EVALUATION AND UTILIZATION OF AIRPLANE FLIGHT LOADS DATA

PART I. TECHNIQUES AND UNCERTAINTIES

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FOREWORD

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ABSTRACT

This report summarizes the important factors to be considered in flight loads investigations. Flight loads are defined as the forces and moments to which an airplane structure is subjected while the vehicle is airborne and are treated here as random phenomena. Evaluation of flight loads then consists of estimating the statistical properties of the underlying processes from sample measurements. Methods appropriate for this purpose are discussed along with the associated estimation errors. Current methods for computing loads from flight data are reviewed and analyzed. Then, the utilization of the computed loads in structural design applications is described. The data requirements for both design criteria and design substantiation are discussed. Operational flight loads data are covered under the latter heading.

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LIST OF SYMBOLS

A	proportionality constant; real valued coefficient
a	linear acceleration
a	system constant
a	lift curve slope
B	dimensional constant
B	data frequency bandwidth
B_f	spectral density resolution bandwidth
b	system constant
b[ ]	bias error of [ ]
C_{hs}	static aerodynamic influence coefficient
C_{m_0}	pitching moment coefficient
C(f)	coincident spectral density function
$\bar{c}$	mean aerodynamic chord
E[ ]	expected value of [ ]
E(f)	Fourier transform of e(t)
e	strain

F	linear force
$F(\ )$	cumulative frequency of ( )
$\mathcal{F}[\ ]$	Fourier transform of [ ]
f	frequency
f_n	undamped natural frequency
f_c	cutoff or folding frequency
f_0	sinusoidal frequency
Δf	frequency interval
$G(f)$	power or cross-spectral density function
g	acceleration produced by gravity
$H(f)$	frequency response function (complex)
$\lvert H(f)\rvert$	gain factor (magnitude of the frequency response function)
h	control point deflection
$h(\tau)$	impulse response function
I	unit impulse
$[I]$	unit matrix
I	moment of inertia
J	total number of terms in a sum
j	$\sqrt{-1}$

K	total number of intervals in a histogram
K_o	static sensitivity of a system
K_g	gust factor for rigid body assumption
K_ϕ	gust factor for flexible assumption
L	loads variable
L_{max}	extreme load amplitude
ΔL	load amplitude interval
M	loads variable
M	moment
M	mass
MS	mean square
$\hat{M}(f)$	ratio of actual to calibrated frequency response functions
m	number of binary digits (bits) in a binary word
$\hat{m}(\tau)$	Fourier transform of $\hat{M}(f)$
$m(y)$	spanwise mass distribution function
N	sample size
N_α	number of crossings of the level α
$N(\alpha)$	rate of crossing for the level α
$N^+(0)$	rate of crossing with positive slope for the level zero
$N(f)$	Fourier transform of $n(t)$

n	ideal system order
n	number of structural inputs
n	load factor
n(t)	extraneous noise in an output measurement
P	probability
Prob [ ]	probability that [ ]
P( )	probability distribution function for ( )
p	rolling velocity
$\dot{p}$	rolling acceleration
p	pressure
p( )	probability density function for ( )
Q(f)	quadrature spectral density function
q	pitching velocity
$\dot{q}$	pitching acceleration
q_d	dynamic pressure
R(τ)	auto- or cross-correlation function
RMS	root-mean-square
r	yawing velocity
$\dot{r}$	yawing acceleration

S	wing area
SE [ ]	standard error of [ ]
s	wing semi-span
T	sample record length
T	time period
t	time
U	total number of usage categories
U_{de}	derived gust velocity
V_e	equivalent airspeed
VGH	velocity-acceleration-altitude data combination
Var [ ]	variance of [ ]
W	aircraft weight
w	normal gust velocity
X	full range or peak value of x
X(f)	Fourier transform of x(t)
x	distance along the X-axis
x	variable which can be measured in flight
Δx	interval between quantizing levels for x
x(t)	input variable

$Y(f)$	Fourier transform of $y(t)$
y	distance along the Y-axis
$y(t)$	output variable
Z	normally distributed variable with zero mean and unit variance
$Z_{\alpha/2}$	100 $\alpha/2$ percentage point of the standardized normal distribution
z	distance along the Z-axis
α	small probability; level of significance
α	amplitude level
α	angle of attack
β	spectral parameter of the peak probability distribution for Gaussian variables
$\gamma^2(f)$	coherence function (real)
δ	additive error term
ϵ	normalized standard error
ζ	system damping factor
$\theta(f)$	argument of the cross-spectral density function
$\theta(f)$	phase factor (argument of the frequency response function)
μ	true mean value
$\nu(t)$	extraneous noise in an input measurement
ρ	air density
ρ_0	sea level air density

σ^2	true variance
σ	true standard deviation
τ	relative time
τ_1	first order system time constant
ϕ	arbitrary statistical property
Ψ^2	true mean square value
$\hat{\Psi}(f)$	reciprocal of the calibrated frequency response function for a system
$\hat{\psi}(\tau)$	Fourier transform of $\hat{\Psi}(f)$

Subscripts

A	axial
a	acceleration
a	aerodynamic
g	gust
h	horizontal tail
i	inertia
i, j, k, ℓ	arbitrary indices
ℓ	service life
N	normal (vertical with respect to the aircraft body axes)
n	last of a set (except for f_n)

p	peak
r	reference
u	aircraft usage index
wf	wing fuselage
X	with reference to the aircraft longitudinal axis
Y	with reference to the aircraft lateral axis
Z	with reference to the aircraft normal axis

Operations

$(\hat{\ })$	estimate or prediction of ( )
$(\dot{\ })$	time derivative of ( )
$(\ )^*$	complex conjugate of ( )
$(\ )*(\ )$	convolution of ( ) and ( )
$\lvert(\ )\rvert$	absolute value of ( )
$\{\ \}$	column matrix
$[\ ]$	rectangular, square, or diagonal matrix

1. INTRODUCTION

Airplane flight loads are usually defined as the forces and moments to which the primary structure is subjected while the vehicle is airborne. The main sources of flight loads, aside from steady state lifting and balancing forces, include pilot-induced control motion and natural atmospheric turbulence. The first of these is said to produce maneuver loads while the second results in gust loads. Either maneuver or gust loads can be evaluated as the gross forces and moments on the entire vehicle or in terms of the detailed shear, bending moment, and torque on structural components. Other sources of flight loads which are not covered in this report include thermal effects, fuel sloshing, vibration, and acoustics. Also, the unstable conditions of flutter and divergence are not considered to be within the scope of this study.

Maneuver loads are sometimes referred to as an induced environment since they result from the action of the aircraft on the medium. In the same context, gust loads are often termed a natural environment due to the action of the medium on the aircraft. In either case the detailed loads on a structural component represent a summation of all externally applied aerodynamic loads and internally generated inertia loads.

In their simplest forms, basic loads consist of the aerodynamic lift required to support an aircraft in straight and level flight and the reacting weight of the vehicle. For any situation of interest, however, much more complicated descriptions of the flight loads are ordinarily required. Not only is it necessary to consider many variables in such a description, but the behavior of these variables must be expressed in meaningful terms. In order to do this, it is generally necessary to make use of statistical methods and random process theory.

Since flight loads do not represent phenomena of a predictable or repeatable nature, they must be classed as random processes. As such, they are described in terms of statistical averages and probability statements. Evaluation of flight loads, then, consists of estimating these properties from sample data records. The particular variables which are investigated and the statistical properties which are computed from the observations depend on the end use of the data. That is, the applicational requirements establish the procedures which are used in a measurement program for loads data acquisition and processing. The major steps in such a program are outlined in the summary flow diagram of Figure 20 at the end of this report.

A certain amount of error is associated with each operation in the evaluation and utilization of flight loads data. At the beginning of a program there may be fundamental errors in assumptions concerning the nature of the phenomenon being investigated. Based on these false assumptions, the wrong variables may be measured, or the data acquisition system may be improperly designed. The error then tends to propagate into the stage where loads are computed from measured variables. If the wrong variables have been measured, or the correct variables measured inaccurately, the computations may amplify these errors. In any event, the reliability of the final results cannot be improved at this stage. In addition, some of the computing procedures may be based on erroneous assumptions. This can easily cause the computing procedures themselves to become a source of error. Finally, it is necessary to have a good understanding of the uncertainties involved in utilizing the data in order to derive the maximum benefit from the computed flight loads.

Various considerations which are pertinent to the evaluation and utilization of airplane flight loads data are covered in the body of this document. Section 2 discusses the statistical and random process concepts which are applicable to the subject study. The general performance characteristics and measurement errors associated with data acquisition systems are then summarized in Section 3. A much more detailed analysis of instrumentation equipment performance and accuracy is provided by Part II of this technical report. Section 4 reviews the procedures used currently for computing flight loads estimates from sample observations. The uncertainties associated with this stage of the evaluation are discussed at length. Finally, the utilization of the data for aircraft design is discussed in Section 5. Both design criteria for new aircraft and design substantiation of existing aircraft are covered by the discussion. Important aspects of this investigation are summarized and the conclusions which can be drawn from the study are presented in the last section of the report.

2. FLIGHT LOADS AS RANDOM PROCESSES

All physical phenomena can be classified broadly as either deterministic or random processes. A deterministic process is one which can be defined by an explicit mathematical relationship. That is, with a knowledge of the appropriate relationship and some set of initial conditions, it is possible to predict precisely the value of a deterministic variable at some future time. Random variables, on the other hand, are not precisely predictable and must be described in terms of probability statements and statistical averages. For example, while the tidal amplitudes of the ocean are essentially deterministic, the wave heights are random. Flight loads, in general, would be defined as random processes, although some functions of the ground-air-ground cycle may be considered deterministic for all practical purposes. In the following treatment, however, the emphasis will be on the random nature of flight loads.

Random processes may be classified as either stationary or nonstationary. A stationary random process is one for which the descriptive properties do not change with time. The descriptive properties of flight loads generally do vary with time. Hence, flight loads would rigorously be classified as a nonstationary random process. It is customary and appropriate, however, to view this nonstationary process as a collection of stationary processes. For example, assume an aircraft in straight and level flight passes through intermittent patches of turbulence. The resulting loads data will be nonstationary since their properties will change radically as the aircraft enters and leaves each turbulence patch. On the other hand, the data produced by segments of the flight in still air will tend to be stationary. Likewise, the data produced by those portions of the flight in turbulent air may also be stationary, although they will have completely different properties from the still air data. By neglecting the transients which are induced as the aircraft enters and leaves turbulence patches, it is reasonably easy to describe the resulting nonstationary random process by two different stationary processes, one applying to the still air portions of the flight and the other applying to the turbulent air portions of the flight. In this same manner, the entire flight loads history for the aircraft can be described by a collection of stationary random processes which, in turn, permits the data to be analyzed by conventional time averaging procedures.

From a statistical viewpoint, any given history obtained for a stationary flight loads phase represents only one physical realization of the stationary random process describing that phase. In other words, a given flight loads history represents only a finite sample of an infinite population. The statistical properties of the flight loads are evaluated empirically by computing "estimates" from these finite samples. Since these estimates are specific functions of the samples, they should be expected to differ in value from the actual population properties. Moreover, the estimates will vary from sample to sample, and the resulting collections of observations will have their own sets of statistical properties. This leads to the concepts of probable estimation error and confidence limits. The estimation error is a function of the sampling procedure and, implicitly, the data acquisition system capabilities.

The pertinent statistical descriptions and the methods for evaluating estimation errors will now be discussed.

2.1 STATISTICAL DESCRIPTIONS

If a random process is to be evaluated, this must be done through statistical measures of random variables. Therefore, it is important at the outset of any random process investigation to define the random variables to be studied. Then, the statistical measures which are appropriate for the variables and the program objectives can be selected.

Time series random processes can be described statistically in terms of amplitude properties and time or frequency properties. Once a random variable which characterizes the process has been defined, the amplitude properties are given by probability functions and statistical moments. In the time-frequency domain, the statistical properties of interest to flight loads analysis include correlation functions, power spectra, and level crossing rates. These subjects will be reviewed here briefly. They are covered in much greater detail in References 1 and 2.

2.1.1 Random Variables

The definition of a random variable is not always as straightforward as it may seem. For example, "normal acceleration" is an incomplete definition requiring further qualification such as: "peak amplitude of." Several such qualifying definitions are applied to statistical descriptions in flight loads work. These are summarized in the following.

a. Instantaneous Amplitude — Any continuous function of time can be considered to have some amplitude value at each instant. This is a clear concept which is quite familiar to engineers. Instantaneous amplitudes are usually associated with easily observed records such as oscillograms. The idea applies equally to discrete samples of continuous variables such as exist in the case of digitized time series data.

Figure 1a is a sketch of the continuous and discrete series of instantaneous amplitudes which define a random loads time history. Statistical descriptions in the time-frequency domain are nearly always applied to the random variable, instantaneous amplitude.

b. Peak Amplitude — If a continuous function of time is evaluated only when it reaches a peak, these values must be considered as observations of a new random variable. A peak occurs at the instant in a time history when the slope is zero and the rate of change of slope is negative as shown in Figure 1b. A trough can be considered the opposite of a peak. A random variable which is required to be a peak cannot be continuous in time but exists only at discrete instants. In general, the statistical properties of the instantaneous amplitudes of a continuous random process are independent of the statistical properties which describe the peak amplitudes in that process.

c. Extreme Amplitude — For any finite sample of a random process, there will be a single point which has an amplitude greater than that of all other points in the sample (Figure 1c). This is known as an extreme amplitude and is a separate random variable for loads studies. An extreme amplitude is always associated with a particular sample size since, for most random processes; it is implicit that larger samples (i. e. more points in the sample) tend to contain extremes of greater magnitudes.

d. Rise and Fall Amplitude — In some flight loads applications, the random variable of interest is the difference in amplitude between a time history trough and the following peak (rise) and the difference between a peak and the subsequent trough (fall). This is illustrated in Figure 1d.

e. Level Crossing Time — The sequence of times at which a continuous function crosses arbitrary amplitude levels constitutes a random variable of interest in certain types of loads analysis. Of particular importance is the sequence

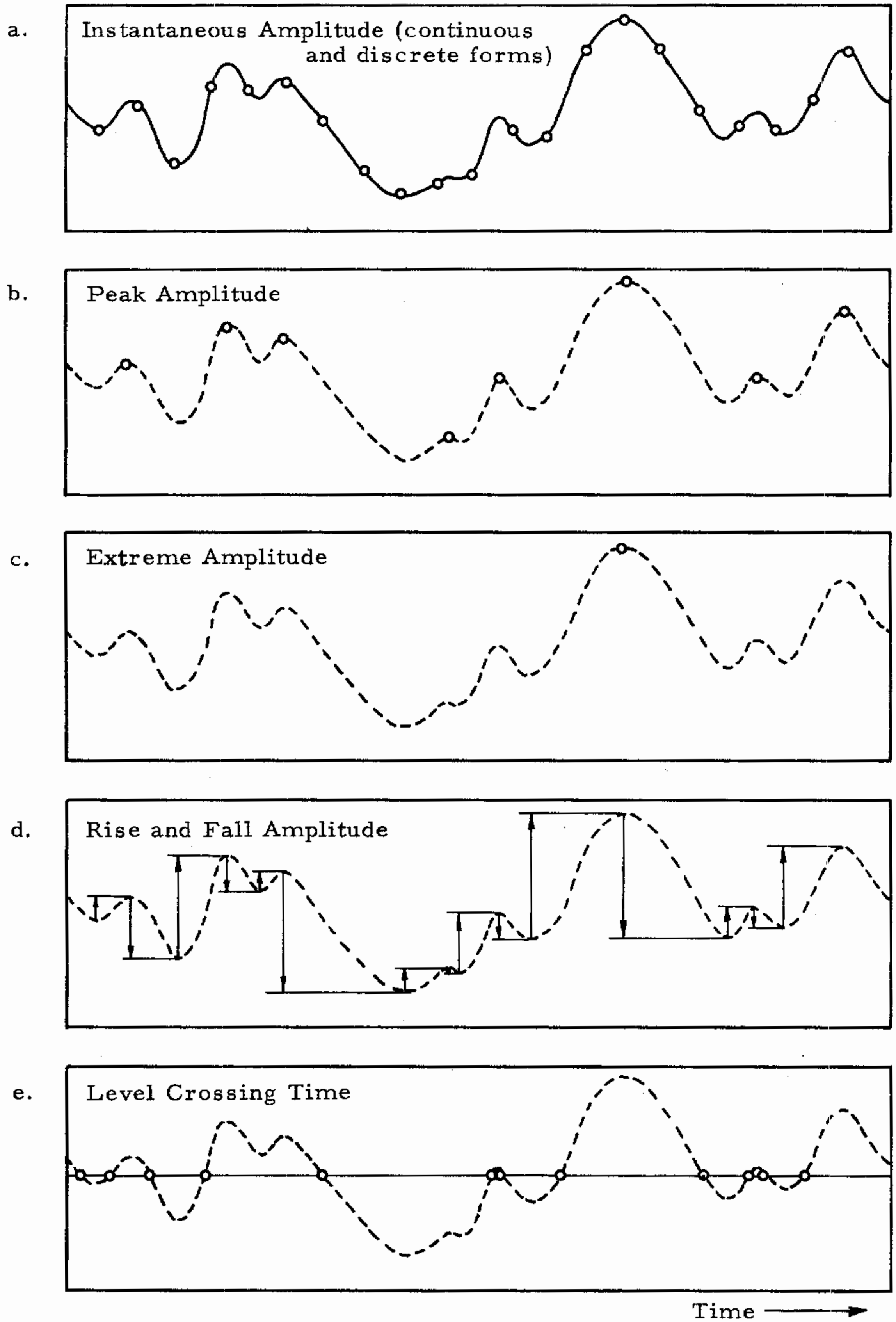


Figure 1. Types of Random Variables Used in Statistical Loads Descriptions

of times defining the crossing of the zero amplitude level. This is illustrated in Figure 1e.

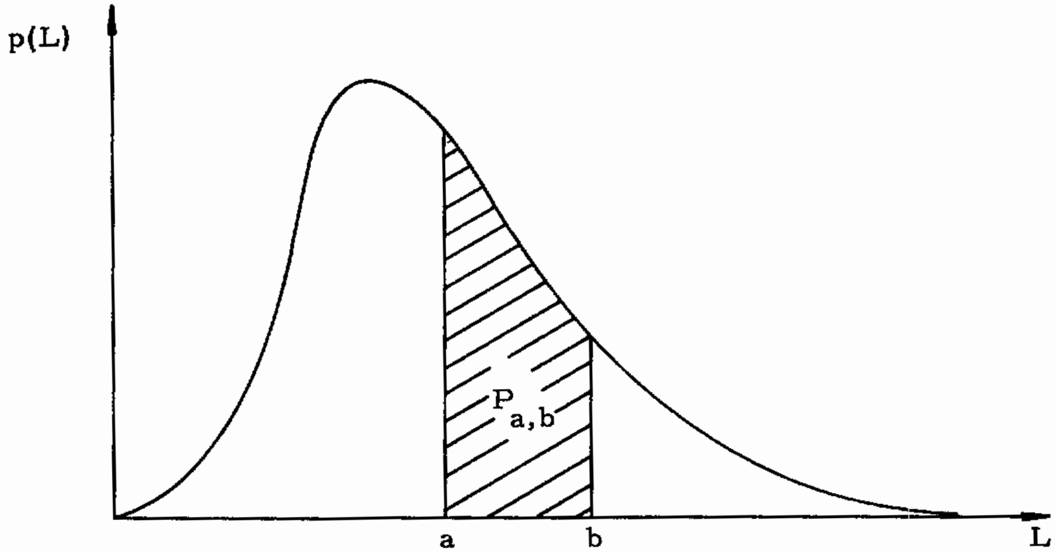
f. Sample Estimates — Estimates of the statistical properties of a random process are computed from samples which consist of unique sets of observations. Therefore, the estimates themselves become random variables and must be described statistically.

2.1.2 Amplitude Properties of Random Variables

Even though a future value for a random variable cannot be predicted precisely, it is possible to talk about the probability of an observed amplitude value being within specific limits. The basic descriptor for this purpose is the probability density function. For a stationary loads variable L, the probability density function is defined by Eq. (1).

$$p(L) = \lim_{\Delta L \to 0} \frac{\text{Prob}\,[L < L_i < L + \Delta L]}{\Delta L} \quad (1)$$

This function has the following properties, as illustrated in Figure 2. The area under the curve p(L) versus L between any two values of L equals the probability that any future observed value chosen at random will lie between those limits. The area under the entire curve is equivalent to a certainty that the variable will have a value, and is equal to one. The integral under the curve from minus infinity to any arbitrary value equals the probability that a future observation will not exceed the selected value.



$$\text{Prob}\left[a < L_i < b\right] = \int_a^b p(L)\, dL = P_{a,b}$$

$$\text{Prob}\left[-\infty < L_i < \infty\right] = \int_{-\infty}^{\infty} p(L)\, dL = 1$$

Figure 2. Properties of the Probability Density Function

The last property can be utilized to generate a function giving the probability that any level will not be exceeded if a random observation is made. This is the probability distribution function, sometimes called the cumulative probability, and is defined as

$$P(L) = \text{Prob}\left[L_i \leq L \right] \qquad 0 \leq P(L) \leq 1$$

$$= \int_{-\infty}^{L} p(\mathcal{L})\, d\mathcal{L} \tag{2}$$

The properties of first order probability functions as just discussed can be extended to cover the joint probability relationships of two or more random loads variables. For example, a two-dimensional joint probability distribution function would be expressed as follows.

$$P(L, M) = \text{Prob}\left[L_i \leq L \text{ and } M_j \leq M \right] \qquad 0 \leq P(L, M) \leq 1$$

$$= \int_{-\infty}^{L} \int_{-\infty}^{M} p(\mathcal{L}, \mathcal{M})\, d\mathcal{M}\, d\mathcal{L} \tag{3}$$

where p(L, M) is a two-dimensional probability density function for the loads variables L and M. An n-dimensional integration of the joint probability density function for n variables over the complete range of each variable produces a value of unity, or a certainty that all the variables will have values. Another property of the joint probability density function is that it equals the product of the probability density functions for each component variable if all variables are independent.

For a stationary random loads variable L with known probability density function p(L), a set of descriptive parameters called moments is defined by the following relationship:

$$E\left[L^k\right] = \int_{-\infty}^{\infty} L^k \, p(L) \, dL \tag{4}$$

where the kth moment of the distribution is the expected value E[ ] of the kth power of L. An expected value for a random variable can be interpreted as the long time average, or population average. This is distinguished from a sample average where the averaging operation is performed on a finite quantity of data. As applied to Eq. (4), for continuous and discrete random loads variables respectively,

$$E\left[L^k\right] = \lim_{T\to\infty} \frac{1}{T} \int_0^T L^k(t)\, dt \qquad \text{(continuous)} \tag{5a}$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} L_i^k \qquad \text{(discrete)} \tag{5b}$$

The first moment which can be derived from Eq. (4) is the mean value and is commonly denoted by μ_L. In a physical sense, this can be interpreted as the center of gravity of the area under the probability density function.

The second moment defined by Eq. (4) is the mean square value Ψ_L^2. Ordinarily, however, moments of higher order than the first are expressed as moments about the mean, or central moments. The kth central moment is defined as:

$$E\left[(L - \mu_L)^k\right] = \int_{-\infty}^{\infty} (L - \mu_L)^k \, p(L) \, dL \qquad (6)$$

Note that this transformation makes the first central moment equal to zero. The second central moment is the variance σ_L^2. The variance provides a measure of dispersion of the distribution and can be thought of as the area moment of inertia under the probability density function. The positive square root of the variance is the standard deviation of the distribution, σ_L.

The three descriptive parameters just discussed are related in the following way:

$$\Psi_L^2 = \mu_L^2 + \sigma_L^2 \qquad (7)$$

Equation (7) shows that the mean square value is really dependent upon both the mean and the variance. This relationship will be applied usefully in the subsequent material to describe many aspects of random process applications.

The next two central moments are called the skewness and kurtosis. In a few cases, these two statistical parameters will be of interest in loads analysis. However, no reference material has been found describing the application of statistical moments beyond the fourth. All subsequent discussion in this section will be limited to first and second order moment descriptors.

2.1.3 Time-Frequency Properties of Random Variables

Certain flight loads applications require a description of amplitude characteristics as related to dimensions of time or frequency. The functions which describe the statistical properties of a random loads variable in the time domain can generally be used interchangeably with those of the frequency domain. Therefore, both types will be discussed in this section. In all cases, it will be assumed that the random processes under consideration are stationary.

Correlation Functions. The general dependence of the amplitude value of a random loads variable at one time upon the value at another time is described by the autocorrelation function. This function, with respect to an arbitrary time series loads variable, $L(t)$, is written as

$$R_L(\tau) = E[L(t)\ L(t+\tau)] \tag{8}$$

where τ is the time delay between any two observations of L.

The autocorrelation function has the following properties. It is always a real valued, even function with a maximum at $\tau = 0$ and may be either positive or negative. For the special case $R_L(0)$, the autocorrelation function equals the mean square value of the distribution of

L as defined in Eq. (7). If the variable has a zero mean value, $R_L(0)$ is simply the variance. The autocorrelation function for a periodic signal will also be periodic. Another property is that in most cases the value of the autocorrelation function for a random variable approaches the square of the mean value μ_L^2 as $|\tau|$ becomes arbitrarily large. These properties are illustrated in Figure 3.

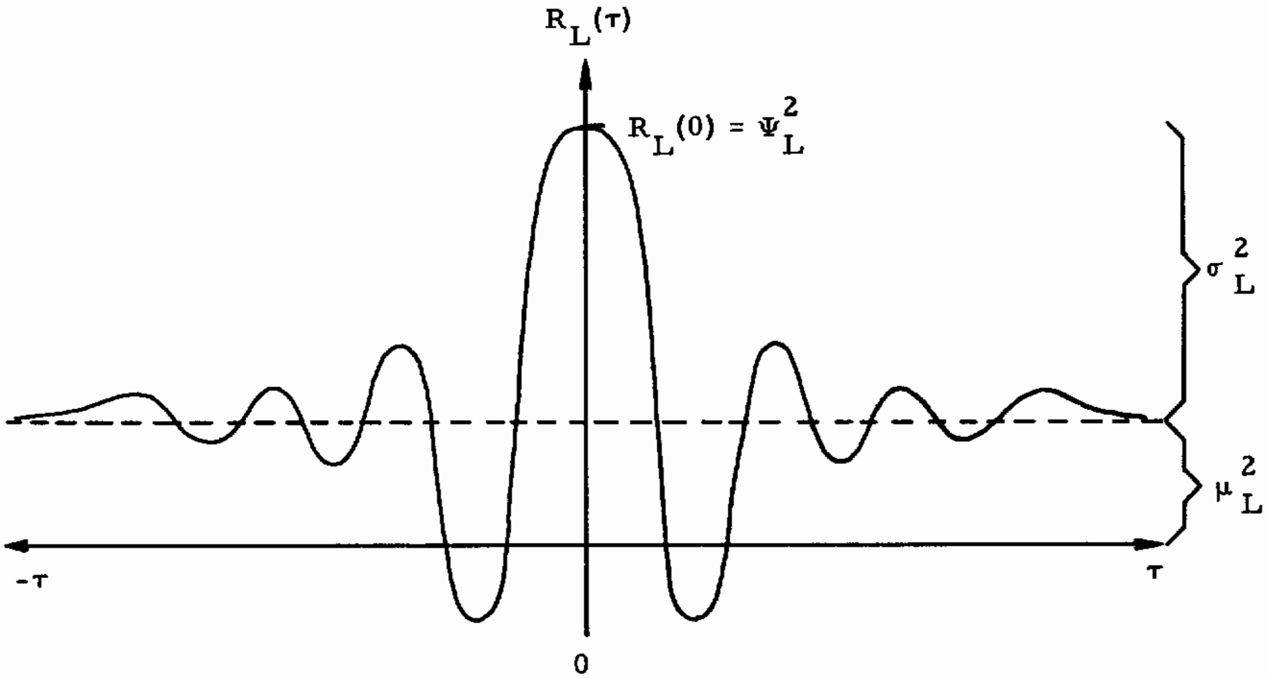


Figure 3. Properties of the Autocorrelation Function

The cross correlation function between two time series random loads variables L(t) and M(t) is defined by

$$R_{LM}(\tau) = E[L(t)\ M(t+\tau)] \tag{9}$$

The cross correlation function is always real valued and may be either positive or negative. However, $R_{LM}(\tau)$ does not necessarily have a maximum at $\tau = 0$, as shown in Figure 4. Also indicated in the figure is the fact that as $|\tau|$ becomes arbitrarily large, the cross correlation function approaches the product of the mean values of the two variables. This may be either positive or negative.

