AN IMPLICIT FOURIER TRANSFORM METHOD FOR NONLINEAR DYNAMIC ANALYSIS WITH FREQUENCY DEPENDENT DAMPING

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ABSTRACT

The SILFD (Step-by-step Incremental Linearization Frequency Domain) method for the frequency domain analysis of nonlinear structural systems with frequency dependent damping, described in Venancio-Filho and Claret [1989] is implemented in this work through the IFT (Implicit Fourier Transform) algorithm, Venancio-Filho and Claret [1991]. A new and more efficient process for the consideration of the initial conditions in the SILFD method is presented. Numerical examples are presented which show the applicability of the proposed method.

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INTRODUCTION

A very efficient and accurate method for the treatment of structural dynamics engineering problems with frequency dependent damping is based in the frequency domain solution of the motion equations. Physical and geommetrical nonlinearities, when present, should be considered in these problems. Only recently methods of nonlinear dynamic structural analysis in the frequency domain have been adressed. Several researchers have presented contributions in this subject. Kawamoto [1983] described a method called Hybrid Frequency-Time Domain, abreviated HFTD, for nonlinear analysis in frequency domain. Wolf and Darbre [1986] presented the segmenting approach of HFTD method and obtained its convergence properties. Hilmer and Schmid [1988] describe a technique similar to the segmenting approach using Laplace Transform which computationally differs from Fourier Transform only in the treatment of initial conditions.

All these methods present some problems related to its applicability to real situations in structural engineering. Two problems are adressed in this work. The first refers to the computational effort in nonlinear analysis in the frequency domain where the conventional process needs numerous executions of direct and inverse Fourier transforms of complex series with a great number of terms. Consequently, the memory allocation and the computational effort is normally very high. The second problem is the treatment of initial conditions by a segmenting approach. Hilmer and Schmid [1988] state that the treatment of non null initial conditions through Fourier Transforms is numerically unfavorable because, in general, step functions cause great errors in transformed functions.
The SILFD method, described by Venancio-Filho and Claret [1989], combined with the Implicit Fourier Transform Algorithm for dynamic response in frequency domain, Venancio-Filho and Claret [1991], solves efficiently the first problem. The second problem is treated here using the physical significance of initial conditions and transforming the original problem in another with null initial conditions.

THE IMPLICIT FOURIER TRANSFORM ALGORITHM

The dynamic response of a SDOF system in the frequency domain can be expressed by the following equations, Clough and Penzien [1982]:

$$v(t_n) = \frac{\Delta\bar{\omega}}{2\pi} \sum_{m=0}^{N-1} H(\bar{\omega}_m)\, P(\bar{\omega}_m)\, e^{i2\pi\frac{mn}{N}} \tag{1}$$

and

$$P(\bar{\omega}_m) = \Delta t \sum_{n=0}^{N-1} p(t_n)\, e^{-i2\pi\frac{mn}{N}}. \tag{2}$$

The total time interval T_p in which the response is to be calculated is divided into N equal time intervals given by

$$\Delta t = \frac{T_p}{N} \tag{3}$$

and the discrete times in which the load is defined are given by

$$t_n = n\,\Delta t = n\,\frac{T_p}{N} \qquad (0 \le n \le N-1). \tag{4}$$

The frequency range is likewise divided into N equal intervals $\Delta\bar{\omega}$ expressed as

$$\Delta\bar{\omega} = \frac{2\pi}{T_p} \tag{5}$$

and the discrete frequencies $\bar{\omega}_m$ are taken according Table I (see Appendix 1).

In equation (2), $P(\bar{\omega}_m)$ is the discrete Fourier transform of the load; in equation (1), $H(\bar{\omega}_m)\,P(\bar{\omega}_m)$ is the discrete Fourier transform of the response (or the response in the frequency domain) and $v(t_n)$ is the inverse discrete Fourier transform of the response (or the response in the time domain).

The dynamic response expressed by equations (1) and (2) can be numerically determined by the Fast Fourier Transform algorithm.

Let now

$$\{p\} = \{ p(t_0), p(t_1), p(t_2), \ldots, p(t_n), \ldots, p(t_{N-1}) \} \tag{6}$$

and

$$\{v\} = \{ v(t_0), v(t_1), v(t_2), \ldots, v(t_n), \ldots, v(t_{N-1}) \} \tag{7}$$

be, respectively, the vectors of the load and the response at the discrete times

$$t_n = n\Delta t, \; n = 0,1,2,\ldots,N-1, \tag{8}$$

and let

$$\{P\} = \{ P(\bar{\omega}_0), P(\bar{\omega}_1), P(\bar{\omega}_2), \ldots, P(\bar{\omega}_m), \ldots, P(\bar{\omega}_{N-1}) \} \tag{9}$$

be the vector of the discrete Fourier transform of the load defined at the discrete frequencies $\bar{\omega}_m$ interpreted according to Table I.

