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THE OPTIMUM USE OF UNCONSTRAINED LAYER DAMPING TREATMENTS

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FOREWORD

This report was prepared by the University of Southampton, Department of Aeronautics and Astronautics, Southampton, England, under USAF Contract No. AF 61(052)-504. The contract was initiated under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals" and it was administered by the European Office, Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Research and Technology Division, under the direction of Mr. W. J. Trapp.

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ABSTRACT

Since unconstrained layer damping treatments on vibrating plates dissipate energy by undergoing bending strain, the treatment in the vicinity of lines of contra-flexure contributes little to the total damping. This report investigates the effect of concentrating the treatment in the regions of highest bending moment in order to use all the material more effectively. Three methods of analysis are presented, two of them being approximate but relatively rapid in use. The third has been used in extensive digital computer calculations to determine the effectiveness of given quantities of treatment, having given stiffness, when distributed over different proportions of the length of a vibrating, simply supported strip. The effectiveness has been assessed using certain criteria which apply to different problems, e.g. to the problems of reducing harmonic resonant vibration amplitudes, or inertia forces exerted by randomly vibrating panels, etc. Different criteria relate to each problem. It has been shown that very considerable increases of effectiveness are obtained by concentrating the treatment over a proportion of the length or area. For a given quantity and stiffness of treatment, the effectiveness may be maximised by covering certain optimum proportions. The optimum proportions corresponding to different criteria differ only slightly, permitting curves to be drawn showing the 'mean' optimum coverage required for a wide range of given quantities of treatment, and for a range of treatment stiffness.

An experiment is described which confirms the validity of the theoretical approaches. The effects of temperature changes on the criteria values are also briefly considered, and it is concluded that the temperature bandwidth of the treatment when partially covering the plate is not significantly different from that when uniformly covering the plate.

This technical documentary report has been reviewed and is approved.

W. J. TRAPP
Chief, Strength and Dynamics Branch
Metals and Ceramics Division

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

a	length defining coverage of treatment
e_s	maximum amount of strain energy stored per unit length in the covered portion of the strip, per cycle
e_u	maximum amount of strain energy stored per unit length in the uncovered portion of the strip, per cycle
e_d	the energy dissipated per unit length of coated region, per cycle
E_d	real part of the complex Young's Modulus of the treatment
E_m	Young's Modulus of the material
EI	Flexural rigidity of the strip at any point
$(EI)_c$	flexural rigidity of the strip over the covered portion
$(EI)_u$	flexural rigidity of the strip over uncovered portion
f_c	normalised displacement function of fundamental modes of vibration of treated strip
f_o	normalised displacement function of fundamental modes of vibration of untreated strip
I_d	second moment of area of section of damping treatment about the composite beam neutral axis
I_m	second moment of area of basic strip section about the composite beam neutral axis
k	$(EI)_c \div (EI)_u$
$K(\alpha)$	a function of α and k (equation 14a)
ℓ	the length of the strip, or distance between nodes
m_o	mass per unit length of basic strip section
m	mass per unit length of treated portion of strip
$m(x)$,	mass per unit length of treated strip at any point
M	bending moment in the strip at any point

LIST OF SYMBOLS (Cont'd)

M_o, M_c	bending moment in the untreated and treated strip (respectively) per unit generalised displacement in the mode of vibration
p	generalised force ratio (equation 13)
p_c	percentage of strip length, or plate dimension, covered
$S(\alpha)$	a function of α and k (equation 14b)
x	lengthwise co-ordinate
y	the deflection of the strip at any point
y_o	the generalised deflection of the strip in a given mode
α	$2a/\ell$
δ	damping ratio increment due to treatment $\frac{\text{actual damping}}{\text{critical damping}}$
δ_d	the normalised damping ratio (i.e. δ/δ max.)
δ_{max}	the maximum damping ratio increment obtainable from the treatment
δ_t	the damping ratio increment obtainable from a uniform covering of treatment of thickness t
ε	energy ratio (equation 9)
η_d	loss factor of damping treatment (i.e. complex Young's Modulus is $E_d(1+i\eta_d)$)
μ	generalised mass of treated strip ÷ generalised mass of untreated strip
ρ	mass per unit length of treated portion ÷ mass per unit length of untreated portion
ξ	non-dimensional lengthwise co-ordinate (= x/ℓ)
σ	generalised stiffness of treated strip ÷ generalised stiffness of untreated strip (equation 11)
τ	thickness of treatment ÷ thickness of strip (thickness ratio)
ω	circular frequency
Y_o	the generalised deflection of the strip in a given mode

1. INTRODUCTION

Unconstrained layer damping treatments have been used for many years as a means of alleviating vibration problems of thin plates, and of attenuating the associated transmitted sound. Recent developments of these treatments have produced materials which provide very large increases of the damping ratios of the structures to which they are added and, furthermore, very considerable increases of the structural stiffness. The latter is particularly true when the structure consists basically of a thin aluminum alloy plate, such as in an aeroplane fuselage. In this context the use of damping treatments must be subjected to very careful considerations of weight economy. The optimum usage of the treatment must therefore be sought to provide the maximum attenuation of vibration, sound transmission etc. for a given weight of the treatment.

An unconstrained damping layer dissipates vibrational energy by virtue of the bending strains to which it is subjected. For any particular mode of plate flexural vibration, it is evident that the treatment in the vicinity of strain nodes (and hence of displacement nodes) contributes little, if anything, to the damping of that mode. On the other hand the material at the strain (and displacement) anti-node makes the largest contribution. Removing the material from the nodal regions to the anti-nodal regions should therefore result in more material being used more effeciently, with a consequent increase in the damping. This technique is known as "the anti-nodal treatment". If this process is continued indefinitely a very large thickness of the treatment exists over a very small region, but for two main reasons the damping then becomes very small, approaching zero. In between these two extremes of total coverage (100% of length) and very small coverage (0%), there must obviously lie an optimum at which the damping is maximum. A calculation of this maximum has already been carried out (reference 1) where one particular problem was considered with one given quantity of damping material. The calculation was directed towards finding the maximum "damping ratio" or "loss factor" of a simply-supported beam vibrating in its fundamental flexural mode only, and it was found that the maximum occurred when the centre 40% of the beam was covered. The damping ratio was then about 40% greater than that of the beam covered over the whole length.

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This report presents experimental confirmation of the validity of the theoretical methods used in the previous work. In view of the lengthy nature of the experiment, this confirmation was obtained, again, considering one particular beam and one given quantity of a commercial damping treatment. (Aquaplas K 102). Furthermore, extensive computer calculations have been carried out to investigate, theoretically, the optimum usage of different unconstrained layer treatments applied to strips and plates. The results are presented in this paper. A wide range of treatment stiffnesses have been considered, together with a wide range of given initial weights of treatment. The calculations are based on the following assumptions:

(a) The damping and stiffness of a plate may be represented by that of a thin strip, simply supported at its ends. This is justifiable if the plate is vibrating in a mode having a longitudinal wavelength much greater than the lateral wavelength.

(b) The strip vibrates in its fundamental mode (equal to, or approximating to a half sine wave).

(c) The damping layer is of constant thickness, and is applied on one side of the strip only; when the strip is partially covered, the material is applied symmetrically about the centre.

(d) The damping treatment acts as a linear hysteretic material. The effect of frequency change on the complex stiffness, $E_d\ (1 + i\ \eta_d)$ is ignored.

(e) The bending stress distribution at all points in the strip is given by the simple beam theory - even up to the change in cross-section of the composite beam where the damping treatment ends. Stress diffusion effects are therefore ignored.

The general method adopted in the calculations was as follows :

With a given quantity of damping treatment, the damping ratio, the generalised stiffness and generalised mass were calculated for the strip having several different proportions of its length covered. The damping ratio and stiffness were found by estimating the bending moment distribtuion along the strip, and from this, by estimating the bending strain energy stored in the covered and uncovered portions. The bending moment distribution obviously depends upon the mode of vibration and the mass distribution, and each of these change as the treatment coverage changes.

Three different methods have been used to take account of this:

(i) The Constant Bending Moment Method. Here, the change of mass distribution and mode are assumed to have a negligible effect upon the bending moment distribution, which is considered to be sinusoidal for all treatment coverages.

(ii) The Complementary Energy Method. The change of mode is ignored, and the bending moment is computed by double integration of the inertia forces on the known distributed mass vibrating in a sine mode. This gives a better approximation to the bending moment than (i).

(iii) Stodola's Method of Mode Calculation. An iterative method is used to compute the correct mode and the correct bending moment distribution.

Methods (i) and (iii) were used in reference 1. Method (ii) has been investigated in this report in an effort to find a more rapid method than (iii), yet a more accurate method than (i). Stodola's iterative method is readily programmed for solution on a digital computer, and the results of computer calculations are presented in this report.

Several different values of E_d were considered for the treatment, covering the range 100,000 lb. per in.^{2} to 1,500,000 lb. per in.^{2}. E_d for Aquaplas falls in between these extremes. One density of treatment only was considered, (that of Aquaplas K 102), as changes in density should not affect the optimization process to the same extent as changes in stiffness or changes in the initial given quantity of treatment. The different given quantities ranged from an amount equivalent to the whole strip being covered by a thickness of treatment equal to 0.1 times the strip thickness to 6.4 times the strip thickness. The basic strip was considered to be of aluminium alloy ($E = 10 \times 10^6$ lb. per in.^{2}). It has been shown in reference 3 that the magnitude of the damping ratio increment provided by a damping treatment is, in itself, an insufficient criterion by which to judge the effect or efficiency of the treatment. The criterion to be used depends on the problem to be solved, i.e. a stress to be reduced, transmitted sound pressure to be attenuated, vibration amplitude to be cut down, etc. The efficiency of the treatment in relation to each of these different quantities must be assessed by a certain (different) criterion involving the stiffness and mass increments, together with the damping ratio increment. Furthermore, if the mode of vibration changes appreciably due to the addition of the treatment, the generalised exciting force corresponding to the mode will also change. The efficiency of the 'partial-covering' treatment has therefore been assessed using some of the criteria developed in reference 3, taking into account, where applicable, the effect of the change of the generalised force.

2. THE DAMPING OF UNIFORMLY COVERED STRIPS

As certain of the steps in the partial-coverage calculations involve the damping of a completely covered strip, the results of theory relating to this will be outlined here.

The damping ratio, δ, of a system vibrating in a single mode may be obtained from the relationship (reference 1):-

$$4\pi\delta = \frac{\text{Energy Dissipated by the Damping Treatment per cycle}}{\text{Maximum Strain Energy Stored during the cycle}} \quad . \qquad (1)$$

Using this expression to calculate the damping ratio for a uniform beam vibrating in flexure, we find

$$\delta = \frac{\eta_d}{2} \cdot \frac{E_d I_d}{E_m I_m + E_d I_d} \quad , \qquad (2)$$

where η_d is the loss factor of the damping material, E_d and E_m are the Young's Moduli of the damping material, and metal strip respectively, and I_d and I_m are the second moments of area, of the damping material and metal strip respectively, about the composite neutral axis of the whole section.

The maximum damping ratio obtainable occurs when $E_d I_d$ is very much greater than $E_m I_m$, and this is evidently $\delta_{max} = \eta_d/2$.

Equation 2 may then be written

$$\frac{\delta}{\delta_{max}} = \frac{E_d}{E_m} \cdot \frac{I_d}{I_m + \frac{E_d I_d}{E_m}} \qquad (3)$$

This gives an equation for the damping which depends only on the Young's Modulus Ratio, and the geometry of the strip section. The term δ/δ_{max} will be referred to as the "normalised damping ratio".

3. THE DAMPING OF PARTIALLY COVERED STRIPS

3.1 Derivation of Relevant Formulae

We consider a strip of length ℓ, coated with a thickness, t, of damping treatment over the portion of its length from x = a to x = -a. The origin of x is at the centre of the strip.

We now define the term δ_t by analogy with equation 1, as follows:

$$4\pi\delta_t = \frac{\text{Energy Dissipated per unit length of coated region per cycle } (= e_d)}{\text{Maximum Energy Stored per unit length of coated region during the cycle } (= e_s)} \qquad (4)$$

δ_t is therefore the damping ratio increment for a strip coated over its whole length with thickness t.

The total energy dissipated per cycle in the damping treatment is

$$\int_{-a}^{+a} e_d \, dx = 4\pi\delta_t \int_{-a}^{+a} e_s \, dx \qquad (5)$$

The total maximum energy stored in the strip is $\int_{-a}^{+a} e_s \, dx + 2\int_{a}^{\ell/2} e_u \, dx$ (6)

where e_u is the maximum energy stored per cycle per unit length of the uncovered portion. Now $e_s = \frac{M^2}{2EI}$, where M is the amplitude of the local bending moment, EI is the flexural rigidity at any point, and has two different values, of course, depending on whether the point is within or outside the covered region. Let its value within the covered region be $(EI)_c$, and elsewhere be $(EI)_u$. Let $k = (EI)_c/(EI)_u$. Then

$$e_s = \frac{M^2}{2k(EI)_u} \quad . \tag{7}$$

Likewise,

$$e_u = \frac{M^2}{2(EI)_u} \quad . \tag{8}$$

Equation 1 may still be used to determine the damping ratio of the partially covered beam. If we write

$$\varepsilon = \frac{\int_{-a}^{+a} M^2/k \, dx}{2\int_{a}^{\ell/2} M^2 dx + \int_{-a}^{+a} M^2/k \, dx} \quad , \tag{9}$$

then it may readily be shown (by substituting expressions 4 to 9 into equation 1) that

$$\delta/\delta_{max} = \varepsilon . \frac{\delta_t}{\delta_{max}} \quad . \tag{10}$$

δ_{max} is still defined by $\eta_d/2$, as in par. 2. We call δ/δ_{max} the 'Normalised Damping Ratio', and ε the 'Energy Ratio', since it represents the ratio of the maximum energy stored in the covered region during one cycle to the total maximum energy stored in the whole strip.

k is readily determined by using the elementary composite beam theory, ignoring the effect of the imaginary part of the complex stiffness of the treatment. This effect has been discussed briefly in reference 1, and should not be of any significance for most practical problems. When the bending moment, M, is known for all points along the strip, the integrals of equation (5) may be evaluated, leading to the calculation of the damping ratio, δ, from equation 10.

In addition to the damping ratio, a generalised stiffness ratio, a generalised mass ratio, and a generalised force ratio are also required. The generalised stiffness ratio, σ, is defined by the relationship

$$\sigma = \frac{\text{Generalised Stiffness of Fundamental Mode of Treated Strip}}{\text{Generalised Stiffness of Fundamental Mode of Untreated Strip}} .$$

Writing the generalised stiffness in terms of the strain energy stored per unit generalised displacement, this equation becomes

$$\sigma = \frac{\int_{-a}^{+a} M^2/k \, dx + 2\int_{a}^{\ell/2} M^2 dx}{\int_{0}^{\ell} M_o^2 \, dx} \tag{11}$$

where M_o is the bending moment in the untreated strip, and M the moment in the treated strip; these moments correspond to equal displacements at the centre of the strip when vibrating at resonance.

The generalised mass ratio, μ , is the generalised mass of the coated strip divided by the generalised mass of the uncoated strip

$$\text{i.e.} \quad \mu = \frac{\int_{-a}^{+a} \rho f_c^2 \, dx + 2\int_{a}^{\ell/2} f_c^2 \, dx}{\int_{0}^{\ell} f_o^2 \, dx} . \tag{12}$$

In this, f_c and f_o are the normalised displacement functions corresponding to the fundamental modes of vibration of the coated and uncoated strips respectively. ρ is the mass per unit length of the coated region divided by the mass per unit length of the uncoated strip.

