COUPLED MODES RESOLUTION BY AN EXPONENTIAL WINDOW

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ABSTRACT

This paper deals with the identification of coupled modes; the viscous damping ratio can be estimated with good precision by the complex signals generated via the Hilbert transform.

The proposed technique consists in windowing the time domain data with an increasing exponential window in order to decouple the modes by decreasing their half power bandwidths.

Natural frequencies and damping ratios have been estimated in the time domain because of the possible lack of frequency resolution or truncation.

In noise free data the method works quite well, whereas some troubles arise when noisy data is considered; for low noise levels, acceptable estimations can be obtained using an exponential window truncated before the time when the noise level becomes comparable or greater than the signal.

The limits and validity of the proposed approach have been explored considering different amplitudes and different damping ratios of two coupled modes.

1. INTRODUCTION

The identification and resolution of coupled modes is an important topic in the general field of modal analysis and it is a basic point in the aerospace structural design.

The viscous damping ratio estimation, for one or several modes, could be carried out with good precision from the complex signals generated by the Hilbert transform. The Hilbert transform technique permits, starting from the impulse response of a system, to get a complex signal, the real part of which is the original function and the imaginary one is its Hilbert transform.

Filtering one mode at a time, the modulus of the complex signal represents the envelope that exponentially decays, while its argument is the instantaneous phase. The slope of the envelope, in the semi-log plane, is the decay rate, whereas the damped angular frequency is derived from the slope of the instantaneous phase.
It is straightforward to get the natural angular frequency and the viscous damping ratio from the above mentioned estimated parameters.

The method, as said before, only works on a single degree of freedom system, and therefore, when two modes (or more) are excited as considered in this paper, it is necessary to filter the one of interest. This signal processing leads to a distorted envelope because of both the filtering itself [1] and the tail of the rejected mode.

Sometimes, for very tightly coupled modes, it is quite impossible to filter the mode of interest; actually two coupled modes may also appear as a single mode.

In these cases the data in the time domain can be multiplied by an increasing exponential window; the decreasing of the bandwidth of the two coupled modes allows one to filter each mode and then to evaluate with acceptable errors the natural angular frequency and the viscous damping ratio. Nevertheless this technique, as pointed out by Dossing [2], yields unacceptable errors if the parameters estimation is carried out in the frequency domain with the half power method. In fact the new signal could result highly truncated at the end of the observation window and then the half power method leads to an overestimation of the damping ratio [3,4] ; on the other hand, the half power bandwidth could become so narrow that it results filled by an insufficient number of spectral lines. In addition the random noise, always present in the experimental time data, is amplified by the exponential window, that, for this reason, must be limited at a time when the impulse response level is still higher than the noise [5].

The parameters evaluation, using the Hilbert transform technique, is carried out from the instantaneous envelope and phase by linear least square regressions, which average out the residual random noise.

2. THEORETICAL BASIS

The impulse response of a system, with N modes excited, can be written as follows:

$$h(t) = \left[\sum_{k=1}^{N} |r_k| \exp(-\sigma_k t) \, \sin(\omega_{dk} + \alpha_k)\right] u(t) \tag{1}$$

where $|r_k|$ is the residue magnitude of the k-th mode, α_k its phase, σ_k the decay rate and ω_{dk} the damped angular frequency (u(t) states the causality of the function).

Hereafter only two real modes will be considered, then h(t) reduces to:

$$h(t) = [|r_1| \exp(-\sigma_1 t)\sin(\omega_{d1}t) + |r_2| \exp(-\sigma_2 t)\sin(\omega_{d2}t)]u(t) \quad (2)$$