With the definition of equations (6) and (9), equation (2) can be casted in matrix form as

$$\{P\} = \Delta t \, [E^*] \, \{p\} \tag{10}$$

where the (NxN) matrix $[E^*]$ is defined as the matrix whose generic term E_{mn} is

$$E^*_{mn} = e^{-imn\alpha} \tag{11}$$

or, explicitly,

$$\underset{\sim}{E}^{*} = \begin{bmatrix} e^{0} & e^{0} & e^{0} & \dots e^{0} & \dots e^{0} \\ & e^{-i\alpha} & e^{-i2\alpha} & \dots e^{-in\alpha} & \dots e^{-i(N-1)\alpha} \\ & & e^{-i4\alpha} & \dots e^{-i2n\alpha} & \dots e^{-i2(N-1)\alpha} \\ & & & \ddots & \\ & & & e^{-imn\alpha} & \dots e^{-im(N-1)\alpha} \\ & \text{Symmetric} & & & \vdots \\ & & & & e^{-i(N-1)\alpha} \end{bmatrix} \qquad (12)$$

where $\alpha = (2\pi/N)$. By the same token, the response from equation (2) is written in matrix form as

$$\{v\} = \frac{\Delta\bar{\omega}}{2\pi} [E] [H] \{p\} \qquad (13)$$

where [E] is the matrix defined in equation (11) with positive signs in the exponentials instead of negative ones, and [H] is the diagonal matrix formed with the complex frequency response functions calculated at the discrete frequencies of Table I. The typical term of [H] is given by

$$H(\bar{\omega}_m) = (k - m\bar{\omega}_m^2 + i\bar{\omega}_m c)^{-1}, \quad (0 \le m \le m-1) \qquad (14)$$

where k, m, and c are the stiffness, mass, and damping of the SDOF system, respectively. Substituting now {P} from equation (10) into equation (13), the following equation is obtained:

$$\{v\} = \frac{1}{N} [E] [H] [E^{*}] \{p\} \qquad (15)$$

Equation (15) expresses the matrix formulation of the dynamic analysis of SDOF systems in the frequency domain. The calculation of the structural response in the frequency domain through this equation is the IFT algorithm.

THE SILFD METHOD

Consider the SDOF system of Fig. 1 submitted to an arbitrary excitation p(t). The spring stiffness k depends on the displacements v due to the system non-linearity and the damping coefficient depends on the frequency of the excitation, $\bar{\omega}$. The problem is then to integrate the dynamic equilibrium equation

$$m\ddot{v} + c(\omega)\dot{v} + k(v)v = p(t). \tag{16}$$

As the damping coefficient is ω dependent a frequency-domain analysis has to be performed and, as the stiffness depends on the displacement, a linearization technique must be employed. Consequently the present method is a Step-by-step Incremental Linearization in the Frequency Domain (SILFD) method. In each linearized step a secant stiffness is considered.

In order to calculate the response of the system governed by Eq. 1 two approximations are made. The first one is the approximation of the given load by piecewise linear segments. The total time interval in which the response is to be calculated is divided in intervals $\Delta t_j = t_j - t_{j-1}$; p_j and p_{j-1} are the values of p(t) in the times t_j and t_{j-1}, respectively, and $\Delta p_j = p_j - p_{j-1}$, Fig. 2a. The load variation in time interval Δt_j is given by, Fig. 2a,

$$p(\tau) = p_{j-1} + \frac{\Delta p_j}{\Delta t_j}\tau \tag{17}$$

where τ is the current time in Δt_j ($0 \le \tau \le \Delta t_j$). The second approximation refers to the spring force versus displacement curve. This curve is also approximated by piecewise linear segments as indicated in Fig. 2c. The levels of these two approximations depend on the accuracy with which the load and the stiffness variation can have a good representation.

The response of the system is calculated through the linearized steps along the time intervals Δt_j in which the spring is considered linear with stiffness k_j, Fig. 2b. the linearized dynamic equilibrium equation in time interval Δt_j

is

$$m\ddot{v} + c(\bar{\omega})\dot{v} + k_j v = p(\tau) \tag{18}$$

with the initial conditions v_{j-1} and $\dot{v}_{j-1}$, Fig. 2a. Herein the treatment of the initial conditions departs from Venancio-Filho and Claret [1989] in order to circumvent the errors in the transformed functions to the step functions.

The displacement response in time interval Δt_j due to the applied load obtained through the IFT algorithm is

$$\{v_j\}_L = \frac{\Delta\bar{\omega}}{2\pi} [E] [H] [E^*] \{p_j\}_L \tag{19}$$

where $\{p_j\}_L$ is the load vector in the time interval Δt_j.