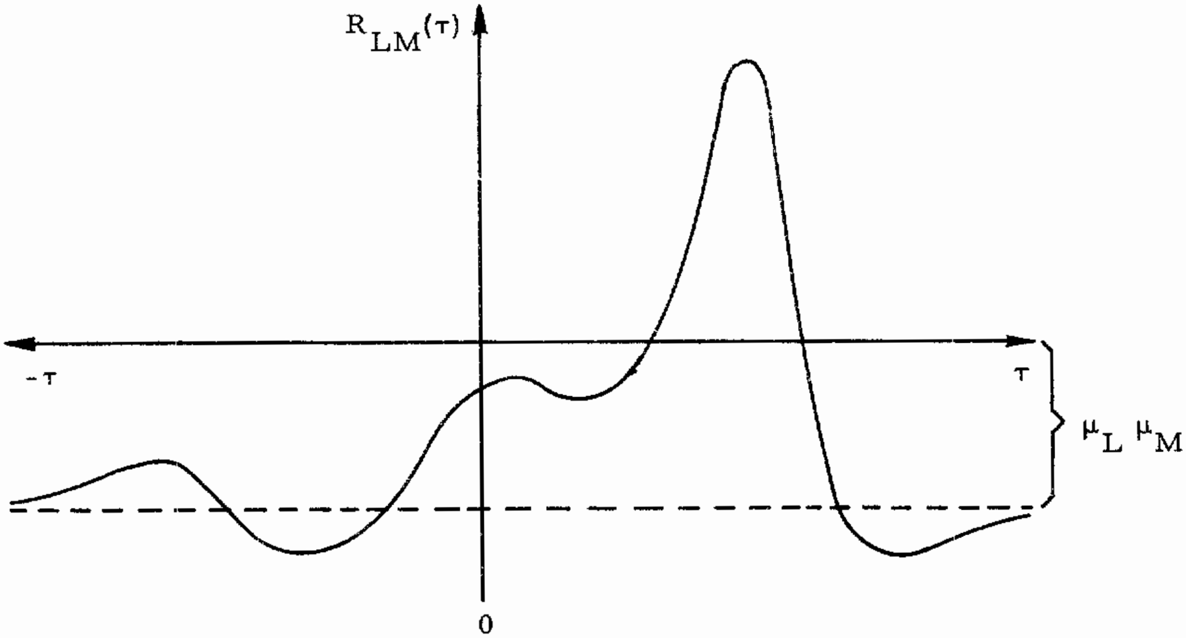


Figure 4. Properties of the Cross Correlation Function

Spectral Density Functions. The relationship between amplitude and frequency for a random loads variable is best described by the power spectral density function. For an arbitrary time series loads variable L(t), the power spectral density function $G_L(f)$ can be written as

$$G_L(f) = \lim_{\Delta f \to 0} \frac{E[L^2(t, f, \Delta f)]}{\Delta f} \tag{10}$$

where $L(t, f, \Delta f)$ is that part of L(t) which exists in the frequency range from f to $f + \Delta f$, and f is defined here as nonnegative. In other words, the power spectral density function defines the mean square value of L(t) in a contiguous set of frequency bands as the band widths go to zero. It follows that the sum of the mean square values in each band equals the overall mean square value of L(t). This property is expressed by Eq. (11).

$$\int_0^{\infty} G_L(f)\, df = \Psi_L^2 \tag{11}$$

Since the mean square contains both the mean and the variance, these can be separated by selective integration of the power spectrum as follows:

$$\mu_L = \left[\int_{0^-}^{0^+} G_L(f)\, df \right]^{1/2} \qquad (12a)$$

$$\sigma_L^2 = \int_{0^+}^{\infty} G_L(f)\, df \qquad (12b)$$

Equation (12) points out the static and dynamic character of the mean and variance as applied to time series variables. It will be shown later how this difference can be used to advantage in data evaluation. Since the square of the mean value is a delta function on the frequency scale, this can be noted graphically as shown in Figure 5.

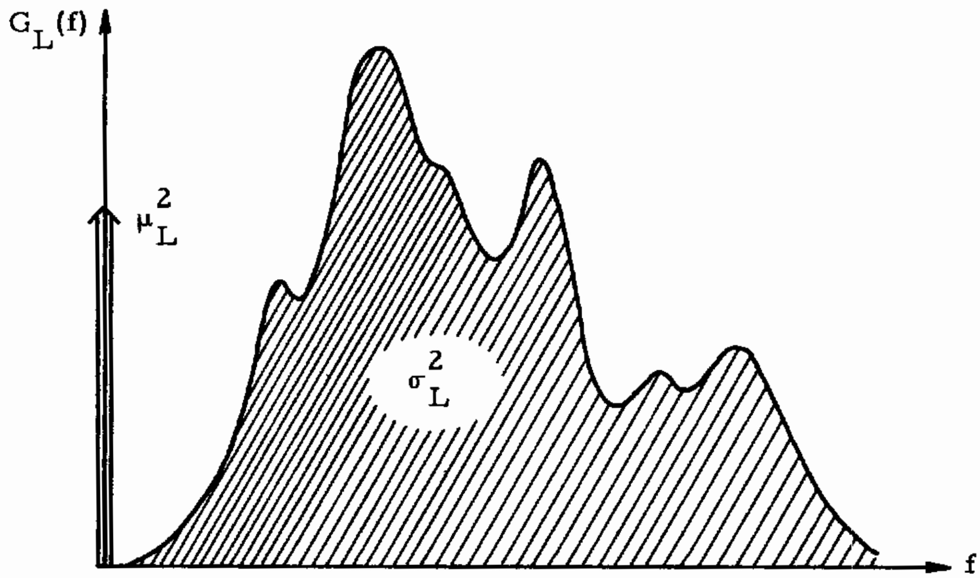


Figure 5. Properties of the Power Spectral Density Function

The power spectral density function has the properties of being a real valued, nonnegative function. Another important property is its relationship to the autocorrelation function. The two functions are Fourier transform pairs, and since the autocorrelation function is even, the transformation is given by Eq. (13).

$$G_L(f) = 4 \int_0^{\infty} R_L(\tau) \cos 2\pi f \tau \, d\tau \tag{13}$$

The cross spectral density function between two time series random loads variables L(t) and M(t) is defined by

$$G_{LM}(f) = C_{LM}(f) + j\, Q_{LM}(f) \tag{14}$$

where

$$C_{LM}(f) = \lim_{\Delta f \to 0} \frac{E[L(t, f, \Delta f)\ M(t, f, \Delta f)]}{\Delta f}$$

$$Q_{LM}(f) = \lim_{\Delta f \to 0} \frac{E[L(t, f, \Delta f)\ M^{o}(t, f, \Delta f)]}{\Delta f}$$

and

$$M^{o}(t, f, \Delta f) = M(t, f, \Delta f) \qquad \text{shifted } 90^{o} \text{ in phase}$$

Just as the power spectral density function was shown to be the Fourier transform of the autocorrelation function, the cross spectral density function is related to the cross correlation function by this linear transformation. However, since the cross correlation function is not even, the cross spectral density function is complex. In Eq. (14), the real part $C_{LM}(f)$ is called the coincident (or co) spectral density function, and the imaginary part is the quadrature (or quad) spectral density function.

For many engineering applications, it is preferable to express the cross spectrum in complex polar notation as follows.

$$G_{LM}(f) = \left| G_{LM}(f) \right| e^{j\theta_{LM}(f)} \tag{15}$$

Then, if only power relationships between L and M are desired, just the magnitude term $\left| G_{LM}(f) \right|$ needs to be considered. For example, a measure of the dependence of the power in one variable upon that of a second variable can be obtained as a function of frequency by dividing the cross spectrum magnitude squared by the product of the individual power spectra. The result is termed the coherence function and is written

$$\gamma^2_{LM}(f) = \frac{\left| G_{LM}(f) \right|^2}{G_L(f)\, G_M(f)} \tag{16}$$

The coherence function can have values from zero to one. If $\gamma^2_{LM}(f)$ is zero at any frequency, L and M must be considered incoherent, or uncorrelated at that frequency. If the function is zero at all frequencies, the variables are said to be independent. A value of one indicates that the variables are fully coherent.

Level Crossing Rates. The rate at which a continuous time series random loads variable L(t) crosses arbitrary amplitude levels α is defined by

$$N_L(\alpha) = E\left[\frac{1}{t_{n+1} - t_n}\right]_\alpha \tag{17}$$

where t_n and t_{n+1} are any two subsequent level crossing times for the level α. The definition of Eq. (17) is general and applies to zero crossings as well.

In the event that L(t) has a Gaussian distribution and zero mean value, the rate at which the time history crosses zero with positive slope is related to the power spectrum by Eq. (18).

$$N_L^+(0) = \left[\frac{\int_0^\infty f^2\, G_L(f)\, df}{\int_0^\infty G_L(f)\, df}\right]^{1/2} \tag{18}$$

This relationship has special significance in the application of data to the gust loads problem.

2.2 ESTIMATION OF STATISTICAL PROPERTIES

The great majority of flight loads measurement programs are concerned with providing empirical estimates of the statistical properties discussed in Sections 2.1.2 and 2.1.3. These properties define the characteristics of the underlying random loads processes and are based on theoretically infinite numbers of observed points. Since every point making up the process cannot be observed in practice, the true values of the statistical properties can only be estimated. Estimation then consists of determining and evaluating the properties of finite numbers of observations called samples, and the sample size becomes an important parameter in the evaluation.

Sample size is a term defined here as the number of independent observations N of a random variable used to compute an estimate of a statistical property of that variable. For a continuous time series random variable, the sample actually consists of an infinite number of continuous correlated observations. In many cases, however, this continuous sample may be interpreted as an equivalent number of independent observations. One approach is to use the sampling theorem which states that a continuous time history can be described by a series of discrete values occurring at time intervals equal to $1/2B$, where B is the frequency bandwidth of the time history. Thus, over an abservation time T, the time history could be described by $2BT$ observations. These observations, however, would be independent only for the special case where the time history is from a Gaussian random process with a uniform power spectrum over the bandwidth B. For any other case, the equivalent number of independent observations represented by the sample time history would be less than $2BT$. Hence, $N = 2BT$ can be used as an upper limit on the number of independent observations. Furthermore, in certain special cases an appropriate value for the equivalent number of observations can be calculated if the power spectrum of the data is known. For example, the equivalent number of independent observations for a variance estimate from a sample time history with a power spectrum of G(f) is given in Reference 3 by

$$N = 2T \frac{\left[\int_0^\infty G(f)\, df\right]^2}{\int_0^\infty G^2(f)\, df} \qquad (19)$$

For the case of single point observations made at different points in time (separated in time by $\tau >> 1/2B$), at different points on the aircraft structure, or on different aircraft, N is simply the total number of observations.

Estimates, or sample values, generally differ from the true values by some amount. Therefore, measured statistical quantities must never be confused with the corresponding values of the underlying process in any given engineering application. The difference between an estimate and the true value is called the estimation error. Since the estimates are random variables, this error will have random characteristics and can best be described by statistical methods.

The appropriate statistical medium for describing the properties of sample values is the sampling distribution with its first and second moments. If ϕ_L is some statistical measure of a random loads variable L, estimates computed from sample observations are denoted by $\hat{\phi}_L$, and the sampling distribution would be $P(\hat{\phi}_L)$. The first moment of the sampling distribution (by analogy with Eq. (4)) is $E\left[\hat{\phi}_L\right]$, the expected value of $\hat{\phi}_L$. In most cases, this is identical with the true value ϕ_L. The exception to this rule occurs when $\hat{\phi}_L$ is a biased estimator. The second central moment of the sampling distribution is the variance denoted by $\mathrm{Var}\left[\hat{\phi}_L\right]$, and its positive square root is called the standard error $SE\left[\hat{\phi}_L\right]$. The standard error in estimating ϕ_L is a function of sample size, and its value, in most cases, approaches zero as the sample size becomes arbitrarily large. This is indicated for the case of mean value estimation in Figure 6 where it should be noted that $P(\hat{\mu}_L) = P(L)$ for a sample of size 1. The quantity $SE\left[\hat{\phi}_L\right]$ divided by $E\left[\hat{\phi}_L\right]$ is called the normalized standard error or coefficient of variation ϵ.

Estimation error analysis is of interest during two phases of a flight loads measurement program. These are the planning phase and the data evaluation phase. The approaches to estimation error in the two phases are in opposite directions. During the planning phase, the emphasis is on determining the required sample size for each measurement. Since this analysis precedes data acquisition, a great deal must be assumed regarding the characteristics of each random variable of interest. This introduces a certain amount of variation and must be considered an imprecise operation. In the data evaluation phase, however, the sample sizes have all been fixed, and the sample properties are directly available. Then, it is a matter of determining the accuracy of the estimates. Aspects of the estimation error problem which are related to these two phases are summarized briefly in the following material.

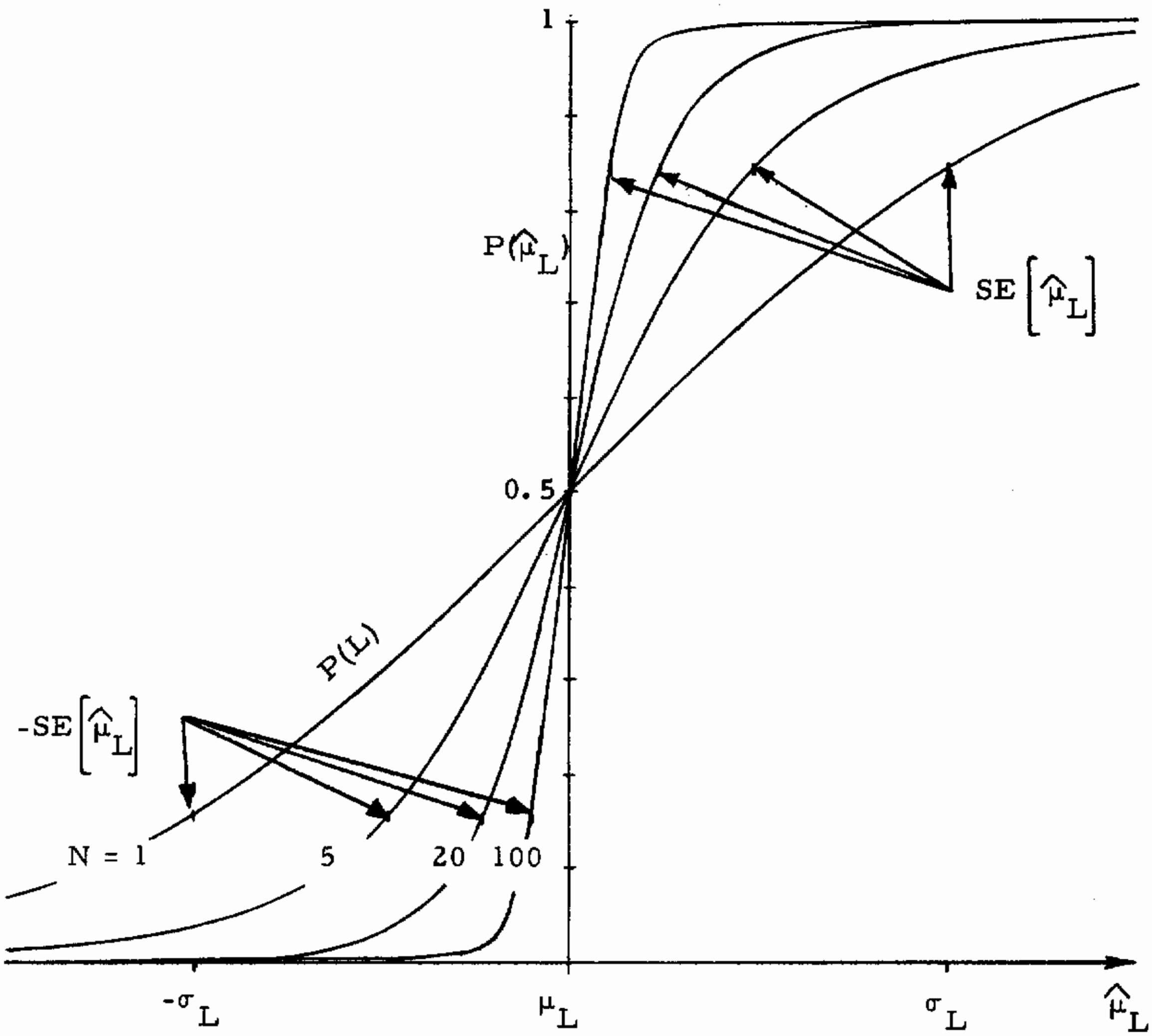


Figure 6. Illustration of the Effect of Sample Size on the Distribution of Sample Mean Values

2.2.1 Sampling Considerations

One of the economic tradeoffs in acquiring flight loads data is the relationship between estimation accuracy and the proportion of the total environment observed. Since it is impractical to consider measuring all variables of interest 100% of the time on every aircraft in service, it is necessary to determine ways in which a limited number of observations can be used without unacceptably degrading the quality of the results.

This tradeoff forms the basis for any sampling program and is reviewed here as applied to flight loads investigations.

Three sampling regimes are of interest in flight loads work. First, it is necessary to estimate the statistical properties of the environment at single points on the vehicle (References 1 and 4). Sample size considerations are mainly functions of time in this case. Second, it is often of interest to sample a random process spatially at multiple points (Reference 5). If the points are widely spaced and selected randomly, the sample size is defined by the number of measurement points. Finally, many properties cannot be determined without multiple aircraft sampling, and the sample size depends upon the number of instrumented aircraft which are operating under the same nominal set of conditions (Reference 6). The procedural approach to sampling in all three regimes is approximately the same with the principal difference being definition of the random variables.

The initial step in the sampling procedure is to establish a level of error which is considered acceptable when viewed in the perspective of program objectives and the other likely error sources. This error must be stated probabilistically since any other description would be meaningless. An appropriate expression is given below.

$$\text{Prob}\left[\,\left|\hat{\phi} - \phi\right| \leq \delta\,\right] \geq 1 - \alpha \qquad (20)$$

The above relationship simply says that the error in future estimates should not exceed $\pm\delta$ in $100(1 - \alpha)\%$ of all trials. The problem then is to determine the minimum sample size which will assure this. In order to be precise, it is necessary to have a priori knowledge of both the underlying distribution of the random variable and the sampling distribution for each statistical quantity of interest. However, the limited accuracy required in measurement planning justifies the assumption that sample values are normally distributed. Then, a general purpose model for sample size determination can be written and evaluated for different underlying distributions.

For reasonably large samples (say $N > 10$), a variable Z, which is approximately normal with zero mean and unit variance, can be defined by

$$Z = \frac{\hat{\phi} - \phi}{SE\left[\hat{\phi}\right]} \tag{21}$$

The acceptable error stated by Eq. (20) can then be substituted in Eq. (21) to produce the bounding condition

$$Z_{\alpha/2} = \frac{\delta}{SE\left[\hat{\phi}\right]} \tag{22}$$

where $Z_{\alpha/2}$ is the $100\,\alpha/2$ percentage point of the standardized normal distribution. By substituting the appropriate expressions for $SE\left[\hat{\phi}\right]$ in Eq. (22), the sample sizes required for estimating various statistical properties of interest with the specified accuracy can be determined.

It can be shown that all of the descriptions covered in Sections 2.1.2 and 2.1.3 are, at least from the measurement planning standpoint, variations of three properties: mean, variance, and probability. Analytical solutions for the standard errors in estimating these three properties have been presented in References 1 and 5 as follows.

$$SE\left[\hat{\mu}\right] = \frac{\sigma}{\sqrt{N}} \tag{23a}$$

$$SE\left[\hat{\sigma}^2\right] = \frac{\sqrt{2}\,\sigma^2}{\sqrt{N - 1}} \tag{23b}$$

$$SE\left[\hat{P}\right] = \sqrt{\frac{P(1 - P)}{N - 1}} \tag{23c}$$

The corresponding sample size expressions are then given by Eq. (24).

Mean Value Estimates:
$$N = \frac{\sigma^2 Z^2_{\alpha/2}}{\delta^2} \tag{24a}$$

Variance Estimates:
$$N = \frac{2\sigma^4 Z^2_{\alpha/2}}{\delta^2} + 1 \tag{24b}$$

Probability Estimates:
$$N = \frac{P(1 - P) Z^2_{\alpha/2}}{\delta^2} + 1 \tag{24c}$$

Specific applications of the above formulas depend upon the underlying distributions being sampled. In general, the variance is unknown, and a suitable value must be estimated on the basis of experience. In addition, the distribution, and hence the error, may not be symmetrical. In this instance, the worst case δ is usually the criterion employed. The sample size required for probability estimates is particularly sensitive to the underlying distribution as illustrated by Eq. (24c). In some cases, a conservative approach is taken here by using the maximum value of $P(1 - P)$ equal to $1/4$.

Sampling relationships have been developed for several well known distributions. Among these, Reference 5 considers the normal and log normal distributions as applied to spatial sampling on an aircraft structure. Reference 4 develops the standard error expressions for the random variable "peak value" of a zero mean, Gaussian random process with arbitrary frequency content. Reference 4 also discusses the conservative non-parametric sampling approach which requires no assumptions about underlying process distributions.

2.2.2 Estimation Uncertainties

Estimates of flight loads statistics are inherently inaccurate because of practical limitations associated with data acquisition and reduction. Errors of both the systematic, or bias, and variability types can occur. These may be interpreted as the average and fluctuating components of the error, respectively, and are related to the mean square error by

$$\begin{aligned} \text{MS error} &= E\left[(\hat{\phi} - \phi)^2\right] \\ &= b^2\left[\hat{\phi}\right] + \text{Var}\left[\hat{\phi}\right] \end{aligned} \tag{25}$$

where the bias error is defined as

$$b\left[\hat{\phi}\right] = E\left[\hat{\phi} - \phi\right] \tag{26}$$

and the variability error is measured in terms of the variance of the estimate

$$\mathrm{Var}\left[\hat{\phi}\right] = E\left[\hat{\phi}^2\right] - E^2\left[\hat{\phi}\right] \tag{27}$$

These errors can be stated in either absolute or relative terms. The relative bias error is obtained from the absolute form by dividing Eq. (26) by ϕ. The relative variability error is usually given by the normalized standard error e.

Variability errors in statistical loads estimates are the result of practical sample size limitations. Bias errors are caused by finite resolution limitations inherent in practical data reduction procedures and occur in the evaluation of some statistical functions. Where bias errors exist, they can be reduced only at the expense of increasing the variability error. These matters are illustrated by comparing the equations which define the properties and the formulas by which the estimates are computed.

Estimates of the mean value of a continuous or discrete random loads variable L are computed from sample observations using the following formulas, where T is the sample record length.

$$\hat{\mu}_L = \frac{1}{T}\int_0^T L(t)\,dt \qquad \text{(continuous)} \qquad (28a)$$

$$= \frac{1}{N}\sum_{i=1}^{N} L_i \qquad \text{(discrete)} \qquad (28b)$$

Note the difference between Eq. (28) and Eq. (5) with k equal to one. Essentially this describes the difference between a sample mean value and an expected value. Since sample size is the only practical limitation here, mean value estimates are unbiased and subject only to variability errors.

Estimates of the variance of a continuous or discrete random loads variable are computed in a similar fashion according to the following

$$\hat{\sigma}_L^2 = \frac{1}{T}\int_0^T \left[L(t) - \overline{L}\right]^2 dt \qquad \text{(continuous)} \qquad (29a)$$

$$= \frac{1}{N-1}\sum_{i=1}^{N} (L_i - \overline{L})^2 \qquad \text{(discrete)} \qquad (29b)$$

Again it should be noted that the practical restriction on estimation accuracy is only a function of sample size, and therefore, sample variance is unbiased.

Current techniques for processing flight loads data ordinarily result in discrete amplitude values. These are sorted into a finite number of amplitude intervals K each ΔL wide, and are often presented as frequency histograms or cumulative frequencies. Sample probability density and distribution function estimates are then computed according to the following discrete formulas.

$$\hat{p}(k\Delta L) = \frac{\hat{P}(\Delta L)_k}{\Delta L} = \frac{N_k/N}{\Delta L} \qquad k = 0,\ 1,\ 2,\ \ldots,\ K \tag{30a}$$

$$\hat{P}(k\Delta L) = \sum_{\ell=0}^{k} \hat{p}(\ell\Delta L)\,\Delta L = \sum_{\ell=0}^{k} \frac{N_k}{N} \qquad \ell = 0,\ 1,\ 2,\ \ldots,\ k \tag{30b}$$

The ratio N_k/N is the proportion of the total number of sample points with amplitude values in the kth ΔL interval. In any practical situation, ΔL will not approach zero as in the limiting case of Eq. (1), and therefore, probability function estimates are subject to both bias and variability errors.

The autocorrelation function for a continuous, time dependent random variable $L(t)$ is computed from a sample record of length T using Eq. (31).

$$\hat{R}_L(\tau) = \frac{1}{T - \tau} \int_0^{T-\tau} L(t)\, L(t+\tau)\, d\tau \qquad \tau \ll T \tag{31}$$

It can be seen that sample autocorrelation depends only on sample size. Therefore, the estimates are unbiased.

The sample power spectral density function for a continuous random variable L(t) is computed using the following formula

$$\hat{G}_L(f) = \frac{1}{B_f T} \int_0^T L^2(t, f, B_f)\, dt \tag{32}$$

where B_f is a finite resolution bandwidth associated with computation of the power spectrum. B_f has a minimum theoretical value of $1/T$. Therefore, neither one of the limiting operations indicated by Eq. (10) is achieved in practice. The finite sample size limitation leads to variability errors while the fact that Δf becomes B_f instead of approaching zero introduces bias errors.

Estimates of level crossing rates are obtained from sample observations according to the practical equivalent of Eq. (17) as follows

$$\hat{N}_L(\alpha) = \frac{N_\alpha - 1}{t_{N_\alpha} - t_1} \tag{33}$$

where N_α is the total number of observed crossings of the level α. It can be seen that no resolution requirement is involved here, and the empirical results are subject only to the errors associated with finite sample size.

Variability errors which are caused by practical sample size limitations are summarized in the following table for the statistical properties just discussed. Derivations of these expressions are given in References 1 and 5. Bias errors are covered in Section 4.2.2 as computational problems.

Table 1. Summary of Variability Errors

Statistical Property, ϕ	$\epsilon = SE\left[\hat{\phi}\right]/\phi$
mean, μ	$\frac{1}{\sqrt{N}}\frac{\sigma}{\mu}$
variance, σ^2	$\sqrt{\frac{2}{N-1}}$ $(\mu = 0)$
probability, P	$\sqrt{\frac{1}{N-1}\frac{(1-P)}{P}}$
autocorrelation, $R(\tau)$	$\sqrt{\frac{1}{N-1}\left(1+\frac{R^2(0)}{R^2(\tau)}\right)}$ $(\mu = 0)$
power spectral density, $G(f)$	$\sqrt{\frac{1}{B_f T}}$
level crossing rate, $N(\alpha)$	generally unknown

3. DATA ACQUISITION SYSTEMS

The instrumentation system used to acquire flight loads data represents an important link in the overall loads evaluation problem. The design of such a system depends on the variables to be measured, the sampling plan, and measurement accuracy requirements. In this section, the general performance characteristics and error expressions associated with flight instrumentation are summarized with respect to the overall system. Performance and accuracy of individual system components are described in detail in Part II of this technical report.

3.1 SYSTEM PERFORMANCE CHARACTERISTICS

When selecting components and designing systems to be used for acquiring flight loads data, several important questions must be answered. For example, how can one determine whether an instrument was designed to do the required job? Can the performance be quantified, or can it at least be compared to that of another instrument? To help resolve these questions, it is convenient to speak of a limited number of performance characteristics which can be related to numerical values of the system parameters.

The idea of a system parameter is usually associated with an assumed mathematical model which would describe the system behavior if things were ideal. Since things are not ideal, a great amount of effort usually must be spent on determining the nature and magnitudes of real system deviations before sound judgments on the above questions can be rendered. The general considerations which are pertinent to this problem will now be discussed briefly.

3.1.1 Ideal Systems

An ideal system is one which has time invariant parameters and is linear between two clearly defined points of interest called the input and the output (Reference 1). It is worthwhile here to study ideal systems in order to understand and manage deviations from the ideal exhibited by all real systems.

The performance characteristics of an ideal system can be described by any of several related methods. Three of the more commonly employed approaches include:

a. Solution of a linear differential equation with constant coefficients which relates the system output to the input.

b. The impulse response function which is the output of the system resulting from a unit impulse input.

c. The complex frequency response function of the system.

In using the differential equation approach, it is assumed that the relationship between a time varying input x(t) and the corresponding output y(t) of an "nth order" system can be expressed in the form

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = bx \qquad (34)$$

where the a and b coefficients are constant and incorporate the necessary engineering units.

Employing the differential operator, $D = d/dt$, Eq. (34) becomes the algebraic expression

$$\left(a_n D^n + a_{n-1} D^{n-1} + \ldots + a_1 D + a_0\right) y = bx \qquad (35)$$

The complete solution of Eq. (35) includes both the complementary and particular parts of the solution. For an nth order differential equation, it is necessary then to have n initial conditions.

The impulse response function $h(\tau)$ for a system in itself supplies a complete description of the static and dynamic characteristics of the system. This function is defined as the output of a system at any time produced by a unit impulse input applied to the system τ units of time before. A unit impulse consists of an input having infinite amplitude, zero duration, and an area of one under the time history.

The impulse response function can be applied usefully to determine a system output $y(t)$ corresponding to any time varying input $x(t)$ through the convolution integral of Eq. (36).

$$y(t) = \int_0^{\infty} h(\tau)\, x(t - \tau)\, d\tau \qquad (36)$$

By performing a linear transformation of Eq. (36) from the time domain to the frequency domain, an equivalent input-output relationship is derived in terms of frequency. This is accomplished by taking the Fourier transform of both sides of Eq. (36) noting that the Fourier transform of convolved terms equals the product of their transforms.

$$\mathcal{F}\left\{y(t)\right\} = \mathcal{F}\left\{h(\tau)\right\} \mathcal{F}\left\{x(t)\right\} \qquad (37)$$

The Fourier transform of the impulse response function is called the frequency response function H(f). Denoting the transforms of x(t) and y(t) by X(f) and Y(f), Eq. (37) is given in simplified notation by

$$Y(f) = H(f)\ X(f) \tag{38}$$

The frequency response function is a complex quantity which can be written in vector notation as

$$H(f) = \left| H(f) \right| \ e^{j\theta(f)} \tag{39}$$

where

$$\left| H(f) \right| = \sqrt{H\,H^*} = \text{gain factor} \qquad \text{(amplitude ratio)}$$

$$\theta(f) = -\tan^{-1} \frac{\text{Im}\,(H)}{\text{Re}\,(H)} = \text{phase factor} \qquad \text{(phase lag)}$$

$$H^* = \text{complex conjugate of } H$$

It should be noted that a frequency response function is not a unique property of a given system but depends on the nature of the input and output variables.

Frequency response functions can be derived directly from the differential equation for the input-output relationship. In this case it is convenient to select a unit impulse I for the input. Then Eq. (35) would be written

$$\left(a_n D^n + a_{n-1} D^{n-1} + \ldots + a_1 D + a_0\right) h(\tau) = bI \qquad (40)$$

The Fourier transform of Eq. (40) can then be taken term by term, noting that $y(t)$ has now been defined as $h(\tau)$, the response to a unit impulse. This operation produces

$$\left[a_n(j2\pi f)^n + a_{n-1}(j2\pi f)^{n-1} + \ldots + a_1(j2\pi f) + a_0\right] H(f) = b \qquad (41)$$

or

$$H(f) = \frac{b}{a_n(j2\pi f)^n + \ldots + a_1(j2\pi f) + a_0}$$

It is interesting to study Eq. (41) as applied to several example systems commonly encountered in flight loads instrumentation. These will be defined in terms of the order of differential equation necessary for a description of the system characteristics.