If the modes are close together, calling:

$$\Delta\omega_d = (\omega_{d2} - \omega_{d1}) \quad (3)$$

with $\omega_{d2} > \omega_{d1}$, the damped angular frequencies are:

$$\begin{cases} \omega_{d1} = \bar{\omega}_d - \dfrac{\Delta\omega_d}{2} \\ \omega_{d2} = \bar{\omega}_d + \dfrac{\Delta\omega_d}{2} \end{cases} \quad (4)$$

where $\bar{\omega}_d$ is their average value:

$$\bar{\omega}_d = (\omega_{d1} + \omega_{d2})/2 \quad (5)$$

The impulse response (2) can be also expressed with the product:

$$h(t) = \{M(t)\sin[\bar{\omega}_d t + \varphi(t)]\}u(t) \quad (6)$$

wherein the modulation function is equal to:

$$M(t) = \{|r_1|^2 \exp(-2\sigma_1 t) + |r_2|^2 \exp(-2\sigma_2 t) + 2|r_1||r_2| \exp[-(\sigma_1+\sigma_2)t]\cos(\Delta\omega_d t)\}^{1/2} \quad (7)$$

and the phase is:

$$\varphi(t) = \operatorname{arctg}\left\{\frac{|r_2|\exp(-\sigma_2 t) - |r_1|\exp(-\sigma_1 t)}{|r_2|\exp(-\sigma_2 t) + |r_1|\exp(-\sigma_1 t)} \operatorname{tg}\left(\frac{\Delta\omega_d}{2}t\right)\right\} \quad (8)$$

Let us consider the Bedrosian theorem [6,7] valid, at least in limit sense [8], thus the Hilbert transform of the impulse response is given by the following relationships:

$$\check{h}(t) \cong -\{|r_1| \exp(-\sigma_1 t)\cos(\omega_{d1}t) + |r_2|\exp(-\sigma_2 t)\cos(\omega_{d2}t)\} \quad (9)$$

or

$$\check{h}(t) \cong -M(t)\cos\left[\bar{\omega}_d t + \varphi(t)\right] \quad (10)$$

The complex signal z(t) formed by the original impulse response and its Hilbert transform:

$$z(t) = h(t) + j\check{h}(t) \quad (11)$$

permits to obtain the average value of the damped angular frequency from the slope of the total phase:

$$\mathrm{arctg}\left[\frac{\check{h}(t)}{h(t)}\right] = \bar{\omega}_d t + \varphi(t) - \frac{\pi}{2} \tag{12}$$

in fact, the values assumed by $\varphi(t)$ are in general much smaller than those ones of the linear term; while from the envelope:

$$|z(t)| = \{h^2(t) + \check{h}^2(t)\}^{1/2} \tag{13}$$

it is possible to get a global decay rate and after dividing by $\bar{\omega}_d$, a damping ratio ζ_c from which the window parameter can be derived:

$$\sigma_w \cong \frac{1}{2}\zeta_c\bar{\omega}_d \tag{14}$$

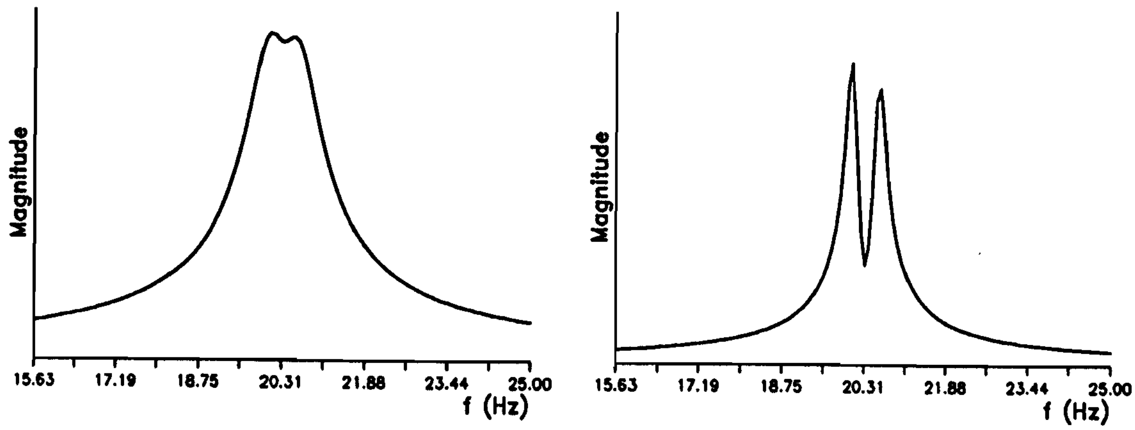
When the increasing exponential window:

$$w(t) = \left[\exp(\sigma_w t)\right] u(t) \tag{15}$$

is multiplied by the impulse response:

$$h_w(t) = h(t)\ w(t) \tag{16}$$

the half power bandwidths of the two coupled modes reduce to $[2(\sigma_1 - \sigma_w)]$ and $[2(\sigma_2 - \sigma_w)]$ respectively. After this reduction it is possible to evaluate the damping ratios either by the half power method or by the Hilbert transform approach (Figure 1).



(a) (b)

Fig.1 - Magnitudes of Frequency Response Functions derived from unwindowed (a) and windowed time histories (b).

Some troubles can arise with the first method, in fact the bandwidth of the mode of interest could be so narrow that an insufficient number of spectral lines is contained within it, besides for small decay rates the windowed impulse response can be truncated at the end of the observation time and that leads to overestimated damping ratios [3,4].

On the contrary, evaluations, carried out in the time domain through the complex signals obtained by the Hilbert transform, seem not to feel the effects of either the truncation[4] or the poor frequency resolution [5].

Actually this method must be applied to a mode at a time and thus the mode of interest has to be filtered in order to obtain its decay rate from the slope of the envelope (in the semi-log plane) and its damped angular frequency from the slope of the total phase [4,8].

The scheme of the technique used in the numerical tests and based on the Hilbert transform is presented in Figure 2.

Some noise is always present in an impulse response derived from experimental tests, therefore when the exponential window is applied,whereas $h_w(t)$ decreases, the additive noise, supposed as incorrelated with the impulse response and with zero mean value,results amplified [2]:

$$h_w^n(t) = h_w(t) + n(t)\ w(t) \tag{17}$$

For this reason the time duration of the increasing exponential window must be limited at the time t_0, when the level of the impulse response is higher than the noise [5]:

$$h_w^n(t;t_0) = h_w^n(t)\,\mathrm{rect}\,[(t-t_0/2)/t_0] \tag{18}$$

Attention must be paid to the length of the observation time, in fact if t_0 is too short the main lobe of the Dirichlet kernel becomes so wide that the coalescence of the two peaks occurs in the frequency domain [9] and neither filtering nor parameters evaluation can be carried out.

The possibility to separate two coupled modes was investigated by Pendered [10], who calculated the critical value of the dimensionless quantity:

$$\tau = \frac{\varepsilon}{2\zeta_1} \quad \text{or} \quad \tau = \frac{\varepsilon}{2\zeta_2} \tag{19}$$

where:

$$\varepsilon = 1 - \left(\frac{\omega_{n2}}{\omega_{n1}}\right)^2 \tag{20}$$

and $\omega_{n2} > \omega_{n1}$. For $\tau < \tau_{cr}$ it is not possible to separate the modes and only one natural frequency can be found. If equal damping ratios are considered, the common critical value can be achieved from the following relationship:

$$\zeta_{cr} = \frac{\Delta\omega_n}{\tau_{cr}\,\omega_{n2}\left[1+\frac{\Delta\omega_n}{2\bar{\omega}_n}\right]} \cong \frac{\Delta\omega_n}{\tau_{cr}\,\omega_{n2}} \tag{21}$$

where $\Delta\omega_n$ is the difference between the two natural frequencies, while $\bar{\omega}_n$ is their mean value (ω_{n2} appears because the mode with the widest half power bandwidth has been considered).