The displacement response due to the initial displacement is equivalent to the response due to a constant force, in the time interval Δt_j, given by

$$\{p_j\}_I = -k_j v_{j-1}\{1\} \tag{20}$$

where {1} is a vector with all elements equal to 1. Consequently the response is obtained from Eq. 19 as

$$\{v_j\}_{v_{j-1}} = - \frac{\Delta\bar{\omega}}{2\pi} [E] [H] [E^*] k_j v_{j-1}\{1\}. \tag{21}$$

The displacement response due to the initial velocity $_{j-1}$ is the response to an impulse m $_{j-1}$ which is obtained from the unit impulse response function as

$$v_j = m\dot{v}_{j-1}h(t). \tag{22}$$

h(t) is the inverse Fourier Transform of the complex frequency response function $H(\bar{\omega})$ and is given by

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(\bar{\omega})\, e^{i\bar{\omega}t} d\bar{\omega}. \tag{23}$$

Considering Eqs. 23, 22, and 15, the response due to $_{j-1}$ is obtained as

$$\{v_j\}_{\dot{v}_{j-1}} = \frac{m\dot{v}_{j-1}\Delta\bar{\omega}}{2\pi} [E]\,[H]\,\{1\}. \tag{24}$$

The total response in time interval Δt_j is the given by the sum of the responses in Eqs. 19, 21, and 24. The result is

$$\{v_j\} = \frac{\Delta\bar{\omega}}{2\pi} [E][H]\Big[[E^*]\,(\{p_j\}_L - k_j v_{j-1}) - m\dot{v}_{j-1}\{1\}\Big]. \tag{25}$$

EXAMPLES

A SDOF system formed by a mass m = 1 kg and by a bilinear spring with constants K_1 = 10000 N/mm and K_2 = 10 N/mm was analysed by Kawamoto [1983] considering undamped vibrations. The same system is now analysed considering the following cases: I)- undamped system; II)- frequency-dependent damping acording to the function $c(\bar{\omega})$ shown in Fig. 3; III)- frequency-dependent damping acording to the function $c(\bar{\omega})$ shown in Fig. 4. The load function is

$$p(t) = 50 \sin(1.5t) + 100 \sin 0.005t \tag{26}$$

which is pictured in Fig. 5. The natural period of vibration is T = 0.063 sec. Kawamoto [1983] considered Δt = 1 sec to perform the analysis of system's response through the HFTD method, and Δt = 0.02 sec using direct integration of equilibrium equations.

In case I, using the SILFD method with the IFT, a time interval Δt = 25 sec is used, and the system's response is shown in Fig. 6. Comparing this response with Kawamoto's one (Kawamoto [1983], Figure 6.94, page 341), it is evident that the proposed method is efficient in predicting the maximum and minimum response of the system. Furthermore, the proposed method is better than HFTD in describing the "true" response of the system, particularly if we consider the accentuated spring softening.

The responses of Cases II and III, Figs. 7 and 8, respectively, show that frequency-dependent damping is treated conveniently by the proposed method. Others types of $c(\omega)$ functions can be considered with no changes in the algorithm. A very small diferences in the moduli of maxima displacements are observed from Case I to Cases II and III. One reason is predominant for this fact: the steady-state response is calculated and the static amplitude $p(t)/K_2$, for such a small value of K_2, is predominant in the system's response.

CONCLUSIONS

The proposed method is efficient for treatment of dynamic nonlinear systems with frequency-dependent damping. In a future work, the computational effort needed will be measured and compared with the cpu time of others methods. Howewer, it is very apparent that the SILFD method combined with the IFTalgorithm is well suited for nonlinear analysis in frequency domain, optmizing computational effort and memory allocation.

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APPENDIX

The discrete frequencies employed in this formulation must be interpreted according Table I. Taking into account the frequencies $\bar{\omega}_m$ from Table I, $H(\bar{\omega}_m)$ and $H(\bar{\omega}_{N-m})$, Eq. 14, are complex conjugate.

Table I. Discrete frequencies (N odd)

m	m or (N-m)	$\bar{\omega}_m$
0	0	0
1	1	$\Delta\bar{\omega}$
2	2	$2\Delta\bar{\omega}$
...	...	...
(N/2-1)	(N/2-1)	$(N/2-1)\Delta\bar{\omega}$
N/2	N/2	$(N/2)\Delta\bar{\omega}$
(N/2+1)	(N/2+1)	$[-(N/2+1)]\Delta\bar{\omega}$
...	...	...
N-2	2	$-2\Delta\bar{\omega}$
N-1	1	$-\Delta\bar{\omega}$