Zero Order Systems. The simplest system which can be described by the model of Eq. (41) is the zero order system for which the frequency response function is

$$H(f) = \frac{b}{a_0}$$

$$|H(f)| = \frac{b}{a_0} = K_o \qquad (42)$$

$$\theta(f) = 0$$

The response of the system is independent of frequency and, therefore, the output would be expected to follow the input in the exact proportion b/a_0 or K_o with zero phase difference between the input and output at all frequencies. The term K_o is sometimes referred to as the static sensitivity of the system. Practical examples of systems which approximate the zero order model over limited ranges include strain gages and potentiometers.

First Order Systems. If the input-output relationship for a system can be modeled satisfactorily using a linear first order differential equation, it is referred to as a first order system. Thus, specializing Eq. (41) for the first order system leads to

$$H(f) = \frac{b}{a_1(j2\pi f) + a_0} \qquad (43)$$

By rearranging terms and making the substitutions $b/a_0 = K_o$, $a_1/a_0 = \tau_1$, the above relationship becomes

$$H(f) = \frac{K_o}{\tau_1(j2\pi f) + 1} \tag{44a}$$

$$\left|H(f)\right| = \frac{K_o}{\sqrt{\tau_1^2(2\pi f)^2 + 1}} \tag{44b}$$

$$\theta(f) = -\tan^{-1} 2\pi f\tau_1 \tag{44c}$$

where K_o is defined again as the static sensitivity, and τ_1 is called the time constant of the system. It can be seen from the plots of Eqs. (44b) and (44c) in Figure 7 that the system sensitivity decreases with increasing frequency and that the phase of the output, which always lags the input, approaches 90^o as the frequency becomes arbitrarily high. Two data system components usually modeled as first order systems are rate integrating gyros and simple RC low pass filters.

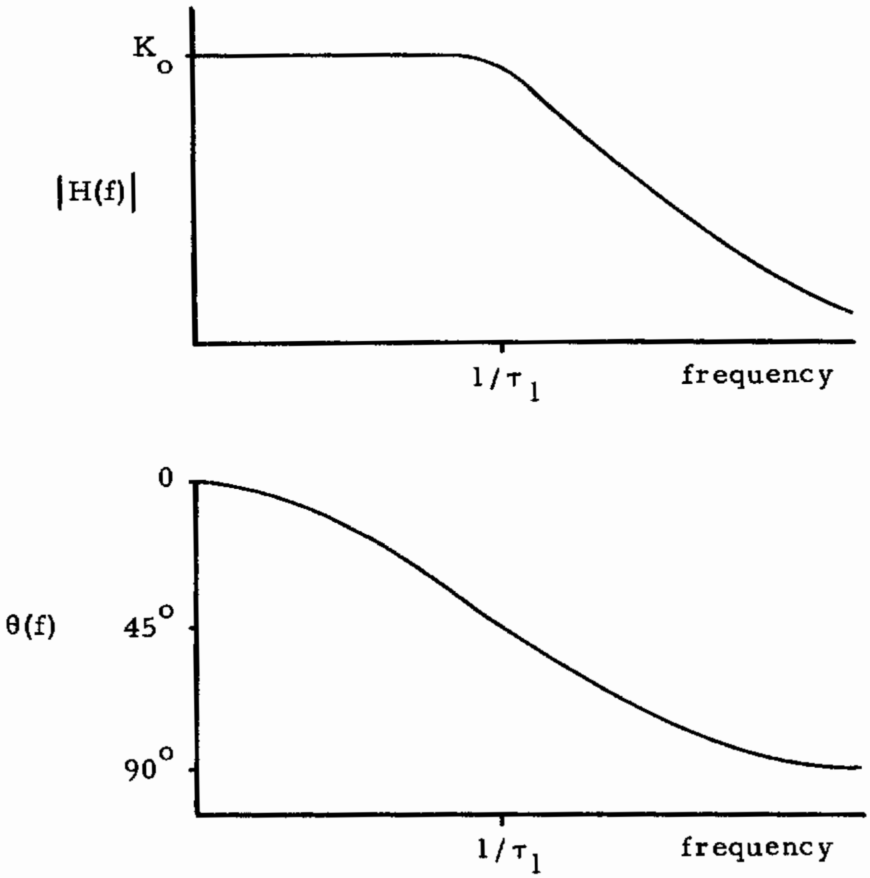


Figure 7. Frequency Response Gain and Phase Factors for a First Order System

Second Order Systems. Following the procedure used above, the frequency response function for an ideal second order system is given by

$$H(f) = \frac{b}{a_2(j2\pi f)^2 + a_1(j2\pi f) + a_0}$$

$$= \frac{b/a_0}{-\frac{a_2}{a_0}(2\pi f)^2 + \frac{a_1}{a_0}(j2\pi f) + 1} \qquad (45)$$

where the following substitutions can be made

$$\frac{b}{a_0} = K_o$$

$$\frac{a_2}{a_0} = \frac{1}{(2\pi f_n)^2}$$

$$\frac{a_1}{a_0} = \frac{\zeta}{\pi f_n}$$

and the two second order system parameters f_n and ζ are defined as follows:

f_n = undamped natural frequency of the system

ζ = system damping factor equal to the ratio of system damping to critical damping

This results in the following useful forms of the frequency response function for a second order system:

$$|H(f)| = \frac{K_o}{\sqrt{\left[1 - (f/f_n)^2\right]^2 + \left[2\zeta f/f_n\right]^2}} \qquad (46a)$$

$$\theta(f) = -\tan^{-1}\left[\frac{2\zeta f/f_n}{1 - (f/f_n)^2}\right] \qquad (46b)$$

Equations (46a) and (46b) are plotted in Figure 8 with the gain and phase factors as functions of the ratio of arbitrary frequencies f to the natural frequency f_n. Clearly, both the sensitivity and the phase lag for a second order system can be influenced greatly in certain frequency ranges by the system damping. Various types of open loop accelerometers can be described as second order systems.

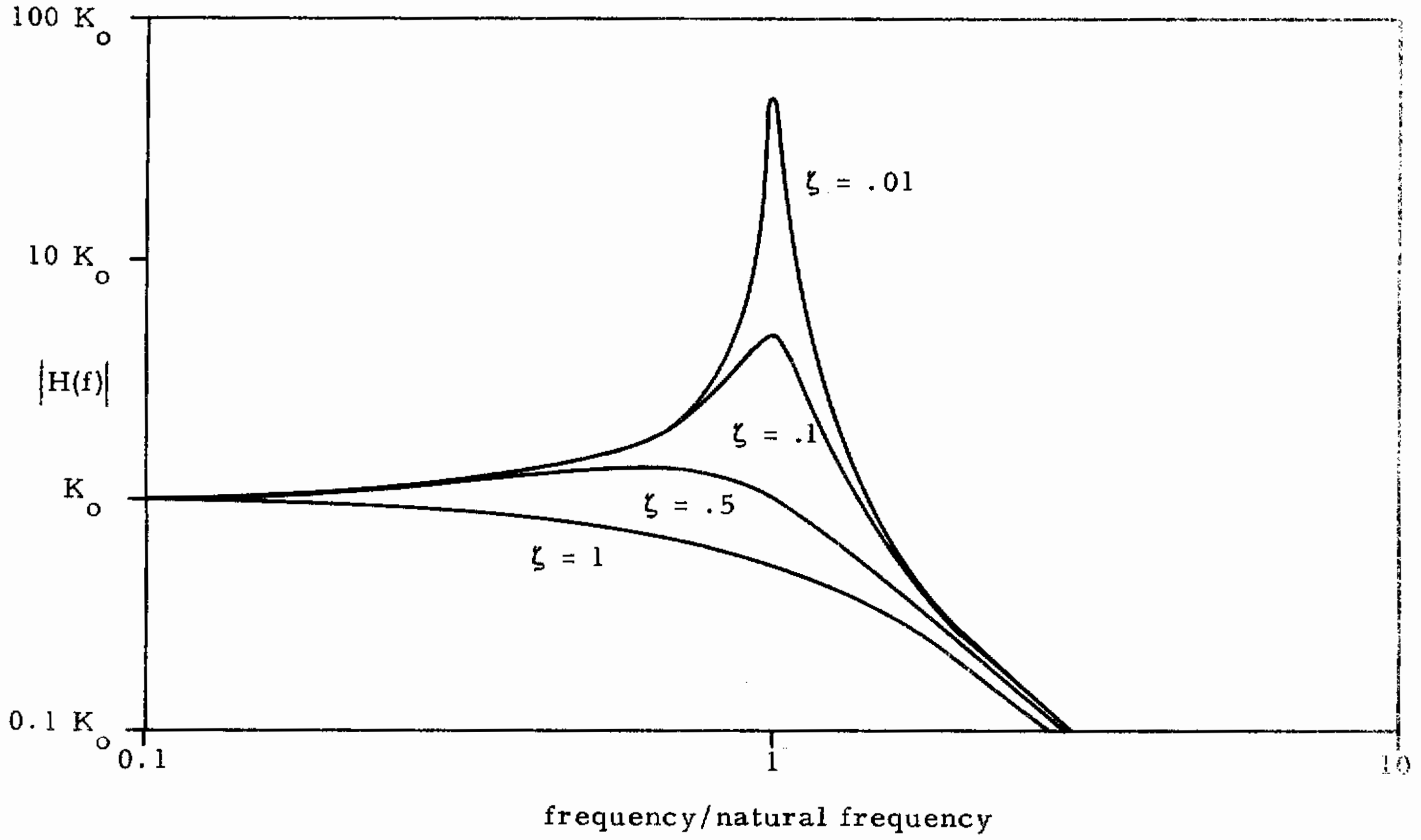
Based on the above discussion of simple systems, it is apparent that for an nth order ideal system, $n + 1$ system parameters are required in order to describe the static and dynamic input-output characteristics. It will be shown in the next section that real systems require additional descriptive factors to account for deviations from ideal performance.

3.1.2 Real Systems

Because of certain fundamental limitations, the performance of a real system can only approach that of an ideal system. The existence of, for example, the second law of thermodynamics and a finite world immediately impose restrictions on the validity of any of the theoretical descriptions presented in the last section. The quality of a real system can be judged then in terms of how closely the performance approaches the ideal.

From the definition of an ideal system given in Section 3.1.1, real system deviations may be classed under:

a. Variation of system parameters with time
b. Variation of system parameters with amplitude
c. Outputs resulting from extraneous inputs



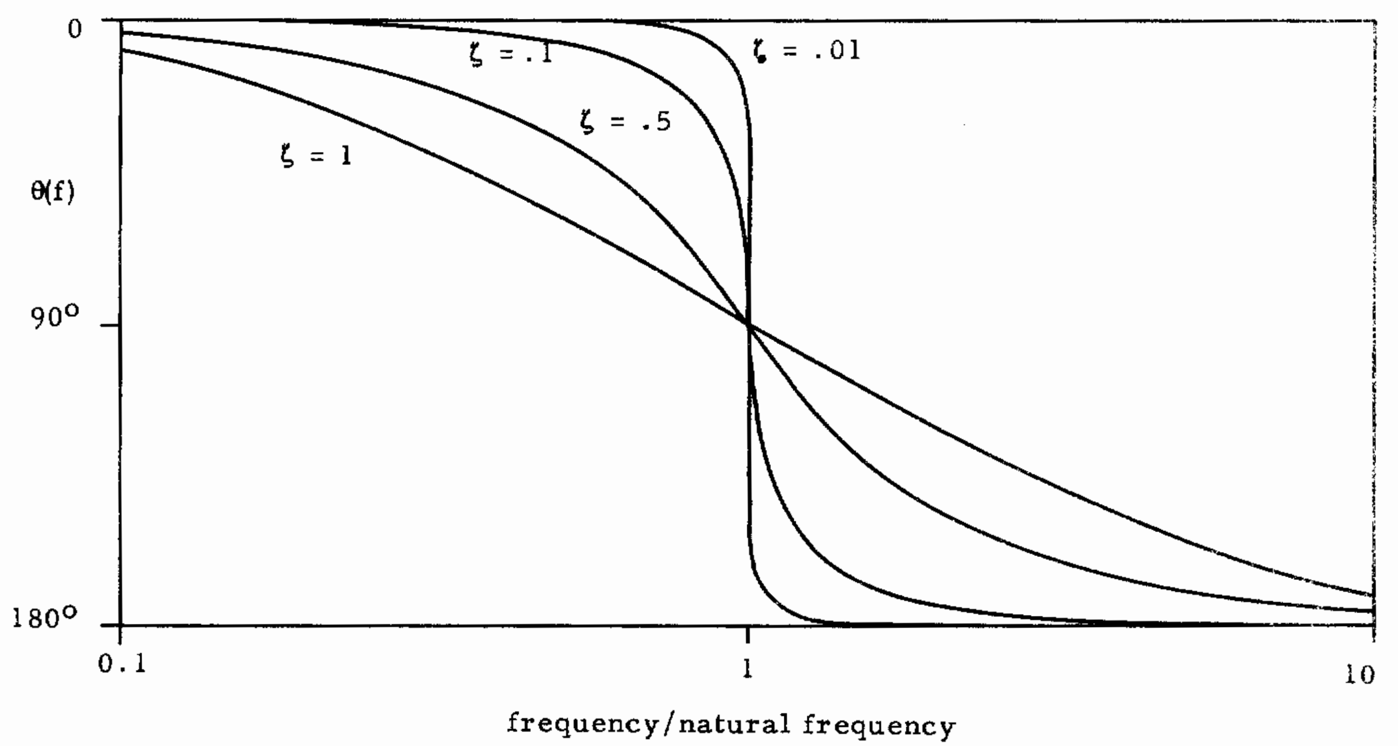


Figure 8. Frequency Response Gain and Phase Factors for a Second Order System

Specific examples of these deviations, as exhibited by real flight loads data systems, will now be reviewed. It is convenient to separate the review into discussions of deviations in static performance and in dynamic performance.

Deviations in Static Performance. The static performance of a system is described by the ratio of the output to the input at zero frequency. Ideally, this relationship is described by a straight line which passes through the origin and has a slope equal to K_o. Deviations from ideal performance, then, include various forms of nonlinearity and changes in the intercept and the slope of the input-output curve.

Consider first, the two degrees-of-freedom which are associated with an ideal linear input-output curve. In a real system, either of these may change in response to time variation of system constants or extraneous environmental effects. This is illustrated in Figure 9,

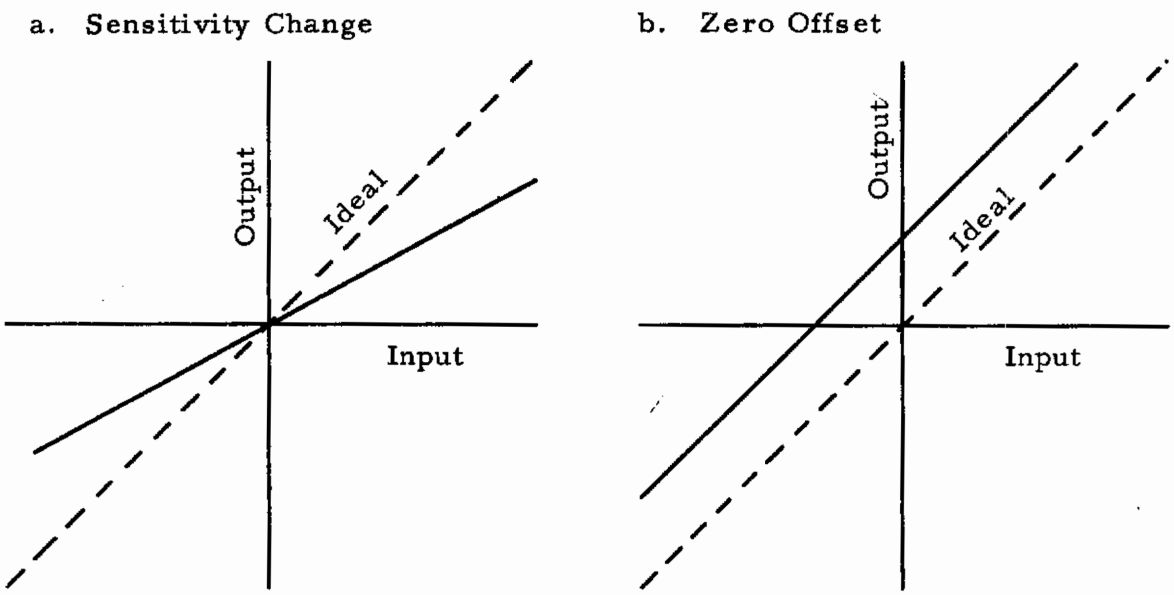


Figure 9. Two Types of System Parameter Variation with Time

where the first case constitutes a change in static sensitivity, and the second is a zero offset. The significance of these deviations is attached to the fact that they vary with time. That is, they are not subject to preflight calibration correction procedures. Specifications of these deviations are in terms of time or temperature coefficients of drift.

Variation of system parameters with amplitude produces static input-output relationships which deviate from the ideal straight line. Five major types of amplitude deviations which are worthy of discussion here are illustrated in Figure 10.

a. Nonlinearity — Nonlinearity is a general term applying to any deviation from a straight line input-output relationship. In this context, it includes the four types of deviations described below, as well as those which are ordinarily not specified separately. In a general description of nonlinearity, the deviations from the straight line input-output function are considered to be of a random nature. Therefore, the specified limits set for nonlinear performance are based on statistical error analyses.

b. Hysteresis — Hysteresis encompasses any of several specific nonlinearities which are manifest in the forms of different input-output curves for increasing amplitudes and for decreasing amplitudes. This deviation is produced by an internal energy dissipating mechanism which causes the output amplitude to lag variations in the input.

c. Finite Threshold — It is characteristic of real systems that there is some point near the origin of the input-output curve which represents the minimum absolute measurable output of the system. For the purpose of quantifying this deviation from the ideal, some specific measurable change in the output is usually given for which the corresponding input value becomes the threshold.

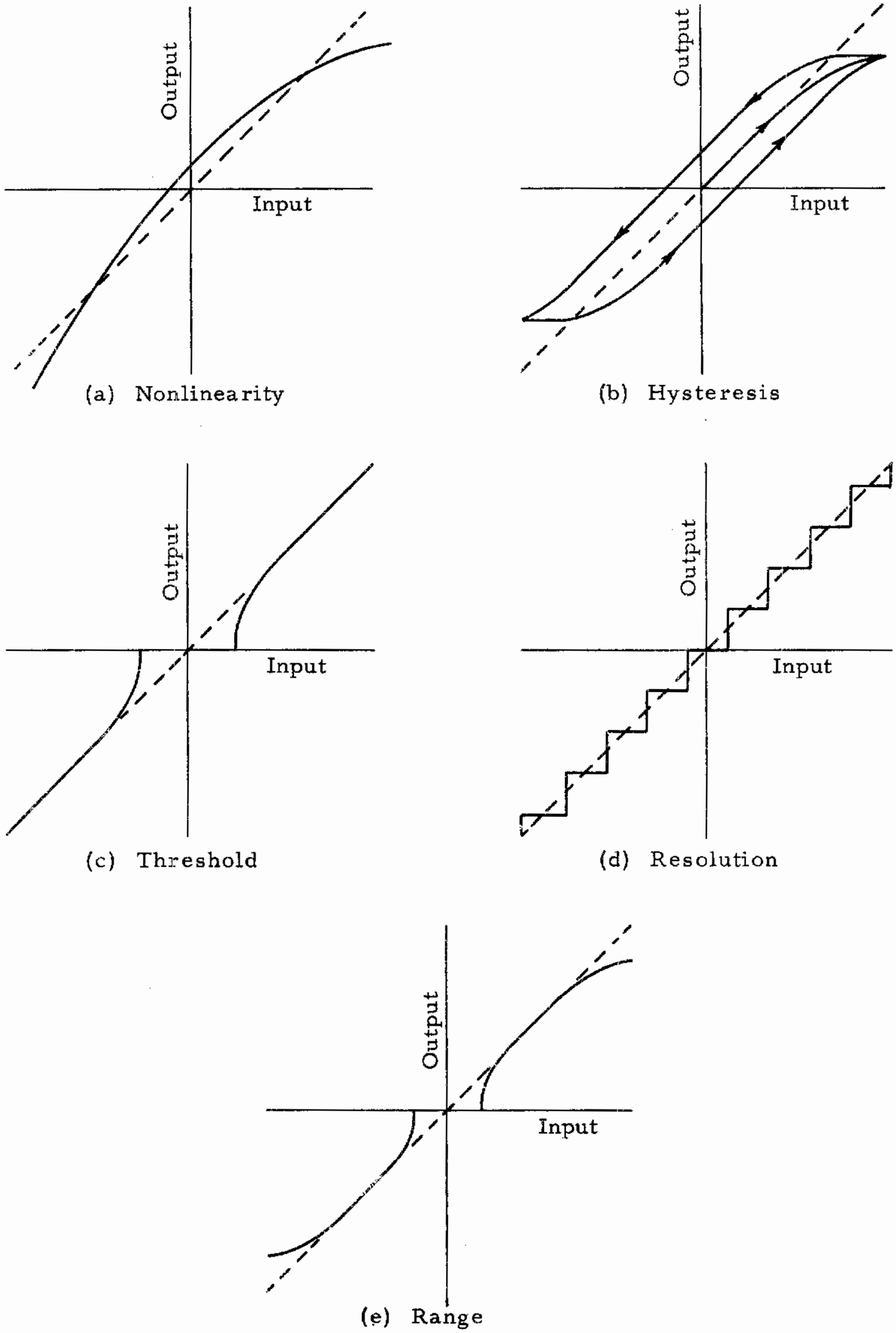


Figure 10. Major Types of Amplitude Deviations

d. Finite Resolution — A similar practical limitation in real systems which can occur at any point on the input-output curve is the minimum measurable change in output for a change in input; this is called the resolution of the system. A specific example of this limiting factor is the finite resolution inherent in digital systems which always have a fixed number of quantization levels. The numerical value of the resolution is equal to that change in the input which causes a measurable change in output.

e. Finite Range — Definitely associated with several of the above deviations from ideal system performance is the concept of a finite range for the input-output relationship. This range may be defined as the linear operating range. In another way, the usable range of static conditions may be that part of the curve falling between specified upper and lower limits of input. These limits can be established on the basis of some finite deviation at each end. In all practical systems, an upper limitation on the range of inputs is caused by the intrinsically finite capability of an instrument to handle large mechanical or electrical loads. The lower limit of this range is typically established on the basis of threshold or resolution limitations.

It is an unfortunate circumstance that real systems which are designed for response to particular physical excitations are usually also sensitive to undesired inputs. For example, a typical system responds to variations in ambient temperature, although it may not have been designed as a temperature sensor. Thermal inputs are generally accepted as the major cause of time varying deviations in static performance of data systems.

Deviations in Dynamic Performance. The dynamic performance of a system is described by the input-output relationship at all frequencies other than zero. In real systems which exhibit deviations in dynamic performance, it is convenient to assume that static performance deviations can be superimposed on the dynamic components across the frequency range. Major items to be considered in the later category include the time variation of system parameters other than K_o and the dynamic response to extraneous inputs.

Dynamic performance deviations which are related to time variation of system parameters are generally connected in some way with variability of the imaginary part of the frequency response function. In first order systems, this is produced by an instability in the time constant. As a result, the frequency response function for a real first order system could exhibit variations as shown in Figure 11. The analogous situation in a second order system is produced by time variation of system damping. Figure 12 illustrates the nature of the dynamic performance deviation caused by this parameter variation. Note especially the sensitivity of $|H(f)|$ in the region of the natural frequency to variation of damping. The shaded region of uncertainty in Figure 12 corresponds to the range of values $0.1 \leq \zeta \leq 1$ shown in Figure 8. Such variations are not unknown in practice.

Perhaps the most troublesome deviations in dynamic performance are those which appear as system response to extraneous inputs. These extraneous dynamic responses can all be lumped under the heading of noise. Noise is defined here as that part of the dynamic output which is unrelated to the desired input. The existence of noise in practical data systems is unavoidable and has the effect of decreasing the amplitude range over which usable information can be recorded. It is proper, then, to speak of the dynamic range of an instrument.

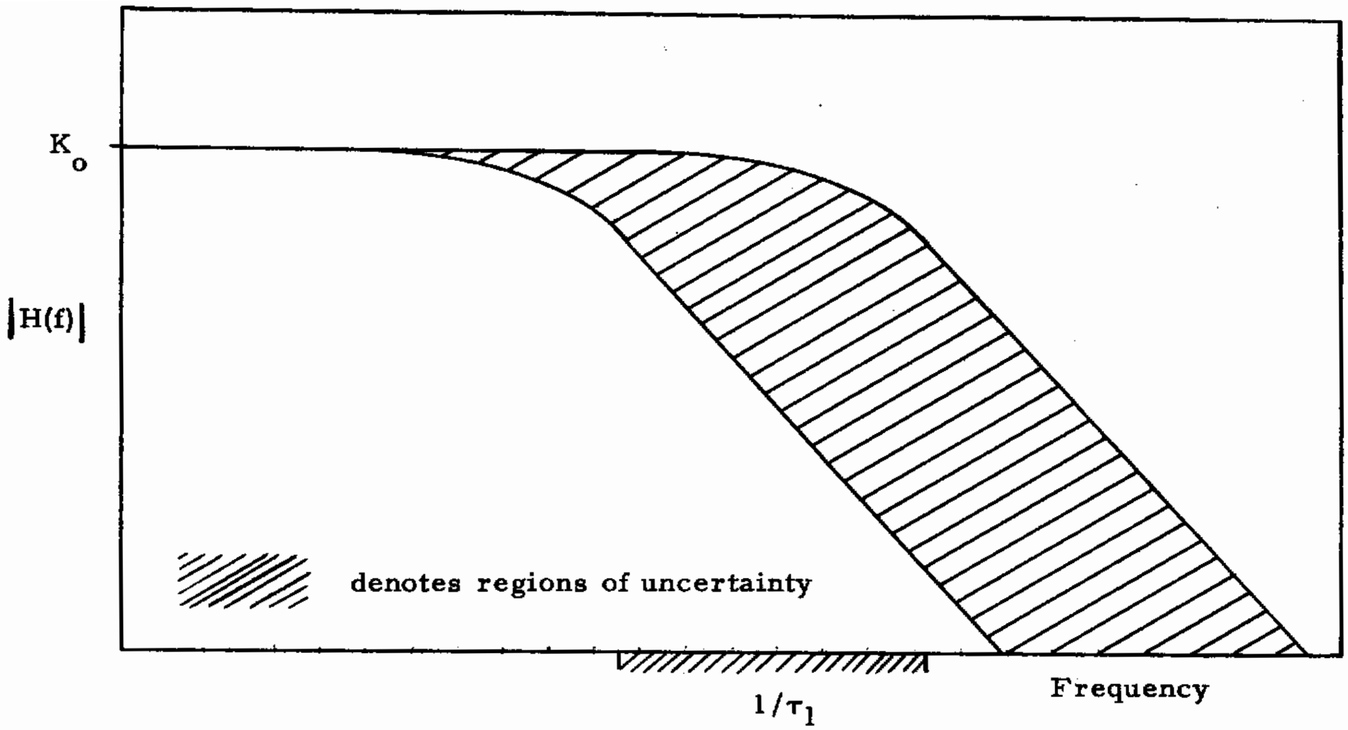


Figure 11. Effect of Time Constant Variation on the Dynamic Performance of a First Order System

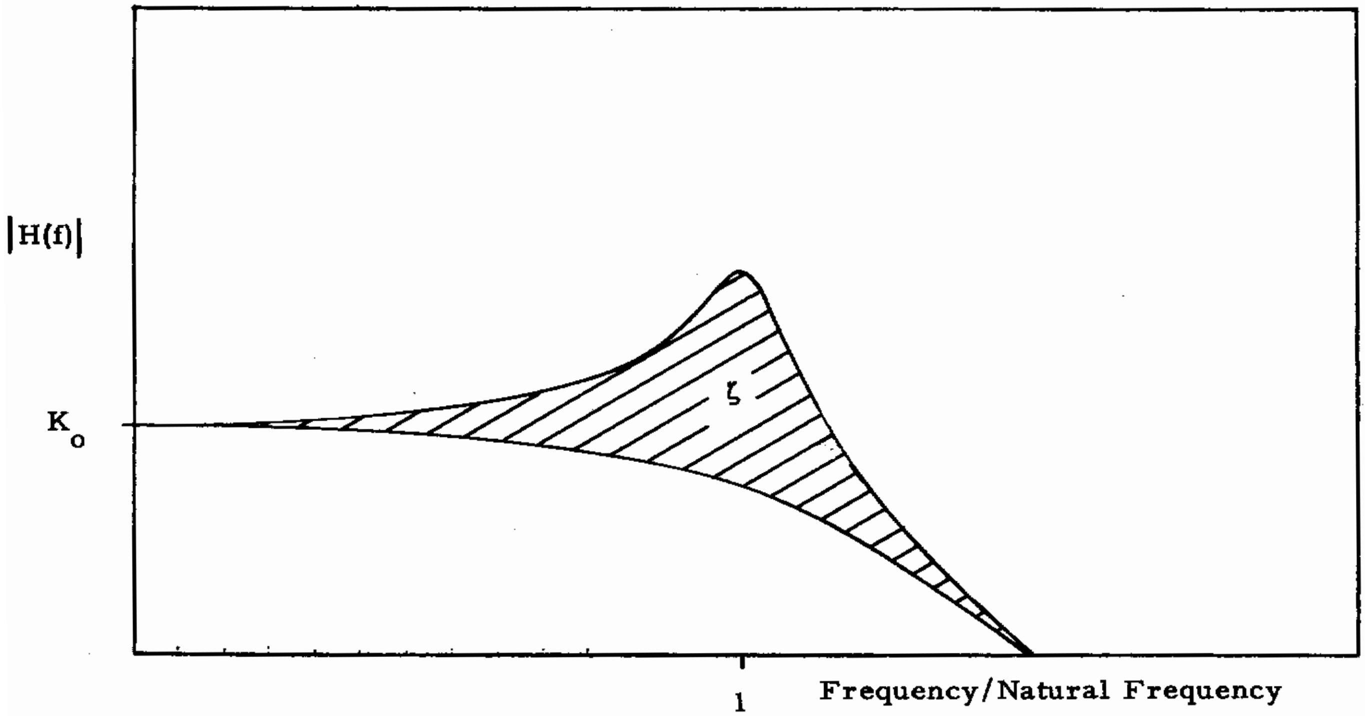


Figure 12. Effect of Damping Factor Variation on the Dynamic Performance of a Second Order System

The dynamic range is usually defined as the logarithmic ratio of maximum to minimum inputs which can be measured with specified accuracy at the output of the system and is expressed in decibels (dB). An alternative, but more ambiguous description of the effects of noise on amplitude range is called the signal-to-noise ratio (S/N). This means different things to different people, and often depends upon whether it is expressed by an instrument manufacturer or a user. Several common definitions include:

a. RMS signal/RMS noise
b. peak signal/RMS broadband noise
c. peak signal/RMS unit bandwidth noise

Here, the RMS or peak signal is chosen according to some nonlinearity criterion such as a specified percentage harmonic distortion of a sine wave. The dynamic range description of system performance is to be preferred over the S/N method. However, in doing this, an allowable error must always be specified.

