An Alternative To FFT For Precise Damping Estimates

Sudhakar M. Pandit
Mechanical Engineering-Engineering Mechanics Department
Michigan Technological University
Houghton, MI 49931

ABSTRACT

The prevalent methods of estimating structural damping are based on the FFT of the test data. The paper discusses the distortion introduced by FFT to show that any post-processing of the FFT will provide a poor estimate of damping, typically overestimating it; this will be the case even with curve fitting in frequency domain, or in time domain after applying inverse FFT. The paper then presents an alternative time domain methodology called Data Dependent Systems (DDS) for precise damping estimation. The DDS methodology uses difference equation models directly fitted to the time domain data for estimating damping ratios, natural frequencies, and mode shapes. After explaining the rationale of why the DDS damping estimates are more precise compared to those from FFT, the paper gives examples to highlight the precision. These examples demonstrate that the DDS is capable of precise system identification even in the presence of high damping, high modal density and considerable noise. In particular, it is shown that the frequency and the damping ratio are correctly identified even when the damping ratio is so high that the corresponding peak disappears from the authospectrum and hence it is impossible to identify it from FFT.

INTRODUCTION

Determination of structural damping is a difficult problem that can be solved only using test data in most cases. Although finite element modeling can be used to determine natural frequencies, modal testing is usually needed to provide the damping estimates necessary in predicting the realistic response from the finite element codes.

Much of the current modal testing hardware and modal analysis software is based on Fast Fourier Transform (FFT) of the test data to take advantage of the computational efficiency of FFT. However, the FFT of test data introduces distortions due to problems such as leakage and resolution, and also errors such as bias and variance (random error). Smoothing of raw FFT by averaging or windowing is necessary to reduce the effect of such errors. Such distortion and smoothing both affect the damping estimates. Therefore, damping estimates obtained by post-processing FFT of test data are usually unreliable; typically FFT overestimates damping.

This paper briefly outlines an alternative time domain methodology called Data Dependent Systems (DDS) for precise damping estimation and reviews some of the results illustrating the precision from references[1-8], which may be referred for more details. Difficulties in damping estimation from FFT of test data are discussed at the beginning. The rationale of more precise estimation by DDS is then explained. Examples from the literature demonstrating the precision of damping estimation by DDS are given at the end.

Why Does FFT Overestimate Damping?

Usually FFT and its post-processing tends to flatten a sharp peak and thus leads to overestimation of damping. The primary causes of such flattening of the peaks are discussed below briefly, more details may be found in texts such as[9-10].

1. Frequency Resolution, Leakage and Aliasing: When the record length say T of the data is not large enough, since the FFT is calculated at multiples of frequency $\frac{1}{T}$, the frequency resolution may not be adequate to capture sharp peaks. Even when the record length is increased, the leakage caused by the absence of integral number of cycles in the record length spreads over neighboring frequencies and flattens the peaks as illustrated in Figure 1. This flattening of peaks persists even when damping is high, as illustrated in Figure 2. Post-processing of FFT by averaging and windowing to reduce leakage and use of antialiasing filtering also adds damping. Figure 1 illustrates the peak flattening caused by truncation-another windowing effect.

2. Bias and Variance (Random Errors): The finite amount of data used in FFT may be considered as the original "infinite" data truncated by a rectangular "box-car" window, which introduces bias in the estimate of the peak. Since this bias is usually negative, it has the effect of increasing the damping. Moreover, when the test data is noisy, the FFT introduces variance (random error) and is consequently erratic and choppy, containing many spurious peaks in addition to the genuine ones. It is therefore usually necessary to smooth

the FFT by averaging, either simple or weighted, which applies another window that also increases damping. To make matters worse, one cannot reduce bias and variance simultaneously, a window that reduces bias inevitably increases variance and vice versa.

PRECISE DAMPING BY DDS

The DDS methodology [1-7] estimates damping by fitting difference equation models of successively higher order until the reduction in the residual sum of squares is statistically insignificant or the variance of the residuals drops below a known noise floor. The eigenvalues or characteristic roots of these models then yield the estimates of natural frequencies and damping ratios. No prior assumptions or conjectures about the model are needed, they can be obtained directly from the data by available computer routines[4]. The difference equation is in the form of an Autoregressive Moving Average model, ARMA (n,n-1):

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + a_t - \theta_1 a_{t-1} - \ldots - \theta_{n-1} a_{t-n+1}$$

where X_t is a single scalar or multiple vector series of data, ϕ_i and θ_i are scalar or matrix parameters and a_t represents scalar or vector white noise. These a_t's model and remove the noise in the data to provide modal parameter estimates from one sample without any need for averaging.