The generalised force ratio is obtained by considering the coated and uncoated strip to be subjected to a uniform exciting pressure. The ratio 'p', is then given by

$$p = \frac{\text{Generalised Force Corresponding to Fundamental Mode of Coated Strip}}{\text{Generalised Force corresponding to Fundamental Mode of Uncoated Strip}}$$

$$= \frac{\int_{0}^{\ell} f_c \, dx}{\int_{0}^{\ell} f_o \, dx} . \tag{13}$$

The next part of the problem is the determination of the bending moment distributions and the modal displacement functions corresponding to the fundamental modes of the strips in the different partially-covered conditions. This is dealt with in the following sections.

3.2. The Constant Bending Moment Method

In this method, it is assumed that the non-uniform mass and stiffness distribution does not change the moment distribution from the sinusoidal form of the uniform simply supported strip. Since it is the integral of M^2 that is required in equation (9), small errors in the form of M should not lead to large errors in ε. In fact, reference 1 shows that in one special case the accuracy of this method was of high order over a considerable range of coverage. In equation (9), M is therefore replaced by $M_c \cos \frac{\pi x}{\ell}$. To determine the generalised stiffness ratio, it is necessary to determine the strain energy stored in the strip, with and without the treatment, for the same central (generalised) strip deflection. When the fractional coverage, $a/(\ell/2)$, is α, and the bending moment is $M_c \cos \frac{\pi x}{\ell}$ (x measured from the centre of the simply supported strip), it is readily shown that the central deflection is

$$\frac{M_c}{(EI)_u} \frac{\ell^2}{\pi^2}\left[\frac{1}{k} + \left(\frac{1}{k} - 1\right)\left\{\frac{\pi}{2}\left(1-\alpha\right) . \sin \frac{\pi\alpha}{2} - \cos\frac{\pi\alpha}{2}\right\}\right] = \frac{M_c \ell^2}{(EI)_u \pi^2} . K(\alpha) \tag{14a}$$

Furthermore, the strain energy stored may be shown to be

$$\frac{M_c^2}{(EI)_u} \cdot \frac{\ell}{8}\left[1 + \frac{1}{\pi}\left(\frac{1}{k} - 1\right)\left(\pi\alpha + \sin\pi\alpha\right)\right] = \frac{M_c^2}{(EI)_u} \cdot \frac{\ell}{8} S(\alpha) \tag{14b}$$

where $K(\alpha)$, $S(\alpha)$ are defined by these equations.

The central deflection of an untreated strip (k = 1) is clearly

$$\frac{\ell^2}{\pi^2} \cdot \frac{M_c}{(EI)_u}$$

whence the central bending moment per unit central displacement is

$\frac{\pi^2 (EI)_u}{\ell^2}$. The corresponding strain energy stored per unit central displacement is $\frac{\pi^4 (EI)_u}{8\ell^3}$.

Similarly, the central bending moment per unit central displacement of the treated strip is $\frac{\pi^2 (EI)_u}{\ell^2} \frac{1}{K(\alpha)}$, and the corresponding strain energy stored is

$$\frac{\pi^4 (EI)_u}{8\ell^3} \cdot \frac{S(\alpha)}{[K(\alpha)]^2} .$$

The generalised stiffness ratio, σ, is therefore given by

$$\frac{S(\alpha)}{[K(\alpha)]^2} .$$

3.3 The Complementary Energy Method

An approximate mode of vibration is assumed, of a form which can be easily manipulated and integrated. From this, the inertia loading, shear force and bending moment distributions are found by successively integrating. M^2 is then integrated to give the terms required for $\mathcal{E}$ (equation 9). For the simply supported strip a cosine mode of deflection may be assumed, with its origin at the middle of the strip. The deflection at any point is then given by

$$y = y_o \cos \frac{\pi x}{\ell} , (-\ell/2 \leq x \leq \ell/2).$$

When undergoing flexural vibrations in this mode, at the frequency, ω, the inertia loading is

$$\omega^2 y \rho m_o$$

With the strip coated with damping treatment over the middle portion from x = -a to x = a, the shear force in the beam at any station is given by

$$\omega^2 y_o m_o \int_o^x \rho \cos\left(\frac{\pi x}{\ell}\right) dx \qquad (0 < x < a)$$

and $$\omega^2 y_o m_o \left[\int_o^a \rho \cos\left(\frac{\pi x}{\ell}\right) dx + \int_a^x \cos\left(\frac{\pi x}{\ell}\right) dx \right] (a < x < \frac{\ell}{2}).$$

(Note: Both ρ and k have unit value for $a < x < \frac{\ell}{2}$).

The bending moment at any station may then be shown to be

$$M = \omega^2 y_o m_o \left[(\rho-1) \left(\frac{\ell}{2}-a\right)\frac{\ell}{\pi} \sin\frac{\pi a}{\ell} - (\rho-1)\frac{\ell^2}{\pi^2} \cos\frac{\pi a}{\ell} + \rho\frac{\ell^2}{\pi^2} \cos\frac{\pi x}{\ell}\right], (0 < x < a) \quad (15)$$

and $$M = \omega^2 y_o m_o \left[(\rho-1)\left(\frac{\ell}{2}-x\right)\frac{\ell}{\pi} \sin\frac{\pi a}{\ell} + \frac{\ell^2}{\pi^2}\cos \frac{\pi x}{\ell}\right], (a < x < \frac{\ell}{2}) . \quad (16)$$

Substituting (15) and (16) into the integrals of equation (9), we have

$$\frac{1}{\omega^4 y_o^2 m_o^2}\int_{-a}^{+a} \frac{M^2}{k} dx = \frac{2}{k}\left[(\rho-1)^2\left\{\left(\frac{\ell}{2}-a\right)\frac{\ell^2}{\pi^2}\left[\left(\frac{\ell}{2}-a\right)\sin^2\frac{\pi a}{\ell} - \frac{2\ell}{\pi}\sin\frac{\pi a}{\ell}\cos\frac{\pi a}{\ell}\right]a + \frac{\ell^4}{\pi^4}\cos^2\frac{\pi a}{\ell} . a\right\} + \right.$$

$$\left. + \rho^2\frac{\ell^4}{\pi^4}\left(\frac{a}{2} + \frac{\ell}{4\pi}\sin\frac{2\pi a}{\ell}\right) + (\rho-1)\rho . 2\frac{\ell^4}{\pi^4}\left[\left(\frac{\ell}{2}-a\right)\sin^2\frac{\pi a}{\ell} - \frac{\ell}{\pi}\cos\frac{\pi a}{\ell} . \sin\frac{\pi a}{\ell}\right]\right] \quad \ldots . (17)$$

and

$$\frac{1}{\omega^4 y_o^2 m_o^2}\int_{a}^{\ell/2} M^2 dx = (\rho-1)^2\left\{\frac{1}{3}\left(\frac{\ell}{2}-a\right)^3\frac{\ell^2}{\pi^2}\sin^2\frac{\pi a}{\ell}\right\} - (\rho-1)\left\{(\ell-2a)\frac{\ell^4}{\pi^4}\sin^2\frac{\pi a}{\ell} - 2\frac{\ell^5}{\pi^5}\sin\frac{\pi a}{\ell} . \cos\frac{\pi a}{\ell}\right\} +$$

$$+ \frac{\ell^4}{\pi^4}\left\{\frac{\ell}{4} - \frac{a}{2} - \frac{\ell}{4\pi}\sin\frac{2\pi a}{\ell}\right\} . \quad \ldots . . (18)$$

Having chosen values for a , and knowing the corresponding values of ρ and k, the integrals 17 and 18 may be evaluated, and the energy ratio (equation 9) determined. ρ and k, of course, vary with the value of α and with the initial amount of damping treatment. Equations 17 and 18 may also be used to determine the generalised stiffness ratio, σ (equation 11), but ω^2 must first be found by equating the total complementary energy to the maximum kinetic energy,

$$\frac{y_o}{2}\int_{-\frac{\ell}{2}}^{+\frac{\ell}{2}} \rho m_o \omega^2 \cos^2\frac{\pi x}{\ell} dx .$$

The generalised mass ratio, μ , (equation 12) is found by putting $f_c = \cos \pi x/\ell$. As this is also equal to f_o, a less accurate value of μ will be found from this 'Complementary Energy' Method, than from a method which employs a more accurate estimate of the displacement function f_c .

The generalised force ratio (equation 13) will obviously have unit value if $f_c = \cos \pi x/\ell$, and will again be of less accuracy than that given by a more accurate displacement function.

3.4 The Iterative Method

In this method, Stodola's method of calculating torsional modes of vibration (reference 2) is adapted to the calculation of the fundamental flexural mode of the coated strip. The differential equation for the modes of flexural vibration of a beam is

$$\frac{d^2}{dx^2}\left[EI \cdot \frac{d^2y}{dx^2} \right] = \omega^2 . m(x) . y \tag{19}$$

where m(x) is the mass per unit length

Integrating this four times

$$y = \omega^2 \iint \frac{1}{EI} \iint m(x) . y . \; dxdxdxdx \; . \tag{20}$$

If we now guess the fundamental mode (y_1 , say an 'approximate' mode) and substitute into the right hand side, the integral will give a mode, y_2, which is a better approximation to the fundamental mode than y_1. If y_2 is now substituted into the right hand side, the result will be a mode which is more accurate still. If this process is continued the mode calculated in successive stages rapidly converges on the correct fundamental mode.

The integral of equation 20 may be non-dimensionalised by writing $EI = k(EI)_u$, $m = \rho m_o$, $x = \xi \ell$, and $y = y_o f(\xi)$. Equation 20 may then be written

$$f(\xi) = \frac{\omega^2 m_o \ell^4}{(EI)_u} \iint \frac{1}{k} \iint f(\xi) \; d\xi d\xi d\xi d\xi \; . \tag{21}$$

This is the form used for the computational purposes of this paper. The multiplier outside the integral may, in fact, be ignored for the purpose of mode calculation.

Now if equation (19) is integrated twice an equation for the bending moment is obtained. Use is made of this in calculating the energy ratio.

Having integrated numerically to obtain the accurate mode (which was obtained after 2 iteration processes) the accurate mode may then be integrated twice to get the bending moment distribution. This is then used in equation (9) to calculate the energy ratio.

This process being ideal for a digital computer, a programme was developed for a simply supported beam, using 30 ordinates equi-spaced throughout the length, to define the mode shape. Assuming symmetry about the centre, it was possible to obtain the energy ratio for each of 15 values of percentage coverage. The integration process was repeated three times, this being more than sufficient to converge on the correct mode to the degree of accuracy required. The generalised force ratio, and generalised stiffness ratio are also easily obtainable by this method, and their calculation was also included in the computer programme. The generalised mass ratio is also very readily obtainable by appropriate integration.

4. THE VARIATION WITH COVERAGE OF THE EFFECTIVENESS OF A DAMPING TREATMENT

4.1 Criteria for Assessing the Effectiveness

In the former, limited, investigation into the effect of partial coverage (reference 1) the effect on the damping ratio only was considered. It has been pointed out in reference 3 that in general, the damping ratio increment is an insufficient criterion by which to judge the effectiveness of a treatment, except when the effect on harmonic resonant inertia forces is concerned. It is shown in reference 3 that when the effect on other response quantities is being investigated, the contribution to the generalised stiffness and mass from the treatment must also be taken into account, as the response is obviously dependent upon these quantities as well as upon the damping ratio. The manner of the dependence of different response quantities on the damping ratio, generalised stiffness and mass has been determined, considering both harmonic and random exciting forces. Certain criteria, containing the damping ratio, stiffness ratio (σ) and mass ratio (μ), have been derived by which the effectiveness of the damping treatment should be judged in relation to the various response quantities. Some of the more important response quantities and the associated criteria are listed below:

(i) Harmonic resonant inertia forces - δ (as above)

(ii) Harmonic resonant vibration amplitude - $\sigma\delta$

(iii) Random vibration amplitudes (r.m.s., assuming one dominant mode only in the response)

(iv) Random acceleration amplitudes (r.m.s.) - $\mu^{5/4} \sigma^{-\frac{1}{4}} \delta^{\frac{1}{2}} p^{-1}$. This criterion relates only to that component of the acceleration deriving from the 'resonant' response of one mode of the system.

(v) Random inertia forces (r.m.s., assuming one dominant mode) $\mu^{\frac{1}{4}}.\sigma^{\frac{1}{4}}.\delta^{\frac{1}{2}}.p^{-1}$. This may be related to the reaction forces at the edge of a plate, vibrating randomly in one dominant mode. A practical application is therefore to jet-excited random loads on the rivets attaching a fuselage skin 'panel' to its boundary members.

(vi) Harmonic, resonant sound transmission - $\mu\delta p^{-1}$. This relates to the simplified system of the sound pressure transmitted through a finite plate set in an infinite rigid wall.

Reference 3 gives a detailed derivation of these criteria, and a detailed description of their applicability. The term p^{-1} was not, however, incorporated in this work, as it was assumed that the addition of the damping treatment did not significantly change the mode of vibration. It should be noted that the expression for p (equation 13) is only true when the strip (or plate) is being excited by pressures which, at any instant, are uniform over the surface of the plate. If the source of excitation is a point force at the centre of the strip (where $f_o = f_c = 1$), then $p = 1$. For other pressure and force distributions p may be determined in a straightforward way.

4.2 Computed Criteria for Different Coverages

Calculations of the criteria have been made in the following way :

(i) A particular value of E_d/E_m was chosen, together with an amount of damping treatment sufficient to cover the strip uniformly on one side to a thickness of τ x the strip thickness. τ is referred to as the 'initial thickness ratio', and therefore represents the amount of treatment used.

(ii) Using the methods of section 3, values of δ / δ_{max}, σ, μ and p were calculated with the quantity τ of treatment distributed over different proportions of the strip. The criteria have then been calculated from the sets of the ratios obtained for each fractional coverage. δ / δ_{max} may be used instead of δ in the criteria since it is assumed that the damping treatment properties do not change significantly with the frequency change associated with the increase of μ and σ.

(iii) A range of values of τ (from 0.1 to 6.4) and a range of values of E_d/E_m (from .01 to .15) were taken, and the criteria were calculated for all combinations of these values, and at fifteen different fractional coverages.

Figure 5 shows a typical set of curves relating the criteria to the percentage coverage. These correspond to the values.

$$E_d/E_m = .07 , \quad = 0.8$$

It will be seen that as the percentage coverage is varied, a maximum value is obtained for each criterion. These maxima do not all occur at the same value of percentage coverage. Consideration of similar sets of curves corresponding to other values of E_d/E_m and shows that different maximum values of the criteria are obtained (as expected), at different values of the percentage coverage. These maximum values of each criterion are plotted against in figures 6 to 11, and the coverage at which these maxima occur are shown in figures 12 to 15. The 'average' values of these maximum coverages for given quantities and stiffnesses of treatment are shown in figure 16.

5. THE DAMPING OF PARTIALLY COVERED PLATES

5.1 A Simplified Treatment

The determination of the damping ratio, stiffness ratio and generalised mass ratio of a partially covered plate involves the integration of the bi-harmonic plate equation. The difficulties inherent in applying either the complementary energy method or the Stodola integration method (as used for the strip, par. 3) are obviously very great. A computer programme to cover all possible plate sizes (length-breadth ratios) and all possible coverages of treatment (different percentages in both directions) would be very lengthy, as also would be the presentation of the results. The following simplified treatment is therefore put forward, on the basis of which calculations have been carried out.

