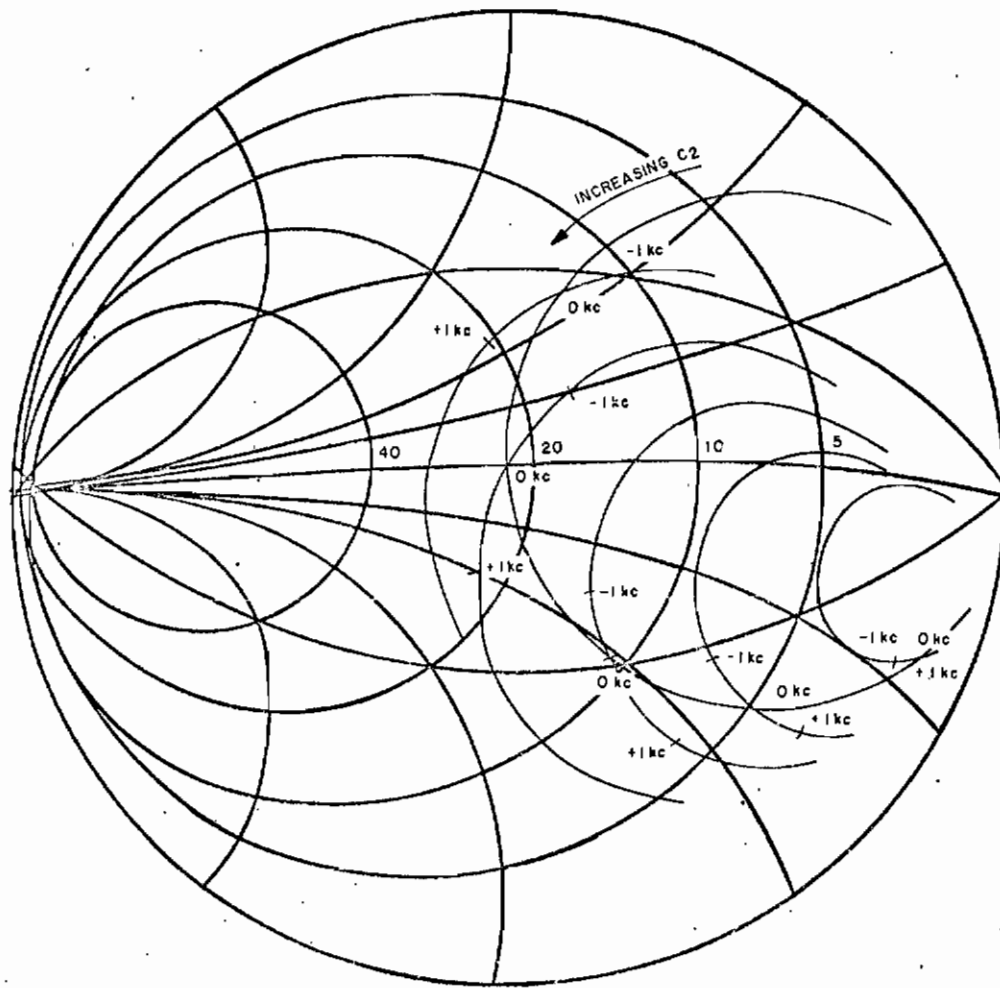


Figure 22. Admittance Circles of a Series-Parallel Circuit Consisting of a Fixed Susceptance in Parallel with a 200-Megacycle Crystal and Alternately Five Different Series Susceptances

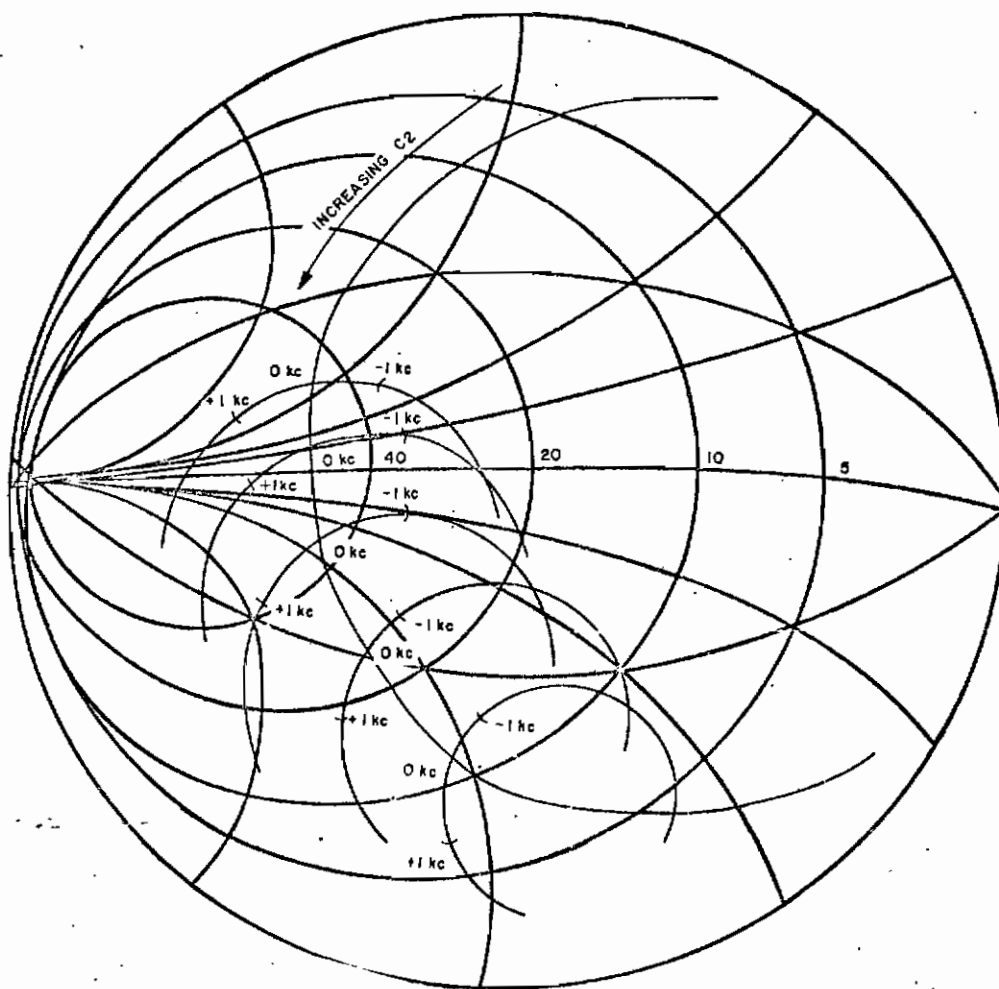


Figure 23. Admittance Circles of a Series-Parallel Circuit Consisting of a Larger Fixed Susceptance than in Figure 22 in Parallel with a 200-Megacycle Crystal and Alternately Five Different Series Susceptances

The final step in the design procedure given in Section V is to choose a suitable transformation to place the admittance of the crystal branch at a calculated value.