The natural frequency and damping ratio corresponding to each mode are obtained from the characteristic roots or eigenvalues λ_i defined in the scalar case by

$$(1 - \phi_1 B - \phi_2 B^2 \ldots . -\phi_n B^n) = (1-\lambda_1 B)(1-\lambda_2 B) \ldots (1-\lambda_n B).$$

If λ_i and λ_i^* are a complex conjugate pair of roots, then the corresponding damping and natural frequency terms are obtained by

$$2\zeta\omega_n = \ln (\lambda_i \lambda_i^*)/\Delta$$

$$\omega_n = \tan^{-1} \left[\frac{Im \lambda_i}{Re \lambda_i}\right] / \Delta$$

where Δ is the sampling interval and Im and Re represent imaginary and real parts respectively. The mathematical background, procedure and expressions for obtaining other parameters such as mode shapes and mass, damping and stiffness matrices from the response data may be found in references[3-7].

The number of parameters in the model fitted by DDS is limited by the actual effective degrees of freedom reflected in the response data. Therefore, as the number of data points is increased, the additional information improves the accuracy of the limited number of parameters such as mode shapes and damping ratios. This is in contrast to the FFT, where such additional information from the increased number of data points is spread over the increased number of frequencies, and the accuracy of estimates at any given frequency does not improve; subsequent processing by averaging, smoothing or windowing adds its own distortion as explained earlier. This is the basic reasons why the DDS methodology provides far more precise damping estimates than the FFT.

ILLUSTRATIVE EXAMPLES

The relative precision of DDS compared to FFT is already clear graphically from Figures 1-2. These figures show that the DDS estimates of damping almost exactly match with the actual, whereas the FFT always overestimates it. This is true at low damping, Figure 1, as well as high damping, Figure 2. We will now give examples with numerical estimates to demonstrate the precision.

Table 1 gives results from reference[3] on a simulated 3-degrees-of-freedom system using 500 data points and scalar models. Note that the results are as precise with noise as without it; the only difference is that the model order needs to be higher for noisy data to model the noise modes. Results in Table 2 from the same reference[3] then show that closely spaced mode shapes can also be resolved if high enough model order is used. Similar results on a 2-degree-of-freedom system in Table 3, taken from reference[3], show that this precision extends to mass, damping and stiffness matrices [m], [C] and [K].

Table 1: Comparison of Estimates With and Without 17% Noise

	Theoretical	Noise-Free ARMA(12,11)	Noisy ARMA(26,25)
Natural Frequency (Rad/Sec)	0.58569 1.2319 1.6003	0.58567 1.2323 1.6002	0.58503 1.2312 1.5999
Damping Ratio ζ	0.03122 0.16404 0.10206	0.03122 0.16412 0.10200	0.03118 0.16412 0.10202
Mode Shapes (Relative Amplitudes)	2.4829 3.7044 1.0420 2.0963 1.0000 0.7122	2.4788 3.7023 1.0493 2.0959 0.9988 0.7149	2.4912 3.6976 1.0493 2.0651 1.0009 0.7111

Table 2: Comparison With Closely Spaced Modes

	Theoretical	ARMA(28,27)
Natural Frequency (Rad/Sec)	0.96697 1.8881 2.0000	0.96698 1.88888 2.0009
Damping Ratio ζ	0.15512 0.07944 0.07500	0.15503 0.07965 0.07471
Mode Shapes (Relative Amplitudes)	1.0669 1.0000 -1.4366 1.0000 0.0 -1.0	1.0667 1.0009 -1.4359 1.0071 0 -1.0010

Table 3: Comparison of Parameter Matrices

	[m]	[C]	[K]
Theoretical:	$\begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$	$\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1.5 \end{bmatrix}$	$\begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix}$
DDS Estimated:	$\begin{bmatrix} 5.001 & 0 \\ 0 & 10.001 \end{bmatrix}$	$\begin{bmatrix} 1.000 & -0.5002 \\ -0.5002 & 1.500 \end{bmatrix}$	$\begin{bmatrix} 4.000 & -2.001 \\ -2.001 & 6.001 \end{bmatrix}$

Tables 4 and 5 present results from reference[7] obtained using vector models for a 2-degree-of-freedom system with light and heavy damping respectively. Note that since the vector models use multiple series of data simultaneously, their estimates are even more precise than the scalar models with the same number of data points per series. The precision of damping ratio estimates is quite good in both low and high damping environment.