In addition to the range reduction effects, noise also contributes errors to the instantaneous data amplitudes and to some statistical functions computed later. Methods for describing and evaluating these errors are covered in the next section.

3.2 SYSTEM INDUCED MEASUREMENT ERRORS

Since it has been shown that modification of the data characteristics between the input and the output can occur with both ideal and real systems, the important consideration here is whether or not this modification produces measurement errors. In a broad way, the problem can be separated into two parts. First there are those errors which result mainly from improper system design. In other words, a data system may be expected to perform operations for which it was never designed. Since it is possible to predict exactly the way in which an ideal system will affect a known input once the system performance characteristics are completely defined, this source of error should be considered manageable.

The great majority of measurement errors, however, can be related to a lack of complete knowledge about the system. Uncertainty concerning the way in which a system performs is associated directly with time variation of system parameters and response to extraneous inputs. Since, by definition these uncertainties are nondeterministic, they must be described statistically. The appropriate methods for this are similar to those used in Section 2.2 for evaluating the accuracy of estimated statistical properties. Both the bias and the variability aspects of system induced errors are considered here.

The ideal system input-output relationships in the time and frequency domains are given by Eqs. (36) and (38). In a real system these relationships may deviate considerably from the ideal, but the system can still be defined empirically by calibration. However, such a real system definition cannot be expected to remain constant with time. Thus, an initial description of a practical flight loads data system which is based on preflight calibrations actually represents a sample estimate of a random process. The sample system performance is then used in conjunction with outputs observed at any future time to estimate the behavior of the corresponding inputs. These outputs must be assumed to include system response to extraneous inputs as well as the desired inputs. Since the extraneous part of the output, or noise, is unknown it becomes a contributing factor to the overall error in the estimated input. The three stages in this estimation procedure are illustrated schematically below in Figure 13.

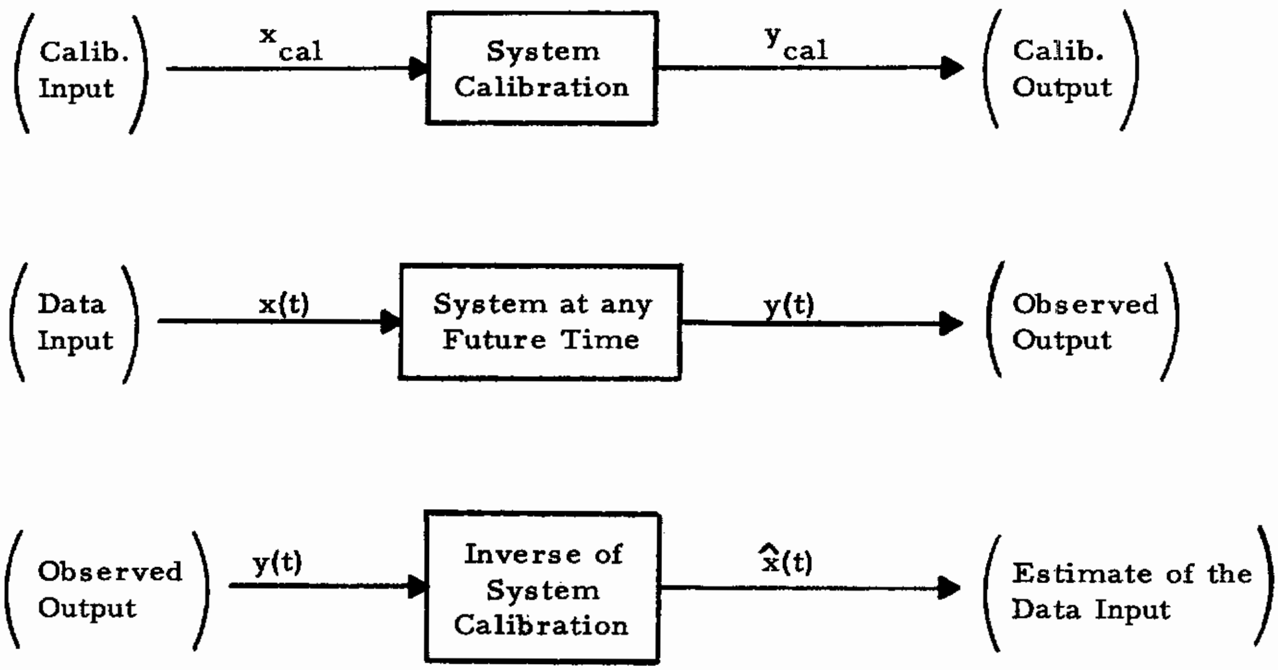


Figure 13. Procedure for Estimating Data System Inputs

The sequence of operations outlined in Figure 13 is expressed by the following formulas.

$$y_{cal} = \int_0^\infty \hat{h}(\tau)\, x_{cal}(t - \tau)\, d\tau \tag{47a}$$

$$y(t) = \int_0^\infty h(\tau)\, x(t - \tau)\, d\tau + n(t) \tag{47b}$$

$$\hat{x}(t) = \int_0^\infty \hat{\psi}(\tau)\, y(t - \tau)\, d\tau \tag{47c}$$

The data system impulse response function estimated initially through calibrations is shown in Eq. (47a) as $\hat{h}(\tau)$. The equivalent calibrated frequency response function is $\hat{H}(f)$. At any time following calibration, this function is denoted by $h(\tau)$ as in Eq. (47b). The spurious noise output which becomes a part of the total observed output $y(t)$ is symbolized by $n(t)$. The estimated input is then derived from the total observed output by Eq. (47c). The term $\hat{\psi}(\tau)$ in Eq. (47c) represents the impulse response function for a system having characteristics which are the inverse of those determined at the time of calibration. These relationships are somewhat easier to visualize in the transformed frequency domain. Thus, Eqs. (47b) and (47c) can be represented by

$$Y(f) = H(f)\, X(f) + N(f) \tag{48a}$$

$$\hat{X}(f) = \hat{\Psi}(f)\, Y(f) \tag{48b}$$

$$= \frac{Y(f)}{\hat{H}(f)}$$

Where $\hat{\Psi}(f)$ is the frequency response function corresponding to the impulse response function $\hat{\psi}(\tau)$ given in Eq. (47c). The actual input is then related to its estimate by the following

$$\begin{aligned} \hat{X}(f) &= \frac{H(f)}{\hat{H}(f)} X(f) + \frac{N(f)}{\hat{H}(f)} \\ &= \hat{M}(f)\, X(f) + \hat{\Psi}(f)\, N(f) \end{aligned} \tag{49}$$

The simplified expression given by Eq. (49) is not suitable directly for error analysis since, in the general case, x(t) does not possess a Fourier transform. This technicality is not pertinent to the development however, because a transformable input could be specified which would lead to the same final result. Then, transforming Eq. (49) back into the time domain results in an expression for the equivalent erroneous input which reflects the system induced errors.

$$\hat{x}(t) = \int_0^{\infty} \hat{m}(\tau)\, x(t - \tau)\, d\tau + \int_{0^+}^{\infty} \hat{\psi}(\tau)\, n(t - \tau)\, d\tau \tag{50}$$

The bias error associated with the above estimate of x(t) is equal to the difference between the expected value of the estimate and that of the actual input. Therefore, it is a function only of deviations in static performance. This is shown below in Eq. (51).

$$b\left[\hat{x}(t)\right] = E\left[\hat{x}(t)\right] - E\left[x(t)\right]$$

$$= E\left[\int_0^\infty \hat{m}(\tau)\, x(t-\tau)\, d\tau + \int_0^\infty \hat{\psi}(\tau)\, n(t-\tau)\, d\tau\right] - E\left[x(t)\right]$$

$$= E\left[\hat{M}(0)\right] \mu_x + E\left[\hat{\Psi}(0)\right] \mu_n - \mu_x \tag{51}$$

$$= \left(\frac{E\left[K_o\right]}{\hat{K}_o} - 1\right) \mu_x + \frac{\mu_n}{\hat{K}_o}$$

The ratio of the static sensitivity at any time given by K_o to the static sensitivity determined in calibration $\hat{K}_o$ would be expected to equal one on the average. Also, the mean value of the extraneous output μ_n, which reflects various additive random nonlinearities and zero offset of the system, would ordinarily be zero. Therefore, $\hat{x}(t)$ is usually assumed to be an unbiased estimate of $x(t)$ unless there is specific information to the contrary.

The variance of the estimate $\hat{x}(t)$ is a function of the variance of $x(t)$ as well as the variance associated with the random system parameters. Then, the variance of the erroneous measurement is derived by the following steps:

$$\mathrm{Var}\left[\hat{x}(t)\right] = E\left[\hat{x}^2(t)\right] - E^2\left[x(t)\right]$$

$$= E\left[\left(\int_0^\infty \hat{m}(\tau)\, x(t-\tau)\, d\tau + \int_0^\infty \hat{\psi}(t)\, n(t-\tau)\, d\tau\right)^2\right]$$

$$- E^2\left[\int_0^\infty \hat{m}(\tau)\, x(t-\tau)\, d\tau + \int_0^\infty \hat{\psi}(\tau)\, n(t-\tau)\, d\tau\right]$$

$$= \int_{0^+}^\infty E\left[\hat{M}(f)\, \hat{M}^*(f)\, X(f)\, X^*(f) + \hat{\Psi}(f)\, \hat{\Psi}^*(f)\, N(f)\, N^*(f)\right] df \qquad (52)$$

$$= \int_{0^+}^\infty \left[\left|\hat{M}(f)\right|^2 G_x(f) + \left|\hat{\Psi}(f)\right|^2 G_n(f)\right] df$$

$$= \int_{0^+}^\infty \frac{|H(f)|^2}{|\hat{H}(f)|^2}\, G_x(f)\, df + \int_0^\infty \frac{G_n(f)}{|\hat{H}(f)|^2}\, df$$

Evaluation of the integral expressions in the above result can only be performed after the various functions of frequency are known. The ratio of the frequency response function $|H(f)|$ at any time to that determined during calibration $|\hat{H}(f)|$ may differ considerably from unity as indicated by Figures 11 and 12. Also, if any noise is present in the output, the variance of the noise, given at the input by the second integral, will add directly to the total variance. These factors demonstrate the effects of system dynamic performance deviations on the accuracy of input estimates. The mean square value of the estimate according to Eq. (25), then consists of the sum of the variance from Eq. (52) and the square of the bias given by Eq. (51).

4. LOADS COMPUTATIONS

Flight loads originate with external aerodynamics and internal mass reaction. Since aerodynamic pressures act at all points on the exterior of an aircraft, and inertia forces can be generated in any particle of the mass, these loads are continuously distributed. As such, they are not suitable for direct measurement. Therefore, it is necessary to measure other variables which are known to be correlated with the desired loads, and to provide a means for bridging the gap between the measured variables and the results required for a particular application. In most cases this is done using a semi-empirical computational model. Such a model permits time history or statistical measures of a loads variable of interest to be computed on the basis of corresponding values for one or more measurable variables.

The accuracy of computed loads depends upon the accuracy of the computational models and computing techniques as well as the accuracy of any measured quantities. Model accuracy is a function of the uncertainty of basic assumptions and the completeness of the expressions. Errors associated with incomplete models can be related to flight instrumentation requirements in that there is an implied trade-off between the number of transducers installed on a vehicle and the accuracy of the computed loads for any particular application. Thus, the efficiency of data acquisition is largely a function of the computational models used to reduce the data. Several aspects of this relationship are brought out in the following review and analysis of the more commonly employed models currently in use.

4.1 COMPUTATIONAL MODELS AND VARIABLES

In keeping with current practice, it is assumed that a separable loads function for any particular set of flight loads exists which can be stated in general form as follows:

$$\left\{L\right\} = \left[A\right] \left\{x\right\} \tag{53}$$

Here $\left\{L\right\}$ represents a matrix of computed loads, $\left\{x\right\}$ is a matrix of variables or combinations of variables which can be measured in flight, and $\left[A\right]$ is a transfer matrix relating loads to measured quantities. This transfer matrix may include auxiliary geometric, mass, and aerodynamic variables, or it may consist of calibration factors or influence coefficients. In any case, the values assigned to the elements are assumed to be predictable during flight. As will be shown subsequently, the accuracy of some computed loads is highly sensitive to the accuracy of these predictions. Actually, the A's should be thought of as random variables. As such, they will contribute to the overall uncertainty in loads computation.

The basic form of Eq. (53) can be expanded to include analytical models of ever increasing complexity which produce loads estimates of corresponding refinement. Some of the models in current use and the variables required for their application will now be discussed. This discussion is broken down into approaches based on rigid body and flexible aircraft response models.

4.1.1 Rigid Body Response Models

The use of rigid body models requires the assumption that the aerodynamic forces and moments applied externally to an aircraft can be described completely by the relative motion between the aircraft body coordinates, with origin at the center of gravity, and a set of inertial coordinates. It also must be assumed that all inertia loads can be computed from measured accelerations through and about the center of gravity, known geometry, and known mass distribution. Aeroelastic dynamic effects are ignored. That is, the velocity and acceleration of all parts of the aircraft relative to the body axes are taken equal to zero although quasi-static elastic deflections are permitted under the rigid body assumption.

The geometric and aerodynamic notation in this section will generally follow the ASA standard (Reference 7). Figure 14 illustrates this convention for the aircraft body axes, angular velocities, and external forces and moments. Note that the axial and normal forces are opposite in sense to the x and z coordinates respectively. Moments and angular velocities about the center of gravity are all clockwise when looking in the positive direction of the axes.

The simplest expansion of Eq. (53) which is of current interest defines the total aerodynamic normal force F_N acting through the aircraft center of gravity in terms of vehicle weight W and normal acceleration a_N.

$$F_N = \frac{W}{g} a_N \qquad (54)$$

The normal acceleration is often divided by the earth's gravitational constant g for presentation as normal load factor n_N (or n_Z). In this case Eq. (53) has only the one coefficient W. If the actual weight of the aircraft is known, and the normal acceleration measurement is truly made at the center of gravity, Eq. (54) provides an adequate rigid body model for determining the total aerodynamic normal force. This

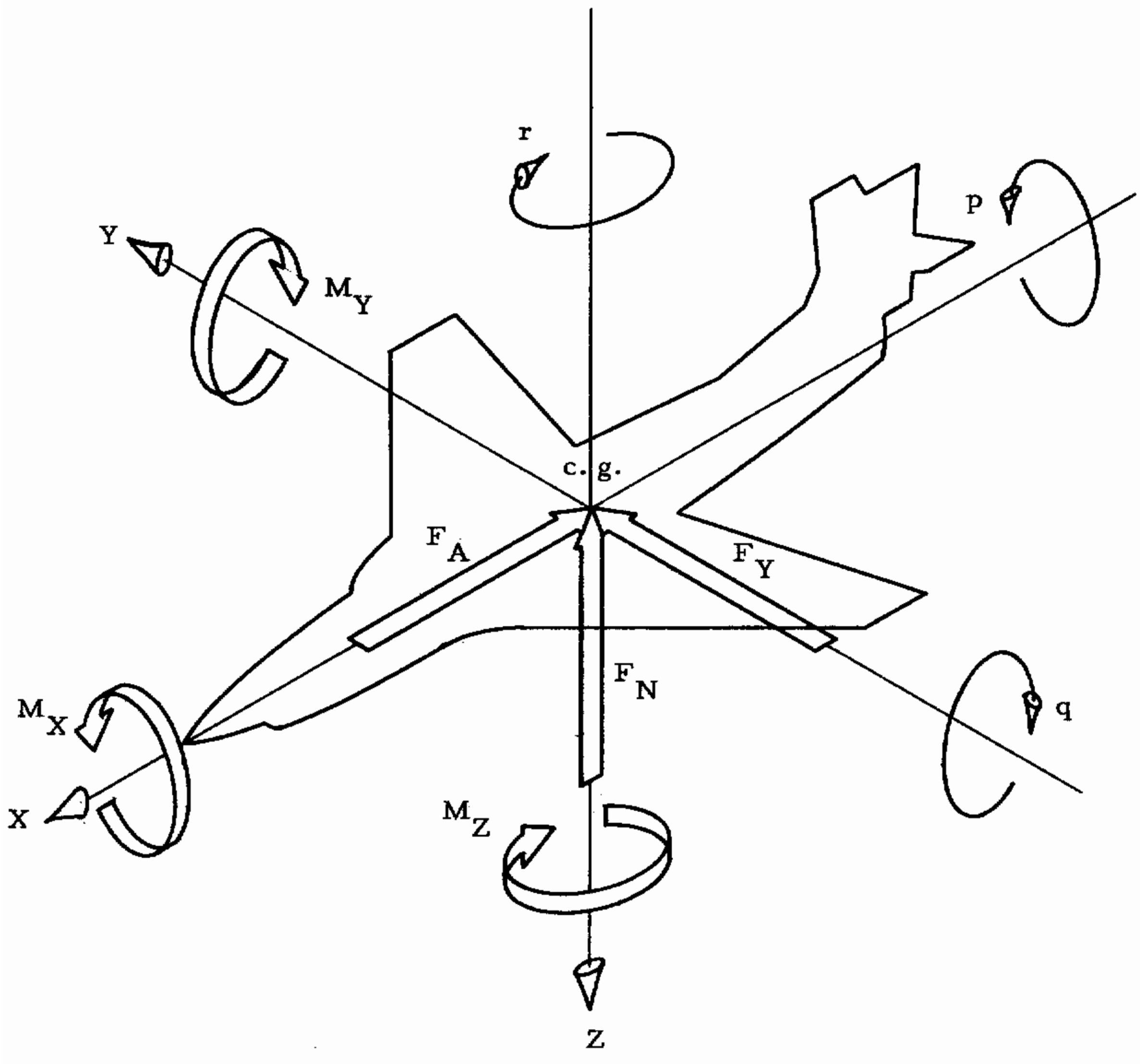


Figure 14. Sign Convention for Body Axes, Forces, Moments and Angular Velocities Used in Loads Computation

relationship, however, has limited application to detailed structural loads analysis.

The usefulness of normal acceleration data can be enhanced somewhat by adding the variables airspeed and altitude to the measurement list. These two variables are actually derived from pressure measurements but the details of the conversion are not considered essential to this discussion. The additional data can be utilized in the following three ways:

a. As an aid to the identification of flight activity

b. As an indication of changes in aerodynamic pressure distributions

c. As a requirement for computing normal gust velocities to which the aircraft is responding

Airspeed and altitude measurements permit the grouping of data into statistically significant classes. These classes may be identified merely in terms of airspeed and altitude intervals or they may describe mission type or mission segment. In addition, a trained data analyst can utilize airspeed and altitude information in conjunction with normal acceleration data to identify with reasonable certainty the maneuvers through which an aircraft has gone.

Although center of gravity normal acceleration is proportional to the net aerodynamic loading integrated over the entire aircraft, it provides no measure of air load distribution. Knowledge of the Mach number, derived from the same pressures used for airspeed and altitude, will at least supply a rudimentary indication that major shifts in centers of pressure have occurred. These shifts are due to different air flow characteristics in the supersonic and subsonic flight regimes.

One principal application for velocity-acceleration-altitude (VGH) data is to provide a crude estimate of the gust environment through which an aircraft has flown. A normal component of gust velocity acts as an effective angle of attack modulator, and the resulting fluctuations in wing lift are assumed to bear a linear relationship to the velocities. In keeping with linearity and rigid body assumptions, it follows that the maximum normal acceleration is also directly proportional to the maximum gust velocity. A standard model of this relationship, often referred to as the discrete gust equation (Reference 8), is given by Eq. (55).

$$U_{de} = \frac{2W}{a\,\rho_0\,S\,K_g}\;\frac{a_{N_{max}}}{V_e} \tag{55}$$

where

U_{de} = derived normal gust velocity (peak value)

$a = \dfrac{dC_L}{d\alpha}$, the lift curve slope

ρ_0 = sea level atmospheric density

S = wing area

K_g = gust alleviation factor ($0 < K_g < 1$)

V_e = equivalent airspeed (a function of altitude and true airspeed)

The term K_g provides an adjustment to allow for the fact that actual aircraft acceleration response lags a transient change in angle of attack by a finite amount and never quite achieves the theoretical maximum. Curves for this function to be used in military aircraft design are given in Reference 9 for subsonic and supersonic speeds. A somewhat different function is used if an aircraft is assumed to have flexible response.

A more useful form of Eq. (54) is made possible by the addition of pitching acceleration $\dot{q} = dq/dt$ (or pitching velocity which can be differentiated during data reduction) to the measurements and the assumption that all aircraft motion is symmetrical with respect to the XZ-Plane. Under these conditions, the normal force through the center of gravity can be broken down into the sum of the normal force associated with the wing-fuselage combination $F_{N, wf}$ and the normal force on the horizontal tail $F_{N, h}$.

$$F_N = F_{N, wf} + F_{N, h} \tag{56}$$

Then, with the three variables normal acceleration, pitching acceleration, and dynamic pressure q_d (derived from pressure measurements), the two components of normal force can be written in the following forms:

$$F_{N, wf} = \underbrace{\frac{x_h}{x_h - x_{wf}} \frac{W}{g}}_{A_{11}} a_N - \underbrace{\frac{I_Y}{x_h - x_{wf}}}_{A_{12}} \dot{q} + \underbrace{\frac{C_{m_0, wf} S \bar{c}}{x_h - x_{wf}}}_{A_{13}} q_d \tag{57a}$$

$$F_{N, h} = \underbrace{- \frac{x_{wf}}{x_h - x_{wf}} \frac{W}{g}}_{A_{21}} a_N + \underbrace{\frac{I_Y}{x_h - x_{wf}}}_{A_{22}} \dot{q} - \underbrace{\frac{C_{m_0, wf} S \bar{c}}{x_h - x_{wf}}}_{A_{23}} q_d \tag{57b}$$

where

x_h = coordinate distance along the X-Axis from the center of gravity to the center of aerodynamic pressure for the horizontal tail

x_{wf} = coordinate distance along the X-Axis between the center of gravity and the center of aerodynamic pressure for the wing-fuselage combination

I_Y = aircraft moment of inertia in pitching

$C_{m_0, wf}$ = pitching moment coefficient for the wing-fuselage with zero lift

S = wing area

$\bar{c}$ = mean aerodynamic chord

It can be seen that the normal forces defined in Eqs. (57a) and (57b) are heavily influenced by the elements A_{ij} containing geometric, mass, and aerodynamic quantities not measured in flight. Thus, the ability to predict these quantities becomes fairly important to the overall accuracy.

The complete wing-fuselage and horizontal tail normal force expressions for arbitrary motion (except for engine gyroscopic moments, thrust vector eccentricity moments, and a few insignificant loads) are adapted from Reference 10 as follows.

$$F_{N,wf} = \frac{x_h}{x_h - x_{wf}} \frac{W}{g} a_N - \frac{I_Y}{x_h - x_{wf}} \dot{q} + \frac{I_Z - I_X}{x_h - x_{wf}} pr + \frac{I_{XZ}}{x_h - x_{wf}} (r^2 - p^2) + \frac{C_{m_0, wf} S \bar{c}}{x_h - x_{wf}} q_d \tag{58a}$$

$$F_{N,h} = - \frac{x_{wf}}{x_h - x_{wf}} \frac{W}{g} a_N + \frac{I_Y}{x_h - x_{wf}} \dot{q} - \frac{I_Z - I_X}{x_h - x_{wf}} pr \qquad (58b)$$

$$+ \frac{I_{XZ}}{x_h - x_{wf}} (r^2 - p^2) - \frac{C_{m_0, wf} S \bar{c}}{x_h - x_{wf}} q_d$$

where

p = angular velocity about the X-Axis (rolling velocity)

r = angular velocity about the Z-Axis (yawing velocity)

I_X = aircraft moment of inertia in rolling

I_Z = aircraft moment of inertia in yawing

I_{XZ} = aircraft product of inertia

Computation of the wing-fuselage and horizontal tail normal forces from Eq. (58) requires in-flight measurement of two additional variables, p and r. In many cases these variables contribute little to the accuracy of the computation and are ignored. They are essential, however, for critical unsymmetrical loads evaluation.

Similar equations for the major aerodynamic forces and moments in the different axes are given in Reference 10. Some simplifications which were found to be applicable to a particular fighter-bomber are listed in Reference 11. Models of the complexity described in both of these references require a maximum of eight in-flight measured variables plus a time reference. The eight variables are:

$$\left.\begin{array}{l} a_X\ (-a_A) \\ a_Y \\ a_Z\ (-a_N) \end{array}\right\}\ \text{center of gravity linear accelerations}$$

$$\left.\begin{array}{l} p\ (\text{or } \dot{p}) \\ q\ (\text{or } \dot{q}) \\ r\ (\text{or } \dot{r}) \end{array}\right\}\ \text{angular velocities (or accelerations)}$$

$$\left.\begin{array}{l} p_1 \\ p_2 \end{array}\right\}\ \text{pressures for: } \begin{array}{l}\text{airspeed, altitude, air density} \\ \text{dynamic pressure, and Mach number}\end{array}$$

With reference to flight recorder requirements, this combination is often called "eight channel data." These variables are adequate for the derivation of gross aerodynamic loads and, in conjunction with known mass distribution, detailed inertia loads.

The inertia loads induced by the motion of an aircraft under the influence of air loads are determined by the distributions of mass and acceleration in the vehicle. Under the rigid body assumption, the acceleration at any point is strictly a function of geometry and the first six variables listed above. Thus, the accelerations in each of the coordinate directions associated with an arbitrary particle of mass are given by the following expressions (Reference 11).

$$a_X(x, y, z) = a_X(0, 0, 0) - x(q^2 + r^2) - y(\dot{r} - pq) + z(\dot{q} + rp) \tag{59a}$$

$$a_Y(x, y, z) = a_Y(0, 0, 0) - y(r^2 + p^2) - z(\dot{p} - qr) + x(\dot{r} + pq) \tag{59b}$$

$$a_Z(x, y, z) = a_Z(0, 0, 0) - z(p^2 + q^2) - x(\dot{q} - rp) + y(\dot{p} + qr) \tag{59c}$$

Although eight channel data do not define aerodynamic pressure distributions, these can often be estimated on the basis of wind tunnel test data or actual flight test pressure surveys. Given this additional input, it is possible to compute detailed structural loading conditions.

As an example of a detailed loads computation, consider the shear on a conventional wing at any given station y_1 for symmetrical motion of the aircraft. Let $\phi(y)$ represent the normalized spanwise distribution of lifting forces on the wing-fuselage, and assume $m(y)$ to be the distributed mass function. Then, at any instant of time the shear $V(y_1)$ at the given wing station is obtained by integrating the aerodynamic and inertia forces from station y_1 to the wing tip as follows:

$$V(y_1) = F_{N,wf} \int_{y_1}^{s} \phi(y)\, dy + \int_{y_1}^{s} m(y)\, a_N(y)\, dy \qquad (60)$$

where $F_{N,wf}$ and $a_N(y)$ are obtained from Eqs. (57a) and (59c) respectively, and s is the wing semispan.

The rigid body response models discussed to this point have been limited to the use of kinematic measurements made at the aircraft center of gravity. If however, the attitudes of the lifting and control surfaces with respect to the wind stream are also measured, a somewhat better method of quasi-static loads evaluation is available. This requires inputs from the eight-channel data discussed previously plus the following additional in-flight measured variables: angle of attack, angle of side slip, and control surface positions. The new variables are employed in an aeroelastic model to determine the aerodynamic forces (local surface pressure integrals) at a set of points on the vehicle surface called control points. The principal element in the required transfer matrix for this model is the aerodynamic influence coefficient (AIC). AIC's define the relationships between a unit control point deflection and the aerodynamic forces which this induces at the set of control points. A general form of the model which defines this relationship for quasi-static loading conditions can be written

$$\left\{F_a\right\} = B \left[C_{hs}\right] \left\{h\right\} \tag{61}$$

where

F_a = aerodynamic force acting at each control point

B = factorable dimensional constants

C_{hs} = static aerodynamic influence coefficient

h = control point deflection

The form of Eq. (61) implies the need for evaluating deflections arising from both rigid body motion and quasi-static elastic deformation. However, with the appropriate ground test data it is possible to rewrite this expression in terms more adaptable to in-flight data acquisition. The required ground measurements are those needed to determine a matrix of structural influence coefficients $\left[A\right]$. These coefficients define the relationships between a unit control point force and the elastic deflections which this induces at the set of control points. Then, from Reference 12, the matrix of aerodynamic forces at the control points resulting from a set of deflections expressable in terms of the additional in-flight variables is given by

$$\left\{F_a\right\} = \left(\left[I\right] - \frac{q_d S}{\bar{c}} \left[C_{hs}\right]\left[A\right]\right)^{-1} \frac{q_d S}{\bar{c}} \left[C_{hs}\right]\left\{h\right\} \tag{62}$$

where $\left[I\right]$ is a unit matrix. The aerodynamic influence coefficients for quasi-static loading conditions are real valued and can be obtained from wind tunnel tests as functions of airspeed, altitude and Mach number.