It is assumed that the length of the rectangular simply-supported plate is more than, say, three times its breadth. The dynamic characteristics of the fundamental mode of the plate, may then be regarded as being proportional to those of an elemental strip of span equal to the width of the plate. We next assume that the damping treatment is applied such that it covers p_c% of the width and p_c% of the length of the plate. If the amount of treatment is such as to cover the whole plate to a thickness of t, then the thickness when p_c% of the length and breadth are covered is t. $10^4/p_c^2$. It is now assumed that the uncovered regions at the two ends of the plate contribute very little to the potential energy of vibration of the system, and that energy associated with longitudinal strain is small compared with that associated with lateral (width-wise) strain. This assumption should be satisfactory provided that the thickness of treatment, , is not excessive and that p_c is not too small. Under these assumptions, it is evident that the plate problem may be solved approximately by regarding it as a beam (or strip), having p_c% of its span covered by a thickness of $\tau t.10^4/p_c^2$ of treatment. The methods of par. 3 may now be applied on this basis.

It might be thought that a 'constrained' modulus, $E/(1-\nu^2)$, should be used instead of the usual modulus E, (ν = Poisson's Ratio). It will be assumed, however, that the values of ν_2 for the plate and treatment are equal, in which case the factor $(1 - \nu^2)$ will cancel out (except at the lengthwise extremities of the treatment. This will be a local effect, and will be ignored).

5.2 Computed Criteria for Different Coverages

Calculations were carried out for the simplified plate theory, following the approach outlined in par. 4.2 for the strip theory. These calculations were computed on a digital computer, using the Stodola integration method. Only two criteria (δ_d, and $\mu\delta_d/p$) were computed for this plate theory, using the same range of variables, as were used for the strip theory. Direct comparison of the results is made in figures 17 to 19 and par. 7.6. In figure 20, the "average" optimum coverage graph is reproduced for the plate theory, and curves from the strip theory, (figure 16) are superimposed for comparison.

6. EXPERIMENTAL VERIFICATION OF THEORIES USED

6.1 Details of Experiment

Experiments have been conducted as an experimental verification of the theories used, and of the validity of the assumptions. In particular, the effect of assumption (e) (see par. 1) on the calculation of the energy ratio at small coverages required investigation. The experiment consisted of vibrating a free-free beam in its fundamental mode and measuring the natural frequency and damping ratio. The beam was initially covered uniformly on one side, with 0.1 in. of 'Aquaplas K102', after which the coverage was successively reduced, while the thickness of the treatment was correspondingly increased. A free-free beam was used in order to minimise support damping. Calculations using the constant bending moment distribution, and the iterative method were carried out for the free-free beam to predict, theoretically, the damping ratios and natural frequencies.

Measurements of the damping ratio, and the resonant frequency, were taken at several values of percentage coverage. The beam was a top-hat section stringer with damping treatment on the crown surface. It was supported on thin wires at the two nodes of the fundamental mode. Two coils were fixed to the centre of the beam, and positioned between the poles of permanent magnets. One was used for exciting the vibration, and the other for measuring the vibration velocity amplitude. As the visco-elastic properties of Aquaplas were temperature sensitive, the rig was enclosed in an asbestos-lined box and thermostatically maintained at $68^{\circ}F \pm 2^{\circ}F$. The damping ratios of the treated beams were found by using the well known technique of measuring the width of the frequency response curve in the neighbourhood of resonance, the beam being excited by a force of constant amplitude. As the damping was found by other means to be virtually linear, this method can be relied upon to give results of sufficient accuracy.

6.2 Comparison of Experimental Results with Theoretical Predictions

Figure 1 shows the variation with percentage coverage of the measured and calculated damping ratios for the stringer-beam. Very good agreement is seen to exist between 50% and 100% coverage. The existence of a maximum value of the damping ratio is clearly borne out by experiment and calculation.

The Stodola Integration Method over-estimates slightly the peak value of the damping, and predicts an optimum coverage which is rather less than that indicated by the test.

This is due to the invalidity of assumption (e) (par.1) when the damping layer is thick over a limited region of the beam. The ends of the layer are not, in fact, being strained as much as is assumed by the elementary beam theory, and consequently do not contribute as much to the damping as the Stodola Integration method predicts. As the length of the layer decreases with decreasing coverage, the ineffective ends of the layer form a greater proportion of the total amount of the damping treatment, with the result that the actual (measured) damping ratio falls more and more below the Stodola Curve. This is seen to occur below 40% coverage. The constant bending moment distribution method gives the maximum at the correct coverage, but the maximum damping ratio is under-estimated. At lower coverages, the damping ratio exceeds the measured values by a small amount. This is not to say that this method is, in general, more accurate for handling the lower coverage conditions than the Stodola integration method. The apparent improvement in accuracy is due to the cancellation of two sources of error inherent in the approximations, i.e. the error due to the invalidity of assumption (e), and that due to the bending moment distribution being inaccurate at low coverages. With different quantities of material or different material properties there may be a wider discrepancy between measured values and 'constant bending moment method' values. It does appear, however, that the method gives a good approximation to the maximum damping ratio obtainable, and also to the percentage coverage at which this occurs.

The calculations and experimental results demonstrate that by suitably re-distributing the damping treatment, the damping ratio may be made as much as twice that of a uniformly covered beam.

7. DISCUSSION OF THEORETICAL RESULTS

7.1 Comparison of the Results from the Complementary Energy and the Iterative Methods

The results from the complementary energy method are not shown graphically. However, the normalised damping ratios found by each method were found to agree extremely well, even under conditions of large τ and high E_d/E_m, where the approximate energy method would be expected to give the least accurate results. The agreement between the stiffness ratios was not, on the whole, as close, but the maximum values of the stiffness ratio, and the values of the percentage coverage at which it occurred, did agree closely. It can be concluded therefore, that the complementary energy method may be used with confidence to establish the optimum coverage and the corresponding (approximate) maximum value of the required criterion, although values of the criteria at off-optimum coverages may be subject to some considerable error. It may be noted that the complementary energy method is exactly the same as the iterative method, terminated after one complete stage of the multiple-integration process. It does not, however, provide an "improved mode" which would permit a more accurate estimate of the generalised mass or generalised force. These quantities are always derived in this method by integration of the initially chosen mode.

7.2 The Variation of the Stiffness, Mass and Damping Ratios with Coverage

Figure 4 shows typical curves of p, μ and σ' plotted against percentage covered. At 0% coverage, the beam (or strip) is virtually a uniform beam with a concentrated mass at the centre. If this mass is small compared with the total beam mass, the mode of vibration differs only very slightly from that of a uniform beam, and the generalised stiffnesses scarcely differ, i.e. The stiffness ratio, σ , will be very close to unity. The generalised mass ratio will be considerably larger, since the mass of the treatment is now concentrated at the centre of the beam where the amplitude is greatest.

At 100% coverage, the mass and stiffness ratio are identical with the ratios of the local mass per unit length and flexural stiffness, respectively, of the treated and untreated beams, the modes being identical in each case. Between 0 and 100% the existence of the maximum value of μ may be understood by reference to the curve of p, and also to figure 3 which shows a typical modal variation with coverage. When the coverage is such that p is close to its maximum value, the mode exhibits a pronounced flatness (figure 3). With the correspondingly greater extent of the beam undergoing a displacement close to that of the centre (reference) displacement, and with the mass of the treatment spread over this region, the generalised mass reaches its maximum value.

The existence of the maximum value of σ follows from the fact that as the coverage is reduced from 100%, the treatment is at first being redistributed to the regions of higher bending moment, which are therefore being stiffened up by the increased thickness. The regions at the ends carry little bending moment, and the removal of the redistributed material is of relatively little consequence. The nett effect is therefore an overall generalised stiffening of the beam. At very low coverages, the very large depth of treatment, ideally, gives a very great stiffness over the very small region; the remainder of the beam has the stiffness of the untreated beam. In the limit, as the coverage approaches zero, the generalised stiffness of the treated beam approaches that of the untreated beam, apart from a small factor arising from the difference in mode of the beam in the two conditions. The generalised stiffness of the beam therefore decreases as the coverage decreases to zero. Since the generalised stiffness also decreases as the coverage increases to 100% , a maximum must exist at some intermediate coverage.

In practice, the theoretical maximum stiffness will not be achieved, owing to the stress diffusion and concentration at the abrupt change of section, where the treatment is terminated. An experimental stiffness ratio curve would be expected to drop below the theoretical curve, more so at lower coverages, than at high. At both 0% and 100%, however, the experimental and theoretical curves must agree.

The variation of the damping ratio with coverage follows a similar trend to that of the stiffness, and has been discussed in reference 1. It should be noted, however, that whereas the stiffness ratio approaches a value slightly greater than unity as the coverage approaches 0%, the damping ratio approaches zero. This follows from the damping ratio being proportional to the energy stored in the damping treatment, and the latter energy approaching zero as the coverage is made very small.

It will be observed that the maximum value of the normalised damping ratio in figure 5 is not as great in proportion to the 100% value as the maximum stiffness ratio is to its 100% value. Now the damping ratio is proportional to the maximum energy stored per cycle in the treatment $\div$ the maximum energy stored per cycle in the whole beam, and the energy stored in the whole beam is proportional to the generalised stiffness. The damping ratio of the non-uniformly covered beam is therefore proportional to the energy stored in the treatment $\div$ the stiffness ratio. Thus, it is not to be expected that the proportions of the stiffness ratio curve of figure 4 and of the normalised damping ratio curve of figure 5 would be the same. The energy stored in the treatment increases at first as the coverage is reduced from 100%, due to a greater thickness of it existing over the region of greater bending moment. However, as the coverage decreases and the thickness increases, the local flexural stiffness increases to the extent that the curvature , and hence the energy stored in the treatment decreases.

It should be noted that the criterion $\sigma\delta$ is the value of the generalised hysteretic damping coefficient of the beam, and is proportional only to the energy dissipated (or stored) in the treatment. It is really a more fundamental property of the system than the damping ratio, which is $\frac{1}{2} \times \frac{\text{the hysteretic damping coefficient}}{\text{the generalised stiffness coefficient}}$ The explanation in the last paragraph of the behaviour of the energy stored in the treatment may be applied directly, therefore, to the behaviour of the curve of $\sigma\delta$ in figure 5.

7.3 The Variation of the Criteria with Coverage

The criteria plotted on figure 5 all behave in a similar way as the coverage changes. In particular, it is noteworthy that with one exception, the optimum coverages for the maximum values of the criteria are very close. The exception relates to the r.m.s. inertia force, which criterion contains the stiffness ratio raised to a negative power. As the coverage is reduced from 100% the increasing stiffness ratio therefore has a detrimental effect on the criterion preventing it from rising to the same extent as the other criteria and also causing the optimum coverage to be greater than that for the other criteria.

The r.m.s. sound pressure criterion also contains σ raised to a negative power. In this case, the increase of the generalised mass ratio (raised to the power of 5/4) helps to oppose the detrimental effect of the increasing stiffness, and the optimum coverage is of the same order as that for the other criteria.

No curves of criteria are shown for values of E_d and τ other than those of figure 5. However, it may be stated that as these quantities are increased, there comes a point when the maximum value of δ_d occurs at 100% coverage. The value of τ is then close to that at which the characteristic δ vs. treatment thickness curve for a uniformly covered beam reaches a maximum (see, for example, Oberst's curves, reference 4). Increasing the thickness of treatment over the covered portion (by redistribution) can then only reduce δ_t, (equation 10); the damping ratio of the non-uniformly covered beam will then also be lower than the uniformly covered value, since the energy ratio, ε, cannot be greater than unity.

The maximum in the characteristic δ vs. thickness curve for a uniformly covered strip also partially explains the smallness of the increase in damping ratio in figure 5 as the coverage is reduced. As the coverage is reduced the thickness of the treatment approaches the value for maximum δ_t and no further increase in δ_d is obtained. The lower the value of E_d, however, the higher is the optimum thickness for maximum δ_t (reference 4); consequently for lower E_d, the optimum coverage for maximum δ_d is lower.

7.4 The Effects of E_d and Initial Thickness of Treatment on the Maximum Values of the Criteria

The effects of E_d and initial thickness on the maximum criteria values are shown in figures 6 - 11. In interpreting these curves, it must be remembered that materials having different Young's Moduli, E_d, also have different loss factors, η_d. The normalising process, whereby a normalised damping ratio, δ_d, has been used in the criteria, has effectively provided criterion values corresponding to the same value of loss factor for all E_d. The normalised damping ratio is the actual damping ratio divided by δ_{max} (the maximum possible value of the damping ratio obtainable with the treatment) and this is known to be $\eta_d/2$. In order to incorporate the effect of different η_d's in the curves of figures 6 - 11, therefore, it is necessary to multiply each curve by the appropriate value of $\eta_d/2$. Furthermore, materials having different E_d's will, in general, have different densities, whereas the curves of Figures 6 - 11 have all been calculated for the same density of treatment. The correction that must be made to allow for different densities is discussed at the end of this section.

The correction of the curves for the appropriate values of η_d can only be carried out when particular materials have been specified. It is useful, however, firstly to consider the effect on the criteria of E_d and thickness ratio together, assuming that η_d is the same for each curve. The curves of figures 6 - 11 may be set into three categories:

(i) Figures 7 and 8, in which the criteria increase monotonically with both thickness and E_d. These curves relate to harmonic and random displacements respectively, and to a certain extent may be related to the bending stresses that occur at the centre of a vibrating panel.

(ii) Figures 9 and 11, in which the criteria increase monotonically with thickness; at low thickness, the criteria increase with E_d, but at high thickness they decrease with E_d. These criteria relate to random and harmonic transmitted sound pressures, respectively.

(iii) Figures 6 and 10, in which the criteria rise to a maximum as the thickness increases and then fall off. At low thicknesses, the criteria increase with E_d but at high thickness they decrease with E_d. These criteria relate to harmonic and random inertia forces, respectively.

It will be noticed that the criteria of group (i) have σ raised to a positive power, whereas those of groups (ii) and (iii) have σ raised to a negative power. Since σ is small at small values of the thickness ratio, it has a small effect upon each of the criteria in this range, and E_d affects the criteria mainly through the damping ratio δ_d. Oberst's work has shown that the highest damping ratio for a uniformly covered beam is obtainable with treatments of highest stiffness, E_d, and it is this which explains the effect of E_d on the criteria at low thicknesses. At high thicknesses, however, the stiffness ratio becomes very large, the higher E_d's giving rise to the largest stiffness ratios. This is favourable for the criteria of group (i) (positive power of σ). It follows from this that at the high thicknesses the group (i) criteria increase with E_d, whereas the others decrease with E_d.

The characteristic maximum of the criteria of group (iii) follows from the asymptotic behaviour of the damping ratio of a uniformly covered beam as the treatment thickness increases. At the same time, the stiffness increases monotonically. The r.m.s. inertia force criterion, containing the negative power of σ , therefore decreases beyond the point at which δ reaches its maximum. The group (ii) criteria, although having negative powers of σ , have μ raised by powers of one greater than those of group (iii). It is this mass effect at high thicknesses that causes these criteria to increase monotonically with thickness rather than to fall off after passing through a maximum.

Consider now the effect of changing the density of the treatment. If the stiffness of the treatment (E_d), the thickness ratio and the percentage coverage are maintained constant, the mode of vibration of the beam changes due to the new mass distribution. This, in turn, results in a change of the generalised stiffness ratio, σ , and also in the normalised damping ratio, δ_d. It is evident, therefore, that it is not strictly permissible to apply a simple correction only to the mass ratio to obtain the new values of the criteria. The new density must be incorporated in the mode calculations from the start. However, as this means that the already extensive numerical calculations should be repeated for a range of treatment densities, it is suggested that for the time being the density effect be included by a simple mass ratio correction. Let the density for which the calculations have already been carried out be ρ_{d1}, and that of the new treatment under investigation be ρ_{d2}. If the calculated generalised mass ratio using density ρ_{d1} was μ , then the mass ratio using the new density may be taken as $1 + \frac{\rho_{d2}}{\rho_{d1}}(\mu - 1)$

This is a good approximation to the correct value provided that the mode of vibration does not change appreciably with change of density.