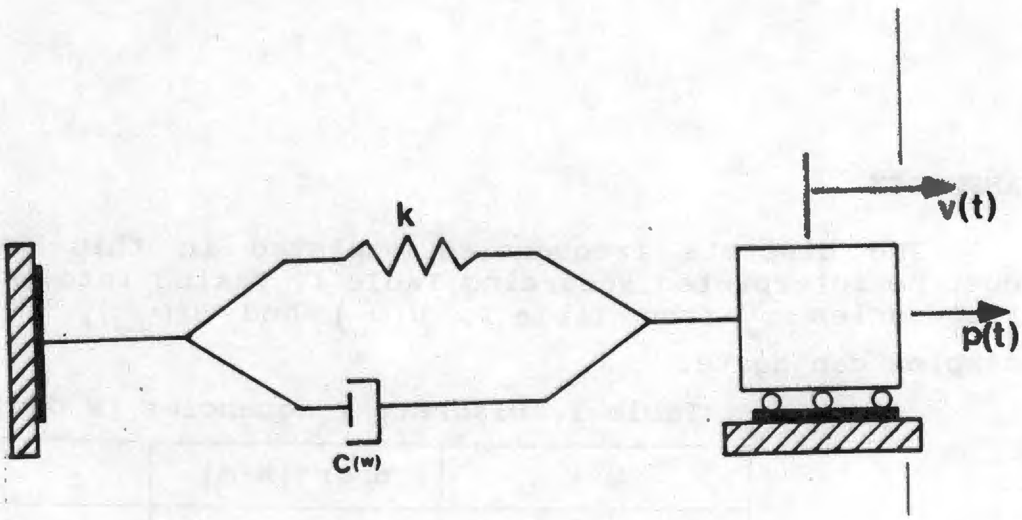

Fig. 1- SDOF system.

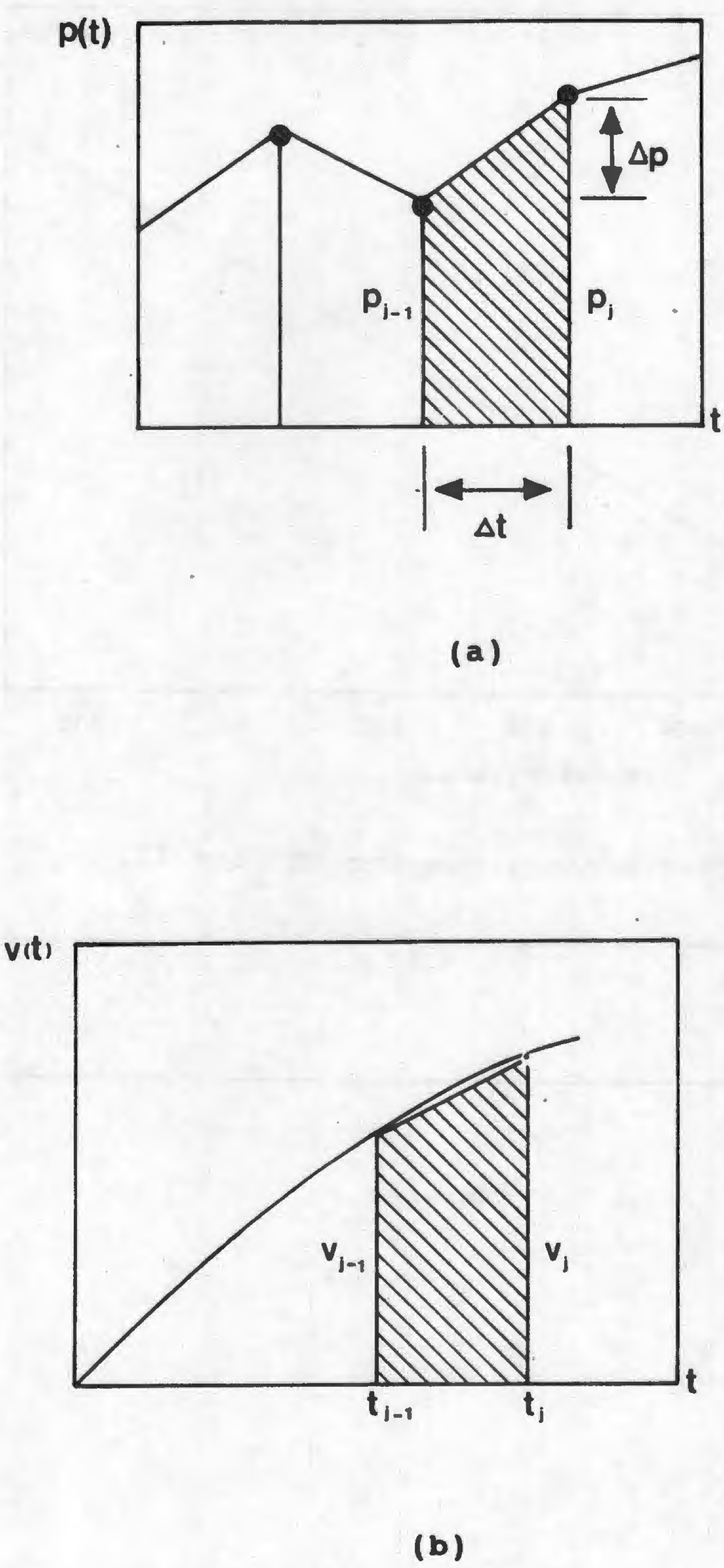

Fig. 2- (a) Load variation; (b)- displacement response.