An alternative to Eq. (62) would be the direct measurement of surface pressure at each control point. Although this approach is inherently more accurate, it increases the number of measurements greatly and is only practical on special test aircraft.

The inertia force associated with each control point is given by the product of the local mass M and the control point acceleration computed using Eq. (59). The matrix representation for the inertia forces is written

$$\left\{F_i\right\} = \left[M\right]\left\{a\right\} \tag{63}$$

Finally, the detailed loads on various structural components are computed from the total control point forces using transformation matrices established by ground calibration procedures.

4.1.2 Flexible Response Models

Computational models which apply under the flexibility assumption often represent extensions into the frequency regime of the corresponding static analyses used with rigid body assumptions. Looked at another way, these models can be considered as general purpose tools and then specialized for quasi-static loading by making frequency terms approach zero in the proper ways. Thus, it may sometimes be advantageous to use flexible response models for computation of quasi-static loads where there is no penalty involved in doing this.

If the total response of an aircraft to flight loads includes a significant proportion of elastic mode participation, the rigid body assumptions used in the last section can no longer be considered adequate. This is the case with large and medium bomber, reconnaissance, and transport aircraft. Elastic mode response can be excited by various inputs which are generally classed as dynamic loads environments. Three of the more important of these are:

a. Atmospheric turbulence
b. Landing impact
c. Runway roughness

Since this presentation is concerned with the measurement of flight loads, only the first of these environments is examined. Note that pilot induced maneuvers are not usually considered to be in the class of dynamic loads since aircraft control systems are intentionally designed to minimize elastic mode response.

Gust loads on a flexible aircraft are typically evaluated by measuring some aircraft response function and employing a model to compute the statistical properties of the input gust velocity. A simplified model in current use requires only VGH data and transfer terms similar to those of Eq. (55). Although the necessary in-flight measurements are quite unsophisticated, it is usually difficult to evaluate the unmeasured part of the model with reasonable accuracy.

A good description of this approach, which has been referred to as the power spectral method, is given in Reference 8. The basic assumptions are that the gust velocity environment is:

a. Isotropic

b. Homogeneous

c. Gaussian with zero mean value

d. Described by a particular power spectral density function at each altitude

Given the above assumptions, the gust velocity environment can be completely described by the standard deviation or RMS value of the instantaneous vertical (normal) velocity σ_w. This is obtained from measured RMS normal acceleration σ_N through the following model.

$$\sigma_w = \frac{2W}{a\,\rho_0\,S\,K_\phi}\;\frac{\sigma_N}{V_e} \qquad (64)$$

where the terms are the same as those defined for Eq. (55) except that the gust factor K_ϕ includes both rigid body and flexible response effects. This factor can be defined by expanding Eq. (64) in terms of the input gust velocity power spectrum $G_w(f)$ and the aircraft frequency response function magnitude $\left|H_{wN}(f)\right|$ between vertical gust velocity w and normal acceleration response a_N at the center of gravity.

$$K_\phi = \frac{2W}{a\,\rho_0\,S\,V_e}\left[\frac{\int_0^\infty G_w(f)\,\left|H_{wN}(f)\right|^2\,df}{\int_0^\infty G_w(f)\,df}\right]^{1/2} \qquad (65)$$

A gust frequency response function is established during the development phase for each aircraft configuration and set of flight conditions. An appropriate computational model which is based upon measurement of instantaneous value gust velocity and normal acceleration data is given by

$$\left|H_{wN}(f)\right| = \frac{\left|G_{wN}(f)\right|}{G_w(f)} \qquad (66)$$

The term $\left|G_{wN}(f)\right|$ is the magnitude of the cross spectral density function between $w(t)$ and $a_N(t)$.

Once a set of gust frequency response functions has been determined, it is only necessary to measure VGH data and estimate the aircraft weight and configuration at any time in order to describe the vertical gust environment. Note that this description should apply to axial and lateral gusts as well as to vertical gusts if the isotropic assumption is valid.

Instantaneous value gust data such as required in the use of Eq. (66) are computed from in-flight measurements as follows.

$$w(t) = V\left[\alpha(t) - \bar{\alpha}\right] - V\int_0^t \left[q(\tau) - \bar{q}\right] d\tau + \int_0^t \left[a(\tau) - \bar{a}\right] d\tau + V\left[\alpha'(t) - \bar{a}'\right] \tag{67}$$

where

- w = true vertical gust velocity
- V = air speed
- α = indicated angle of attack
- q = pitch rate
- a = vertical acceleration at the angle of attack sensor on nose boom
- α' = incremental angle of attack resulting from nose boom bending

Note that Eq. (67) consists entirely of in-flight measured quantities. Therefore, it is not necessary to make any assumptions concerning the particular aircraft involved. The first term on the right side of Eq. (67) represents the uncorrected gust velocity measurement and the remaining terms correct for aircraft attitude and movement. The second term is an aircraft pitch angle correction which can be obtained from stable platform measurements, vertical gyro measurements, or as indicated, by integration of pitch rate. The third term corrects for aircraft vertical velocity at the gust sensor. This correction is most easily obtained from acceleration measurements and takes into account the modal response of the fuselage. The last term is of minor importance and corrects for angle of attack errors induced by boom flexibility. This quantity can be measured as a function of the bending strain of the boom.

The gust velocity descriptions which can be computed from Eqs. (64) and (67) are one-dimensional in nature. That is, at any instant in time it is assumed that the spanwise distribution of vertical velocity is uniform. Semi-empirical analytical models do exist which provide two-dimensional gust descriptions. However, these models generally contain terms which are difficult to evaluate either analytically or empirically. (See, for example, Reference 12.) Attempts to measure the spanwise distribution of vertical gusts directly have so far been unproductive. Therefore, current practice is to accept the one-dimensional descriptions of the gust environment computed from measurements on the fuselage center line.

An independent approach to dynamic (and static) flight loads evaluation which is in common use for experimental and structural demonstration flight tests involves direct observation of the elastic deformation of the structure. This deformation is described in terms of deflections and strains. Although deflections are sometimes measured using photographic techniques, the principal variable of interest here is strain. It should be noted, however, that either method may prove superior for a specific application.

Strain measurements are particularly valuable in the evaluation of detailed loading conditions on conventional tail structures and medium to high aspect-ratio wings. Unfortunately, serious problems have arisen when attempting to apply this technique to low aspect-ratio, highly swept wings and some fuselage structures. These difficulties result from the multiple load paths inherent in redundant structures and are manifest as gross nonlinearities. In the absence of severe problems, however, strain measurements provide the most accurate medium for loads evaluation since they can be calibrated directly in terms of precisely measured forces and moments. The practical drawbacks to this approach concern the number and reliability of transducers required for a complete survey, as well as the recorder capacity requirements.

Computational models used to derive loads from strain measurements are of the general form

$$\left\{ L \right\} = \left[A \right] \left\{ e \right\} \qquad (68)$$

where $\left[A \right]$ is a matrix of calibration coefficients obtained from ground tests by least squares fitting of the strain responses to a set of known loads.

4.2 COMPUTATIONAL UNCERTAINTIES

Errors which are functions of the methods used to compute loads must be considered in addition to the other known error sources when evaluating overall uncertainty in the results. These errors can be classified broadly into two categories. First, there are those errors which are associated with incorrect assumptions in defining and evaluating the computational models, and then there are the errors which are introduced or modified by the computing techniques.

4.2.1 Model Errors

It can be said that the accuracy of computed loads is no greater than the accuracy of the computational models employed. In this context, accuracy has several meanings. First, there is the accuracy of basic assumptions about the nature of the phenomenon being studied. For instance, the question of whether or not the loading conditions can be assumed to be quasi-static, and thus suitable for computation using a rigid body model, bears directly on the quality of the results. This is pointed out in Reference 13 where it was found that the rigid body assumption for a large flexible aircraft led to errors as great as 100% in wing strain measurements.

Even if the correct type of model has been chosen, two other major areas of inaccuracy must be considered. These are the model complexity which is appropriate in each case and the values which are assumed for the transfer terms. It is difficult to define many of these uncertainties by explicit equations for the general problem. However, functional expressions have been developed which can be utilized in specific cases if the appropriate information is available.

Model Complexity. Several of the computational models in current use have been reviewed in Sections 4.1.1 and 4.1.2. These vary in complexity over a wide range, where the degree of complexity is implicitly a function of the number of variables measured in flight. One of the criteria considered when planning any loads measurement program, then, is to establish the simplest acceptable model for which in-flight variables must be measured. The acceptability of a model depends upon the particular loads which are to be evaluated and the amount of error which is introduced by ignoring specific in-flight variables. For example, if only the total normal aerodynamic force F_N is to be computed from flight data according to Eq. (54), it would be redundant to require eight-channel data for this purpose. Only normal acceleration values at the center of gravity would be necessary in this case, and no error would be introduced by ignoring seven of the eight-channel variables. On the other hand, eight-channel data do not define the distribution of aerodynamic loads on a vehicle. Thus, detailed loads computed from eight-channel data are always somewhat in error because in-flight measurement of the pressure distribution has been neglected.

It is usually desirable to delete from the measurement list those variables which are of negligible importance in the computations. However, this should not be done indiscriminately. In any particular situation, the relative importance of excluding each variable can be assessed by analysis of the resulting error. A functional expression relating model complexity and error should be developed along the following lines.

An arbitrary loads variable L_i can be defined in terms of Eq. (53) by the series

$$L_i = \sum_j A_{ij} x_j + \sum_k A_{ik} x_k \tag{69}$$

where the first summation refers to the components of the total load derived from actual in-flight measured variables x_j. The second summation includes all measurable effects on L_i which are to be neglected. It is assumed that a "complete" solution of Eq. (69) would result in true loads.

Simplification of the "complete" model produces a bias error in the estimate of L_i which is given by

$$b\left[\hat{L}_i\right] = E\left[L_i - \sum_j A_{ij} x_j\right] \tag{70}$$

Assuming for the present that the various transfer terms are known constants, this error can be expressed as a relative percent by Eq. (71).

$$\frac{b\left[\hat{L}_i\right]}{E\left[L_i\right]} = \frac{100\sum_k A_{ik}\, E\left[x_k\right]}{\sum_j A_{ij}\, E\left[x_j\right] + \sum_k A_{ik}\, E\left[x_k\right]}\ \% \tag{71}$$

The expected values required for any particular application can usually be estimated from past data, and the transfer terms would be functions of the particular aircraft being studied. This approach is especially well suited to rigid body models of the type given in Eq. (58).

<u>Transfer Terms</u>. Since the transfer terms A_{ij} in Eq. (53) are actually not known constants in any practical set of circumstances, they must be treated as random variables. Uncertainties associated with these quantities can lead to errors in the computed loads which are greater than those from any other source.

Transfer term uncertainties are introduced at two stages during a measurement program. First, the statistical nature of the empirical methods used in evaluating various geometric, mass, aerodynamic, and structural constants implies that the nominal values for these terms are actually sample estimates of underlying random variables. Therefore, a small amount of uncertainty exists in what might be considered the initial values of the model parameters. Then, during the period of data acquisition, the values of these constants must be predicted as they change with time and related flight parameters. Uncertainty in the predicted values is ordinarily much greater than in the nominal values. However, the relative effects of these two error sources on the computed loads can be a function of model complexity. In complex models which employ aerodynamic influence coefficients or multiple strain measurements, for instance, the dominant error may be in the nominal values of the coefficients, although the total error is often quite small.

It is usually desirable to assume that the errors resulting from uncertain model parameters are unbiased. Then, the appropriate way of expressing the variability error is in the relative statistical form of normalized standard error e.

Using Eq. (53) to define the model, an estimate of the loads variable L_i would be computed according to the series

$$\hat{L}_i = \sum_j \hat{A}_{ij} x_j \tag{72}$$

where uncertainty in the A's is now considered as the only error source. The expected value and variance of $\hat{L}_i$ are given by

$$E\left[\hat{L}_i\right] = \sum_j E\left[\hat{A}_{ij}\right] \mu_{x_j} \tag{73a}$$

$$\mathrm{Var}\left[\hat{L}_i\right] = \sum_j \left(\mathrm{Var}\left[\hat{A}_{ij}\right] \left(\mu_{x_j}^2 + \sigma_{x_j}^2 \right) + E^2\left[\hat{A}_{ij}\right] \sigma_{x_j}^2 \right) \tag{73b}$$

Dividing the square root of Eq. (73b) by Eq. (73a) produces the normalized standard error in terms of the model parameter estimates and the moments of the measurable random variables.

$$e\left[\hat{L}_i\right] = \frac{100\left[\sum_j\left(\mathrm{Var}\left[\hat{A}_{ij}\right]\Psi_{x_j}^2 + E^2\left[\hat{A}_{ij}\right]\sigma_{x_j}^2\right)\right]^{1/2}}{\sum_j E\left[\hat{A}_{ij}\right]\mu_{x_j}}\ \% \tag{74}$$

If the nominal values of the A_{ij}'s have been determined with reasonable accuracy, they can be used as first order approximations for the expected values $E\left[\hat{A}_{ij}\right]$. Estimates of the variances $\mathrm{Var}\left[\hat{A}_{ij}\right]$ must come from experience. Reference 10 discusses and tabulates the expected errors for a few useful coefficient terms. However, many factors are involved here. Probably the most important of these is the proximity between the time when the nominal values for the constants are determined and the time of data acquisition. If, for example, the weight, weight distribution, and aerodynamic configuration for an aircraft are recorded just prior to a data flight (known conditions), Reference 10 indicates a general reduction in computational error of about 5 to 1 over the case where these events occur at independent times (unknown conditions). Another factor influencing the uncertainty in model parameters is the accuracy of pilot log records. These records can be of great help in correlating loads with changes in weight, weight distribution and configuration.

The statistical moments of the measurable random variables required for the error analysis of Eq. (74) can be estimated from data acquired in previous studies of loads on similar aircraft. Sample probability density functions would provide the approximate information for computation of the moments by Eqs. (4), (6) and (7).

4.2.2 Computing Errors

Errors are both introduced and transformed while operating on measured quantities to compute flight loads. The errors which originate in computing are generally functions of the procedures and are under the control of the program management. That is, procedure related errors can usually be reduced by making various compromises. Some aspects of the tradeoffs are discussed here. Also of interest are the errors in the computed loads which can be traced back to errors in the original measurements. These errors are transformed along with transformations of the measured variables during computations.

Procedure Related Errors. A certain amount of error is generated during flight loads computation which is strictly a function of the computing procedures employed. It is usually possible to minimize such errors through some form of procedural optimization. The penalty for increased accuracy ordinarily involves an increase in the time required to process a given amount of data.

A detailed discussion of all the errors which can be traced to the data processing procedures would be as extensive as the total number of procedures. Therefore, only a few major points which are particularly relevant to flight loads data processing are covered here. The objective is to point out the compromises which can be made in the procedures to improve the accuracy of statistical flight loads estimates.

As pointed out in Section 2. 1. 2. 2, practical procedures for computing sample probability density and spectral density functions lead to biased estimates. Bias errors result from the fact that the functions are estimated in finite intervals instead of at every point. This type of bias is a smoothing error produced by the joint effects of interval width and the shape of the function being estimated. Figure 15 illustrates

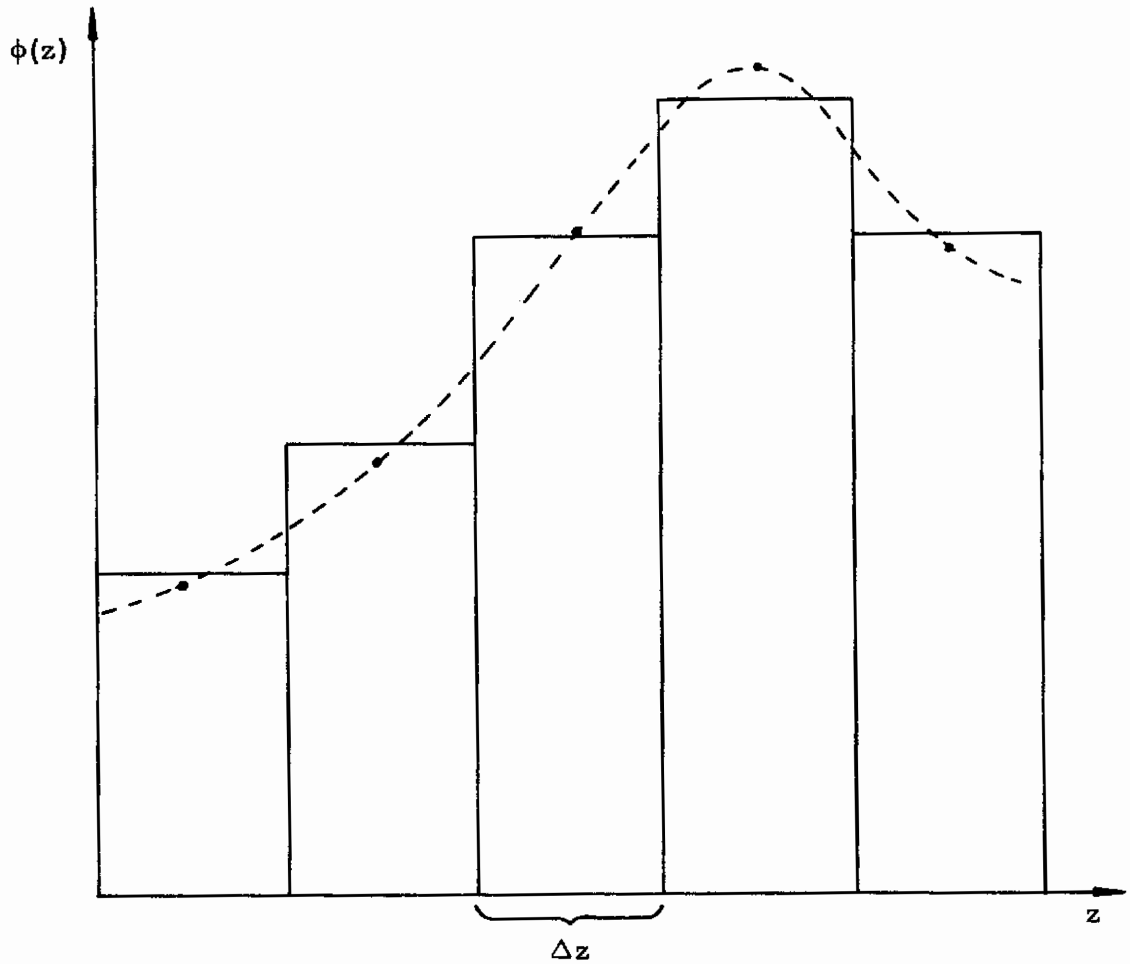


Figure 15. Finite Resolution Bias Error

the nature of the problem. The dashed curve represents a probability density or spectral density function which is being estimated on the basis of finite interval approximations. That is, the value of the function at an interval midpoint is estimated on the basis of the expected value for that interval. Ignoring variability errors for the moment, the bias error is defined as the difference between the expected value of the function over the interval and the value at the interval midpoint. The following is a general approach which can be used in evaluating these bias errors.

The expected value of a density function estimate, $\hat{\phi}(z)$, associated with a finite interval Δz is equal to the average value of the ordinate over that interval. In functional form, this is given by

$$E\left[\hat{\phi}(z)\right] = \frac{1}{\Delta z}\int_{z-\frac{\Delta z}{2}}^{z+\frac{\Delta z}{2}} \phi(\xi)\, d\xi \qquad (75)$$

The function $\phi(\xi)$ can be expanded in a Taylor series about the point $\xi = z$ as follows.

$$\phi(\xi) = \phi(z) + (\xi - z)\,\phi'(z) + \frac{(\xi - z)^2}{2}\,\phi''(z) + \ldots \qquad (76)$$

In a term-by-term integration of Eq. (76), the even terms all equal zero.

Neglecting fourth-power and higher order terms of the series, the expected value of the estimate is approximated by

$$E\left[\hat{\phi}(z)\right] \approx \phi(z) + \frac{\Delta z^2}{24}\phi''(z) \tag{77}$$

where the second term represents the bias error in the estimate. For probability density and spectral density functions respectively, this can be expressed in the relative error forms

$$\frac{b[\hat{p}(L)]}{p(L)} = 100\,\frac{\Delta L^2\, p''(L)}{24\, p(L)}\ \% \tag{78a}$$

$$\frac{b[\hat{G}_L(f)]}{G_L(f)} = 100\,\frac{B_f^2\, G_L''(f)}{24\, G_L(f)}\ \% \tag{78b}$$

Equation (78) implies that the amplitude and frequency intervals used for computing estimates of these functions should be made as narrow as possible. However, another factor must be considered here. This is the effect of interval width on variability errors in the estimates. From the summary of normalized standard errors given in Section 2.2.2, it can be shown that for a given sample size, the variability errors in probability density and power spectral density estimates are approximately proportional to the inverse root of their respective analysis intervals. That is,

$$e[\hat{p}(L)] \sim \frac{1}{\sqrt{\Delta L}} \qquad (79a)$$

$$e[\hat{G}_L(f)] \sim \frac{1}{\sqrt{B_f}} \qquad (79b)$$

Clearly, from a comparison of Eqs. (78) and (79), it can be seen that a reduction in bias error tends to produce an increase in variability error. Thus there is an error tradeoff which, within limits, is strictly a function of the computational procedures.

There is one procedural technique which has been specifically designed to obviate this conflict in the case of spectral density estimation. This is called "pre-whitening," and consists of operating on the time history of a random variable in such a way that the spectral content is almost evenly distributed, or white. Then, the analysis frequency interval B_f can be greatly increased without significantly biasing the estimate. Following analysis, the results are "post-darkened" in order to restore the original spectral shape.

There is a definite trend in flight loads data processing toward complete automation. Therefore, several procedure related errors which are peculiar to digitized data should be discussed here. These are concerned principally with the resolution and rate at which analog data signals are quantized.

At the present time, nearly all of the transducers used for measuring in-flight variables produce continuous analog data. If these data are to be processed on a digital computer, they must first be converted into discrete numbers. This process is called quantization, and the numbers which result are composed of binary digits (bits). Quantization accuracy is then a function of the number of bits which can be used to define a data value. The conversion error is determined as follows.

Consider a continuous in-flight measurement $x(t)$ which is to be converted into binary words of m bits each. This means that there are 2^m possible numerical values for $x(t)$. Then, if the full range of the variable is X, the quantization levels will be separated by equal intervals

$$\Delta x = \frac{X}{2^m} \tag{80}$$

Since quantization is ordinarily performed at a sequence of equally spaced times, the value of $x(t)$ which must be converted can fall anywhere within one of the Δx intervals. However, since it is necessary to give all such values in each interval the same numerical value, quantization produces a variability error in the data. If the difference between the continuous and the numerical values is denoted by δ, then the variance of the error can be shown to be

$$\sigma_\delta^2 = \frac{(\Delta x)^2}{12} \tag{81}$$

from which the standard deviation, or RMS quantizing noise is

$$\sigma_\delta = \frac{\Delta x}{\sqrt{12}}$$

$$= \frac{X}{2^m \sqrt{12}} \approx 0.29 \frac{X}{2^m} \qquad (82)$$

From the above, it is evident that σ_δ can be reduced very quickly by increasing the number of bits in the binary words used to describe the data values. For example, the RMS error associated with 8-bit words would be one-fourth that for 6-bit words. In effect, the signal-to-noise ratio is increased by 6 dB for every additional bit.

The time interval between quantizations is a parameter of the data processing which influences error magnitudes in at least two cases. First, there is the problem of aliasing. Aliasing errors are confined to the area of spectral density estimation. When a continuous time dependent random variable is quantized at a sequence of equally spaced times Δt apart, it has been shown that a special frequency defined by $f_c = 1/2\Delta t$ has the following significance. For any frequency f in the range 0 to f_c, a power spectral density estimate will consist of the power originally at that frequency plus the power at a series of frequencies defined by $2nf_c \pm f$ $(n = 1, 2, 3, \ldots)$. This can be thought of as a folding of the power at higher frequencies back into the range 0 to f_c. Therefore, f_c has been called a folding frequency or Nyquist frequency. Addition of the folded power to the existing power in the range of interest produces an aliased estimate of the power spectrum. This is illustrated by the sketch of Figure 16. The actual magnitude of the error at any frequency depends entirely on the ratio of the aliased power to the power which actually exists at that frequency.

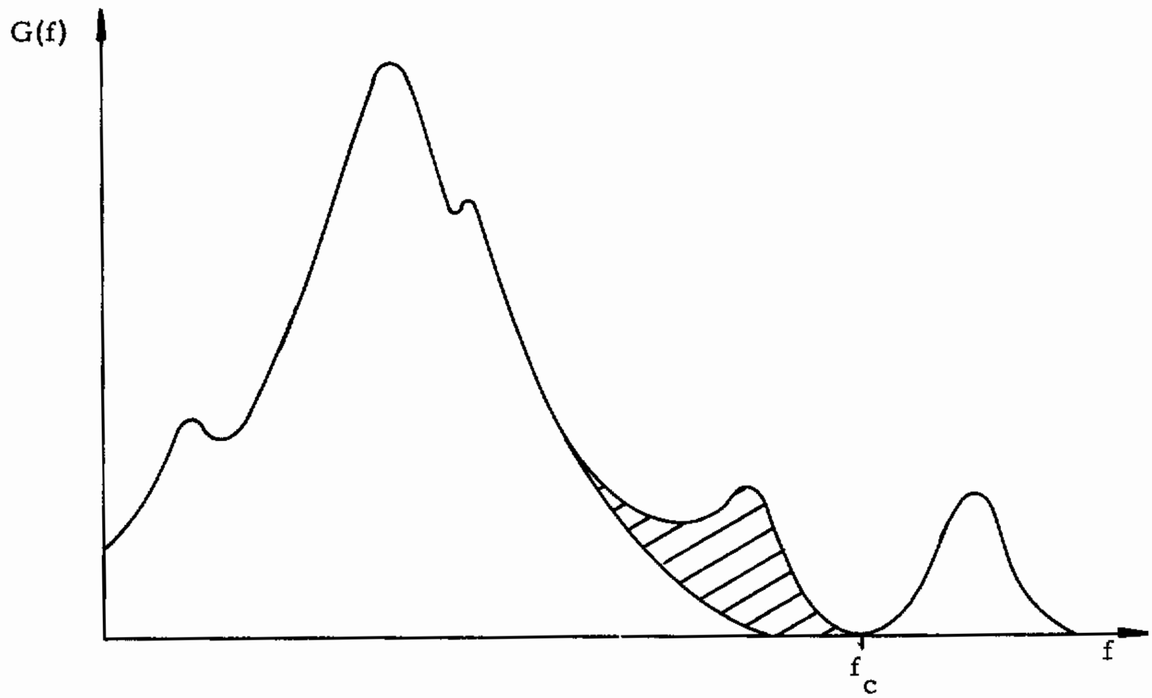


Figure 16. Aliased Power Spectrum with High Frequency Power Folded into the Power Below f_c

There are two things which can be done procedurally to reduce aliasing error. First, since the folding frequency is a function of the quantizing rate, this rate can be set high enough so that all of the power in x(t) will be included in the range 0 to f_c. If this approach is not desirable, the analog data can be low-pass filtered prior to quantization in order to attenuate the power at frequencies higher than f_c. Either method is effective, and both are usually under the control of the program management.

Another effect of the quantizing rate concerns the related error in estimating peak values of a continuous random variable. An exact analysis of the effect is not available. However, it is possible to make the following generalization.

Whenever a peak value of the random variable $x(t)$ is estimated by the maximum of a set of periodically quantized values x_i, the expected value of the estimate will always be less than the expected value of the peak. The difference constitutes a bias error. A relationship between this error and the quantizing rate can be formulated for the special case of a sinusoidally varying $x(t)$. This result can then be extrapolated qualitatively for application to random data.

Assume that the function $x(t)$ defined by

$$x(t) = X \sin 2\pi f_0 t \tag{83}$$

is quantized at equal time intervals Δt. It is desired to estimate the peak amplitude X on the basis of a maximum in the observed sequence x_i. Since the peak must occur during a time interval Δt, the expected value of the estimated peak is given by

$$E[\hat{X}] = \frac{X}{\Delta t} \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \cos (2\pi f_0 \tau)\, d\tau \tag{84}$$

$$= X \frac{\sin \pi f_0 \Delta t}{\pi f_0 \Delta t}$$

It follows that the bias error in the estimate of the peak value of a sine wave over the frequency range $0 \leq f_0 \leq f_c = \frac{1}{2\Delta t}$ can be expressed in the following absolute and relative forms:

$$b[\hat{X}] = X \left(\frac{\sin \frac{\pi}{2} \frac{f_0}{f_c}}{\frac{\pi}{2} \frac{f_0}{f_c}} - 1 \right) \tag{85a}$$

$$\frac{b[\hat{X}]}{X} = 100 \left(\frac{\sin \frac{\pi}{2} \frac{f_0}{f_c}}{\frac{\pi}{2} \frac{f_0}{f_c}} - 1 \right) \% \tag{85b}$$

This result cannot be used directly with broadband random data for obvious reasons. However, the qualitative aspects are noteworthy. For example, Eq. (85) indicates that the bias error approaches zero as the frequency ratio f_0/f_c becomes small. Even for frequency ratios as large as 0.5, the indicated bias error is only about 10%. The worst case error exists for a sine wave frequency equal to the folding frequency, and is approximately 36%. Extrapolation of these results for use with peak estimates of random variables involves a certain amount of interpretation. For instance, the narrowband random response of a lightly damped structure to turbulence excitation may look quite a bit like a sine wave, and it will have an error relationship similar to that given by Eq. (85). For random data of much increased bandwidth, the peak estimation error for any f/f_c will tend to become much smaller than this result.

Errors in Transformed Variables. When computing the values of flight loads variables by any of the models discussed in Section 4.1, it is necessary to perform various transformations on the data. If the in-flight variables have been measured accurately, errors in the results depend primarily on the sampling methods, model accuracy, and computing procedures. However, if the individual measurements are in error, data transformations will also transform these errors. This can be illustrated by considering some specific cases.