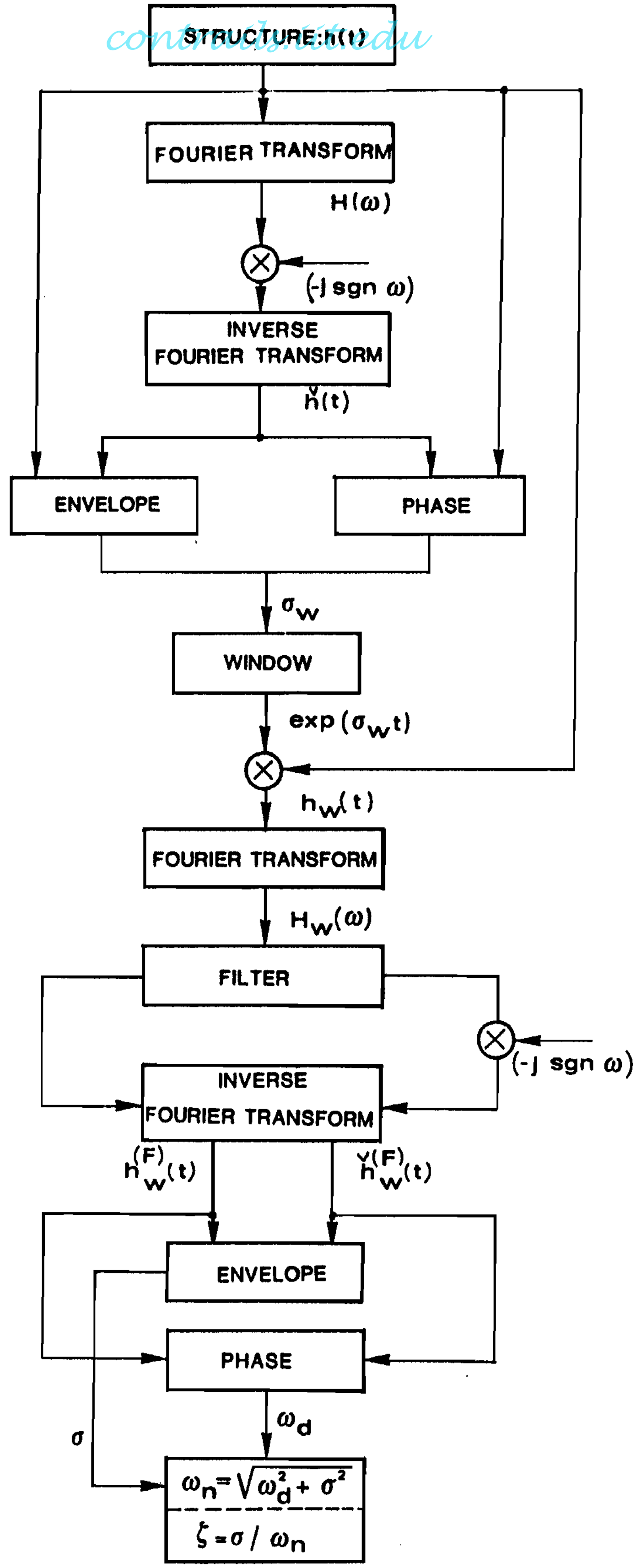


Fig.2 - Procedure for the window parameter estimation, for the natural frequency and damping ratio evalution of two coupled modes.

As shown by Pendered, τ_{cr} depends on the method used to analyse the frequency response function, in particular the maximum frequency spacing technique, introduced by Kennedy and Pancu [11], is the most sensible, i.e. it has the lowest τ_{cr} and then it permits to single out the presence of two modes when the other methods fail.

3. NUMERICAL TESTS AND COMPARISONS

Numerical simulations have been carried out using the Hilbert approach, that seems to give more reliable results [4,8].

As it is necessary to filter the spectrum of the impulse response, the discrete Fourier transform has been used in order to get the discrete Hilbert transform; for this purpose a null buffer, the length of which was equal to the data block, has been added at the end of the time samples so that the circularity of convolution is avoided.

In Table 1 estimations derived from two coupled modes, with one of the residue decreasing are shown.