7.5 The Optimum Coverage for Maximum Values of the Criteria

Figure 5 and Figures 12 - 15 show clearly that a different optimum coverage is required for each criterion, for each initial thickness and for each different value of E_d. It is to be noted that the highest optimum coverages are required for the group (iii) criteria, and the lowest coverages for the group (i) criteria. This has already been explained in para. 7.3, in which figure 5 was discussed.

The differences between the optimum coverages for the various criteria are not so great as to cause one criterion to be appreciably below its maximum value if the optimum coverage corresponding to another is used. In particular, if an average is taken of the different optimum coverages, it will be found that the corresponding value of each of the criteria is still very close to its maximum value. This suggests that this average value may be given as the optimum percentage coverage corresponding to the particular initial thickness and material stiffness. Accordingly, average values of coverage have been determined from many such sets of curves as figure 5 and have been plotted in figure 16.

Figures 12 - 15 and figure 16 show that the greater amount of treatment (i.e. the greater the initial thickness ratio) the greater must be the coverage for maximum effect. Similarly, the greater the value of E_d, the greater must be the coverage. That this should be so follows from the fact of the asymptotic nature of the δ vs. τ curve for a uniformly covered beam. The stiffer materials have the lower values of τ at which δ reaches a maximum. Consequently, as the percentage coverage of a non-uniformly treated beam is reduced and the thickness of the given amount of treatment over the beam centre is increased, the maximum value of δ is reached sooner (i.e. at a higher coverage) with the stiffer material. Also, if a greater amount of treatment is given, the maximum value of δ will be reached sooner than with a lesser amount.

Figures 12 to 15 also show that for a given amount of treatment the difference between the optimum coverages required to maximise the different criteria is greater for the larger values of E_d and initial thickness. This follows from the value of σ increasing with both of these quantities. Since some of the criteria contain negative powers of σ and some contain positive powers, the difference in the behaviour and optimum requirements for these criteria would be expected to diverge as σ and therefore E_d and thickness increase.

7.6 Comparison of Results from Strip Theory and Simplified Plate Theory

In general the simplifed plate theory gives higher maximum criteria values than the strip theory, and at higher optimum percentage coverages. This is to be expected, as, for the same initial thicknesses of treatment, the thickness over the covered region of a partially covered plate is greater than the thickness over the covered region of a partially covered strip. Hence the maximum criterion value is higher. Also, the damping ratio of a partially covered plate approaches its asymptotic value sooner than that of a strip, as p_c is reduced. It follows from this that the optimum coverage for a plate will be higher than for a strip.

It will be noticed in figure 17 that the plate theory curves fall away more rapidly than the beam theory curves on either side of the maximum values. Hence if a coverage is used which is slightly different from the optimum coverage, a plate criterion will be a smaller proportion of the maximum value than will be the corresponding strip criterion. The "average" optimum coverage given by the curves of figure 20 differ, of course, from the optimum coverage for any specific criterion. Figures 12 to 16 show that it is the Harmonic and RMS Displacement and RMS Inertia Force Criteria which have optimum coverages differing most from the average coverage. It follows therefore that the Harmonic and RMS Displacement and RMS Inertia Force Criteria for a plate are likely to be considerably below their maximum (optimised) value when the "average" coverage is used, more so, in fact, than the criteria for a strip.

7.7 The Applicability of the Theoretical Results

The Curves of figures 16 or 20 may be used to find the optimum coverage for a given amount of material of known stiffness. The calculations leading to these curves are based on the assumption that the mode of vibration is the fundamental of a simply supported plate (or strip). There is evidence to suggest that an aeroplane fuselage panel responds in such a mode when excited by jet efflux pressures, provided that the panel is bounded on two sides by open section stringers (reference 5). In this case, optimum damping would be obtained with an unconstrained layer treatment by treating the centre p_{opt}% of the panel width and length with a uniform layer of the treatment. Some evidence also exists to suggest that fuselage stringers vibrate in flexural modes having nodes at the frame attachment points. The stringer could then be regarded as a simply supported beam, of span equal to the frame spacing, and theoretical methods of this paper could be used to obtain the optimum coverage of treatment. The vibration of automobile door panels is sometimes found to be primarily in the fundamental mode, and again the anti-nodal treatment may be used.

When the vibration is not in the fundamental mode, but is primarily in one particular overtone, then it may be possible to use the anti-nodal treatment by considering each inter-nodal area as a single simply supported panel, or beam, vibrating in its fundamental mode. The optimum coverage is then p_{opt}% of the inter-nodal distance, each separate inter-nodal area being treated. It is unlikely, however, that it would be necessary to damp the overtone and not the fundamental mode and other overtones. When all the modes need to have increased damping, uniform coverage over the whole panel (or beam) is required. The anti-nodal treatment is therefore of most value when only one mode (preferably the fundamental) requires increased damping.

When treating a panel the edges of which are completely restrained against rotation, it is obvious that no treatment is required in the vicinity of the points, or lines, of contraflexure of the mode to be damped. If the areas between the lines of contraflexure are regarded as simply supported at the lines of contraflexure, then the curves of figure 20 may be used to assess the extent of the optimum coverage over these areas. The area between a fixed boundary and a line of contra-flexure may be regarded in an analogous way, but since the plate bending moment is high at the boundary, the treatment should cover a region adjacent to the boundary, of width equal to p_{opt}% of the distance between the boundary and the line of contraflexure.

8. CONCLUSIONS

The three methods which have been set out to determine the optimum coverage of the strip by the damping treatment have shown good agreement, insofar as the calculated coverages and corresponding maximum criterion values are concerned. The constant bending moment method gives results most rapidly, but is not as accurate as the complementary energy method which is rather more tedious in its application. The Stodola integration method really requires a high speed (digital) computer, and is the most accurate method of all.

It has been established that the anti-nodal treatment does indeed improve the efficiency of an unconstrained layer damping treatment. The efficiency must be judged by criteria which contain the factors by which the stiffness and mass of the plate are increased, together with the damping ratio increment. The results have further proved that the damping ratio increment alone is an insufficient criterion by which to judge.

The results have shown that the coverage required to maximize the different criteria are not appreciably different from one another, for a given quantity and stiffness of treatment. Accordingly, it has been possible to draw up curves of the 'mean' optimum coverage for a wide range of given quantities of treatment and a range of material stiffnesses. In general, the greater the quantity of material to be used, and/or the greater the stiffness of the material, the greater is the optimum coverage. Ultimately, with very large amounts of treatment, or with very stiff materials, the coverage must be 100% for maximum effect. According to some criteria, there are optimum quantities for maximum effect, as well as optimum coverages. According to other criteria the greatest effect is obtained with the greatest amount of treatment.

In the experiment to measure the damping ratio of a vibrating beam, which was covered over different proportions of its length with a given amount of 'Aquaplas' damping treatment, the calculations of the Stodola integration and Constant Bending Moment methods were confirmed over the range 50% to 100% coverage. The measured maximum damping ratio was slightly lower than that predicted by the Stodola method, but this was to be expected in view of the stress diffusion effects at the ends of the damping layer, which were ignored in the calculation. The measured optimum coverage was slightly greater than that predicted by the Stodola method, for the same reason.

The effect of a change of temperature on certain criteria has also been investigated, (see Appendix), temperature characteristics for two specific treatments being used. It has been shown that changing from uniform to anti-nodal treatment does not significantly change the temperature bandwidth over which the criteria exceed one half of the maximum values which occur at the optimum temperature.

Appendix

A.1 The Effect of Temperature on the Criteria

The purpose of this investigation was to assess the effects of the anti-nodal technique on the temperature bandwidth of the criteria. The temperature effects on the Harmonic Sound Transmission Criterion only are considered, but the method is applicable to all the criteria. The criterion varies with temperature, due to the variation of both the Young's Modulus and Loss Factor of the treatment, but as the normalised criterion $\mu\delta_d/p$ depends only on the Young's Modulus and is independent of ηd, the problem simplifies into two steps:

(a) The determination of the value of $\mu\delta d/p$, for a range of temperatures, due to variation of Young's Modulus.
(b) The multiplication of $\mu\delta d/p$ by $\eta_d/2$ at the corresponding temperatures, giving $\mu\delta/p$ for each temperature.

Both E_d and η_d vary with frequency, but this effect is ignored in this present paper.

A2 The Effect of Young's Modulus and Loss Factor on the Criteria

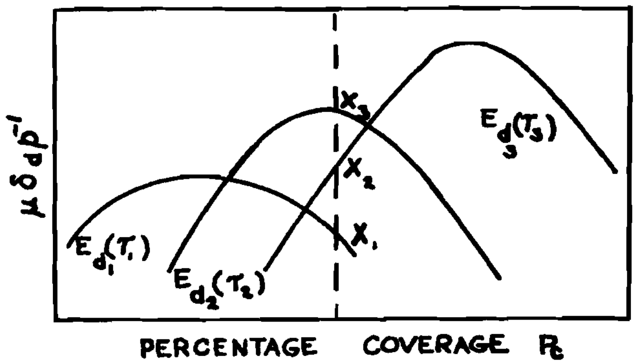

FIG. A

Figure A represents the variation of a criterion with coverage, in the region of the maximum values. The three curves relate to the same thickness ratio, τ, but to a different temperature, T. At a certain temperature, T_2, the material has a Young's Modulus of E_{d2}, and the corresponding optimum coverage is p_{c2}% (point X_2 in figure A).

Suppose the "normal operating temperature" is T_2. The coverage chosen for the treatment will obviously be p_{c2}% (at X_2). If this coverage and thickness have been used, and the temperature rises, say to T_1, E_{d_2} will drop to E_{d1}, and the value of the criterion will now be determined by point X_1. As this point is generally below X_2, the value of the criterion will drop.

If the temperature drops, say to T_3, E_{d2} will rise to E_{d3}, and the value of the criterion will be determined by point X_3. If the change $E_{d3} - E_{d2}$ is sufficiently small, X_3 will be slightly above X_2, but if the change in E_d is larger, X_3 will be below X_2.

Thus from a graph similar to figure A, but which contains curves for many values of E_d, we can determine the variation of the criterion with the variation of E_d. If we know the variation of E_d with temperature, we can hence determine the variation of the normalised criterion $\mu \delta d/p$, with temperature.

Now we may regard the loss factor, η_d, as varying with temperature, independently of the Young's Modulus, E_d. Knowing the variation of η_d with temperature, we may combine the correct value of η_d with the normalised criterion to obtain the variation with temperature of the "unnormalised" criterion, $\mu \delta /p$.

A.3 <u>The Variation of the Criteria with Temperature, using Two Specific Treatments</u>

Dr. H. Oberst of Farbwerke Hoechst has supplied data relating E_d, η_d and temperature for two different damping materials:-

(a) A "broad band" Vermiculite filled treatment, having optimum properties at about 50°C according to Oberst's criterion**, and a temperature band width* of about 90°C.

(b) A conventional, Vermiculite filled treatment, having optimum properties at 40°C according to Oberst's criterion. The temperature band width of these properties is about 32°C.

Using the "Simplified Plate Theory" values of Harmonic Sound Transmission Criterion ($\mu \delta d/p$) for several values of E_d (at a given p_c%), in conjunction with Oberst's data, figures 21, 22 were calculated. These show the variation of the criterion with temperature for three different conditions of coverage:-

(1) Anti-Nodal Treatment, with an initial thickness ratio $\tau = 0.8$, partially covered to maximise the criterion $\mu \delta /p$ at the temperature for the material optimum properties (i.e. 5°C and 40°C respectively).

(2) Uniform coverage (100%) with a thickness ratio $\tau = 0.8$.

(3) Uniform coverage, but with a higher thickness ratio, chosen to give, at the temperature for the material optimum properties, the same value of the criterion as the anti-nodal treatment, (1).

*The temperature band width is defined as that temperature range over which the damping properties are equal to, or greater than, one half of the maximum damping property.

**The criterion used by Oberst, to establish the temperature at which the treatment has its optimum damping properties, is the damping ratio of a simply supported strip, covered, uniformly (100%) on one side with the damping treatment, having a treatment weight 20% of the weight of the metal strip, at a resonant frequency of 200 c.p.s.

A.4 Conclusions

Throughout the range of temperature, considered in figures 21, 22, the anti-nodal treatment gives higher values of the criterion that the uniform (100% coverage) treatment of the same weight. For a given weight of treatment it is therefore always advantageous to use the anti-nodal treatment. It will be seen that condition (3), giving the same criterion values at certain optimum temperatures as condition (1), has a slightly greater bandwidth than condition (1). However the difference in the bandwidths is hardly sufficient to justify the additional weight of treatment required by condition (3).

R E F E R E N C E S

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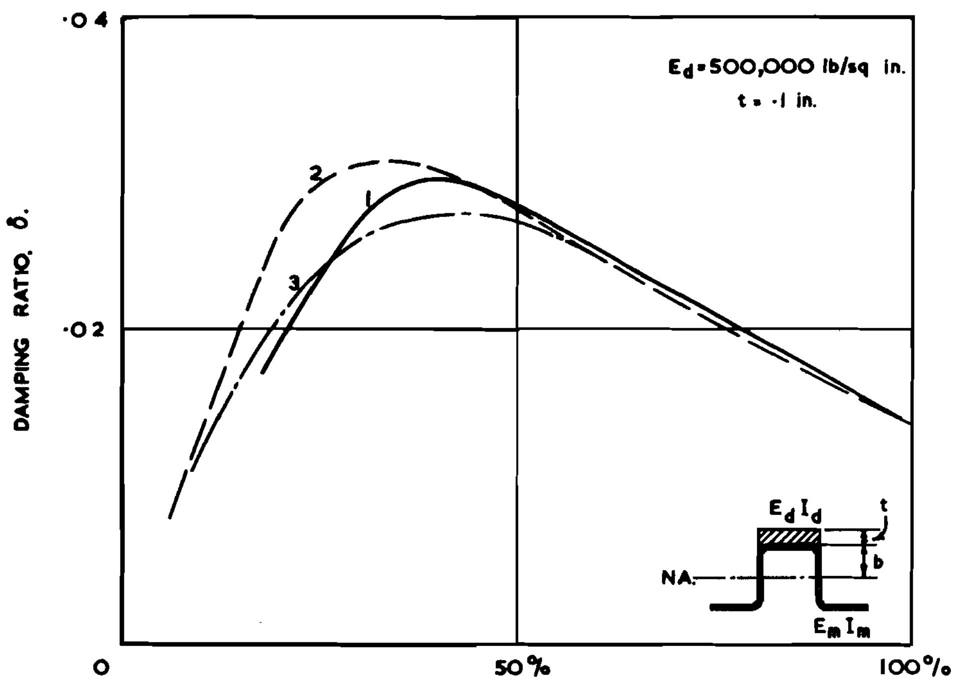
2. Den Hartog — "Mechanical Vibrations" p.155, 4th Edition, McGraw-Hill Book Co.

3. D.J. Mead — "Criteria for Comparing the Effectiveness of Damping Treatments", University of Southampton Report U.S.A.A. 125.

4. H. Oberst, K. Frankenfeld — "Über die Dämpfung der Biegeschwingungen dunner Bleche durch festhaftende Belage I" Acustica, Akustische Beihafte, 4, 1952.

5. B.L. Clarkson, R.D. Ford — "Experimental Study of the Random Vibrations of an Aircraft Structure Excited by Jet Noise" University of Southampton Report No. U.S.A.A. 128



CURVE 1. EXPERIMENTAL CURVE.

CURVE 2. STODOLA INTEGRATION CURVE.

CURVE 3. CONSTANT B.M. DISTRIBUTION CURVE.