Let x represent any continuous or discrete random variable which has been sampled in flight, and assume that the sample is in error by the amount δ. That is, the observed value $\hat{x}$ is given by

$$\hat{x} = x + \delta \tag{86}$$

where δ is also a random variable defined here to be independent of x. The expected value and variance of $\hat{x}$ are then, respectively,

$$\begin{aligned} E[\hat{x}] &= E[x] + E[\delta] \\ &= \mu_x + \mu_\delta \end{aligned} \tag{87a}$$

$$\text{Var}[\hat{x}] = E\left[(x+\delta)^2\right] - E^2[x+\delta] = \sigma_x^2 + \sigma_\delta^2 \quad (87b)$$

Note that the mean value of the error term δ is added directly to the true mean value of the variable x. The mean value of the error may be either positive or negative. Similarly, the variance of the error term adds directly to the true variance. However, since variance is a squared quantity, σ_δ^2 is always positive. Therefore, the variance of $\hat{x}$ is always greater than the true variance. This can be thought of as a bias error in estimating σ_x^2.

Now it is of interest to study the effects of these errors on the computed loads which are obtained by transforming the erroneous in-flight variables. The true and error portions of the results are separated as in Eq. (87).

Consider first the linear transformation $L = Ax$, where A is a constant.

$$\hat{L} = A\hat{x} \quad (88)$$

The expected value of $\hat{L}$ is determined as follows.

$$E[\hat{L}] = E[A\hat{x}] = A\,E[x] + A\,E[\delta] = A\mu_x + A\mu_\delta \quad (89a)$$

As might be expected, the mean value of the error is subjected to the same linear transformation as the true mean. Again, since μ_δ can be either positive or negative, the expected value of $\hat{L}$ may be either greater than or less than the true mean of L given by $A\mu_x$.

Assuming that x and δ are independent random variables, the variance of the computed load is obtained according to the standard variance expression given in Eq. (27) as follows.

$$\begin{aligned} \text{Var}\,[\hat{L}] &= E\left[(A\hat{x})^2\right] - E^2[A\hat{x}] \\ &= A^2\left(E\left[(x+\delta)^2\right] - E^2[x+\delta]\right) \\ &= A^2\sigma_x^2 + A^2\sigma_\delta^2 \end{aligned} \tag{89b}$$

The variance of the error transforms in the same way as the true variance. The two are added in the computations, and thus, the variance of $\hat{L}$ is always greater than that of L if an error exists in the original measurement.

A logical extension of the previous example is given by the following linear combination of transformed flight measurements.

$$\hat{L} = \sum_j A_j \hat{x}_j \tag{90}$$

By similarity with Eqs. (89a) and (89b), it follows that the expected value and variance of the combination are given by

$$E\left[\hat{L}\right] = \sum_j A_j \mu_{x_j} + \sum_j A_j \mu_{\delta_j} \tag{91a}$$

$$\mathrm{Var}\left[\hat{L}\right] = \sum_j A_j^2 \sigma_{x_j}^2 + \sum_j A_j \sigma_{\delta_j}^2 \tag{91b}$$

Next, consider the quadratic transformation $L = Ax^2$. This operation must be performed when using models such as those given in Eq. (58). A load estimate computed on the basis of an erroneous measurement would be

$$\hat{L} = A\hat{x}^2 \tag{92}$$

The expected value of $\hat{L}$ is obtained as follows.

$$\begin{aligned} E[\hat{L}] &= E\left[A\hat{x}^2\right] \\ &= A\,E\left[(x+\delta)^2\right] \\ &= A\left(\sigma_x^2 + \mu_x^2\right) + A\left(\sigma_\delta^2 + \mu_\delta^2 + 2\mu_x\mu_\delta\right) \end{aligned} \tag{93a}$$

Notice that variance terms appear in the expected value expression. This results from the fact that the squaring operation makes all positive and negative fluctuations of both the true and error portions of x positive. In fact, the last error term in Eq. (93a) is the only one which can ever assume negative values. This tends to bias the expected value of $\hat{L}$ somewhat toward the positive side.

The variance of the computed load is given by

$$\begin{aligned}
\mathrm{Var}\,[\hat{L}] &= E\left[(A\hat{x}^2)^2\right] - E^2\left[A\hat{x}^2\right] \qquad (93b)\\
&= A^2\left(E\left[(x+\delta)^4\right] - E^2\left[(x+\delta)^2\right]\right)\\
&= A^2\left(\alpha_{4_x}\,\sigma_x^4 + 4\alpha_{3_x}\,\sigma_x^3\mu_x + 4\sigma_x^2\mu_x^2 - \sigma_x^4\right)\\
&\quad + A^2\Big(4\alpha_{3_x}\,\sigma_x^3\mu_\delta + 8\sigma_x^2\mu_x\mu_\delta + 4\sigma_x^2\sigma_\delta^2 + 4\sigma_x^2\mu_\delta^2 + 4\sigma_\delta^2\mu_x^2 + 4\mu_x^2\mu_\delta^2\\
&\quad + 4\alpha_{3_\delta}\,\sigma_\delta^3\mu_x + 8\sigma_\delta^2\mu_\delta\mu_x + \alpha_{4_\delta}\,\sigma_\delta^4 + 4\alpha_{3_\delta}\,\sigma_\delta^3\mu_\delta + 4\sigma_\delta^2\mu_\delta^2 - \sigma_\delta^4\Big)
\end{aligned}$$

This is a very complicated expression which includes the third and fourth moment descriptors α_3 and α_4 defined as follows.

$$\alpha_{3_x} = \frac{E\left[(x - \mu_x)^3\right]}{\sigma_x^3} \qquad \text{(skewness coefficient)} \tag{94a}$$

$$\alpha_{4_x} = \frac{E\left[(x - \mu_x)^4\right]}{\sigma_x^4} \qquad \text{(kurtosis coefficient)} \tag{94b}$$

Some of the terms in Eq. (93b) can be simplified if δ is assumed to be normally distributed about μ_δ. Under such an assumption, $\alpha_{3_\delta} = 0$ and $\alpha_{4_\delta} = 3$. In general, x cannot be assumed to have the normal distribution, and these simplifications would not apply to the measured variable.

A useful extension of this second order transformation relates to the product of two in-flight variables, each of which contains independent error terms. That is,

$$\begin{aligned} \hat{x}_1 &= x_1 + \delta_1 \\ \hat{x}_2 &= x_2 + \delta_2 \end{aligned} \tag{95}$$

and the transformation of the product can be written as

$$\hat{L} = A\hat{x}_1\,\hat{x}_2 \tag{96}$$

The expected value of the load, computed on the premise that x_1 and x_2 are independent, is given by

$$E[\hat{L}] = E[A\hat{x}_1 \hat{x}_2] \tag{97}$$

$$= A E[(x_1 + \delta_1)(x_2 + \delta_2)]$$

$$= A\mu_{x_1} \mu_{x_2} + A\left(\mu_{x_1} \mu_{\delta_2} + \mu_{x_2} \mu_{\delta_1} + \mu_{\delta_1} \mu_{\delta_2}\right)$$

The effect of independence between x_1 and x_2 is evident in Eq. (97) where no variance terms are to be seen. That is, on average, the fluctuations in both the true and error portions of the measurements cancel out, leaving only cross products of the various mean values. Therefore, as a generality, the error in the expected value of $\hat{L}$ is just as likely to be positive as negative.

The variance which corresponds to the transformation of Eq. (96) may be developed in two stages. First, the basic relationship for the variance of a product can be written as

$$\mathrm{Var}\left[\hat{x}_1 \hat{x}_2\right] = \hat{\sigma}_1^2 \hat{\sigma}_2^2 + \hat{\sigma}_1^2 \hat{\mu}_2^2 + \hat{\sigma}_2^2 \hat{\mu}_1^2 \tag{98}$$

Then, by using the definitions of Eq. (95), the following substitutions can be made in Eq. (98).

$$\hat{\mu}_1 = \mu_{x_1} + \mu_{\delta_1}$$
$$\hat{\mu}_2 = \mu_{x_2} + \mu_{\delta_2}$$
$$\hat{\sigma}_1^2 = \sigma_{x_1}^2 + \sigma_{\delta_1}^2 \qquad (99)$$
$$\hat{\sigma}_2^2 = \sigma_{x_2}^2 + \sigma_{\delta_2}^2$$

This results in the following variance expression for $\hat{L}$.

$$\mathrm{Var}\left[\hat{L}\right] = A^2 \, \mathrm{Var}\left[\hat{x}_1 \, \hat{x}_2\right]$$
$$= A^2 \sigma_{x_1}^2 \sigma_{x_2}^2 + A^2 \sigma_{x_1}^2 \left(\sigma_{\delta_2}^2 + \mu_{\delta_2}^2 + 2\mu_{\delta_2} \mu_{x_2} + \mu_{x_2}^2\right)$$
$$+ A^2 \sigma_{x_2}^2 \left(\sigma_{\delta_1}^2 + \mu_{\delta_1}^2 + 2\mu_{\delta_1} \mu_{x_1} + \mu_{x_1}^2\right) \qquad (100)$$
$$+ A^2 \sigma_{\delta_1}^2 \left(\mu_{\delta_2}^2 + 2\mu_{\delta_2} \mu_{x_2} + \mu_{x_2}^2\right)$$
$$+ A^2 \sigma_{\delta_2}^2 \left(\mu_{\delta_1}^2 + 2\mu_{\delta_1} \mu_{x_1} + \mu_{x_1}^2\right)$$

Since x_1 and x_2 were assumed independent, the above variance is given entirely in terms of the first and second moments of the true and error portions of the two measurements.

It is sometimes convenient to substitute the computed time derivative of a variable measured in flight for actual measurement of the derivative. As an example, angular velocity measurements are often differentiated with respect to time when angular acceleration is required for a loads computation. If an error exists in the original measurement, the estimated load computed as a function of time is given by the following transformation.

$$\hat{L}(t) = A \frac{d\,\hat{x}(t)}{dt} = A\dot{\hat{x}} \qquad (101)$$

For any stationary random variable, that is, one with a time invariant mean value, the expected value of the derivative is zero. However, for this analysis it is assumed that x(t) may have nonstationary characteristics, but that δ(t) is stationary. Then the expected value of $\hat{L}(t)$ is given by

$$\begin{aligned} E\left[\hat{L}\right] &= E\left[A\dot{\hat{x}}\right] \\ &= A\,E\left[\dot{x} + \dot{\delta}\right] \qquad (102a) \\ &= A\,\mu_{\dot{x}} \end{aligned}$$

The term $\mu_{\dot{x}}$ is the mean value of the time derivative of x(t), and as mentioned above, will always equal zero if x(t) has a stationary mean value. Under the assumption of a stationary random error component in the original measurement, there is no error in the expected value of $\hat{L}$.

The variance of $\hat{L}(t)$ is derived according to the following operations.

$$\begin{aligned} \mathrm{Var}\left[\hat{L}\right] &= E\left[(A\hat{\dot{x}})^2\right] - E^2\left[A\hat{\dot{x}}\right] \\ &= A^2\left(E\left[(\dot{x} + \dot{\delta})^2\right] - E^2\left[\dot{x} + \dot{\delta}\right]\right) \\ &= A^2\,\sigma_{\dot{x}}^2 + A^2\,\sigma_{\dot{\delta}}^2 \end{aligned} \tag{102b}$$

The above variance expression can be rearranged into the following interesting form

$$\mathrm{Var}\left[\hat{L}\right] = A^2\,\sigma_{\dot{x}}^2\left(1 + \frac{\sigma_{\dot{\delta}}^2}{\sigma_{\dot{x}}^2}\right) \tag{103}$$

where the quotient of the variances inside the parentheses can be thought of as the square of the RMS noise/RMS signal ratio (inverse of the S/N ratio discussed in Section 3.1.2). It is often assumed that differentiation of a time dependent variable increases the magnitude of this ratio, and thus the variance of $\hat{L}$, but this is not always true. Since the variance equals the integral of the power spectrum as defined by

Eq. (12b), the factor which determines whether differentiation increases or decreases the noise/signal ratio is the relationship between the power spectra of these two components. This factor can be demonstrated as follows.

Given that the power spectra for the true and error components of the original measured variable are defined over the finite frequency range 0 to f_c by $G_x(f)$ and $G_\delta(f)$ respectively, the variance ratio for the undifferentiated measurement is

$$\frac{\sigma_\delta^2}{\sigma_x^2} = \frac{\int_{0^+}^{f_c} G_\delta(f)\, df}{\int_{0^+}^{f_c} G_x(f)\, df} \tag{104}$$

Differentiation of x then leads to the following variance ratio computed from the power spectra of the component derivatives.

$$\frac{\sigma_{\dot{\delta}}^2}{\sigma_{\dot{x}}^2} = \frac{4\pi^2 \int_{0^+}^{f_c} f^2\, G_\delta(f)\, df}{4\pi^2 \int_{0^+}^{f_c} f^2\, G_x(f)\, df} \tag{105}$$

A cursory examination of Eqs. (104) and (105) will quickly show that the noise/signal ratio does not change with differentiation for the special case of uniform power spectra for both the true and error components.

In the typical situation, however, the power spectrum of $x(t)$ decreases with frequency while that of $\delta(t)$ is either uniform or an increasing function of frequency. Thus, differentiation does usually tend to aggravate the noise problem. This effect is especially pronounced when the spectrum of the noise has not been cut off sharply at the upper limit f_c.

The reverse of the previous example, that is, integration of a measured time dependent variable, is employed when it is easier to do this than to measure the integrated variable directly. Transformation through integration of a measured variable containing an error component is described by the following expression:

$$\hat{L}(t) = A \int_0^t \hat{x}(\tau)\, d\tau \tag{106}$$

The expected value of $\hat{L}(t)$ is then given by

$$\begin{aligned} E\left[\hat{L}\right] &= E\left[A \int_0^t \hat{x}(\tau)\, d\tau\right] \\ &= A\,E\left[\int_0^t \left[x(\tau) + \delta(\tau)\right] d\tau\right] \\ &= A \int_0^t \left(E\left[x\right] + E\left[\delta\right]\right) d\tau \\ &= A \int_0^t (\mu_x + \mu_\delta)\, d\tau \end{aligned} \tag{107}$$

The result of Eq. (107) shows that integration of a nonzero mean variable with respect to time produces a nonstationary process for which the expected value is time dependent. This is described in terms of a nonstationary mean value by the following

$$E\left[\hat{\mu}_L(t)\right] = A\mu_x t + A\mu_\delta t \tag{108}$$

Note that the nonstationarity would exist even if the error were zero. One method of avoiding this difficulty is indicated by Eq. (67). If the mean value of the observed variable is subtracted from all instantaneous values, $\hat{x}(t)$ is transformed into a zero mean variable. Thus, problems of nonstationarity are eliminated, and there is no computational error in the expected value of the estimate $\hat{L}(t)$.

The variance of the load estimate computed according to Eq. (106) is defined in the conventional way. However, it is convenient to develop the variance expression in terms of the frequency domain transforms. Then,

$$\begin{aligned}
\mathrm{Var}\left[\hat{L}(t)\right] &= E\left[\left(A\int_0^t \hat{x}(\tau)\,d\tau\right)^2\right] - E^2\left[A\int_0^t \hat{x}(\tau)\,d\tau\right] \\
&= E\left[\iint_{0^+}^{\infty} \mathcal{F}\left\{A\int_0^{t_1} \hat{x}(\tau_1)\,d\tau_1\right\}\mathcal{F}^*\left\{A\int_0^{t_2} \hat{x}(\tau_2)\,d\tau_2\right\} df_1\,df_2\right] \\
&= E\left[\iint_{0^+}^{\infty} A^2 \frac{\hat{X}(f_1)}{j2\pi f_1}\,\frac{\hat{X}(f_2)}{(-j2\pi f_2)}\,e^{j2\pi(f_1 t_1 - f_2 t_2)}\,df_1\,df_2\right]
\end{aligned} \tag{109}$$

which, if $\hat{x}(t)$ is stationary, reduces to

$$= \frac{A^2}{4\pi^2} \int_{0^+}^{\infty} E\left[\frac{\hat{X}(f)\ \hat{X}^*(f)}{f^2}\right] df$$

$$= \frac{A^2}{4\pi^2} \int_{0^+}^{\infty} \frac{G_x(f)}{f^2}\, df + \frac{A^2}{4\pi^2} \int_{0^+}^{\infty} \frac{G_\delta(f)}{f^2}\, df$$

Since the power spectra in the above integrals are divided by the frequency squared, high frequency noise is generally of less concern in the integrated load than in the original measurement. A new problem is introduced however, by low frequency error components. This is related to the fact that division by the frequency amounts to amplification of all spectral components below one frequency unit (Hz, radians/sec, radians/ft, etc.). This presents no problem with regard to the first term in the result of Eq. (109) since it is assumed that L(t) is a differentiable variable. Thus, any low frequency amplification of x(t) produced by integration exactly cancels the low frequency attenuation associated with differentiation of L(t). However, if δ(t) contains any spectral components near zero frequency (caused, e.g., by data system drift), the second integral in the equation tends to blow up. This may result in very large values for the variance and comparably large low frequency excursions in $\hat{L}(t)$. The problem is one of great concern in the measurement of gust velocity inputs using Eq. (67). Reference 14 reports and analyzes an incident of this wherein the aircraft pitch attitude correction was obtained by integrating the measured pitch rate. Low frequency gyro drift, with a period of approximately 240 seconds, produced errors in the computed gust velocities which were nearly four times as great as the true values.

An effective method of preventing the second integral of Eq. (109) from blowing up is to high-pass filter $\hat{x}(t)$ prior to integration. The ideal filter would have the characteristics of a differentiator in the frequency range 0 to 1 and would be flat at frequencies above one unit. This operation, of course, merely stabilizes the integration process and does nothing to improve the evaluation of L(t) in the low frequency region. To do this, it is necessary to measure the integrated variables directly rather than computing them.

5. UTILIZATION OF FLIGHT LOADS DATA

The data acquired in structural flight loads measurement programs are utilized in two main applicational areas: generation of design criteria and substantiation of new designs. In the first application, the objective is to characterize the loads environment to which current and future aircraft will be subjected during their operational lifetimes. This is done by acquiring large quantities of flight loads data using existing aircraft operating under differing flight conditions. Various processing operations are then performed on the data in order to make them generally useful for aircraft other than just those upon which the measurements were made. Data acquired for the second application are utilized primarily with regard to the particular design being tested. This testing spans the development and operational phases for a new vehicle and includes initial engineering tests, demonstration tests for the customer, long term fatigue certification after delivery, operational damage monitoring, and diagnosis of special problems that arise under operational conditions. Test results are recirculated into revision and updating of the design criteria and often are used to generate the criteria for future designs.

In the following discussion, the utilization of loads data is considered under these two main classifications with the emphasis on the data requirements for each. Understanding these requirements permits test planning with a much better overall perspective.

5.1 STRUCTURAL DESIGN CRITERIA

Before a new aircraft structure can be designed, there must exist a definition of the flight loads to which it will be subjected. Ordinarily, this definition is put forth in the form of loads requirements in a design criteria specification. The specified loads are typically derived from a combination of the new aircraft performance requirements, past experience with similar aircraft, and reliability factors.

The most significant past experience which can be utilized in estimating the loads requirements for a new design comes from the results of flight loads surveys on aircraft which have been in operation for some time. Since no single set of operating conditions can describe the intended usage for a new aircraft, the predicted environment which forms the basis for the new requirements must be compiled from past data which have been gathered under many different conditions. Ideally, all of the relevant conditions will have been investigated, and a breakdown of the expected usage for the new design will have been established. This breakdown should be such that each of a finite number of activities is characterized by a single stationary random loads process. Then this collection of stationary processes can be combined in the appropriate way to predict the overall loads environment for the entire service life.

The main statistical descriptions for this application include variations of the amplitude probability density and distribution functions previously discussed. For the present, let $\hat{P}(L)$ stand for an arbitrary estimated loads probability distribution function. Then, if a particular aircraft activity or usage is identified by the index u, the distribution function estimated for each activity can be denoted by $\hat{P}_u(L)$.

The function $\hat{P}_\ell(L)$ which describes the predicted overall loads probability for the service life is then derived as a weighted sum of the components $\hat{P}_u(L)$ as follows.

$$\hat{P}_\ell(L) = \sum_{u=1}^{U} \frac{\hat{T}_u}{T_\ell} \hat{P}_u(L) \qquad u = 1, 2, \ldots, U \tag{110}$$

where $\hat{T}_u/T_\ell$ is the fraction of the total service life T_ℓ predicted for each of the U different usage categories. Note that the sum of these fractions for all u is unity, and that T_ℓ is assumed to be a constant.

The function $\hat{P}_u(L)$ in Eq. (110) is predicted by semi-empirical methods on the basis of previously acquired data which have been separated appropriately into groups corresponding to each aircraft usage. Two basically different approaches are in current use for applying these data. The first method is based on the assumption that the major aerodynamic loads which will be experienced by the new design are about the same as those determined previously for another aircraft engaged in the same activity. Then the probability function for a particular air load, which was computed from the previous in-flight measurements is substituted directly for $\hat{P}_u(L)$ in Eq. (110).

The second method assumes that the in-flight measured variables themselves have characteristics which will be about the same on the new aircraft as observed on the older design for the same type of usage. This approach is the one most often used, but it involves a more complex prediction procedure which is performed along the following lines.

Assume that the major aerodynamic loads which are to be predicted for a given activity can be modeled after Eq. (53). In this case, the loads computation will be based on an estimated set of design constants $\hat{A}$ for the new aircraft and previously acquired in-flight measurements $\hat{x}$. Typically, the only environmental information available for this purpose consists of first order sample probability estimates for the x's. In order to proceed, it is necessary to know also the joint probability relationships between the x's, or else to make the assumption that these variables are independent. The independence assumption is usually acceptable when there is no evidence to the contrary.

The probability density and distribution functions predicted for a particular air load by this method are, respectively,

$$\hat{p}_u(L_i) = \hat{p}_u\left(\sum_j L_{ij}\right) = \hat{p}_u\left(\sum_j A_{ij}\, x_j\right) \qquad j = 1, 2, \ldots, J \tag{111a}$$

$$\hat{P}_u(L_i) = \int_{-\infty}^{L_i} \hat{p}_u(\mathscr{L}_i)\, d\mathscr{L}_i \tag{111b}$$

The problem in Eq. (111a) is to obtain the probability density function for the sum of a finite series of random loads variables. The predicted distribution function follows directly from this in accordance with Eq. (111b). The probability density function for a sum of random terms is obtained by multiple convolution of the probability density functions for all the terms. In this case the density function for each term is given by the transform

$$\hat{p}_u(L_{ij}) = \frac{\hat{p}_{x_j}\left(\dfrac{L_{ij}}{A_{ij}}\right)}{\left|\hat{A}_{ij}\right|} \tag{112}$$

where the numerator on the right represents previously measured probability density functions for the in-flight variables. Then $\hat{p}_u(L_i)$ is computed by multiple convolution as noted in the following.

$$\hat{p}_u(L_i) = \hat{p}_u(L_{i1}) * \hat{p}_u(L_{i2}) * \cdots * \hat{p}_u(L_{iJ}) \qquad (113)$$

The method of multiplying characteristic functions can be used for solving Eq. (113) in either the continuous or discrete forms (see, e.g., Reference 15). Then, the distribution function is computed as in Eq. (111b) for substitution in Eq. (110).

The operations described functionally in Eq. (110) are illustrated by the sketch of Figure 17 for the case of a service life prediction based on three usage categories. Note that the overall loads probability distribution function $P_\ell(L)$ spans the range 0 to 1 only if $\Sigma \hat{T}_u / T_\ell = 1$. That is, the entire service experience must be accounted for.

5.1.1 Specific Loads Requirements

Structural loads requirements are specified in terms of design limit and repeated loads. Design limit loads reflect the worst conditions predicted to occur during the aircraft service life. Requirements for the minimum static strength of a structure are established directly from these by applying an ultimate factor of safety. Repeated loads requirements predict the less severe, but more numerous, cyclic loading conditions which produce fatigue damage. These are given as loads spectra with amplitudes specified as percentages of design limit load. The particular loads probability function which is appropriate for the prediction of each of these requirements using Eq. (110) will now be discussed.

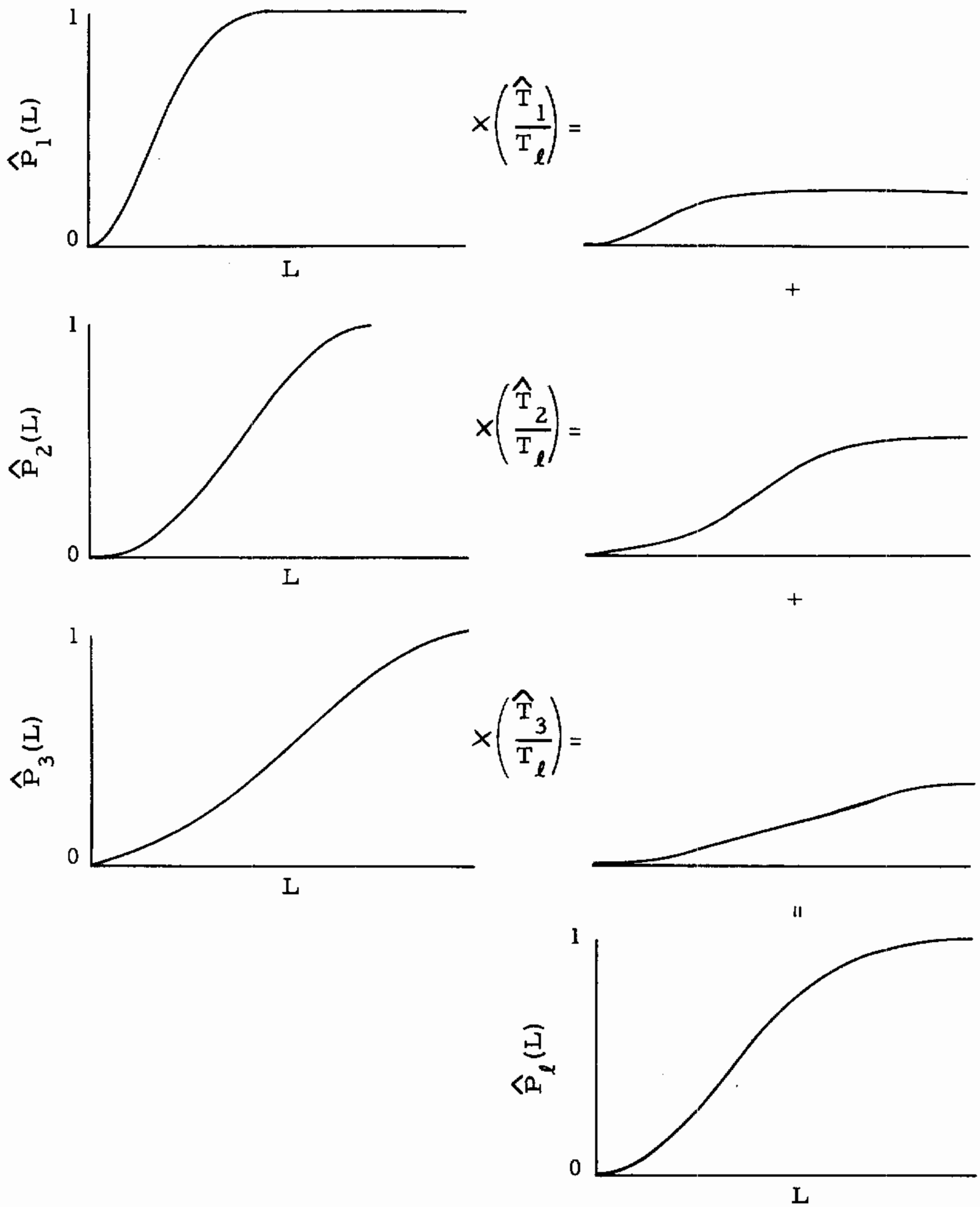


Figure 17. Derivation of a Predicted Overall Loads Probability Distribution Function from Three Component Environments

Limit Loads. Limit loads are based primarily on estimates of the most severe loads to be expected during the projected service life for a new aircraft. Since each particular load can be considered to occur a finite number of times (finite sample size) during the service life, it is appropriate to establish limit loads on the basis of extreme amplitude concepts. An extreme amplitude is defined as the single point in a finite sample of a random process which has an amplitude greater than that of all other points in the sample. That is, for N independent values of a loads variable L the one extreme amplitude is defined by

$$L_{max}(N) = \max \left[L_1, \; L_2, \; \ldots, \; L_N \right] \tag{114}$$

Extreme amplitudes taken from an infinite collection of size N samples are distributed according to the extreme amplitude probability distribution function $P\left[L_{max}(N)\right]$. This function defines the probability that a given load amplitude will not be exceeded in a sample of N flight load occurrences. By relating sample size to service life, limit loads are established which will be exceeded by only a small percentage of all aircraft of a given design during the service life period.