In Section III it was assumed that a transistor amplifier could be represented by the equivalent circuit given in Figure 5. This is a two-port parameter characterization of a linear active network where a transistor and its bias circuit form the network. The parameters of a linear active network are determined from a set of measurements made at the ports and no direct relation to the physical, internal properties is considered. At given frequency, temperature, and bias point, the two-port parameters exactly characterize the transistor. A two-port network representation of a linear active network is given in Figure 24.

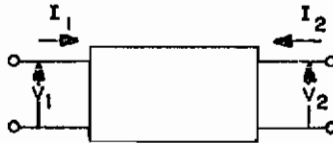


Figure 24. Two-Port Network

In general, any of six possible sets of parameters can be used to describe the network. (See Gartner.³) The admittance set of parameters is chosen to simplify the oscillator design calculations and because the y parameters are easily measured on the general radio admittance bridge. The admittance parameters of the general two-port network of Figure 24 are defined as follows:

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0} \quad = \text{input admittance for radio frequency short-circuited output.}$$

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1 = 0} \quad = \text{reverse transfer admittance for radio frequency short-circuited input.}$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2 = 0} \quad = \text{forward transfer admittance for radio frequency short-circuited output.}$$

$$y_{22} = \frac{I_2}{V_2} \left| \begin{array}{l} = \text{output admittance for radio frequency short-} \\ \text{circuited input.} \\ V_1 = 0 \end{array} \right.$$

An equivalent circuit using the above admittance parameters is given in Figure 25. If y_{12} is zero, the circuit of Figure 25 reduces to the circuit given in Figure 5.

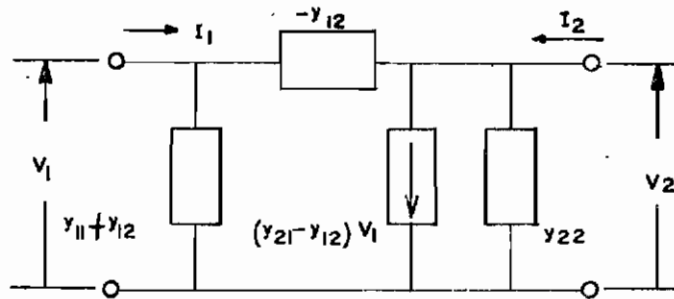


Figure 25. Transistor Equivalent Circuit

The operating point of the transistor is chosen such that the transistor has the desired radio frequency parameters and such that these parameters are linear for small voltage and current variations. The operating point will change from the design point since the operating point is a function of quantities having unavoidable variations. These variations occur with changes in bias supply voltage and temperature and with component aging. Changes in temperature produce changes in the transistor characteristics I_{CO} , α_0 , and V_{eb} ; and changes in the bias resistors which in turn affect the operating point of the transistor. The small signal parameters also vary with temperature even if the operating point does not change. This strong tendency of the transistor parameters to vary would prohibit the use of transistors in high stability oscillators without special biasing circuits. Shea²⁹ has described a single battery bias network for stabilization of the operating point which is employed in the oscillator circuits described in Section V. Small variations will still occur in the transistor parameters unless the environment is carefully controlled.

In a transistor oscillator circuit any change in the amplifier y parameters requires a corresponding change in the parameters of the feedback network if the circuit continues to oscillate. This change in

feedback network parameters is accomplished by a change in frequency since the impedance of reactive elements in the network are functions of frequency. In a crystal controlled oscillator the high Q crystal provides nearly all of the required reactance change in the feedback network and does so with a frequency change that is only slight. Utilization of the design conditions giving maximum frequency stability as derived in Section II reduces the frequency change to a minimum.

The admittance parameters of the transistor amplifier can be modified by adding an admittance in one or more of the terminal leads. (See Gartner.³) Such a modification can be used to help satisfy the conditions for maximum frequency stability found in Section II. Formulas for the modified parameters y'_{11} and y'_{21} for the admittance (y) positioned as shown in Figure 26 are calculated below.

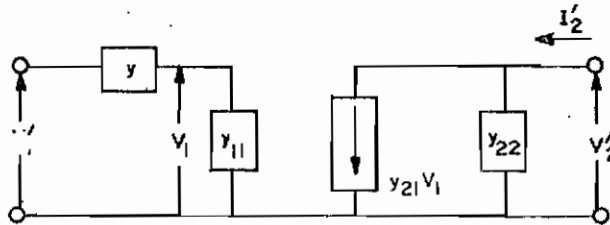


Figure 26. Modified Transistor Equivalent Circuit

The input admittance with the output short-circuited is

$$y'_{11} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0} = \frac{1}{1/y + 1/y_{11}}$$

or

$$y'_{11} = \frac{y y_{11}}{y_{11} + y} \quad (32)$$

The defining relation for the forward transfer admittance is

$$y'_{21} = \left. \frac{I_2}{V_1} \right|_{V_2 = 0}$$

When $V_2 = 0$, it follows that $I_2 = y_{21}V_1$. Consequently,

$$y_{21} = \frac{y_{21}V_1}{V_1} \quad (33)$$

From Figure 26 it can be seen that

$$\frac{V_1}{V_1} = \frac{y}{y_{11} + y} \quad (34)$$

Substituting Equation (34) into Equation (33) gives

$$y_{21} = \frac{y_{21}y}{y_{11} + y} \quad (35)$$

Equations (32) and (35) can be used to calculate the modified y_{11} and y_{21} parameters of the two-port network. y_{22} is unchanged by this modification.