$|r_2| = 10$; $f_1 = 20$ (Hz); $f_2 = 20.5$ (Hz); $\zeta_1 = \zeta_2 = 0.005$

$\lvert r_1\rvert$	$\hat{f}_1$	$\lvert\varepsilon_{f1}\rvert$ (%)	$\hat{\zeta}_1$	$\lvert\varepsilon_{\zeta 1}\rvert$ (%)	$\hat{f}_2$	$\lvert\varepsilon_{f2}\rvert$ (%)	$\hat{\zeta}_2$	$\lvert\varepsilon_{\zeta 2}\rvert$ (%)
10	20.001599	0.0109	0.004603	7.9317	20.497197	0.0137	0.004912	1.7617
7	19.993841	0.0308	0.004475	10.4954	20.494073	0.0289	0.005021	0.4220
5	19.998928	0.0054	0.004724	5.5280	20.503985	0.0194	0.004763	4.7323
3	19.977730	0.1113	0.004402	11.9654	20.500705	0.0034	0.004674	6.5168
1	20.019244	0.0962	0.005918	18.3534	20.492994	0.0342	0.004669	6.6256

Table 1 - Coupled modes with the same damping ratio ($\zeta_1 = \zeta_2 = 0.005$) and one variable residue

The errors in the damping ratio evaluation increase as the ratio $|r_1| / |r_2|$ decreases, in fact the two half power bandwidths remain constant and then the interaction between the greater mode and the smaller one becomes more significant.

The same cases are presented in Table 2, where natural frequencies and damping ratios are evaluated applying the method of the increasing exponential window; the errors are always negligible and however much lesser than those ones obtained from a direct estimation.

$\lvert r_2\rvert = 10;\ f_1 = 20$ (Hz); $f_2 = 20.5$ (Hz); $\zeta_1 = \zeta_2 = 0.005$								
$\lvert r_1\rvert$	$\hat{f}_1$	$\lvert\varepsilon_{f1}\rvert$ (%)	$\hat{\zeta}_1$	$\lvert\varepsilon_{\zeta 1}\rvert$ (%)	$\hat{f}_2$	$\lvert\varepsilon_{f2}\rvert$ (%)	$\hat{\zeta}_2$	$\lvert\varepsilon_{\zeta 2}\rvert$ (%)
10	19.997749	0.0113	0.005023	0.4685	20.502273	0.0111	0.004990	0.1957
7	19.997583	0.0121	0.005025	0.5077	20.501585	0.0077	0.005017	0.3300
5	19.996528	0.0174	0.004981	0.3769	20.501337	0.0065	0.005003	0.0526
3	19.995084	0.0246	0.005118	2.3586	20.500651	0.0032	0.005014	0.2765
1	19.986315	0.0684	0.005003	0.0589	20.500233	0.0011	0.004993	0.1469

Table 2 - Coupled modes with variable amplitudes (increasing exponential window)

In Tables 3 and 4 similar numerical simulations are shown; due to the higher damping ratios, worse evaluations than those in Table 1 have been obtained, nevertheless also in this case the use of the proposed technique permits to obtain good results.

$\lvert r_2\rvert = 10;\ f_1 = 20$ (Hz); $f_2 = 20.5$ (Hz); $\zeta_1 = \zeta_2 = 0.01$								
$\lvert r_1\rvert$	$\hat{f}_1$	$\lvert\varepsilon_{f1}\rvert$ (%)	$\hat{\zeta}_1$	$\lvert\varepsilon_{\zeta 1}\rvert$ (%)	$\hat{f}_2$	$\lvert\varepsilon_{f2}\rvert$ (%)	$\hat{\zeta}_2$	$\lvert\varepsilon_{\zeta 2}\rvert$ (%)
10	19.973086	0.1346	0.008041	19.5950	20.536197	0.1766	0.007115	28.8508
7	20.016101	0.0805	0.007161	28.3903	20.455063	0.2192	0.008976	10.2372
5	20.041772	0.2089	0.009051	9.4935	20.477664	0.1090	0.007062	29.3810
3	19.987569	0.0622	0.006886	31.1412	20.544468	0.2169	0.007089	29.1125