FIG. 1. EXPERIMENTAL AND THEORETICAL VARIATION OF DAMPING RATIO WITH PERCENTAGE COVERAGE

NORMALISED DAMPING RATIO. δ_d.

NORMALISED DAMPING RATIO X GENERALISED STIFFNESS RATIO. $\sigma \delta_d$.

R. M. S. DISPLACEMENT CRITERION. $\mu^{1/4} \sigma^{3/4} \delta_d^{1/2} p^{-1}$.

R.M.S. SOUND PRESSURE CRITERION. $\mu^{3/4} \sigma^{-1/4} \delta_d^{1/2} p^{-1}$.

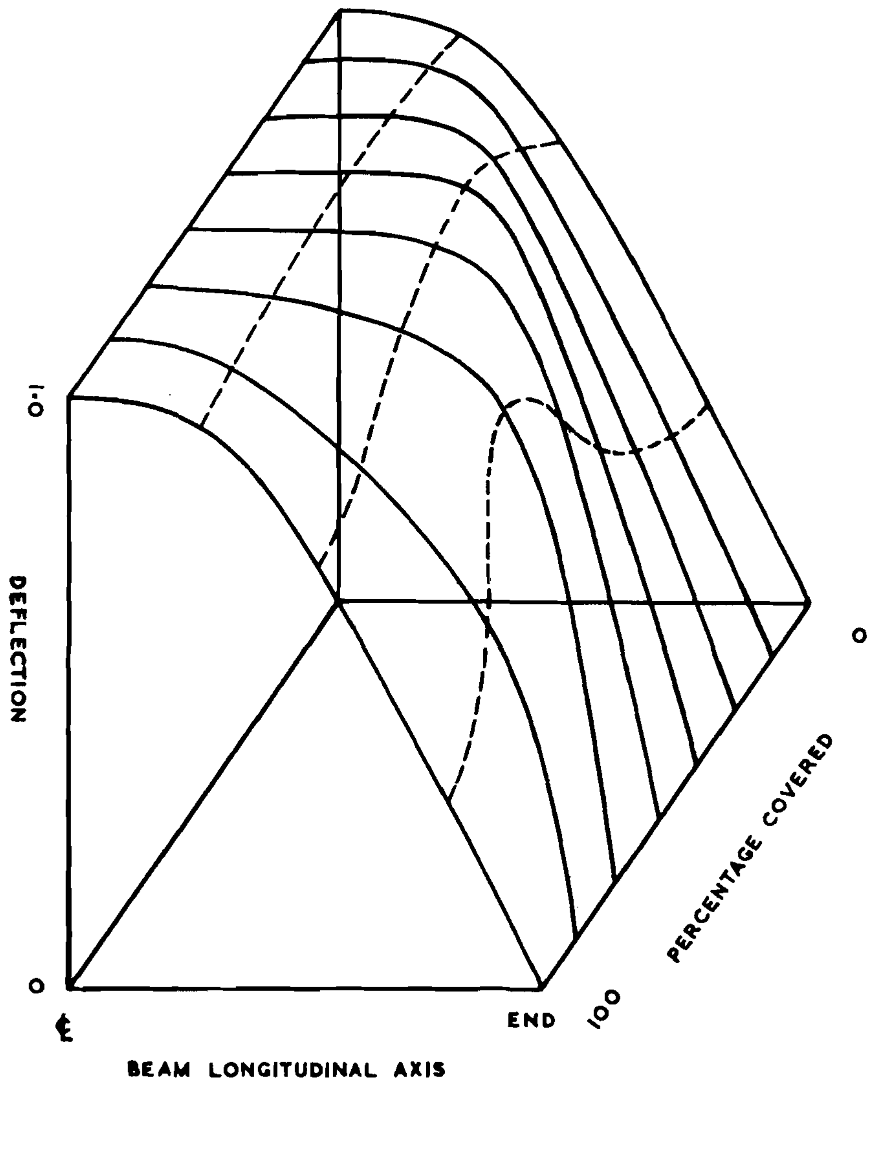
R.M.S. INERTIA FORCE CRITERION. $\mu^{1/4} \sigma^{-1/4} \delta_d^{1/2} p^{-1}$.

HARMONIC SOUND TRANSMISSION CRITERION. $\mu \delta_d p^{-1}$.

μ GENERALISED MASS RATIO.

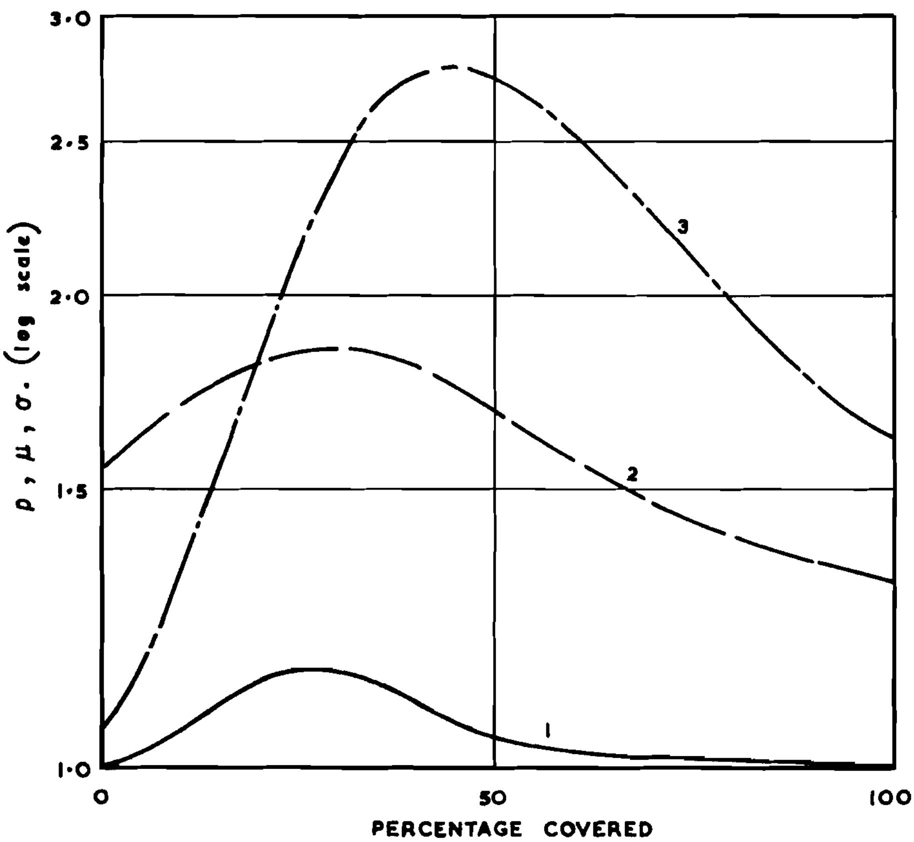
p GENERALISED FORCE RATIO.

FIG. 2. KEY TO GRAPHS OF CRITERIA



$E_d = 1{,}500{,}000$ lb/sq in. $\tau = 6{\cdot}4$

FIG. 3. CHANGE IN MODE SHAPE WITH PERCENTAGE COVERED



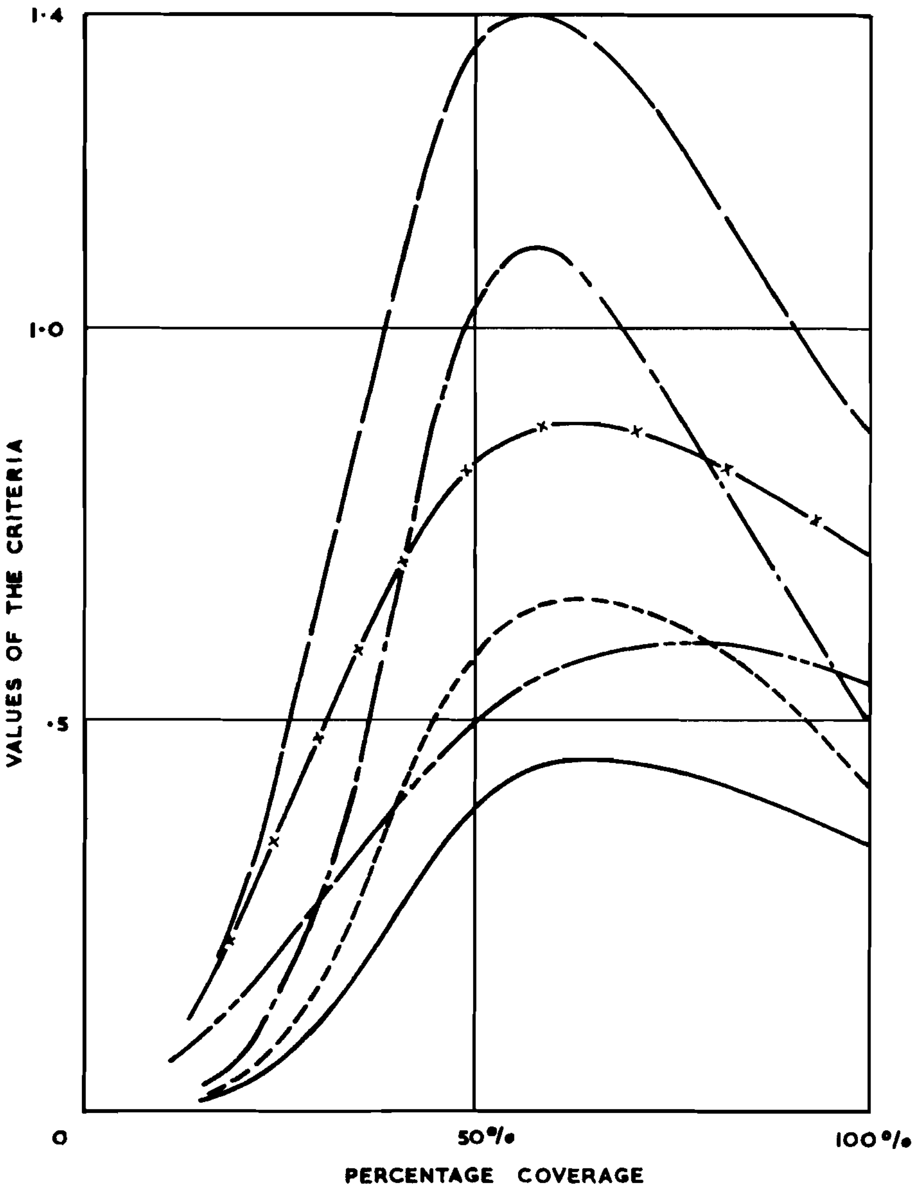
CURVE 1. VARIATION OF GENERALISED FORCE RATIO. p

CURVE 2. VARIATION OF GENERALISED MASS RATIO. μ

CURVE 3. VARIATION OF GENERALISED STIFFNESS RATIO. σ

E_d = 700,000 lb/sq.in. τ = ·8

FIG. 4. VARIATION OF GENERALISED FORCE, MASS AND STIFFNESS RATIO



E_d = 700,000 lb/sq in. τ = ·8. FOR KEY SEE FIG. 2.

FIG. 5. VARIATION OF CRITERIA WITH PERCENTAGE COVERAGE.

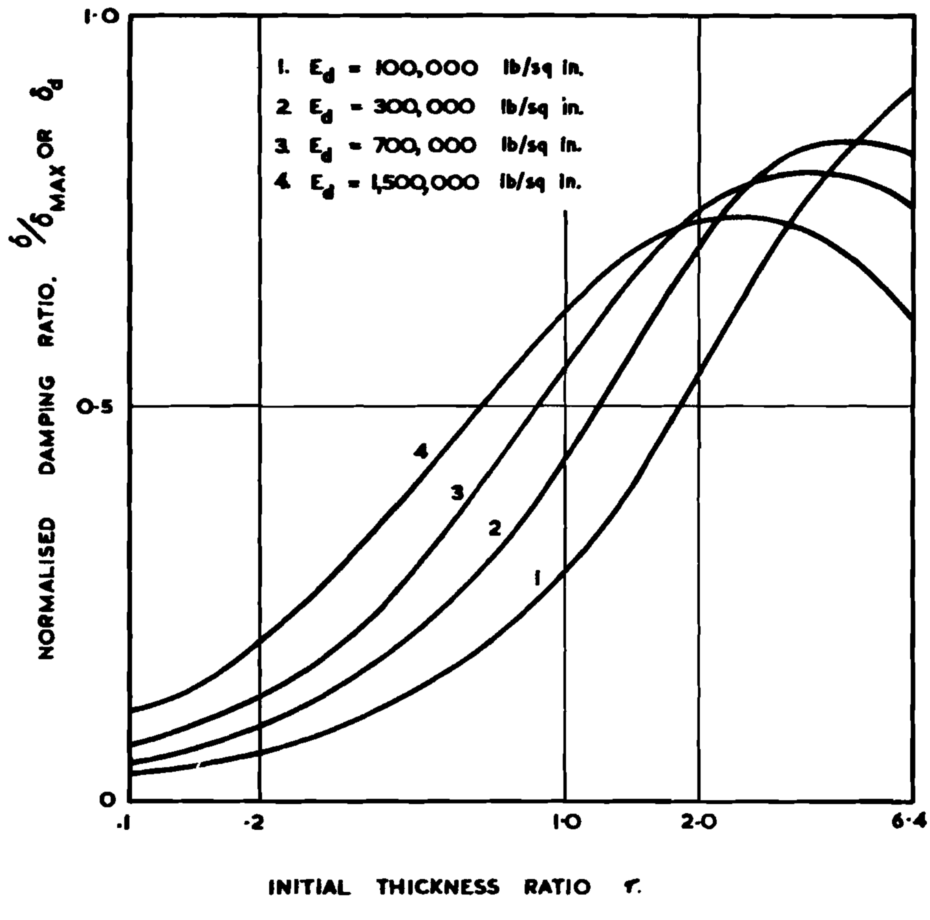


FIG. 6. VARIATION OF MAXIMUM VALUE OF NORMALISED DAMPING RATIO WITH THICKNESS OF TREATMENT

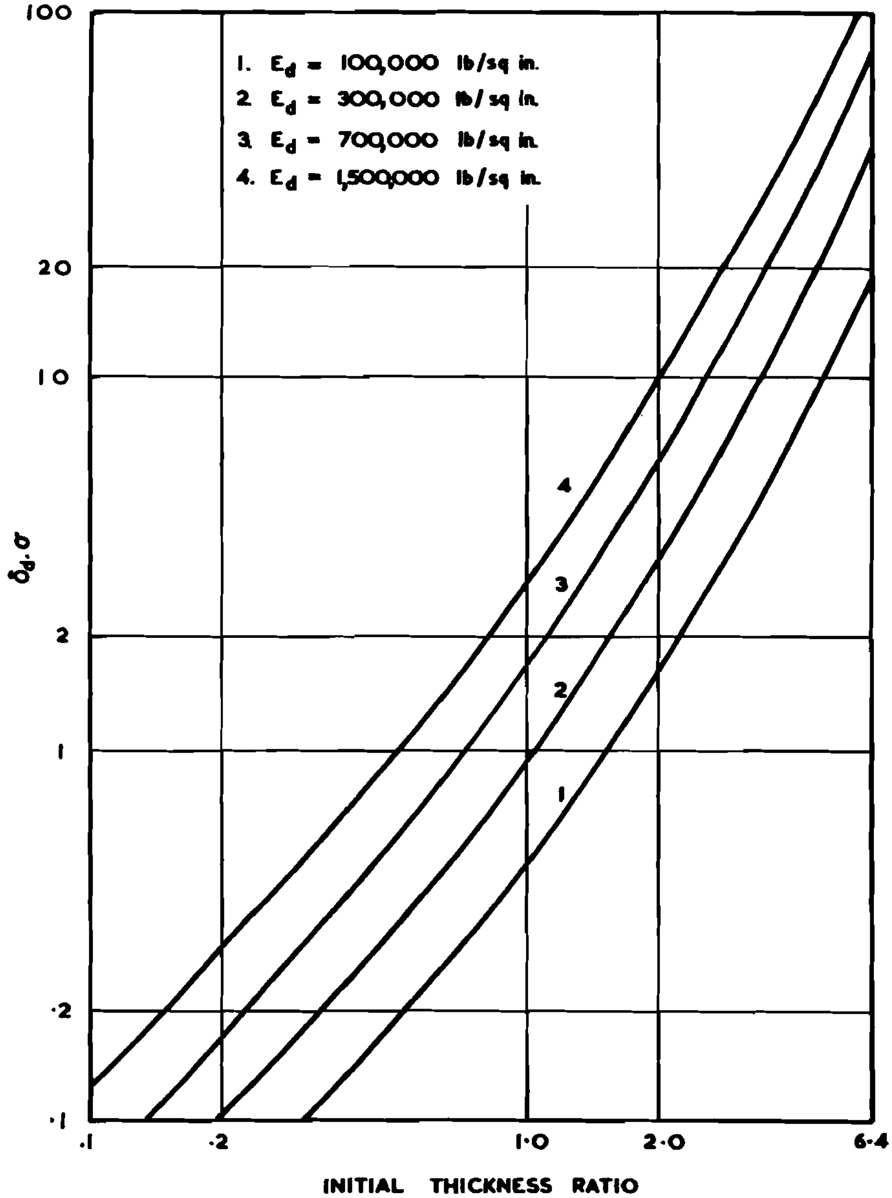


FIG. 7. VARIATION OF MAXIMUM VALUE OF PRODUCT $\delta_d \sigma$ WITH THICKNESS OF TREATMENT.