For any given flight loads variable, an extreme amplitude distribution is estimated for each type of usage, which is a function of the particular sample size associated with that usage. This is denoted by $\hat{P}_u\left[L_{max}(\hat{N}_u)\right]$. In turn, the estimated sample size $\hat{N}_u$ is dependent upon the predicted time allocated for each usage and the estimated frequency with which loads occur under those conditions. Thus, the sample size can be estimated by

$$\hat{N}_u = \hat{f}_u \hat{T}_u \tag{115}$$

where $\hat{f}_u$ is the estimated rate at which the particular load of interest will occur in the uth usage condition. Then, the component extreme amplitude distributions are combined using the basic form of Eq. (110) in order to derive the predicted overall extreme amplitude distribution upon which to base the limit requirements for each load. This can be expressed as

$$\hat{P}_\ell \left[L_{max} (\hat{N}_\ell) \right] = \sum_{u=1}^{U} \frac{\hat{T}_u}{T_\ell} \hat{P}_u \left[L_{max} (\hat{N}_u) \right] \tag{116}$$

The various components which make up the overall environment are derived from probability estimates for both maneuver and gust loads. Extreme amplitude distributions for maneuver loads are based on similar distributions for either the computed loads or the in-flight variables measured during previous loads surveys. In the event that the only past information available is in the form of amplitude distribution functions $P_u(L)$, extreme amplitude distributions for maneuver loads can be approximated by

$$\hat{P}_u \left[L_{max} (\hat{N}_u) \right] \approx \left[\hat{P}_u(L) \right]^{\hat{N}_u} \tag{117}$$

Estimation of extreme amplitude distributions in the case of gust loads is somewhat more complicated. Since turbulence inputs excite dynamic response of an aircraft structure, the detailed loads on structural components are functions of the parameters of the structure as well as those of the input. This means that the estimated extreme gust loads required for design are more in the nature of output criteria than environmental descriptions. This is shown clearly by the following expression from Reference 16 which bounds the extreme response amplitude distribution function for a zero mean Gaussian input.

$$\hat{P}_u \left[L_{max} \left(\hat{N}_L^+(0)_u \hat{T}_u \right) \right] \leq 1 - \hat{N}_L^+(0)_u \hat{T}_u \exp \left[\frac{- L_{max}^{\;2}}{2 \hat{\sigma}_L^2} \right]_u \qquad (118)$$

The term $\hat{N}_L^+(0)$ in Eq. (118) is the estimated rate at which the variable $L(t)$ crosses the zero amplitude level with positive slope and is mainly a characteristic of the structure. This can be estimated, for a nonzero mean variable, in terms of the incremental variable $\delta L(t) = L(t) - \mu_L$. The zero crossing (or mean value crossing) rate is given by Eq. (18) as a function of the predicted structural response to turbulence. That is, $G_L(f)$ is the power spectral density function for a local shear, bending moment, or torque as estimated from the relationship

$$\hat{G}_L(f) = \left| \hat{H}_{gL}(f) \right|^2 \hat{G}_g(f) \qquad (119)$$

where

$\left|\hat{H}_{gL}(f)\right|$ = magnitude of the frequency response function between a gust velocity input and a load response

$G_g(f)$ = power spectral density function for the input gust velocity

A value of $\hat{\sigma}_L^2$ for use in Eq. (118) is then obtained by integrating Eq. (119) over the required frequency range.

Methods currently in use (see Reference 8) for estimating the input gust power spectral density function needed in Eq. (119) are based on the assumption that the function is completely determined by three parameters. These are the intensity as measured by σ_g^2, the shape defined by a turbulence scale parameter L, and the airspeed V. For any particular aircraft usage, then, it is only necessary to predict values for these parameters in order to estimate the gust input.

Repeated Loads. Repeated loads specifications cover those loads which are not individually severe enough to produce catastrophic failures, but which eventually cause fatigue damage if repeated often enough. The requirements are usually given in the form of discrete frequency histograms defining the number of peak load occurrences at each of several load amplitudes for the service life of the aircraft. Therefore, the appropriate statistical description of the loads environment associated with each type of usage consists of a nonnormalized load peak distribution function. This function is an estimate of the actual number of times that the peak value of a load would exceed arbitrary amplitudes if the entire service life of the aircraft were spent in that particular activity.

Multiplication of this distribution function by the fraction of the service life estimated for that usage results in the predicted component loads environment. These components are then added according to the relationship of Eq. (110) as follows.

$$\hat{F}_{\ell}(L_p) = \sum_{u=1}^{U} \frac{\hat{T}_u}{T_{\ell}} \hat{F}_u(L_p) \qquad (120)$$

The function $\hat{F}_u(L_p)$ predicts the load peak distribution which would be characteristic of an entire service life under the uth set of conditions. The overall load peak distribution estimate which results from this summation must then be differentiated in order to determine a loads spectrum. Ordinarily this is done at a set of discrete load amplitudes by counting the number of load peaks which exceed one level but do not exceed the next. The number of levels selected establishes the resolution of the loads spectrum.

The component frequency distributions which are weighted and summed in Eq. (120) are derived separately for maneuver and gust loads. Peak distributions for maneuver loads are predicted on the basis of distributions measured in previous surveys under similar flight conditions. Gust load peak distributions, however, must be computed in conjunction with predicted structural dynamic characteristics.

Since the turbulence phenomenon can be assumed, with some justification, to be representative of a Gaussian random process, the response of a structure to gust inputs can also be assumed Gaussian. Then, the distribution of peaks is a function only of the intensity and frequency content of the response. This is given by the following expression adapted from Reference 4.

$$\hat{F}_u\left(\frac{L_p}{\hat{\sigma}_L}\right) = \hat{N}_{p_u}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{L_p/\hat{\sigma}_L}{\sqrt{1-\hat{\beta}^2}}} e^{-\frac{\xi^2}{2}}\,d\xi\right.$$

$$\left. + \hat{\beta}\, e^{-\frac{L_p^2}{2\hat{\sigma}_L^2}}\left(1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\hat{\beta}L_p/\hat{\sigma}_L}{\sqrt{1-\hat{\beta}^2}}} e^{-\frac{\xi^2}{2}}\,d\xi\right)\right]_u \tag{121}$$

where the total number of peaks N_{p_u} which would be experienced during a service life under the given set of conditions is predicted in terms of the particular estimated response power spectrum for that usage by

$$\hat{N}_{p_u} = T_\ell\left[\frac{\int_0^\infty f^4\,\hat{G}_L(f)_u\,df}{\int_0^\infty f^2\,\hat{G}_L(f)_u\,df}\right]^{1/2} \tag{122}$$

The spectral influence on the distribution is included in the term

$$\hat{\beta} = \frac{\int_0^{\infty} f^2\, \hat{G}_L(f)\, df}{\int_0^{\infty} \hat{G}_L(f)\, df \int_0^{\infty} f^4\, G_L(f)\, df} \qquad (123)$$

and the estimated power spectral density of the structural response $G_L(f)$ is obtained from Eq. (119). These relationships clearly show the need for a priori knowledge of the structural dynamics in order to derive the repeated gust loads requirements.

5.1.2 Criteria Uncertainties

Design criteria, by their very nature, must be considered imprecise. Sources of variance exist not only in the background data upon which the criteria are founded but also in the ability to predict the various ways in which a new aircraft will be used. Because of these uncertainties, reliability margins are ordinarily applied to the environmental estimates in order to establish the specific loads requirements. Then, additional factors of safety are added to account for the uncertainties in designing and verifying a structure for a given set of loads requirements.

Distribution functions derived according to Eq. (110), which predict the overall loads environment for an aircraft service life can be considered as unbiased estimates of the true environment. That is,

$$P_{\ell}(L) = E\left[\hat{P}_{\ell}(L)\right] \qquad (124)$$

where

$$E\left[\hat{P}_{\ell}(L)\right] = \sum_{u=1}^{U} E\left[\frac{\hat{T}_u}{T_{\ell}}\right] E\left[\hat{P}_u(L)\right] \tag{125}$$

The uncertainty in a loads environment prediction is then only a function of the variance of $\hat{P}_{\ell}(L)$. This in turn is dependent on the variances of the two major input terms.

For any particular usage activity, the variance of the service life-probability product is given by the basic relationship of Eq. (98) as

$$\begin{aligned} \mathrm{Var}\left[\frac{\hat{T}_u}{T} \hat{P}_u(L)\right] &= \hat{\sigma}_u^2 \\ &= \hat{\sigma}_T^2 \hat{\sigma}_P^2 + \hat{\sigma}_T^2 \hat{\mu}_P^2 + \hat{\sigma}_P^2 \hat{\mu}_T^2 \end{aligned} \tag{126}$$

where

$$\hat{\sigma}_T^2 = \mathrm{Var}\left[\frac{\hat{T}_u}{T_{\ell}}\right]$$

$$\hat{\sigma}_P^2 = \mathrm{Var}\left[\hat{P}_u(L)\right]$$

$$\hat{\mu}_T = E\left[\frac{\hat{T}_u}{T_{\ell}}\right]$$

$$\hat{\mu}_P = E\left[\hat{P}_u(L)\right]$$

Note that the variance of the service life fraction is weighted as heavily as that of the loads distribution function in the combined uncertainty. Also, as indicated by Eq. (23c), the variance of an estimated probability distribution function varies continuously over the function.

Even though the service life fraction for a given usage is not independent of that for any other usage (since their sum must equal one), it is assumed that the number of usage categories employed in the prediction procedure is sufficient to minimize the ill effects of dependence. This permits the variance of the overall loads probability function to be estimated from the sum of the individual usage variances by

$$\mathrm{Var}\left[\hat{P}_{\ell}(L)\right] = \sum_{u=1}^{U} \hat{\sigma}_u^2 \qquad (127)$$

Then the overall variance is applied to the results of Eqs. (116) and (120) in conjunction with the appropriate reliability criteria to establish the actual loads requirements.

5.2 STRUCTURAL DESIGN SUBSTANTIATION

Since it is necessary to make many assumptions in the process of designing a new aircraft structure, the design must eventually be substantiated empirically. This is accomplished through both laboratory and flight investigations. These investigations begin with material and component tests and extend through the development and operational phases of the aircraft. Aspects of design substantiation which are related to flight measurements are discussed in this section.

Tests which substantiate a new design can essentially be separated by type into those conducted prior to and after acceptance of the design by the customer. The first type of test is performed during the development phase when the objectives are to verify certain design assumptions and to demonstrate the structural integrity of prototype aircraft. Although few test specimens are involved in this phase, the instrumentation requirements for each specimen are extensive. The second type of testing, which is conducted during the operational phase, has the dual objectives of substantiating environmental assumptions used in the design criteria and certifying the structure for long-time exposure to repeated loads. The flight instrumentation requirements for this type of testing are reduced to the minimum on each aircraft, and a relatively large number of aircraft are involved.

5.2.1 Development Phase

A high degree of accuracy is one of the foremost requirements for flight loads measurements made during the development phase of a new aircraft. This implies that complex computational models requiring a large number of in-flight measurements must be employed. The actual number of data channels recorded simultaneously in current development programs may range from a minimum of approximately 60 for a small trainer (Reference 17) to as many as 150 for a large transport (Reference 18). These data are used principally to substantiate analytical predictions of structural response to control inputs (maneuver loads) and turbulence inputs (gust loads).

The response of the structure to maneuvering control inputs is measured in terms of the strains at an array of points on the structure. The inputs are monitored by a separate set of test condition variables.

Reference 19 specifies that these shall include the eight-channel data previously discussed, plus control forces, control surface positions, and angles of attack and sideslip. Determination of the dynamic structural response to turbulence inputs requires, in addition to strain measurements, the measurement of acceleration at an array of points on the lifting and stabilizing surfaces. The corresponding turbulence inputs are usually evaluated on the basis of angle of attack and angle of sideslip data in the manner of Eq. (67).

The above measurements are used to define the characteristics of the structure when viewed as a multiple input-multiple output system. The principal structural descriptor for this purpose is the complex frequency response function between any particular input and a responding point on the structure. Such a function provides information about the frequency dependent amplitude ratio and phase difference between the input and the output. A different frequency response function exists for every pair of input-output variables at every location of interest, and probably for every set of flight conditions. Note that the frequency response function applies throughout the range from static conditions to the highest response frequency of interest. Thus, one method of describing structural behavior is suitable for both maneuver and gust loads.

Experimentally determined frequency response functions are computed from measured spectra of the input and output variables. The complexity of the computations and the amount of information required for this depends on the design assumptions which are to be substantiated. If the structure is assumed to be an ideal system, as defined in Section 3.1.1, then the frequency response function is determined exactly by the observed input and output values. Such an ideal structure is sketched in Figure 18.

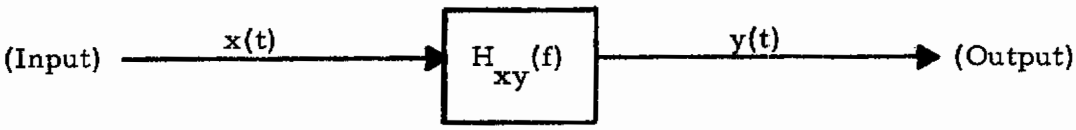


Figure 18. Model of an Ideal Structure with a Single Input

The input-output relationship for this structure is the same as that for the ideal data system described by Eqs. (36) and (38). Therefore, a frequency response function gain factor can be estimated from the power spectra of two in-flight measurements as follows.

$$\left|\hat{H}_{xy}(f)\right| = \left[\frac{\hat{G}_y(f)}{\hat{G}_x(f)}\right]^{1/2} \tag{128}$$

In order for the above computed gain factor to be an unbiased estimate of $\left|H(f)\right|$, it is necessary that the following conditions be met.

a. The structure between the two measurement points must be a constant parameter, linear system.

b. The power spectral density estimates $\hat{G}_x(f)$ and $\hat{G}_y(f)$ must be unbiased.

c. The measured input $x(t)$ must be the only input contributing to the observed output $y(t)$.

Since these ideal conditions do not exist in any practical situation, frequency response function gain factors estimated according to Eq. (128), typically will be biased. In practice, both the input and output power spectra are biased by error components (noise) in their respective measurements, and structural outputs are usually the result of more than one input. This problem can be diagramed as shown below in Figure 19.

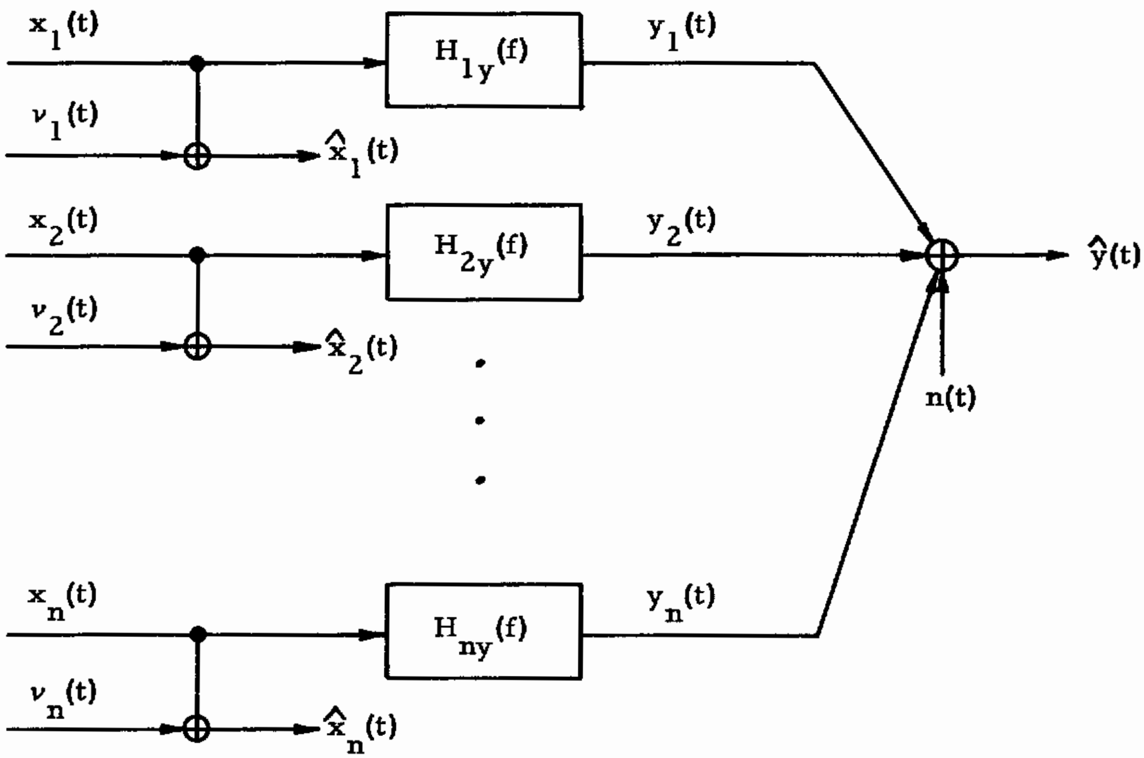


Figure 19. Multiple Input Model of a Structure with Noise Added to the Input and Output Measurements

Figure 19 underlines the comment made earlier that a different frequency response function is required for every input-output combination. In this case, there is a single output at a point on the structure resulting from n inputs. Therefore, n frequency response functions are required for a complete description of the structure with regard to this one point. Noise in the input and output measurements is shown in the diagram as $\nu(t)$ and $n(t)$ respectively. If it is assumed that $n(t)$ is completely independent of any measured input $\hat{x}_j(t)$, and that the x's are independent of each other, then each frequency response function can be estimated independently from measured quantities by the following relationship,

$$\hat{H}_{jy}(f) = \frac{\hat{G}_{jy}(f)}{\hat{G}_j(f)} \tag{129}$$

where $\hat{G}_j(f)$ is the measured power spectral density function for the jth input, and $\hat{G}_{jy}(f)$ is the measured cross spectral density function between the jth input and the response as defined by Eq. (15). From these definitions, it follows that the gain factor and phase factor of each estimated frequency response function are given by

$$\left|\hat{H}_{jy}(f)\right| = \frac{\left|\hat{G}_{jy}(f)\right|}{\hat{G}_j(f)} \qquad \text{(gain factor estimate)} \tag{130a}$$

$$\hat{\theta}_{jy}(f) = \hat{\theta}_{jy}(f) \qquad \text{(phase factor estimate)} \tag{130b}$$

The problem of estimating the structural characteristics is a great deal more complicated if the various inputs are coherent, or dependent upon one another. In this case, it is necessary to evaluate the dependency of the inputs by measuring the cross spectral density function $\hat{G}_{jk}(f)$ between each input and each of the other inputs. This leads to a matrix representation for the frequency response functions between a set of coherent inputs and a single response point. The following equation, presented in Reference 20 and based on the development given in Reference 21, defines the spectral measurements which are required for the estimation procedure

$$\begin{Bmatrix} \hat{H}_{1y}(f) \\ \hat{H}_{2y}(f) \\ \vdots \\ \hat{H}_{ny}(f) \end{Bmatrix} = \begin{bmatrix} \hat{G}_{11}(f) & \hat{G}_{12}(f) & \cdots & \hat{G}_{1n}(f) \\ \hat{G}_{21}(f) & \hat{G}_{22}(f) & \cdots & \hat{G}_{2n}(f) \\ \vdots & \vdots & & \vdots \\ \hat{G}_{n1}(f) & \hat{G}_{n2}(f) & \cdots & \hat{G}_{nn}(f) \end{bmatrix}^{-1} \begin{Bmatrix} \hat{G}_{1y}(f) \\ \hat{G}_{2y}(f) \\ \vdots \\ \hat{G}_{ny}(f) \end{Bmatrix} \qquad (131)$$

The diagonal elements in the inverse square matrix of Eq. (133) are simply the measured power spectral density functions for the n input variables. That is, $\hat{G}_{jj}(f) = \hat{G}_{j}(f)$. The off-diagonal functions denoted by $\hat{G}_{jk}(f)$ are the input cross spectral densities. Note that for the special case of independent multiple inputs, these terms are all zero and the various frequency response functions are estimated by a set of independent equations of the type given in Eq. (129).

Equation (131) provides a useful tool for the definition of structural characteristics during the development of a new aircraft design. Consider for example, the typical problem of attempting to separate maneuver and turbulence effects on the total structural response at an arbitrary point on the vehicle. The total response at any time is not purely a function of either type of input. Furthermore, aeroelastic coupling exists between gust inputs and control surface deflections, and there is a feedback loop between the aircraft response and pilot initiated control inputs. Therefore, the two input sources cannot be considered independent. Then it becomes necessary to evaluate all of the interacting effects in order to estimate either the maneuver or gust frequency response functions for that particular point. If it is assumed, for simplicity, that a single input measurement of each type is sufficient for the desired definitions, then the gust and maneuver frequency response functions are written according to Eq. (130) as follows

$$\hat{H}_{1y}(f) = \frac{\hat{G}_2(f)\,\hat{G}_{1y}(f) - \hat{G}_{12}(f)\,\hat{G}_{2y}(f)}{\hat{G}_1(f)\,\hat{G}_2(f) - \left|\hat{G}_{12}(f)\right|^2} \qquad \text{(gusts)}$$

$$\hat{H}_{2y}(f) = \frac{\hat{G}_1(f)\,\hat{G}_{2y}(f) - \hat{G}_{21}(f)\,\hat{G}_{1y}(f)}{\hat{G}_1(f)\,\hat{G}_2(f) - \left|\hat{G}_{12}(f)\right|^2} \qquad \text{(maneuvers)} \qquad (132)$$

where

$\hat{G}_1(f)$ = measured power spectral density function for the gust input

$\hat{G}_2(f)$ = measured power spectral density function for the control input

$\hat{G}_{12}(f) = \hat{G}^*_{21}(f)$ = cross spectral density function measured between the gust and control inputs

$\hat{G}_{1y}(f)$ = cross spectral density function measured between the gust input and structural response

$\hat{G}_{2y}(f)$ = cross spectral density function measured between the control input and structural response

5.2.2 Operational Phase

The detailed structural descriptions estimated during development testing are usefully applied after the design has been accepted, and production units have gone into operation. The basic measurement objectives in this phase include evaluation of the bending moment, shear, and torque loads on structural components under operational flight conditions. It is necessary, however, to estimate these on the basis of measured flight variables which are suitable for operational aircraft. This restriction eliminates, for example, accurate measurement of angles of attack and sideslip. Also, practical restrictions on operational recorder capacity severely limit the number of structural locations which can be considered in any transducer array. The problem of meeting the measurement objectives is then approached in the following way.

It is reasonable to assume that the detailed flight loads on all structural components of interest can be estimated with a high degree of accuracy during the development phase from strain measurements and computational models of the type

$$\left\{\hat{L}\right\} = \left[\hat{A}\right]\left\{\hat{e}\right\} \tag{133}$$

Here, the real valued transfer term estimates A are obtained through carefully controlled ground calibration procedures. Detailed component loads which are computed from measured flight strains can then be considered to define reference or "true loads" conditions. Concurrent with strain measurements during the development tests, flight variables are recorded which are actually suitable for use with operational data systems. Thus, it is possible to establish a set of input-output relationships between the operationally suitable in-flight variables and the reference strain measurements. These would be typified in the time domain by the sum of the strain responses to a collection of arbitrary measurable inputs as

$$\hat{e}_i(t) = \sum_j \int_0^\infty \hat{h}_{ij}(\tau)\, \hat{x}_j(t - \tau)\, d\tau \tag{134a}$$

$$= \sum_j \hat{h}_{ij} * \hat{x}_j$$

and in the frequency domain by either

$$\hat{E}_i(f) = \sum_j \hat{H}_{ij}(f)\, \hat{X}_j(f) \tag{134b}$$

or

$$\hat{G}_{e_i}(f) = \sum_j \left| \hat{H}_{ij}(f) \right|^2 \hat{G}_{x_j}(f) \tag{134c}$$

In simplified matrix notation, which can be thought of as either a time or frequency functional form, the strains and "other" in-flight variables are related as indicated below.

$$\left\{\hat{e}\right\} = \left[\hat{h}\right]\left\{\hat{x}\right\} \tag{135}$$

Then, Eq. (135) can be substituted into Eq. (133) to produce a model which accomplishes the desired result.

$$\left\{\hat{L}\right\} = \left[\hat{A}\right]\left[\hat{h}\right]\left\{\hat{x}\right\} \tag{136}$$

In acquiring operational flight loads data, it is not necessary to measure exactly the same variables which were used in deriving the design criteria. Since the design limit and repeated loads requirements given in the criteria consist merely of design input information from which to compute allowable static and fatigue strength, any other such information which achieves these same results is adequate. Thus, for example, it is sometimes appropriate to include structural strain in the measurement list during operational fatigue certification. As a generality, establishment of the measurement list should be based upon a multiple regression study conducted during the development phase to determine those flight variables which are the most strongly correlated with known flight loading conditions as established by strain measurements. Then, the maximum number of variables which can be handled adequately by an operational recorder should be selected in order of correlation strength and used in conjunction with Eq. (136) to determine detailed loads. These loads are then used to substantiate the fatigue life estimation assumptions.

A second approach to design substantiation during the operational phase is intended only to evaluate the statistical amplitude properties or the overall damaging effects of the measured variables. A certain amount of data processing is performed in flight which results in loss of the time history information for each variable. In return for this compromise, data acquisition efficiency is greatly increased in terms of space, weight, and cost per channel.

The simplest example of a device which is intended to substantiate the fatigue strength of a new design consists of an inherent fatigue damage monitor. This may take the form of a plastic stress detection coating on a highly stressed structural component, a strip of the same alloy as a component which is attached to the component so that it will experience somewhat higher stress levels, or a fatigue gage. The last of these is an instrument for which the electrical resistance is a function of fatigue damage. It must be said, however, that none of these devices has been proven to be generally reliable under random operational flight loading conditions.

Coarsely resolved amplitude distribution functions for linear acceleration variables are estimated inexpensively using "counting accelerometers." Counting accelerometers are simple electro-mechanical switching devices which record the number of times that a linear acceleration variable exceeds each of several preselected amplitudes. Various reset and time delay features are incorporated to minimize the influence of local structural vibration on the readings. A minimum system ordinarily consists of a single transducer, mounted at the aircraft center of gravity, which registers the number of times that normal acceleration exceeds four preset positive amplitudes. The results are used in conjunction with Eq. (54) to estimate amplitude distribution functions for the normal aerodynamic force. By making certain simplifying assumptions, these results are usually interpreted in terms of wing loads. Other counting accelerometers have been developed which record both positive and negative amplitudes at as many as fifteen levels. None of these switching devices, however, provides for the recording of air speed or altitude at the time of data acquisition. It is also difficult to correlate the aircraft weight with the count registration. Therefore, in addition to sacrificing time history information, the principle of the counting accelerometer is based on a computational model which is inherently imprecise.

A recent innovation in operational data acquisition systems makes use of data compression techniques. Since one of the major limitations of operational systems is the space requirement, this approach is advantageous in that the rate of data recording, and hence the cumulative recording time, depends on the significance of the measurements. This significance is computed by the data acquisition system, and may involve the joint behavior of a moderately large number of in-flight variables. An additional feature of some current systems is the ability to store information temporarily during periods of peak activity when significant data are being generated at a rate which exceeds the maximum recording rate. Clearly, the irregular time intervals at which flight measurements are sampled makes it difficult, if not impossible, to reconstruct the time history for any particular variable. Therefore, these data are utilized mainly when the rigid body response assumption applies. That is, input-output relationships such as those given in Eq. (134) are valid here only for quasi-statically varying loads.

Verification of the assumptions used to design an aircraft involves both the loads and the usage. As was pointed out in Section 5.1.2, estimation accuracy depends as heavily on the accuracy of usage estimates as that of loads estimates. Therefore it is necessary to monitor the operational usage and to correlate this with any recorded flight data.

The statistical quantities which are assumed in the design criteria are tested for equivalence with the sample estimates measured during the operational phase. When the assumed and measured quantities differ significantly, the design criteria are revised, and there may be a need for redesigning the structure and conducting additional development tests.

5.2.3 Substantiation Uncertainties

The uncertainties associated with substantiating a new structural design are of lesser degree than those discussed under design criteria in Section 5.1.2. Primarily, this is because the relevant data apply directly to the particular design instead of being extrapolated from other, similar aircraft. The sources of uncertainty are then reduced to the areas of statistical sampling, flight instrumentation, and computing.

In the development phase, the sampling, instrumentation, and computational errors are almost completely within the control of the program management. That is, by acquiring adequate samples of a sufficient number of inflight variables under various test conditions, the detailed structural response to control and turbulence inputs can be estimated with nearly any degree of accuracy desired. The subject of estimation accuracy for frequency response functions has received detailed development in References 20, 21 and 22. Some of the results of these studies are reviewed here briefly.

Frequency response functions are most accurately measured by applying a single frequency periodic input to a system and measuring the vector amplitude and phase of the output relative to the input. The operation must be repeated at all frequencies of interest in order to generate magnitude and phase functions. This technique has limited application, however, in development phase flight testing. Although it may be possible to employ single frequency periodic control inputs for estimating the structural response to maneuver loads, gust frequency response functions must be estimated on the basis of random data. Several equations have been presented in Section 5. 2. 1 which may be employed in this estimation procedure. The errors involved in the use of each of these will now be discussed.

The conditions under which Eq. (128) provides unbiased estimates of a frequency response gain factor are listed following the equation. If any of these conditions is violated, the resulting estimate will contain a bias error. Consider for example, a single input-single output version of the system sketched in Figure 19. The power spectra of the input and output measurements include the effects of noise such that

$$\begin{aligned} \hat{G}_x(f) &= G_x(f) + G_\nu(f) \\ \hat{G}_y(f) &= G_y(f) + G_n(f) \end{aligned} \qquad (137)$$

Substitution of these relationships in Eq. (128) indicates the quantities which actually make up the gain factor estimate. However, the true gain factor is given by

$$\left|H_{xy}(f)\right| = \left[\frac{G_y(f)}{G_x(f)}\right]^{1/2} = \frac{\left|\hat{G}_{xy}(f)\right|}{G_x(f)} \tag{138a}$$

or, from the definition of the coherence function given in Eq. (16), by

$$\left|H_{xy}(f)\right| = \frac{\hat{\gamma}_{xy}(f)\sqrt{\hat{G}_x(f)\ \hat{G}_y(f)}}{G_x(f)} \tag{138b}$$

Then, the relative bias error in the gain factor estimate of Eq. (128) can be expressed as follows

$$\frac{b\left[\left|\hat{H}_{xy}(f)\right|\right]}{\left|H_{xy}(f)\right|} = 100\left(1 - \frac{G_x(f)}{\hat{\gamma}_{xy}(f)\ \hat{G}_x(f)}\right)\ \% \tag{139}$$

Bias errors in gain factor estimates are greatly reduced if the cross spectral density function between the measured input and the measured output is utilized as in Eq. (130a). The following bias error expression taken from Reference 22 applies in this case.

$$\frac{b\left[\left|\hat{H}_{jy}(f)\right|\right]}{\left|H_{jy}(f)\right|} = 100\left(1 - \frac{1}{1 + G_{\nu_j}(f)\ /\ G_{x_j}(f)}\right)\ \% \tag{140}$$

Note that noise in the output measurement does not contribute to the error of the estimate. This is because the output noise is assumed to be unrelated to the input, and therefore, does not influence the cross spectral density function between the two points. Thus, a gain factor estimate computed according to Eq. (130a) will only be biased if the input measurement is in error. Also, from Eq. (130b) it can be seen that phase factor estimates based upon cross spectral density functions are unbiased.