Table 3 - Coupled modes with one variable amplitude and higher damping ratios

$\|r_2\| = 10$; $f_1 = 20$ (Hz); $f_2 = 20.5$ (Hz); $\zeta_1 = \zeta_2 = 0.01$								
$\|r_1\|$	$\hat{f}_1$	$\|\varepsilon_{f1}\|$ (%)	$\hat{\zeta}_1$	$\|\varepsilon_{\zeta1}\|$ (%)	$\hat{f}_2$	$\|\varepsilon_{f2}\|$ (%)	$\hat{\zeta}_2$	$\|\varepsilon_{\zeta2}\|$ (%)
10	19.998091	0.0096	0.010012	0.1223	20.501640	0.0080	0.010009	0.0892
7	19.997747	0.0113	0.009988	0.1167	20.500241	0.0012	0.010017	0.1704
5	19.997330	0.0133	0.010055	0.5468	20.500887	0.0043	0.009986	0.1391
3	19.998745	0.0063	0.010091	0.9120	20.500416	0.0020	0.010003	0.0341

Table 4 - Increasing exponential windows applied to coupled modes with one variable amplitude

When modes with variable coupling due to different damping ratios are considered (Table 5), better evaluations have been achieved, expecially for the mode with the smaller damping ratio, because it gets predominant while its half power bandwidth narrows. For these reasons the tail of the rejected mode, present in the bandpass filter used before the processing that permits to get the envelope and phase, is smaller and smaller and therefore an acceptable estimation of ζ_2 and at the same time an improvement of ζ_1 have been obtained for the last case.

$\|r_1\| = \|r_2\| = 10$; $f_1 = 20$ (Hz); $f_2 = 20.5$ (Hz); $\zeta_1 = 0.01$								
ζ_2	$\hat{f}_1$	$\|\varepsilon_{f1}\|$ (%)	$\hat{\zeta}_1$	$\|\varepsilon_{\zeta1}\|$ (%)	$\hat{f}_2$	$\|\varepsilon_{f2}\|$ (%)	$\hat{\zeta}_2$	$\|\varepsilon_{\zeta2}\|$ (%)
0.01	19.973075	0.1346	0.007971	20.2923	20.538392	0.1873	0.007670	23.2974
0.007	19.950628	0.2469	0.007976	20.2324	20.531764	0.1588	0.005662	19.1099
0.003	19.962001	0.1900	0.008379	16.2111	20.501625	0.0079	0.029280	2.3914

Table 5 - Coupled modes with variable coupling

The use of an exponential window leads to very good results (Table 6) if the damping ratios of the two modes are similar, whereas if the damping ratios are different enough as in the case $\zeta_1 = 0.01$ and $\zeta_2 = 0.003$ it is convenient to apply a first window, the parameter of which is determined from the initial part of the envelope where the most damped mode is still present, in order to estimate ζ_1 and a second window, derived from the second part of the envelope, for ζ_2. Actually for the second mode it could be possible to evalutate the damping ratio directly, without any window, from the envelope of the two coupled modes when the first of them is damped out.