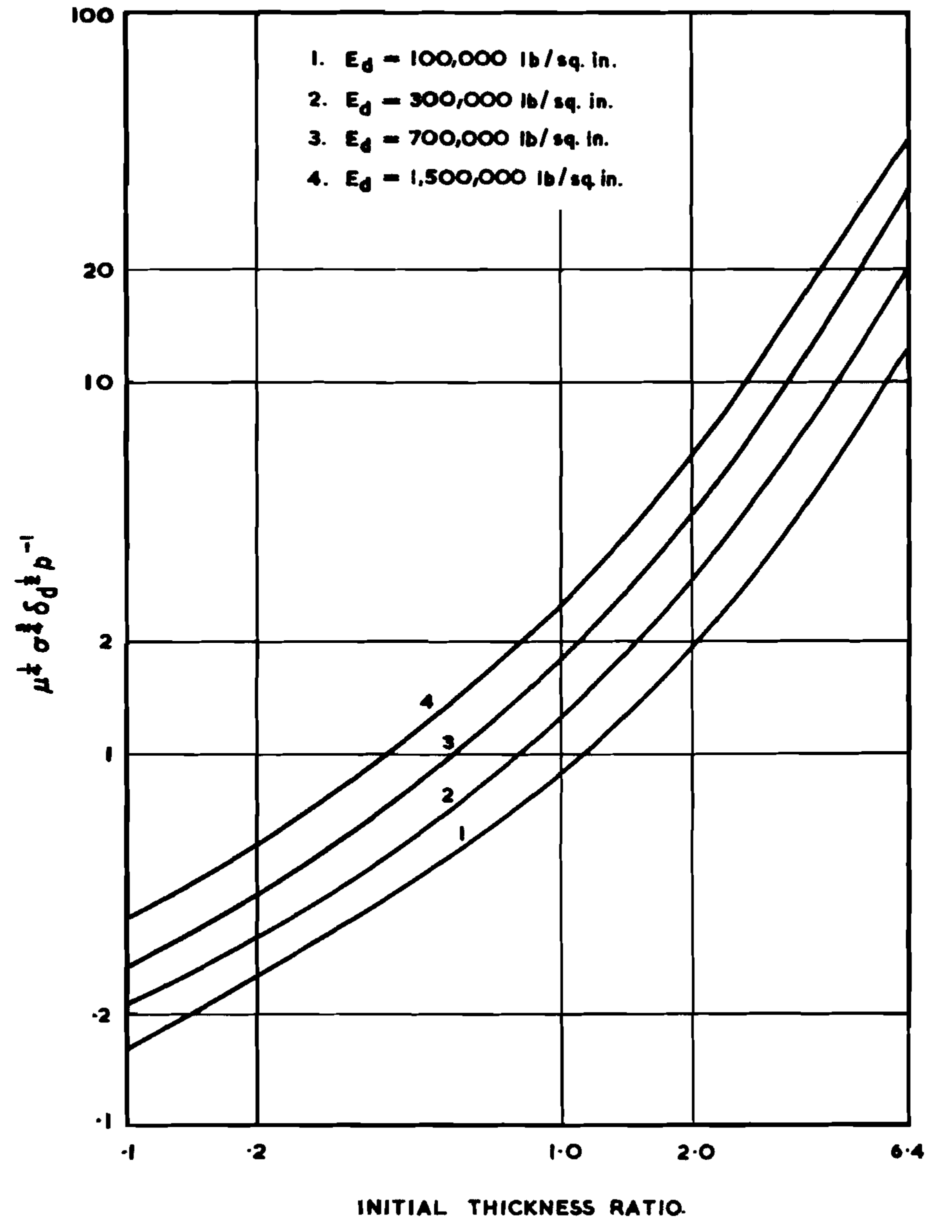


FIG. 8. VARIATION OF MAXIMUM VALUE OF R.M.S. DISPLACEMENT CRITERION, $\mu^{\frac{1}{4}} \sigma^{\frac{3}{4}} \delta_d^{\frac{1}{4}}/p$, WITH THICKNESS OF TREATMENT.

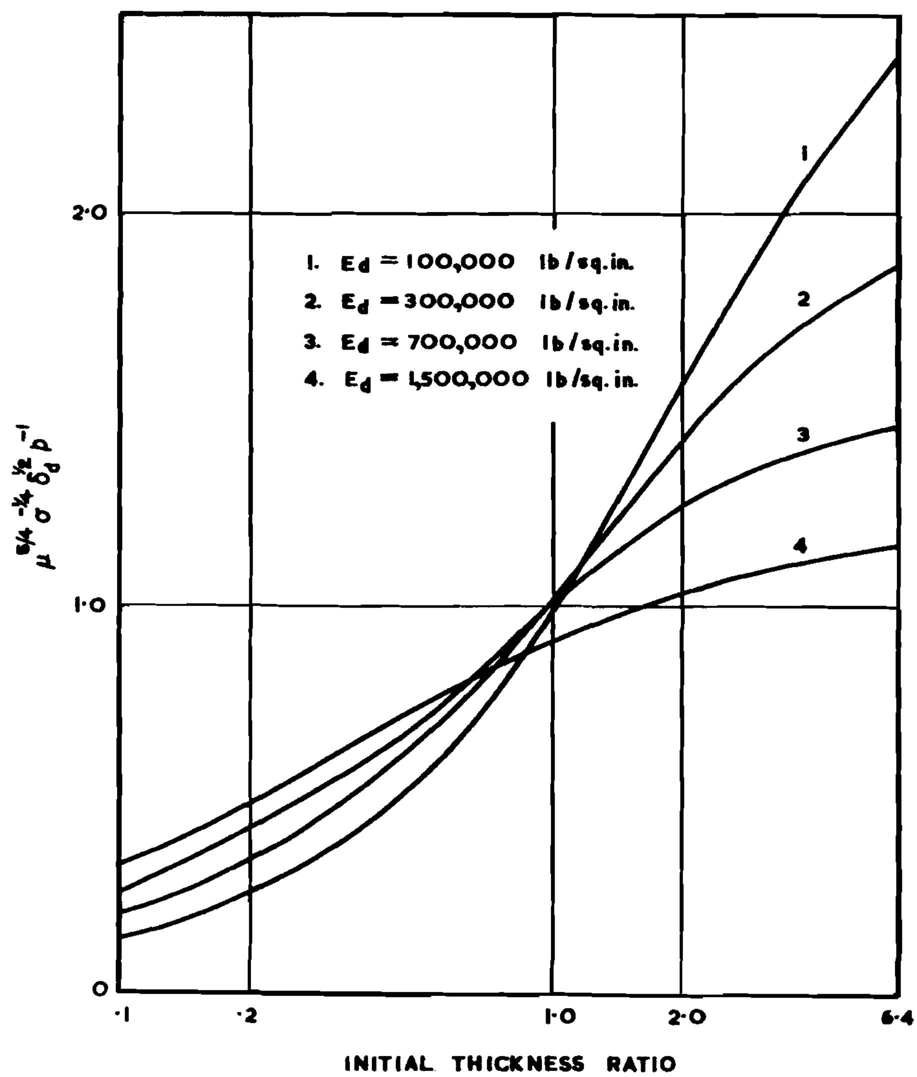


FIG. 9. VARIATION OF MAXIMUM VALUE OF R.M.S. SOUND PRESSURE CRITERION, $\mu^{5/4}\sigma^{-1/4}\delta_d^{1/2}/p$, WITH THICKNESS OF TREATMENT.

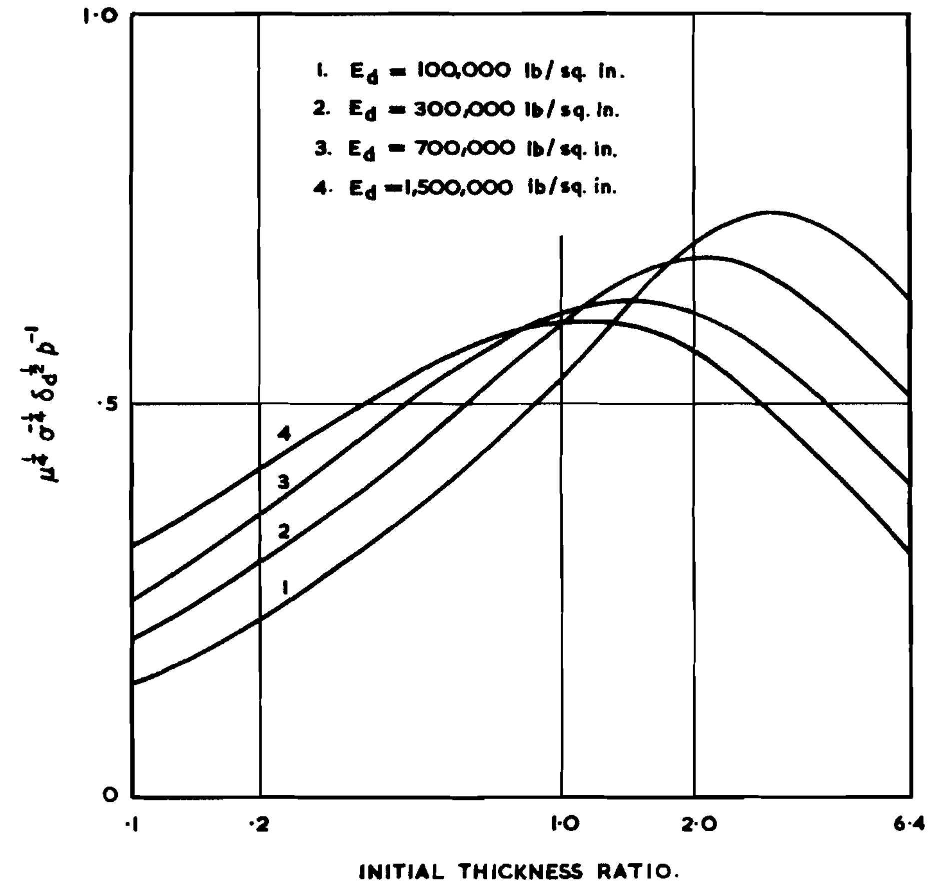


FIG. 10. VARIATION OF MAXIMUM VALUE OF R.M.S. INERTIA FORCE CRITERION, $\mu^{\frac{1}{4}}\sigma^{-\frac{1}{4}}\delta_d^{\frac{1}{2}}/p$, WITH THICKNESS OF TREATMENT.

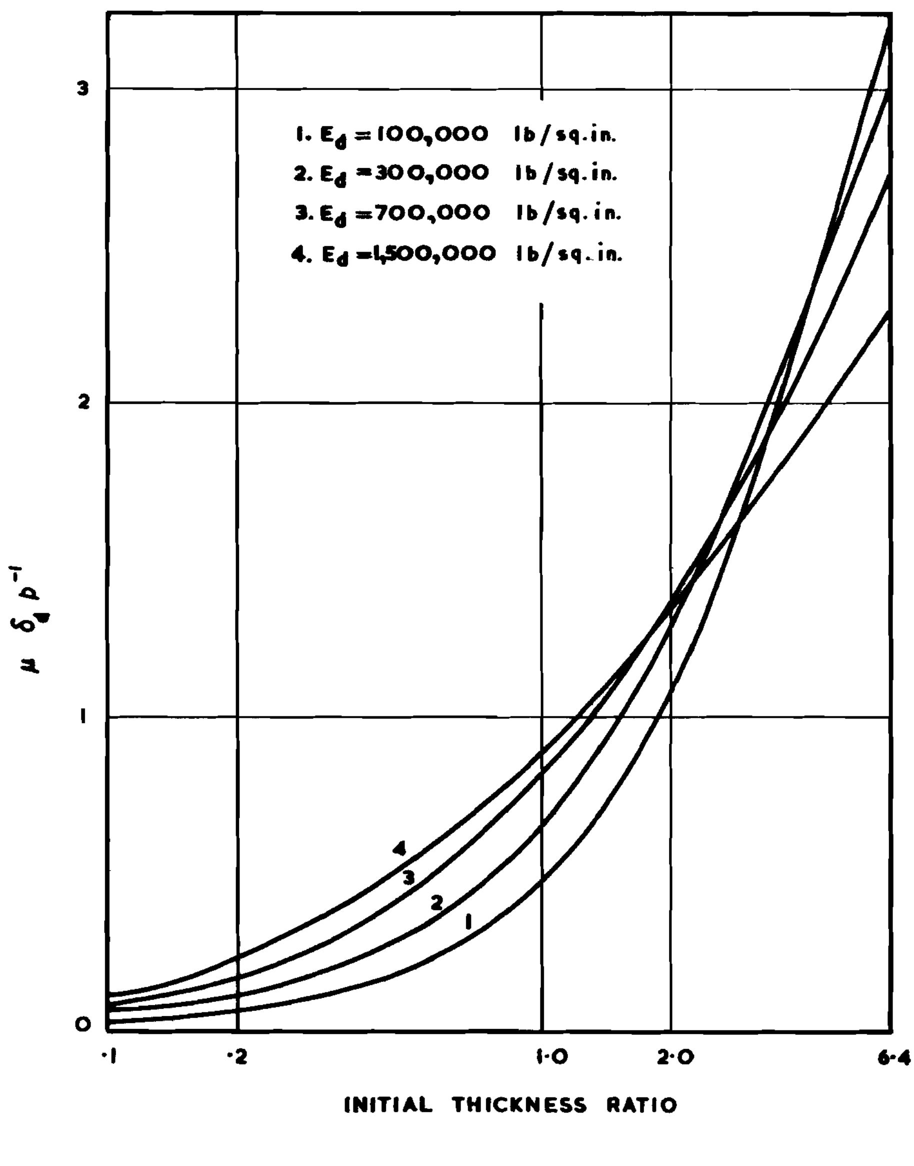
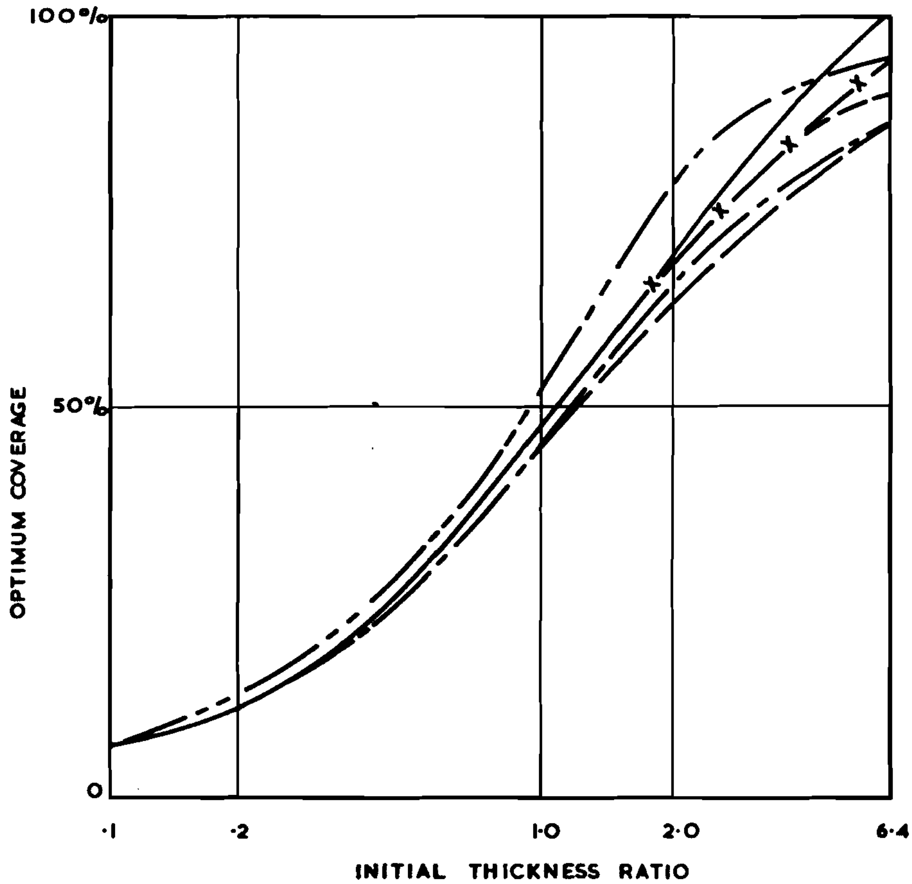