Table 4: Comparison of DDS and Theoretical Mode Shapes and Natural Frequencies for ACSL-Simulated 2 DOF System With Light Damping

PARAMETER	THEORETICAL		DDS	
1ST MODE SHAPE	Mag. 1.0 3.14603	Phase(deg) 0.0 -2.57608	Mag. 1.0 3.14573	Phase(deg.) 0.0 -2.5750
2ND MODE SHAPE	1.0 0.159754	0.0 175.714	1.0 0.159753	0.0 175.7140
1ST NATURAL FREQUENCY (Hz)	68.3069		68.3067	
2ND NATURAL FREQUENCY (Hz)	114.298		114.298	
1ST DAMPING RATIO	0.0216419		0.0216420	
2ND DAMPING RATIO	0.0601704		0.0601704	

Table 5: Comparison of DDS and Theoretical Mode Shapes and Natural Frequencies for ACSL-Simulated 2 DOF System With Heavy Damping

PARAMETER	THEORETICAL		DDS	
1ST MODE SHAPE	Mag. 1.0 2.50746	Phase(deg) 0.0 14.8578	Mag. 1.0 2.50745	Phase(deg.) 0.0 14.8570
2ND MODE SHAPE	1.0 0.245211	0.0 163.0810	1.0 0.245211	0.0 163.0807
1ST NATURAL FREQUENCY (Hz)	68.7122		68.7122	
2ND NATURAL FREQUENCY (Hz)	113.624		113.624	
1ST DAMPING RATIO	0.212392		0.212392	
2ND DAMPING RATIO	0.606938		0.606936	

Some results of DDS modeling of tool vibrations from reference[8] are reproduced in Table 6. The second mode of tool vibration that is sensitive to tool wear is small compared to other modes and hence difficult to use for tool wear using FFT. Its damping increases with tool wear and the peak corresponding to it altogether disappears from the FFT plot. However, as Table 6 indicates, the DDS model continues to track it even at nearly 30% damping.

Table 6: Tool Vibration Modes With Increasing Tool Wear

Wear (mm)	First Mode		Second Mode	
	Nat. Freq. (Hz)	Damp. Ratio	Nat. Freq. (Hz)	Damp. Ratio
0.000	4340	0.007	8616	0.028
0.175	4320	0.007	8596	0.030
0.225	4276	0.007	8578	0.036
0.288	4328	0.006	8700	0.051
0.338	4354	0.009	8996	0.163
0.400	4526	0.007	9308	0.279

Finally, a comparison of DDS and FFT modal analysis of disc-brake rotor vibrations is partially reproduced from reference[6] in Figure 3. Overestimation of damping by FFT is evident for every model. This study further shows the capability of DDS in resolving repeated roots, characteristic of symmetric structures like rotors, by closely spaced modes that were illustrated by simulation in Table 2.

REFERENCES

[1] Pandit, S.M., 1977, Shock & Vibration Bulletin, 47, 161-174. Analysis of vibration records by data dependent systems.

[2] Pandit, S.M., 1977, Trans. ASME Journal of Dynamic Systems, Meausrement and Control, 99G, 221-226. Stochastic linearization by data dependent systems.

[3] Pandit, S.M., Mehta, N.P., 1984, Proceedings of the 2nd International Modal Analysis Conference, 35-43. Data dependent systems approach to modal parameter identification.

[4] Pandit, S.M., Wu, S.M., 1983, Time Series & System Analysis With Applications, New York: John Wiley.

[5] Pandit, S.M., Mehta, N.P., 1985, Trans. ASME, Jour. of Dyn. Systems, Measurement and Control, 107, 132-138. Data dependent systems approach to modal analysis via state space.

[6] Pandit, S.M., and Jacobson, E.N., 1988, Journal of Sound and Vibration, 122(2). Data Dependent Systems Approach to Modal Analysis, Part II: Applications to Structural Modifications of a Disc-Brake Rotor.

[7] Pandit, S.M., Helsel, R.J., and Evensen, H.A., 1986, Proceedings of the 4th Internaitonal Modal Analysis Conference, 414-421, Modal Estimation of Lumped Parameter Systems Using Vector Data Dependent System Models.

[8] Pandit, S.M., Suzuki, H. and Kahng, C.H., 1980, ASME Journal of Mechanical Design, 102, 233-241, Application of Data Dependent Systems to Diagnostic Vibration Analysis.