Section V. DESIGN OF SOLID STATE CRYSTAL OSCILLATORS

The crystal oscillator design method is divided into design of the transistor amplifier and design of the feedback and output networks. The conditions for maximum frequency stability derived in Section II, the transformations of the crystal admittance circle described in Section IV, and the modification of the transistor amplifier parameters described in Section IV are applied.

The following procedure is for oscillators where the crystal is in the Y_3 branch and the maximum frequency stability conditions $B_1 = B_2 = -B_f$ and $G_1, G_2,$ and G_f small are used. The same general procedure could be followed if the crystal were in the Y_1 and Y_2 branches. In that case, other maximum frequency stability conditions would be used and Y_3 would be interchanged with the branch admittance containing the crystal everywhere they appear in the following procedure.

1. Amplifier Design

- 1) A transistor is selected having high gain at the desired frequency of oscillation. The gain will be sufficient if the maximum frequency of oscillation of the transistor is three or four times higher than the desired oscillator frequency. Either silicon or germanium transistors may be used, however, silicon transistors are less sensitive to temperature changes.
- 2) The manufacturer's data sheet usually gives a bias point which may be used in the oscillator. A bias point may also be chosen by observing the transistor characteristics on a curve tracer and picking a point on the curves where β is high and the curves are linear.
- 3) Either a grounded base or grounded emitter amplifier configuration is chosen.
- 4) The transistor admittance parameters are measured at the bias point selected above and at the desired oscillator frequency. The amplitude of the signal applied to the transistor by the measuring instrument should be less than 10 millivolts.
- 5) A bias network required to establish the bias point is then designed. Several different bias networks may be used. A good type network and a simple procedure for calculating the resistor and supply voltage values are given in Section VI.

- 6) The transistor amplifier parameters are modified to make G_1 and G_f smaller by placing the admittance (y) in series with the amplifier input. y_{11} and y_{21} are calculated using Equations (32) and (35) respectively.

2. Feedback and Output Network Design

- 1) The design condition $B_1 = -B_f$ is first applied. The forward transfer admittance is defined by $Y_f = y_{21} = G_f + jB_f$. The admittance Y_1 , as defined in Section II, is the output admittance of the transistor amplifier plus the total admittance (Y_O) which is connected directly across the output of the amplifier. The oscillator load is included in Y_O .

$$Y_1 = Y_{22} + Y_O$$

or in terms of the conductive and susceptive components

$$Y_1 = G_{22} + G_O + jB_{22} + jB_O \quad (36)$$

The design condition requires

$$B_1 = B_{22} + B_O = -B_f$$

$$B_O = -B_f - B_{22} \quad (37)$$

The value required for B_O is calculated using Equation (37). G_O should be small for best frequency stability; however, as G_O is made smaller, power output decreases. Ten millimhos was found to be a reasonable value for G_O in the oscillators built.

- 2) Once G_O and B_O are determined the output network is adjusted to the desired admittance. One type of output network is given in Figure 27.

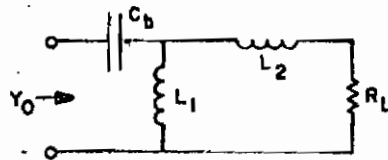


Figure 27. Output Network

The capacitor C_b is a blocking capacitor. R_L is the load resistance. If R_L is known, the required values for L_1 and L_2 can be determined with the aid of a Smith chart. First, the load resistance point and the Y_0 point are marked on the Smith chart normalized with respect to R_L as shown in Figure 28. A circle is drawn in on the chart having the same radius as the unit constant resistance circle on which R_L lies and tangent to the outermost circle at the zero reactance point. The intersection of this drawn-in circle and the constant conductance circle on which Y_0 lies determines the value for L_1 . The susceptance of L_1 is equal to the difference between the susceptance at the point of intersection and the susceptance of Y_0 . Next, draw a line, as shown in Figure 28, through the point of intersection and the center of the chart. The point of intersection of this line with the constant resistance circle on which R_L lies determines the required reactance of L_2 . In some cases L_1 and/or L_2 may need to be replaced by capacitances.

L_1 and L_2 are measured on an admittance bridge and then Y_0 is measured on the admittance bridge after the network is connected. The circuit susceptances can be varied over a wide range by varying the lead lengths to obtain the desired value. The output network should be connected into the circuit with the same lead lengths and position of components as when the admittance measurements were made.

- 3) Y_1 is calculated using Equation (36).
- 4) The admittance connected across the amplifiers input needs to be only a susceptance, therefore