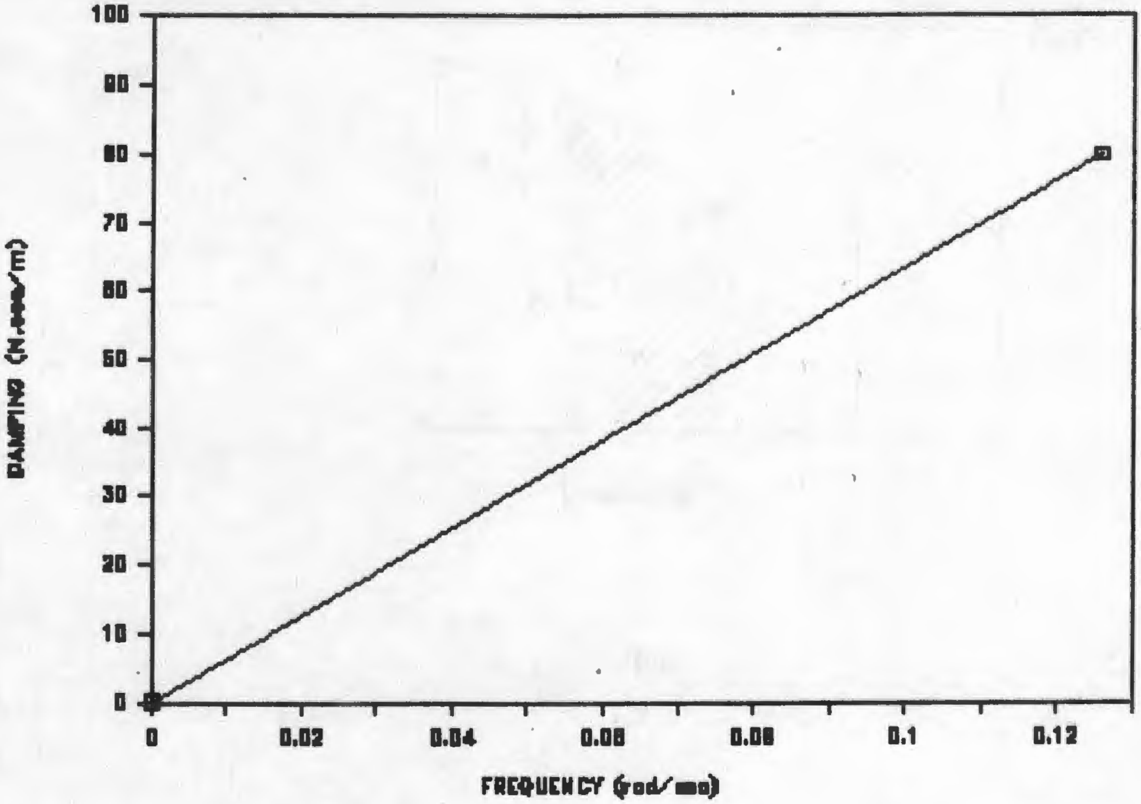

Fig. 3- Frequency-dependent damping in case II.