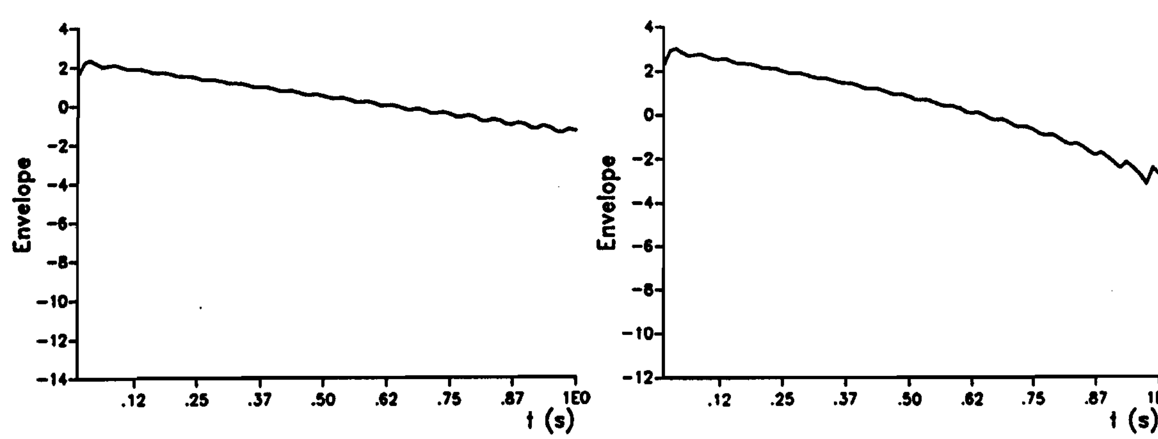
$\lvert r_1\rvert = \lvert r_2\rvert = 10$; $f_1 = 20$ (Hz); $f_2 = 20.5$ (Hz); $\zeta_1 = 0.01$								
ζ_2	$\hat{f}_1$	$\lvert\varepsilon_{f1}\rvert$ (%)	$\hat{\zeta}_1$	$\lvert\varepsilon_{\zeta 1}\rvert$ (%)	$\hat{f}_2$	$\lvert\varepsilon_{f2}\rvert$ (%)	$\hat{\zeta}_2$	$\lvert\varepsilon_{\zeta 2}\rvert$ (%)
0.01	19.998411	0.0079	0.009992	0.0815	20.501839	0.0090	0.009954	0.4574
0.007	19.999648	0.0018	0.010003	0.0318	20.502988	0.0146	0.007006	0.0852
0.003	20.022103	0.1105	0.011088	10.8792	20.501602	0.0078	0.003004	0.1457

Table 6 - Estimation obtained with the proposed technique for the examples of Table 5

In Table 7 both the damping ratios are variable and the estimations have been only carried out by the exponential window approach. The results are always acceptable also in that case ($\zeta_1 = \zeta_2 = 0.03$) wherein it is not possible to discern the presence of two modes from the frequency response function.

This damping ratio is critical, for the theoretical study carried out by Pendered, for all the methods used in the frequency domain, except for the Kennedy-Pancu technique that permits, only with an accurate study of the frequency spacing, to single out that two close natural frequencies are present.

Actually the presence of two modes could be revealed from the envelopes [12], in fact if only one mode is present in the frequency response function its envelope in the semi-log plane is represented by a straight line, on the contrary for two modes the slope is not constant due to the modulation, see relationship (7), Figure 3.



(a) (b)

Fig. 3 - Envelopes of a single mode (a) and of two tightly coupled modes (b)

$\zeta_1=\zeta_2$	$\hat{f}_1$	$\lvert\varepsilon_{f1}\rvert$ (%)	$\hat{\zeta}_1$	$\lvert\varepsilon_{\zeta 1}\rvert$ (%)	$\hat{f}_2$	$\lvert\varepsilon_{f2}\rvert$ (%)	$\hat{\zeta}_2$	$\lvert\varepsilon_{\zeta 2}\rvert$ (%)
$\lvert r_1\rvert = \lvert r_2\rvert = 10$; $f_1 = 20$ (Hz); $f_2 = 20.5$ (Hz)								
0.015	19.999378	0.0031	0.014857	0.9565	20.499313	0.0034	0.015079	0.5293
0.02	20.004927	0.0246	0.020207	1.0343	20.499645	0.0017	0.020230	1.1476
0.025	20.016885	0.0844	0.023802	4.7916	20.468004	0.1561	0.024475	2.0996
0.03	19.977632	0.1118	0.026540	11.5340	20.507453	0.0364	0.026589	11.3712

Table 7 - Two modes with different damping ratios

The window parameter, evaluated with the abovementioned technique, resulted too small and therefore the modes remained rather coupled and errors greater than 10% , in the last case, are due to the high influence of the tails within the filter. An interactive procedure permits to get much better results, in fact using σ_w = 3.2 (rad/s) the following results have been obtained: $\hat{\zeta}_1 = 0.03008$ and $\hat{\zeta}_2 = 0.02966$.