$$Y_2 = y_{11} + jB_{c2} \quad (38)$$

or

$$Y_2 = G_{11} + jB_{11} + jB_{c2}$$

The design condition requires

$$B_1 = B_{11} + B_{c2} = -B_f$$

$$B_{c2} = -B_f - B_{11} \quad (39)$$

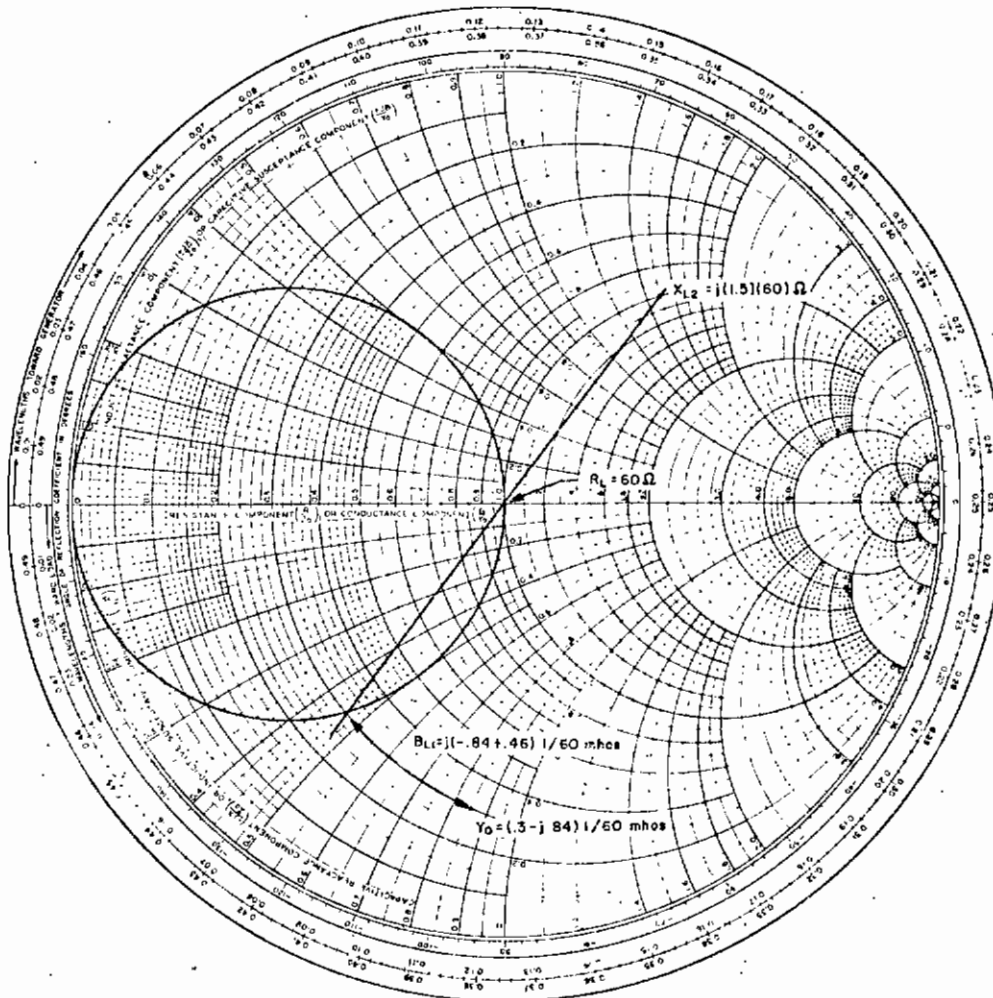


Figure 28. Graphical Construction for Design of an Output Network

The value required for B_{C_2} is calculated using Equation (39). The susceptance B_{C_2} is measured on an admittance bridge and adjusted to the required value. The comments above on measurements and connections apply equally here.

- 5) Y_2 is calculated using Equation (38).
- 6) Y_3 is calculated using Equation (15).
- 7) Y_3 is measured on the admittance plotter and adjusted to the value calculated above. Y_3 is the transformed crystal admittance. The discussion on transformations of the crystal admittance circles and Figures 20 through 23 are used as aids in obtaining the desired transformation.
- 8) Y_1 , Y_2 , and Y_3 are connected to complete the oscillator circuit.
- 9) The circuit will oscillate near the crystal resonant frequency. Adjust the frequency slightly by varying the series element in the crystal network.

Section VI. DESIGN EXAMPLES

The design procedure given in Section V was used in the construction of a Pierce oscillator. Figure 29 shows the oscillator circuit. A T2028 transistor, in a common emitter configuration, was chosen as the active element for the oscillator. The T2028 is a germanium PNP transistor designed for use as a VHF or UHF amplifier, and has a maximum frequency of oscillation of typically 1600 megacycles. The quartz crystal is a ninth overtone 200-megacycle crystal manufactured by the McCoy Electronics Company.

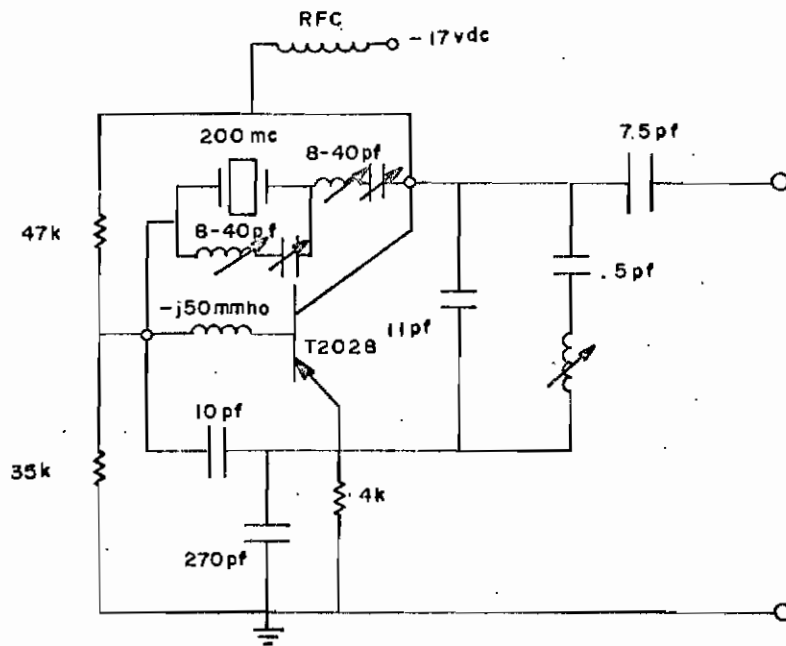


Figure 29. 200-Megacycle Crystal Oscillator

The transistor was biased by the circuit shown in Figure 30. Shea²⁹ described this type bias circuit, which helps stabilize the transistor operating point against changes in temperature and supply voltage. The bias point recommended by the transistor manufacturer ($V_{ce} = -11v$, $I_C = 1.5 ma$) was chosen. To determine the bias resistors and supply voltage values required to establish this bias point, the Thevenin equivalent for E_{dc} and the voltage divider $R_a - R_b$ was first found.

The Thevenin voltage (E_1) is

$$E_1 = \frac{E_{dc}R_b}{R_a + R_b} \quad (40)$$

The Thevenin resistance (R_1) is

$$R_1 = \frac{R_aR_b}{R_a + R_b} \quad (41)$$

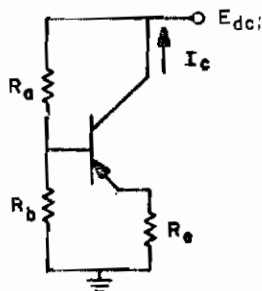


Figure 30. Shea Bias Circuit

Figure 30 is redrawn in Figure 31 using the Thevenin equivalent for E_{dc} and the voltage divider portion of the circuit.