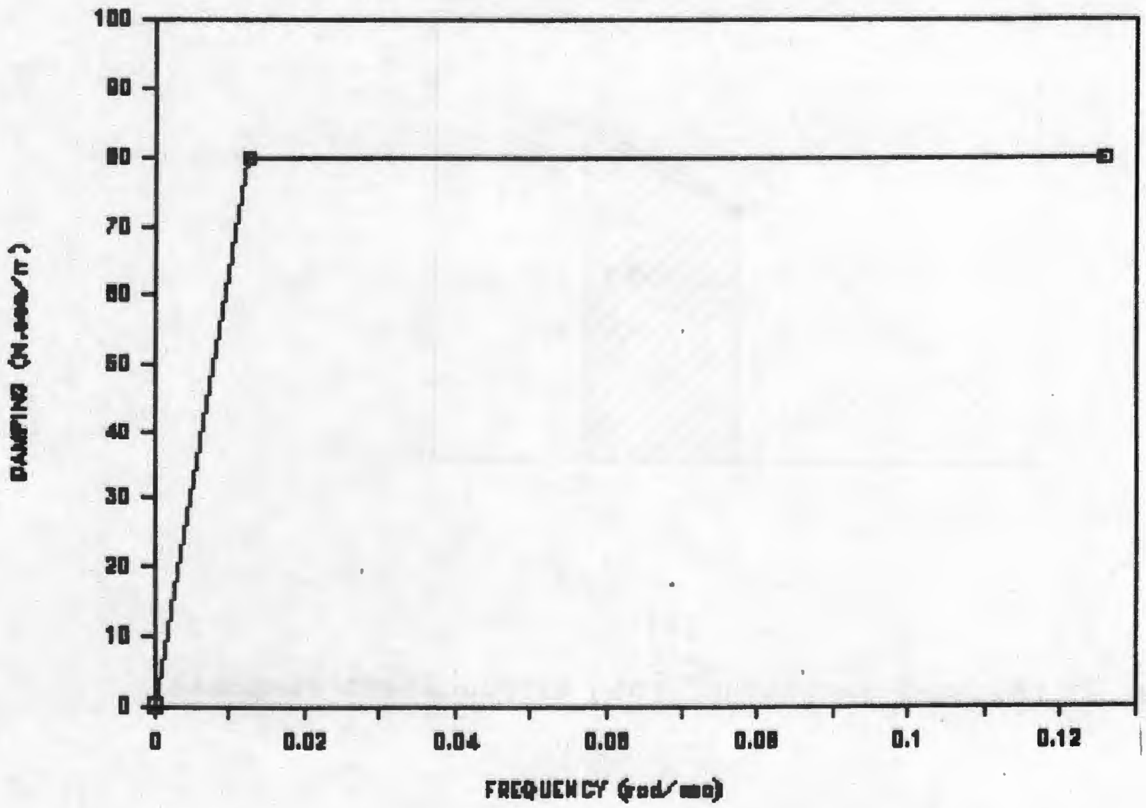

Fig. 4- Frequency-dependent damping in case III.

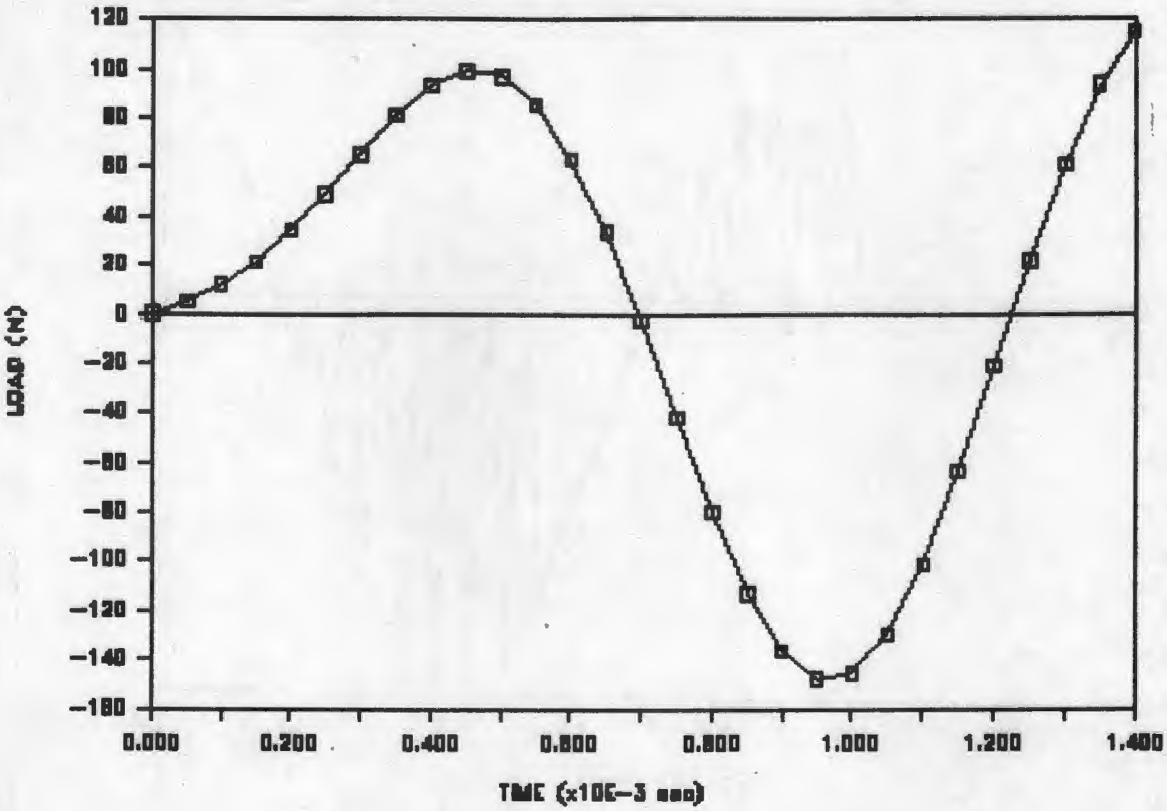

Fig. 5- Load function.

