FOREWORD

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ABSTRACT

The behavior of a dynamic damper in a simple structural environment is studied. The structural model used is a stretched string under harmonic loading at the ends. The aim of the analysis is to establish conditions under which maximum damping is obtainable. The unsymmetrical modes are undamped and the analysis is concerned only with the damping of the symmetrical modes.

This technical documentary report has been reviewed and is approved.

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ILLUSTRATIONS

SYMBOLS

A, B	Arbitrary constants.
c	Velocity of transverse waves in string $=\sqrt{(T/\rho)}$.
$f_n(\alpha)$	n th root of $\cos(\xi) + (\alpha/\xi)\sin(\xi) = 0$, arranged in ascending values of ξ.
F	Restoring force produced by damper at $x = 0$.
G	Real part of complex shear modulus $G(1 + i\mu)$ of damping material.
h	Thickness of layer of damping material.
i	$\sqrt{-1}$.
k	ω/c. Wave number.
L	Length of string.
m, n	Integers.
Q	Amplification factor defined as ratio maximum resonant amplitude to maximum static amplitude under same spacewise loading conditions.
S	Load carrying area in shear of damping material.
t	Time.
T	Tension in string.
W(x)	Transverse displacement of string at point x.
W(0)	Transverse displacement of string at $x = 0$.
x	Distance from center of string.
X	Maximum transverse displacement of string supports.
α	$G\ S\ L\ /\ 4Th$.
δ	(1/2Q). Effective damping ratio.
Δ	$2x/L$.
μ	Loss factor of damping material.
ξ	$kL/2$.
ρ	Linear density of string (mass per unit length).
ω	Frequency

INTRODUCTION

A dynamic damper in the form of a mass, connected through a viscoelastic layer acting in shear to an antinodal part of a vibrating structure has recently attracted some interest. However, the mass required is often so large that the weight penalty has been unacceptable. It is sometimes possible to consider, instead, a device with an effectively infinite mass i.e. a connection through the shear damping material layer straight to a fixed point in the structure. In such a case it is of interest to examine the amount of damping that can be introduced in this way, and the conditions for the damping to be an optimum. Since the damping is zero when the static stiffness of the layer is both zero and infinite, the optimum must exist for some finite value of the static stiffness of the material. In the present note, this problem is discussed with reference to a simple system comprising a finite string, with this type of damper at its center, and vibrating in a symmetrical mode. The damper will not, of course, contribute any damping to the unsymmetrical modes in this position.

THEORY

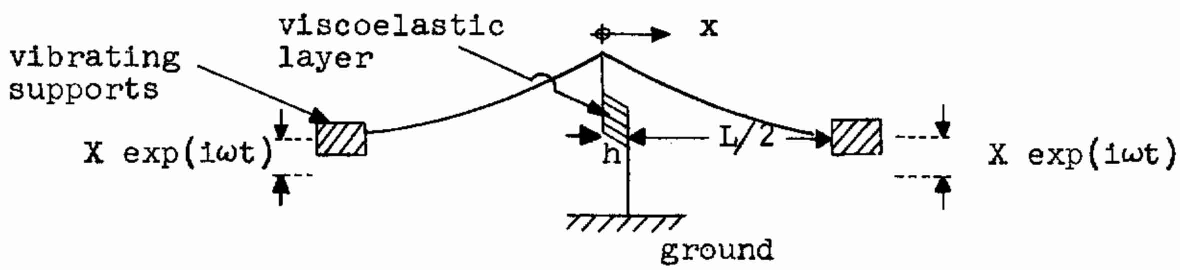

Figure 1. Illustration of Vibrating System

Consider a string of length L fixed to two oscillating supports, each having equal amplitudes X exp($i\omega t$). The damping device is attached at the center of the string. Let the thickness of the layer be h and the load supporting area S. The complex shear modulus of the damping material is $G(1 + i\mu)$ where μ is the loss factor. It is assumed that the mass of the connecting pieces shown is small compared with that of the string (although this is difficult to realize in practice for a string, it will be more representative in this respect of plate or shell structures where the mass added can be made small). The equation of motion of the string is:

$$(d^2W/dx^2) + k^2W = 0 \qquad (1)$$

where $$k = \omega/c \text{ and } c = \sqrt{T/\rho} \qquad (2)$$

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with T the string tension, ρ the linear density of the string and ω the frequency. The solution of (1) is:

$$W(x) = A \operatorname{Cos}(kx) + B \operatorname{Sin}(kx) \qquad (3)$$

The boundary condition at x = L/2 is W = X. The condition at x = 0 is that $2T(\partial W/\partial x)_{x=0}$ is equal to the force produced by the damping layer. Now for the damping layer material:

$$\text{Stress/Strain} = G(1 + i\mu)$$

$$\frac{F/S}{W(0)/h} = G(1 + i\mu)$$

$$\therefore \quad F = (GS/h)(1 + i\mu)\, W(0) \qquad (4)$$

where W(0) is the displacement of the string at x = 0 and F is the restoring force produced by the damper. The solution of (1) which satisfies both these boundary conditions is:

$$W(x) = \left[X - (\alpha/\xi)(1 + i\mu) \operatorname{Sin}(\xi)\, W(0)\right] \operatorname{Cos}(kx)/\operatorname{Cos}(\xi)$$
$$+ \quad (\alpha/\xi)(1 + i\mu)\, W(0) \operatorname{Sin}(kx) \qquad (5)$$

$$\text{with } \xi = kL/2 \text{ and } \alpha = GSL/4Th \qquad (6)$$

If we now put x = 0 in equation (5), we obtain a linear first order equation for W(0), the solution of which is:

$$W(0) = \frac{X}{\operatorname{Cos}(\xi) + (\alpha/\xi)(1 + i\mu)\operatorname{Sin}(\xi)} \qquad (7)$$

Putting (7) back into (5) then gives the equation for W(x) in the form:

$$W(x) = X \left[\frac{\operatorname{Cos}(\xi\Delta) + (\alpha/\xi)(1 + i\mu) \operatorname{Sin}(\xi\Delta)}{\operatorname{Cos}(\xi) + (\alpha/\xi)(1 + i\mu) \operatorname{Sin}(\xi)}\right] \qquad (8)$$

$$\text{with } \Delta = 2x/L \qquad (9)$$

From equation (8), it is seen that the condition for resonance is:

$$\operatorname{Cos}(\xi) + (\alpha/\xi) \operatorname{Sin}(\xi) = 0$$

$$\text{i.e.} \quad \xi = f_n(\alpha) \qquad (10)$$

where n is the number of the symmetrical mode being investigated and $f_n(\alpha)$ is the n th root of $\operatorname{Cos}(\xi) + (\alpha/\xi)\operatorname{Sin}(\xi) = 0$. Note that ξ lies between $\pi/2$ and π for the first mode (n = 1), as α varies from 0 to ∞. From (8) the amplitude at resonance is seen to be:

$$\frac{|W(x)|}{X} = \frac{\sqrt{[\cos(\xi\Delta) + (\alpha/\xi)\sin(\xi\Delta)]^2 + (\alpha^2\mu^2/\xi^2)\sin(\xi\Delta)}}{(\alpha\mu/\xi)\,(\xi/\sqrt{\xi^2+\alpha^2})} \tag{11}$$

where ξ is given by equation (10) and $\sin(\xi) = -\xi/\sqrt{\xi^2+\alpha^2}$ from (10). If μ is less than 0.2 or so, the term involving μ^2 will be negligible in equation (11) for all values of α and, to a sufficient degree of approximation, (11) then becomes:

$$\frac{\xi\alpha\mu|W(x)|}{X\sqrt{\xi^2+\alpha^2}} = \left[\cos(\xi\Delta) + (\alpha/\xi)\sin(\xi\Delta)\right]\xi \tag{12}$$

$$= \sqrt{\xi^2+\alpha^2}\,\sin\left[\xi(1-\Delta)\right] \tag{13}$$

Therefore $W(x)$ is a maximum when:

$$\xi(1-\Delta) = m\,\pi/2$$

$$\therefore \quad \Delta = 1 - m\,\pi/2\xi = 1 - m\,\pi/2f_n(\alpha) \tag{14}$$

where m is an odd integer. The number of values of m for which $0 \leq \Delta \leq 1$ is limited e.g. for n = 1, only one value of m exists subject to this requirement, namely m = 1. From equation (14) it is seen that $\Delta = 0$ when $\alpha = 0$ and $\Delta = 0.5$ when $\alpha = \infty$ for n = 1. A graph of Δ against α is plotted in Figure (2) for several values of n and m. Putting (14) back into (12) now gives the peak value of $|W(x)|$ for given α in the form:

$$\left[\frac{\xi\alpha\mu\,|W(x)|}{X(\xi^2+\alpha^2)^{1/2}}\right]_{max} = (\xi^2+\alpha^2)^{1/2}$$

$$\text{i.e.} \quad |W(x)|_{max} = X\left[f_n^2(\alpha) + \alpha^2\right]/\alpha\mu\,f_n(\alpha) \tag{15}$$

Now at zero frequency, $\xi = 0$ and so $W = X$ everywhere, by equation (8). The maximum value of $|W(x)|$ at zero frequency is therefore X. The amplification factor Q, defined as the ratio of the peak resonant amplitude to the peak static amplitude, therefore becomes:

$$Q = \left[f_n^2(\alpha) + \alpha^2\right]\Big/\alpha\mu\,f_n(\alpha) \tag{16}$$

For small amounts of damping corresponding to small values of μ ($\mu < 0.2$ or so), the equivalent damping ratio is defined as for viscous damping i.e. as 1/2Q.

$$\delta = 1/2Q = \alpha\mu\,f_n(\alpha)\Big/\,2\left[f_n^2(\alpha) + \alpha^2\right] \tag{17}$$

We note from equation (17) that $\delta \to 0$ as $\alpha \to 0$ and as $\alpha \to \infty$. The optimum value of δ occurs for an intermediate value of α. Figure (3) shows the variation of $4\delta/\mu$ with α for the first few modes. For the first mode, it is seen that $\alpha \sim 2.3$ for an optimum.

The physical significance of α is of great importance. Consider the static displacement of the string of length L by an amount W(0) due to a force F at the center. The displacement function for the static case will vary linearly from W(0) at the center to zero at the ends. The restoring force due to the string tension will therefore be:

$$F = 2T(\partial W/\partial x)_{x = 0}$$

i.e. $$F = 2T\,W(0)/(L/2) = (4T/L)\,W(0)$$

i.e. the static stiffness of the string under a point load at the point of application of the damping device is 4T/L in this case. Similarly, for the viscoelastic layer, equation (4) shows that the static stiffness of the damper is GS/h.

$$\alpha = GSL/4Th = (GS/h)\,/\,(4T/L)$$

$$= \frac{\text{static stiffness of damper}}{\text{static stiffness of string}}$$

i.e. the static stiffness of the damper must be in a fixed ratio to that of the string (referred to the point of application of the damper) for the damping to be a maximum. This conclusion will be relevant to far more complex systems than this, although the exact relationship between the two stiffnesses need not be maintained (Figure 4).

CONCLUSIONS

Although the mathematical model used to represent the structure on which the damper operates was so simple, the analysis has highlighted a number of features which, at least qualitatively, can be expected to occur for far more complex structures and has provided some insight into the parameters likely to be involved.

An optimum ratio of damper stiffness to structure stiffness at the point of application of the damper is found to exist, for which the damping ratio is a maximum. For dampers much stiffer than this optimum, the antinodal point at which the damper is placed becomes more and more constrained and eventually becomes fixed i.e. a nodal point. This is, of course, physically obvious but it is useful to know what damper stiffness to give consideration to when designing experiments on more complex structures, so as to avoid this possibility, and this was the aim of the present analysis.