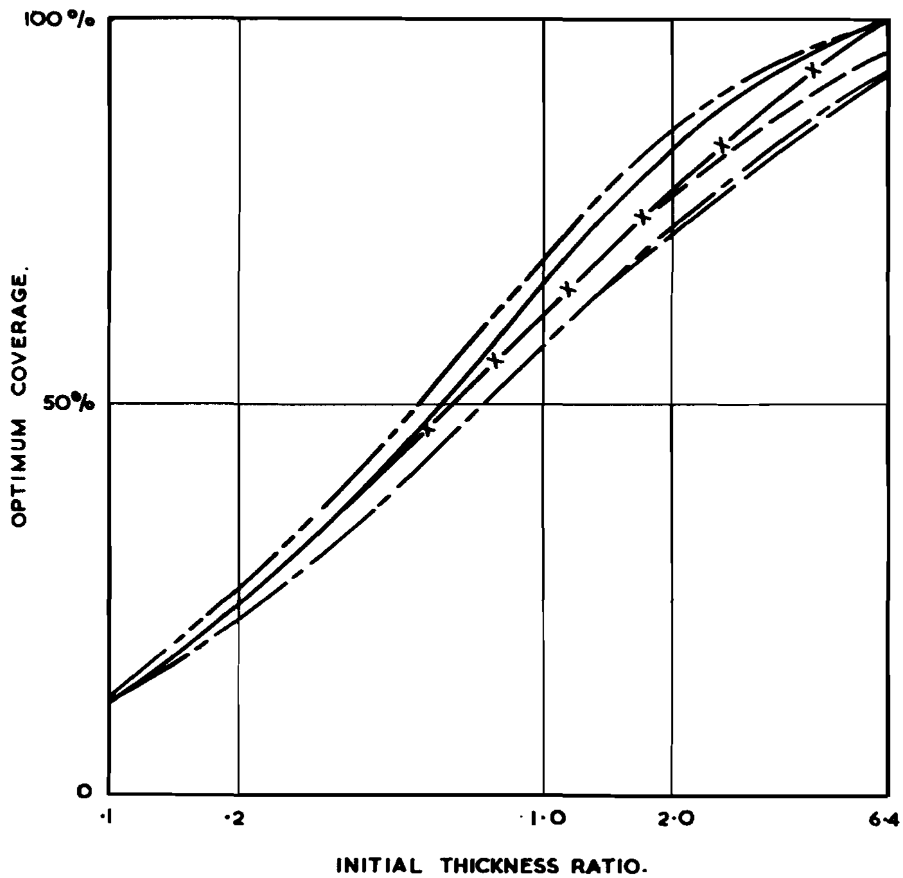
FIG. 11. VARIATION OF MAXIMUM VALUE OF HARMONIC SOUND TRANSMISSION CRITERION, $\mu\ \delta_d / p$, WITH THICKNESS OF TREATMENT.



E_d = 100,000 lb/sq.in.

FOR KEY SEE FIG. 2.

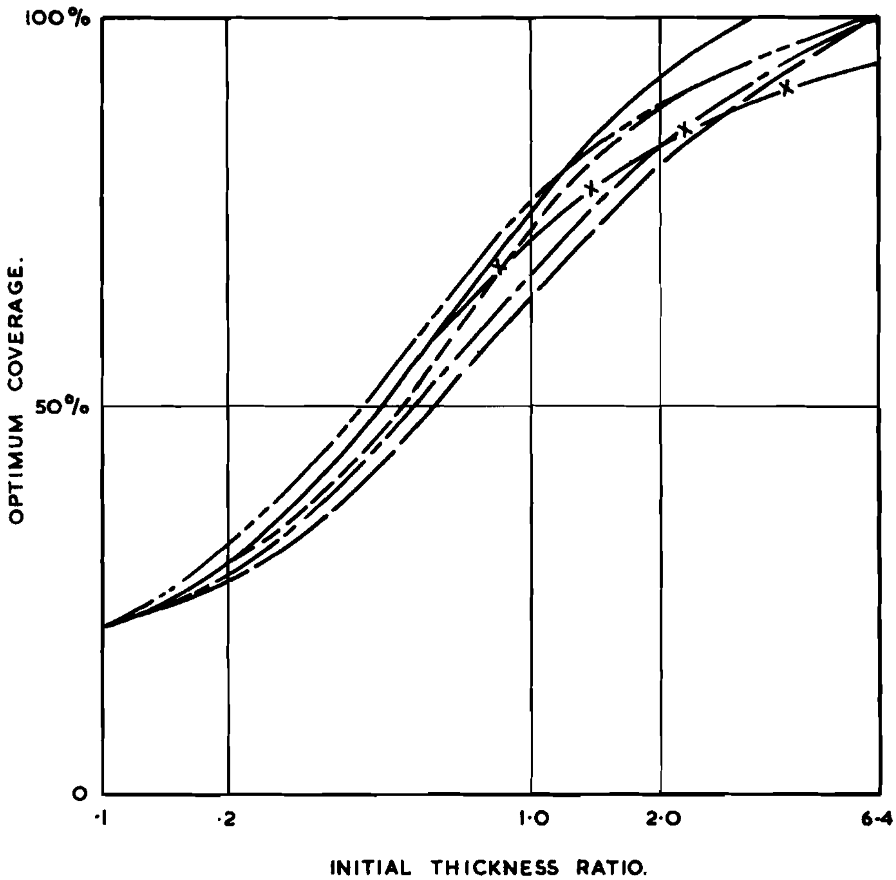
FIG. 12. OPTIMUM COVERAGE FOR THE MAXIMUM VALUES OF THE CRITERIA



E_d = 300,000 lb/sq. in.

FOR KEY SEE FIG. 2.

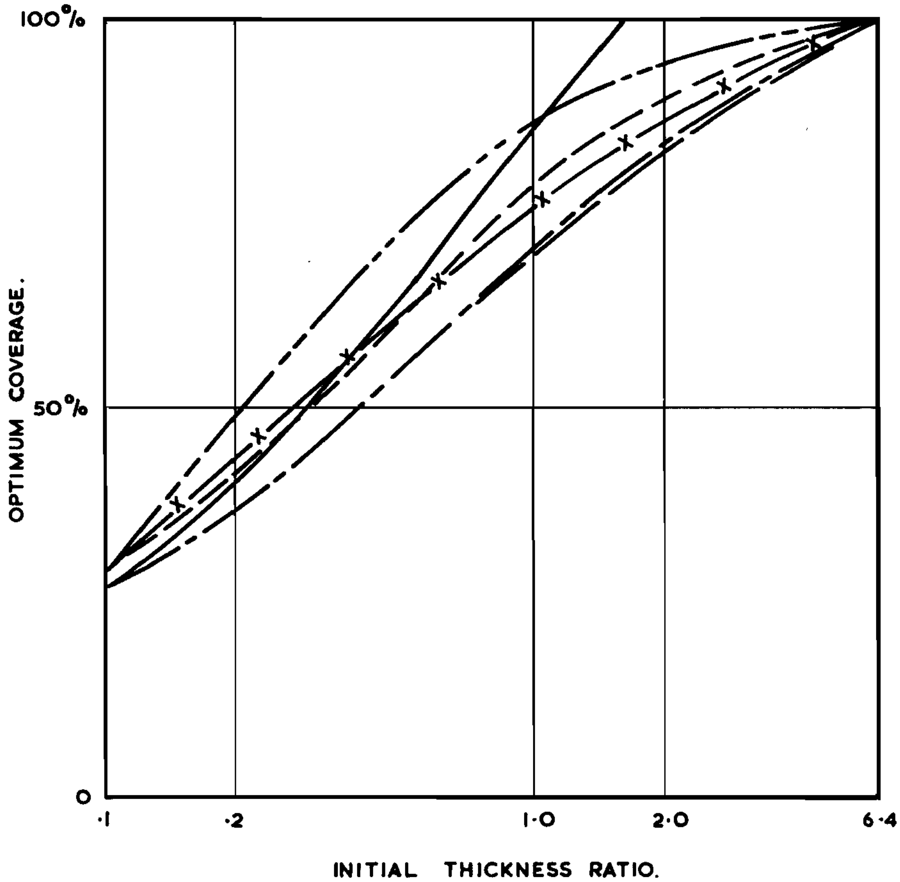
FIG. 13. OPTIMUM COVERAGE FOR THE MAXIMUM VALUES OF THE CRITERIA.



E_d = 700,000 lb/sq. in.

FOR KEY SEE FIG. 2.

FIG. 14 OPTIMUM COVERAGE FOR THE MAXIMUM VALUES OF THE CRITERIA.



E_d = 1,500,000 lb/sq. in.

FOR KEY SEE FIG. 2.

FIG. 15. OPTIMUM COVERAGE FOR THE MAXIMUM VALUES OF THE CRITERIA.

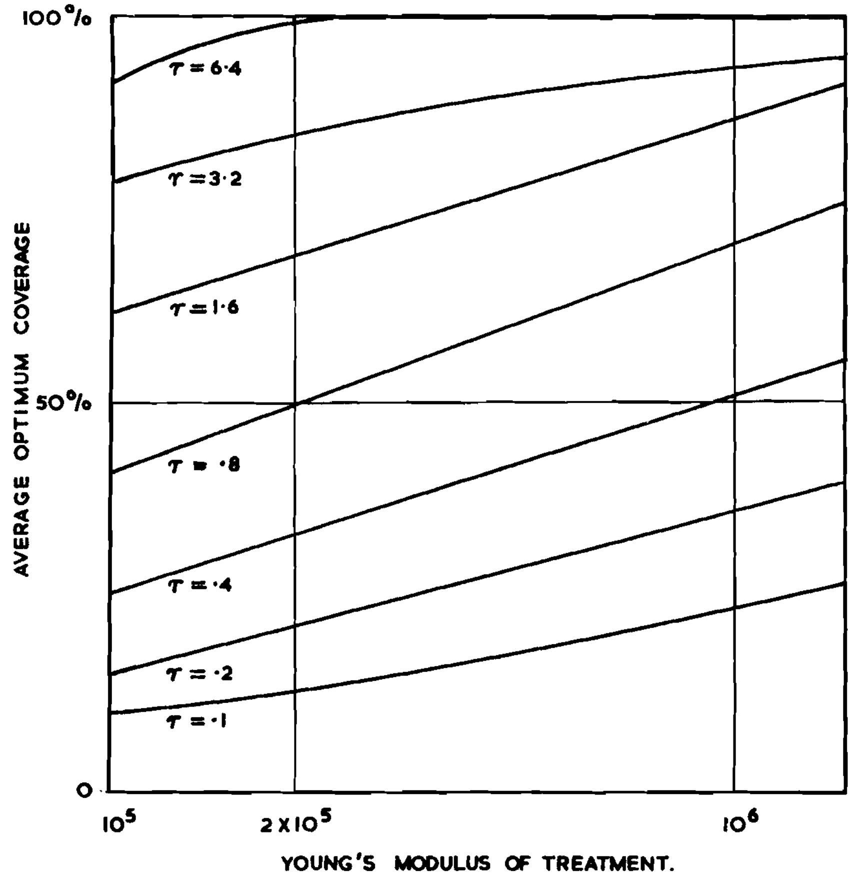
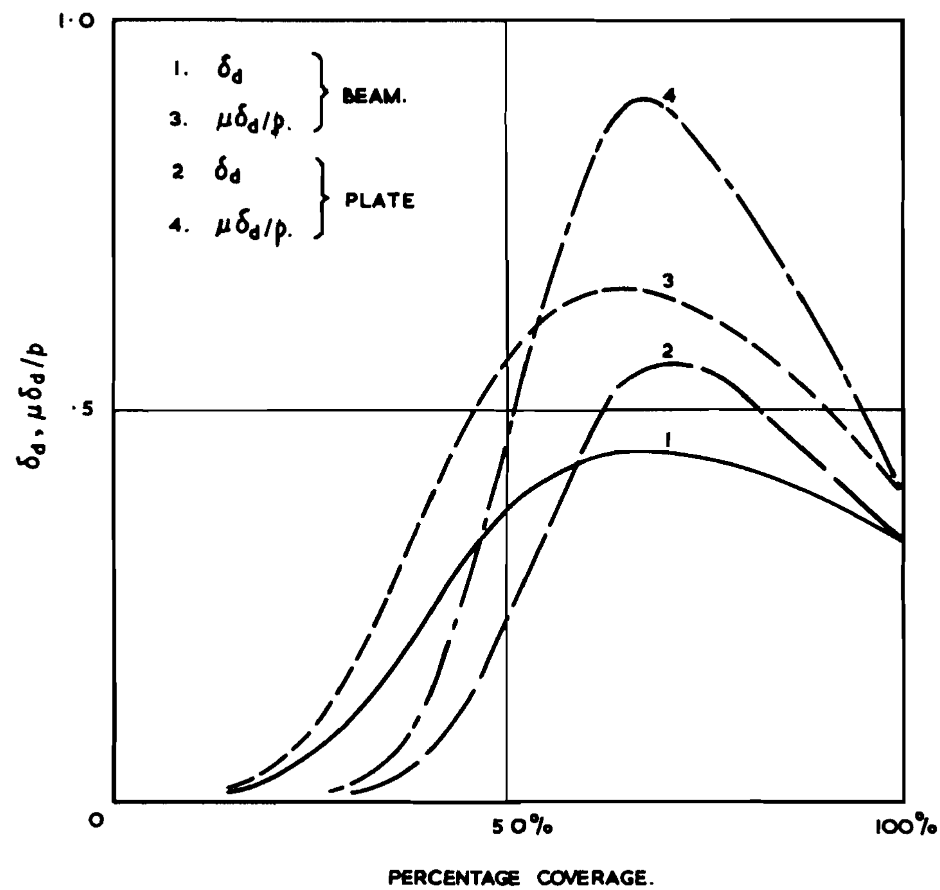


FIG. 16. VARIATION OF THE AVERAGE OPTIMUM COVERAGE WITH THE YOUNG'S MODULUS OF THE TREATMENT.