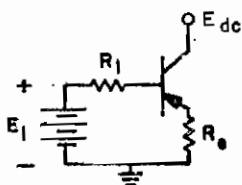


Figure 31. Equivalent Bias Circuit

Angelo³⁰ has shown that the ratio $\frac{R_1}{R_e}$ should be small for good bias point stability. Accordingly, a value of five was chosen for the ratio.

So that no appreciable conductance would be added across the input of the amplifier by the bias circuit, a value of 20 kilohms was chosen for R_1 . Any conductance added across the amplifier input will reduce frequency stability, as shown by the conditions for maximum frequency stability, and will reduce the gain of the amplifier by allowing part of the input current to flow through the added conductance rather than into the base of the transistor.

R_e was calculated by substituting $R_1 = 20$ kilohms into the ratio

$$\frac{R_1}{R_e} = 5$$

The resulting value for R_e was four kilohms.

Substitution of the circuit quantities into the equation

$$E_{dc} \approx R_e I_c + V_{ce}$$

resulted in a value of -17 volts for the supply voltage (E_{dc}).

E_1 was calculated using the equation

$$E_1 = R_e I_c + I_b R_1$$

E_1 was found to be -7.2 volts. The E_1 and E_{dc} values were substituted into Equation (40), and the R_1 value was substituted into Equation (41). These two expressions were solved for R_a and R_b . The required resistance values were found to be

$$R_a = 47.3 \text{ K}$$

and

$$R_b = 34.8 \text{ K}$$

Next, the transistor y parameters were measured at 200 megacycles with the transistor properly biased. The measured parameters were:

$$y_{11} = 10 + j8.4 \text{ millimhos}$$

$$y_{21} = 10.2 - j26 \text{ millimhos}$$

$$y_{22} = j1.4 \text{ millimhos}$$

$$y_{12} = 0$$

Contrails

The parameters were modified to help satisfy the conditions for maximum frequency stability by placing an admittance of $-j50$ millimhos in the base lead. The modified parameters were:

$$y_{21} = 4.48 - j32.3 \text{ millimhos}$$

and

$$y_{11} = 13.6 + j6.8 \text{ millimhos.}$$

Since smaller conductive components are desired for y_{21} and y_{11} , the modification gives some improvement by reducing the conductive component of y_{21} . However, part of the advantage of the modification is offset by the increase in the conductive component of y_{11} .

Following the procedure given in Section V, Y_2 was adjusted to be $13.6 + j32.3$ millimhos in order to meet the $B_2 = -B_f = 32.3$ millimhos condition for maximum frequency stability.

With a 50-ohm oscillator load the output network was adjusted to a value of $10 + j30.9$ millimhos at the amplifier output. Y_1 was then equal to $10 + j32.3$ millimhos which satisfies the $B_1 = -B_f = 32.3$ millimhos condition for maximum frequency stability.

The admittance of the crystal branch (Y_3) was calculated, using Equation (15), and adjusted to the value $.5 - j27.5$ millimhos by varying the variable capacitors and inductors in the crystal branch. The circuit oscillated at 199,987,500 cycles per second when Y_3 was adjusted to this value.

A second Pierce oscillator was constructed using the given design method. The active element was a 2N917 transistor (NPN) in a grounded emitter configuration. Stable oscillations were observed at 200,005,500 cycles per second when the ninth overtone 200-megacycle quartz crystal was connected.

Section VII. RESULTS

The oscillator described in Section VI was tested to determine its short-time stability, frequency change with temperature, frequency change with supply voltage, and spectral purity. Tests were also made with the circuit altered from the maximum frequency stability condition to demonstrate the poorer stability of other designs.

Figure 32 shows the short-time frequency stability of the oscillator during the first 44 seconds after the supply voltage was connected. A Hewlett-Packard frequency counter, having an accuracy of $\pm 7\text{pp}10^{10}$ per second, counted the frequency over a two-second period. The major cause of instability was probably temperature change of the crystal, since it can be seen from Figure 33 that a change of only 0.1°C would change the frequency by $3.3\text{pp}10^8$.