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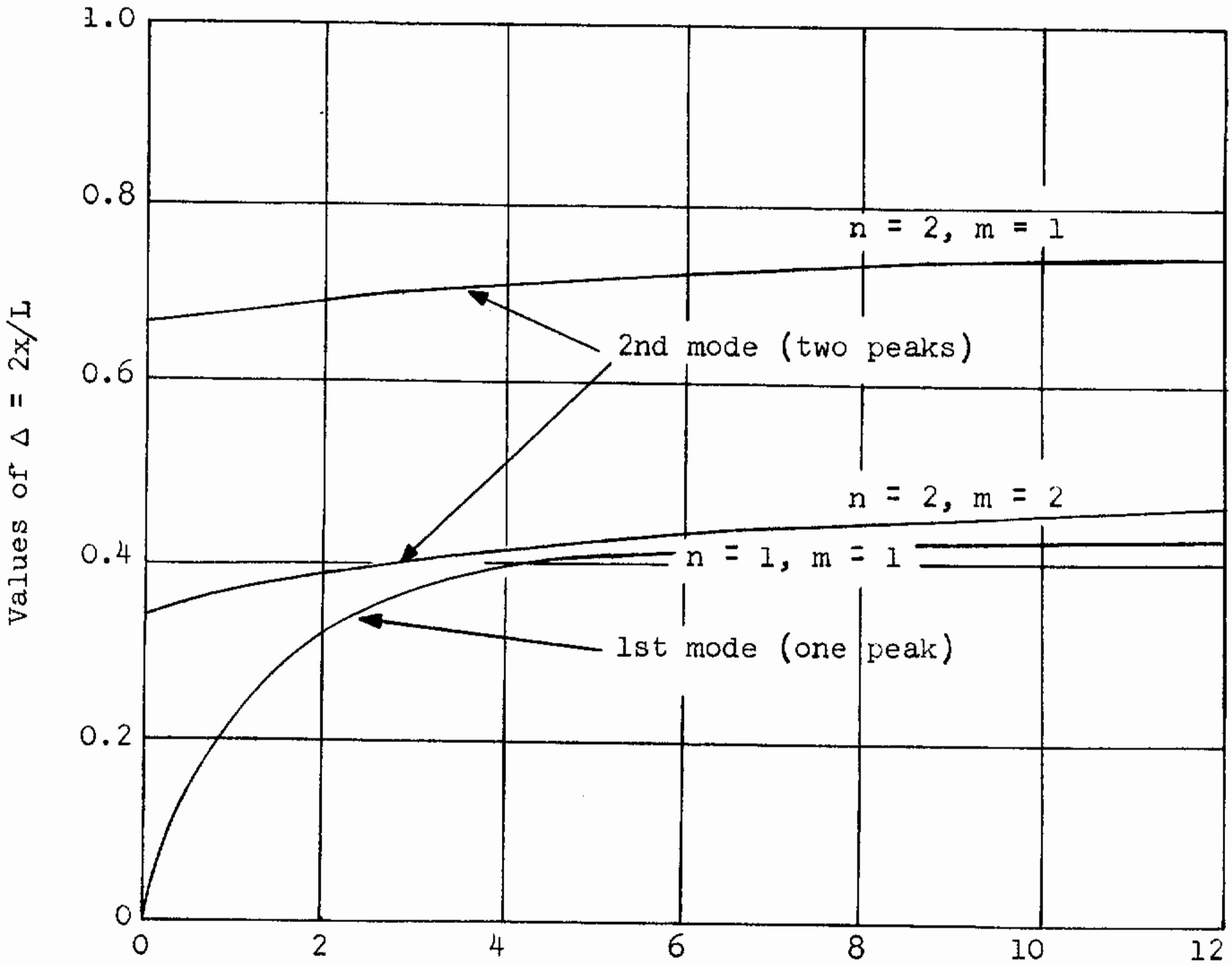
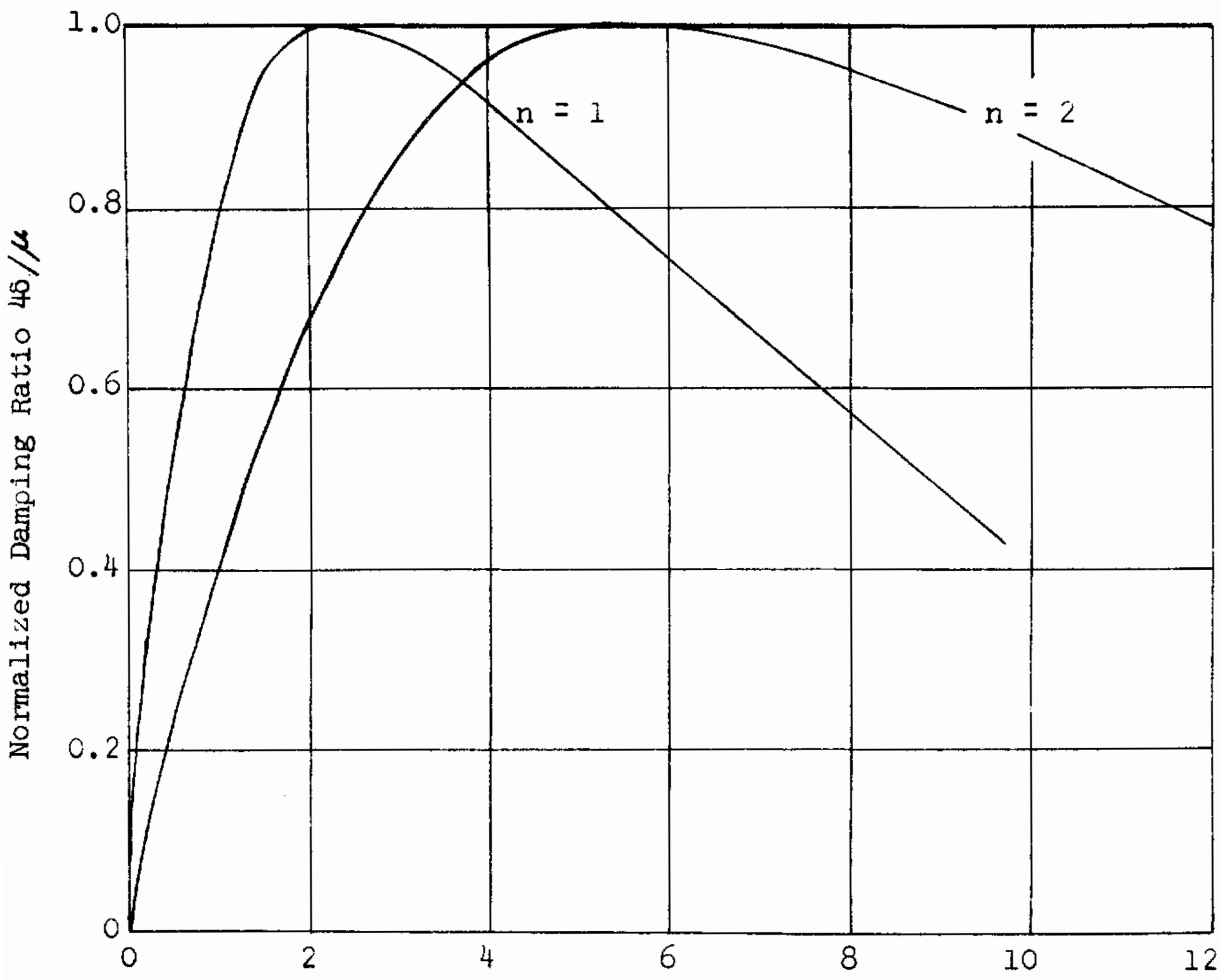