E_d = 700,000 lb/sq. in. $\quad T$ = ·8

FIG. 17. VARIATION OF THE VALUE OF δ_d, AND $\mu\,\delta_d/p$, WITH PERCENTAGE COVERAGE. COMPARISON OF PLATE AND BEAM THEORIES.

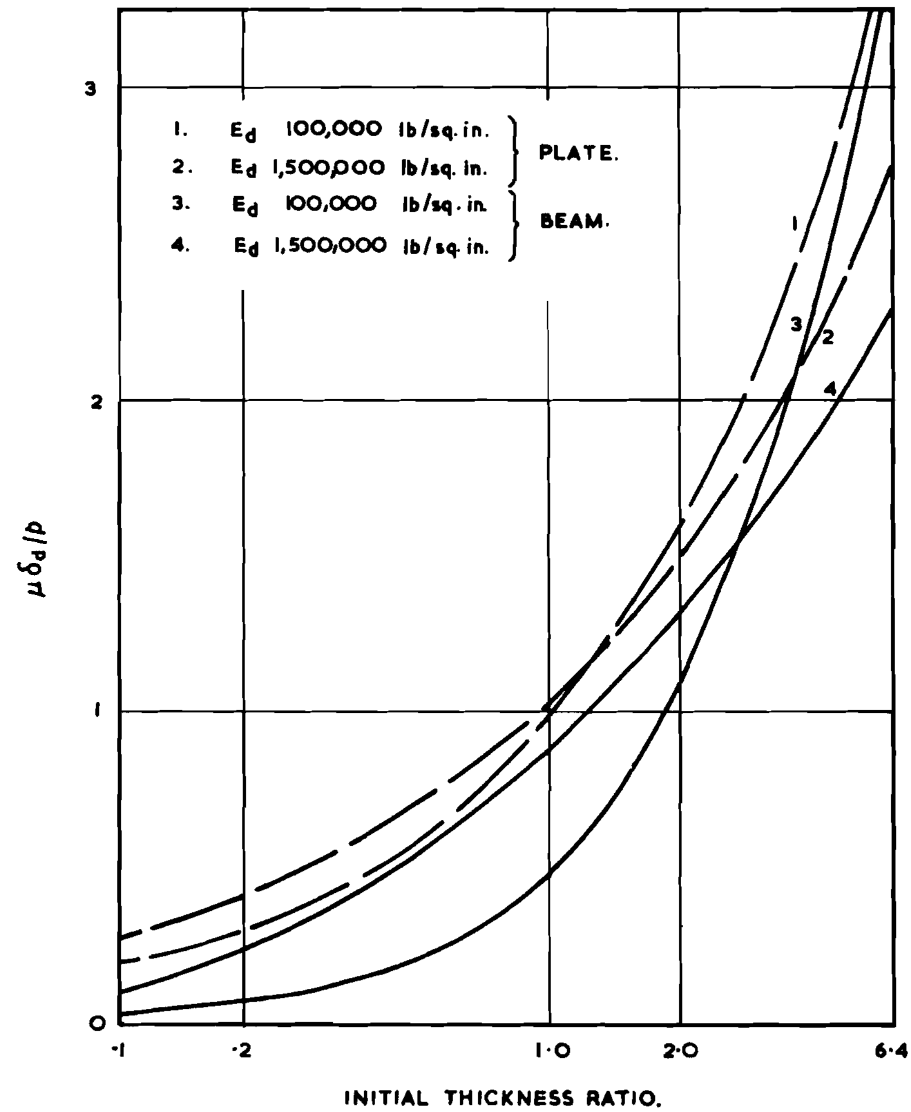
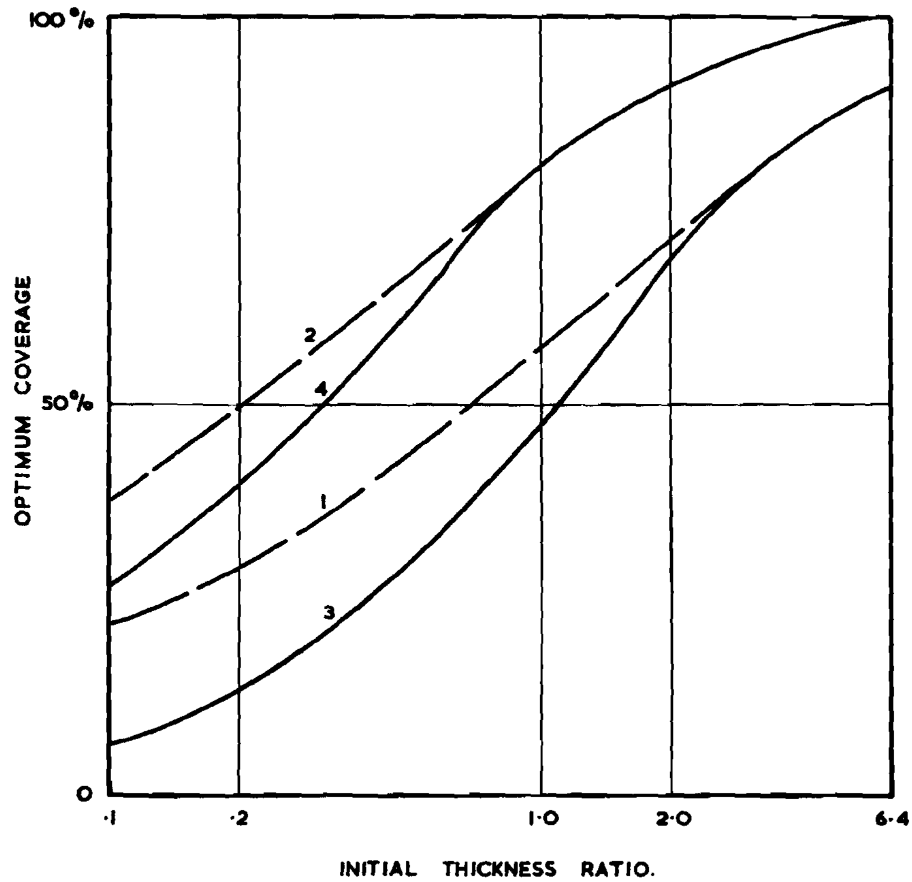
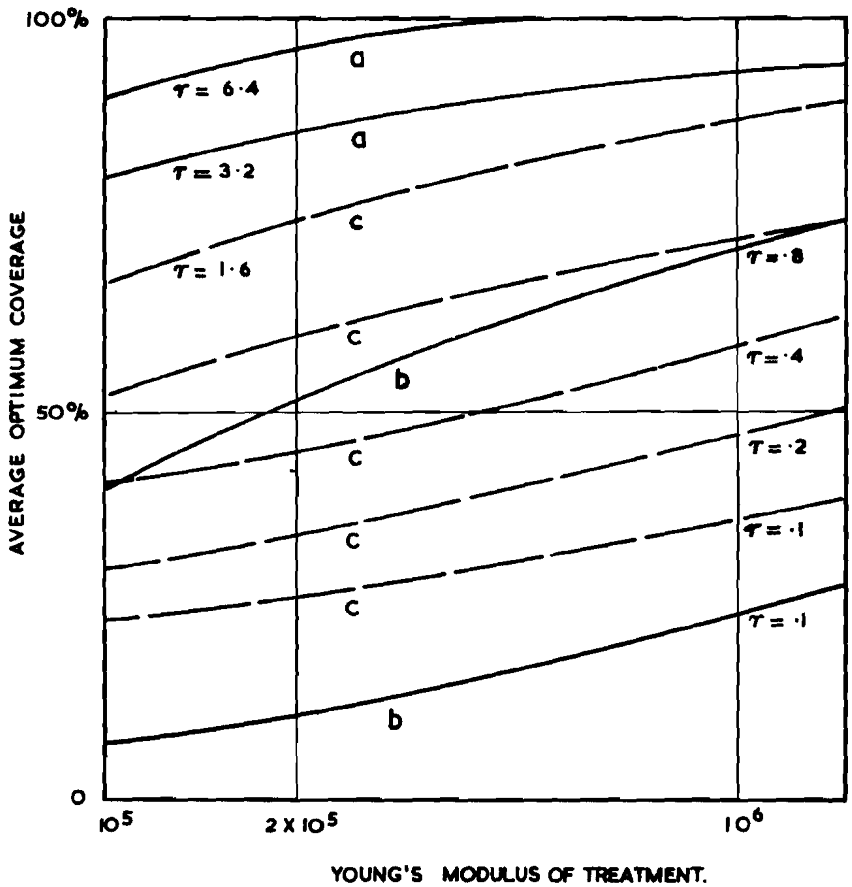


FIG. 18. VARIATION OF MAXIMUM VALUE OF HARMONIC SOUND TRANSMISSION CRITERION, $\mu\delta_d/p$, WITH THICKNESS OF TREATMENT. COMPARISON OF PLATE AND BEAM THEORIES.



FOR KEY SEE FIG. 18.

FIG. 19. OPTIMUM COVERAGE FOR THE MAXIMUM VALUES OF THE HARMONIC SOUND TRANSMISSION CRITERION. COMPARISON OF PLATE AND BEAM THEORIES.



a...PLATE AND BEAM b....BEAM c.... PLATE.

FIG. 20. VARIATION OF THE AVERAGE OPTIMUM COVERAGE WITH THE YOUNG'S MODULUS OF THE TREATMENT. COMPARISON OF PLATE AND BEAM THEORIES.

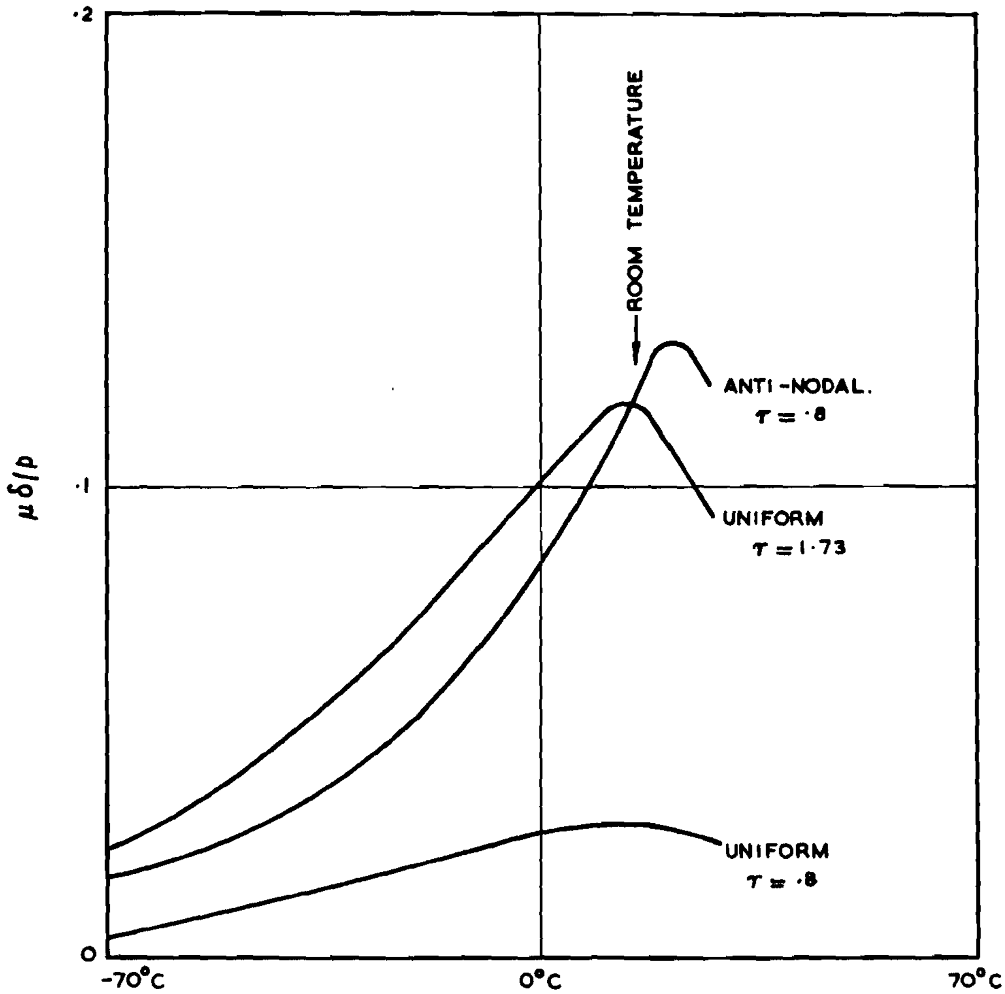


FIG. 21. VARIATION OF HARMONIC SOUND TRANSMISSION CRITERION WITH TEMPERATURE, FOR A WIDE BAND-WIDTH TREATMENT, WITH THE CRITERION OPTIMISED AT ROOM TEMPERATURE FOR ANTI-NODAL TREATMENT.

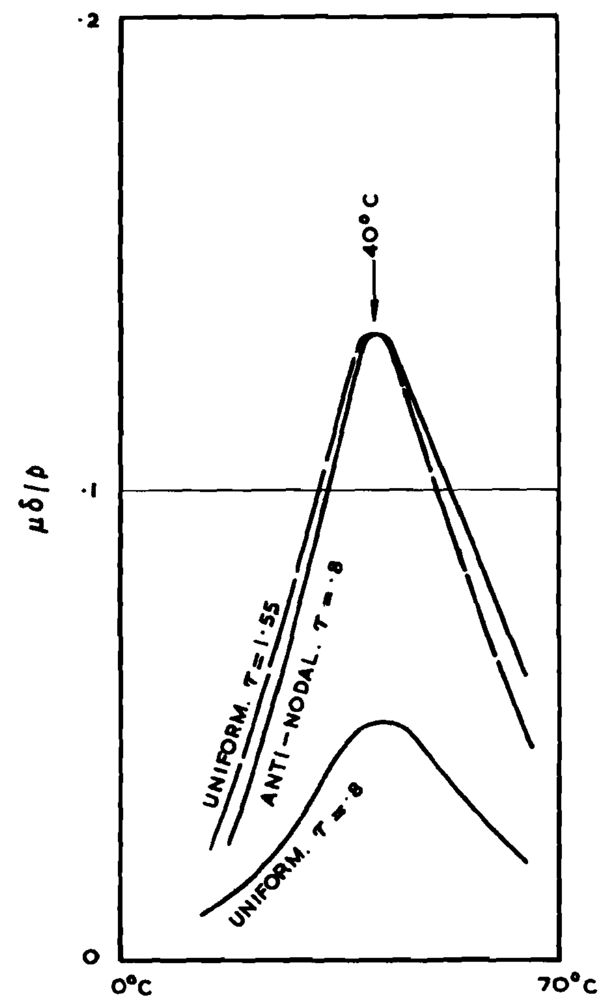


FIG. 22. VARIATION OF HARMONIC SOUND TRANSMISSION CRITERION WITH TEMPERATURE, FOR A NARROW BAND-WIDTH TREATMENT, WITH THE CRITERION OPTIMISED AT 40°C FOR THE ANTI-NODAL TREATMENT.