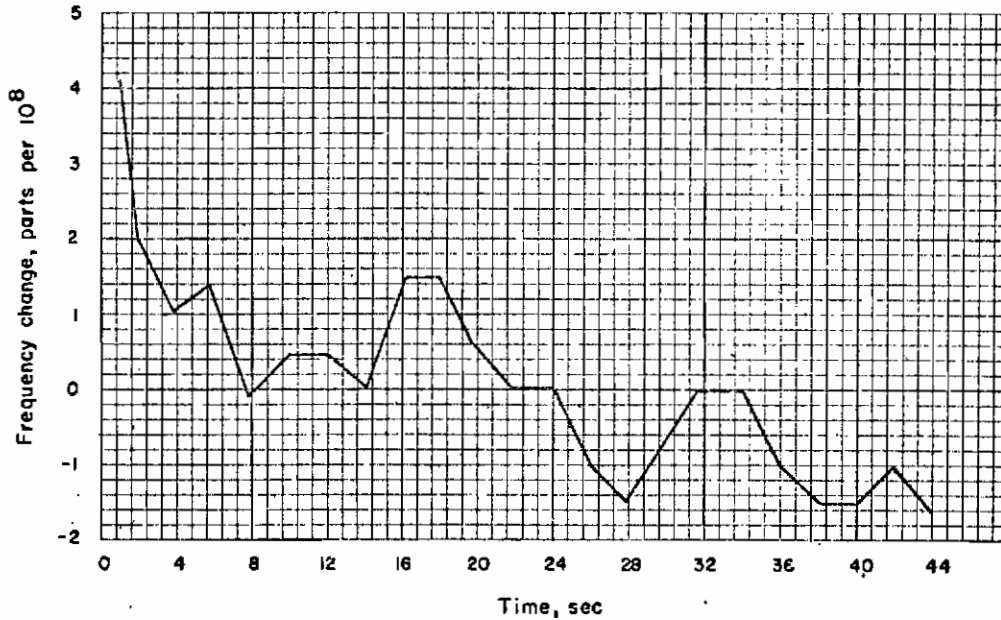


Figure 32. Short-Time Frequency Stability of the 200-Megacycle Oscillator

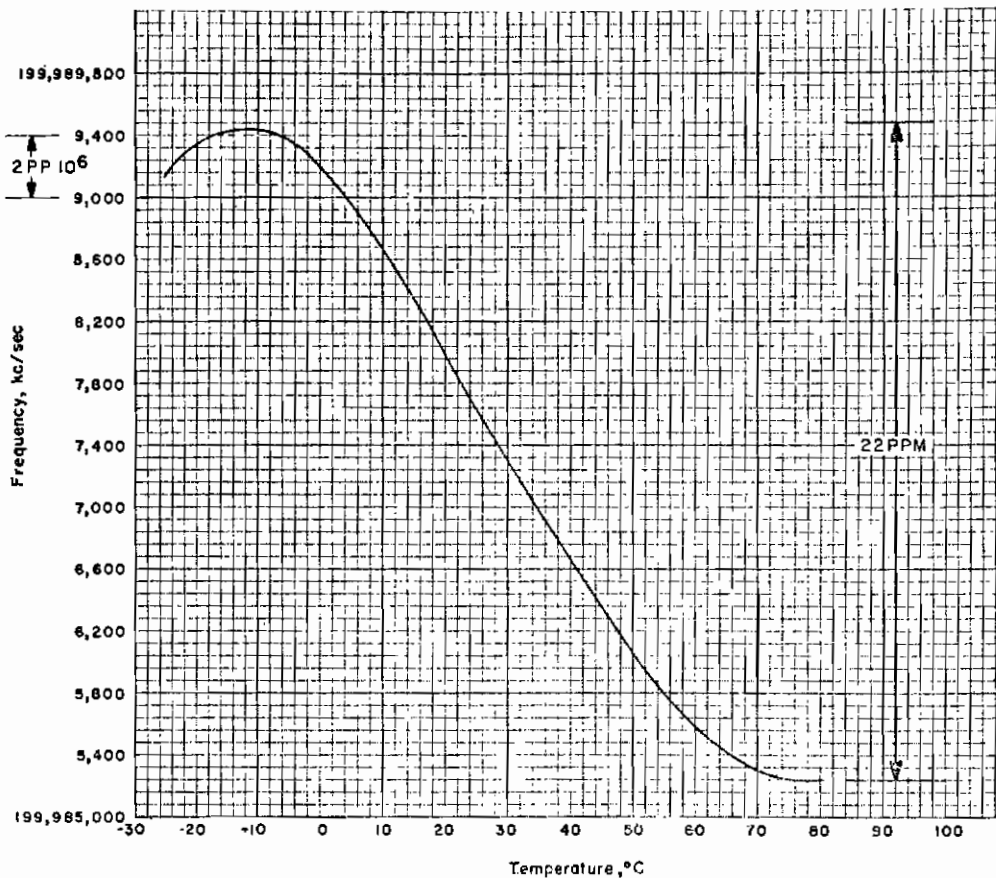


Figure 33. Frequency Versus Temperature for the 200-Megacycle Oscillator

The measured frequency of oscillation as a function of ambient temperature is plotted in Figure 33. The oscillator was placed inside a temperature chamber and temperature was measured by a thermocouple taped to the oscillator. Frequency was counted as in the short-time stability test. The curve in Figure 33 is almost identical to curve two in Figure 19, which is a plot of the crystal resonant frequency as a function of temperature. Thus, practically all of the oscillator frequency change with temperature was apparently caused by the crystal.

Figure 34 shows the oscillator frequency spectrum photographed on the Hewlett-Packard model 8551A/851A spectrum analyzer. The measurement was made with a resolution of 1 kilocycle and a dispersion of 100 kilocycles. It can be seen that sideband noise is at least

approximately 50 decibels below the center frequency at 15 kilocycles away from the center frequency.

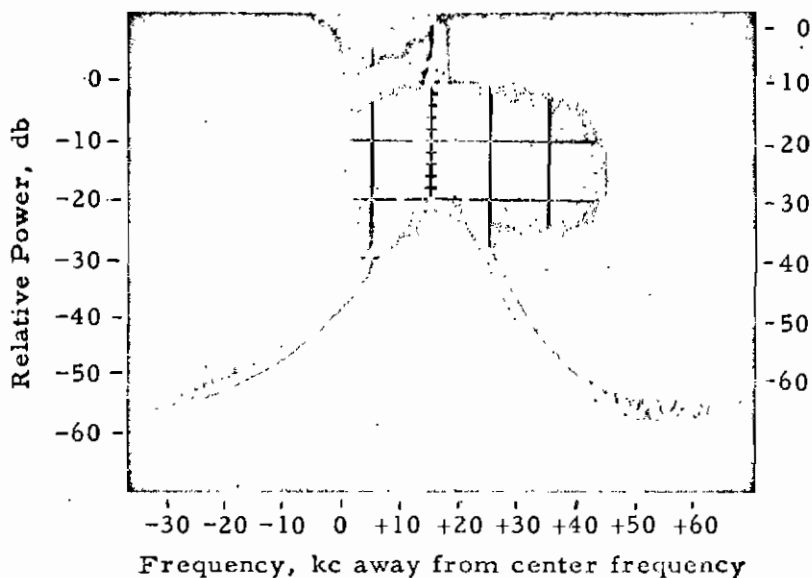


Figure 34. Frequency Spectrum of the 200-Megacycle Oscillator

Figure 35 shows the frequency change with changes in the supply voltage for five different values for B_1 . Curves are given for B_1 values both above and below $j32.3$ millimhos, the predicted value for maximum frequency stability. It can be seen that the slope of the $B_1 = 29.4$ millimhos curve is the smallest. This is the curve having B_1 nearest to the predicted optimum value.