An evalutation from modes with low frequencies and damping ratios is presented in Table 8. It is interesting to note that, even if the theoretical critical value of the damping ratio is far from the ones considered, the half power bandwidths of the two modes equals $(2\sigma) \cong 0.0025$ (rad/s), whereas the angular frequency spacing is $\delta\omega = 0.0049$ (rad/s) and therefore it is impossible to use the half power method.

	$\hat{\zeta}_1$	$\lvert\varepsilon_{\zeta 1}\rvert$ (%)	$\hat{\zeta}_2$	$\lvert\varepsilon_{\zeta 2}\rvert$ (%)
$\lvert r_1\rvert = \lvert r_2\rvert = 10$; $f_1 = 0.2$ (Hz); $f_2 = 0.201$ (Hz); $\zeta_1 = \zeta_2 = 0.001$				
raw data	0.000849	15.1337	0.000873	12.6637
windowed data	0.000991	0.8626	0.001043	4.2762

Table 8 - Coupled modes with low frequencies and damping ratios

In order to simulate the noise present in an actual acquisition, a random noise with zero mean value and standard deviation given in per cent of the residue magnitude has been added to the impulse response. If an increasing exponential window is applied, the impulse responses still result exponentially damped, whereas the noise is amplified and therefore if the modes are damped out before the end of the observation time and all the data block is employed a useless frequency response function is obtained. For this reason the time window must be limited at an instant when the noise level is lower than the function. In Table 9 examples with different σ_N of the added noise are presented, errors less than 10% have been obtained also in the worst condition. Actually the considered noise is relatively low, therefore in a high noise environment it is necessary to process the impulse response in order to decrease the noise level.

$|r_1| = |r_2| = 10$; $f_1 = 20$ (Hz); $f_2 = 20.5$ (Hz); $\zeta_1 = \zeta_2 = 0.01$

σ_N (%of|r|)	$\hat{f}_1$	$\|\varepsilon_{f1}\|$ (%)	$\hat{\zeta}_1$	$\|\varepsilon_{\zeta1}\|$ (%)	$\hat{f}_2$	$\|\varepsilon_{f2}\|$ (%)	$\hat{\zeta}_2$	$\|\varepsilon_{\zeta2}\|$ (%)
0.03	19.981120	0.0944	0.009719	2.8063	20.535809	0.1747	0.009445	5.5506
0.07	19.957235	0.2138	0.009537	4.6306	20.547187	0.2302	0.009425	5.7464
0.10	19.945065	0.2747	0.009410	5.8980	20.552418	0.2557	0.009341	6.5945

Table 9 - Estimations from noisy impulse responses

4. CONCLUDING REMARKS

A precise evaluation of natural frequencies and damping ratios of tightly coupled modes can be obtained by windowing the impulse response with an increasing exponential function.

The numerical tests have been carried out using the complex signals generated via the Hilbert trasform because estimations in the frequency domain could have been affected by poor frequency resolution and truncation effects.

The method suggested in this paper has leaded, in all the considered cases, to good results and for many examples the errors in the damping ratio estimations have been reduced to immaterial values.

Troubles can arise when very different damping ratios are considered, in fact in this case it is convenient to use a first window, in order to obtain an acceptable damping ratio estimation for the most damped mode and a second one for the other mode.

Also in presence of an additive random noise the method gave acceptable results; due to the amplification of the noise by the increasing exponential window, it is necessary to limit its duration up to the time when the noise level is less than the signal. This implies a preprocessing of the time data in order to increase the signal to noise ratio when high noise measurements are encountered.

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6. AKNOWLEDGEMENTS

This paper was sponsored both by the M.P.I. research project "L'uso della trasformata di Hilbert nella stima dello smorzamento" and by the C.N.R. grant 86.02037.11 "Identificazione caratteristiche dinamiche strutture con analisi modale".