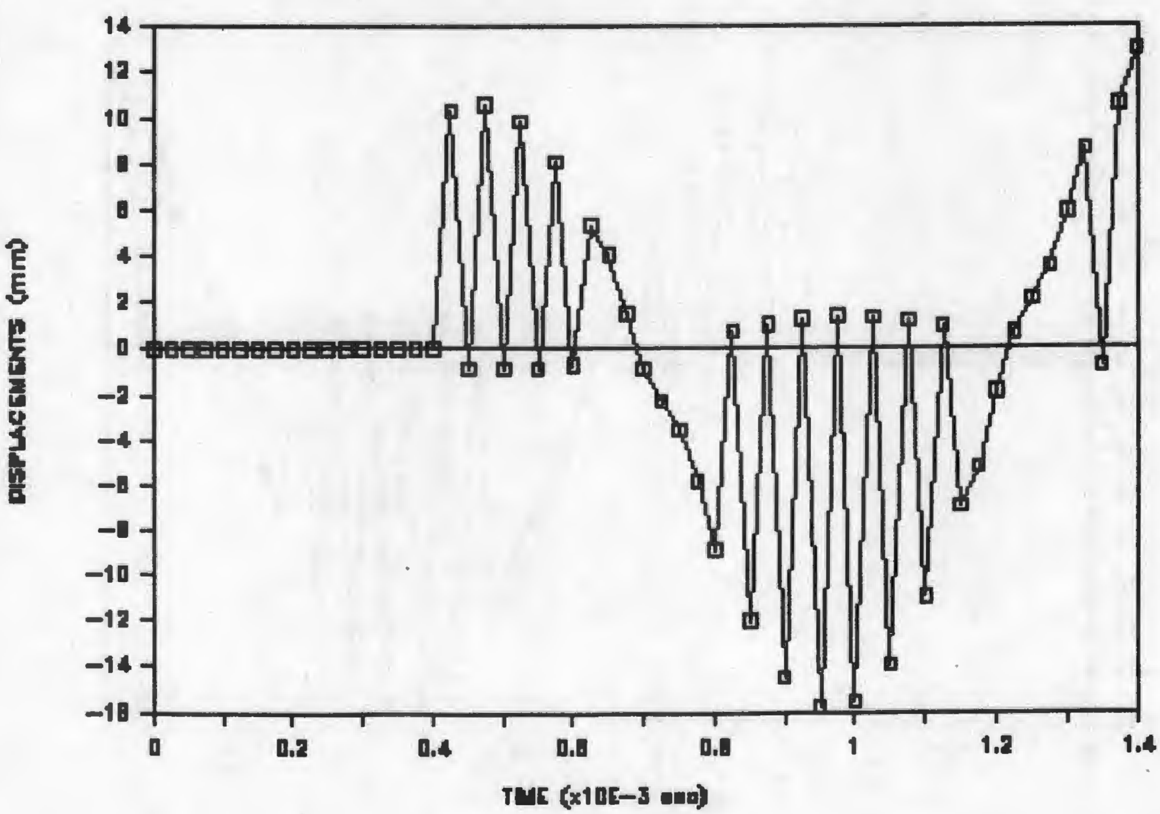

Fig. 6- System response in case I.