The frequency change with change in supply voltage for five different values of B_2 is shown in Figure 36. Curves are given for B_2 values both above and below $j32.3$ millimhos, the predicted value for maximum frequency stability. Again the experimental curves have a minimum slope near the predicted value for maximum frequency stability.

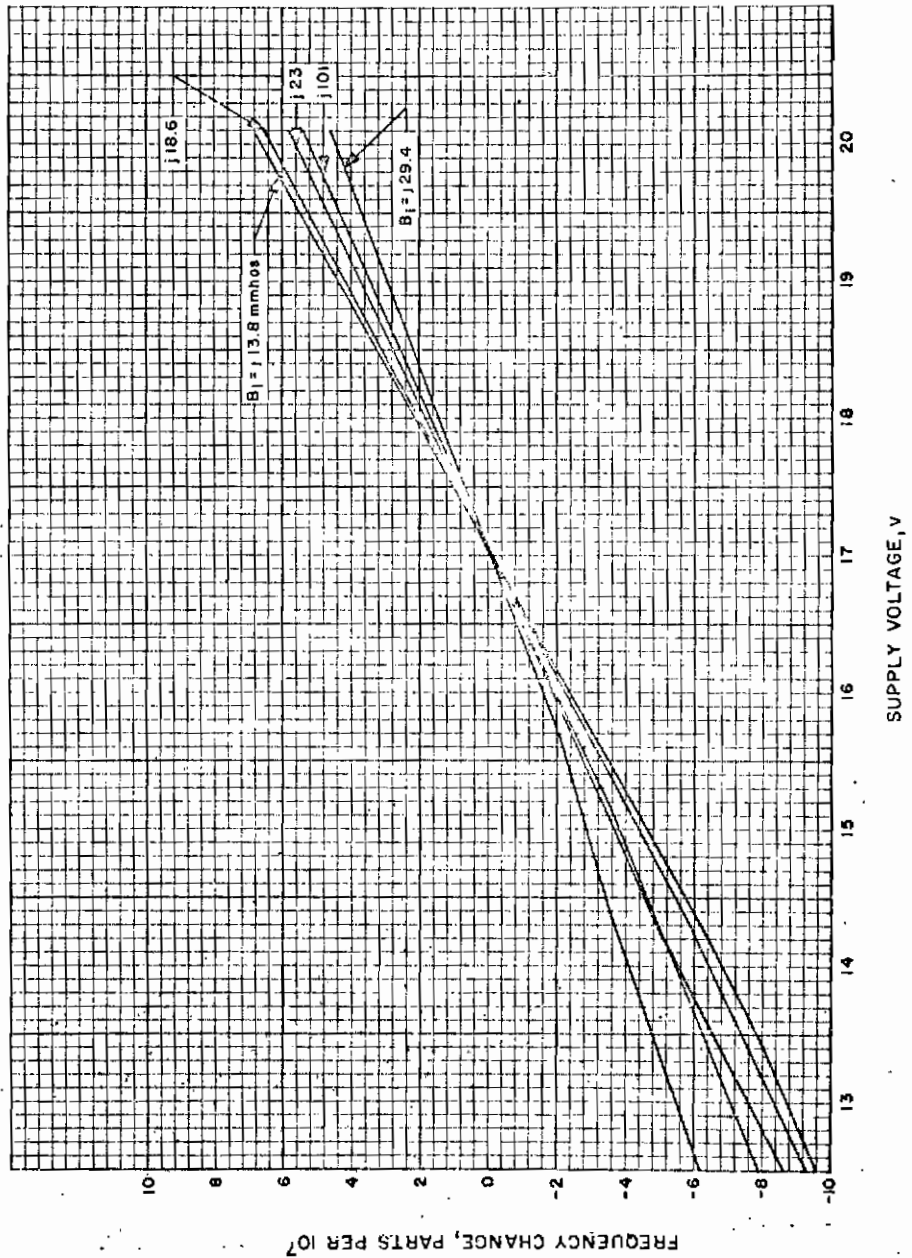


Figure 35. Frequency Change Versus Supply Voltage for Different B₁ Values

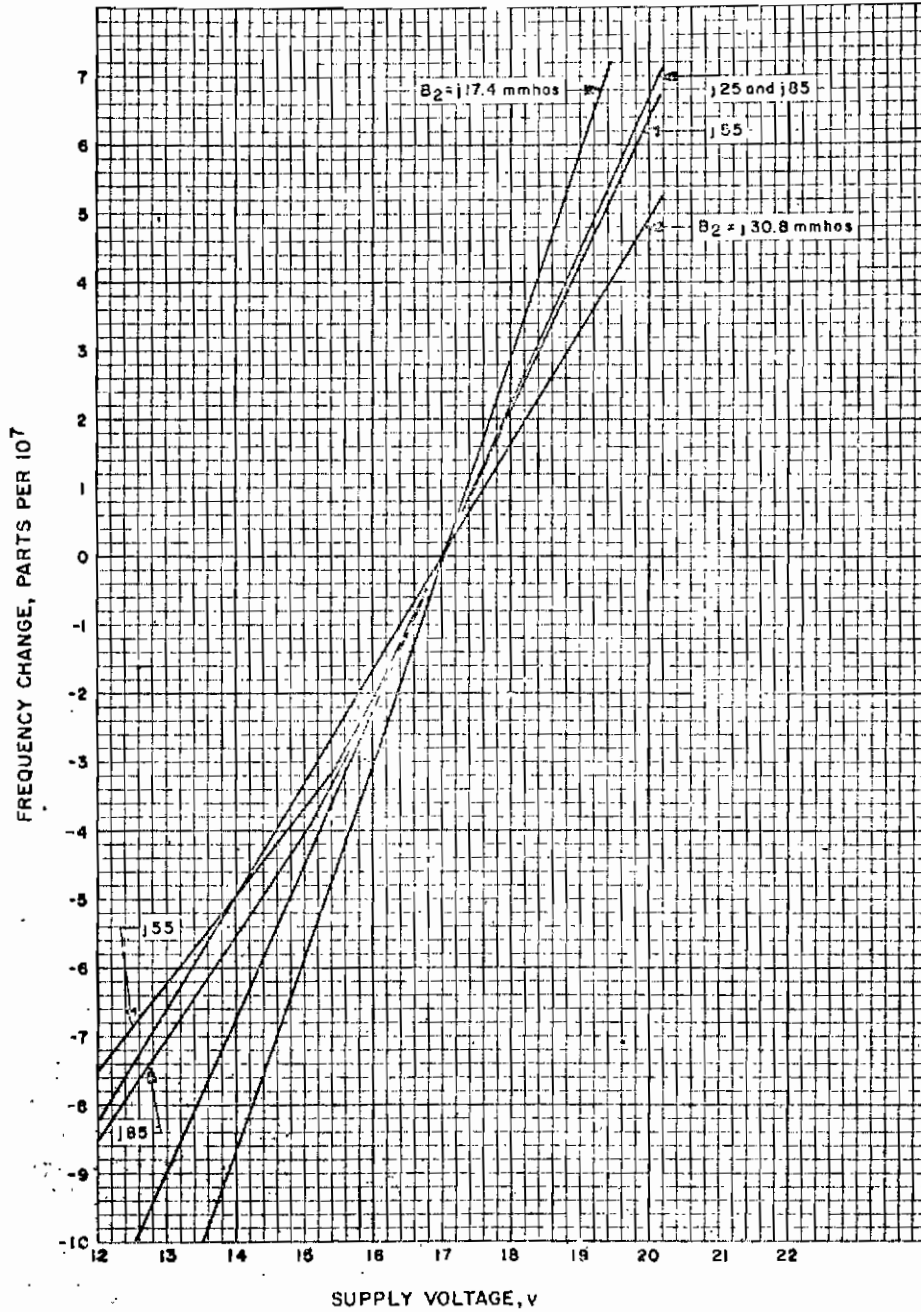


Figure 36. Frequency Change Versus Supply Voltage for Five Different B₂ Values

Section VIII. CONCLUSION

The method of designing transistorized, crystal oscillators at VHF was found to be accurate and practical. The experimental results show that when the derived design conditions for maximum frequency stability, $B_1 = B_2 = -B_f$, are met the oscillator frequency is least sensitive to changes in the supply voltage. This indicates that the oscillator has maximum frequency stability with respect to changes in the circuit parameters, Y_1 , Y_2 , and Y_f , since the effect of a supply voltage change is to change the transistor parameters.

There was an improvement by a factor of three in frequency stability with temperature and supply voltage change over oscillators designed by Firth and Yope.²³ Very little other experimental data on VHF crystal oscillators has been published which can be compared with the data given in Section VII.

A crystal oscillator operating at 200 megacycles was designed using the method described. This frequency was chosen because it is the highest frequency that crystal manufacturers specify for operation of their crystals. However, the design method could be used at higher