[9] Jenkins, G.M., and Watts, D.G., 1968, Spectral Analysis and Its Application, Holden-Day, San Francisco.

[10] Priestley, M.B., 1981, Spectral Analysis and Time Series, Academic Press, New York.

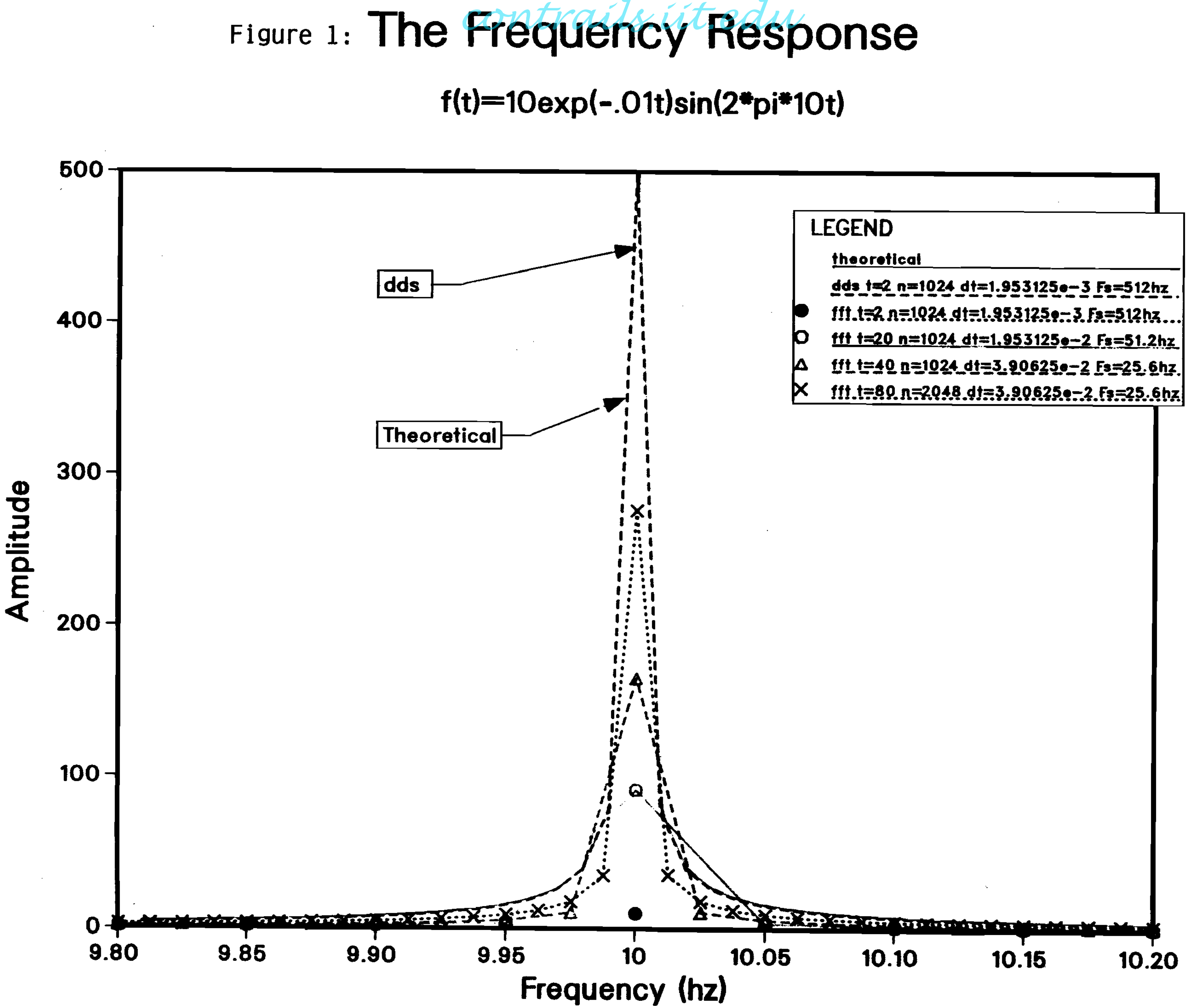
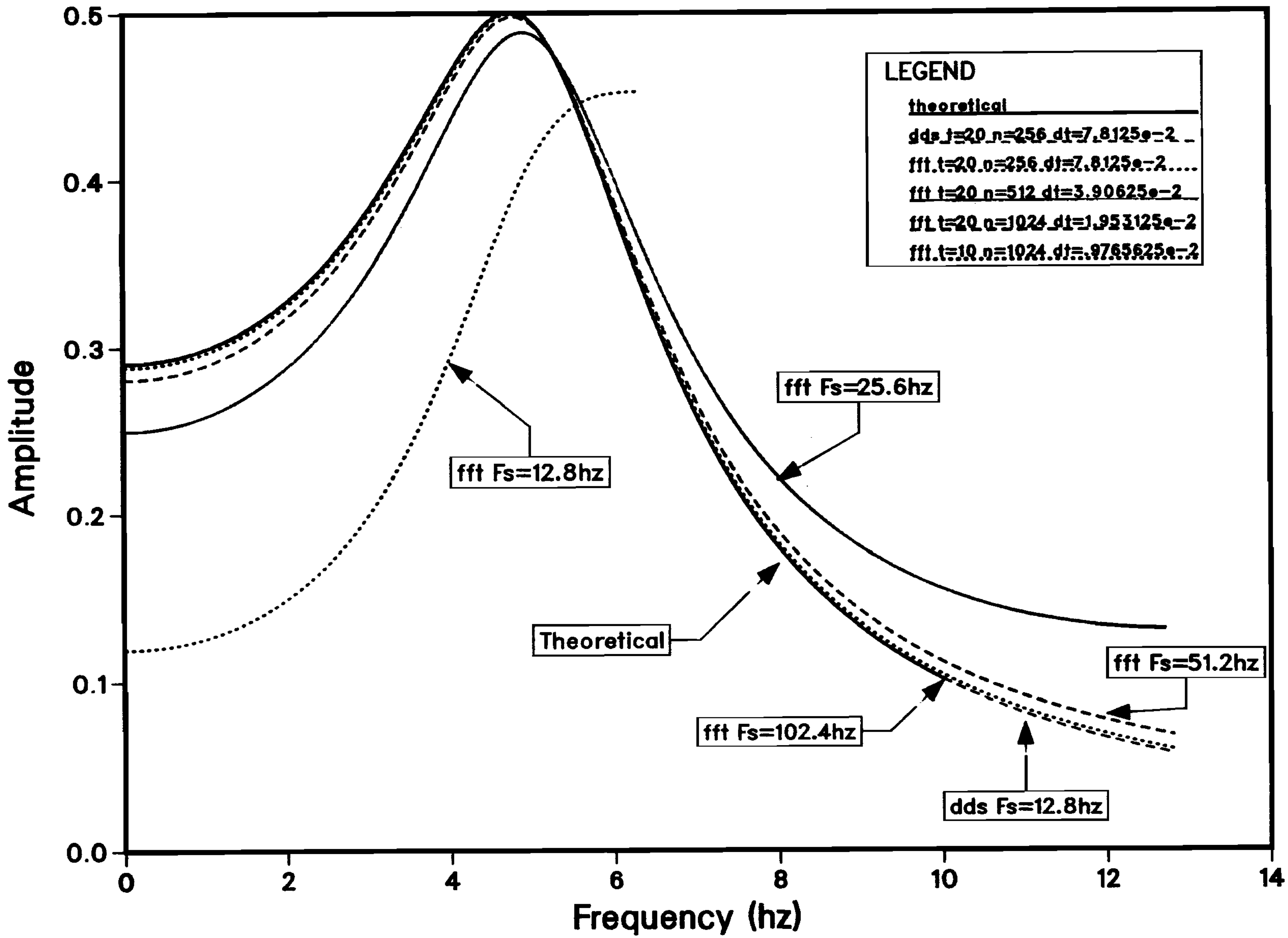


Figure 1: The Frequency Response
f(t)=10exp(-.01t)sin(2*pi*10t)
Amplitude
Frequency (hz)
dds
Theoretical
LEGEND
theoretical
dds t=2 n=1024 dt=1.953125e-3 Fs=512hz
fft t=2 n=1024 dt=1.953125e-3 Fs=512hz
fft t=20 n=1024 dt=1.953125e-2 Fs=51.2hz
fft t=40 n=1024 dt=3.90625e-2 Fs=25.6hz
fft t=80 n=2048 dt=3.90625e-2 Fs=25.6hz
500
400
300
200
100
0
9.80
9.85
9.90
9.95
10
10.05
10.10
10.15
10.20

Figure 2: **The Frequency Response of f(t)=10*exp(-10t)*sin(2*pi*5t)**



CBB-10

Figure 3: Comparison of DDS and FFT Modal Analysis of Disc Brake Rotor