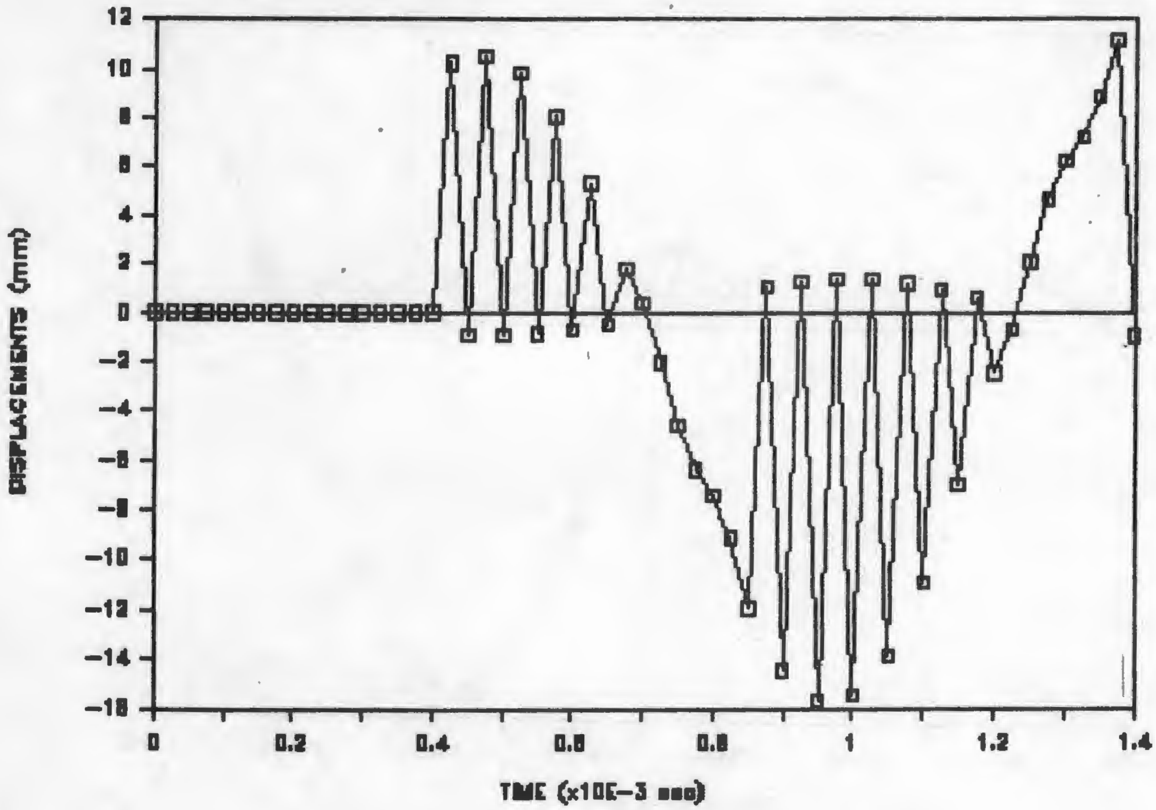

Fig. 7- System response in case II.

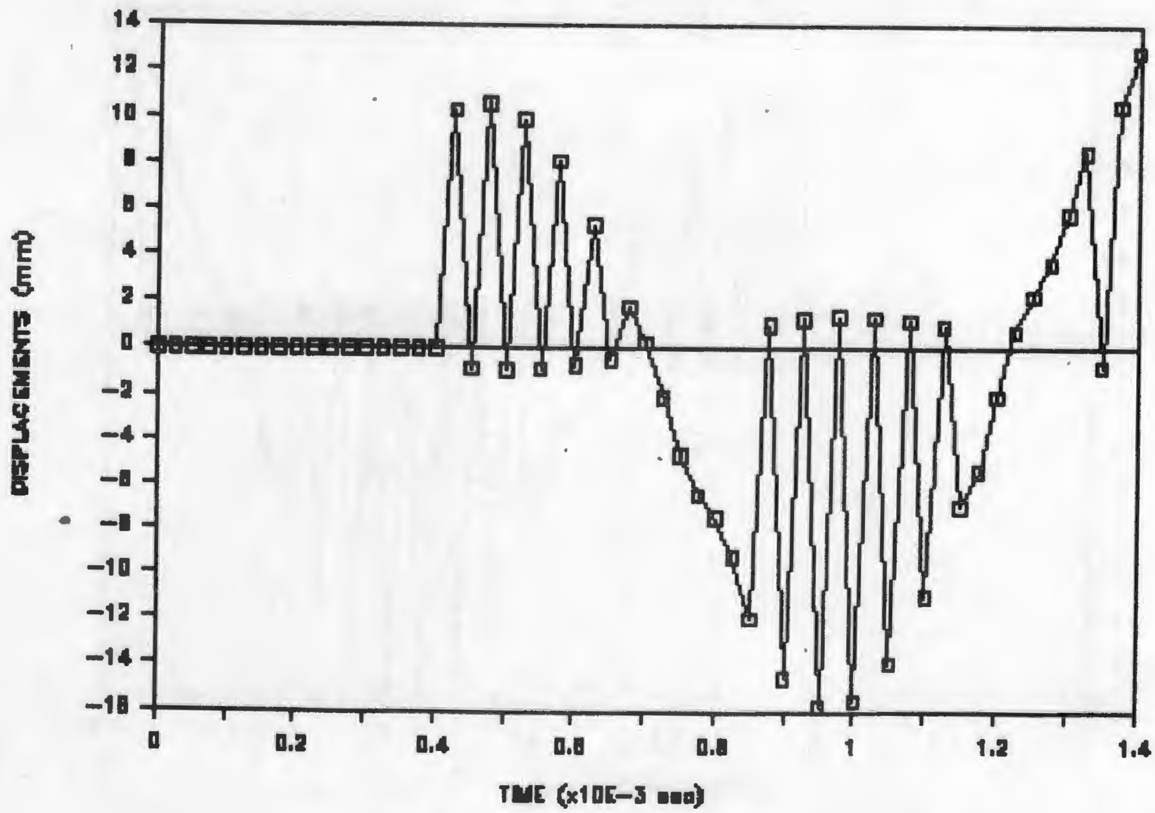

Fig. 8- System response in case III.