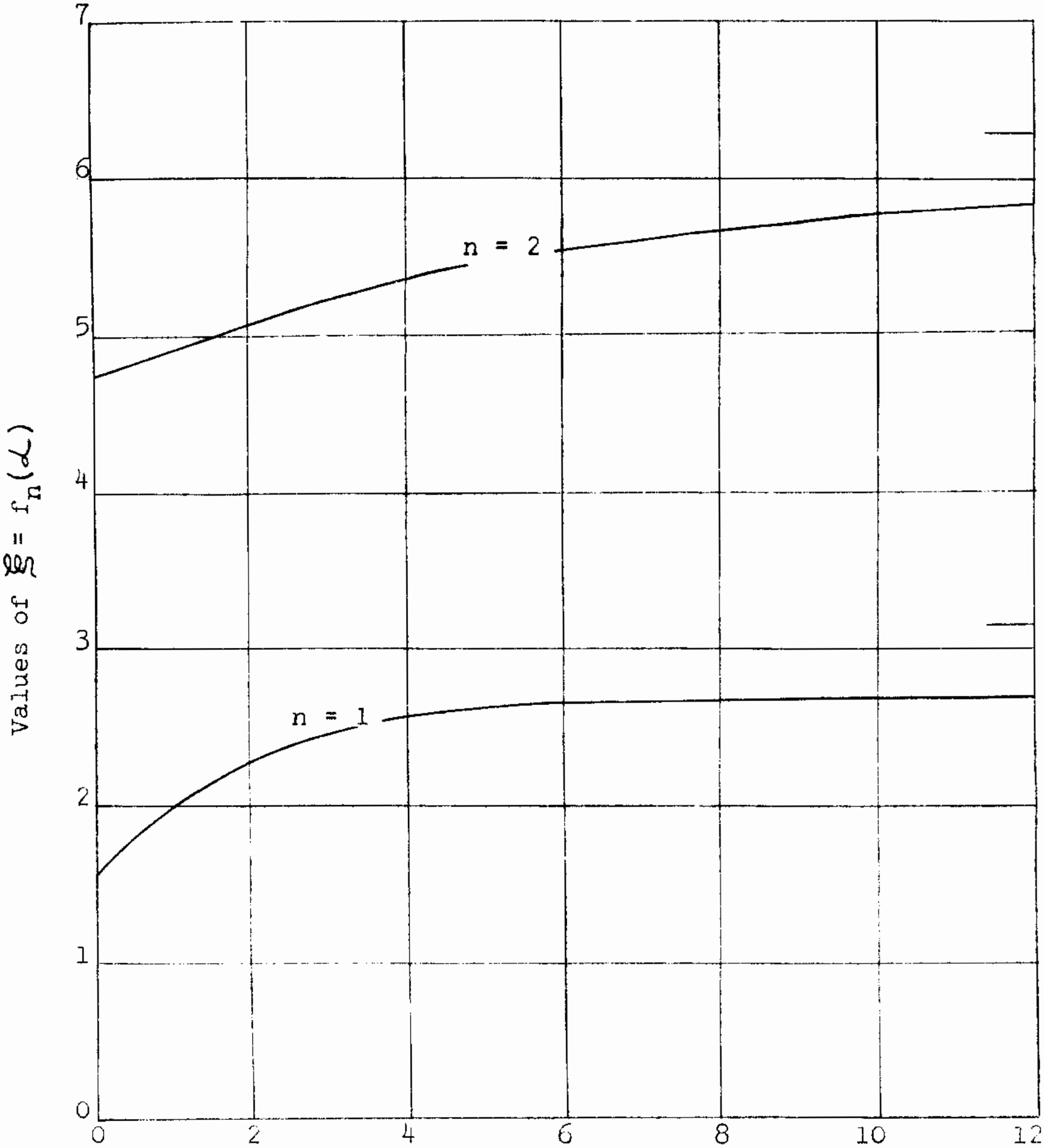
Static stiffness of damper / Static stiffness of string = α

Figure 2. Variation of positions of peak resonant amplitude with damper stiffness.



Static stiffness of damper / Static stiffness of string = α

Figure 3. Effect of damper stiffness on effective damping ratio.



Static stiffness of damper / Static stiffness of string = α

Figure 4. Roots of $\cos(\xi) + (\alpha/\xi)\sin(\xi) = 0$.