In the typical situation where the structural response at a point results from multiple inputs, the frequency response functions between any of these inputs and the given output are estimated from Eq. (131). If the various inputs are independent, the cumulative effect on the output of all inputs but one can be lumped under the heading of extraneous output noise n(t). Then, the bias error expression given by Eq. (140) is applicable for each input. A corresponding bias error relationship for the case of multiple coherent inputs is generally unknown. However, if it is assumed that coherence between the inputs produces errors in the input measurements, Eq. (140) implies that this increases the bias errors in gain factors estimated according to Eq. (130a).

The variability of frequency response function estimates is often much greater than the bias. Variability in the estimates is a function of the sample sizes used for computing the required spectral density estimates, the amount of noise in the measurements, and the linearity of the structure.

Variance expressions have been developed in Reference 21 which are applicable to the single input case of Eq. (129) and the case of multiple inputs described by Eq. (131). It is shown that both gain and phase factor estimates are subject to variability errors.

As demonstrated by Eqs. (128) through (131), frequency response functions consist of the ratios of spectral densities. Since spectral densities are, in reality, measures of variance, and the ratio of sample variance is distributed according to the F statistic, it follows that the variability in sample gain and phase factors is a function of the dispersion of the F-distribution P(F). This dispersion depends on two degrees of freedom, N_1 and N_2, and the desired measures are given by the mean and standard deviation of P(F). From Reference 1,

$$\mu_F = \frac{N_2}{N_2 - 2} \qquad N_2 > 2$$

$$\sigma_F = \left[\frac{2N_2^2(N_1 + N_2 - 2)}{N_1 (N_2 - 2)^2 (N_2 - 4)} \right]^{1/2} \qquad N_2 > 4 \tag{141}$$

Reference 21 then shows that the sample variance of a gain factor estimate based on Eq. (130a) is given at any frequency by

$$\hat{\sigma}^2_{|H|}(f) = \frac{2}{B_f T - 1} (\mu_F + \sigma_F) \left[1 - \hat{\gamma}^2_{xy}(f) \right] \frac{\hat{G}_y(f)}{\hat{G}_x(f)} \tag{142a}$$

where the appropriate degrees of freedom for μ_F and σ_F are

$N_1 = 2$

$N_2 = 2B_f T - 2$

and

B_f = resolution bandwidth of the spectral density analyses

T = sample record length for the spectral density measurements

$\hat{G}_x(f)$ = sample measurement of the power spectral density function for the input, $x(t)$

$\hat{G}_y(f)$ = sample measurement of the power spectral density function for the response, $y(t)$

$\hat{\gamma}^2_{xy}(f)$ = sample estimate of the coherence function between $y(t)$ and $x(t)$

The corresponding sample variance for the phase factor estimate is

$$\hat{\sigma}^2_\theta(f) = \left[\sin^{-1} \frac{\hat{\sigma}_{|H|}(f)}{|\hat{H}(f)|} \right]^2 \tag{142b}$$

Two features of Eq. (142) which are of interest concern the equivalent sample size 2BT and the coherence function $\hat{\gamma}^2(f)$. Note, for example, that the variability at any frequency in terms of $\hat{\sigma}(f)$ approaches zero as the corresponding coherence approaches unity. Thus, if the spectral measurements are noise-free, the structure is linear, and the output results only from the one input, the variability of the estimated gain and phase factors will be zero. In the typical case, however, both the input and output measurements contain errors, the structure is somewhat nonlinear, and the response consists of the sum of responses to several unmeasured inputs. Some idea of the magnitude of the error involved is given in Reference 13 with regard to estimation of the frequency response function between vertical gust velocities and center of gravity normal acceleration for a large flexible aircraft. Significant reductions in the coherence function were attributed to such factors as the vertical response to axial and lateral gusts, spanwise variation of gusts, and pilot control inputs. Measured coherence functions averaged about 0.7 in the frequency range 0 to 3 Hz. Apparently, some of this was due to errors in reading oscillograph records. However, a large part of the reduction was blamed on the other mentioned factors.

The most practical method of reducing variability errors in frequency response function estimates is to increase the sample size. As noted in Eq. (142), the variance is inversely proportional to the sample size. Therefore, if the coherence between the input and output cannot be brought close to unity by careful experimental technique, the only other way of reducing the variability error is to acquire large quantities of data.

In the case of multiple coherent inputs, where the frequency response functions are determined according to Eq. (132), the sample variance is determined for each of the estimated gain factors by Eq. (143) and used in Eq. (142b) for the phase factor variances.

$$\hat{\sigma}^2_{|H|_j}(f) = \frac{n}{B_f T - n} \left(\mu_F + \sigma_F \right) \frac{\left[1 - \hat{\gamma}^2_{y \cdot x}(f)\right] \hat{G}_y(f)}{\left[1 - \hat{\gamma}^2_{j \cdot x}(f)\right] \hat{G}_j(f)} \qquad (143)$$

where

n = total number of measured inputs

$N_1 = 2n$

$N_2 = 2(B_f T - n)$ } (μ_F and σ_F from Eq. (141))

B_f = resolution bandwidth of the spectral density analyses

T = sample record length for the spectral density measurements

$\hat{G}_j(f)$ = sample measurement of the power spectral density function for the jth input, $x_j(t)$

$\hat{G}_y(f)$ = sample measurement of the power spectral density function for the response, $y(t)$

$\hat{\gamma}^2_{y \cdot x}(f)$ = sample estimate of the multiple coherence function between the response and the measured inputs

$\hat{\gamma}^2_{j \cdot x}(f)$ = sample estimate of the multiple coherence function between the jth input and the other measured inputs

The multiple coherence function is a measure of the linear relationship between the response and all the affecting inputs or between one input and all the other inputs. A multiple coherence function estimate can be obtained from sample measurements of the spectral density functions for input and output variables as follows.

$$\hat{\gamma}^2_{i \cdot x}(f) = 1 - \frac{1}{\hat{G}_i(f)\ \hat{G}^i(f)} \qquad (144)$$

where $\hat{G}_i(f)$ is the power spectral density function for the ith sample measurement from a collection of k sample measurements and $\hat{G}^{i}(f)$ denotes the ith diagonal element of the inverse matrix

$$\begin{bmatrix} \hat{G}_{11}(f) & \hat{G}_{12}(f) & \cdots & \hat{G}_{1k}(f) \\ \hat{G}_{21}(f) & \hat{G}_{22}(f) & \cdots & \hat{G}_{2k}(f) \\ \vdots & \vdots & & \vdots \\ \hat{G}_{k1}(f) & \hat{G}_{k2}(f) & \cdots & \hat{G}_{kk}(f) \end{bmatrix}^{-1}$$

Since it must be assumed that $B_f T$ in Eq. (143) is much larger than n, this indicates that the variability error in multiple frequency response function estimates is approximately proportional to the number of measured inputs and inversely proportional to the sample size. The most important features of Eq. (143), however, involve the two sample coherence functions. Note that increased multiple coherence between the output and the several inputs decreases the variability as might be expected. However, as the inputs themselves become more coherent, the coherence function in the denominator approaches unity and the variability error blows up. Thus, it appears that the redundant information provided by too many coherent inputs degrades the accuracy of the estimated frequency response functions. It follows that the variability error can be minimized by an optimum choice of measured variables such that nearly all of the inputs affecting a particular output are accounted for without too much overlap. Once the measurement list has been established, the variability is only a function of sample size.

In the operational phase, the various compromises made necessary by data system limitations lead to increased uncertainty in the results. For example, it is evident from Eq. (136) that the errors in operational flight loads estimates must always be equal to or greater than the errors in estimating the structural parameters. Errors of the later type are associated mainly with variability in the frequency response function and impulse response function estimates just discussed. The real valued structural constants denoted by A in Eq. (136) can be considered error-free by comparison. Error in the computed loads is then compounded by errors in the measured variables. Aside from these uncertainties there are bias errors of the type described by Eq. (71) which are introduced by neglecting the measurement of important variables.

Estimation of a particular flight load from operational data according to Eq. (136) can be expressed in the following form

$$\hat{L} = \sum_i \hat{A}_i \left(\sum_i \hat{h}_{ij} * \hat{x}_j + \sum_k \hat{h}_{ik} * \hat{x}_k \right) \tag{145}$$

where the $\hat{x}_j$'s are in-flight variables actually measured during the operational phase, and the $\hat{x}_k$'s are those which are neglected. Then, the resulting bias error in the estimate $\hat{L}$ is given in the absolute and relative forms by

$$\begin{aligned} b\left[\hat{L}\right] &= E\left[L - \sum_i \hat{A}_i \sum_j \hat{h}_{ij} * \hat{x}_j\right] \\ &= \sum_i E\left[\hat{A}_i\right] \sum_k E\left[\hat{h}_{ik} * \hat{x}_k\right] \end{aligned} \tag{146}$$

$$\frac{b\left[\hat{L}\right]}{E\left[L\right]} = \frac{100 \sum_i E\left[\hat{A}_i\right] \sum_k E\left[\hat{h}_{ik} * \hat{x}_k\right]}{\sum_i E\left[\hat{A}_i\right] \left(\sum_j E\left[\hat{h}_{ij} * \hat{x}_j\right] + \sum_k E\left[\hat{h}_{ik} * \hat{x}_k\right] \right)} \%$$

The bias error in the power spectrum of $\hat{L}$ can be expressed in a similar way based on the notation of Eq. (134c). By analogy with Eq. (146) above, the relative bias error as a function of frequency would be

$$\frac{b\left[\hat{G}_L(f)\right]}{\hat{G}_L(f)} = \frac{100 \sum_i E\left[\hat{A}_i^2\right] \sum_k \left|\hat{H}_{ik}(f)\right|^2 \hat{G}_k(f)}{\sum_i E\left[\hat{A}_i^2\right]\left(\sum_j \left|\hat{H}_{ij}(f)\right|^2 \hat{G}_j(f) + \sum_k \left|\hat{H}_{ik}(f)\right|^2 \hat{G}_k(f)\right)} \ \% \tag{147}$$

where it is temporarily assumed that the estimates of the power spectra and frequency response functions are error-free. That is, the observed functions are substituted in this expression for their expected values.

The variability of an operational flight load estimated according to Eq. (136) can best be analyzed in the frequency domain. Thus, if the power spectrum of $\hat{L}$ is estimated by

$$\hat{G}_L(f) = \sum_i \hat{A}_i^2 \sum_j \left|\hat{H}_{ij}(f)\right|^2 \hat{G}_j(f) \tag{148}$$

then the variance in the estimate would be obtained by summing the variance of the gain factor-power spectrum product for all j and subsequently summing the variance of the second product for all i. Actually, this can be simplified by assuming that the variance of the A_i's is relatively small in comparison to that of the other terms and merely dealing with the first product variance. Denoting this variance by the following,

$$\text{Var}_j\left[\left|\hat{H}(f)\right|^2 \hat{G}(f)\right] = \hat{\sigma}_H^2\,\hat{\sigma}_G^2 + \hat{\sigma}_H^2\,\hat{\mu}_G^2 + \hat{\sigma}_G^2\,\hat{\mu}_H^2 \tag{149}$$

where

$$\hat{\sigma}_H^2 = \text{Var}\left[\left|\hat{H}_{ij}(f)\right|^2\right]$$

$$\hat{\sigma}_G^2 = \text{Var}\left[\hat{G}_j(f)\right]$$

$$\hat{\mu}_H = E\left[\left|\hat{H}_{ij}(f)\right|^2\right]$$

$$\hat{\mu}_G = E\left[\hat{G}_j(f)\right]$$

then the variance as a function of frequency for the flight load power spectrum would be as given below.

$$\text{Var}\left[\hat{G}_L(f)\right] = \sum_i E^2\left[\hat{A}_i^2\right]\sum_j \text{Var}_j\left[\left|\hat{H}(f)\right|^2 G(f)\right] \tag{150}$$

Operational data which have been partially processed in flight are subject to additional uncertainties. First, it must be noted that the preceding development applies here only if the data represent quasi-static loading conditions. Then a frequency response function is treated as that of a zero order system, and the sequence of discrete measured values for a variable is considered to represent separate observations of a constant. If a measured variable includes dynamic response of the aircraft there may be considerable error in the estimated amplitude distribution function. This is especially true if the random variable under investigation is the peak value of the measured time history.

Most of the in-flight variables which are suitable for measurement under operational conditions must be transformed into computed loads by some rigid body response model. This operation, in effect, transforms the distribution function. Such transformations are practical under certain conditions. For example, note the summation of distribution functions described by Eq. (110), and the multiple convolution operation defined by Eq. (113). In both these cases, however, it is assumed that the data are quasi-static and a rigid body model is being used. The contributions of dynamic response at a point render these assumptions invalid. Then, an appropriate transformation for an arbitrary time dependent random variable must be developed. Even though this is no problem for a Gaussian process, and the response to turbulence has been assumed Gaussian, the combined response at a point to all loading inputs will typically not be Gaussian. Also, the loss of time history information makes it impossible to separate the gust response from the maneuver response.

In order to transform an observed non-Gaussian distribution for a variable in such a way that the distribution of a particular load can be determined, it is necessary to know a great deal about the correlation structure of the time history. Clearly, this information is not available in the recorded format. Therefore, violation of the quasi-static assumption will lead to unspecified errors in distribution functions derived from the observed distributions.

6. SUMMARY AND CONCLUSIONS

This study represents part of a continuing investigation aimed at improving the evaluation and utilization of airplane flight loads data. A summary of the important aspects of the problem is given below, followed by a discussion of the conclusions which can be drawn from this part of the investigation.

6.1 SUMMARY

The major steps in the evaluation and utilization of flight loads data are outlined in Figure 20. As indicated in the outline, the requirements associated with a given application must be established at the beginning of a program. These requirements define the computational models and the computing procedures to be used in deriving the desired loads information from measured in-flight variables. The pertinent variables are determined at this time as well as the statistical properties which are to be estimated. These items of information make up the inputs for actual measurement program planning. A list of measurements corresponding to the selected computational models is established by this operation. A sampling plan is also formulated at this point which defines the sample sizes required for estimating the desired statistical properties within specified limits of acceptable error. The measurement list and the sampling plan jointly form the basis for the airborne data system design criteria. These criteria answer questions concerning the capacity and accuracy of the system. Data are then acquired in flight by this system according to the established measurement list and sampling plan. Flight loads are computed from these data using the models and procedures defined by the applicational requirements. Finally, the computed results are utilized for the specific applications. Mainly these include the generation of structural design criteria for new airplanes and the substantiation of existing structural designs.

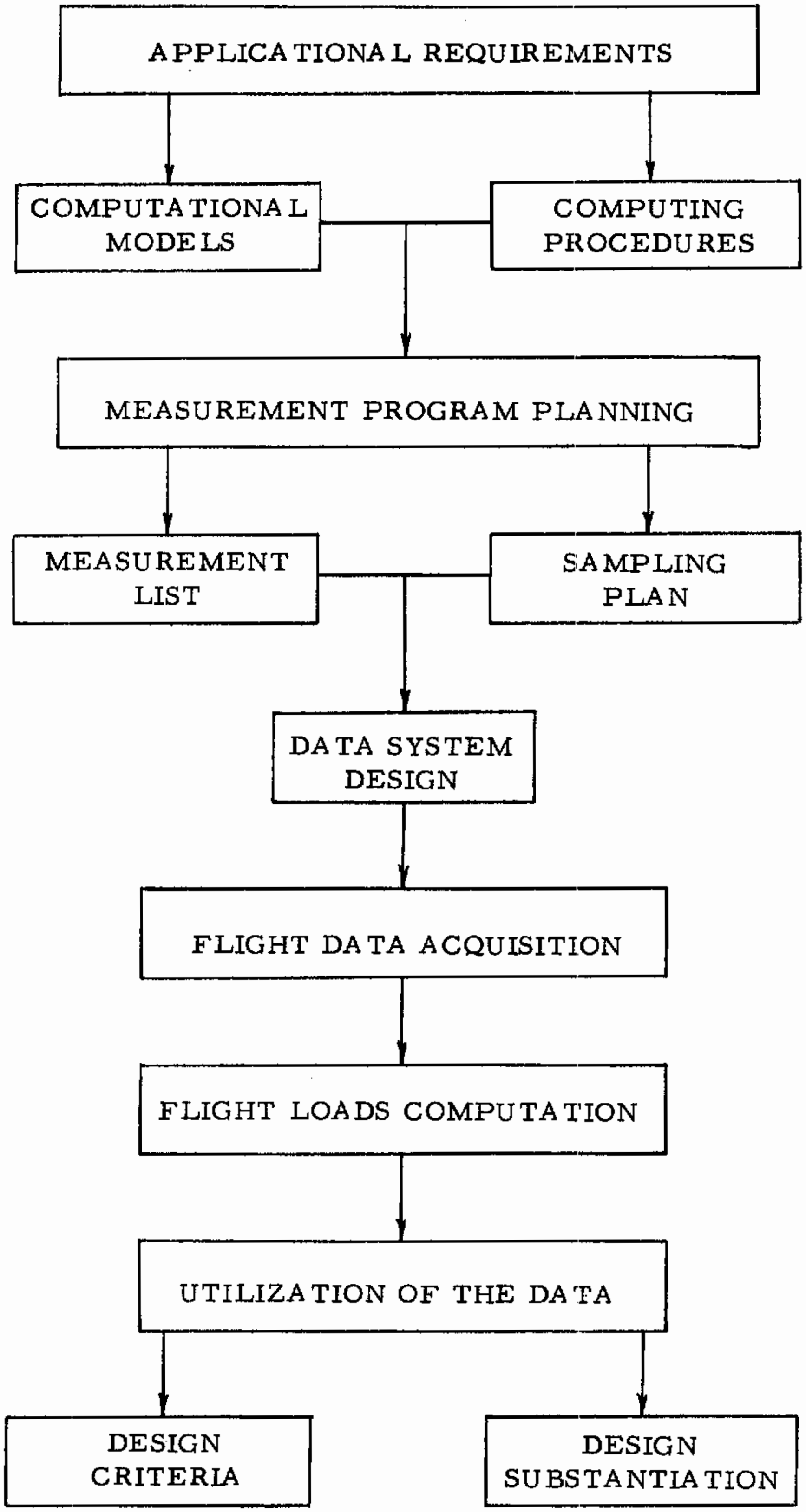


Figure 20. Outline of the Overall Flight Loads Data Evaluation and Utilization Process

6.2 CONCLUSIONS

Each of the steps summarized in Figure 20 has an associated uncertainty which reflects on the accuracy of the final results. In the course of studying this process, it has become apparent that some steps are more uncertain than others. The study was begun under the assumption that airborne data system inaccuracy was probably the major contributor to the overall error. This notion has since been modified. Following a review of current flight loads evaluation and utilization methods, two other areas can be seen as the major sources of uncertainty in this process. The first of these concerns the art of selecting an appropriate computational model to characterize a given loads phenomenon. The second involves current approaches to generating structural design criteria.

Shortcomings in the models used for computing flight loads from measured data are related to the misuse and poor design of some models. A classic example of a poorly designed model is given by Eq. (57b) in the text. This relationship is supposed to provide computed values for the normal force on a horizontal tail from normal and pitching acceleration measurements. In practice, however, it is generally conceded that use of this model can easily lead to errors in the estimated loads on the order of 100%. This is caused principally by the variance in the distance between the center of gravity and the center of lift for the wing-fuselage, denoted by x_{wf}. The model is poorly designed because the actual position of the aircraft center of gravity at any time is one of the least certain parameters of an airplane in flight. On any particular airplane, the range of center of gravity position during flight is roughly of the same magnitude as the distance x_{wf}. It would appear that models of this type which exhibit a high error sensitivity have been designed without taking into account all of the practical considerations. Before any computational model is used, the variance which is to be expected in the transfer terms should be investigated and an analysis of the overall effect conducted. Equation (74) is a useful tool in this regard.

A suggested alternate to Eq. (57b) might be a model which makes use of normal acceleration measured near the horizontal tail center of pressure. If an acceleration transducer were located reasonably close to the body X-axis, it could be used in conjunction with the center of gravity normal acceleration to record both normal and pitching accelerations without increasing the number of measurements. From this investigation, it is concluded that energies devoted to improving computational models will be better spent than those used to increase instrument accuracy.

As mentioned in Section 5.2.2, the measurement list for an operational flight data system should be based on the degree of correlation between measurable variables and the desired loads. The current approach to establishing an operational measurement list seems to be based more on tradition than on this correlation. The traditional measurement of normal acceleration at the center of gravity undoubtedly has merit. It must be observed, however, that this variable is known to be strongly correlated with wing bending loads. Since this is an environment of great concern, the measurement is a very important one. From the reference material, though, it is not as immediately apparent that some of the other commonly measured variables are as strongly correlated with particular loads. The list of eight-channel measurements is a good example. The conclusion which can be reached here is that no specific measurement list for the operational phase of a new aircraft should be established until tests are conducted during the development phase. Then by regression and correlation analysis, any of the many variables which are required for structural definition purposes may turn out to be the best ones for use in the operational phase.

There is a current tendency to attempt a compromise between dynamic and static loads evaluation on some flexible aircraft. That is, data acquisition systems are often specified such that components of flexible gust response become a part of the measured data, and then the data are used in conjunction with rigid body models. This constitutes misuse of models which may otherwise be well designed. An example of this is given by the use of counting accelerometer data in conjunction with Eq. (54) to derive loads distributions. Some counting accelerometer transducers employed in the past have been capable of responding to acceleration inputs at frequencies as high as 20 cps. In contrast, References 23 and 24 indicate that rigid body response seldom exceeds 1 cps. In attempting to decrease the frequency sensitivity of counting accelerometers, various delays have been built into the count registration logic. These methods have not been completely effective, and if for no other reason, past counting accelerometer data should be viewed as the poorest source of loads information for new design criteria.

As a generality, it can probably be said that variables which are intended for use with rigid body models should be measured using data systems with sharp frequency roll-off characteristics somewhere between 1 and 1.5 cps. On the other hand, if a flexible response model based on time history information is to be used, the data system must be capable of accurately recording instantaneous values to frequencies well beyond 10 cps. There is no meaningful compromise between these requirements. Accurate recording of dynamic signals requires that the system gain factor be uniform over the range 0 to f_c, and that the phase factor be at least linear with frequency. Then none of the spectral components will be attenuated with respect to the other components, and each component will be delayed equally in time. An additional requirement for digital systems is that the quantizing rate must be adequate for the intended data application. For spectral density function estimates, this rate should exceed $2f_c$. If peaks in the time history of a variable are to be evaluated, Eq. (85) indicates that the rate should be in excess of $4f_c$. However, if only the variance is desired from a measurement, for use with an expression such as Eq. (64), the quantizing rate is immaterial as long as the variable remains stationary. One suggestion in this regard might be to compute the variance of a signal continuously in flight and sample only the computed values at a very low rate.

Generation of structural design criteria from measured flight loads data represents a second area of large uncertainty. This is partly due to inaccurate or poorly presented data, but also involves some of the current methods for establishing the loads requirements.

If the loads requirements for a new design are to be established with reasonable accuracy, it is necessary to weight previously acquired data in terms of the intended usage for the new vehicle. This is done to some degree in all design criteria. However, current methods make use of relatively few usage categories. Gust loads criteria, for example, are based on calm air conditions and storm conditions in a set of altitude bands. Categories for maneuver loads are generally separated on the basis of mission type. With the available information, it would seem possible to refine this breakdown to a much greater degree.

Another facet of the problem concerns a tendency on the part of specification writers to place great importance on statistical sample sizes to the exclusion of other factors. Unfortunately, the flight loads data which exist in the greatest quantities are usually the least refined.

This is a natural consequence of the economic tradeoffs involved in data acquisition. The result is that vast quantities of counting accelerometer data, moderately large samples of VGH data, and only small amounts of data from present generation recorders are available. Many design criteria are then heavily biased by the inclusion of poorly refined and inaccurate counting accelerometer data. Needless to say, the usage breakdown for this type of data is very coarse. The presentations of even VGH data do not ordinarily provide descriptions of aircraft configuration. Only the more recent reports on VGH and eight-channel data elaborate on these factors. One outstanding result of this shortcoming is that there are no published data upon which to base accurate predictions of tail loads.

Since past data are not reported with the desired refinement, current loads specification writers generally have a conservative viewpoint. It is not uncommon for a loads requirement to be derived by forming an envelope over all existing data which apply. This approach almost completely ignores whatever usage information may be available. The general philosophy is that it is better to overdesign than to underdesign. However, in terms of mission capability, this may not always be true. A more analytical approach would be one which weighs the consequences of overdesign against those of underdesign.

An important point which is relevant to the design criteria application concerns the optimum presentation formats for the measured variables. Typical presentations consist of first order probability functions for individual variables. In some of the more recent reports, the pertinent bivariate distributions are also presented, but very little information on extreme amplitude distributions can be found in the references.

Sample distributions for important products of two variables are also in short supply. These distributions cannot, in general, be derived from the distributions of the variables being multiplied.

There is still a basic question, however, as to the appropriateness of previously acquired data for any given new design. The approach currently used most often assumes that various kinematic variables which have been evaluated previously for another aircraft can be used directly with new designs. In many cases, the performance of a new design is quite a bit different from that of the aircraft supplying the data. At present, there do not seem to be adequate weighting factors to account for these differences. This is an important area which deserves investigation.

REFERENCES

1. Bendat, J. S., and Piersol, A. G., Measurement and Analysis of Random Data, New York, John Wiley & Sons, Inc., 1966.

2. Rice, S. O., "Mathematical Analysis of Random Noise," in Noise and Stochastic Processes, ed. Nelson Wax, New York, Dover Publications, Inc., 1954.

3. Blackman, R. B., and Tukey, J. W., The Measurement of Power Spectra, New York, Dover Publications, Inc., 1958.

4. Bendat, J. S., Beckwith, R. E., and Choi, S. C., "Estimation of Peak Probability Functions and Structural Fatigue Damage," AFFDL-TR-66-201, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, May 1967.

5. Choi, S. C., "Design of Sampling Procedures for Flight Loads Studies," AFFDL-TR-66-202, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, May 1967.

6. deCani, J. S., Tremblay, H. G., and Fischer, C. F., "Design of Sampling Program for Aircraft Loads Study," Aeronautical Computer Laboratory Report No. NADC-AC-5902, U.S. Naval Air Development Center, Johnsville, Pennsylvania, June 1959.

7. American Standard No. Y10.7-1954, "Letter Symbols for Aeronautical Sciences," New York, The American Society of Mechanical Engineers, 1954.

8. Houbolt, J. C., Steiner, R., and Pratt, K. G., "Dynamic Response of Airplanes to Atmospheric Turbulence Including Flight Data on Input and Response," National Aeronautics and Space Administration TR R-199, June 1964.

9. Military Specification, Airplane Strength and Rigidity Flight Loads, MIL-A-8861(ASG), May 1960.

10. Mayer, J. P., Stone, R. W., Jr., and Hamer, H. A., "Notes on a Large-Scale Statistical Program for the Establishment of Maneuver-Loads Design Criteria for Military Airplanes," National Advisory Committee for Aeronautics, NACA RM L57E30, July 1957.

11. Berens, A. P., Clay, L. E., and Giessler, F. J., "A Statistical Model for Application of Maneuver Flight Loads Data to Structural Design Criteria," Report No. 970-65-4, Technology Incorporated, Dayton, Ohio, prepared for National Aeronautics and Space Administration under Contract No. NASw-970, August 1965.

12. Rodden, W. P., and Revell, J. D., "The Status of Unsteady Aerodynamic Influence Coefficients," Institute of the Aerospace Sciences, S. M. F. Fund Paper No. FF-33, January 1962.

13. Coleman, T. L., Press, H., and Meadows, M. T., "An Evaluation of Effects of Flexibility on Wing Strains in Rough Air for a Large Swept-Wing Airplane by Means of Experimentally Determined Frequency-Response Functions with an Assessment of Random-Process Techniques Employed," National Aeronautics and Space Administration TR R-70, 1960.

14. Peloubet, R. P., and Haller, R. L., "Application of a Power Spectral Gust Design Procedure to Bomber Aircraft," AFFDL TR-66-35, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, June 1966.

15. Papoulis, A., Probability, Random Variables, and Stochastic Processes, New York, McGraw-Hill Book Co., Inc., 1965.

16. Gray, C. L., "First Occurrence Probabilities for Extreme Random Vibration Amplitudes," Shock and Vibration Bulletin No. 35, Part 4, February 1966.

17. Barrett, J. O., "T-38 Gust Loads, Flight Test Results," Northrop Corporation, Norair Division, Report NOR-60-303, June 1962.

18. Lockheed-Georgia Company, "C-141A Engineering Flight Test Results of the Structural Dynamic Response Tests," Engineering Report ER-5247, October 1966.

19. Military Specification, Structural Criteria, Piloted Airplanes, Structural Tests, Flight, MIL-S-5711 (USAF), August 1947.

20. Gray, C. L., and Piersol, A. G., "Methods for Applying Measured Data to Vibration and Acoustic Problems," AFFDL-TR-65-60, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, June 1965.

21. Goodman, N. R., "Measurement of Matrix Frequency Response Functions and Multiple Coherence Functions," AFFDL-TR-65-56, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, June 1965.

22. Enochson, L. D., "Frequency Response Functions and Coherence Functions for Multiple Input Linear Systems," National Aeronautics and Space Administration CR-32, April 1964.

23. Mayer, J. P., and Hamer, H. A., "The Frequency Content of the Control Input and Airplane Response Obtained During Service Operations of Fighter Airplanes," National Advisory Committee for Aeronautics Report No. RM L57E21, Washington, D. C., July 12, 1957.

24. Cornell Aeronautical Laboratory, Inc., "Analysis of Probability Distributions and Power Spectra of Time Histories of Maneuvering Flight," Report No. AC-1428-F-1, December 1960.

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13. ABSTRACT

This report summarizes the important factors to be considered in flight loads investigations. Flight loads are defined as the forces and moments to which an airplane structure is subjected while the vehicle is airborne and are treated here as random phenomena. Evaluation of flight loads then consists of estimating the statistical properties of the underlying processes from sample measurements. Methods which are appropriate for this purpose are discussed in the report along with the associated estimation errors. Current methods for computing loads from flight data are reviewed and analyzed. Then the utilization of the computed loads in structural design applications is described. The data requirements for both design criteria and design substantiation are discussed. Operational flight loads data are covered under the latter heading.

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