frequencies. Crawford³¹ and Witt¹⁹ have observed crystal overtone responses above 200 megacycles. Also, it can be seen from Figure 18 that a Q of 26,000 is possible at 500 megacycles. This is sufficiently high to assure a frequency stability greater than can be had with other types of oscillators at this frequency. Thus, it apparently would be feasible to build crystal oscillators operating at frequencies up to 500 megacycles or possibly higher using the method given. The method might be applied in the microwave region to oscillators controlled by crystalline materials having higher Q's than quartz. Yttrium iron garnet, for example, has a Q almost 10 times higher than can be obtained with quartz at microwave frequencies. (See Gultwein.³²)

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Appendix. ADMITTANCE PLOTTER

The admittance plotter was devised so that the crystal branch admittance could be accurately and speedily determined. With the system shown in Figure 37, the crystal or crystal branch admittance was instantaneously plotted as a function of frequency simply by varying the frequency of the signal generator. Corresponding frequencies were read from the frequency counter. The admittance was plotted on a chart which is slightly different from a Smith chart. The special admittance chart was drawn to make the calibration of the system easier. A complete discussion of the theoretical basis and calibration procedure for the admittance measurement system is given in a previous report.³³

Power applied to the crystal during measurement was less than 0.1 milliwatt.

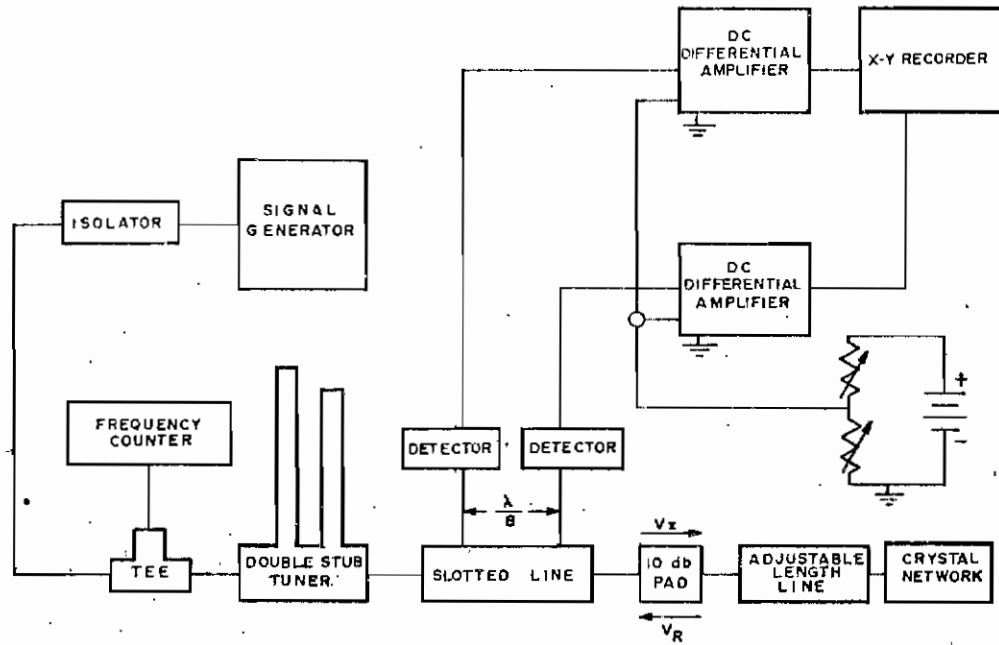


Figure 37. Admittance Measurement System

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